

# MADS-MMS – Mathematics and Multivariate Statistics

## Clustering – Overview

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# Agenda

Motivation

Goals of Clustering

What is Clustering?

Clustering Methods

Ingredients

# Outline

## Motivation

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# Chapter Goals

- ▶ overview on the topic of clustering
- ▶ categorization of methodology
- ▶ understand motivation and application of clustering

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# Goals of Clustering

## Goals:

- ▶ identify clusters (categories / subsets / groups) in datasets
- ▶ instances in the same cluster should be as similar as possible
- ▶ instances in different clusters should have low similarity
- ▶ (identify instances that belong to no group: outliers, noise)
- ▶ NOT: assign instances to known classes

## Context:

- ▶ detect sets of “comparable/similar/close elements”
- ▶ explore and analyze unknown data
- ▶ engineer classes / features
- ▶ semi-automatic – often data scientist has to “judge” and interpret clusterings
- ▶ requires a useful and meaningful distance/similarity function (often individually chosen or designed)

# Applications for Clustering

## Applications:

- ▶ Market segmentation / customer base segmentation
- ▶ Pattern recognition
- ▶ Community discovery in social networks
- ▶ Topic detection in text corpora
- ▶ Tracking of evolutionary steps
- ▶ Identifying common behavior or common interests (e.g. for pdf recommender systems)
- ▶ Identifying common physical properties in sensor data
- ▶ ...

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# Examples

Clusters of different size, form, density, and hierarchical structure



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<sup>1</sup>Source: [1], Abb. 3-1

# Clustering Formally

There is no hard mathematical definition of clustering in general.

## Definition 1 (Clustering)

Clustering comprises (machine learning) methods of **unsupervised learning** to collect data instances into **groups, categories, or classes**, called **clusters**. The set of all clusters is called a **clustering**.

Criteria for the grouping can be

**intra-class similarity:** similarity within a cluster

**inter-class dissimilarity:** dissimilarity between different clusters

# Machine Learning Disciplines

## Supervised

- ▶ labelled data
- ▶ goal: class/prediction of unknown/future data
- ▶ idea: Learn by deriving a model from looking at examples
- ▶ correctness of the training can be assessed (supervised)
- ▶ examples: classification, regression

## Unsupervised

- ▶ unlabelled data
- ▶ goal: Detect patterns (groups, structure) in the data
- ▶ learning is unsupervised, no “correct” result that we can compare to
- ▶ examples: **clustering**, dimensionality reduction

# Clustering Process

## Definition 2 (Clustering Process)

A clustering process comprises the following steps:

- ▶ representation of the data
- ▶ definition of a similarity measure (domain-specific)
- ▶ creating the clusters
- ▶ optionally abstraction of knowledge
- ▶ optionally evaluation of the output

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# Classification of Clustering Methods

## Partitioning Methods (Hard Clustering)

- ▶ determines a partition into disjoint subsets, minimizing a cost function
- ▶ typical parameters: number of clusters  $k$ , distance function

## Density-based Methods

- ▶ adds neighbors to clusters, as long as density does not fall below threshold
- ▶ distinguishes between the cores of clusters, its borders, and noise
- ▶ parameters: minimal acceptable density in a cluster, distance function

## Hierarchical Methods

- ▶ determine a hierarchy of clusters, fuses most similar clusters
- ▶ parameters: distance function

## Other Methods (incomplete)

- ▶ Soft Clustering (Fuzzy Clustering, Overlapping Clustering)
- ▶ Graph-based Methods

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We need:

- ▶ a way to express different kinds of data mathematically, and
- ▶ a way to measure distance/similarity between points
- ▶ all that in many-dimensional realms
- ▶ 2D-visualizations despite multidimensional data
- ▶ algorithms that cluster
- ▶ basic mathematics like logarithms, vector geometry, matrices

 Exercises 1



# References



M. Ester and J. Sander.

*Knowledge Discovery in Databases.*

Springer-Verlag, Berlin/Heidelberg, 2000.