

# **Simplicial complexes associated with point clouds II**

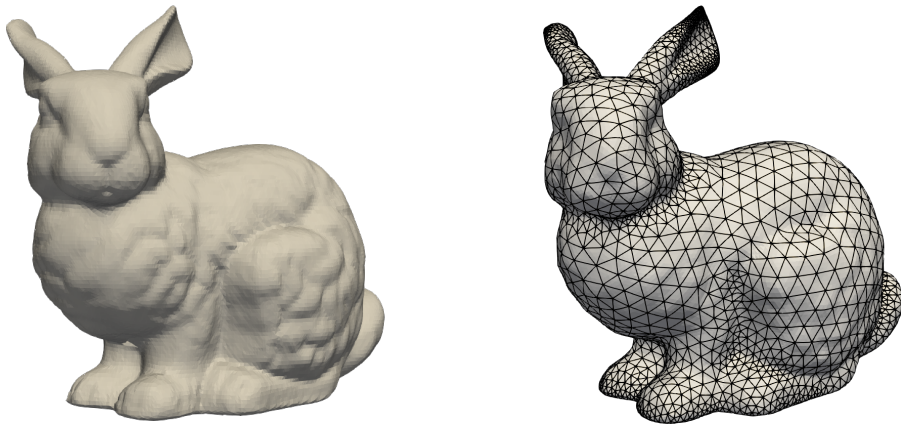
Seminar on Topological Data Analysis

Tim van Beeck

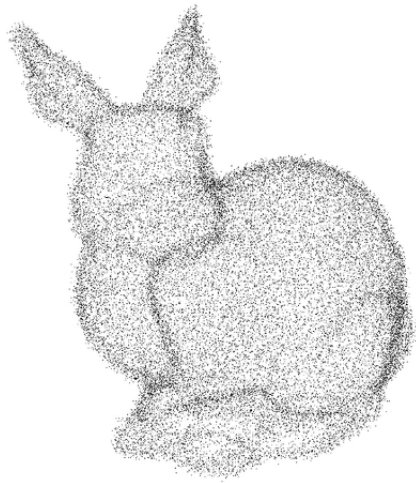
May 17, 2022

University of Göttingen

# Simplicial complexes



**Figure 1:** Stanford bunny, smooth and approximated by a mesh [8].



**Figure 2:** Point cloud representation of the bunny [2].

Voronoi diagrams

Delaunay triangulations

Alpha complexes and Filtrations

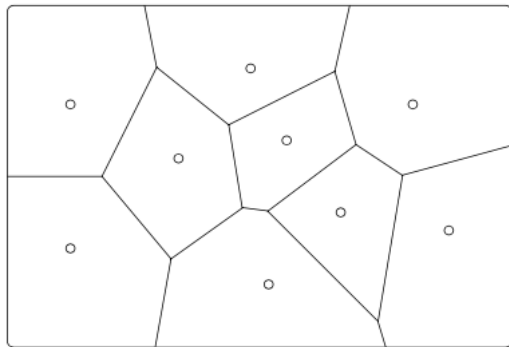
Collapses

Critical and regular events

# Voronoi diagrams

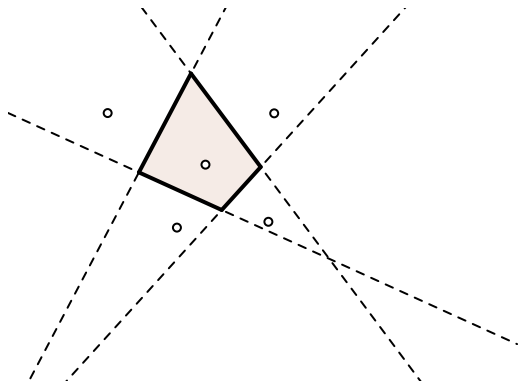
---

## Example of a Voronoi diagram



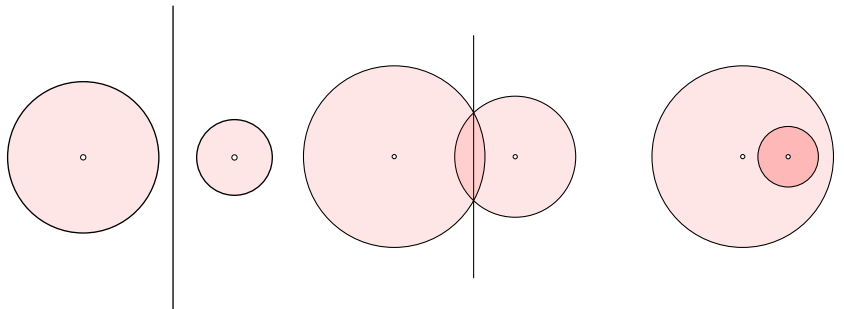
**Figure 3:** Voronoi diagram [1].

## Voronoi cell as intersection of halfplanes



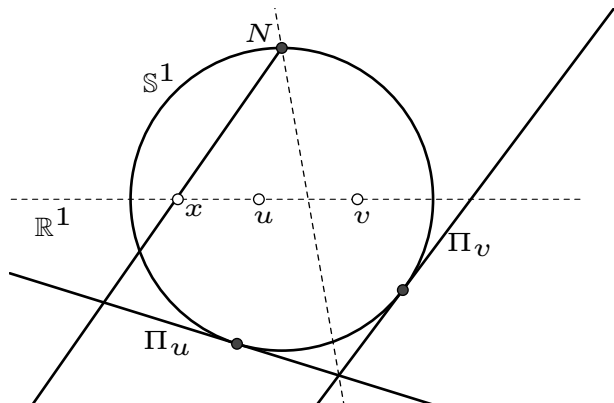
**Figure 4:** A Voronoi cell as intersection of halfplanes.

## Weighted bisectors - Example

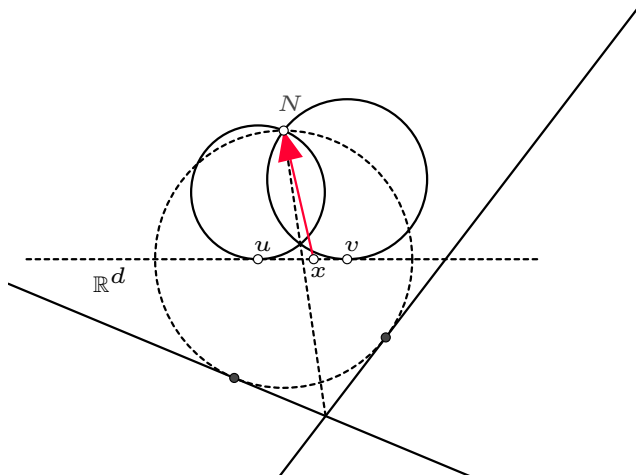


**Figure 5:** Weighted bisectors [1].





**Figure 6:** Lifting of Voronoi cells [inspired by [1]].

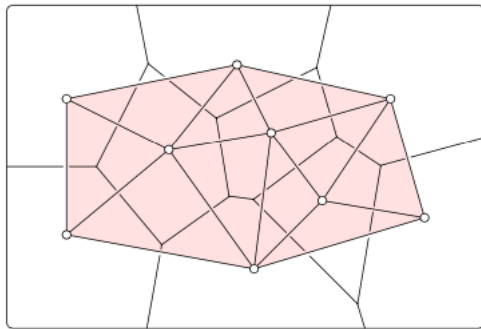


**Figure 7:** Lifting of Voronoi cells [inspired by [1]].

# Delaunay triangulations

---

## Example of a Delaunay triangulation

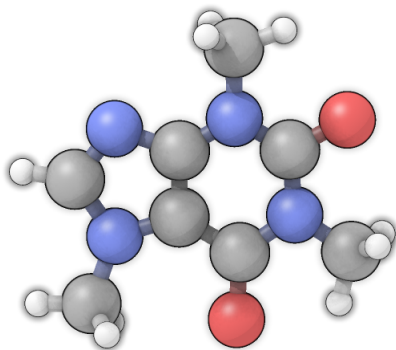


**Figure 8:** Delaunay triangulation [1].

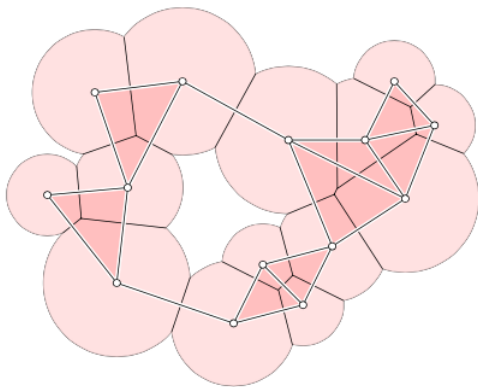
# Alpha complexes and Filtrations

---

## Weighted alpha complexes - Motivation

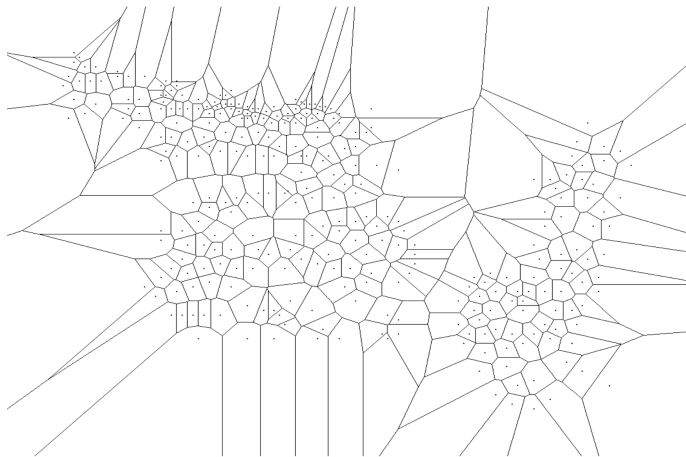


**Figure 9:** Sketch of a caffeine molecule [3].



**Figure 10:** A weighted alpha complex [1].

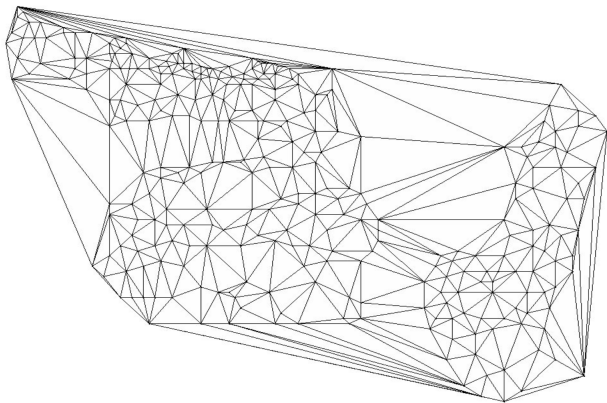
## Location of trees - Voronoi diagram



**Figure 11:** Voronoi diagram of the location of trees [4].

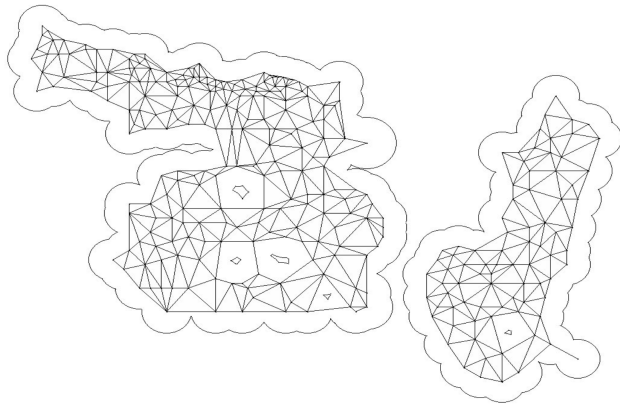


## Location of trees - Delaunay triangulation



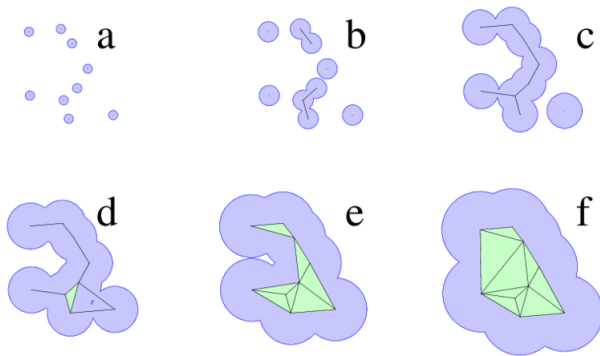
**Figure 12:** Delaunay triangulation of the location of trees [4].

## Location of trees - Alpha complex



**Figure 13:** Alpha complex of the location of trees [4].

## Example Filtration

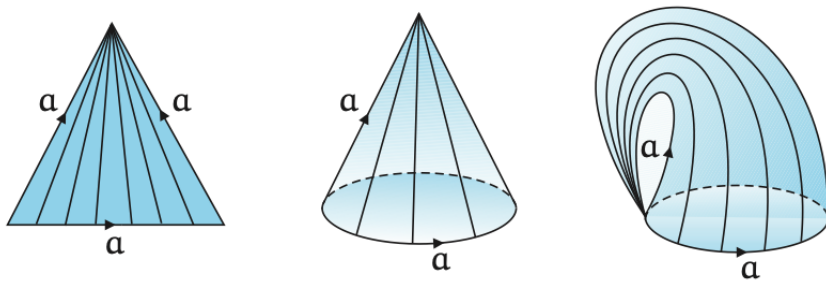


**Figure 14:** Filtration [5].

# Collapses

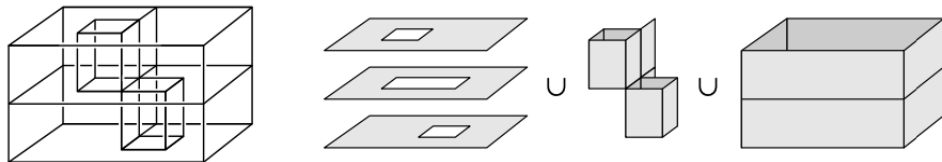
---

## Contractible, but not collapsible!



**Figure 15:** The dunce hat [6].

## Contractible, but not collapsible!

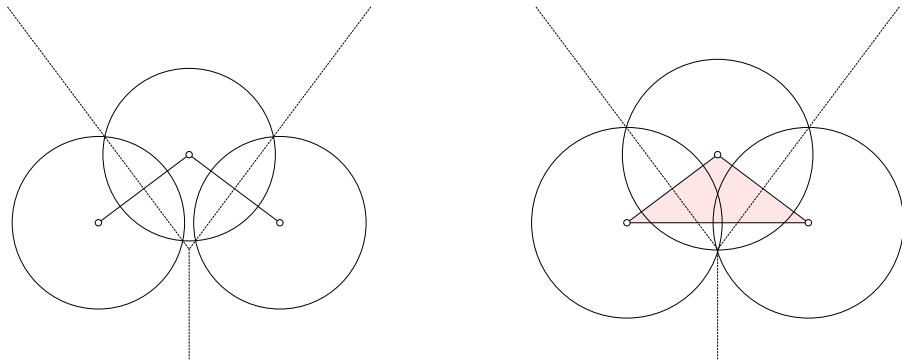


**Figure 16:** Bing's house with two rooms [7].

# Critical and regular events

---

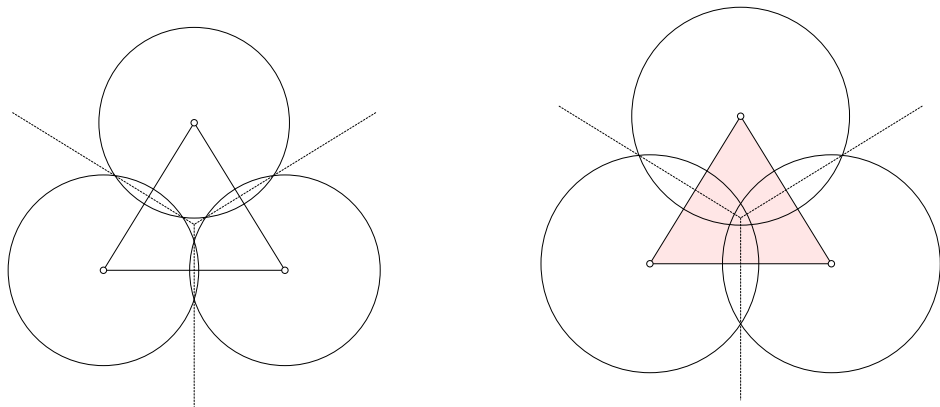
## Regular events - Example



**Figure 17:** Two edges appear before a regular event adds the triangle [inspired by [1]].



## Critical events - Example



**Figure 18:** Three edges appear before a critical event adds the triangle [inspired by [1]].

## References

- [1] H. Edelsbrunner and J.L. Harer. *Computational Topology: An Introduction*. Miscellaneous Bks. American Mathematical Society, 2022.
- [2] Patrick O'Neil and Thomas Wanner. *Analyzing the Squared Distance-to-Measure Gradient Flow System with k-Order Voronoi Diagrams*. Jan 2019
- [3] [https://upload.wikimedia.org/wikipedia/commons/d/df/Caffeine\\_ballandstick.png](https://upload.wikimedia.org/wikipedia/commons/d/df/Caffeine_ballandstick.png)
- [4] H. Edelsbrunner. *Shape Reconstruction with Delaunay Complex (Invited Paper)*. Springer-Verlag Berlin Heidelberg. 1998.
- [5] Jie Liang. *Computation of protein geometry and its applications: Packing and function prediction*. 2008.
- [6] Ronald Brown. *Topology and Groupoids*. 2006.
- [7] Allen Hatcher. *Algebraic Topology*. 2001.
- [8] Lederer PL, Lehrenfeld C, Schöberl J. *Divergence-free tangential finite element methods for incompressible flows on surfaces*. Int J Numer Methods Eng. 2020.