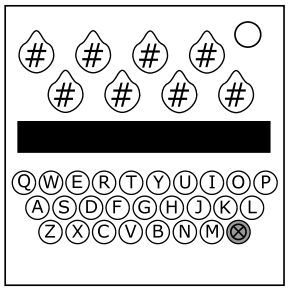
On the Subject of The Hill Cycle



Not the kind you need a mountain bike for.

This module consists of a screen, eight dials with blue labels, and a QWERTY keyboard.

The labels on the dials, when decrypted and read from left to right, spell out an eight letter word.

The eight dials can be split into four pairs of adjacent dials, each with one upper and one lower dial.
The number of 72° rotations, starting from north, of each pair is interpreted are interpreted as two digit base 5 numbers.
These numbers give the entries of a 2x2 matrix in reading order.

The alphabetic positions of the letters on these pairs of dials modulo 26 become entries in 2x1 column vectors.
Each vector is multipled by the matrix and taken modulo 26 to produce the a vector whose entries are the alphabetic positions of the encrypted letters.
(with Z=0)

Once deciphered, find the word in the table below, the word written below it is the word that should be entered.

Apply the same encryption to the response word, and type out the encrypted response word using the keys.

The word is automatically submitted when eight keys are pressed.

The red button can be pressed at any time before the eighth key is pressed to cancel the input.

Inputting any of the eight letters incorrectly will cause a strike to be issued and reset the module.

Decrypting a Hill cipher

The encryption process utilises matrix multiplication:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} X' \\ Y' \end{bmatrix}$$

To undo this process, the inverse matrix must be found:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$$

First, find the adjugate of the key matrix:

$$\text{adj}\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

Then, find the multiplicative inverse of the determinant of the matrix, N:

$$\det\begin{bmatrix} A & B \\ C & D \end{bmatrix} = AD - BC$$

$$N(AD - BC) \bmod 26 = 1$$

(Multipicative inverses must also be found to decrypt affine enciphered messages.)

The inverse matrix is the adjugate matrix multiplied by the multiplicative inverse:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = N\begin{bmatrix} D & -B \\ -C & A \end{bmatrix}$$

The alphabetic positions of each encrypted pair of letters can be multiplied by the inverse matrix to retrieve the original message.

Keyword Table

ADVERTED	ADVOCATE	ALLOCATE	ALTERING	BINORMAL	BINOMIAL	BULKHEAD	BULLETED
COMMANDO	ENCIPHER	NUMERALS	CIPHERED	HAZARDED	MULTIPLY	DECEIVED	ULTRAHOT

CIPHERED	CIRCUITS	COMPILER	COMMANDO	DECIMATE	DECEIVED	DISCOVER	DISPOSAL
GATEWAYS	TRICKIER	DECIMATE	OBSERVED	ORDERING	YOURSELF	NUMEROUS	UNDERWAY

ENCIPHER	ENTRANCE	EQUATORS	EQUALISE	FINALISE	FINNICKY	FORTRESS	FORWARDS
WHATNESS	QUADRANT	TOGETHER	QUARTICS	ROTATORS	KILOBYTE	WHATNOTS	JIGGLING

GAUNTLET	GAMBLING	GATEPOST	GATEWAYS	HAZARDED	HAZINESS	HUNGRIER	HUNTRESS
ENTRANCE	EQUALISE	INCOMING	ALLOCATE	RELAYING	UNDERLIE	ADVOCATE	VICINITY

INCOMING	INDIRECT	ILLUSION	ILLUMINE	JIGSAWED	JIGGLING	JUNCTION	JUNKYARD
PROGRESS	QUARTILE	HUNGRIER	HUNTRESS	REVOLVED	MONOGRAM	YOKOZUNA	VOLITION

KILOWATT	KILOBYTE	KNOCKOUT	KNOCKING	LINGERED	LINEARLY	LINKAGES	LINKWORK
ZUGZWANG	PROJECTS	JIGSAWED	BULLETED	ILLUSION	GAUNTLET	BINOMIAL	ADVERTED

MONOGRAM	MONOMIAL	MULTIPLY	MULTITON	NANOGRAM	NANOWATT	NUMEROUS	NUMERALS
LINKWORK	ZIPPERED	STANDOUT	QUADRICS	MONOMIAL	BULKHEAD	PROPHASE	JUNCTION

ORDINALS	ORDERING	OBSERVED	OBSCURED	PROGRESS	PROJECTS	PROPHASE	PROPHECY
OBSCURED	DISCOVER	KNOCKING	FINNICKY	REVERSED	TOGGLING	ZYGOMATA	CIRCUITS

QUADRANT	QUADRICS	QUARTILE	QUARTICS	REVERSED	REVOLVED	ROTATORS	RELAYING
VOLATILE	KNOCKOUT	STOPPING	ULTRARED	STOPWORD	LINKAGES	HAZINESS	ZYMOGENE

STANZAIC	STANDOUT	STOPPING	STOPWORD	TRIGONAL	TRICKIER	TOGGLING	TOGETHER
ILLUMINE	YEASAYER	MULTITON	NANOWATT	WHATSITS	PROPHECY	COMPILER	GAMBLING

UNDERWAY	UNDERLIE	ULTRAHOT	ULTRARED	VICINITY	VICELESS	VOLITION	VOLATILE
GATEPOST	LINEARLY	KILOWATT	BINORMAL	JUNKYARD	TRIGONAL	INDIRECT	ALTERING

WHATNESS	WHATSITS	WHATEVER	WHATNOTS	YEARLONG	YEASAYER	YOKOZUNA	YOURSELF
DISPOSAL	NANOGRAM	FORTRESS	YEARLONG	WHATEVER	ORDINALS	STANZAIC	LINGERED

ZIPPERED	ZYGOMATA	ZUGZWANG	ZYMOGENE
EQUATORS	FINALISE	FORWARDS	VICELESS