Question 1

$$H(D) = 100\,G(60, 5, D) + 20\,G(180, 20, D)$$
$$= 100\exp\left(-\frac{(D-60)^2}{2\times 5^2}\right) + 20\exp\left(-\frac{(D-180)^2}{2\times 20^2}\right)$$

Note, $G(\mu, \sigma, D) = \exp\left(\frac{(D-\mu)^2}{2\sigma^2}\right)$

As mentioned, you can find minima by solving $\frac{dH(D)}{dD} = 0$ and testing each root.

I get $D = 87$

I would not ask a question with hard derivatives and roots like this because this is not a Calculus course. However, I do expect you to know how to differentiate and integrate polynomials and sines and cosines.

I solved the above numerically in MATLAB.

Question 2

There is too much subjectivity in this question and I would ask something where the thresholds are easier to determine.

Assuming the ball has gray level 1, 2, 3 (dark) and 12, 13, 14 (light)

then N = # of pixels in ball

$= H(1) + H(2) + H(3) + H(12) + H(13) + H(14)$

$= 3750$

I think the author meant that "black" is not pure black in image, but dark, and white is actually light.

Let ΔA = area of pixel

$N\Delta A$ = area of circle corresponding to soccer ball

A_c = Area of circle $= \pi\left(\frac{d}{2}\right)^2$

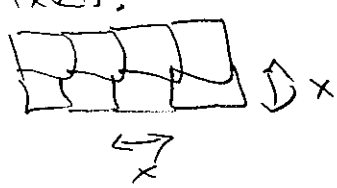
where d = diameter

$$A_c = \pi\left(\frac{230\,\mu m}{2}\right)^2 \approx 4.15 \times 10^{4}\,\mu m^2$$

$$\therefore N\Delta A = A_c = 4.15 \times 10^{4}\,mm^2$$

Assume pixel, is isotropic, i.e., pixel is square:

$$\Delta A = x^2 \quad \text{where } x = \text{pixel spacing}$$



$$\therefore Nx^2 \approx 4.15 \times 10^{4}\,mm^2$$

$$x \approx 3.3\,mm$$