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Sigma Notation: Introduction to Summation

Video companion

1 Sigma notation (Σ)

Examples that will be seen in this video:

$$\sum_{i=1}^{4} i^2 = 30$$

$$\sum_{i=1}^{5} (2i+3) = 45$$

$$\sum_{j=3}^{7} \frac{j}{2} = \frac{25}{2}$$

2 First example

Example:

$$\begin{aligned}\sum_{i=1}^{4} i^2 &= 1^2 + 2^2 + 3^2 + 4^2 \\ &= 30\end{aligned}$$

$i = 1$ on bottom tells us to *start* with $i = 1$.
4 on top tells us to *finish* with $i = 4$.
Implicitly know that you increment by 1.

For each number i that you count,

$i = 1 : i^2 = 1^2$
$i = 2 : i^2 = 2^2$
$i = 3 : i^2 = 3^2$
$i = 4 : i^2 = 4^2$

then the Σ tells you to *sum* the results.

3 Second example

Example:

$$\sum_{i=1}^{5}(2i+3) = (2(1)+3)+(2(2)+3)+(2(3)+3)+(2(4)+3)+(2(5)+3)$$
$$= 45$$

Work for problem:

$$i=1 : 2i+3 = 2(1)+3$$
$$i=2 : 2i+3 = 2(2)+3$$
$$i=3 : 2i+3 = 2(3)+3$$
$$i=4 : 2i+3 = 2(4)+3$$
$$i=5 : 2i+3 = 2(5)+3$$

4 Third example

Example:

$$\sum_{j=3}^{7}\frac{j}{2} = \frac{3}{2}+\frac{4}{2}+\frac{5}{2}+\frac{6}{2}+\frac{7}{2} = \frac{25}{2}$$

$$\sum_{r=3}^{7}\frac{r}{2} = \frac{25}{2}$$

j and r are "dummy indices," symbols for counters.

$$\sum_{☺=3}^{7}\frac{☺}{2} = \frac{25}{2}$$

Common choices for indices:
i, j, k, l, r, m, n