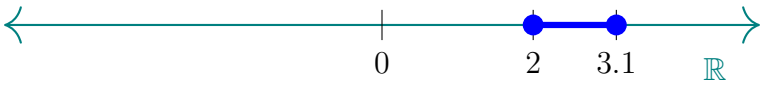
Numbers: Intervals and Interval Notation

Video companion

1 Closed intervals



Real number line is an infinite set. There are also infinite subsets.

$[2, 3.1] = \{x \in \mathbb{R} : 2 \leq x \leq 3.1\}$

$2.3 \in [2, 3.1]$ because $2 \leq 2.3 \leq 3.1$
$3 \in [2, 3.1]$
$3.1 \in [2, 3.1]$
$1 \notin [2, 3.1]$ because $2 \not\leq 1 \leq 3.1$

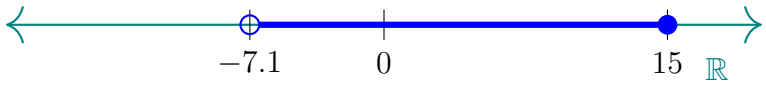
2 Open intervals

$(5, 8) = \{x \in \mathbb{R} : 5 < x < 8\}$

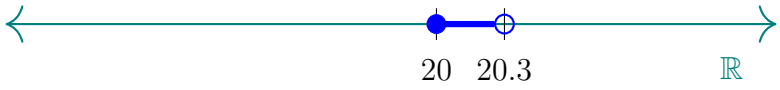
$5.5 \in (5, 8)$ because $5 < 5.5 < 8$
$5.0001 \in (5, 8)$
$5 \notin (5, 8)$ because $5 \not< 5 < 8$

The intervals $[5, 8]$ and $(5, 8)$ differ at exactly two numbers: 5 and 8.

3 Half-open intervals



$(-7.1, 15] = \{x \in \mathbb{R} : -7.1 < x \leq 15\}$

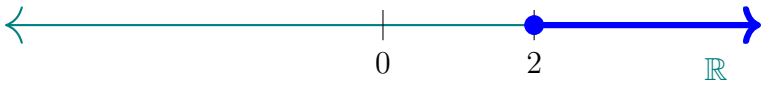


$[20, 20.3) = \{x \in \mathbb{R} : 20 \leq x < 20.3\}$

4 Recap vocabulary

- Closed intervals $[2, 3.1]$
- Open intervals $(5, 8)$
- Half-open intervals $(2, 3]$, $[20, 20.3)$

5 Rays



$[2, \infty) = \{x \in \mathbb{R} : x \geq 2\}$

Another example:
$(-\infty, 7.1) = \{x \in \mathbb{R} : x < 7.1\}$

6 What does an "answer" mean?

Solving an equality gives you a number:

$$\begin{aligned} x + 5 &= 10 \\ x &= 5 \end{aligned}$$

Solving an inequality give you an interval:

$$\begin{aligned} 1 \leq x + 5 &< 10 \\ -4 \leq x &< 5 \\ x &\in [-4, 5) \end{aligned}$$