MADS-ML – Machine Learning

Decision Trees

Prof. Dr. Stephan Doerfel





Moodle (WiSe 2024/25)

''if-then-else''

Outline

Basics

Construction

Overfitting

Decision Trees in Python

Basic Idea 1/2

▶ think of the game 20 questions

Basics 2 / 27

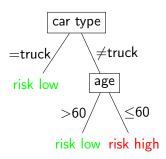
Basic Idea 1/2

- ▶ think of the game 20 questions
- drawback of kNN: No explanation on how which features yield which decision

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Motivation

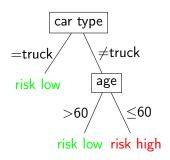
ID	age	car type	risk
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4	68	family	low
5	32	truck	low



Basics 3 / 27

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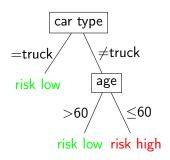


decision trees find explicit knowledge

Basics 3 / 27

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- decision trees find explicit knowledge
- decision trees provide explanations for their decisions (white box)

Basics 3 / 27



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Definition 2 (Rooted Tree)

A rooted tree is a graph in which one node is designated the **root**. The root induces a direction on the edges, i.e. all edges point away from the root.

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- ▶ one node A with all its descendants forms another binary rooted tree, called the branch or subtree of A

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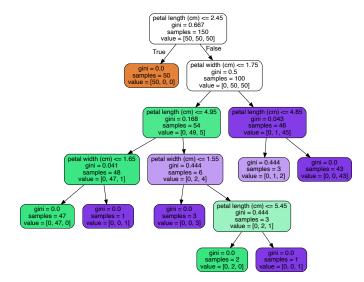
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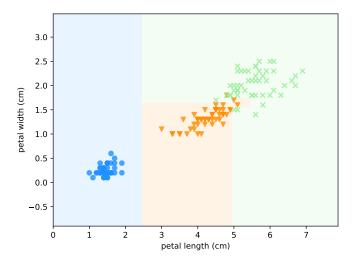
predict the class corresponding to that leaf.

An Example – The Iris Dataset (Features 2 and 3)



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Remark: Notice the combination of rectangular regions!

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- ▶ no more split attributes available

Decision Tree: Example

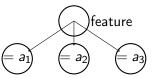
day	forecast	temperature	humidity	wind	tennis?
1	sunny	hot	high	weak	no
2	sunny	hot	high	strong	no
3	overcast	hot	high	weak	yes
4	rain	mild	high	weak	yes
5	rain	cool	normal	weak	yes
6	rain	cool	normal	strong	no

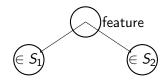
Is this a day to play tennis? Let's try a split by forecast, . . .

Split Types

Categorical Features:

- ightharpoonup split condition of the form "feature = a" or "feature in \in set"
- ► many possible subsets





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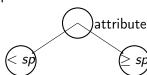
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 - ► It depends only on the largest class among the training instances, the distribution on the other classes is ignored.

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• entropy(T) = 1 for k = 2 with $p_i = \frac{1}{2}$.

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 maximizing information gain is equivalent to minimizing the subtrahend

$$\sum_{i=1}^{m} \frac{|T_i|}{|T|} \cdot entropy(T_i)$$

Split Strategy: Gini-Coefficient 1/2

▶ Gini-Coefficient for a set T of training instances w.r.t. a set of classes c_1, c_2, \ldots, c_k is defined as:

$$gini(T) = \sum_{i=1}^k (p_i) \cdot (1-p_i) = 1 - \sum_{i=1}^k p_i^2$$

- ▶ measures the expected risk of miss-classification (an element of class i occurs with probability p_i and is classified as not i with probability $1 p_i$)
 - ► small gini-coefficient ⇔ small risk
 - ► high gini-coefficient ⇔ high risk

Split Strategy: Gini-Coefficient 2/2

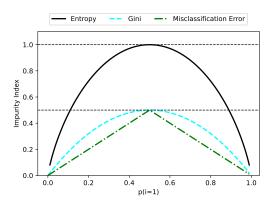
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$$gini_A(T) = \sum_{i=1}^m \frac{|T_i|}{|T|} \cdot gini(T_i)$$

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Gini vs. Entropy

- ► Entropy and Gini-Coefficient are similar
- ► Gini is computationally less expensive.
- ▶ Information gain is biased towards many-valued attributes



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Split Strategies: Example - Tennis dataset

∆ day	F	∆ outlook =	≜ temp =	▲ humidity =	≜ wind =	✓ play =
D1		Sunny	Hot	High	Weak	No
D2		Sunny	Hot	High	Strong	No
D3		Overcast	Hot	High	Weak	Yes
D4		Rain	Mild	High	Weak	Yes
D5		Rain	Cool	Normal	Weak	Yes
D6		Rain	Cool	Normal	Strong	No
D7		Overcast	Cool	Normal	Strong	Yes
D8		Sunny	Mild	High	Weak	No
D9		Sunny	Cool	Normal	Weak	Yes
D10		Rain	Mild	Normal	Weak	Yes
D11		Sunny	Mild	Normal	Strong	Yes
D12		Overcast	Mild	High	Strong	Yes
D13		Overcast	Hot	Normal	Weak	Yes
D14		Rain	Mild	High	Strong	No

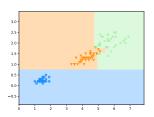
Compare a possible split by humidity to one by wind.

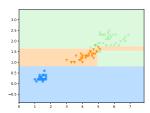
Notebook 04 1 decision trees information gain example

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Stability of a tree

- Decision trees are very sensitive towards small changes in the training data
- ► Two different ways of sampling training data might create completely different trees.





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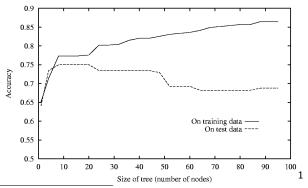
Overfitting

Decision Trees in Python

Problem: Overfitting²

Overfitting in the construction of a decision tree occurs, when a smaller tree (less depth) has

- ▶ a lower classification quality on the training data
- but a higher classification quality on the test data.



¹source

https://jmvidal.cse.sc.edu/talks/decisiontrees/allslides.html

²More detail on overfitting later Overfitting

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- ▶ Idea 2: Prune the tree after it has been constructed

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- → drawback: influence on overfitting not measured, hyper-parameter has to be optimized on separate data subset (validation set).

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Basics

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Overfitting

Decision Trees in Python

Decision Trees in Python

sklearn.tree.DecisionTreeClassifier

- supports only numerical attributes! (encoding of categorical attributes later in this course)
- parameters for stopping criteria and cost complexity pruning
- ► graphical output via graphviz³

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# The following module allow
# accessing Graphviz via Python:
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Notebook 04_2_plot_tree
```

Notebook 04_3_decision_tree_digits

