MADS-MMS – Mathematics and Multivariate Statistics

Representing Tabular Data

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Moodle (WiSe 24/25)

Agenda

Motivation

Data

Vectors

Outline

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- Linear algebra abstracts the similarities of those examples
- Analytical geometry helps modeling notions like distances or angles in abstract spaces

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- understand measurement levels of data
- mathematical modeling of tabular data
- basics of vectors and matrices
- preparation for clustering, SVMs, neural nets, ...

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Example: Alice has height: 1.85, gender: female, MBTI: INTJ, age: 32y, eye-color: blue, education: M. Sc., children: 2, comment: "food allergy"; Bob has height: 179, gender: m, MBTI: ESFP, age: 56y, eye-color: green, education: High School, comment: "likes death metal".

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- How to represent the data?

Data Representation Example 2/2

To address these issues, among others:

- ask domain experts (! a lot!)
- transform the data
- impute missing data
- drop features
- drop data points
- recollect or supplement data
- find a representation that suits the data AND the methods you are going to apply to it

Look at the data (plots, distributions, . . .)!

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Question the data and your and others assumptions about it!

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All the above steps are OUR choices. Different choices might yield different outcome.

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card.	cat.	card.	cat./ord.	cat./ord.

Similar features can have different levels. Different data science methods require different levels.

Representations for Data Operations

Each data point can be represented as a row in a table or as a tuple – a list of fix length with one entry for each feature. Example:

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- Distance functions exist for data on different measurement levels.
- Many distances work in vector spaces.

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Vectors

Vector Space

Definition 1 (Real-valued Vector Space)

A **real-valued vector space** $\mathcal{V}$ consists of a set V and operations
$+ : V \times V \to V : (\boldsymbol{v}, \boldsymbol{w}) \mapsto \boldsymbol{v} + \boldsymbol{w}$ and
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8. $(s + t) \cdot \boldsymbol{v} = s \cdot \boldsymbol{v} + t \cdot \boldsymbol{v}$ (distributivity)

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- In this lecture, we focus thus on a less general abstraction.
- The elements of $\mathbb{R}$ are called **scalars**, the elements of V **vectors**.

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1. $0 \in \mathbb{R}$ is the zero-vector (neutral element of $+$)
2. $u + (v + w) = (u + v) + w$
3. $v + w = w + v$
4. $v + (-1)v = 0$
5. $1v = v$
6. $s(tv) = (st)v$
7. $s(v + w) = sv + sw$
8. $(s + t)v = sv + tv$

Example 1 – $\mathbb{R}^n$

The most commonly know and most frequently used vector space is $(\mathbb{R}^n, +, \cdot)$ with $n \in \mathbb{N}$ and componentwise $+$ and $\cdot$.

- elements of $\mathbb{R}^n$ have n entries: $\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$
- addition: $\boldsymbol{x} + \boldsymbol{y} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ \vdots \\ x_n + y_n \end{pmatrix}$
- scalar multiplication: $s \cdot \boldsymbol{x} = \begin{pmatrix} s \cdot x_1 \\ s \cdot x_2 \\ \vdots \\ s \cdot x_n \end{pmatrix}$
- $0 = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$

Application Examples

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- In a dataset on persons with features size, weight, income, number of children, data instances can be interpreted as vectors in $\mathbb{R}^4$

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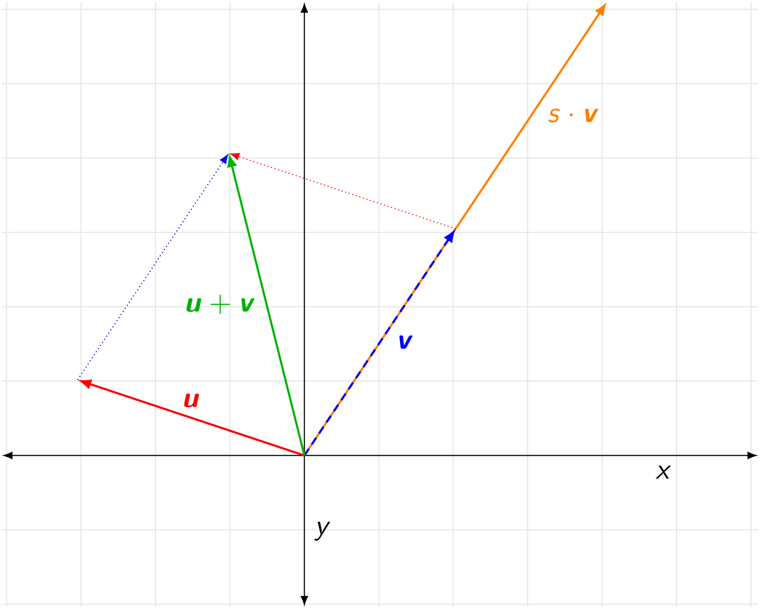
- the measurements of sensors in a machine at each point in time is a vector in $\mathbb{R}^n$ where n is the number of sensors
- bag of words: in text mining: represent each text by a vector of word frequencies; each entry represents one word (dimensionality = number of words in corpus)

Example 1 – $\mathbb{R}^2$ – Geometric Interpretation



Example 1 – $\mathbb{R}^2$ – Geometric Interpretation

- The geometric interpretation works similarly in $\mathbb{R}^n$ with $n > 2$.
- It allows computing various geometric entities, like planes, volumes, distances, angles, ...

Notebook 04_1_vectors_in_python, Cells 1–6

Example 1 – $\mathbb{R}^n$: Vectors vs. Tuples

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- E.g. polynomials form a vector space (infinite dimensions), but its elements are no tuples.
- E.g. tuples with categorical attributes, tuples with integer attributes (can be interpreted as vectors, but form no vector space)

Example 2 – $\mathbb{R}^{n \times m}$

Definition 2 (Matrix)

Let $m, n \in \mathbb{N}$. An $m \times n$ dimensional matrix M is a tuple with $m \cdot n$ elements m_{ij} $(1 \leq i \leq m,\ 1 \leq j \leq n)$.

Matrixes are usually displayed as two-dimensional arrays:

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

- addition and multiplication elementwise

Notebook 04_1_vectors_in_python, Cells 7–11

Exercises 1–3

Handling Tabular Data

- Pandas library

Notebook 04_2_tabular_data