

# INTRODUCTION TO DIGITAL IMAGE PROCESSING

## POSSIBLE SOLUTIONS TO PRACTICE PROBLEMS – ALGEBRAIC OPERATIONS

### Problem 1

Cost function is:

$$C = \sum_{i=1}^N [I(x_i, y_i) - a_0 - a_1 x_i - a_2 y_i - a_3 x_i^2 - a_4 y_i^2 - a_5 x_i y_i]^2$$

For minimum,

$$\frac{\partial C}{\partial a_0} = 0, \quad \frac{\partial C}{\partial a_1} = 0, \quad \dots, \quad \frac{\partial C}{\partial a_5} = 0$$

$$\frac{\partial C}{\partial a_0} = 0$$

$$\sum_{i=1}^N 2[I(x_i, y_i) - a_0 - a_1 x_i - a_2 y_i - a_3 x_i^2 - a_4 y_i^2 - a_5 x_i y_i] = 0$$

$$a_0 \sum_{i=1}^N 1 + a_1 \sum_{i=1}^N x_i + a_2 \sum_{i=1}^N y_i + a_3 \sum_{i=1}^N x_i^2 + a_4 \sum_{i=1}^N y_i^2 + a_5 \sum_{i=1}^N x_i y_i = \sum_{i=1}^N I(x_i, y_i)$$

$$a_0 N + a_1 S_x + a_2 S_y + a_3 S_{x^2} + a_4 S_{y^2} + a_5 S_{xy} = S_I$$

$$\frac{\partial C}{\partial a_1} = 0$$

∴ (done in class; see your class notes)

$$a_0 S_x + a_1 S_{x^2} + a_2 S_{xy} + a_3 S_{x^3} + a_4 S_{xy^2} + a_5 S_{x^2 y} = S_{xI}$$

$$\frac{\partial C}{\partial a_2} = 0$$

$$\sum_{i=1}^N 2[I(x_i, y_i) - a_0 - a_1 x_i - a_2 y_i - a_3 x_i^2 - a_4 y_i^2 - a_5 x_i y_i] (y_i) = 0$$

$$a_0 \sum_i y_i + a_1 \sum_i x_i y_i + a_2 \sum_i y_i^2 + a_3 \sum_i x_i^2 y_i + a_4 \sum_i y_i^3 + a_5 \sum_i x_i y_i^2 = \sum_i y_i I(x_i, y_i)$$

$$a_0 S_y + a_1 S_{xy} + a_2 S_{y2} + a_3 S_{x2y} + a_4 S_{y3} + a_5 S_{xy2} = S_{yI}$$

$$\frac{\partial C}{\partial a_3} = 0$$

$$\sum_{i=1}^N 2[I(x_i, y_i) - a_0 - a_1 x_i - a_2 y_i - a_3 x_i^2 - a_4 y_i^2 - a_5 x_i y_i] (x_i^2) = 0$$

$$a_0 \sum_i x_i^2 + a_1 \sum_i x_i^3 + a_2 \sum_i y_i x_i^2 + a_3 \sum_i x_i^4 + a_4 \sum_i x_i^2 y_i^2 + a_5 \sum_i x_i^3 y_i = \sum_i x_i^2 I(x_i, y_i)$$

$$a_0 S_{x2} + a_1 S_{x3} + a_2 S_{x2y} + a_3 S_{x4} + a_4 S_{x2y2} + a_5 S_{x3y} = S_{x2I}$$

$$\frac{\partial C}{\partial a_4} = 0$$

$$\sum_{i=1}^N 2[I(x_i, y_i) - a_0 - a_1 x_i - a_2 y_i - a_3 x_i^2 - a_4 y_i^2 - a_5 x_i y_i] (-y_i^2) = 0$$

$$a_0 \sum_i y_i^2 + a_1 \sum_i x_i y_i^2 + a_2 \sum_i y_i^3 + a_3 \sum_i x_i^2 y_i^2 + a_4 \sum_i y_i^4 + a_5 \sum_i x_i y_i^3 = \sum_i y_i^2 I$$

$$a_0 S_{y2} + a_1 S_{xy2} + a_2 S_{y3} + a_3 S_{x2y2} + a_4 S_{y4} + a_5 S_{xy3} = S_{y2I}$$

$$\frac{\partial C}{\partial a_5} = 0$$

$$\sum_i 2[I(x_i, y_i) - a_0 - a_1 x_i - a_2 y_i - a_3 x_i^2 - a_4 y_i^2 - a_5 x_i y_i](-x_i y_i) = 0$$

$$a_0 \sum_i x_i y_i + a_1 \sum_i x_i^2 y_i + a_2 \sum_i x_i y_i^2 + a_3 \sum_i x_i^3 y_i + a_4 \sum_i x_i y_i^3 + a_5 \sum_i x_i^2 y_i^2 = \sum_i x_i y_i I(x_i, y_i)$$

$$a_0 S_{xy} + a_1 S_{x^2 y} + a_2 S_{x y^2} + a_3 S_{x^3 y} + a_4 S_{x y^3} + a_5 S_{x^2 y^2} = S_{xy I}$$

Matrix equation is  $[C]\{a\} = \{k\}$   
where

$$[C] = \begin{bmatrix} N & S_x & S_y & S_{x^2} & S_{y^2} & S_{xy} \\ S_x & S_{x^2} & S_{xy} & S_{x^3} & S_{xy^2} & S_{x^2 y} \\ S_y & S_{xy} & S_{y^2} & S_{xy^2} & S_{y^3} & S_{xy^2} \\ S_{x^2} & S_{x^3} & S_{x^2 y} & S_{x^4} & S_{x^2 y^2} & S_{x^3 y} \\ S_{y^2} & S_{xy^2} & S_{y^3} & S_{x^2 y^2} & S_{y^4} & S_{xy^3} \\ S_{xy} & S_{x^2 y} & S_{xy^2} & S_{x^3 y} & S_{xy^3} & S_{x^2 y^2} \end{bmatrix}$$

$$\{k\} = \begin{Bmatrix} S_I \\ S_{xI} \\ S_{yI} \\ S_{x^2 I} \\ S_{y^2 I} \\ S_{xyI} \end{Bmatrix}$$

$\{a\}$  is as given in asst.

## **Problem 2**

See code `imAve.m` on OWL site.

To try the code, type the following at the MATLAB prompt:

```
>> imAve('moon.tif', 16);
```

You will see a noisy image and an image after averaging:



An example noisy image generated by adding noise to 'moon.tif'.



After averaging 16 noisy images.