Fast Growth, Slow Growth: Using Integer Exponents

Video companion

1 Positive Integer Exponents

$$\begin{aligned} 9 &= 3 \cdot 3 && = 3^2 \\ 27 &= 3 \cdot 3 \cdot 3 && = 3^3 \\ 81 &= 3 \cdot 3 \cdot 3 \cdot 3 && = 3^4 \end{aligned}$$

Exponents count how many times factors repeat in a number. 3^4 is pronounced "three to the fourth power" or "three to the fourth."

Example

$$248,832 = 12 \cdot 12 \cdot 12 \cdot 12 \cdot 12 = 12^5$$

A note on pronunciation $4 \cdot 4 = 4^2$ can be pronounced "four to the second"—but also "four squared." Similarly, $4 \cdot 4 \cdot 4 = 4^3$ can be pronounced "four to the third"—but also "four cubed."

2 Zero as an Exponent

$$1^0 = 1 \qquad (2\pi)^0 = 1$$

$$2^0 = 1 \qquad \left(\frac{1}{x^3}\right)^0 = 1$$

$$3^0 = 1$$

Definition By the definition of exponents, any number, except for zero, raised to the zeroth power is one. Note that 0^0 is undefined.

3 Negative Integer Exponents

$$2^{-1} = \frac{1}{2^1} = \frac{1}{2} \qquad \frac{1}{2^{-1}} = 2^1 = 2$$
$$2^{-2} = \frac{1}{2^2} = \frac{1}{4} \qquad \frac{1}{2^{-2}} = 2^2 = 4$$
$$2^{-3} = \frac{1}{2^3} = \frac{1}{8} \qquad \frac{1}{2^{-3}} = 2^3 = 8$$

General rule

$$x^{-n} = \frac{1}{x^n}$$
$$\frac{1}{x^{-n}} = x^n$$

4 Scientific Notation

Mass of the Earth (kg)

$$5,972,000,000,000,000,000,000,000 = 5.972 \times 10^{24}$$

Mass of an electron (kg)

$$0.00000000000000000000000000009109 = 9.109 \times 10^{-31}$$

Keep the significant digits, and there is always one digit to the left of the decimal.