Linear Algebra
Exercises

Exercise 1. (Linear Combinations)

1. Write $(1,2,3)$ as linear combination of vectors $(1,0,0)$, $(0,1,0)$, and $(0,0,1)$!
2. Consider the set of three vectors $A = \{\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}2\\1\\1\end{pmatrix}, \begin{pmatrix}3\\1\\1\end{pmatrix}\}$. Express each of the vectors $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$ as linear combination of A!

   Verify your results in Python.
3. Verify that the vectors $\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}2\\1\\1\end{pmatrix}, \begin{pmatrix}3\\1\\1\end{pmatrix}$ form a base of the vector space $\mathbb{R}^3$!
4. Compute the linear combinations $r_1(2,4,1,7) + r_2(3,2,1,6) + r_3(2,12,2,16)$ for
   - $r_1 = 2, r_2 = 3, r_3 = 1$
   - $r_1 = 6, r_2 = 1, r_3 = 0$
   - What do you observe?
   - Which conclusion can be drawn about linear (in)dependence of the three vectors?
   - Write **0** as non-trivial linear combination of the three vectors.

Exercise 2. (Transposition)
Explain the transpose operation on a matrix.

Exercise 3. (Matrix Multiplication)
Consider $A = \begin{pmatrix}8 & 6 & 4\\2 & 3 & 4\\2 & 2 & 8\end{pmatrix}$ and $B = \begin{pmatrix}6 & 5\\3 & 7\\7 & 6\end{pmatrix}$.
Compute $A \cdot B$ and $B \cdot A$. Verify your results in Python.

Exercise 4. (Invertible Matrix)
For the matrix $M = \begin{pmatrix}1 & 2 & 3\\0 & 1 & 1\\1 & 1 & 1\end{pmatrix}$

1. verify that the inverse of the matrix exists
2. compute the inverse
3. verify its properties according to definition

Exercise 5. (Transformation Matrixes)
Create the following array:

```
v=np.array([[1,2,3,4,5,1,2,3,4,5,1,2,3,4,5],
            [1,1,1,1,1,2,2,2,2,2,3,3,3,3,3]])
```

1. Explain, how the array can be interpreted as points in a two-dimensional space.
2. plot the points into a diagram using equally spaced axes

   ```
   fig = plt.figure()
   ax = fig.add_subplot()
   ax.set_aspect(’equal’, adjustable=’box’)
   ```

3. Multiply each point using the matrix $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and plot the result into the same diagram (using another color). Describe the result.
4. Multiply each point using the matrix $A = \begin{pmatrix} 0.3 & 0 \\ 0 & 1 \end{pmatrix}$ and plot the result into the same diagram (using another color). Describe the result.
5. Multiply each point using the matrix $A = \begin{pmatrix} \cos(\alpha) & -sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix}$ for different values of α, e.g. $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, or π. Plot the result into the same diagram (using different colors). Describe the result.