MADS-ML – Machine Learning

Regression

Prof. Dr. Stephan Doerfel

FACHHOCHSCHULE KIEL
University of Applied Sciences

Moodle (WiSe 2024/25)

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Examples: stock market prediction, rating prediction

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- → different evaluation measures: instead of counting correct vs. incorrect, we must evaluate close vs. far-off predictions
- → different optimization goal
- same train-test setup: splitting, cross validation
- no simple equivalent for stratification (definition requires classes)

Outline

Evaluation – Mean Squared Error

- MSE – the most popular evaluation metric:

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- in Python: `sklearn.metrics.mean_squared_error`

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- R^2 – standardized version of MSE:

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- interpretation: share of the target's variance that is explained by the estimator (by the predictions $\hat{y}$)

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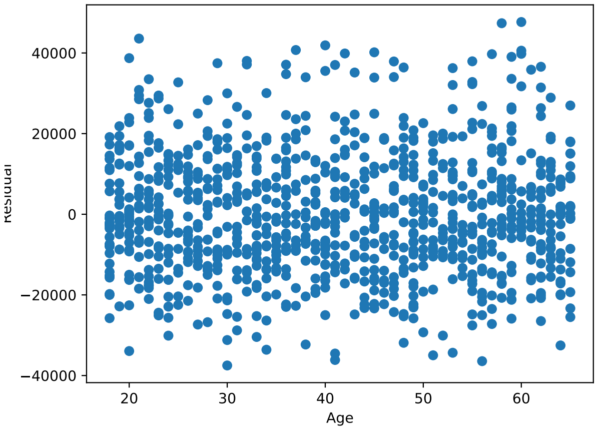
- caveat: only possible if the original value is non-zero

Analysis of the Residual Plot



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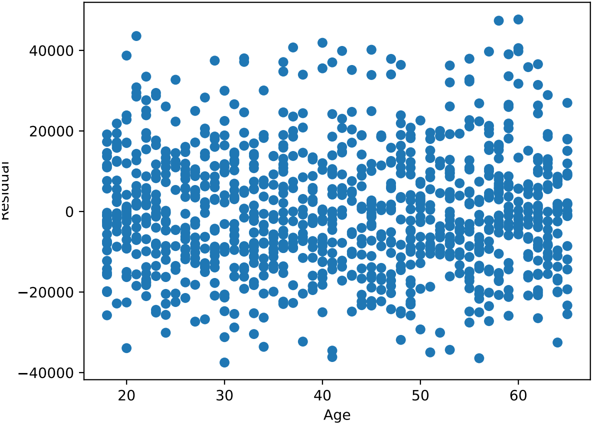

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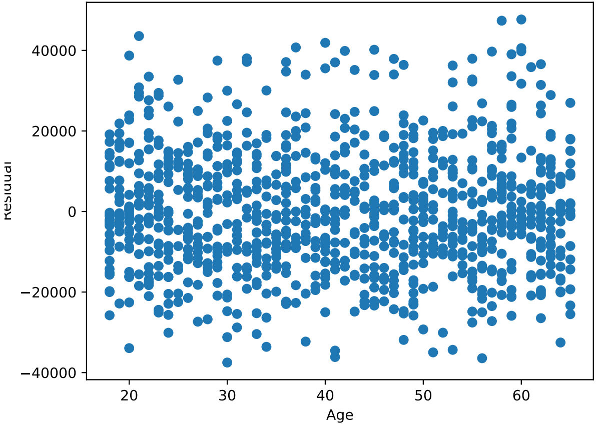

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- the residuals should be distributed at random (no systematic

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- can be used as distance functions between two series of values

Correlation – Pearson's r

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

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- even correlation with low p-value does NOT mean that two variables are actually related – spurious correlations
- Finding correlations with low p-values is a random experiment – the more candidates are tested, the more likely it is that one of them will be significant (low p-value) → **Bonferroni correction**

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  - Coincidence!
- Even when a causality seems plausible at first glance . . . !

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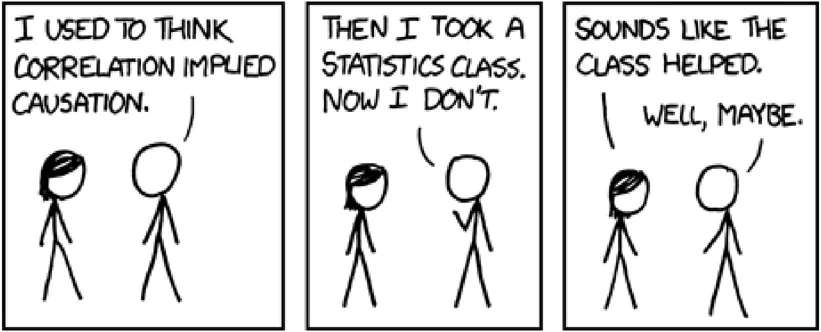

Source: XKCD

Outline

Evaluation Measures

Sidetrack: Correlation

Linear Regression

Interpretation of Population Parameters

More Regression Algorithms

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- works well for linear correlated features

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  - otherwise unexplained influences

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- Initialize parameters at random, compute the loss and use gradient descent to update the parameters

Sidetrack: Gradient Descent 1/2

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General Idea: If we make small enough steps, starting from $\boldsymbol{x}$ we will get closer to (approximately reach) a local minimum of f – provided, one exists.

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Notebook 09_1_gradient_descent_example

Gradient Descent for Linear Regression

$$f : \mathbb{R}^n \to \mathbb{R}$$

is the function that maps a particular choice for the population parameters to the resulting loss (MSE)

$$f : w_0, w_1, \ldots, w_d \mapsto \sum_{i \in D} (y^i - \hat{y}^i)^2$$

$$= \sum_{i \in D} (y^i - (w_0 + w_1 x_1 + \ldots + w_d x_d))^2$$

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This interpretation is a **ceteris paribus** argument, meaning one feature changes and all else is unchanged (all else equal).

Notebook 09_2_linear_regression_wages, Cells 1–21

Outline

- Evaluation Measures
- Sidetrack: Correlation
- Linear Regression
- Interpretation of Population Parameters
- **More Regression Algorithms**

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Notebook 09_2_linear_regression_wages, Cells 22–28

Other Regressors

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