Functions: Composition and Inverse

Video companion

1 Introduction

- Composing two functions

  - Basic identity
  - A warning

- Inverse functions

  - Basic identity
  - A neat picture
  - A warning

2 Composing functions

Definition: Given functions f and g, $(g \circ f)(x) = g(f(x))$, and $(f \circ g)(x) = f(g(x))$

Example:

$$f(x) = x^2$$
$$g(x) = x + 5$$

$$(g \circ f)(x) = g(f(x))$$
$$= g(x^2) = x^2 + 5$$
$$g(f(2)) = g(2^2) = 2^2 + 5 = 9$$
$$(f \circ g)(x) = f(g(x))$$
$$= f(x + 5) = (x + 5)^2 \neq x^2 + 5$$

3   Inverse functions

Example:

$$f(x) = 2x$$
$$g(x) = \frac{1}{2}x$$

$$(g \circ f)(x) = g(f(x)) = g(2x) = \frac{1}{2}(2x) = x$$
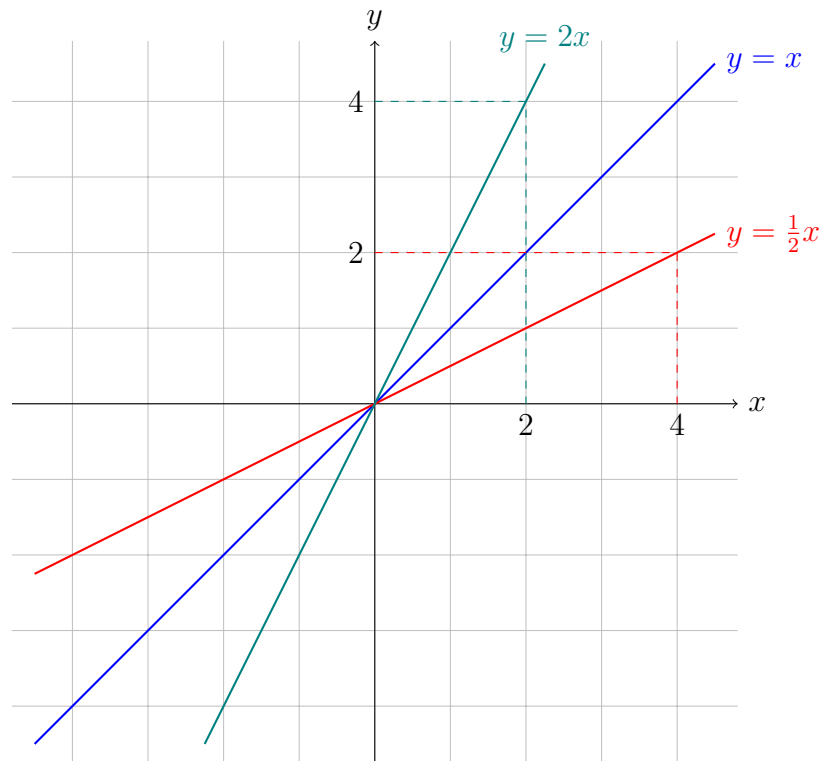
Notice: true for all x

$$(g \circ f)(3) = g(f(3)) = g(2 * 3) = \frac{1}{2}(2 * 3) = 3$$
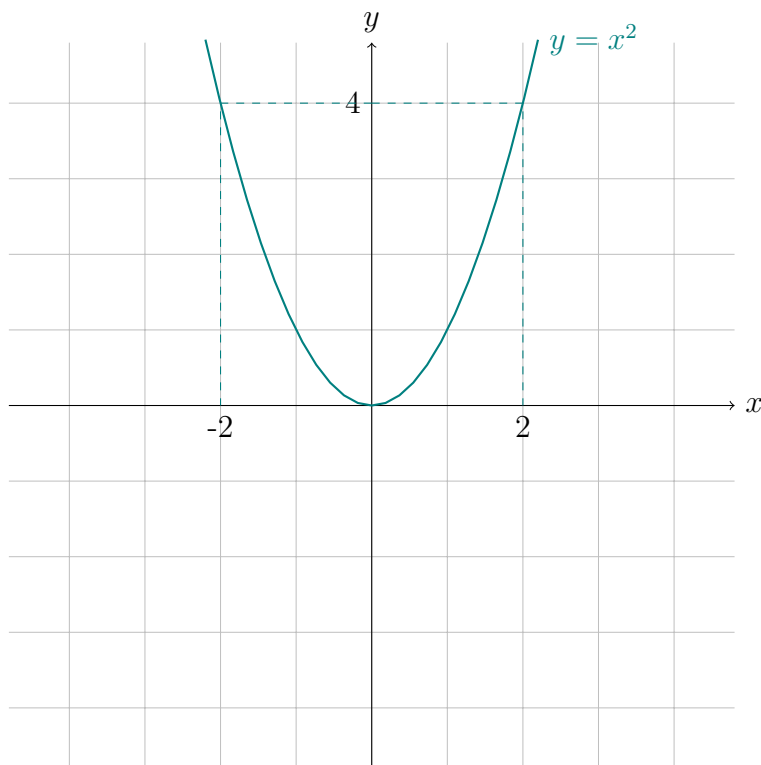$$(g \circ f)(\pi) = g(f(\pi)) = g(2 * \pi) = \frac{1}{2}(2 * \pi) = \pi$$

f and g are *inverses* of each other, i.e. f undoes what g does.

$$g = f^{-1}$$

4   Graphical depiction

Warning: not every function $f : \mathbb{R} \to \mathbb{R}$ has an inverse.

Warning: if the graph of f fails the horizontal line test, then f has no inverse. The only invertible functions are those that are either strictly increasing or strictly decreasing.