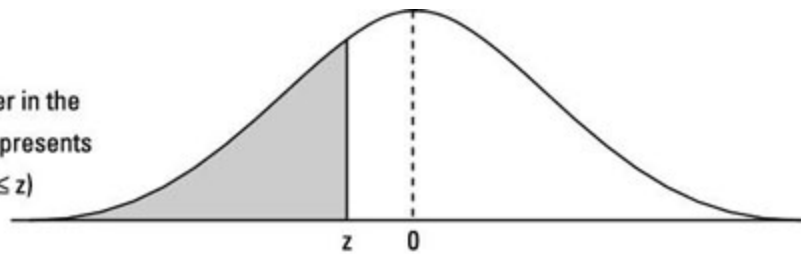


A normal distribution curve is shown with a horizontal axis. The mean is marked as 0 with a vertical dashed line. A point z is marked on the axis to the right of 0. The area under the curve to the left of z is shaded gray.

[illegible]

Number in the
table represents
 $P(Z \leq z)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.6	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

1. A sales manager for a new "super computer" has just returned from visiting five possible clients. She believes that the following table describes the distribution for the number of sales she will make:

y_i	0	1	2	3	4	5
$p(y_i)$	0.25	0.20	0.30	0.10	0.10	0.05

a) Find the probability that she makes more than two sales.

$$p(y_i > 2) = p(y_i = 3) + p(y_i = 4) + p(y_i = 5) = 0.10 + 0.10 + 0.05 = 0.25$$

b) Find the expected number of sales.

$$E = \sum_{i=0}^5 y_i * p(y_i) = 0 * 0.25 + 1 * 0.20 + 2 * 0.30 + 3 * 0.10 + 4 * 0.10 + 5 * 0.05 = 1.75$$

c) Find the variance for the number of sales

$$\text{Var} = \sigma^2 = \sum_{i=0}^5 (y_i - E)^2 * p(y_i) = (0 - 1.75)^2 * 0.25 + (1 - 1.75)^2 * 0.20 + (2 - 1.75)^2 * 0.30 + (3 - 1.75)^2 * 0.10 + (4 - 1.75)^2 * 0.10 + (5 - 1.75)^2 * 0.05$$

d) Find the standard deviation for the number of sales

$$\text{Std} = \sigma = \sqrt{\text{Var}} = \sqrt{\sum_{i=0}^5 (y_i - E)^2 * p(y_i)}$$

2. Phone calls arrive at a customer support center according to a Poisson distribution $P(X = k) = \frac{e^{-\lambda} * \lambda^k}{k!}$ with a rate of 3 ($= \lambda$) calls per minute.

a) Find the probability that there will be less than 2 calls in one minute. $k < 2 \Rightarrow k = 0$ and $k = 1$

$$P(X < 2) = P(X = 0) + P(X = 1) = \frac{e^{-3} * 3^0}{0!} + \frac{e^{-3} * 3^1}{1!} = e^{-3} + 3e^{-3} = 4e^{-3}$$

b) What is the expected time between calls? $E = \frac{1}{\lambda} = \frac{1}{3}$ minutes

c) What is the probability that the time between calls is more than a minute?

Let's denote the random variable Y as the time between calls: We have:

$$P(Y > 1) = 1 - P(Y \leq 1)$$

The cumulative distribution function (CDF) of an exponential distribution is:

$$F = 1 - e^{-\lambda t}. \text{ Substituting } t = 1 \text{ and } \lambda = 3. \text{ So } F = 1 - e^{-3 * 1}$$

3. A packaging device is set to fill detergent packets with a mean weight of 200g. From historical data the distribution of weights is approximately Normal with a standard deviation known to be 6.0g. It is important to check the machine periodically because if it is overfilling it increases the cost of the materials, whereas if it is underfilling the firm is liable to prosecution. A random sample of 9 filled packets is weighed and shows a mean weight of 203.5g. At the 1% significance level, is the process out-of-control?

Step 1: State hypotheses:

- Null hypothesis (H_0): The mean weight of detergent packets (μ) is equal to 200g.

$$H_0: \mu = 200$$

- Alternative hypothesis (H_1): The mean weight of detergent packets (μ) is different 200g. $H_1: \mu \neq 200$

Step 2: State the test statistic and assumptions.

Test statistic: We will use the t-statistic (z-scores) because we are dealing with a small sample size ($n=9$) and the population standard deviation (σ) is known.

$$t \text{ (or } z) = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Assumptions:

- The sample is a random sample.
- The weights are normally distributed.
- The standard deviation of the population is known.

Step 3: State the critical region(s):

Significance level (α) is 1% = 0.01. Since the alternative hypothesis is two-tailed (different from 200g), we have two critical regions, each with an area of $\alpha/2 = 0.5\%$ in the tails of the standard normal distribution, which means our critical region(s) will be in both tails of the distribution.

Step 4: Calculate test statistic:

- Sample mean (\bar{x}) = 203.5 g

- Population mean (μ) = 200 g
- Standard deviation (σ) = 6.0 g
- Sample size (n) = 9

$$t \text{ (or } z) = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{203.5 - 200}{\frac{6}{\sqrt{9}}} = 1.75$$

$$p\text{-value} = 2 * 0.0401 = 0.0802$$

Step 5: Reach conclusions and state in English:

For a two-tailed test at the 1% significance level, we want to find the z-values that correspond to a cumulative probability of 0.005 in the left tail and 0.995 in the right tail. These values divide the distribution into two equal tails, with a total probability of 0.01 in both tails.

Using a standard normal distribution table or a statistical software, we can find the z-values associated with these cumulative probabilities. The critical z-value for a cumulative probability of 0.005 in the left tail is approximately -2.58, and the critical z-value for a cumulative probability of 0.995 in the right tail is approximately 2.58.

These critical z-values indicate the boundaries beyond which the observed test statistic would be considered statistically significant at the 1% significance level in a two-tailed test. If the calculated test statistic falls within this range, we fail to reject the null hypothesis.

Step 6: Calculate the p-value associated with this test. How does this the p-value support your conclusions in Step 5?

$$p\text{-value} = 2 * 0.0401 = 0.0802$$

$$p\text{-value} = 0.0802 > \alpha = 0.01.$$

Therefore, the p-value associated with this test is approximately 0.0802.

The p-value represents the probability of obtaining a test statistic as extreme or more extreme than the one observed, assuming the null hypothesis is true. In this case, the p-value is greater than the significance level of 0.01. This suggests that the observed result is not statistically significant, and we fail to reject the null hypothesis.

Step 7: Suppose samples of 9 packets of detergent are taken every hour to monitor the packaging process (with the target of 200g).

On Monday the following sample means were observed:

202.5, 203.0, 203.5, 201.5, 199.0, 205.0, 202.5.

a) Calculate the appropriate control chart limits

$$\text{sample means: } \frac{202.5 + 203.0 + 203.5 + 201.5 + 199.0 + 205.0 + 202.5}{7} = 202.4285714$$

$$\text{upper control limit: } \bar{x} + 3 * \frac{\sigma}{\sqrt{n}} = 202.428571 + 3 * \frac{6}{\sqrt{9}}$$

$$\text{lower control limit: } \bar{x} - 3 * \frac{\sigma}{\sqrt{n}} = 202.428571 - 3 * \frac{6}{\sqrt{9}}$$

b) Plot the control chart for Monday's sample means

We will plot the sample means on the y-axis and the sample numbers (1, 2, 3, 4, 5, 6, 7) on the x-axis. The control limits will be displayed as horizontal lines on the chart.

c) Comment on your results.

By plotting Monday's sample means on the control chart, we can assess whether the process is within control. If any of the sample means fall outside the control limits, it may indicate an out-of-control process.

Based on the plotted control chart, we observe that all the sample means for Monday's samples fall within the control limits. This suggests that the process is in control, as none of the sample means show any significant deviation from the target mean of 200g. However, it is important to continue monitoring the process and collect more data over time to ensure ongoing process control and identify any potential trends or patterns that may emerge.

4. A manufacturer of battery packs for laptop computers monitors the process for defective packs. Historically, this process averages 4% defective battery packs. Suppose 10 battery packs are randomly selected for testing.

a) Find the probability that exactly 9 battery packs are not defective.

$$P(X = 9) = {}^{10}C_9 * 0.96^9 * 0.04^{(10-9)} = 0.277$$

b) Find the probability that at least two battery packs are defective

$$P(X \geq 2) = 1 - P(X < 2) = 1 - P(X = 0) - P(X = 1)$$

$$P(X = 0) = {}^{10}C_0 * 0.96^{(10-0)} * 0.04^{(0)}$$

$$P(X = 1) = {}^{10}C_1 * 0.96^{(10-1)} * 0.04^{(1)}$$

c) Find the expected number of defective battery packs, the variance and the standard deviation.

$$\text{Mean} = n * p = 10 * 0.04 = 0.4$$

$$\text{Variance} = n * p * (1 - p) = 10 * 0.04 * (1 - 0.04) = 0.384$$

$$\text{Standard deviation} = \sqrt{\text{Var}} = \sqrt{0.384} = 0.620$$

5. In a particular game, a fair die is tossed. If the number of spots showing is a six, you win \$6, if the number of spots showing is a five, you win \$3, if the number of spots showing is 4, you win \$2, and if the number of spots showing is 3, you win \$1. If the number of spots showing is 1 or 2, you win nothing. You are going to play the game twice.

a) What is the probability that you win something on each of the two plays of the game?

- The probability of winning something on any roll is 1 - the probability of winning nothing (1 or 2).
- There are 5 ways to win something (3, 4, 5, or 6) out of 6 possible outcomes on the die.
- Therefore, the probability of winning something on one roll is: $1 - (2/6) = 4/6 = 2/3$.
- So, the probability of winning something on both rolls is: $(2/3) * (2/3) = 4/9$.

b) What is the probability that you win at least \$9 in total on the two plays of the game?

1. Winning \$6 on both rolls: The probability of rolling a six on one roll is $1/6$, so the probability of rolling a six on both rolls is $(1/6) * (1/6) = 1/36$.
2. Winning \$6 on the first roll and \$3 on the second roll: The probability of rolling a six on the first roll is $1/6$, and the probability of rolling a five on the second roll is also $1/6$. Therefore, the probability of this combination is $(1/6) * (1/6) = 1/36$.
3. Winning \$6 on the second roll and \$3 on the first roll: This combination has the same probability as the previous one, which is $1/36$.
4. Winning \$6 on one roll and \$2 on the other: The probability of rolling a six on one roll is $1/6$, and the probability of rolling a four on the other roll is also $1/6$. Hence, the probability of this combination is $(1/6) * (1/6) = 1/36$.
5. Winning \$6 on one roll and \$1 on the other: This combination has the same probability as the previous one, which is $1/36$.
6. Winning \$3 on both rolls: The probability of rolling a five on one roll is $1/6$, so the probability of rolling a five on both rolls is $(1/6) * (1/6) = 1/36$.

Adding up the probabilities of these six winning combinations, we get:

$$(1/36) + (1/36) + (1/36) + (1/36) + (1/36) + (1/36) = 6/36 = 1/6.$$

Therefore, the probability of winning at least \$9 in total on the two plays of the game is $1/6$.

6. An automobile manufacturer gives a 60,000-mile warranty on its drive train. Historically, 7% of this manufacturer's automobiles have required service under this warranty. Recently, a design team proposed an improvement that should extend the drive train's life. A random sample of 200 cars underwent 60,000 miles of road testing; the drive train failed for 12. Has the new designed reduced the proportion of automobiles needing service under this warranty. Test using $\alpha = 0.05$.

Step 1: State hypotheses:

- Null hypothesis (H_0): The proportion of automobiles needing service under the warranty remains the same or has not decreased. $p \leq 0.07$
- Alternative hypothesis (H_1): The new design has reduced the proportion of automobiles needing service under the warranty. $p < 0.07$

Step 2: State the test statistic.

We will use a one-sample proportion z-test because:

- We are testing a proportion (a categorical variable).
- We have a single sample (the 200 cars).
- We know the population proportion (7% from historical data).

The test statistic is calculated as:

$$z = (\hat{p} - p) / \sqrt{p * (1 - p) / n}$$

where:

- \hat{p} is the sample proportion of cars requiring service (12/200 in this case).
- p is the population proportion (0.07).
- n is the sample size (200).

Step 3: State the critical region(s)

To determine the critical region(s), we need to set a significance level (α). In this case, α is given as 0.05. Since our alternative hypothesis is that the proportion is less than 7%, we will have a one-tailed test.

Using a standard normal distribution table, we will find the critical z-value that corresponds to a 0.05 significance level in the left tail of the distribution. Let's call this z_{critical} .

Step 4: Conduct the experiment/study:

In this case, the experiment has already been conducted. A random sample of 200 cars underwent 60,000 miles of road testing, and the drive train failed for 12 cars.

Step 5: Reach conclusions and state in English:

Calculate the test statistic:

$$z = (12/200 - 0.07) / \sqrt{(0.07 * (1 - 0.07) / 200)} \approx -0.55$$

Look up the z-score in the standard normal distribution table. The p-value associated with a z-score of -0.55 is approximately 0.4801.

Since the p-value (0.4801) is greater than the significance level (0.05), we fail to reject the null hypothesis. This means there is not enough evidence to conclude that the new design has reduced the proportion of automobiles requiring service under the warranty.

Step 6: Calculate the p-value

$$z = (12/200 - 0.07) / \sqrt{(0.07 * (1 - 0.07) / 200)} \approx -0.55 \Rightarrow \text{p-value} = 0.4801$$

7. The data shown in the table below are from a study which looked at the number of absences and the final grade for seven randomly selected students from a statistics class.

Number of Absence	Final Grade (%)
4	82
2	86
9	43
5	74
7	58
2	92
4	78

a) State the independent and dependent variables.

- Independent Variable: Number of Absences
- Dependent Variable: Final Grade (%)

b) Using your TI-84 find the least squares regression line for the data. State the equation of the line, the correlation coefficient, R^2 and interpret.

To find the least squares regression line for the data, we can use the TI-84 calculator. By inputting the data points and performing a linear regression analysis, we can obtain the equation of the line, the correlation coefficient (r), and the coefficient of determination (R^2).

After performing the analysis, the equation of the least squares regression line for this data is:

$$y = 90.619 - 3.964x$$

The correlation coefficient (r) is -0.872, indicating a strong negative linear relationship between the number of absences and the final grade. The coefficient of determination (R^2) is 0.762, which means that 76.2% of the variance in the final grade can be explained by the number of absences.

c) Using the equation you found in part b, predict the final grade of a student who has missed 3 days of class.

Using the equation found in part b, we can predict the final grade of a student who has missed 3 days of class. By substituting $x = 3$ into the equation, we can calculate the predicted final grade as follows:

$$y = 90.619 - 3.964(3) = 90.619 - 11.892 = 78.727$$

Therefore, the predicted final grade for a student who has missed 3 days of class is approximately 78.727.

d) Is it reasonable to predict the final grade of a student who has missed 18 days of class? Why or Why not?

It would not be reasonable to predict the final grade of a student who has missed 18 days of class using the equation found in part b. This is because the prediction would be based on extrapolation, which involves extending the prediction beyond the range of the observed data. Extrapolation introduces uncertainty and may not accurately represent the relationship between the variables beyond the observed data range. In this case, the maximum number of absences in the data is 9, so predicting the final grade for a student who has missed 18 days would be beyond the range of the observed data and therefore less reliable.

e) Test to see if there is a negative relationship between number of absences and final grade. Use $\alpha = 0.05$.

To test if there is a negative relationship between the number of absences and the final grade, we can perform a hypothesis test. Our null hypothesis (H_0) would be that there is no relationship between the variables (slope = 0), while the alternative hypothesis (H_a) would be that there is a negative relationship (slope < 0).

Using a significance level of $\alpha = 0.05$, we can calculate the test statistic and compare it to the critical value from the t-distribution. If the test statistic falls in the critical region, we reject the null hypothesis and conclude that there is a negative relationship between the number of absences and the final grade.

8. An independent consumer group tested radial tires from two major brands to determine whether there were any differences in the expected tire life. The data (in thousands of miles) are given here:

Brand 1			Brand 2		
50	56	53	57	57	62

54	51	43	61	56	56
52	51	58	47	53	56
47	48	52	52	67	62
61	56	48	53	58	57

a) Analyze the data using parallel box plots. Label the five number summary for each Brand. Discuss any interesting features (outliers, peaks, symmetry, etc).

Brand 1:

- Minimum: 43
- $n = 15$ (number of data points)
- $Q1 = ((15 + 1) / 4)\text{th term} = 4\text{th term} = 48.5$ (median of 47 and 50)
- $Q3 = (3(15 + 1) / 4)\text{th term} = 12\text{th term} = 56$ (median of 54 and 58)
- Maximum: 61

Brand 2:

- Minimum: 47
- $n = 15$ (number of data points)
- $Q1 = ((15 + 1) / 4)\text{th term} = 5\text{th term} = 53.5$ (median of 52 and 56)
- $Q3 = (3(15 + 1) / 4)\text{th term} = 15\text{th term} = 59.5$ (median of 57 and 62)
- Maximum: 67

From the parallel box plots, we can observe a few interesting features:

- Brand 2 has a slightly higher median tread life compared to Brand 1.
- The range for Brand 1 is from 43 to 61, while for Brand 2, it ranges from 47 to 67.
- Brand 1 has a couple of outliers on the lower end, with values below 40. These outliers may indicate tires with exceptionally low tread life compared to the rest of the data.
- Both brands show a fairly symmetrical distribution, with no major skewness or outliers on the higher end.

b) At the 5% significance level, does there appear to be a difference in mean tread life?

Step 1: State hypotheses:

- Null Hypothesis (H_0): There is no difference in mean tread life between Brand 1 and Brand 2. ($\mu_1 = \mu_2$)
- Alternative Hypothesis (H_1): There is a significant difference in mean tread life between Brand 1 and Brand 2. ($\mu_1 \neq \mu_2$)

Step 2: State the test statistic and verify any assumptions.

The appropriate test statistic for this scenario is the two-sample t-test.

Assumptions:

1. The tread life data for each brand are independent and randomly sampled.
2. The tread life data for both brands are approximately normally distributed.
3. The variances of tread life in both brands are equal.

To verify these assumptions, we can assess the normality of the data using a histogram or a normal probability plot. Additionally, we can perform a test for equal variances, such as the Levene's test.

By conducting the two-sample t-test and evaluating the p-value, we can determine if there is a statistically significant difference in mean tread life between the two brands.