EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

FACULTY OF
SCIENCE
Computer Graphics

Computer Graphics
(Graphische Datenverarbeitung)

- Light Transport -

WS 2021/2022

Corona

- Regular random lookup of the 3G certificates
- Contact tracing: We need to know who is in the class room
  - New ILIAS group for every lecture slot
  - Register via ILIAS or this QR code (only if you are present in this room)

Overview

- Previous lecture
  - Simple shading
  - Light-matter interaction
  - Reflectance function
- Today
  - Physics behind ray tracing
  - Physical light quantities
  - Perception of light
  - Light sources
  - Light transport simulation
- Next Lecture
  - Light Transport II

Describing Light

What is light and how can it be measured?

What is Light ?

- Ray
  - Linear propagation
  - Geometrical optics
- Vector
  - Polarization
  - Jones Calculus: matrix representation
- Wave
  - Diffraction, Interference
  - Maxwell equations: propagation of light
- Particle
  - Light comes in discrete energy quanta: photons
  - Quantum theory: interaction of light with matter
- Field
  - Electromagnetic force: exchange of virtual photons
  - Quantum Electrodynamics (QED): interaction between particles

What is Light ?

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Light in Computer Graphics

- Based on human visual perception
  - Macroscopic geometry
  - Tristimulus color model
  - Psycho-physics: tone mapping, compression, …
- Ray optics
  - Light: scalar, real-valued quantity
  - Linear propagation
  - Macroscopic objects
  - Incoherent light
  - Superposition principle: light contributions add up linearly
  - No attenuation in free space
- Limitations
  - Microscopic structures ($\approx\lambda$)
  - Diffraction, Interference
  - Dispersion
  - Polarization

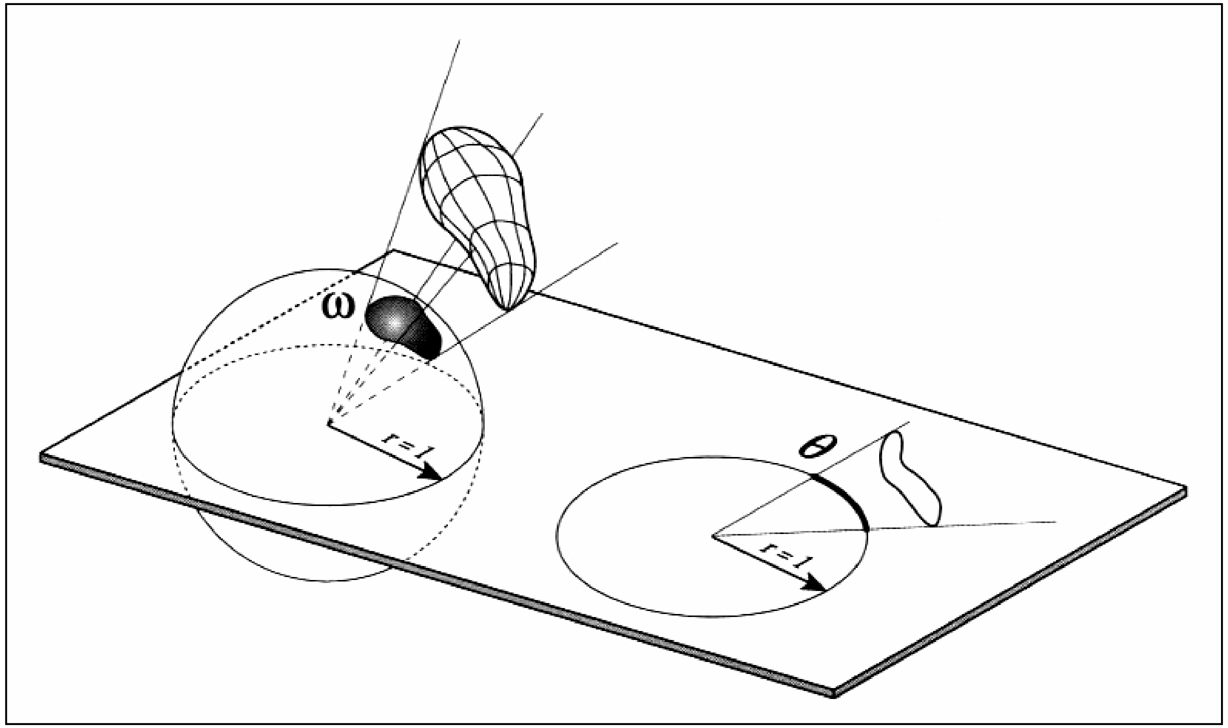
Radiometry

Physical definition of quantities related to light

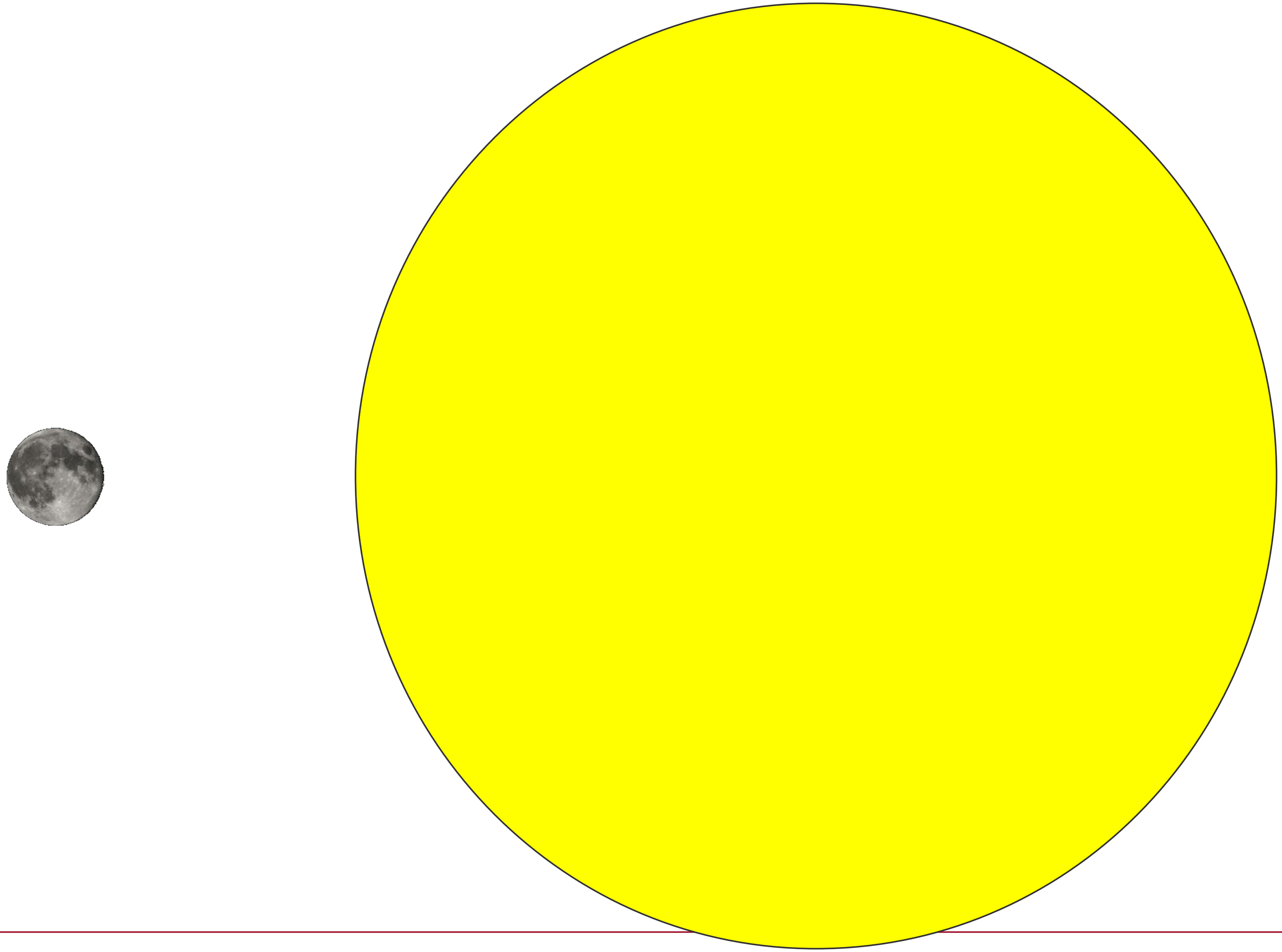
Angle and Solid Angle

θ — the angle subtended by a curve in the plane, is the length of the corresponding arc on the unit circle.

$\Omega, d\omega$ — the solid angle subtended by an object, is the surface area of its projection onto the unit sphere,
Units for measuring solid angle: steradians [sr]



Solid Angle – Solar Eclipse



Solid Angle in Spherical Coordinates

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Infinitesimally small solid angle

$$du = r\,d\theta$$

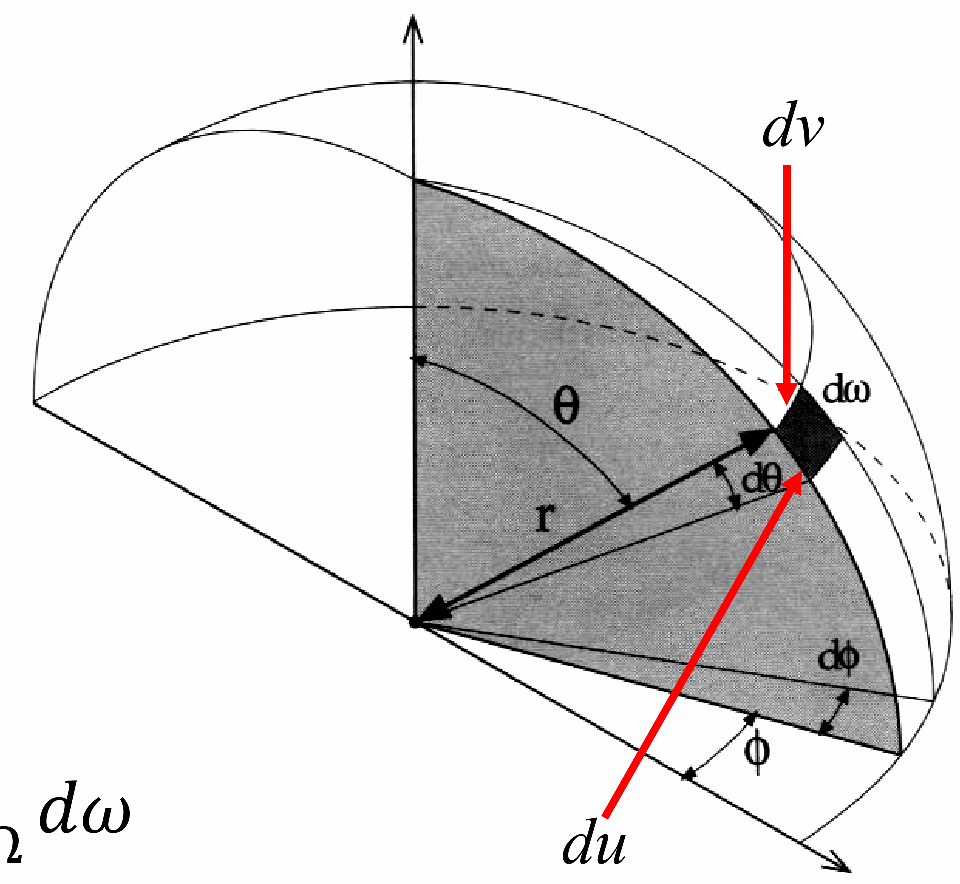
$$dv = r\sin\theta\,d\phi$$

$$dA = du\,dv = r^2\sin\theta\,d\theta\,d\phi$$

$$\Rightarrow \quad d\omega, d\Omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

Finite solid angle

$$\Omega = \int_{\varphi_0}^{\varphi_1}\int_{\theta_0(\varphi)}^{\theta_1(\varphi)} \sin\theta\,d\theta\,d\varphi = \int_\Omega d\omega$$



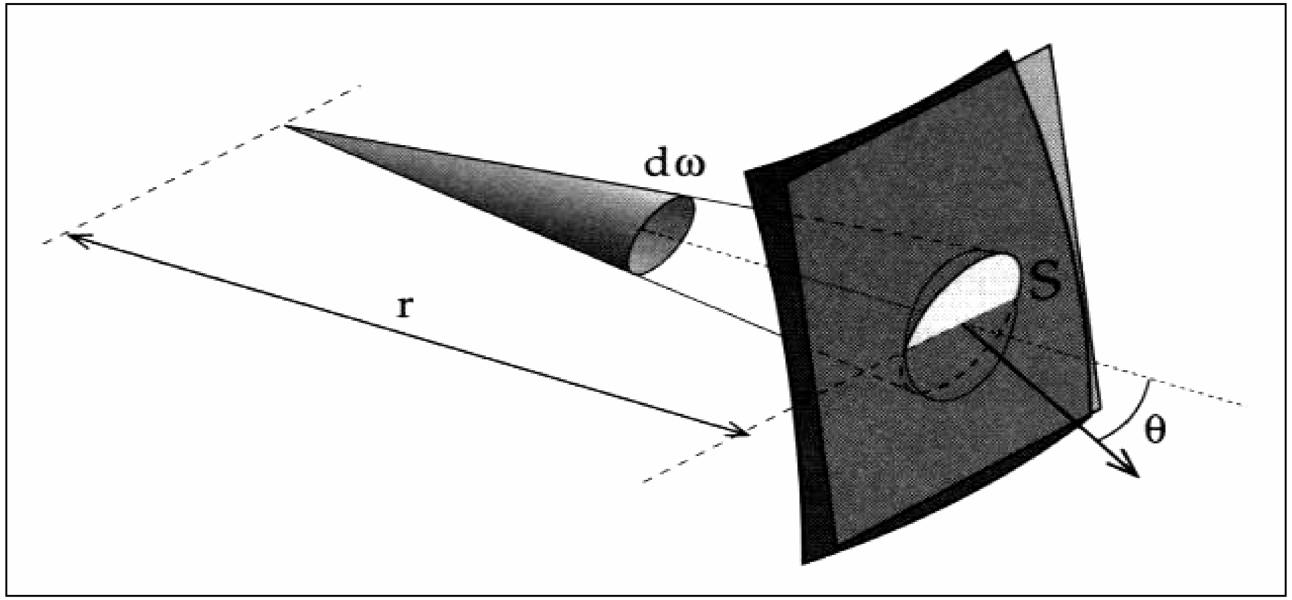
Projected Solid Geometry

The solid angle subtended by a small surface patch S with area ΔA is obtained
(i) by projecting it orthogonal to the vector r to the origin

$$\Delta A \cos\theta$$

(ii) dividing by the square of the distance to the origin:

$$\Delta\Omega \approx \frac{\Delta A \cos\theta}{r^2}$$



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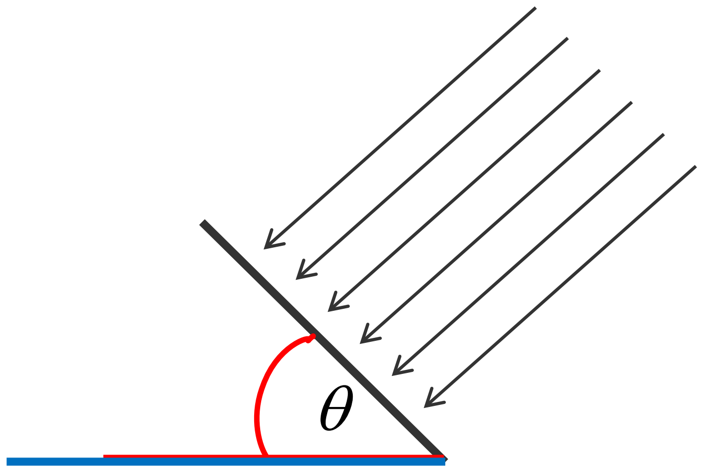
Why cos?



Radiometry

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- Definition:
  - Radiometry is the science of measuring radiant energy transfers. Radiometric quantities have physical meaning and can be directly measured using proper equipment such as spectral photometers.

- Radiometric Quantities

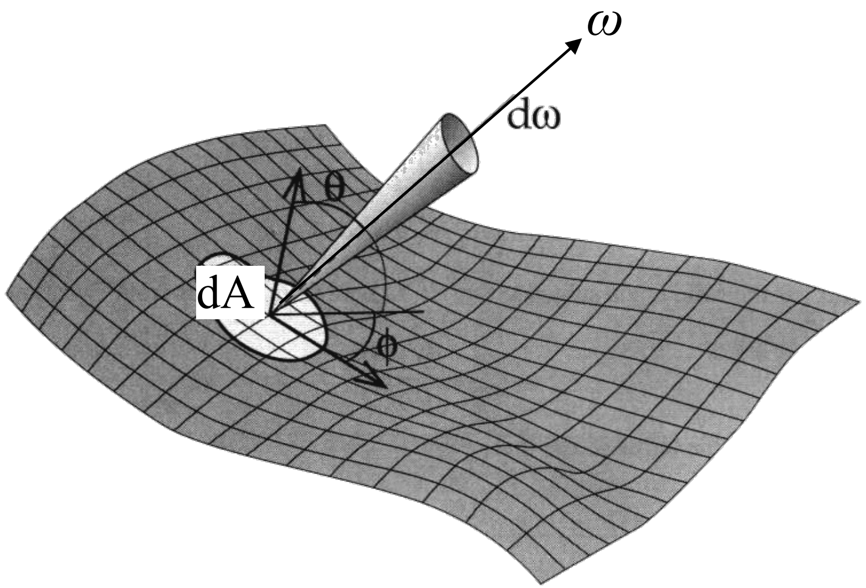
- energy	[watt second]	$n \cdot h\lambda$ (Photon Energy)
- radiant power (total flux)	[watt]	Φ
- radiance	[watt/(m^2 sr)]	L
- irradiance	[watt/m^2]	E
- radiosity	[watt/m^2]	B
- intensity	[watt/sr]	I

Radiometric Quantities: Radiance

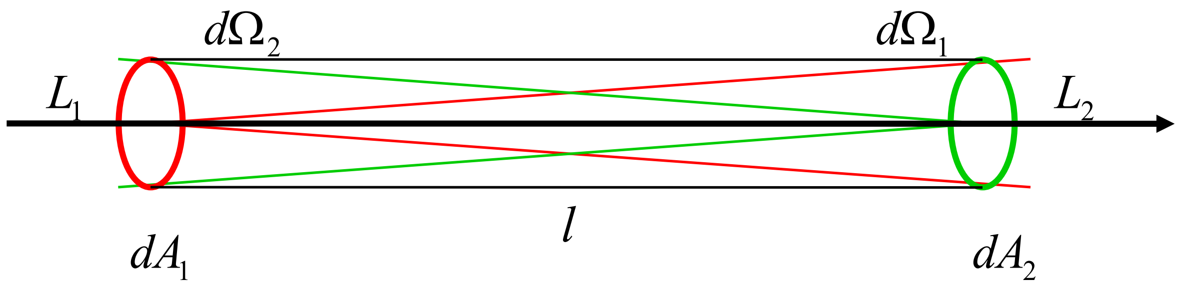
- Radiance is used to describe radiant energy transfer.
- Radiance L is defined as
  - **the power (flux) traveling at some point $\underline{x}$**
  - **in a specified direction $\underline{\omega} = (\theta,\varphi)$,**
  - **per unit area perpendicular to the direction of travel,**
  - **per unit solid angle.**
- Thus, the differential power $d^2\Phi$ radiated through the differential solid angle $d\omega$, from the projected differential area $dA\cos\theta$ is:

$$d^2\Phi = L(\underline{x},\underline{\omega})\, dA\cos\theta\; d\omega$$

$$L(\underline{x},\underline{\omega}) = \frac{d^2\Phi}{dA\cos\theta\, d\omega} \quad \left[\frac{W}{m^2 sr}\right]$$



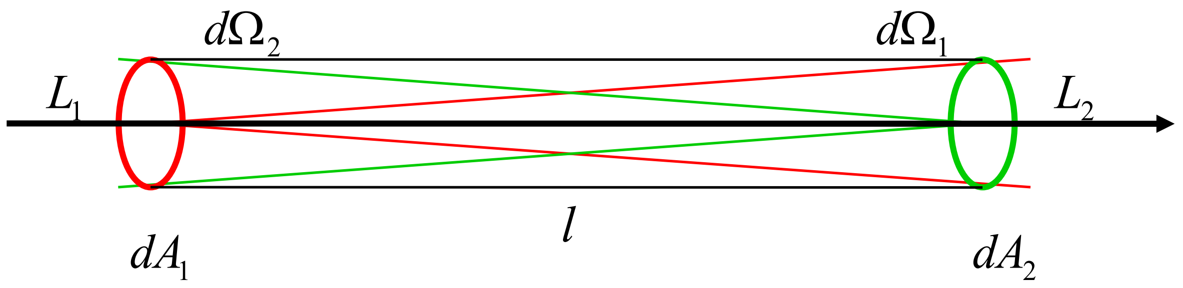
Radiance in Space



Flux leaving surface 1 must be equal to flux arriving on surface 2

$$d^2\Phi = L(\underline{x}, \underline{\omega})\, dA \cos\theta \;\, d\omega$$

Radiance in Space



Flux leaving surface 1 must be equal to flux arriving on surface 2

$$L_1 \cdot d\Omega_1 \cdot dA_1 = L_2 \cdot d\Omega_2 \cdot dA_2$$

$$d^2\Phi = L(\underline{x}, \underline{\omega})\, dA \cos\theta \;\, d\omega$$

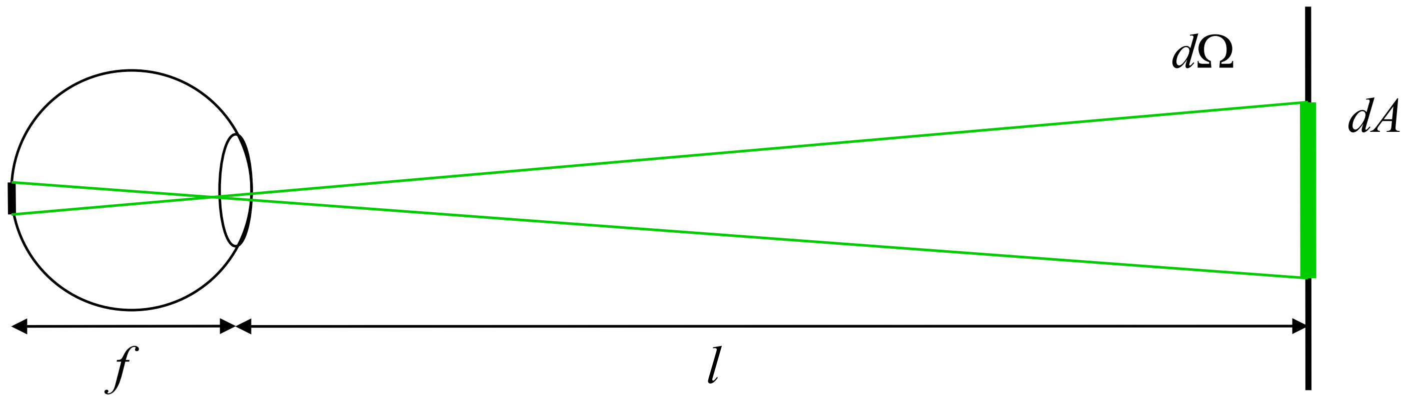
From geometry follows $\quad d\Omega_1 = \frac{dA_2}{l^2} \qquad d\Omega_2 = \frac{dA_1}{l^2}$

Ray throughput $\quad T = d\Omega_1 \cdot dA_1 = d\Omega_2 \cdot dA_2 = \frac{dA_1 \cdot dA_2}{l^2}$

$$L_1 = L_2$$

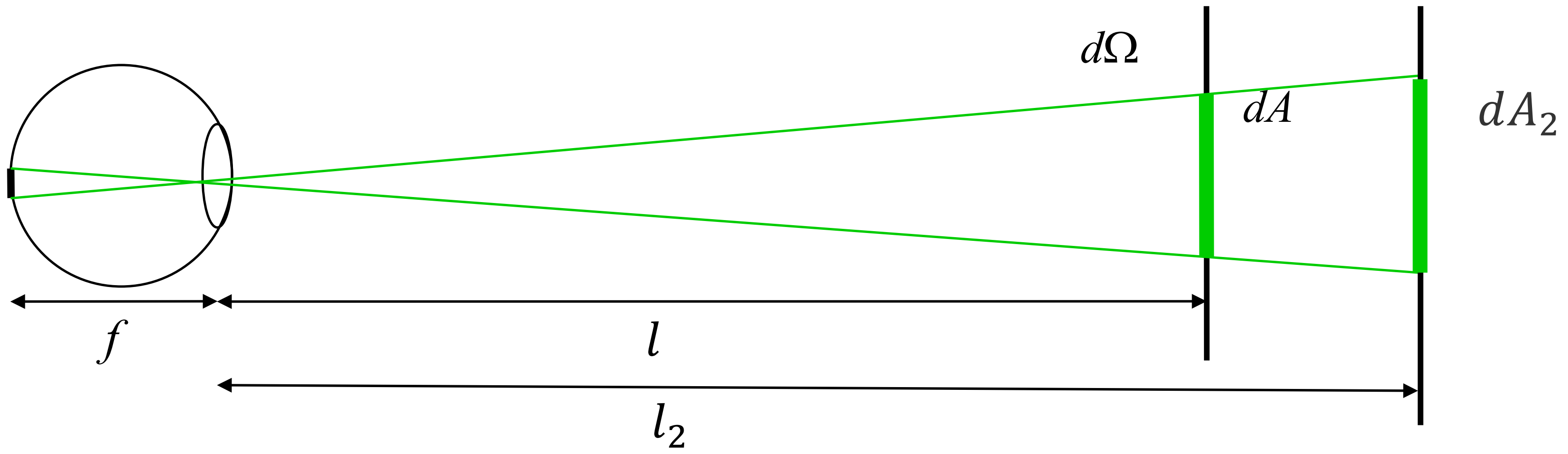
The **radiance** in the direction of a light ray **remains constant** as it propagates along the ray

Perception of Light



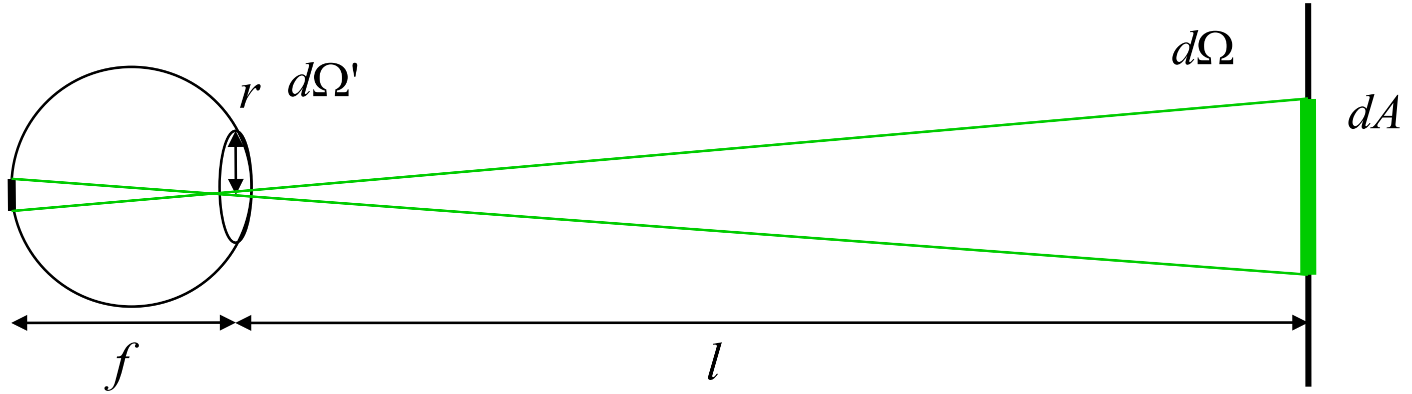
- Looking at a wall the perceived brightness does not change when we vary the distance.

Perception of Light



- Looking at a wall the perceived brightness does not change when we vary the distance
- The wall will not emit / reflect more or less light
- But the area over which we integrate changes

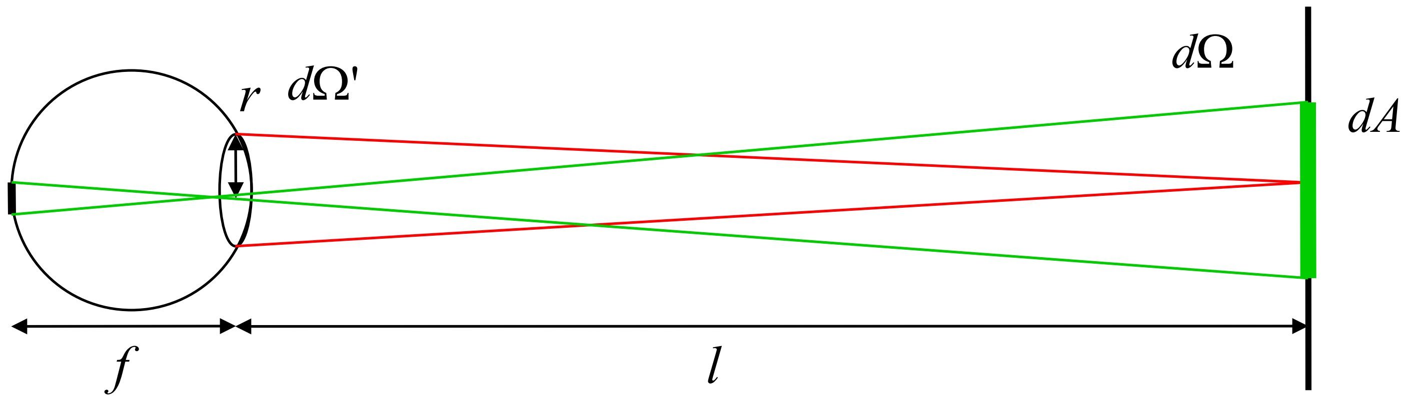
Perception of Light



photons / second = **flux** = energy / time = power	Φ	**rod sensitive to flux**
angular extend of rod = **resolution** ($\approx$ 1 arc minute^2)		$d\Omega$
projected rod size = **area**		$dA \approx l^2 \cdot d\Omega$
angular extend of pupil aperture (r $\leq$ 4 mm) = **solid angle**		$d\Omega' \approx \pi \cdot r^2 / l^2$
flux proportional to area and solid angle		$\Phi \propto d\Omega' \cdot dA$
radiance = flux per unit area per unit solid angle		$L = \dfrac{\Phi}{d\Omega' \cdot dA}$

The eye detects radiance

Perception of Light



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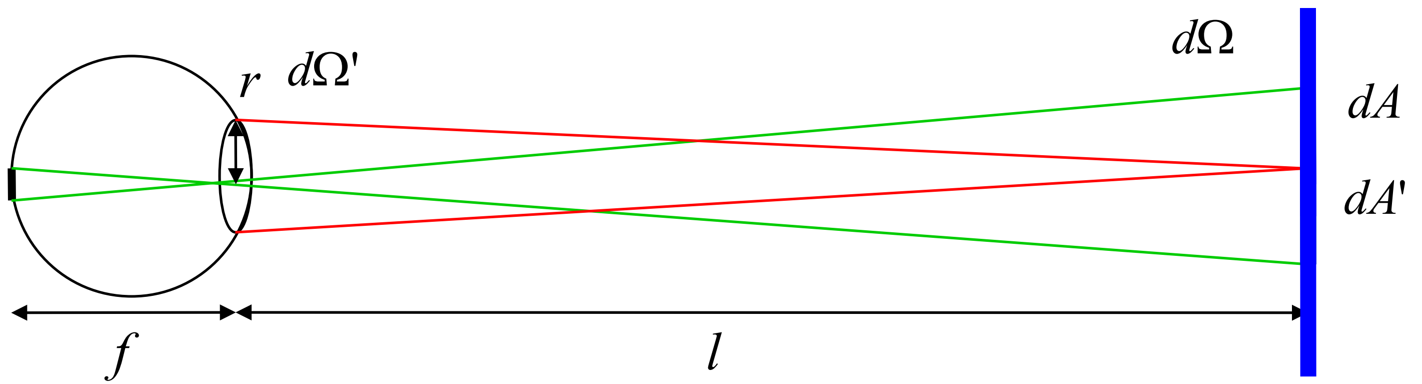
Brightness Perception



As l increases: $\Phi_0 \propto dA \cdot d\Omega' = l^2 d\Omega \cdot \pi \frac{r^2}{l^2} = \text{const}$

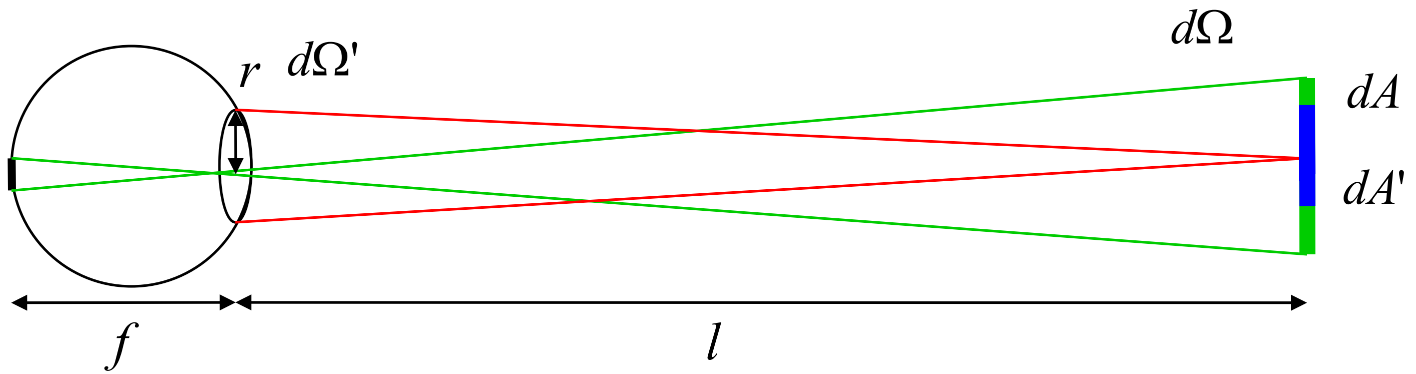
- $dA' > dA$: photon flux per rod stays constant
- $dA' < dA$: photon flux per rod decreases

Where does the Sun turn into a star ?

– Depends on apparent Sun disc size on retina

⇒ Photon flux per rod stays the same on Mercury, Earth or Neptune

⇒ Photon flux per rod decreases when dΩ' < 1 arc minute (beyond Neptune)



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$\Rightarrow$ Photon flux per rod decreases when $d\Omega'$ < 1 arc minute (beyond Neptune)

Radiometric Quantities: Irradiance

Irradiance E is defined as the total power per unit area (flux density) incident onto a surface. To obtain the total flux incident to dA, the incoming radiance L_i is integrated over the upper hemisphere Ω_+ above the surface:

$$E \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[\int_{\Omega_+} L_i(\underline{x}, \underline{\omega}) \cos\theta \, d\omega \right] dA$$

$$E = \int_{\Omega_+} L_i(\underline{x}, \underline{\omega}) \cos\theta \, d\omega = \int_0^{2\pi} \int_0^{\pi/2} L_i(\underline{x}, \underline{\omega}) \cos\theta \sin\theta \, d\theta \, d\phi$$

Radiometric Quantities: Radiosity

Radiosity B is defined as the total power per unit area (flux density) leaving a surface. To obtain the total flux radiated from dA, the outgoing radiance L_o is integrated over the upper hemisphere Ω_+ above the surface.

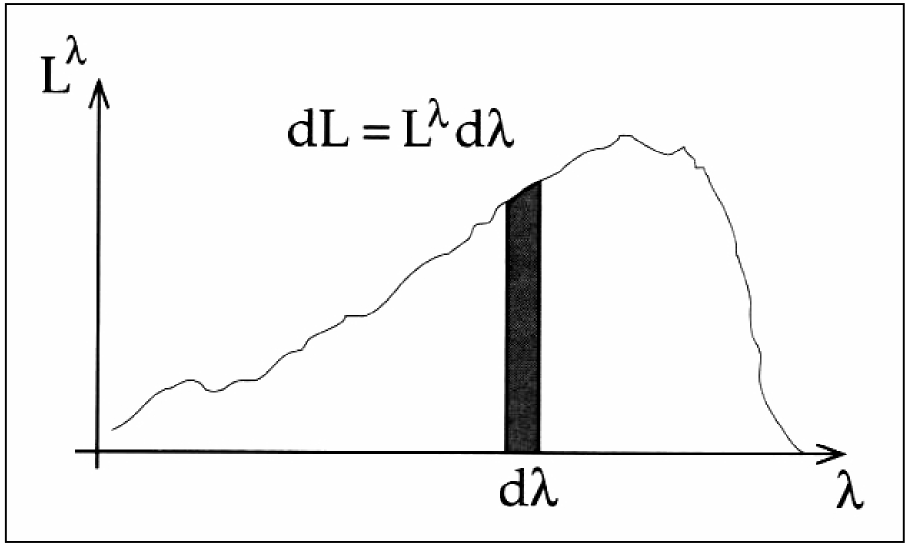
$$B \equiv \frac{d\Phi}{dA}$$

$$d\Phi = \left[\int_{\Omega} L_o(\underline{x}, \underline{\omega}) \cos\theta \, d\omega \right] dA$$

$$B = \int_{\Omega_+} L_o(\underline{x}, \underline{\omega}) \cos\theta \, d\omega = \int_0^{2\pi} \int_0^{\pi/2} L_o(\underline{x}, \underline{\omega}) \cos\theta \sin\theta \, d\theta \, d\phi$$

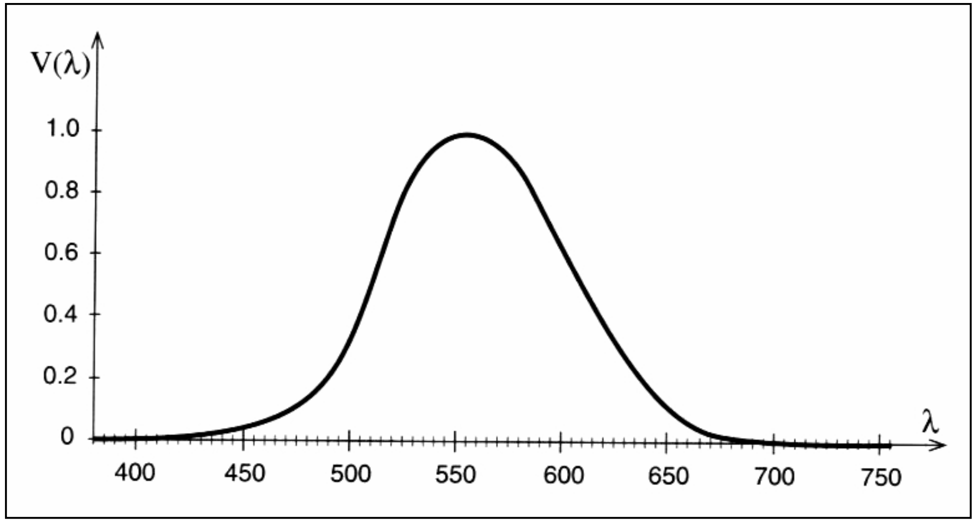
Spectral Properties

- Wavelength
  - Since light is composed of electromagnetic waves of different frequencies and wavelengths, most of the energy transfer quantities are continuous functions of wavelength.
  - In graphics each measurement $L(\underline{x},\underline{\omega})$ is for a discrete band of wavelength only (often some abstract R, B, G)



Photometry

- Photometry:
  - The human eye is sensitive to a limited range of radiation wavelengths (roughly from 380nm to 770nm).
  - The response of our visual system is not the same for all wavelengths, and can be characterized by the luminous efficiency function **V(λ)**, which represents the average human spectral response.
  - A set of photometric quantities can be derived from radiometric quantities by integrating them against the luminous efficiency function V(λ).
  - Separate curves exist for light and dark adaptation of the eye.



Radiometry vs. Photometry

Physics-based quantities **Perception-based quantities**

	Radiometry	→	Photometry	
W	Radiant power	→	Luminous power	Lumens (lm)
W/m^2	Radiosity Irradiance	→	Luminosity Illuminance	Lux (lm/m^2)
$W/m^2/sr$	Radiance	→	Luminance	cd/m^2 ($lm/m^2/sr$)

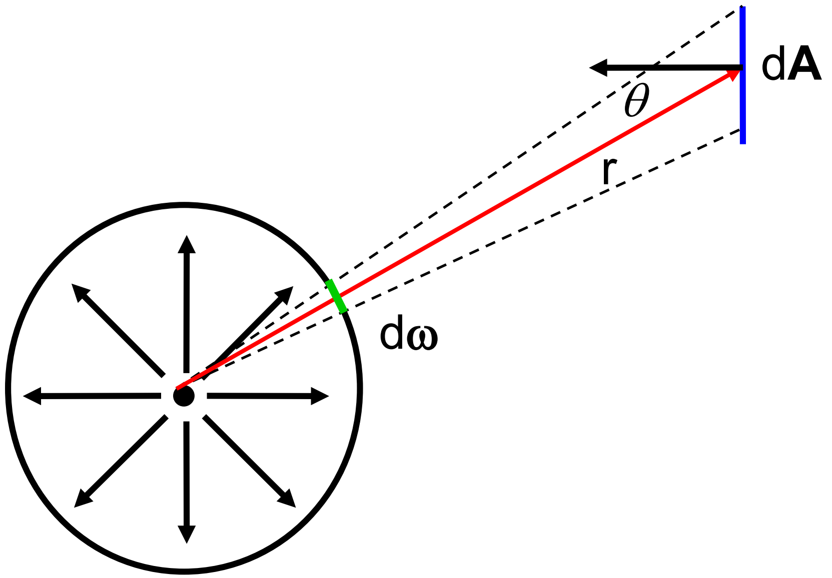
Specifying Light Sources

How to describe the light emitted by a particular source?

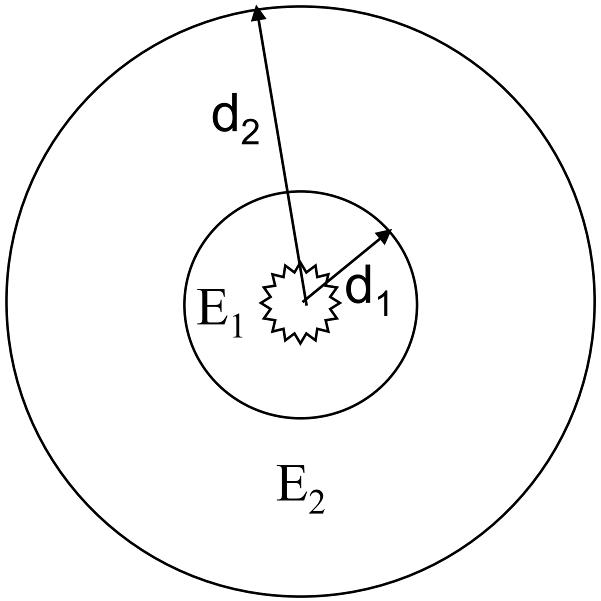
Point Light Source

- Point light with isotropic radiance
  - Power (total flux) of a point light source
    $\forall \Phi_g$ = Power of the light source [watt]
  - Intensity of a light source
    - $I = \Phi_g / (4\pi\,\text{sr})$ [watt/sr]
  - Irradiance on a sphere with radius r around light source:
    - $E_r = \Phi_g / (4\pi r^2)$ [watt/m^2]
  - Irradiance on some other surface A

$$E(x) = \frac{d\Phi_g}{dA} = I\frac{d\omega}{dA} = \frac{\Phi_g}{4\pi}\cdot\frac{dA\cos\theta}{r^2 dA} = \frac{\Phi_g}{4\pi}\cdot\frac{\cos\theta}{r^2}$$



Inverse Square Law



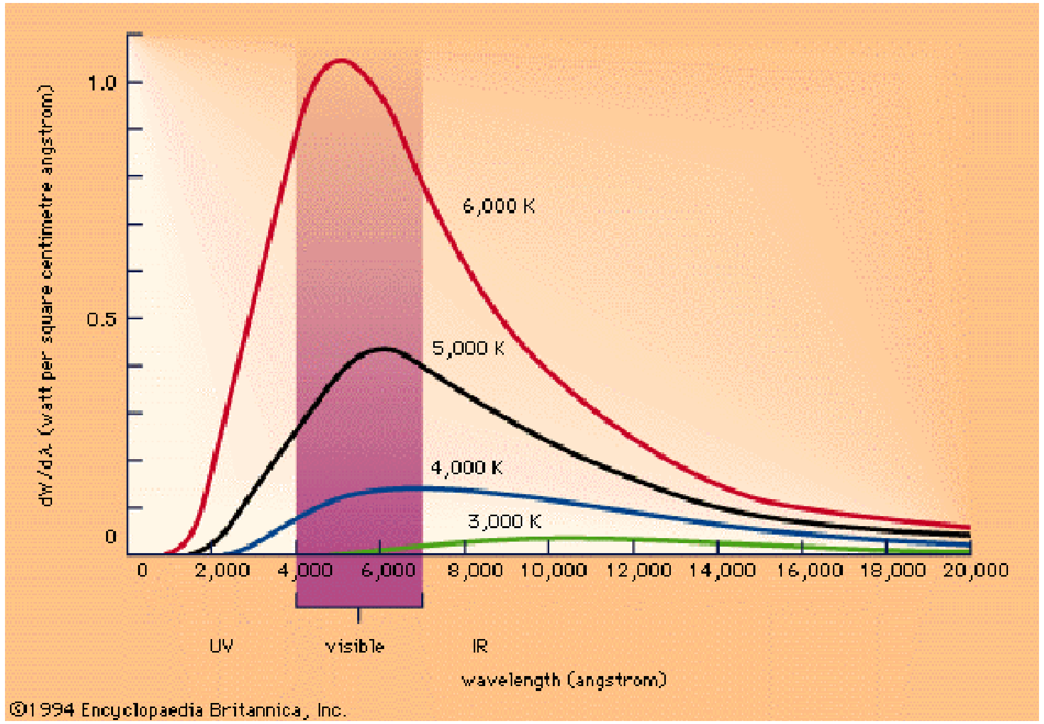
Irradiance E:

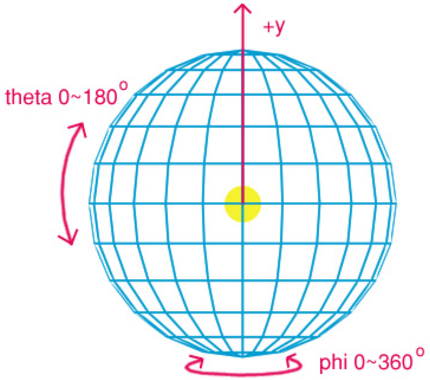
$$\frac{E_1}{E_2} = \frac{d_2^2}{d_1^2}$$

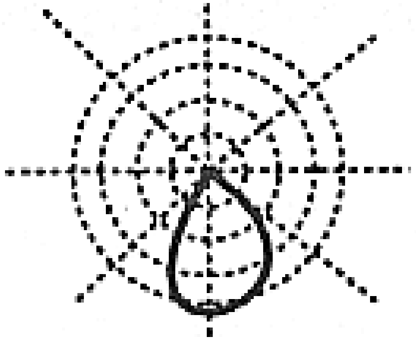
- Irradiance E: power per m^2
  - Illuminating quantity
- Distance-dependent
  - Double distance from emitter: sphere area four times bigger
- Irradiance falls off with inverse of squared distance
  - For point light sources

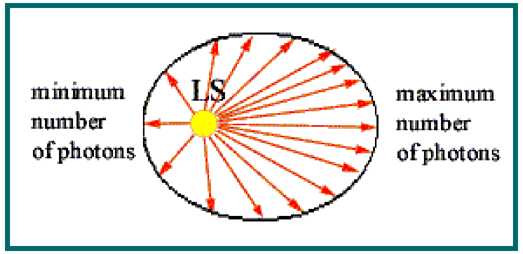
Light Source Specifications

- Power (total flux)
  - Emitted energy / time
- Active emission size
  - Point, area, volume
- Spectral distribution
  - Thermal, line spectrum
- Directional distribution
  - Goniometric diagram









Sky Light

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- Sun
  - Point source (approx.)
  - White light (by def.)
- Sky
  - Area source
  - Scattering: blue
- Horizon
  - Brighter
  - Haze: whitish
- Overcast sky
  - Multiple scattering in clouds
  - Uniform grey

Courtesy Lynch & Livingston

Light Source Classification

Radiation characteristics

- Directional light
  - Spot-lights
  - Projectors
  - Distant sources
- Diffuse emitters
  - Torchieres
  - Frosted glass lamps
- Ambient light
  - “Photons everywhere”

Emitting area

- Volume
  - neon advertisements
  - sodium vapor lamps
- Area
  - CRT, LCD display
  - (Overcast) sky
- Line
  - Clear light bulb, filament
- Point
  - Xenon lamp
  - Arc lamp
  - Laser diode

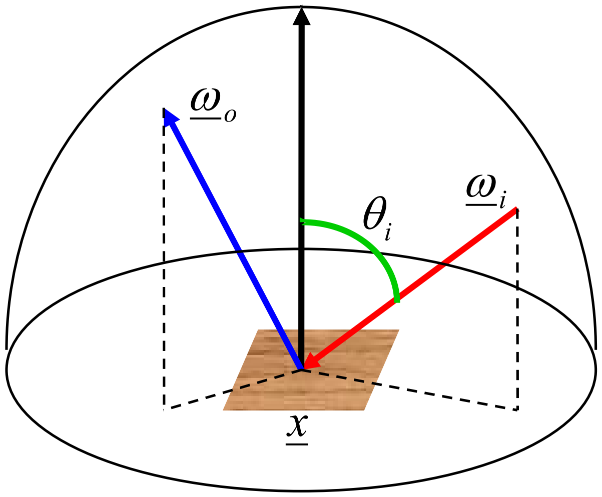
Reflected Radiance

How to calculate the amount of reflected light?

Surface Reflectance

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\Omega} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o)\, L(\underline{x},\underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i$$

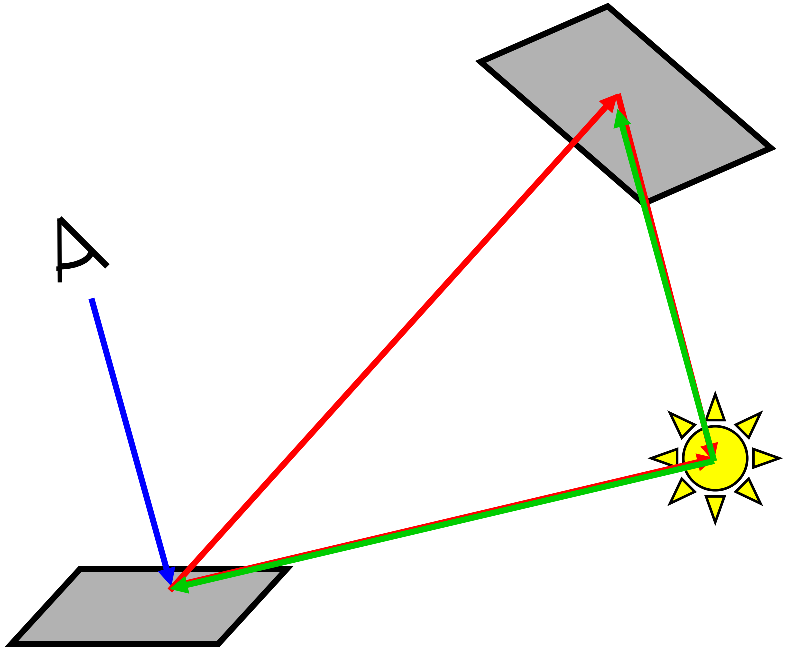
- Visible surface radiance $L(\underline{x},\underline{\omega}_o)$
  - Surface position $\underline{x}$
  - Outgoing direction $\underline{\omega}_o$
  - Incoming illumination direction $\underline{\omega}_i$
- Self-emission $L_e(\underline{x},\underline{\omega}_o)$
- Reflected light
  - Incoming radiance from all directions $L_i(\underline{x},\underline{\omega}_i)$
  - Direction-dependent reflectance (BRDF: bidirectional reflectance distribution function) $f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o)$



Ray Tracing

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\Omega} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o)\, L(\underline{x},\underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i$$

- Simple ray tracing
  - Illumination from light sources only - **local illumination (integral → sum)**
  - Evaluates angle-dependent reflectance function - **shading**
- Advanced Techniques
  - Distribution ray tracing
    - Multiple reflections/refractions (for specular surfaces)
  - Forward/Backward ray tracing
    - Stochastic sampling (Monte Carlo methods)
  - Photon mapping
  - ...



Light Transport in a Scene

- Scene
  - Lights (emitters)
  - Object surfaces (partially absorbing)

- Illuminated object surfaces become emitters, too !
  - Radiosity = Irradiance – absorbed photons flux density
    - Radiosity: photons per second per m^2 leaving surface
    - Irradiance: photons per second per m^2 incident on surface

- Light bounces between all mutually visible surfaces

- Invariance of radiance in free space
  - No absorption in-between objects

- Dynamic Energy Equilibrium
  - emitted photons = absorbed photons (+ escaping photons)

→ **Global Illumination**

The Rendering Equation

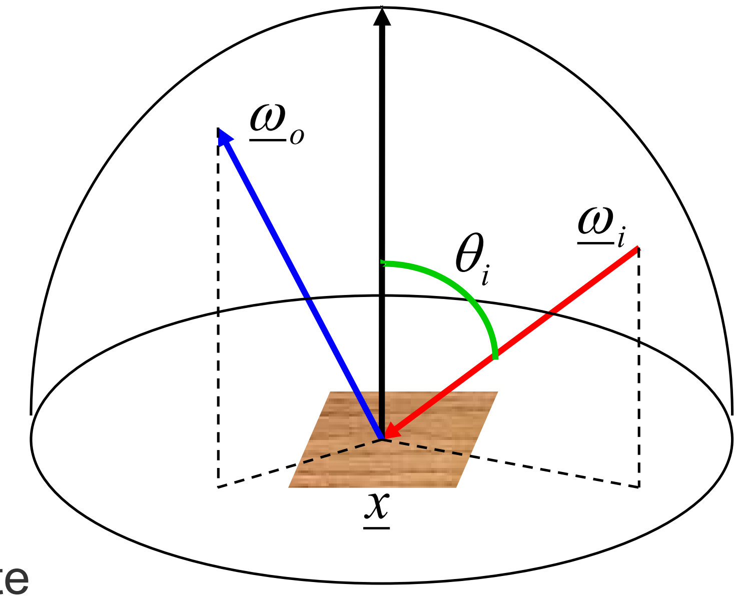
How to express the nature of global illumination?
(The single, most important formula)

(Surface) Rendering Equation

- In Physics: Radiative Transport Equation
- Expresses energy equilibrium in scene

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\Omega} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o)\, L(\underline{x},\underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i$$

- total radiance = emitted radiance + reflected radiance
- First term: emissivity of the surface
  - non-zero only for light sources
- Second term: reflected radiance
  - integral over all possible incoming directions of irradiance times angle-dependent surface reflection function
- Fredholm integral equation of 2nd kind
  - unknown radiance appears on lhs and inside the integral
  - Numerical methods necessary to compute approximate solution



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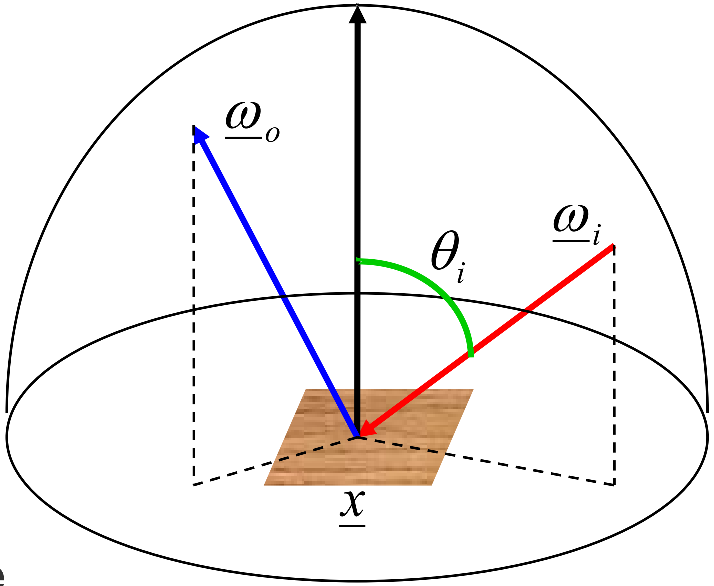


Rendering Equation II

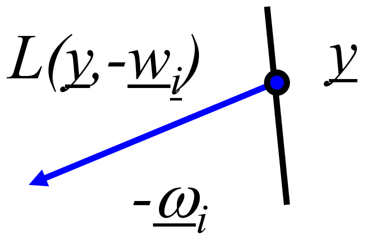
- Outgoing illumination at a point

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + L_r(\underline{x},\underline{\omega}_o)$$

$$= L_e(\underline{x},\underline{\omega}_o) + \int_{\Omega_+} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o) L(\underline{x},\underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i$$

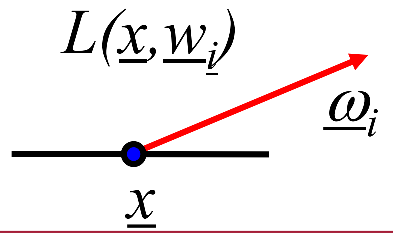
- Linking with other surface points
  - Incoming radiance at x is outgoing radiance at y

$$L_i(\underline{x},\underline{\omega}_i) = L(\underline{y},-\underline{\omega}_i) = L(RT(\underline{x},\underline{\omega}_i),-\underline{\omega}_i)$$



  - Ray-Tracing operator

$$\underline{y} = RT(\underline{x},\underline{\omega}_i)$$



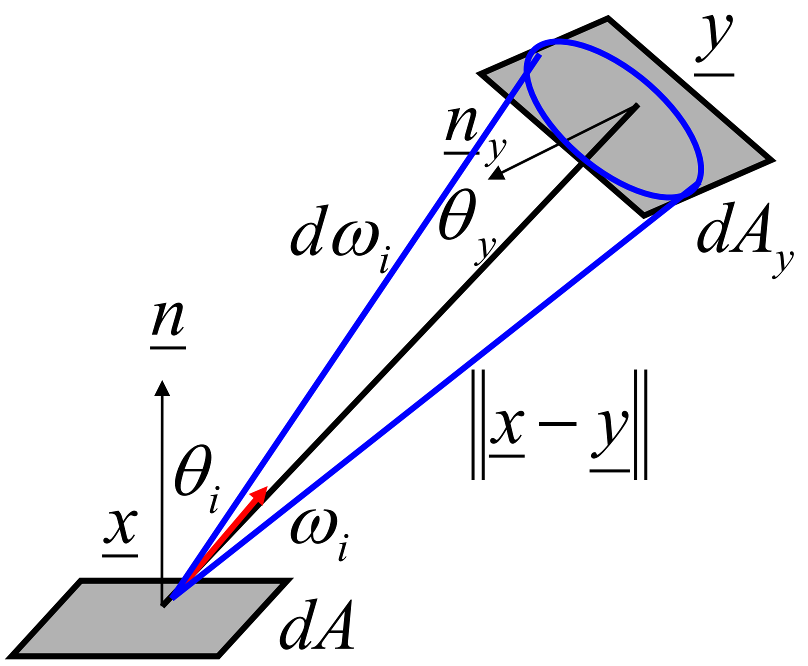
Rendering Equation III

- Directional parameterization

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\Omega_+} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o) L(\underline{y}(\underline{x},\underline{\omega}_i),-\underline{\omega}_i) \cos\theta_i \, d\omega_i$$

- Re-parameterization over surfaces *S*

$$d\omega_i = \frac{\cos\theta_y}{\|\underline{x}-\underline{y}\|^2} \, dA_y$$



$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\underline{y}\in S} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o) \, L(\underline{y},\underline{\omega}_i(\underline{x},\underline{y})) V(\underline{x},\underline{y}) \frac{\cos\theta_i \cos\theta_y}{\|\underline{\mathrm{x}}-\underline{y}\|^2} \, dA_y$$

Rendering Equation IV

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int\limits_{\underline{y}\in S} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o)\, L(\underline{y},\underline{\omega}_i(\underline{x},\underline{y}))\, \mathrm{V}(\underline{\mathrm{x}},\underline{\mathrm{y}}) \frac{\cos\theta_i \cos\theta_y}{\left\|\underline{\mathrm{x}}-\underline{y}\right\|^2}\, dA_y$$

- Geometry term

$$G(\underline{x},\underline{y}) = V(\underline{x},\underline{y}) \frac{\cos\theta_i \cos\theta_y}{\|\underline{x}-\underline{y}\|^2}$$

- Visibility term

$$V(\underline{x},\underline{y}) = \begin{cases} 1 & \text{if visible} \\ 0 & \text{if not visible} \end{cases}$$

- Integration over all surfaces

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int\limits_{\underline{y}\in S} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o)\, L(\underline{y},\underline{\omega}_i(\underline{x},\underline{y}))\, G(\underline{x},\underline{y})\, dA_y$$

Rendering Equation: Approximations

- Using RGB instead of full spectrum
  - follows roughly the eye's sensitivity
- Dividing scene surfaces into small patches
  - Assumes locally constant reflection, visibility, geometry terms
- Sampling hemisphere along finite, discrete directions
  - simplifies integration to summation
- Reflection function model
  - Parameterized function
    - ambient: constant, non-directional, background light
    - diffuse: light reflected uniformly in all directions
    - specular: light of higher intensity in mirror-reflection direction
  - Lambertian surface (only diffuse reflection) → Radiosity
- Approximations based on empirical foundations
  - An example: polygon rendering in OpenGL

Questions

- Why is radiance so important for ray tracing?
- What is described by the rendering equation?
- Which terms does it consist of?
- How does it describe global light transport?

Wrap-up

- Physical Quantities in Rendering
  - Radiance
  - Radiosity
  - Irradiance
  - Intensity
- Light Perception
- Light Sources
- Rendering Equation
  - Integral equation
  - Balance of radiance