PRONALAZAK MINIMUMA VIŠEDIMENZIJSKIH FUNKCIJA BEZ DERIVACIJA

1. TRAŽENJE MINIMUMA NA PRAVCU

$$v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$x_0 + \lambda \cdot v$ → dani smjer

↳ iznos pomaka

↓ početna točka

$f(x_0 + \lambda \cdot v)$ → minimizacija funkcije s jednom varijablom

* ZLATNI REZ

ZAD zadana je $f(x) = (x_1 - 2)^2 + (x_2 + 1)^2$

zadan je smjer pretraživanja $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ i početna točka

$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. $h = 1$, $\varepsilon \leq 0.5$

$$x = x_0 + \lambda \cdot v = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \lambda \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\boxed{\begin{aligned} x_1 &= 0 + 2\lambda \\ x_2 &= 0 + \lambda \end{aligned}}$$

$$F(\lambda) = (2\lambda - 2)^2 + (\lambda + 1)^2 \quad /'$$

ANALITIČKI ODREĐIVANJE IZNOSA POMAKA

$$\cancel{2} \cdot (2\lambda - 2) \cdot 2 + \cancel{2}(\lambda + 1) = 0$$

$$4\lambda - 4 + \lambda + 1 = 0$$

$$5\lambda = 3$$

$$\boxed{\lambda = 0.6}$$

POČETNO: $\lambda = 0 \longrightarrow F(\lambda) = 5$

$$F(\lambda) = (2\lambda - 2)^2 + (\lambda + 1)^2$$

λ	-1	[0	1	2]
$F(\lambda)$	16 >	5 >	4 <	13

$\Rightarrow$ interval $[0, 2]$ (a = 0, b = 2)

a	c	d	b
0	0.764	1.236	2
0	0.472	0.764	1.236
0	0.292	0.472	0.764
0.292	0.472		0.764

$b - a \leq \varepsilon$ KRITERIJ ZAUSTAVLJ. ✓

$\lambda \in [0.292, 0.764]$

$$x \in [x_0 + \lambda_1 v,\ x_0 + \lambda_2 v]$$

$(2, -1)$

$$x \in \left[\begin{bmatrix} 0.584 \\ 0.292 \end{bmatrix}, \begin{bmatrix} 1.528 \\ 0.764 \end{bmatrix} \right]$$

ovo nije minimum, samo iduća točka pretrage po v pravcu, smjeru pretrage

(ne mora prolaziti kroz minimum)

smjerovi v moraju biti NEKOLINEARNI

KOJI SMJER?

TRAŽENJE U SMJERU KOORDINATNIH OSI (KOORDINATNI SPUST)

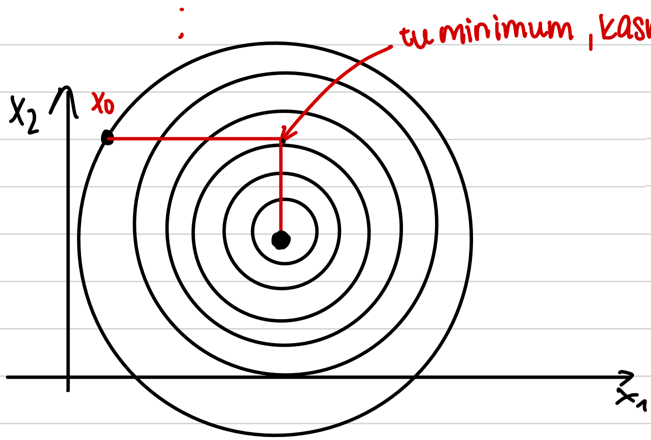
2D: $v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$x_1 = x_0 + \lambda \cdot v_1$

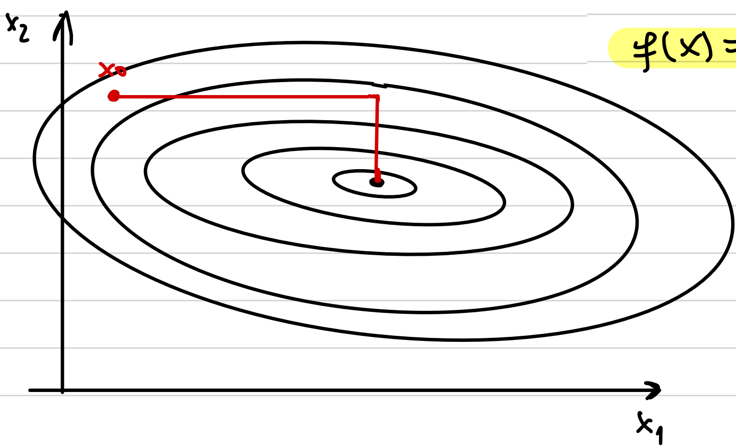
$x_2 = x_1 + \lambda \cdot v_2$

$x_3 = x_2 + \lambda \cdot v_1$

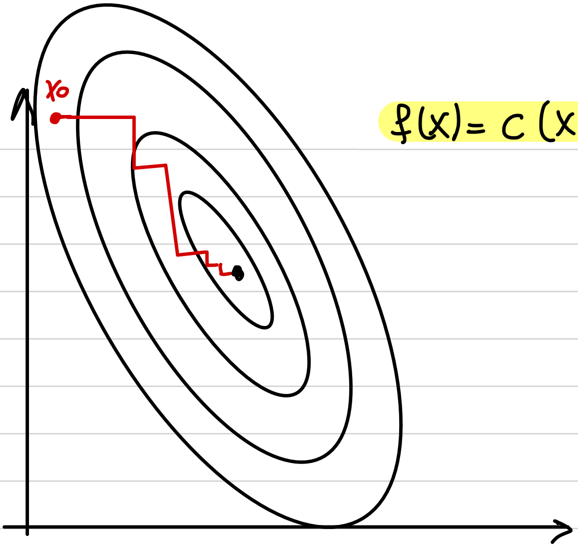
$\vdots$



$f(x) = (x_1 - a)^2 + (x_2 - b)^2$



$f(x) = c(x_1 - a)^2 + d(x_2 - b)^2$



$$f(x) = c(x_1 - a)^2 + d(x_2 - b)^2 + e \cdot x_1 x_2$$

POWELLOV POSTUPAK KONJUGIRANIH SMJEROVA

$$F(x) = c + b^T \cdot x + \frac{1}{2} x^T \cdot A \cdot x$$ ⇒ KVADRATNA FORMA

BILO KOJU KV. FUNKCIJU MOŽEMO TAKO ZAPISATI!

$$x_{MIN} = -A^{-1} b$$

$v_1^T \cdot A \cdot v_2 = 0$ ← UVJET KONJUGIRANOSTI

1. MINIMUM KVADRATNE FUNKCIJE DOBIVA SE TRAŽENJEM U n KONJUGIRANIH SMJEROVA

2. KONJUGIRANI SMJER DOBIVA SE SPAJANJEM MINIMUMA NA 2 PARALELNA PRAVCA

$$\boxed{X_{MIN} = x_0 + \sum_{j=1}^{m} v_j \lambda_j}$$

konjugirani smjerovi

$\boxed{\text{ZAD}}$

problematičan član

početna točka

$$F(x) = x^2 + y^2 + xy \qquad v_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad T_1 = (1,1)$$

X_{min} analitički $= (0,0)$

$$v_1 A v_2 = 0$$

$$T = (1,1) \qquad \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$F(\lambda) = (1+\lambda\cdot 0)^2 + (1+\lambda)^2 + (1+\lambda)\cdot 1$$

$$= 1 + 2\lambda + 1 + \lambda^2 + 1 + \lambda = \lambda^2 + 3\lambda + 3$$

$$\lambda = -\frac{3}{2}$$

$$X_{min_1} = \begin{bmatrix} 1 \\ -\frac{1}{2} \end{bmatrix}$$

$$T = (2,0) \qquad \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \lambda \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$F(\lambda) = (2)^2 + (\lambda)^2 + 2\cdot\lambda = 4 + \lambda^2 + 2\lambda$$

$$2\lambda + 2 = 0$$

$$\lambda = -1$$

$$X_{MIN_2} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$v_2 = X_{min_2} - X_{min_1} = \begin{bmatrix} 2 \\ -1 \end{bmatrix} - \begin{bmatrix} 1 \\ -1/2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1/2 \end{bmatrix} \rightarrow \text{konjugirani smjer}$$

$$v_1^T A v_2 = 0$$

$$F(x) = \frac{1}{2} \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial F}{\partial^2 x} & \frac{\partial F}{\partial x \partial y} \\ \frac{\partial F}{\partial y \partial x} & \frac{\partial F}{\partial^2 y} \end{bmatrix}$$

v_1

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0$$

$$a_1 + 2a_2 = 0$$

$$a_1 = -2a_2$$

$$v_2 = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} -2a_2 \\ a_2 \end{bmatrix}$$

npr. $a_2 = -\frac{1}{2}$

$$v_2 = \begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$$

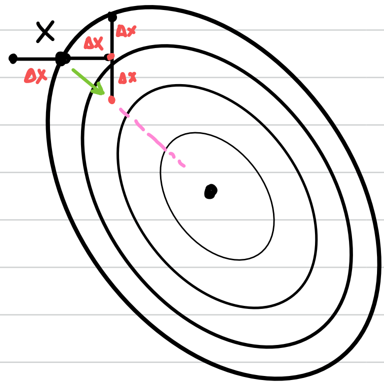
KO ŠTO SMO DOBILI PRIJE

HOOKE - JEEVES

x_0 → POČETNA TOČKA

Δx → početni vektor pomaka

ε → preciznost



POSTUPAK

↳ FAZA ISTRAŽIVANJA Δx

↳ IMA LI NAPRETKA

↳ POMAK U ZADANOM SMJERU $x_N = 2x_1 - x_0$

PRIMJER

$f(x) = x_1^2 + 4 \cdot x_2^2$ $x_0 = (7,3)$ $\Delta x = 1$ $\varepsilon = 0.25$

$x_P = 2x_N - x_B$

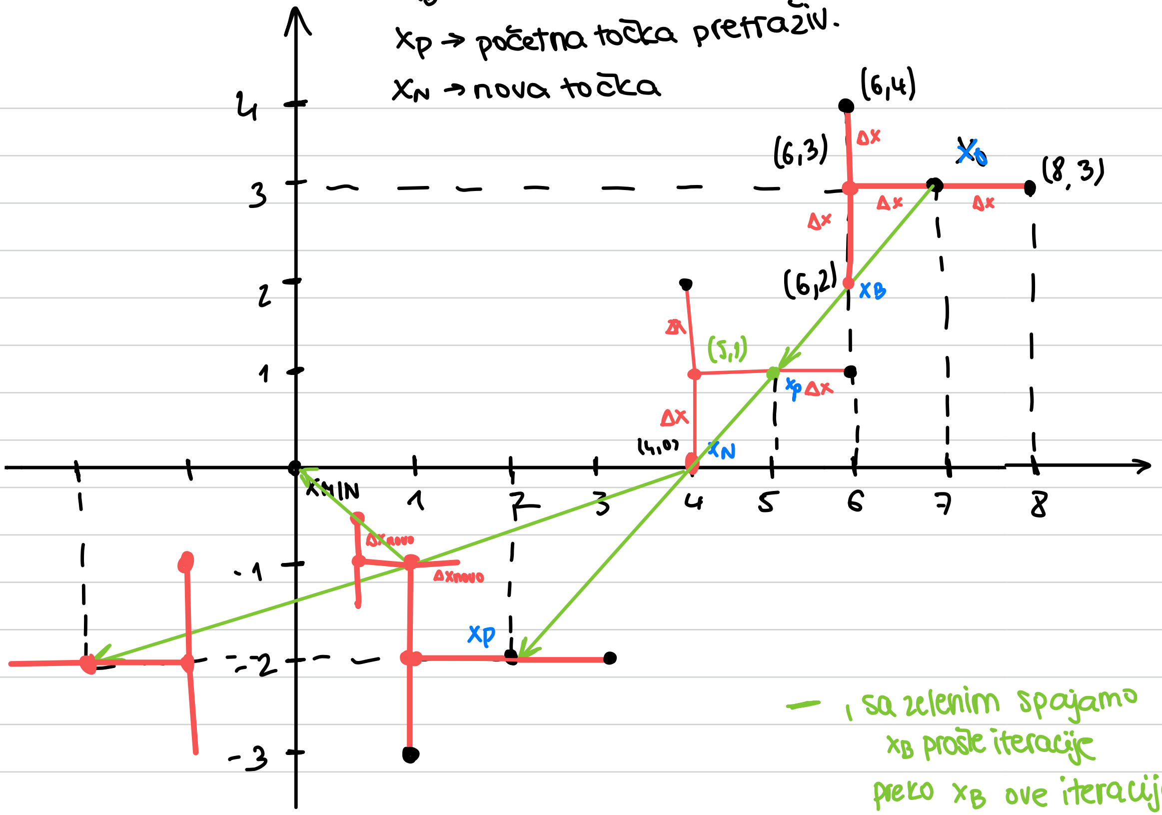
x_B	x_P	x_N	$f(x_N) < f(x_B)$	
(7,3)	(7,3)	(6,2)	✓	
(6,2)	(5,1)	(4,0)	✓	
(4,0)	(2,-2)	(1,-1)	✓	
(1,-1)	(-2,-2)	(-1,-1)	×	$\frac{\Delta x}{2} = 0.5 = \Delta x_{novi}$
(1,-1)	(1,-1)	(0.5,-0.5)	✓	
(0.5,-0.5)	(0,0)	(0,0)	✓	
RJ: (0,0)	(-0.5,0.5)	(0,0)	×	$\frac{\Delta x}{2} = 0.25 = \varepsilon$

ZAUSTAVLJANJE $\Delta x \leq \varepsilon$

$x_B \rightarrow$ bazna točka

$x_P \rightarrow$ početna točka pretraživ.

$x_N \rightarrow$ nova točka

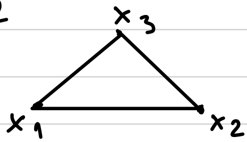


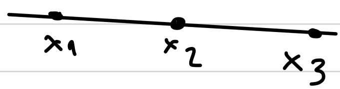
— , sa zelenim spajamo x_B prošle iteracije preko x_B ove iteracije s x_P ove iteracije

NELDER MEAD SIMPLEKS

→ SIMPLEKS → N+1 koje u N-dimenzionalnom prostoru čine N-dimenzijsku tvorevinu

npr. $N = 2$





⇓

ovo nije simpleks, ne rezapinju N-dim. tvorevinu

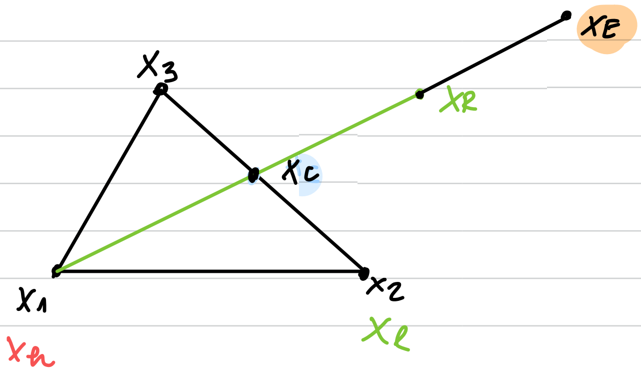
→ PRAVILNI SIMPLEKS

↳ jednake udaljenosti između točaka

→ NEPRAVILNI SIMPLEKS

X_H → NAJGORA TOČKA

X_L → NAJBOLJA TOČKA



CENTROID

$$x_c = \frac{1}{N} \sum x_i$$

↙ ALI BEZ NAJGORE TOČKE

1. REFLEKSIJA → ova operacija se jedina uvijek radi

$$x_R = (1+\alpha) \cdot x_c - \alpha \cdot x_h$$

α - koeficijent refleksije   npr. $\alpha = 1$

2. EKSPANZIJA → ako je x_R bolja od x_L

$$x_E = (1-\gamma) x_c - \gamma \cdot x_R$$

γ - koeficijent ekspanzije   npr. $\gamma = 2$

3. KONTRAKCIJA

→ ako je x_R najgora ili 2. najgora točka

↳ radi se na boljoj točki između

x_n i x_E → približavamo je x_C

$$x_K = (1-\beta) \cdot x_C + \beta x_n$$

$$\beta = 0.5$$

4. SAŽIMANJE

→ ako je x_K lošija

$$x_i = \frac{(x_i + x_e)}{2}$$