Data Science Math Skills
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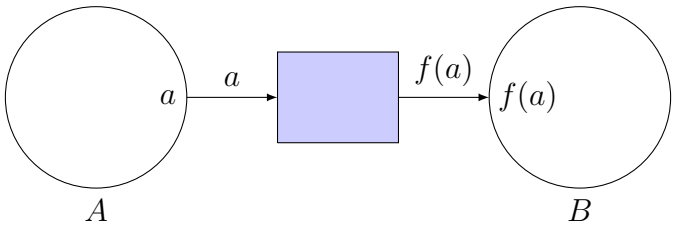
Functions: Graphing in the Cartesian Plane

Video companion

1 Introduction

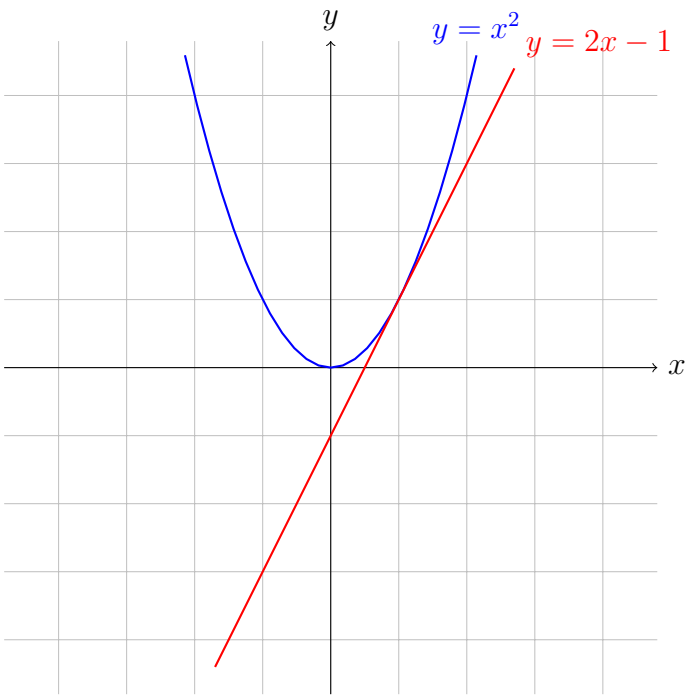
Last time: abstract depiction of a function as a machine

$f : A \to B$



This video: graphs of functions

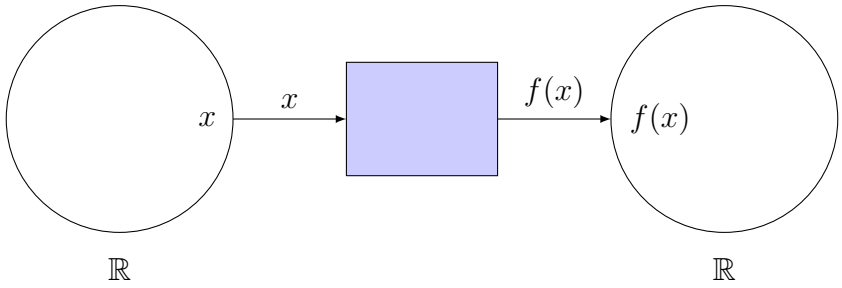
$f : \mathbb{R} \to \mathbb{R}$



You will learn the difference between a *graph* of a function and the function itself.

2 Map real line to real line

$f : \mathbb{R} \to \mathbb{R}$



A function is a formula, a rule for how to operate the machine.

$$\begin{aligned} f(x) &= 2x - 1 \\ f(1) &= 2(1) - 1 = 1 \\ f(0) &= 2(0) - 1 = -1 \\ f(5.1) &= 2(5.1) - 1 = 9.2 \end{aligned}$$

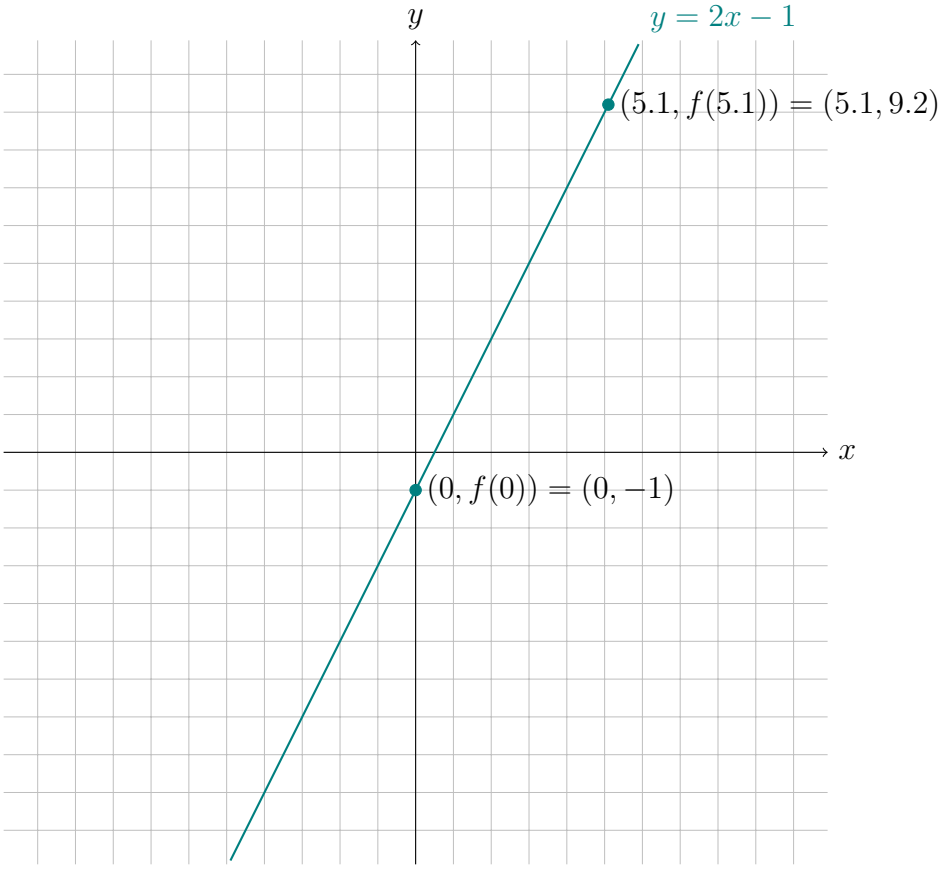
More complicated formulas, like absolute value:

$$\begin{aligned} g(x) &= |x| \\ &= \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases} \end{aligned}$$

Both f and g are functions, with a formula for how to compute the result.

3 What is a graph?

Graph of the function $f : \mathbb{R} \to \mathbb{R}$

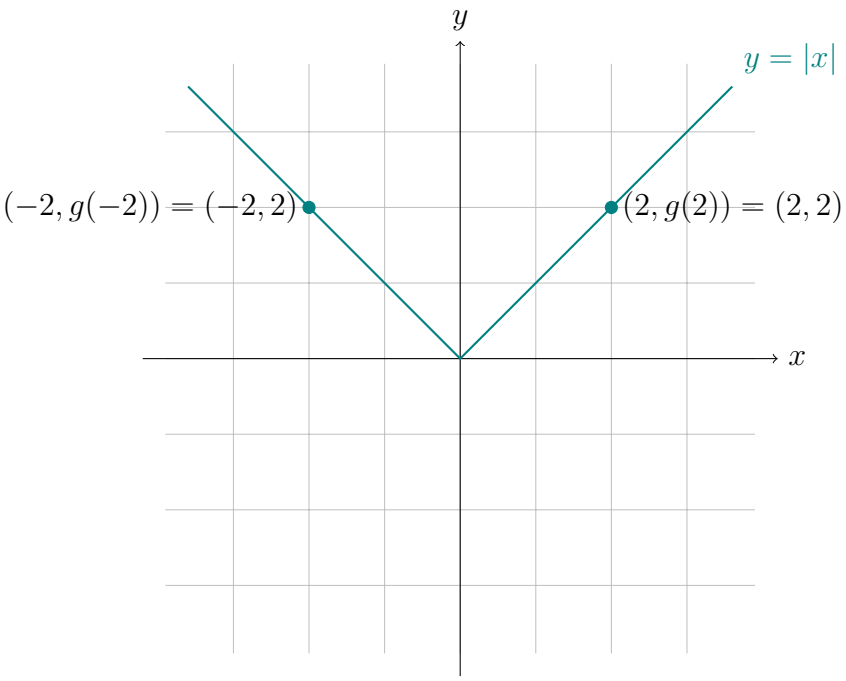


If g is a function $: \mathbb{R} \to \mathbb{R}$, the graph of $g = \{(x, y) \in \mathbb{R}^2 : y = g(x)\}$

4 Examples

Absolute value function

$$g(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



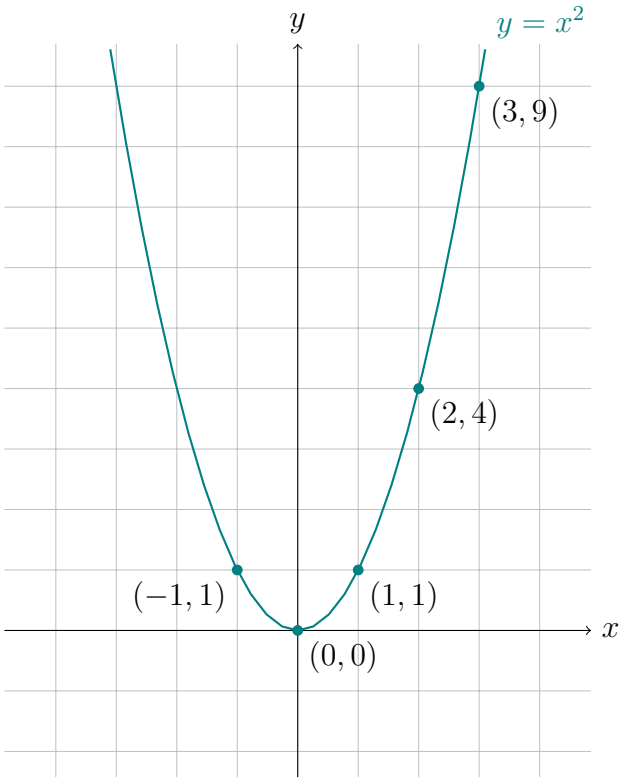
Quadratic function

$h(x) = x^2$

Graph a function by testing input and output pairs, see a pattern, and try to draw a curve through it. This is similar to *querying* in supervised learning.

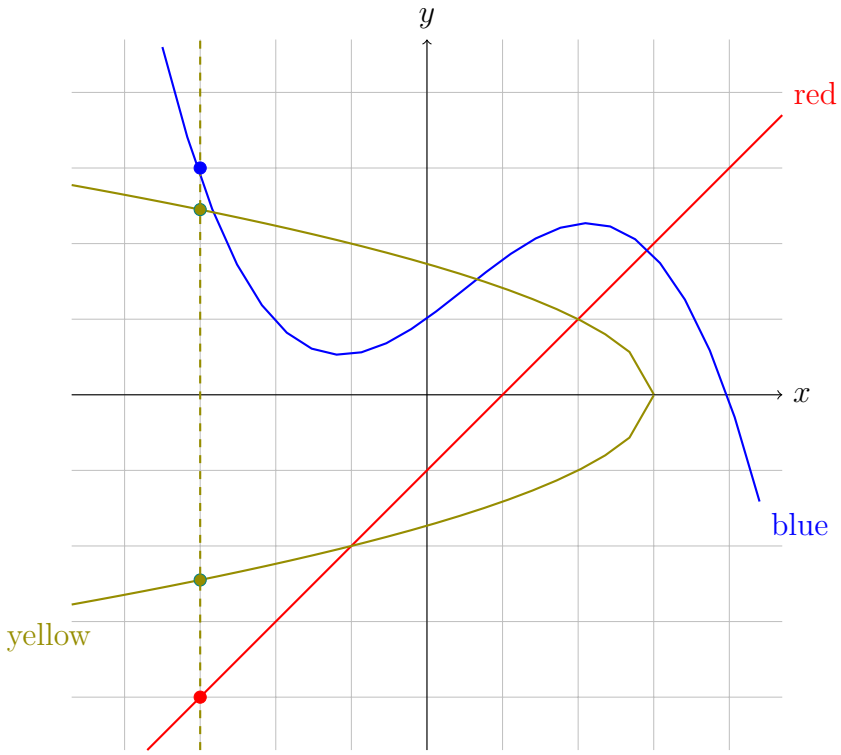
Table of values:

x	$h(x)$
0	$0^2 = 0$
1	$1^2 = 1$
2	$2^2 = 4$
3	$3^2 = 9$
-1	$(-1)^2 = 1$



$h(x) = x^2$ is a *quadratic* function.

5 Vertical line test



Red and blue could be graphs of functions. Yellow could not be the graph of a function because it violates the *vertical line test*, which states that *any vertical line intersects the graph of a function once.*