Shading

What is the Goal?

→ What color should the surface (being hit) be (rendering)

Material Properties → Reflection Properties → Light Reflection Estimation → how much is reflected at Camera

how much light is coming in at the point → Light Reflection Estimation

Content

- Materials
- Light Reflection/Refraction at Intersection Point
  - geometry → Snell's Law
  - Amount of Light → Fresnel
  - Appearance Representation → BRDF (specular, opaque)
- Evaluation of Reflections
  - Light Sources → Point Light, (Area Light)
  - Shading → Flat, Goraud, Phong
  - Reflection Models → Phong, Blinn

Materials

how does Material reflect Light?

→ different Materials will illicit different Reflections

Reflection Types (or Phenomena):

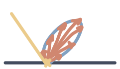
- **Diffuse**: Light is reflected in all directions equally

therefore "surface color" with no other effects visible

like highlights

Also view-independent ≙ equally bright in all directions

- **Glossy**: Light is scattered in a "cone" of directions

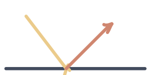


therefore surface color with a "highlight" visible

Also View-Dependent ≙ View Angle affects how bright or dim light is

- **Mirror**: Light is reflected in one direction → one ray that carries all the energy

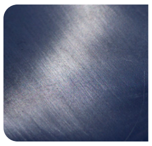
therefore no highlights and such



Also View-Independent

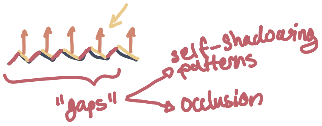
💡 glossy reflection is a "rough" mirror reflection

- Anisoptric: Light texture perpendicular to light direction



so light is reflected somewhat perpendicular to incoming ray
material has surface texture which affects visible light reflected

Also View-Dependent



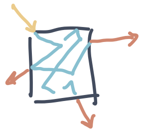
- Translucent: Light enters material and interacts with the "particles" inside of it

$\downarrow$

gets absorbed or reflected

$\downarrow$

then light eventually leaves material
⇒ direction is arbitary due to the unpredictable bounces inside



- Transparent: Light gets reflected in one direction
but also enters material, which leads to change of direction of that ray
then when leaving, ray will change direction again
⇒ leads to a view of the distorted image behind the transparent object

Material Properties

(→ Mechanical, chemical, electrical)

nicht wirklich relevant

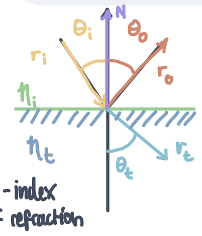
we look at those:
- → Reflection
- → Surface Roughness
- → Geometry / Meso-Structure

⇒ Goal: relightable representations of appearance

Light Reflection/Refraction at Intersection Point

Geometry of Reflection and Refraction

Snell's Law



η - index of refraction

r_i : incident ray → ray of light that hits a surface

θ_i : angle of incidence → the angle between r_i and the surface normal N (perpendicular line to surface)

r_o : reflected ray → ray of light that is reflected by the surface of a r_i

θ_o : angle of reflection → angle between r_m and N

r_t : refracted/transmitted ray → ray of light which is transmitted through the surface of a r_i

θ_t : angle of refraction → angle between r_t and N

η : refraction index of material/medium : quantity describing how fast light travels through the material

$n = \frac{c}{v}$ — c: speed of light in vacuum; v: phase velocity of light in the material

→ determines how much path of light is bent/refracted when it enters the material → Snell's Law :

→ also: amount of light reflected when surface hit

Law of Refraction

for a specular surface θ_o is always equal to θ_i

Law of Conservation of Energy

power of r_i must equal the sum of power in r_o, r_t and any power absorbed in the surface

A ray going from one optical medium (index of refraction → η_i) into another optical medium (index of refraction → η_t) will cause generation of : r_o and r_t

Depending on configuration of several properties (like direction, material, ...)

$\theta_i \Rightarrow \theta_o$ (Law of Refraction) and θ_t change → direction governed by Snell's Law:

$$\eta_i \cdot \sin\theta_i = \eta_t \cdot \sin\theta_t \iff \frac{\sin\theta_i}{\sin\theta_t} = \frac{\eta_t}{\eta_i}$$

So we have : one refracted ray r_t

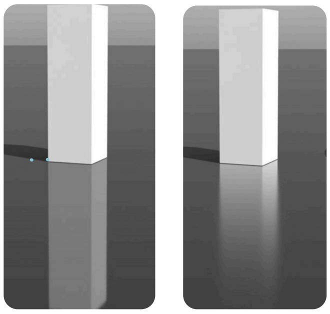
one reflected ray r_o which is the mirror of the incident ray r_i around the surface normal N

And Snell's Law governs the geometry of r_i and r_t (how they look like) as well as allowing us to calc. the geometry of r_o

Amount of Light transmitted \ reflected

Scenario: from air to glass

Fresnel Effect: describes reflection and transmission of light when r_i hits surface



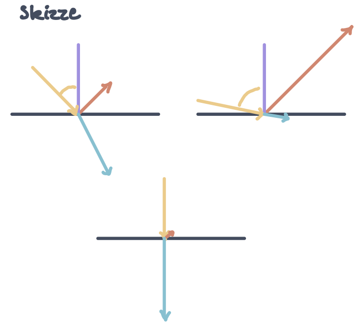
without Fresnel　　Fresnel

→ depending on Θ_i the intensity of r_o and r_t will change (non-linearly)

- Θ_i increasing → reflectance ⬆ refraction ⬇
- Θ_i decreasing → reflectance ⬇ refraction ⬆

⇒ reflectance increases with grazing angle
⇒ refraction decreases with grazing angle

Skizze



Reflectance & Refraction depend on polarisation

polarisation direction of light is either parallel ∥ or perpendicular ⊥ to the plane spanned by r_i and r_t = enegy field

$$R_{\parallel} = \frac{\tan^2(\Theta_i - \Theta_t)}{\tan^2(\Theta_i + \Theta_t)} \qquad T_{\parallel} = \frac{\sin 2\Theta_i \cdot \sin 2\Theta_t}{\sin^2(\Theta_i + \Theta_t)\cdot \cos^2(\Theta_i - \Theta_t)}$$

transmission

$$R_{\perp} = \frac{\sin^2(\Theta_i - \Theta_t)}{\sin^2(\Theta_i + \Theta_t)} \qquad T_{\perp} = \frac{\sin 2\Theta_i \cdot \sin 2\Theta_t}{\sin^2(\Theta_i + \Theta_o)}$$

much stronger surface reflection

Physic formulae → quite costly

better computation →

For unpolarized Light

$$R = \frac{(R_{\parallel} + R_{\perp})}{2}$$

Schlick's approximation based on normal reflection

$\Theta_i = 0$

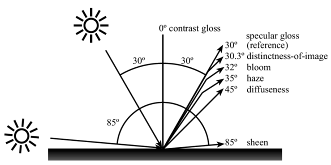
$$R_0 = \left(\frac{n_i - n_2}{n_i - n_2}\right)^2$$

$$\boxed{R \approx R_0 + (1 - R_0)\cdot(1 - \cos\Theta_i)^5}$$

Appearance Representation

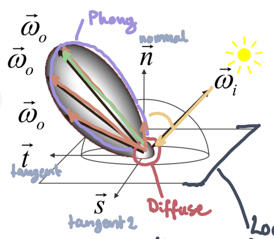
→ Representing / characterizing Reflectance

Gloss Model - 2D



→ with different Θ_i ⇒ Θ_o change of reflectance

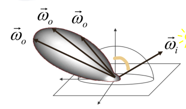
computational model →



Local Coordinate Frame

→ inside model reflectance rays
→ change intensity depending on direction

totally different reflectance values on change of Θ_i

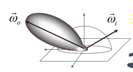


⇓

BRDF

$f_r(\vec{\omega}_i \to \vec{\omega}_o) = \frac{dL(\vec{\omega}_o)}{dE(\vec{\omega}_i)}$

ratio of reflected radiance to incident irradiance



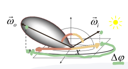
3D

Variant ←

Isotropic

$f_r(\Delta\phi; \theta_i \to \theta_o)$

phi



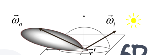
rotating surface around normal

will not change highlight shape ⇒ rotationally invariant

↓ Variant

Spatial Varying

$f_r(\vec{x}; \vec{\omega}_i \to \vec{\omega}_o)$



6D

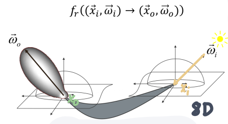
→ account for spatial location x on surface

different point on material has different properties ≙ heterogenous material

Variant ↙

Subsurface Scattering (BSSRDF)

$f_r((\vec{x}_i, \vec{\omega}_i) \to (\vec{x}_o, \vec{\omega}_o))$

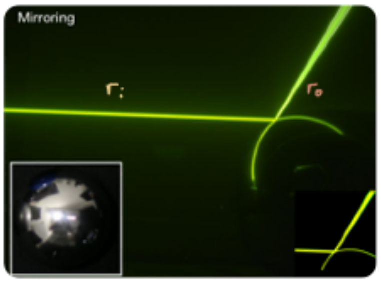


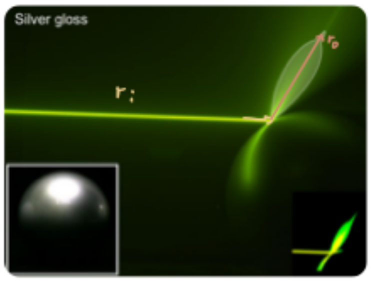
8D

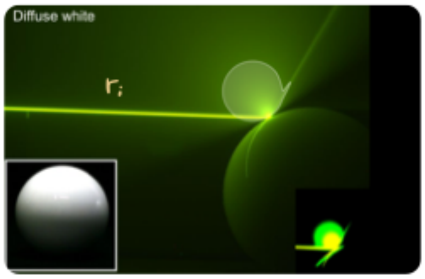
instead of reflectance on r_i hit point light is transported beneath ground and will be reflected on another point x_o

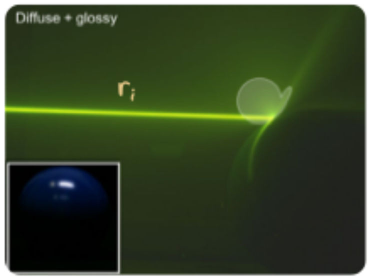
⇒ if heterogenous material even complexer (12D)

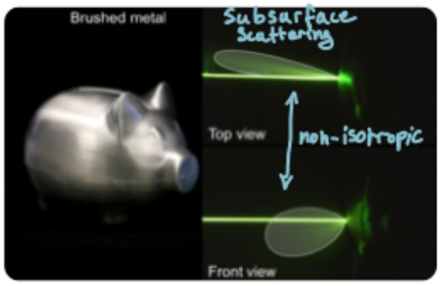
Real World Examples

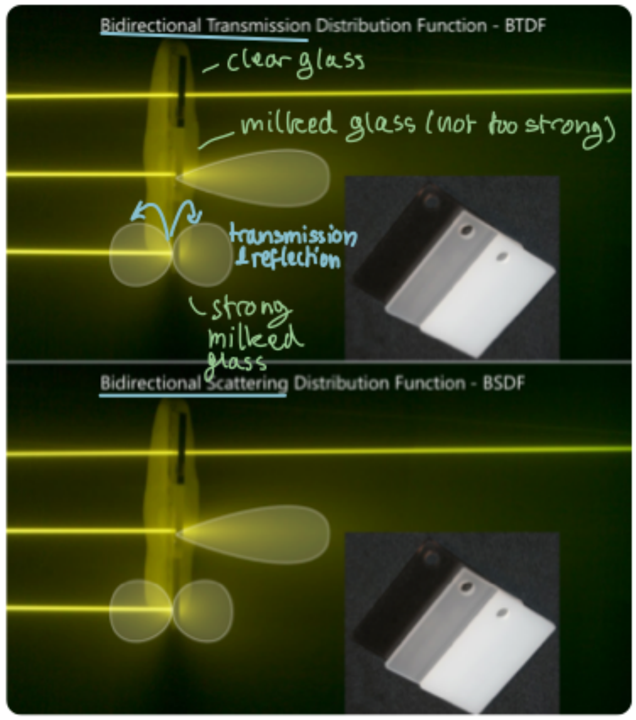












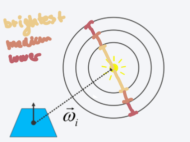
Shading - Evaluation of Reflections

We need:
- Light Source Point
- View Point
- Surface Normal / local coordinate frame (surface)
- Reflectance Model

- **Point Light**
  ↳ like tiny lightbulb
  - Light is uniformly distributed in all directions
  - has a position in 3D-Space
  - the intensity is $\sim \frac{1}{r^2}$ ///

  Distance of Views is not important
  only the angle of viewing.

/// given by how much power light source consumes/emits
the actual intensity hitting a surface depends on
the distance of Surface Point and Light Source



Because energy is shot in all possible directions
it will "hit" surfaces in spherical manner
therefore only partial energy from source is hitting the surface and will decrease with distance
$\rightarrow \frac{1}{r^2}$

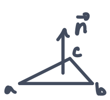
- **Surface Normals**
  - Plane → Hesse form $(\vec{p}, \vec{n})$

Hesse: given a plane
$$ax + by + cz + d = 0$$
and unit normal vector $\hat{n} = \begin{pmatrix} n_x \\ n_y \\ n_z \end{pmatrix}$

$n_x = \frac{a}{\sqrt{a^2+b^2+c^2}}$ $n_y = \frac{b}{\sqrt{\dots}}$ $n_z = \frac{c}{\sqrt{\dots}}$ $-K = \frac{d}{\sqrt{\dots}}$

the Hesse normal form of plane is
$$\hat{n} \cdot x = -K$$

- Triangle
  → Right-Hand-Rule
  → $\vec{n}$ points outwards



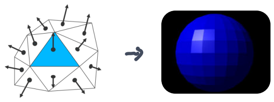
$$\vec{n} = \frac{(c-b)\cdot(a-b)}{\|(c-b)\cdot(a-b)\|}$$ } normalization

• Shading

– Flat Shading

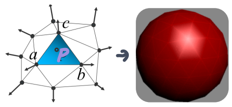
- per surface normal
  ⇒ flat shading
  ⇒ not good for spheres

evaluating with BRDF: every point on surface has the same relative configuration → surface normal, r_i, r_o



– Gouraud Shading

- per vertex normal



calc. normals for all neighbouring triangles
average based on area of all neighbouring triangles (surface normals of polygons meeting at each vertex)
→ evaluate each vertex and interpolate color inside triangle

Evaluating reflectance model at vertices only

$$L_v \approx f(\vec{\omega}_o, \vec{n}_v, \vec{\omega}_i) \cdot L_i$$

Light Source Intensity

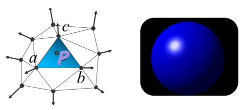
Interpolate vertices Light Source Intensities

$$L_p = \lambda_1 \cdot L_a + \lambda_2 \cdot L_b + \lambda_3 \cdot L_c$$

how much weight of Light at vertex a

⇒ not quite right highlighting:
- – highlight in the middle of of polygon but doesn't spread to vertices
  ⇒ not right render
- – highlight at vertex and spread
  ⇒ correct render at vertex but will render an unnatural spread across all neighbouring polygons (interpolation)

– Phong Shading



→ almost the same as Gouraud but different reflectance model and interpolate Surface Normals of vertices

- Interpolate Surface Normals of Vertices to get Surface Normal of surface hit point P

$$\vec{n}_p = \frac{\lambda_1 \cdot \vec{n}_a + \lambda_2 \cdot \vec{n}_b + \lambda_3 \cdot \vec{n}_c}{\| \lambda_1 \cdot \vec{n}_a + \lambda_2 \cdot \vec{n}_b + \lambda_3 \cdot \vec{n}_c \|}$$

(spherical Interpolation)
↳ using baryocentric coordinates

- evaluate reflectance model at every point

$$L_p \approx f_r(\vec{\omega}_o, \vec{n}_p, \vec{\omega}_i) \cdot L_i$$

→ Phong Reflection Model → for one Light Source

- Cosine power lobe

$$f_r(\vec{\omega}_o, x, \vec{\omega}_i) = k_s(\underbrace{R(I) \cdot V}_{\cos(\theta_{RV})})^{k_e}$$

↳ [0,1]

- Specular Reflection of Lobe

$$L_s = L_i \cdot k_s \cdot \cos^{k_e}(\theta_{RV})$$

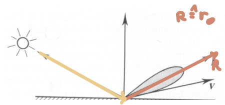
Light Source coming in

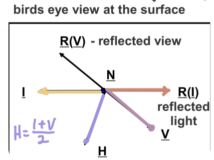
Scale Factor: how much should be reflected overall (usually used to scale down)

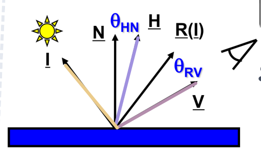
basic Reflectance Distribution

⇒ also not perfect:
- not realistic → no conservation of energy
  non-reciprocal
- plastic-like appearance

We want to create a distribution around the the (mirror direction) ray, the reflected ray r_o of r_i:







Falls off when angle between reflected light R and View Vector V gets larger
→ get angle between (cosine) R and V (two vectors) by dot product
R • V = RV

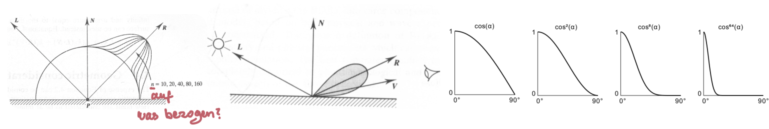
s So RV would be
= 1, if angle = 0°
⇒ V = R "looking into the ray" or R?
= 0, if angle so large that V parallel to Surface
⇒ "ignored", perceiving no light?

this will lead to a "smooth"/Linear effect but want a more focused one (simulating highlight effect)
⇒ use exponent term k_e

Additionally we can steer the strength reflected of our Reflection Lobe by term k_s.

More on Exponent k_e:

→ determines size of highlight → $\lim_{k_e \to \infty} \cos^{k_e}(\alpha)$ = steeper curve ≙ more "line like" look
smaller angular distribution area



Better Version:

Blinn-Phong Model → for one Light Source

use Halfway Vector $H = \frac{I+V}{2}$ and Surface Normal

- Cosine power Lobe

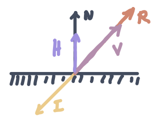
$$f_r(\omega_o, x, \omega_i) = k_s \cdot (H \cdot N)^{k_e}$$

- Specular Reflection of Lobe

$$L_s = L_i \cdot k_s \cdot \cos^{k_e}(\theta_{HN})$$

Why better?

- when N and H aligned, I and V align to R



$\Rightarrow N = H \Leftrightarrow I = V = R$

- for the distribution, just look at how much H deviates from this Alignment
- is energy conserving (up until a certain k_s value
- computation easier

instead of using R and V we use the halfway vector

$H = \frac{I+V}{2}$ average of incident direction and view direction

and the Surface Normal N

for our distribution area, so we use a different angle θ_{HN}, instead of θ_{RV}

⇒ Both Models only simulate glossy reflection for a single light source

→ to fully evaluate the surface point we need to account for multiple/all Light Sources in Scene (later other Light Sources)

Phong-Illumination Model → multiple Light Sources

k_d, k_s = scale

$$L_r = \underbrace{k_a L_{i,a}}_{\text{Ambient}} + k_d \underbrace{\sum_l L_l (I_l \cdot N)}_{\text{Diffuse}} + k_s \underbrace{\sum_l L_l (R(I_l) \cdot V)^{k_e}}_{\text{Specular / glossy}} \quad \text{Phong}$$

$$+ k_s \sum_l L_l (H_l \cdot N)^{k_e} \quad \text{Blinn}$$

physically not plausible constant

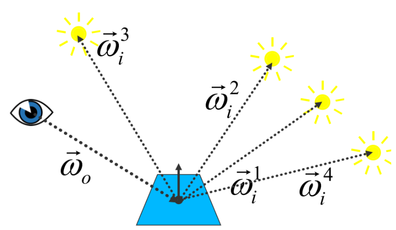
view independent

- is a heuristic Model:
  - → contradicts physics
  - → purely local Illumination
    - direct light from Sources
    - no further reflection on other surfaces
    - constant ambient term

only relies on source dir and viewing dir

Multiple Light Sources

$$L_r = \sum_N f(\vec{\omega}_o, \vec{\omega}_i^k) L_i^k \cos(\theta^k)$$



→ Occlusion

$$L_r = \sum_N f(\vec{\omega}_o, \vec{\omega}_i^k) \boxed{v(p, \vec{\omega}_i^k)} L_i^k \cos(\theta^k)$$

- Point on surface in shadow from some object

➡ trace ray to light source → test for occlusion (collision)

Area Light

→ Sampling