MADS-MMS – Mathematics and Multivariate Statistics

Clustering – Overview

Prof. Dr. Stephan Doerfel

FACHHOCHSCHULE KIEL
University of Applied Sciences

SCAN ME

Moodle (SoSe 2025)

Agenda

Outline

Chapter Goals

- overview on the topic of clustering

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- categorization of methodology

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- overview on the topic of clustering
- categorization of methodology
- understand motivation and application of clustering

Outline

Goals of Clustering

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- semi-automatic – often data scientist has to "judge" and interprete clusterings
- requires a useful and meaningful distance/similarity function (often individually chosen or designed)

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Motivation

Goals of Clustering

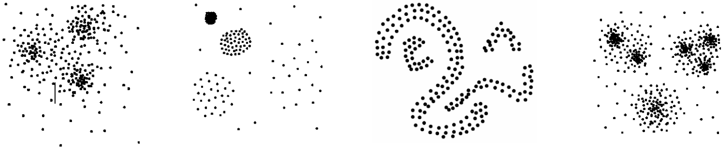
What is Clustering?

Clustering Methods

Ingredients

Examples

Clusters of different size, form, density, and hierarchical structure

 [1]

[1]Source: [1], Abb. 3-1

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Definition 1 (Clustering)

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Criteria for the grouping can be

intra-class similarity: similarity within a cluster

inter-class dissimilarity: dissimilarity between different clusters

Machine Learning Disciplines

Supervised

- labelled data
- goal: class/prediction of unknown/future data
- idea: Learn by deriving a model from looking at examples
- correctness of the training can be assessed (supervised)
- examples: classification, regression

Unsupervised

- unlabelled data
- goal: Detect patterns (groups, structure) in the data
- learning is unsupervised, no “correct” result that we can compare to
- examples: **clustering**, dimensionality reduction

Clustering Process

Definition 2 (Clustering Process)

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- optionally evaluation of the output

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Exercises 1

References

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