Mathematics and Multivariate Statistics   **Prof. Dr. Stephan Doerfel**

MADS-MMS   SoSe 2025

Linear Algebra
Exercises

📊 Please join the 🔗 Participify Room.

Exercise 1. (📊, 📝 + 💻 Linear Combinations)

1. 📊 Write $(1, 2, 3)$ as linear combination of vectors $(1, 0, 0), (0, 1, 0),$ and $(0, 0, 1)$!

2. 📊 Consider the set of three vectors $A = \{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \}$. Express each of the vectors

   $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ as linear combination of A!

   Verify your results in Python.

3. Verify that the vectors $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ form a base of the vector space $\mathbb{R}^3$!

4. Compute the linear combinations $r_1(2, 4, 1, 7) + r_2(3, 2, 1, 6) + r_3(2, 12, 2, 16)$ for

   - $r_1 = 2, r_2 = 3, r_3 = 1$
   - $r_1 = 6, r_2 = 1, r_3 = 0$
   - What do you observe?
   - 📊 Which conclusion can be drawn about linear (in)dependence of the three vectors?
   - Write $\mathbf{0}$ as non-trivial linear combination of the three vectors.

Exercise 2. (📝 Transposition)
Explain the transpose operation on a matrix.

Exercise 3. (📊, 📝 + 💻 Matrix Multiplication)
Consider $A = \begin{pmatrix} 8 & 6 & 4 \\ 2 & 3 & 4 \\ 2 & 2 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} 6 & 5 \\ 3 & 7 \\ 7 & 6 \end{pmatrix}$.
Compute $A \cdot B$ and $B \cdot A$. Verify your results in Python.

Exercise 4. (💻 Invertible Matrix)
For the matrix $M = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

1. verify that the inverse of the matrix exists

2. compute the inverse

3. verify its properties according to definition

Exercise 5. (🖳 Transformation Matrixes)
Create the following array:

```
v=np.array([[1,2,3,4,5,1,2,3,4,5,1,2,3,4,5],
            [1,1,1,1,1,2,2,2,2,2,3,3,3,3,3]])
```

1. Explain, how the array can be interpreted as points in a two-dimensional space.

2. plot the points into a diagram using equally spaced axes

   ```
   fig = plt.figure()
   ax = fig.add_subplot()
   ax.set_aspect('equal', adjustable='box')
   ```

3. Multiply each point using the matrix $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and plot the result into the same diagram (using another color). Describe the result.

4. Multiply each point using the matrix $A = \begin{pmatrix} 0.3 & 0 \\ 0 & 1 \end{pmatrix}$ and plot the result into the same diagram (using another color). Describe the result.

5. Multiply each point using the matrix $A = \begin{pmatrix} \cos(\alpha) & -sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix}$ for different values of α, e.g. $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, or π. Plot the result into the same diagram (using different colors). Describe the result.