Timur Gaimakov

CS 480 Fall 2021 Written Assignment #02

Due: **Friday, October 15th, 11:00 PM CST**
Points: **100**

Instructions:

1. Use this document template to report your answers. Name the complete document as follows:

   LastName_FirstName_CS480_Written02.doc

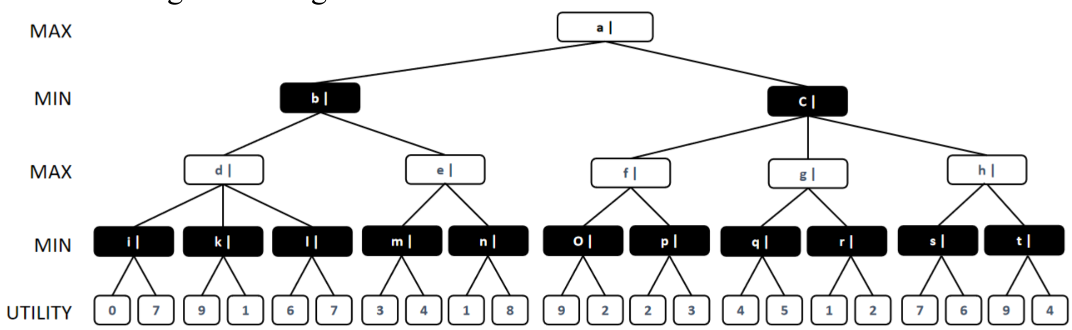
2. Submit the final document to the Blackboard Assignments section before the due date. No late submissions will be accepted.

Objectives:

1. (20 points) Demonstrate your understanding of MinMax games and the α−β pruning algorithm.
2. (80 points) Demonstrate your understanding of propositional logic, its syntax, laws, and inference based on propositional logic.

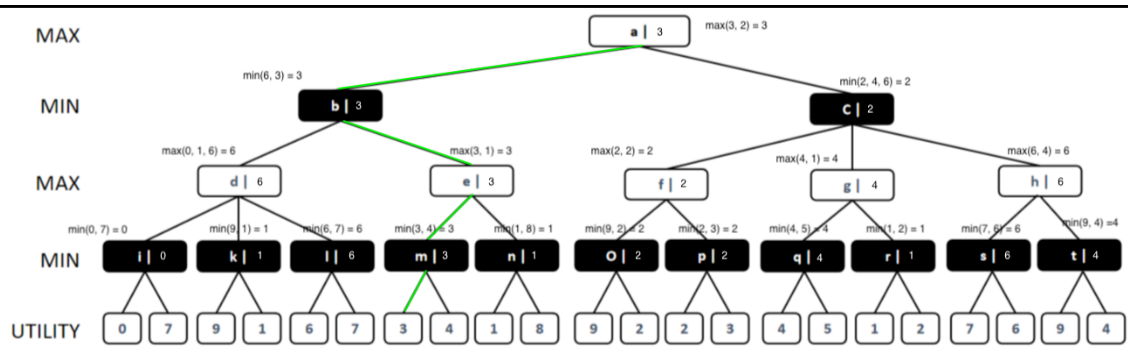
Problem 1 [20 pts]:

Consider the following MinMax game tree



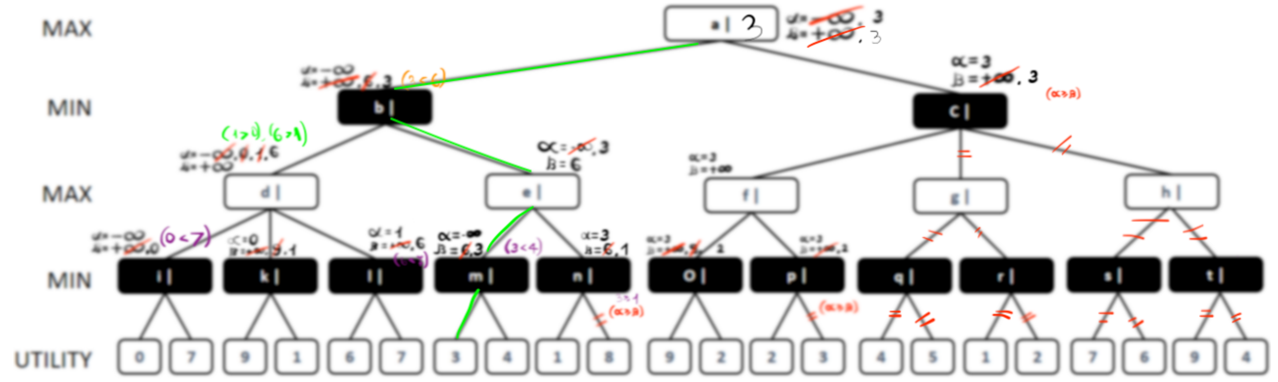
Evaluate MinMax values for all nodes (you can paste in an edited version of this tree below) **[10 pts]**:

Your solution:



Now, apply **alpha-beta (α−β) pruning** to prune some of the tree branches. Show (you can paste in an edited version of this tree below) which sections of the tree will be pruned and **justify your answer** **[10 pts]**:

Your solution:



Problem 2 [20 pts]:

Use **truth tables** to show that the following sentences are **tautologies** **[5 pts]**:

1. ¬(p ∧ q) ⇔ ¬p ∨ ¬q **[5 pts]**

   KB ⇔ ¬(p ∧ q)
   Q ⇔ ¬p ∨ ¬q

p	q	p ∧ q	KB	¬p	¬q	Q	KB => Q
T	T	T	F	F	F	F	**T**
T	F	F	T	F	T	T	**T**
F	T	F	T	T	F	T	**T**
F	F	F	T	T	T	T	**T**

2. p ⇒ q ⇔ ¬q ⇒ ¬p **[5 pts]**

   KB ⇔ p ⇒ q　　　　(‘=>’ has higher precedence over ‘⇔’)
   Q ⇔ ¬q ⇒ ¬p

p	q	KB	¬q	¬p	Q	KB => Q
T	T	T	F	F	T	**T**
T	F	F	T	F	F	**T**
F	T	T	F	T	T	**T**
F	F	T	T	T	T	**T**

3. $((p \Rightarrow q) \wedge (q \Rightarrow p)) \Leftrightarrow (p \Leftrightarrow q)$ **[5 pts]**

KB $\Leftrightarrow ((p \Rightarrow q) \wedge (q \Rightarrow p))$
Q $\Leftrightarrow (p \Leftrightarrow q)$

p	q	$p \Rightarrow q$	$q \Rightarrow p$	KB	Q	KB => Q
T	T	T	T	T	T	**T**
T	F	F	T	F	F	**T**
F	T	T	F	F	F	**T**
F	F	T	T	T	T	**T**

4. $(p \vee q) \wedge (\neg q \vee r) \Rightarrow (p \vee r)$ **[5 pts]**

KB $\Leftrightarrow (p \vee q) \wedge (\neg q \vee r)$
Q $\Leftrightarrow (p \vee r)$

p	q	r	$p \vee q$	$\neg q$	$\neg q \vee r$	KB	Q	KB => Q
T	T	T	T	F	T	T	T	**T**
T	T	F	T	F	F	F	T	**T**
T	F	T	T	T	T	T	T	**T**
T	F	F	T	T	T	T	T	**T**
F	T	T	T	F	T	T	T	**T**
F	T	F	T	F	F	F	F	**T**
F	F	T	F	T	T	F	T	**T**
F	F	F	F	T	T	F	F	**T**

Problem 3 [20 pts]:

Use **deduction** to show (**prove**) that the following sentences are **tautologies**:

1. $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$ **[5 pts]**

Your proof:		
Step	**Resulting sentence**	**Applied law / rule**
1	$(\neg(p \wedge q)) \Leftrightarrow \neg p \vee \neg q$ becomes: $(\neg p \vee \neg q) \Leftrightarrow \neg p \vee \neg q$	De Morgan's Law $\neg(a \wedge b) = \neg a \vee \neg b$
2	$(\neg p \vee \neg q) \Leftrightarrow \neg p \vee \neg q$ becomes:	Equivalence Law $(a \wedge b) \vee (\neg a \wedge \neg b) = (a \Leftrightarrow b)$ Assume that $a = \neg p \vee \neg q$

	$((\neg p \lor \neg q) \land (\neg p \lor \neg q)) \lor (\neg(\neg p \lor \neg q) \land \neg(\neg p \lor \neg q))$	and $b = \neg p \lor \neg q$
3	$((\neg p \lor \neg q) \land (\neg p \lor \neg q)) \lor (\neg(\neg p \lor \neg q) \land \neg(\neg p \lor \neg q))$ becomes: $((\neg p \lor \neg q) \land (\neg p \lor \neg q)) \lor (\neg(\neg p \lor \neg q))$	Idempotent Law $p \land p = p$
4	$((\neg p \lor \neg q) \land (\neg p \lor \neg q)) \lor (\neg(\neg p \lor \neg q))$ becomes: $(\neg p \lor \neg q) \lor (\neg(\neg p \lor \neg q))$	Idempotent Law $p \land p = p$
5	$(\neg p \lor \neg q) \lor (\neg(\neg p \lor \neg q))$ becomes: $(\neg p \lor \neg q) \lor \neg(\neg p \lor \neg q)$	Remove extra parentheses
6	$(\neg p \lor \neg q) \lor \neg(\neg p \lor \neg q)$ becomes: T	Law of Excluded Middle $A \lor \neg A = T$ Assume that $A = \neg p \lor \neg q$
7	So: $\neg(p \land q) \Leftrightarrow \neg p \lor \neg q = T$	We proved that $\neg(p \land q) \Leftrightarrow \neg p \lor \neg q$ is a tautology
Add more rows if necessary | Symbols (copy/paste): T⊥ ⋁ ⋀≡⇔¬⇒∴		

2. $p \Rightarrow q \Leftrightarrow \neg q \Rightarrow \neg p$ **[5 pts]**

Your proof:		
Step	**Resulting sentence**	**Applied law / rule**
1	$p \Rightarrow q \Leftrightarrow \neg q \Rightarrow \neg p$ becomes: $\neg q \Rightarrow \neg p \Leftrightarrow \neg q \Rightarrow \neg p$	Contraposition Law $p \Rightarrow q \Leftrightarrow \neg q \Rightarrow \neg p$
2	$\neg q \Rightarrow \neg p \Leftrightarrow \neg q \Rightarrow \neg p$ becomes: $((\neg q \Rightarrow \neg p) \land (\neg q \Rightarrow \neg p)) \lor (\neg(\neg q \Rightarrow \neg p) \land \neg(\neg q \Rightarrow \neg p))$	Equivalence Law $(a \land b) \lor (\neg a \land \neg b) = (a \Leftrightarrow b)$ Assume that $a = \neg q \Rightarrow \neg p$ and $b = \neg q \Rightarrow \neg p$
3	$((\neg q \Rightarrow \neg p) \land (\neg q \Rightarrow \neg p)) \lor (\neg(\neg q \Rightarrow \neg p) \land \neg(\neg q \Rightarrow \neg p))$ becomes: $((\neg q \Rightarrow \neg p) \land (\neg q \Rightarrow \neg p)) \lor (\neg(\neg q \Rightarrow \neg p))$	Idempotent Law $p \land p = p$
4	$((\neg q \Rightarrow \neg p) \land (\neg q \Rightarrow \neg p)) \lor (\neg(\neg q \Rightarrow \neg p))$ becomes: $((\neg q \Rightarrow \neg p)) \lor (\neg(\neg q \Rightarrow \neg p))$	Idempotent Law $p \land p = p$
5	$((\neg q \Rightarrow \neg p)) \lor (\neg(\neg q \Rightarrow \neg p))$ becomes: $((\neg q \Rightarrow \neg p)) \lor \neg(\neg q \Rightarrow \neg p)$	Remove extra parentheses
6	$((\neg q \Rightarrow \neg p)) \lor \neg(\neg q \Rightarrow \neg p)$ becomes: $(\neg q \Rightarrow \neg p) \lor \neg(\neg q \Rightarrow \neg p)$	Remove extra parentheses
7	$(\neg q \Rightarrow \neg p) \lor \neg(\neg q \Rightarrow \neg p)$ becomes: T	Law of Excluded Middle $A \lor \neg A = T$ Assume that $A = \neg q \Rightarrow \neg p$
8	So: $p \Rightarrow q \Leftrightarrow \neg q \Rightarrow \neg p = T$	We proved that $p \Rightarrow q \Leftrightarrow \neg q \Rightarrow \neg p$ is a tautology
Add more rows if necessary | Symbols (copy/paste): T⊥ ⋁ ⋀≡⇔¬⇒∴		

3. ((p ⇒ q) ∧ (q ⇒ p)) ⇔ (p ⇔ q) **[5 pts]**

Your proof:		
Step	**Resulting sentence**	**Applied law / rule**
1	((p ⇒ q) ∧ (q ⇒ p)) ⇔ (p ⇔ q) becomes: (p ⇔ q) ⇔ (p ⇔ q)	Equivalence Law ((p ⇒ q) ∧ (q ⇒ p)) ⇔ (p ⇔ q)
2	(p ⇔ q) ⇔ (p ⇔ q) becomes ((p ⇔ q) ∧ (p ⇔ q)) ∨ (¬(p ⇔ q) ∧ ¬ (p ⇔ q))	Equivalence Law (a ∧ b) ∨ (¬a ∧ ¬b) = (a ⇔ b) Assume that a = (p ⇔ q) and b = (p ⇔ q)
3	((p ⇔ q) ∧ (p ⇔ q)) ∨ (¬(p ⇔ q) ∧ ¬ (p ⇔ q)) becomes: ((p ⇔ q) ∧ (p ⇔ q)) ∨ (¬(p ⇔ q))	Idempotent Law p ∧ p = p
4	((p ⇔ q) ∧ (p ⇔ q)) ∨ (¬(p ⇔ q)) becomes: ((p ⇔ q)) ∨ (¬(p ⇔ q))	Idempotent Law p ∧ p = p
5	((p ⇔ q)) ∨ (¬(p ⇔ q)) becomes: ((p ⇔ q)) ∨ ¬(p ⇔ q)	Remove extra parentheses
6	((p ⇔ q)) ∨ ¬(p ⇔ q) becomes: (p ⇔ q) ∨ ¬(p ⇔ q)	Remove extra parentheses
7	(p ⇔ q) ∨ ¬(p ⇔ q) becomes: T	Law of Excluded Middle A ∨ ¬A = T Assume that A = (p ⇔ q)
8	So: ((p ⇒ q) ∧ (q ⇒ p)) ⇔ (p ⇔ q) = T	We proved that ((p ⇒ q) ∧ (q ⇒ p)) ⇔ (p ⇔ q) is a tautology
Add more rows if necessary | Symbols (copy/paste): T⊥ ∨ ∧≡⇔¬⇒∴		

4. (p ∨ q) ∧ (¬q ∨ r) ⇒ (p ∨ r) **[5 pts]**

Your proof:		
Step	**Resulting sentence**	**Applied law / rule**
1	(p ∨ q) ∧ (¬q ∨ r) ⇒ (p ∨ r) becomes: ¬[(p ∨ q) ∧ (¬q ∨ r)] ∨ (p ∨ r)	Implication Law
2	¬[(p ∨ q) ∧ (¬q ∨ r)] ∨ (p ∨ r) becomes: ¬(p ∨ q) ∨ ¬(¬q ∨ r) ∨ (p ∨ r)	De Morgan's Law let n = p ∨ q, m = ¬q ∨ r ¬(n ∧ m) ⇔ ¬n ∨ ¬m
3	¬(p ∨ q) ∨ ¬(¬q ∨ r) ∨ (p ∨ r) becomes: (¬p ∧ ¬q) ∨ (q ∧ ¬r) ∨ (p ∨ r)	De Morgan's Law
4	(¬p ∧ ¬q) ∨ (q ∧ ¬r) ∨ (p ∨ r) becomes: (¬p ∧ ¬q) ∨ (q ∧ ¬r) ∨ p ∨ r	Remove parenthesis

5	(¬p ∧ ¬q) ∨ (q ∧ ¬r) ∨ p ∨ r becomes: (¬p ∧ ¬q) ∨ (q ∧ ¬r) ∨ r ∨ p	Commutative Law
6	(¬p ∧ ¬q) ∨ (q ∧ ¬r) ∨ r ∨ p becomes: (¬p ∧ ¬q) ∨ [(q ∨ r) ∧ (¬r ∧ r)] ∨ p	Distributive Law
7	(¬p ∧ ¬q) ∨ [(q ∨ r) ∧ (¬r ∧ r)] ∨ p becomes: (¬p ∧ ¬q) ∨ [(q ∨ r) ∧ T] ∨ p	Law of Excluded Middle
8	(¬p ∧ ¬q) ∨ [(q ∨ r) ∧ T] ∨ p becomes: (¬p ∧ ¬q) ∨ (q ∨ r) ∨ p	Identity Law
9	(¬p ∧ ¬q) ∨ (q ∨ r) ∨ p becomes: (¬p ∧ ¬q) ∨ q ∨ r ∨ p	Remove parenthesis
10	(¬p ∧ ¬q) ∨ q ∨ r ∨ p becomes: [(¬p ∨ q) ∧ (¬q ∨ q)] ∨ r ∨ p	Distributive Law
11	[(¬p ∨ q) ∧ (¬q ∨ q)] ∨ r ∨ p becomes: [(¬p ∨ q) ∧ T] ∨ r ∨ p	Law of Excluded Middle
12	[(¬p ∨ q) ∧ T] ∨ r ∨ p becomes: (¬p ∨ q) ∨ r ∨ p	Identity Law
13	(¬p ∨ q) ∨ r ∨ p becomes: ¬p ∨ q ∨ r ∨ p	Remove parenthesis
14	¬p ∨ q ∨ r ∨ p becomes: ¬p ∨ p ∨ r ∨ q	Commutative Law
15	¬p ∨ p ∨ r ∨ q becomes: T ∨ r ∨ q	Law of Excluded Middle
16	T ∨ r ∨ q becomes: T	Domination Law
Add more rows if necessary | Symbols (copy/paste): T⊥ ∨ ∧≡⇔¬⇒∴		

Problem 4 [20 pts]:

Convert the following sentences into **conjunctive normal form** (**CNF**):

a) p ⇔ q **[6 pts]**

Your conversion steps:		
Step	**Resulting sentence**	**Applied law / rule**
1	p ⇔ q becomes: (p => q) ∧ (q => p)	Equivalence Law
2	(p => q) ∧ (q => p) becomes: (¬p ∨ q) ∧ (¬q ∨ p)	Implication Law

Add more rows if necessary | Symbols (copy/paste): T⊥ ⋁ ⋀≡⇔¬⇒∴

b) p ⋀ q ⇔ p ⋁ q **[6 pts]**

Your conversion steps:		
Step	**Resulting sentence**	**Applied law / rule**
1	p ⋀ q ⇔ p ⋁ q becomes: ((p ⋀ q) => (p ⋁ q)) ⋀ ((p ⋁ q) => (p ⋀ q))	Equivalence Law let n = p ⋀ q, m = p ⋁ q ((n ⇒ m) ⋀ (n ⇒ m)) ⇔ (n ⇔ m)
2	((p ⋀ q) => (p ⋁ q)) ⋀ ((p ⋁ q) => (p ⋀ q)) becomes: (¬(p ⋀ q) ⋁ (p ⋁ q)) ⋀ (¬(p ⋁ q) ⋁ (p ⋀ q))	Implication Law ¬n ⋁ m ⇔ n => m ¬m ⋁ n ⇔ m => n (¬n ⋁ m) ⋀ (¬m ⋁ n) ⇔ n => m ⋀ m => n
3	(¬(p ⋀ q) ⋁ (p ⋁ q)) ⋀ (¬(p ⋁ q) ⋁ (p ⋀ q)) [(¬n ⋁ m) ⋀ (¬m ⋁ n)] becomes: (¬p ⋁ q) ⋀ (p ⋁ ¬q)	Simplification
Add more rows if necessary | Symbols (copy/paste): T⊥ ⋁ ⋀≡⇔¬⇒∴		

c) p ⋀ (p ⇒ q) ⇒ q **[8 pts]**

Your conversion steps:		
Step	**Resulting sentence**	**Applied law / rule**
1	p ⋀ (p => q) => q becomes: ¬(p ⋀ (p => q)) ⋁ q	Implication Law
2	¬(p ⋀ (p => q)) ⋁ q becomes: ¬(p ⋀ (¬p ⋁ q)) ⋁ q	Implication Law
3	¬(p ⋀ (¬p ⋁ q)) ⋁ q becomes: ¬((p ⋀ ¬p) ⋁ (p ⋀ q)) ⋁ q	Distributive Law
4	¬((p ⋀ ¬p) ⋁ (p ⋀ q)) ⋁ q becomes: ¬(⊥ ⋁ (p ⋀ q)) ⋁ q	Contradiction
5	¬(⊥ ⋁ (p ⋀ q)) ⋁ q becomes: ¬(p ⋀ q) ⋁ q	Identity Law
6	¬(p ⋀ q) ⋁ q becomes: (¬p ⋁ ¬q) ⋁ q	De Morgan's Law
7	(¬p ⋁ ¬q) ⋁ q becomes: ¬p ⋁ (¬q ⋁ q)	Associative Law
8	¬p ⋁ (¬q ⋁ q) becomes:	Law of Excluded Middle

	¬p ∨ T	
9	¬p ∨ T becomes: T	Domination Law
Add more rows if necessary | Symbols (copy/paste): T⊥ ∨ ∧≡⇔¬⇒∴		

Problem 5 [20 pts]:

Use **resolution** (convert to CNF first) to show that the:

1. Sentence (p ∨ q) ∧ (¬q ∨ r) ⇒ (p ∨ r) is a **tautology [10 pts]**

Your solution (provide all steps)

By Implication Law:
(p ∨ q) ∧ (¬q ∨ r) ⇒ (p ∨ r)
becomes:
¬[(p ∨ q) ∧ (¬q ∨ r)] ∨ (p ∨ r)

By De Morgan's Law
¬[(p ∨ q) ∧ (¬q ∨ r)] ∨ (p ∨ r)
becomes:
[¬(p ∨ q) ∨ ¬(¬q ∨ r)] ∨ (p ∨ r) ≡ (¬p ∧ ¬q) ∨ (q ∧ ¬r) ∨ p ∨ r

¬q ∨ q ∨ p ∨ r

1. {¬q, q, p, r} clause
2. {¬p} clause
3. {q} clause
4. {p} clause
5. {r} clause
6. {T} 2, 3
7. {T} 4, 6
8. {T} 5, 7

(Problem 4 shows that there is no way to form a CNF from tautology)

2. Sentence ¬(¬hasGas ∧ (hasGas ∨ ¬carCanStart) ⇒ ¬carCanStart) is a **contradiction [10 pts]**

Your solution (provide all steps)

By Distributive Law
¬{[¬hasGas ∧ (hasGas ∨ ¬carCanStart)] ⇒ ¬carCanStart}
becomes:
¬{[(¬hasGas ∧ hasGas) ∨ (¬hasGas ∧ ¬carCanStart)] ⇒ ¬carCanStart}

By Contradiction Law
¬{[(¬hasGas ∧ hasGas) ∨ (¬hasGas ∧ ¬carCanStart)] ⇒ ¬carCanStart}
becomes:
¬{[⊥ ∨ (¬hasGas ∧ ¬carCanStart)] ⇒ ¬carCanStart}

By Identity Law
¬{[⊥ ∨ (¬hasGas ∧ ¬carCanStart)] ⇒ ¬carCanStart} becomes
¬{[¬hasGas ∧ ¬carCanStart] ⇒ ¬carCanStart}

By Implication Law
¬{[¬hasGas ∧ ¬carCanStart] ⇒ ¬carCanStart}
becomes:

¬{¬[¬hasGas ∧ ¬carCanStart] ∨ ¬carCanStart}

By De Morgan's Law
¬{¬[¬hasGas ∧ ¬carCanStart] ∨ ¬carCanStart}
becomes:
¬{[¬¬hasGas ∨ ¬¬carCanStart] ∨ ¬carCanStart}
By Double Negation Law:
¬{[¬¬hasGas ∨ ¬¬carCanStart] ∨ ¬carCanStart}
becomes:
¬{[hasGas ∨ carCanStart] ∨ ¬carCanStart}

By De Morgan's Law
¬{[hasGas ∨ carCanStart] ∨ ¬carCanStart}
becomes:
¬[hasGas ∨ carCanStart] ∧ ¬¬carCanStart

By Double Negation Law
¬[hasGas ∨ carCanStart] ∧ ¬¬carCanStart
becomes:
¬[hasGas ∨ carCanStart] ∧ carCanStart

By De Morgan's Law
¬[hasGas ∨ carCanStart] ∧ carCanStart
becomes:
[¬hasGas ∧ ¬carCanStart] ∧ carCanStart

Remove Parenthesis
[¬hasGas ∧ ¬carCanStart] ∧ carCanStart
becomes:
¬hasGas ∧ ¬carCanStart ∧ carCanStart

Contradiction Law
¬hasGas ∧ ¬carCanStart ∧ carCanStart
becomes:
¬hasGas ∧ ⊥

Domination Law
¬hasGas ∧ ⊥
becomes:
⊥

C ≡ ¬
(Problem 4 shows that there is no way to form a CNF from contradiction too)