

CS 480 Fall 2021 Written Assignment #03

Due: **Wednesday, December 1st, 11:00 PM CST**

Points: **90**

Instructions:

1. Use this document template to report your answers. Name the complete document as follows:

LastName_FirstName_CS480_Written03.doc

2. Submit the final document to the Blackboard Assignments section before the due date. No late submissions will be accepted.

Objectives:

1. (25 points) Demonstrate your understanding of First-Order Logic syntax.
2. (20 points) Demonstrate your understanding of Bayes' Rule.
3. (20 points) Demonstrate your understanding of Bayes Networks.
4. (25 points) Demonstrate your understanding of Decision Networks.

Problem 1 [25 pts]:

Convert English sentences to FOL. Write each of the following English sentences using First Order Logic. Use the following predicates and constants only.

- Occupation(p, o): Predicate. Person p has occupation o. You can also read it as p is o.
- Customer(p1, p2): Predicate. Person p1 is a customer of person p2.
- Boss(p1, p2): Predicate. Person p1 is a boss of person p2.
- Doctor, Surgeon, Lawyer, Actor: Constants denoting some occupations. This list is not comprehensive. There are also other occupations not mentioned in this list.
- Mary, John: Constants denoting some people. This list is not comprehensive. There are also other people not mentioned in this list.

- a) Mary is either a lawyer or an actor [5 pts].

Your solution:
Occupation(Mary, Lawyer) V Occupation(Mary, Actor)

- b) John is a lawyer, but he also holds another job [5 pts].

Your solution:
Occupation(John, Lawyer) ^ (Occupation(John, Doctor) V Occupation(John, Surgeon) V Occupation(John, Actor))

c) All surgeons are doctors [5 pts].

Your solution:

$\forall p \text{ Occupation}(p, \text{Surgeon}) \Rightarrow \text{Occupation}(p, \text{Doctor})$

d) John does not have a lawyer (i.e., John is not a customer of any lawyer.) [5 pts].

Your solution:

$\neg \text{Customer}(\text{John}, \text{Lawyer})$

e) Every surgeon has a lawyer [5 pts].

Your solution:

$\forall p_1 p_2 \text{ Occupation}(p_1, \text{Surgeon}) \Rightarrow \text{Customer}(p_1, p_2) \wedge \text{Occupation}(p_2, \text{Lawyer})$

Problem 2 [20 pts]:

We are given the following joint distribution for variables A, B, and C. Please compute the requested probabilities. **Show each probability distribution as a table/vector.**

A	B	C	P(A, B, C)
T	T	T	0.014
T	T	F	0.126
T	F	T	0.012
T	F	F	0.048
F	T	T	0.392
F	T	F	0.168
F	F	T	0.144
F	F	F	0.096

a) $P(A, C)$ [5 pts]

Your solution:

$P(A = T, C = T) = P(A = T, C = T, B = T) + P(A = T, C = T, B = F) = 0.014 + 0.012 = 0.026$

$P(A = T, C = F) = P(A = T, C = F, B = T) + P(A = T, C = F, B = F) = 0.126 + 0.048 = 0.174$

$P(A = F, C = T) = P(A = F, C = T, B = T) + P(A = F, C = T, B = F) = 0.392 + 0.144 = 0.536$

$P(A = F, C = F) = P(A = F, C = F, B = T) + P(A = F, C = F, B = F) = 0.168 + 0.096 = 0.264$

P(A, C)	C = T	C = F
A = T	0.026	0.174
A = F	0.536	0.264

b) **P(C)** – you can use your answer to part a to compute the answer to this question. [5 pts]

Your solution:

$$P(C = T) = P(A = T, C = T) + P(A = F, C = T) = 0.026 + 0.536 = 0.562$$

$$P(C = F) = P(A = T, C = F) + P(A = F, C = F) = 0.174 + 0.264 = 0.438$$

	C = T	C = F
P(C)	0.562	0.438

c) **P(A|C)** – you can use your answers to parts a and b to compute the answer to this question. [5 pts]

Your solution:

$$P(A|C) = P(A, C) / P(C)$$

P(A, C)	A = T	F = A
C = T	$0.026 / 0.562 = 0.046$	$0.174 / 0.562 = 0.954$
C = F	$0.536 / 0.438 = 0.397$	$0.264 / 0.438 = 0.603$

d) **P(A, B | C)** – you can use your answers from previous parts if they are relevant. [5 pts]

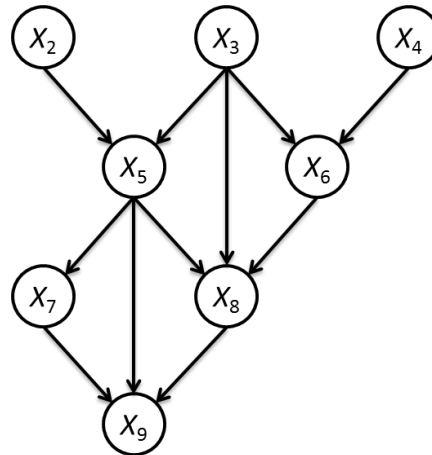
Your solution:

$$P(A, B | C) = P(A, B, C) / P(C)$$

A	B	C	P(A, B C)
T	T	T	$0.014 / 0.562 = 0.025$
T	T	F	$0.126 / 0.438 = 0.288$
T	F	T	$0.012 / 0.562 = 0.021$
T	F	F	$0.048 / 0.438 = 0.11$
F	T	T	$0.392 / 0.562 = 0.698$
F	T	F	$0.168 / 0.438 = 0.384$
F	F	T	$0.144 / 0.562 = 0.256$
F	F	F	$0.096 / 0.438 = 0.219$

Problem 3 [20 pts]:

We are given the following Bayesian network over X_2, X_3, \dots, X_9 . Note that there is no X_1 .



What is the Bayesian network factorization of the joint probability $P(X_2, X_3, \dots, X_9)$?

Your solution:

X_5 is dependent on X_2 and X_3 , so $P(X_5 | X_2, X_3)$

X_6 is dependent on X_3 and X_4 , so $P(X_6 | X_3, X_4)$

X_7 is dependent on X_5 , so $P(X_7 | X_5)$

X_8 is dependent on X_5 , X_3 , and X_6 , so $P(X_8 | X_5, X_3, X_6)$

X_9 is dependent on X_7 , X_5 , and X_8 , so $P(X_9 | X_7, X_5, X_8)$

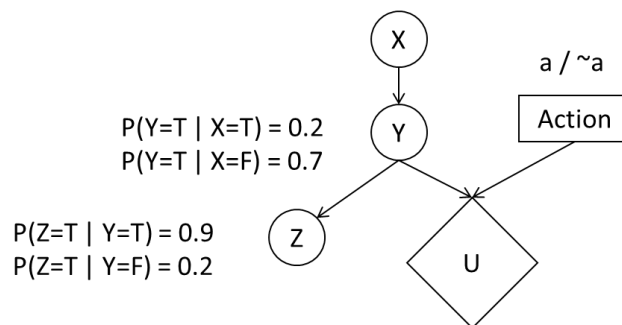
Bayesian network factorization of the joint probability $P(X_2, X_3, \dots, X_9) =$

$= P(X_2) * P(X_3) * P(X_4) * P(X_5 | X_2, X_3) * P(X_6 | X_3, X_4) * P(X_7 | X_5) * P(X_8 | X_5, X_3, X_6) * P(X_9 | X_7, X_5, X_8)$

Problem 4 [25 pts]:

We are given the following decision network:

$$P(X=T) = 0.4$$



Y	Action	U(Y, Action)
T	a	800
T	~a	400
F	a	200
F	~a	1000

a) Which action should be taken? Justify your decision. [5 pts]

Your solution:

$$P(Y = T \mid X = T) = .2$$

$$P(Y = F \mid X = T) = 1 - P(Y = T \mid X = T) = 1 - .2 = .8$$

$$P(Y = T \mid X = F) = .7$$

$$P(Y = F \mid X = F) = 1 - P(Y = T \mid X = F) = 1 - .7 = .3$$

$$EU(\text{Result}(a \mid X = T)) =$$

$$P(Y = T \mid X = T) * U(Y = T, a) + P(Y = F \mid X = T) * U(Y = F, a) =$$

$$.2 * 800 + .8 * 200 = 320$$

$$EU(\text{Result}(a \mid X = F)) =$$

$$P(Y = T \mid X = F) * U(Y = T, a) + P(Y = F \mid X = F) * U(Y = F, a) =$$

$$.7 * 800 + .3 * 200 = 700$$

$$EU(\text{Result}(-a \mid X = T)) =$$

$$P(Y = T \mid X = T) * U(Y = T, -a) + P(Y = F \mid X = T) * U(Y = F, -a) =$$

$$.2 * 400 + .8 * 1000 = 920 \leftarrow \text{this action should be taken (no action)}$$

$$EU(\text{Result}(-a \mid X = F)) =$$

$$P(Y = T \mid X = F) * U(Y = T, -a) + P(Y = F \mid X = F) * U(Y = F, -a) =$$

$$.7 * 400 + .3 * 1000 = 580$$

$$MEU = 920$$

b) What is the value of information about Z? Justify your decision. [5 pts]

Your solution:

$$P(Z = T \mid Y = T) = .9$$

$$P(Z = F \mid Y = T) = 1 - P(Z = T \mid Y = T) = 1 - .9 = .1$$

$$P(Z = T \mid Y = F) = .2$$

$$P(Z = F \mid Y = F) = 1 - P(Z = T \mid Y = F) = 1 - .2 = .8$$

$$EU(\text{Result}(a \mid Y = T)) =$$

$$P(Z = T \mid Y = T) * U(Y = T, a) + P(Z = F \mid Y = T) * U(Y = F, a) =$$

$$.9 * 800 + .1 * 200 = 740$$

$$EU(\text{Result}(a \mid Y = F)) =$$

$$P(Z = T \mid Y = F) * U(Y = T, a) + P(Z = F \mid Y = F) * U(Y = F, a) =$$

$$.2 * 800 + .8 * 200 = 320$$

$$EU(\text{Result}(-a \mid Y = T)) =$$

$$P(Z = T \mid Y = T) * U(Y = T, -a) + P(Z = F \mid Y = T) * U(Y = F, -a) =$$

$$.9 * 400 + .1 * 1000 = 460$$

$$\begin{aligned}
 &EU(\text{Result}(-a \mid Y = F)) = \\
 &P(Z = T \mid Y = F) * U(Y = T, -a) + P(Z = F \mid Y = F) * U(Y = F, -a) = \\
 &.2 * 400 + .8 * 1000 = 880 \leftarrow \text{this action should be taken (no action)} \\
 \\
 &MEU = 880
 \end{aligned}$$

c) What is the value of information about X? Justify your decision. [5 pts]

Your solution:

$$\begin{aligned}
 &P(X = T \mid Y = T) = \\
 &P(Y = T \mid X = T) * P(X = T) / P(Y = T) = \\
 &P(Y = T \mid X = T) * P(X = T) / [P(Y = T \mid X = T) * P(X = T) + P(Y = T \mid X = F) * P(X = F)] = \\
 &(.2 * .4) / [(.2 * .4) + (.7 * .4)] = \\
 &= 2/9
 \end{aligned}$$

$$P(X = F \mid Y = T) = 1 - P(X = T \mid Y = T) = 1 - 2/9 = 7/9$$

$$P(Y = F \mid X = T) = 1 - P(Y = T \mid X = T) = 1 - .2 = .8$$

$$P(Y = F \mid X = F) = 1 - P(Y = T \mid X = F) = 1 - .7 = .3$$

$$\begin{aligned}
 &P(X = T \mid Y = F) = \\
 &P(Y = F \mid X = T) * P(X = T) / P(Y = F) = \\
 &P(Y = F \mid X = T) * P(X = T) / [P(Y = F \mid X = T) * P(X = T) + P(Y = F \mid X = F) * P(X = F)] = \\
 &(.8 * .4) / [(.8 * .4) + (.3 * .4)] = 8/11
 \end{aligned}$$

$$P(X = F \mid Y = F) = 1 - P(X = T \mid Y = F) = 1 - 8/11 = 3/11$$

$$\begin{aligned}
 &EU(\text{Result}(a \mid Y = T)) = \\
 &P(X = T \mid Y = T) * U(Y = T, a) + P(X = F \mid Y = T) * U(Y = F, a) = \\
 &2/9 * 800 + 7/9 * 200 = 3000/9
 \end{aligned}$$

$$\begin{aligned}
 &EU(\text{Result}(a \mid Y = F)) = \\
 &P(X = T \mid Y = F) * U(Y = T, a) + P(X = F \mid Y = F) * U(Y = F, a) = \\
 &8/11 * 800 + 3/11 * 200 = 7000/11
 \end{aligned}$$

$$\begin{aligned}
 &EU(\text{Result}(-a \mid Y = T)) = \\
 &P(X = T \mid Y = T) * U(Y = T, -a) + P(X = F \mid Y = T) * U(Y = F, -a) = \\
 &2/9 * 400 + 7/9 * 1000 = 7800/9 \leftarrow \text{this action should be taken (no action)}
 \end{aligned}$$

$$\begin{aligned}
 &EU(\text{Result}(-a \mid Y = F)) = \\
 &P(X = T \mid Y = F) * U(Y = T, -a) + P(X = F \mid Y = F) * U(Y = F, -a) = \\
 &8/11 * 400 + 3/11 * 1000 = 6200/11
 \end{aligned}$$

d) Given $Z = T$, what is the value of information of X ? Justify your decision. [5 pts]

Your solution:

$$\begin{aligned}
 P(X = T \mid Y = T, Z = T) &= \\
 P(Z = T \mid X = T, Y = T) * P(X = T \mid Y = T) / P(Z = T) &= \\
 P(Z = T \mid X = T, Y = T) * P(X = T \mid Y = T) / [P(Z = T \mid X = T, Y = T) * P(X = T \mid Y = T) &+ \\
 + P(Z = T \mid X = F, Y = T) * P(X = T \mid Y = T) + P(Z = T \mid X = F, Y = F) * P(X = T \mid Y = T)] &= \\
 2/9 / (1) &=
 \end{aligned}$$