Powers and Logarithms
Exercises

Exercise 1. (Computing Powers, Roots, Logarithms)
Solve the following problems:

1. 1.6^5
2. $\log_2 2048$
3. $\log_3 243$
4. $\sqrt[4]{81}$
5. $\log_{\frac{8}{5}} 10.48576$

Exercise 2. (Power and Logarithm Computation Rules)
Express the following terms using $\log x$, $\log y$, and $\log z$.
Note: In situations like this, we do not specifically denote the base. It is implicitly assumed, that all logarithms have the same base.

1. $\log xyz$
2. $\log \sqrt[5]{x^4 y^{-2}}$

Exercise 3. (Equations with Logarithms)
Solve for x!

1. $2^{2x-1} = 512$
2. $\log_a \frac{1}{a} = x$
3. $\log_9 x = -\frac{1}{2}$
4. $\log_3(x + 25) - \log_3(x - 1) = 3$

Exercise 4. (Equation with Logarithms)
Solve for x, analytically and by plotting the solution in Python: $2 \log x = \log 2 + \log(3x - 4)$
For plotting the functions, use the `numpy` command `arange` to create a list of x-values.

Exercise 5. (Epidemic Spreading Model)
Consider the following simplified model of epidemic spreading:

- For a disease, the value R determines the number of persons, a sick person will infect on average.
- In every new step, each currently infected person infects R further persons and then stops being contagious.
- Assume that the disease is discovered, when 100 patients are in their contagious state.

1. Create a mathematical model for the above spreading process – a function for the number of new infections in each step.
2. Plot the number of new infections with every step for different values of R.
3. Describe three qualitatively different behaviors depending on the value for R.
4. Describe the influence of small modifications to R, e.g. 1.7 vs. 1.75.