Sigma Notation: Simplification Rules

Video companion

1 Distributive property

Examples:

$$\sum_{i=1}^{4} i^2 = 30$$

$$\begin{aligned}\sum_{i=1}^{4} 3i^2 &= 3(1)^2 + 3(2)^2 + 3(3)^2 + 3(4)^2 \\ &= 3[1^2 + 2^2 + 3^2 + 4^2] \\ &= 3\left[\sum_{i=1}^{4} i^2\right]\end{aligned}$$

$$\sum_{r=4}^{25} 18r^3 = 18\left[\sum_{r=4}^{25} r^3\right]$$

This is due to the *distributive property*:

$$a(b+c) = ab + ac$$

In other words, constants inside the summed expression can be pulled outside.

2 Commutative property

$$\begin{aligned}\sum_{i=1}^{4}(i^2+2i) &= \big(1^2+2(1)\big)+\big(2^2+2(2)\big)+\big(3^2+2(3)\big)+\big(4^2+2(4)\big)\\ &= \big(1^2+2^2+3^2+4^2\big)+(2(1)+2(2)+2(3)+2(4))\\ &= \left(\sum_{i=1}^{4} i^2\right)+\left(\sum_{i=1}^{4} 2i\right)\end{aligned}$$

This is due to the *commutative property*:

$$a+b=b+a$$

In other words, we can add the terms in any order.

3 Summation of constants

Examples:

$$\begin{aligned}\sum_{k=1}^{10} 5 &= 5+5+5+5+5+5+5+5+5+5\\ &= 10\cdot 5\\ &= 50\end{aligned}$$

$$\begin{aligned}\sum_{r=1}^{7} 8 &= 8+8+8+8+8+8+8\\ &= 7\cdot 8\\ &= 56\end{aligned}$$

When summing constants, you can multiply the constant by the number of indices you count.