Perception: Psychophysics and Modeling

02 | Linear Systems, Fourier transform and optics

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Literature

De Valois, R. L. and De Valois, K. K. (1988). *Spatial Vision*. Oxford University Press.
Relevant is ch. 1, Linear Systems Analysis, pp. 3-22.

B.A. Wandell (1995). *Foundations of Vision*. Sinauer Associates, Inc.
Relevant are ch. 2, Image Formation, pp. 13-44, and Appendix A, Shift-Invariant Linear Systems, pp. 405-412.

Science

Essentially a prediction game—understanding principles qualitatively should allow us to predict hitherto unseen events (and be able to understand how and why the prediction comes about!).

This is obviously true in physics with all its “natural laws”, but equally in psychology:

- What is the detectability of x quanta of light? (Sensory psychology)
- How does context information influence reasoning ability? (Cognitive psychology)
- What is the influence of the size of a group on individual decisions? (Social psychology)
- Which therapy helps fastest for anxiety sufferers? (Clinical psychology)

We want to abstract away from individual cases and find regularities (“laws”) in behaviour. If successful, we can use answers to the above questions to predict future behaviour!

1. Strong hope, that such general (“universal”) laws exist in psychology.
2. How to derive them from a limited number of measurements?

Linear Systems

If an input-output system is linear we can use linear systems methods to characterize it. If successful, a small number of measurements allows us to predict the response to any input.

To test whether the system is linear, we need to test its response, *r*, to a number of inputs:

1. homogeneity (= proportionality): $r(\alpha x) = \alpha r(x)$
2. additivity: $r(x_1 + x_2) = r(x_1) + r(x_2)$

Superposition is the fulfilment of both homogeneity and additivity: $\alpha r(x_1) + \beta r(x_2) = r(\alpha x_1 + \beta x_2)$

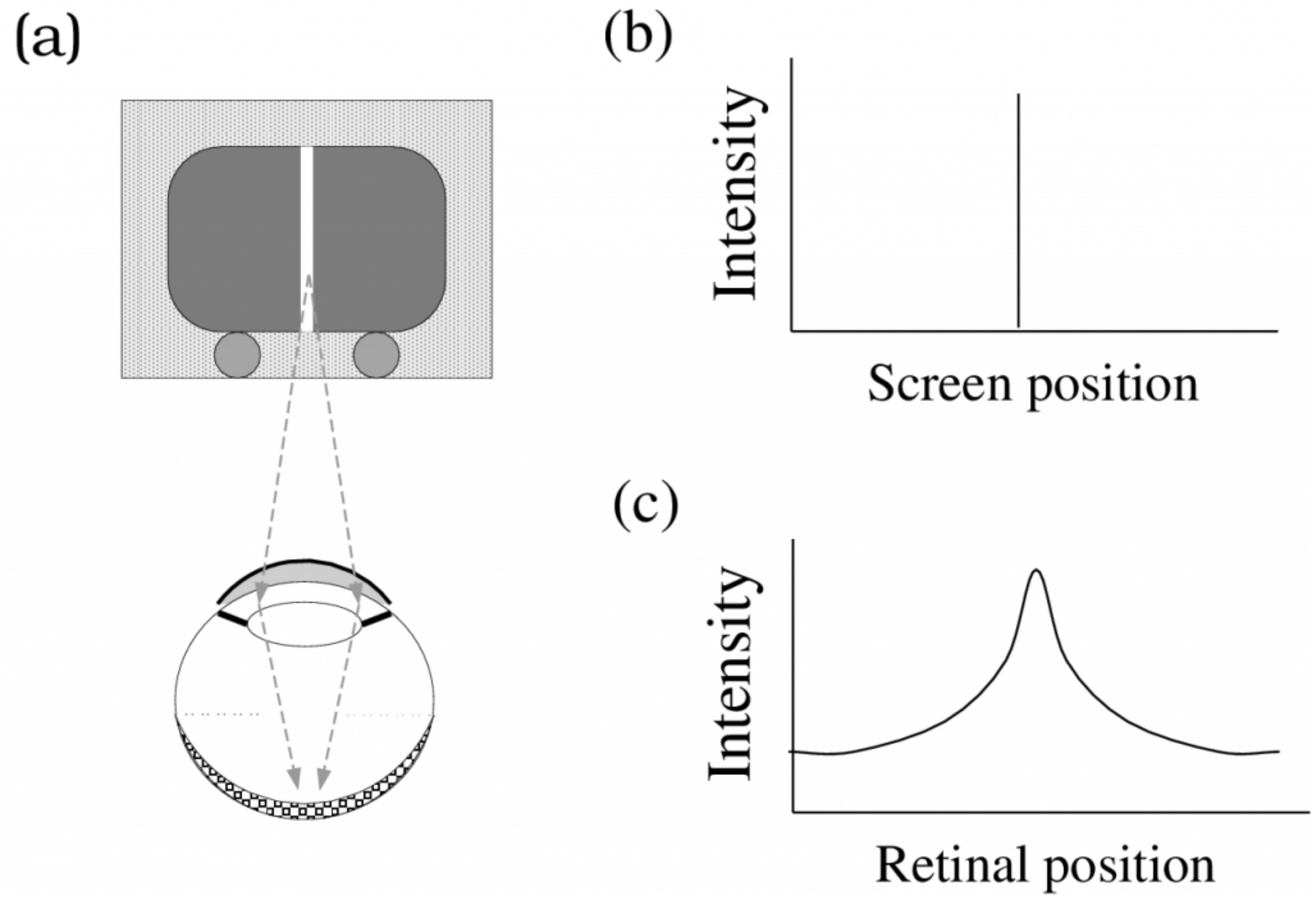
Linear Systems

Both requirements are fulfilled for a number of important linear operators Φ :

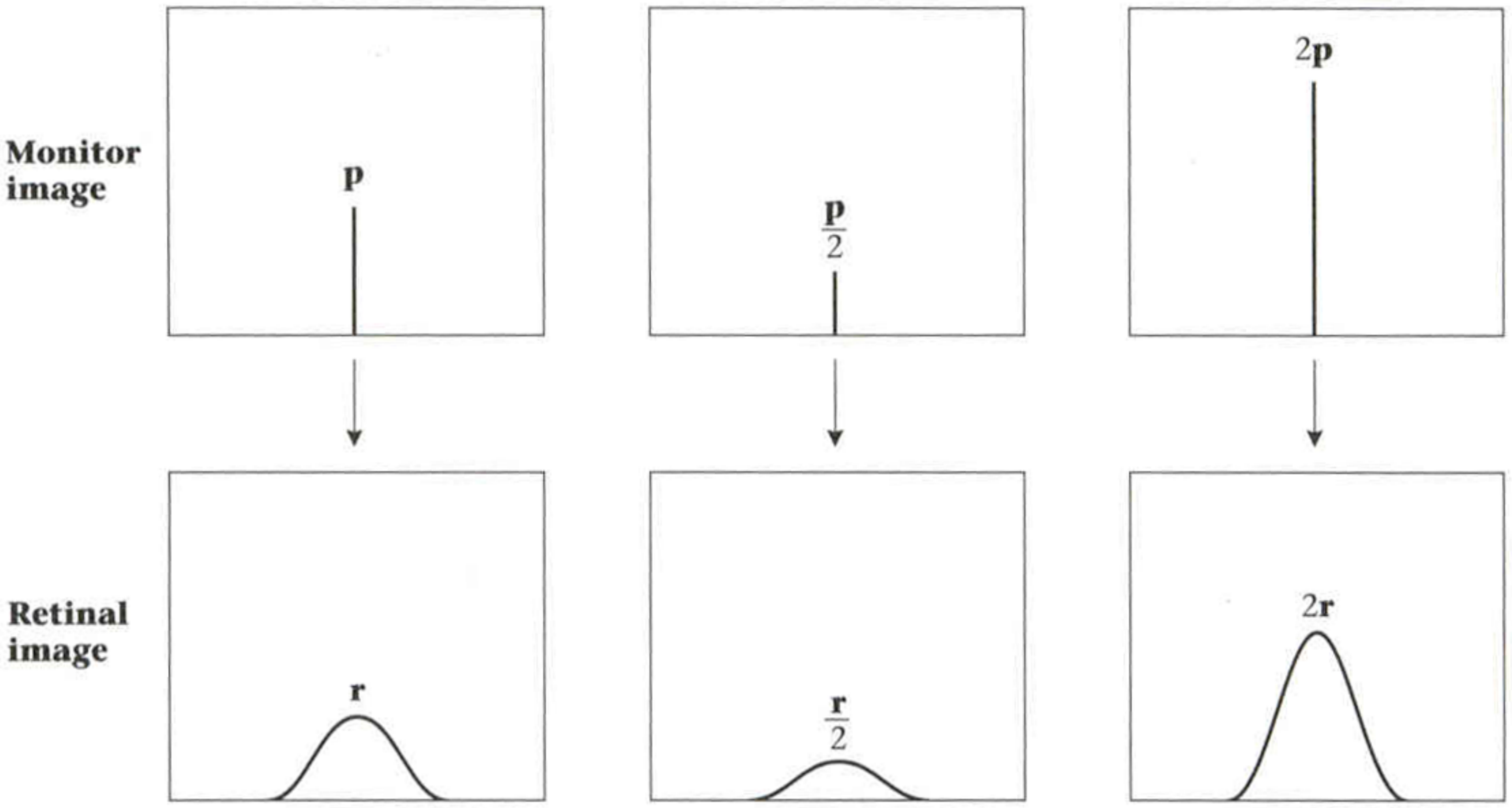
- Any derivative, or combination of derivatives of any order
- An integral expression.
- A convolution with some fixed waveform.
- Any combination or concatenation of the above.

Application to Optics

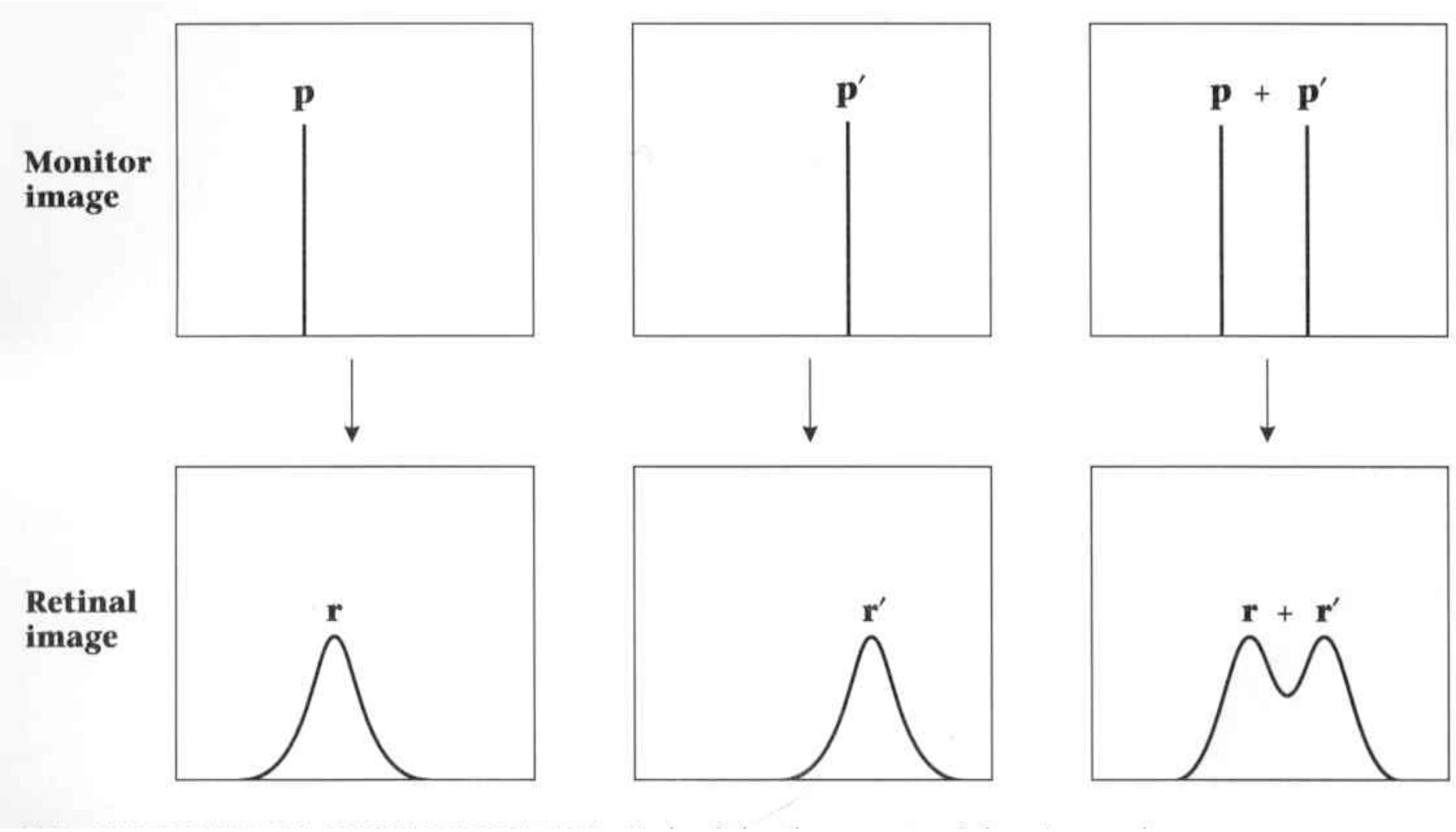
Understand how to predict the retinal image from the scree image: what gets is encoded by the retina



The Principle of Homogeneity (Proportionality)

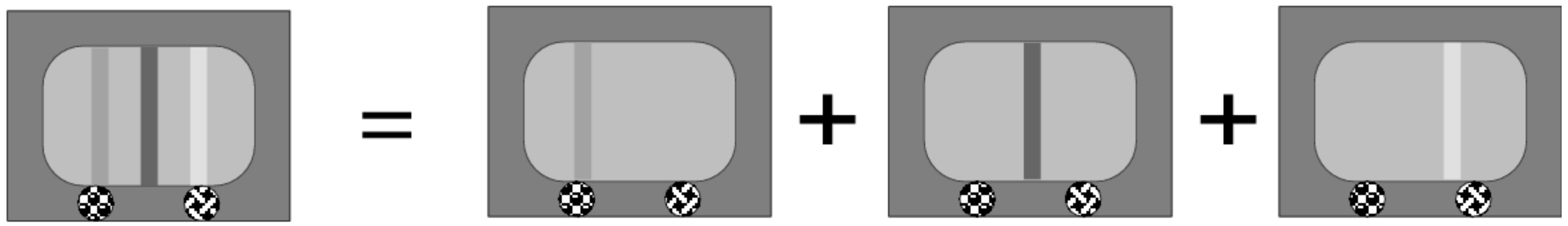


The Principle of Additivity

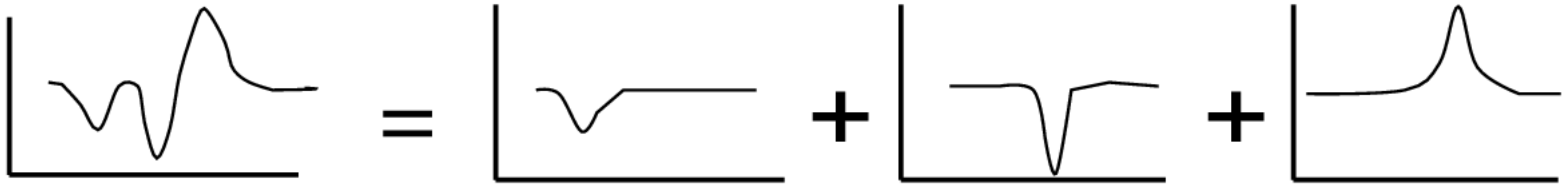


Application of Homogeneity and Additivity

(a)



(b)



(c)

$$\begin{pmatrix} \\ \mathbf{r} \\ \\ \end{pmatrix} = p_1 \begin{pmatrix} \\ \mathbf{r}_1 \\ \\ \end{pmatrix} + p_2 \begin{pmatrix} \\ \mathbf{r}_2 \\ \\ \end{pmatrix} + p_3 \begin{pmatrix} \\ \mathbf{r}_3 \\ \\ \end{pmatrix}$$

Shift-Invariant Linear Systems

If the response r to an input x is the same over time (no memory!) and/or space, then the system is shift-invariant.

Shift-invariant linear systems are even easier to deal with than linear systems in general: if we measure $r(x)$ at one position, and now shift the input x (in space and/or time), we get the same r but shifted (in space and/or time).

In our previous example: measurement of r_1 would have been enough, we can deduce the individual responses r_2 and r_3 from r_1 (shift-invariance & homogeneity) as well as the total response (additivity).

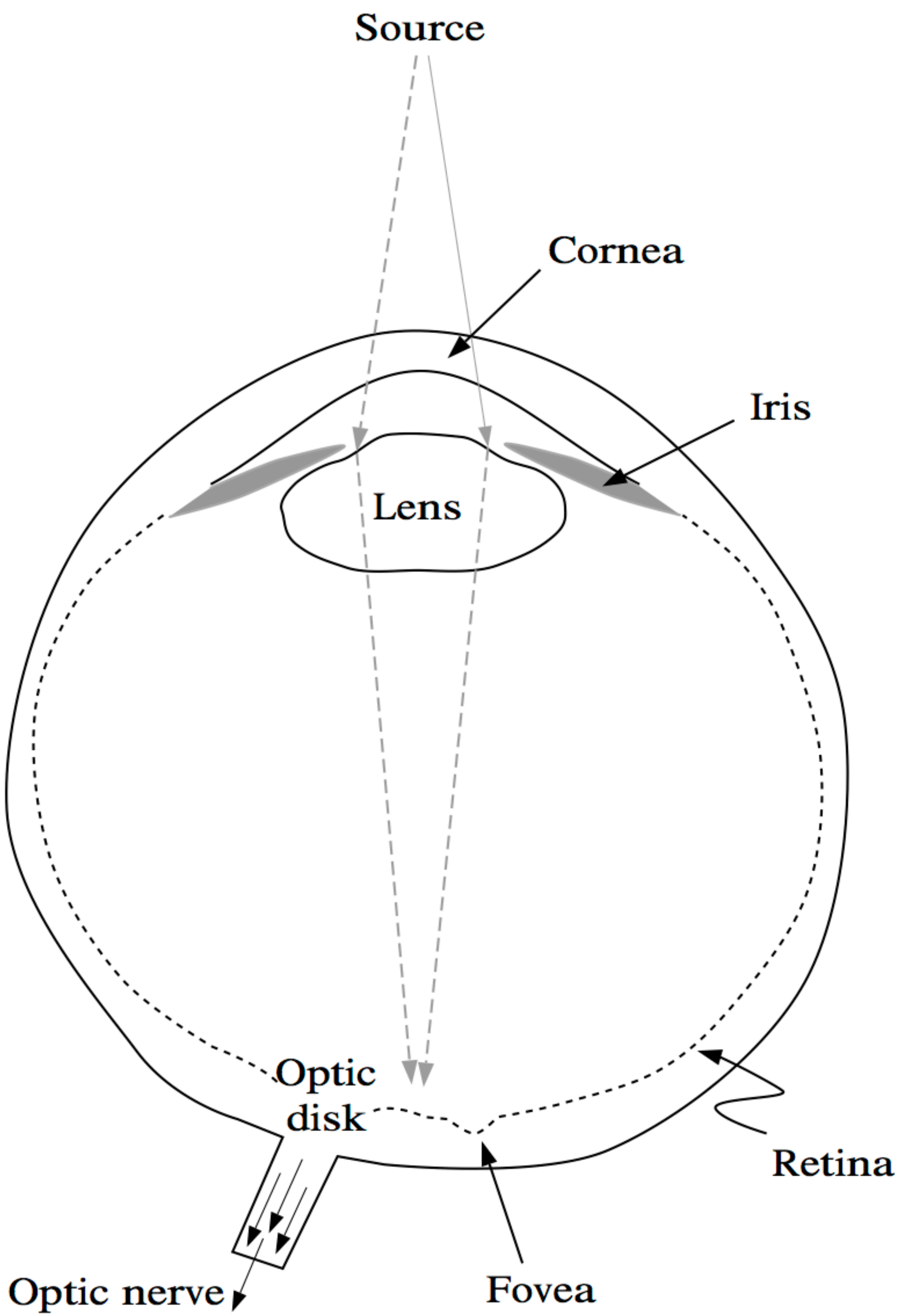
Special Stimuli for Linear Systems Analysis

What stimulus to use?
Most common answer in engineering: An impulse! (Dirac Delta function)

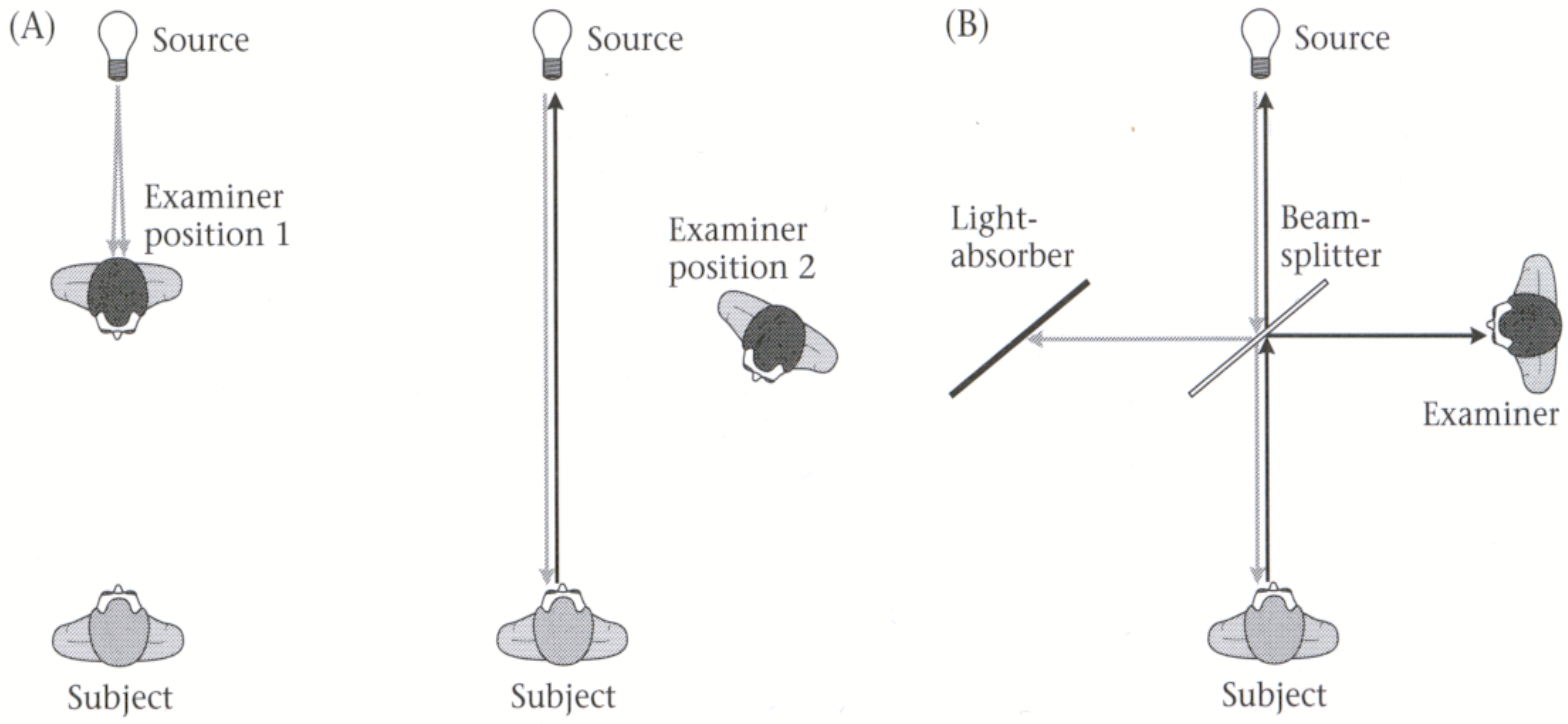
Why?
Because everything else can be seen as the superposition (sum) of very many (shifted) impulses—impulses form a **basis** to describe all possible stimuli (functions).

Eigenfunction of a linear system:
Consider the linear system Φ. Every linear system has a set of special inputs x_0 such that the response r of the system to the stimulus x_0 is simply a (scaled version) of the input: $r = \Phi(x_0) = \lambda x_0$
(The scale factor is in this case typically denoted by λ)

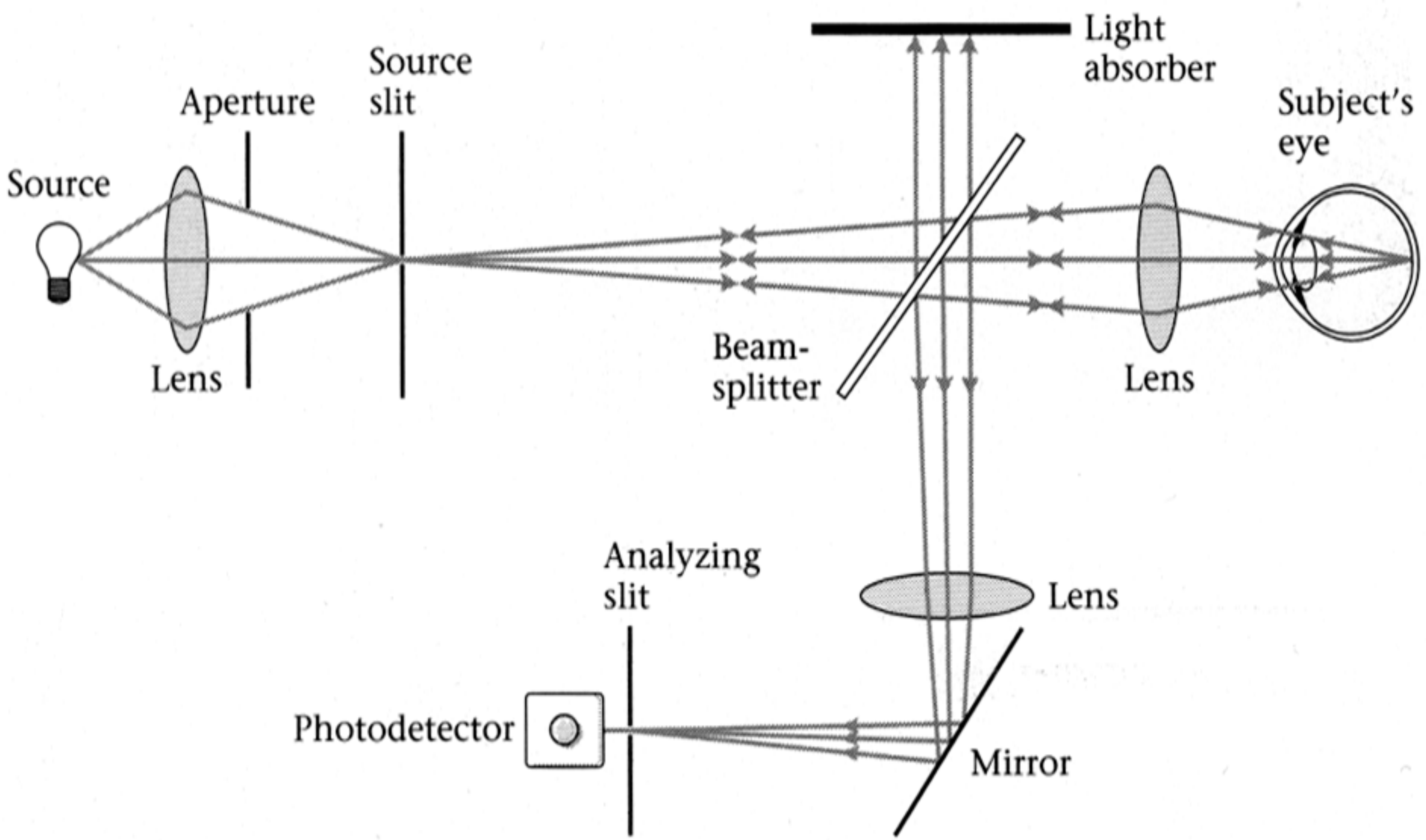
Sine waves (sin and cos) are eigenfunctions of shift-invariant linear systems, thus that makes them special, too, when analysing (shift-invariant) linear systems; all that changes is the amplitude and, perhaps, the phase of the sine wave going into a SILS.


Source
Cornea
Iris
Lens
Optic
disk
Retina
Optic nerve
Fovea

Measuring Light Reflected From the Eye



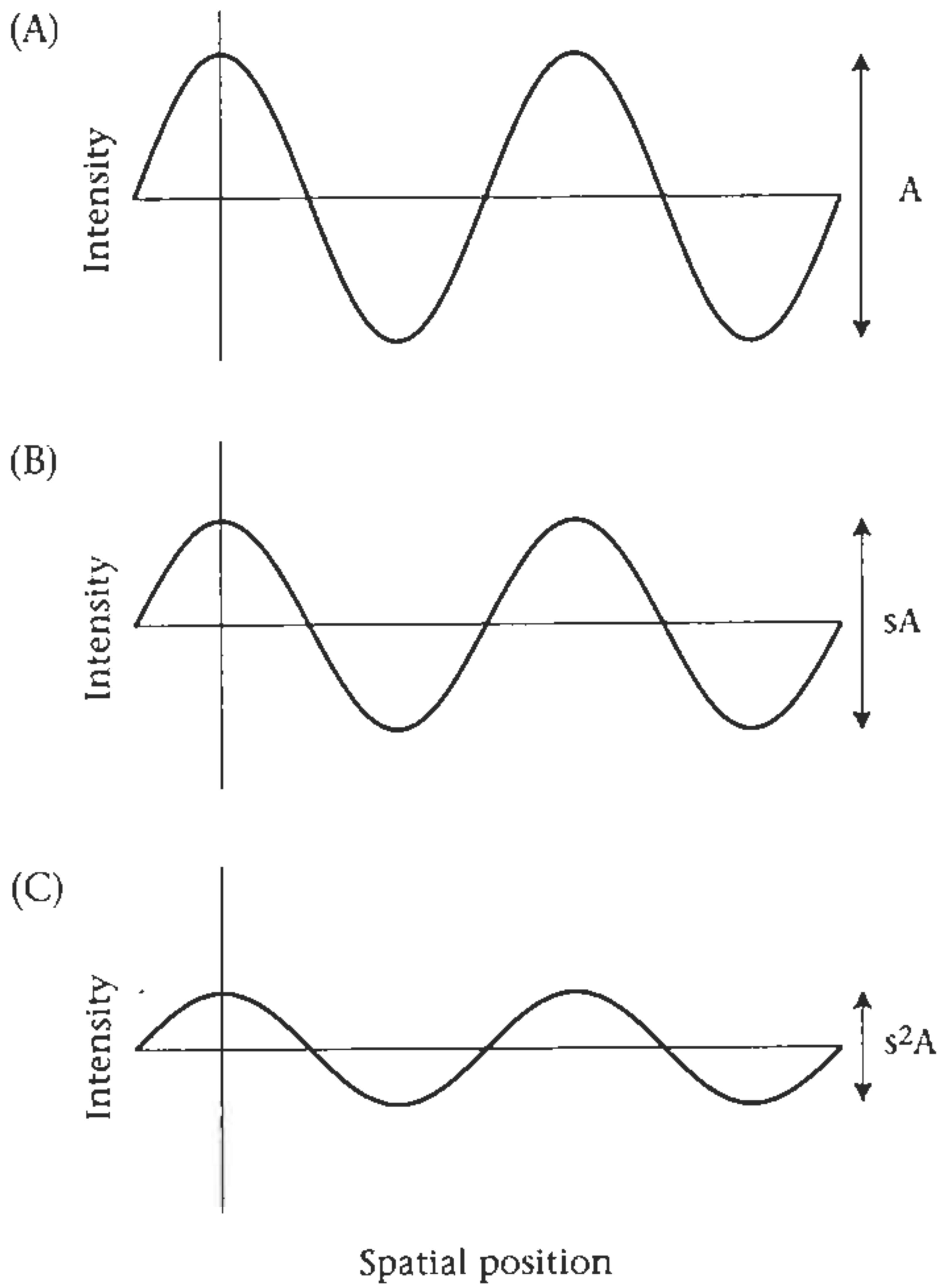
Modified Ophthalmoscope



Sinusoids and Double Passage

Optical systems are (to an approximation) shift-invariant linear systems:

we can simply "calculate out" the effect of the double-passage



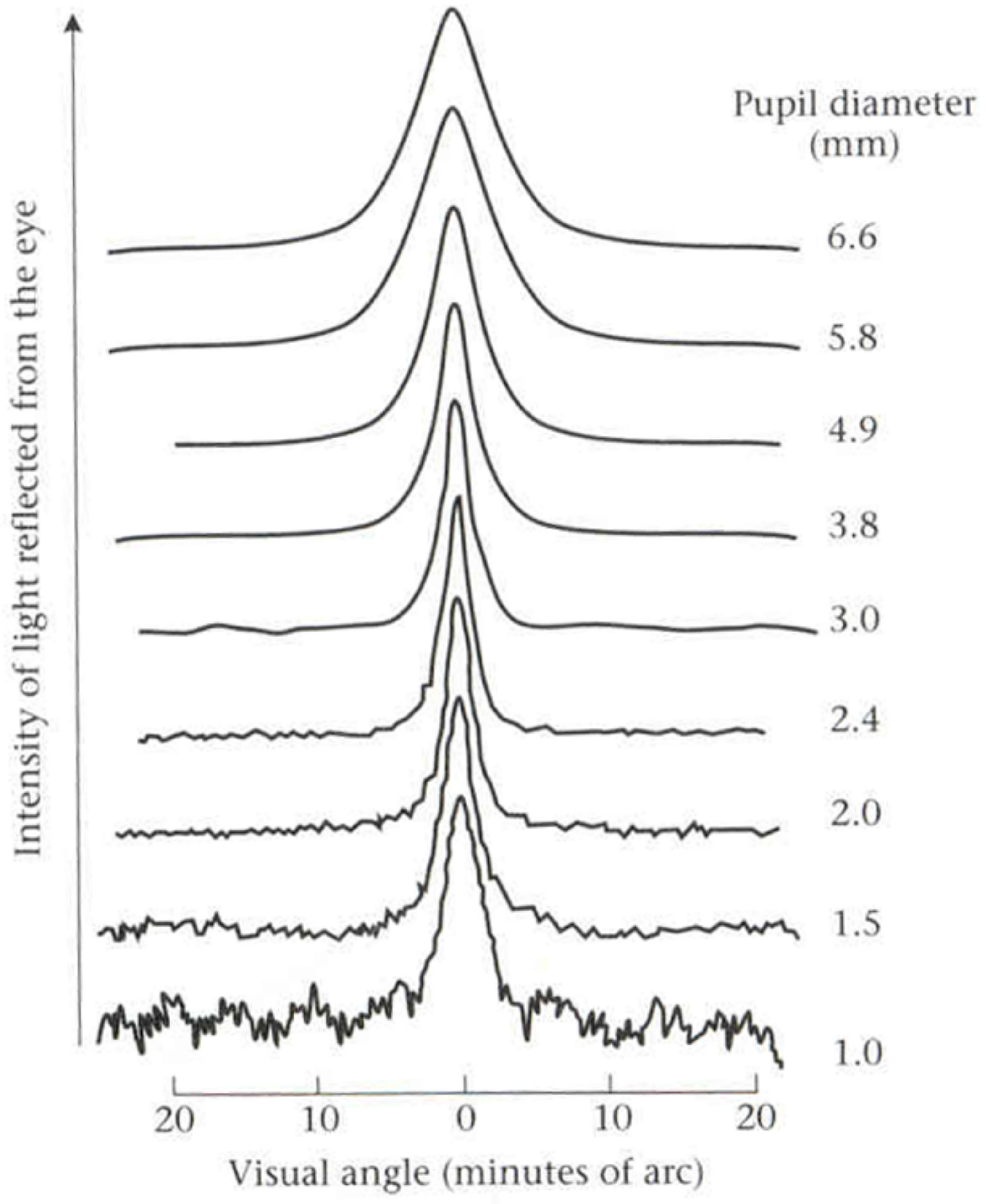
Experimental Measurements

raw data:

1. small pupil diameter means less light on the retina: noisy measurements

2. large pupil diameters show more blur — lens imperfections (aberrations) show up

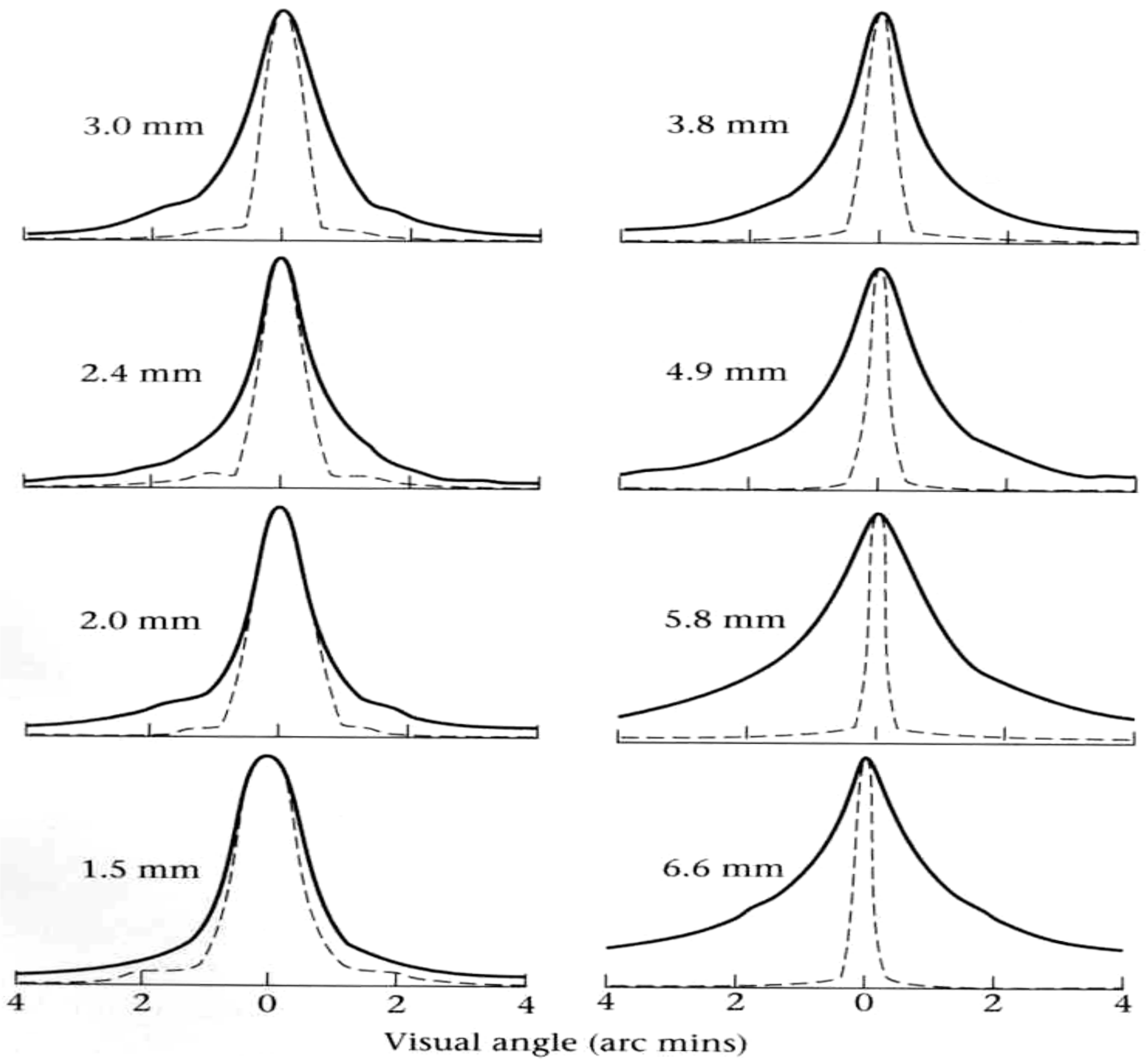
3. double-pass: the light measured had to go through the optics twice



Human LSF

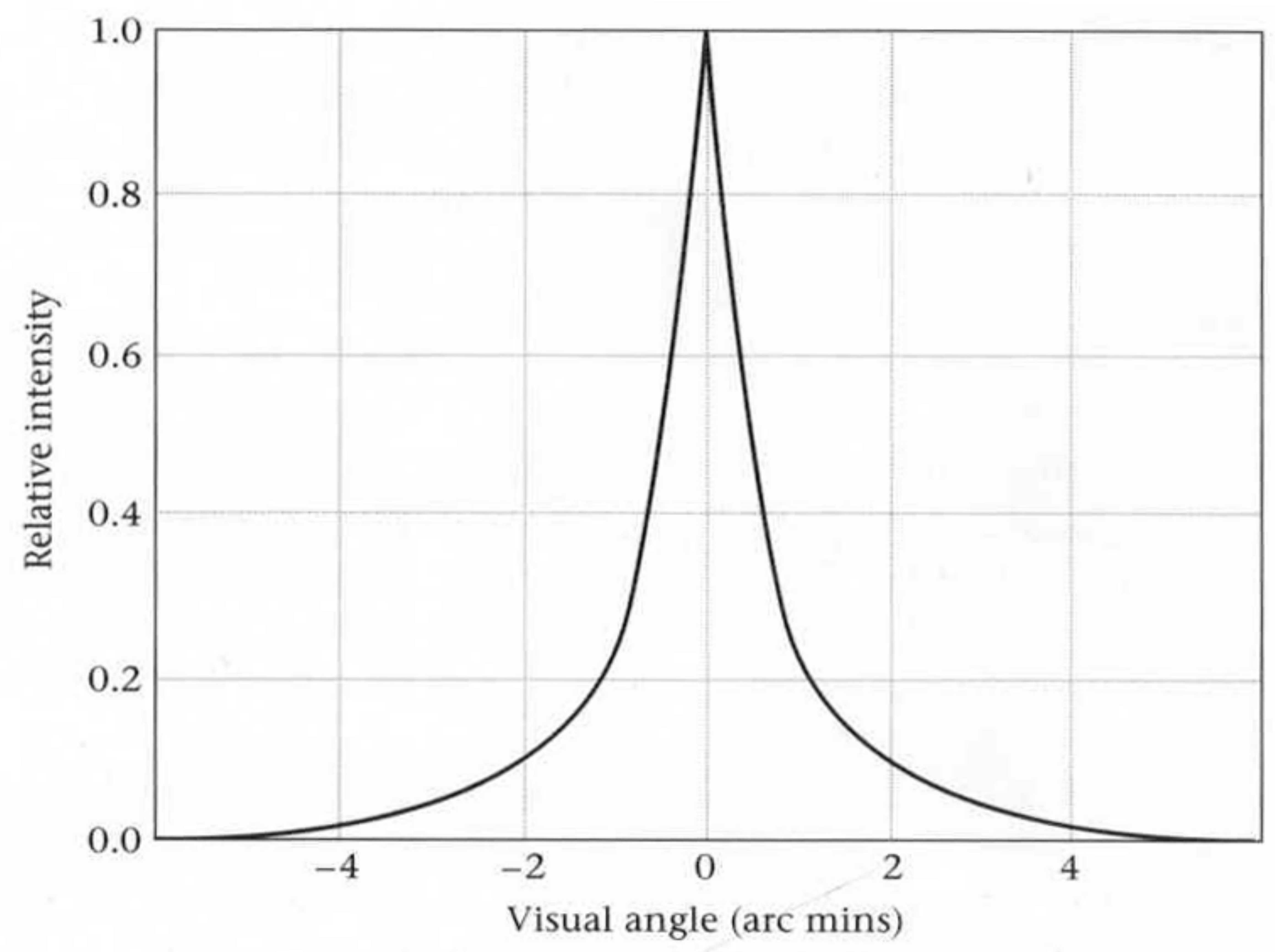
Campbell & Gubisch (1966) data of the linespread function as a function of pupil diameter.

1. much worse for large pupil diameters (to be expected from lens imperfections)
2. worse for very small diameters (< 2.4 mm: diffraction-limited!)
3. Dashed lines show theoretical best for purely diffraction limited system



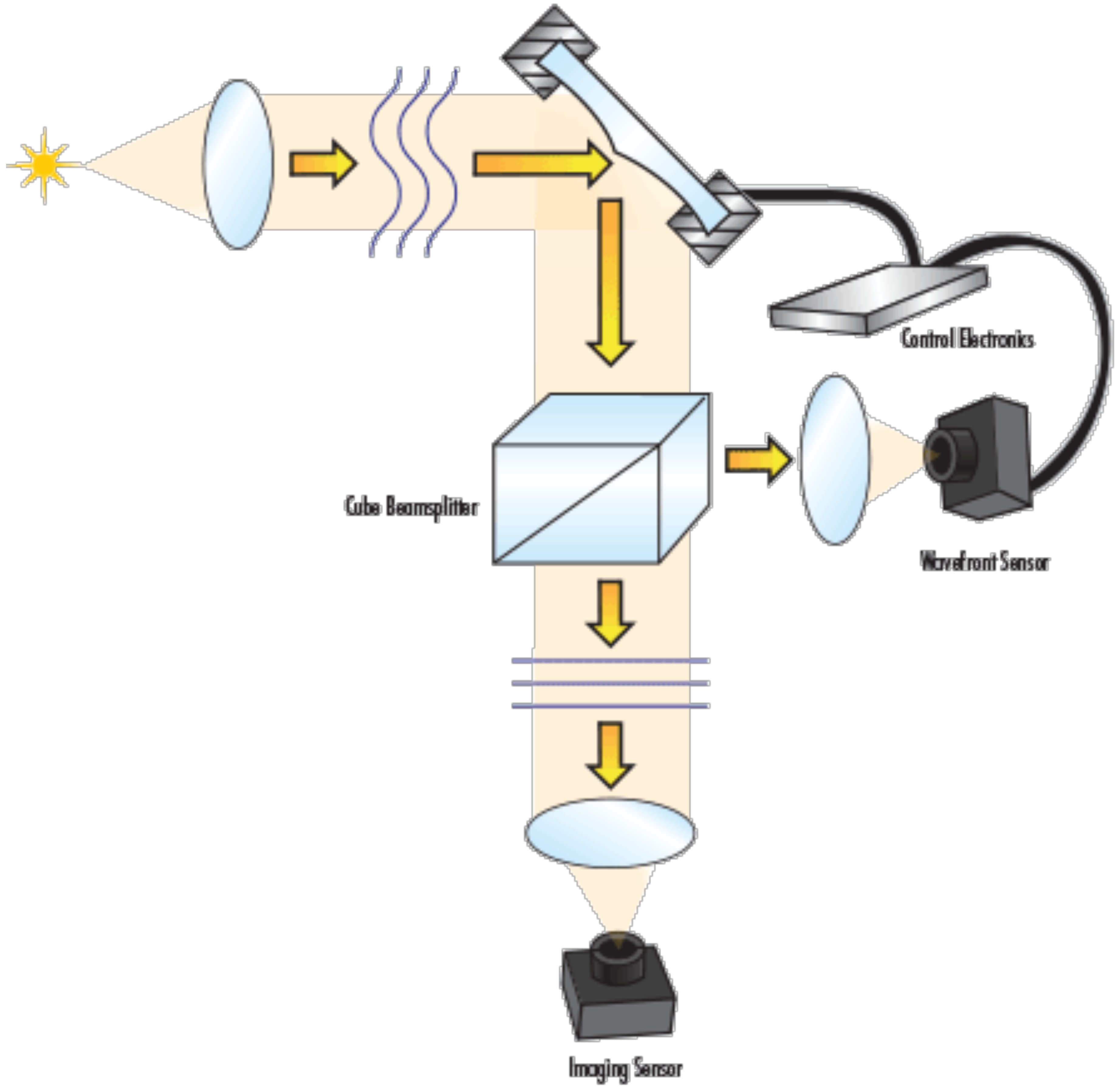
Human LSF

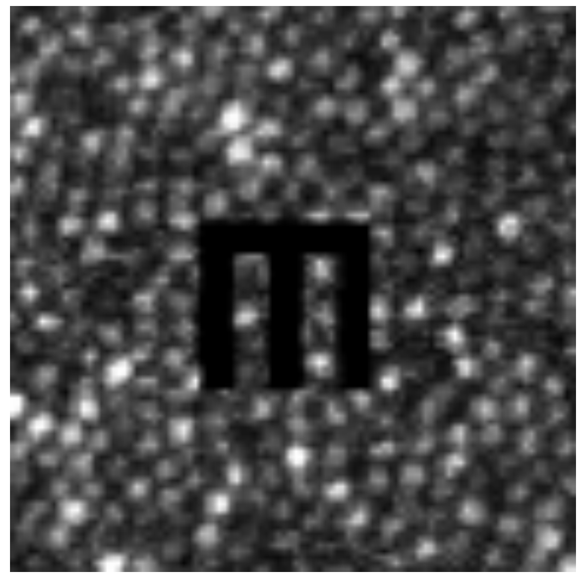
Impulse view of a linear system in the space domain

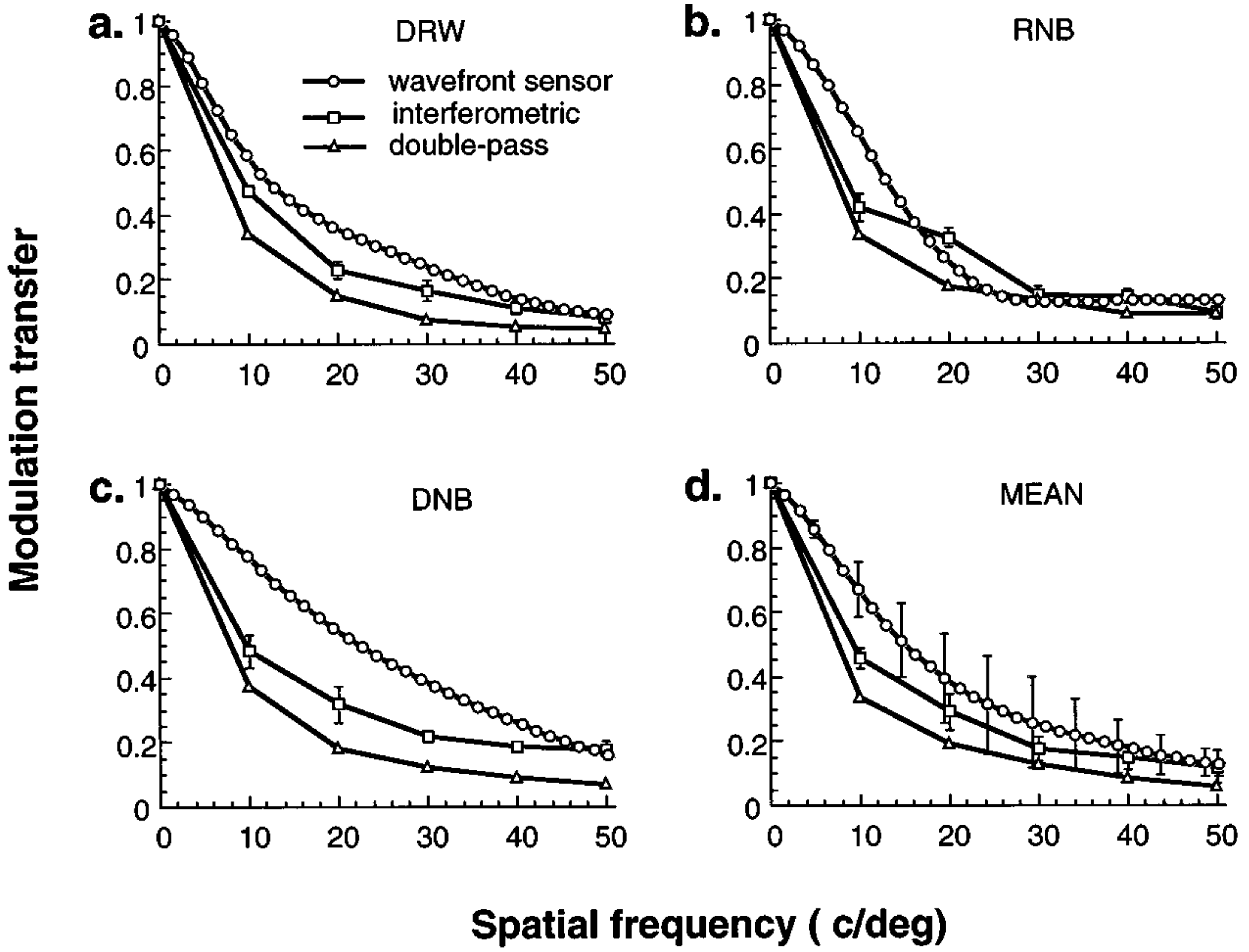


Modern measurement by Westheimer (1986), Williams et al. (1984; interferometry) and Liang & Williams (1997; wave-front sensor methods)

Adaptive optics







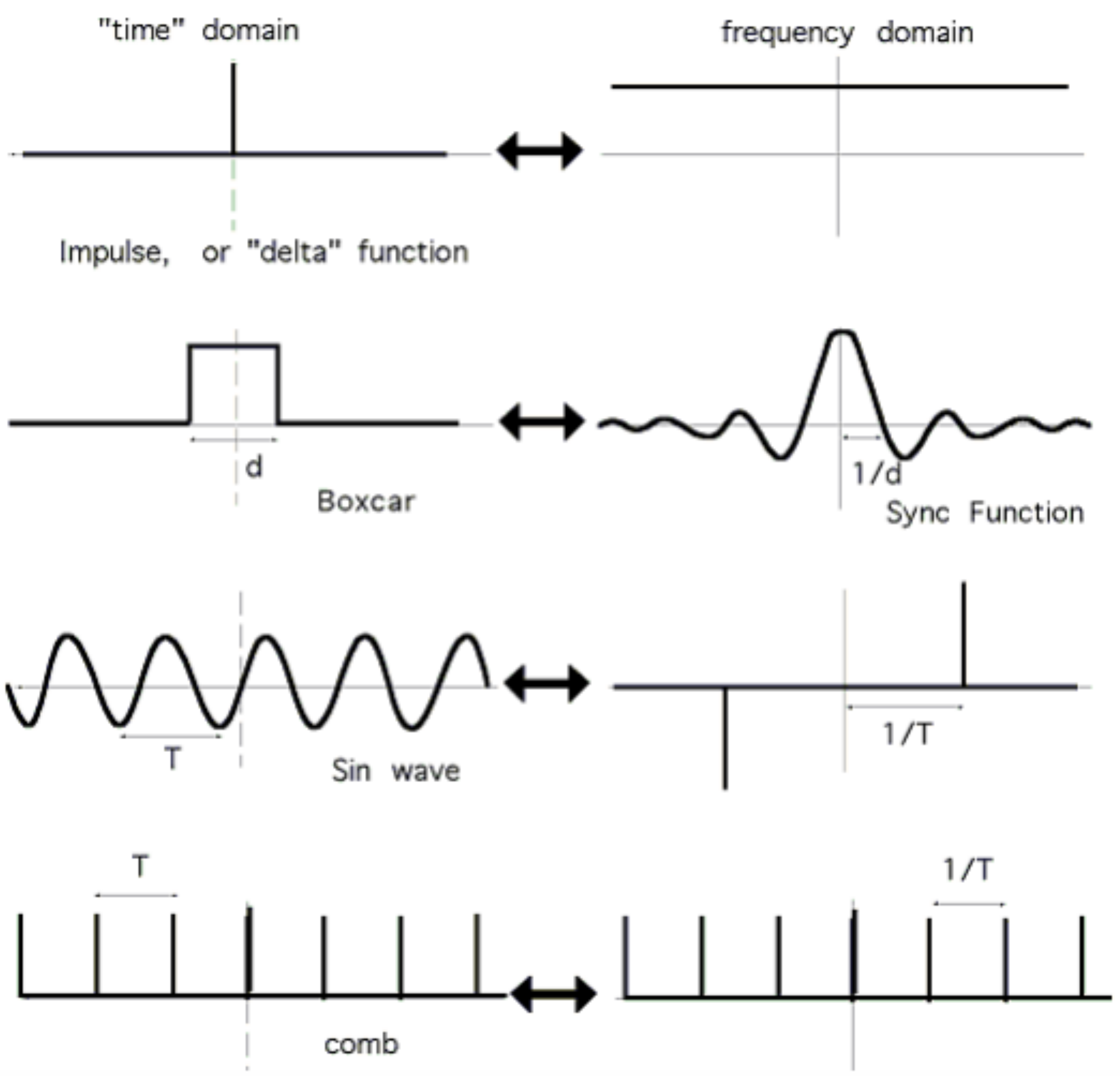
Basis & Fourier Transform

How complex or simple a structure is depends crucially upon the way we describe it. Most of the complex structures found in the world are enormously redundant, and we can use this redundancy to simplify their description. But to use it, to achieve the simplification, we must find the right representation.
(Herbert A. Simon, The Sciences of the Artificial, MIT Press, 1968)

The Fourier transform, named for Joseph Fourier, is a mathematical transform that expresses a mathematical function of time [... space ...] as a function of frequency. For instance, the transform of a musical chord made up of pure notes without overtones, expressed as loudness as a function of time, is a mathematical representation of the amplitudes and phases of the individual notes that make it up. The function of time is often called the time domain [... space domain ...] representation, and the function of frequency is called the frequency domain representation. The inverse Fourier transform expresses a frequency domain function in the time domain [... space domain ...]. Each value of the function is usually expressed as a complex number (called complex amplitude) that can be interpreted as an absolute value and a phase component [... not directly; absolute value and phase can be calculated from the real and imaginary components ...].

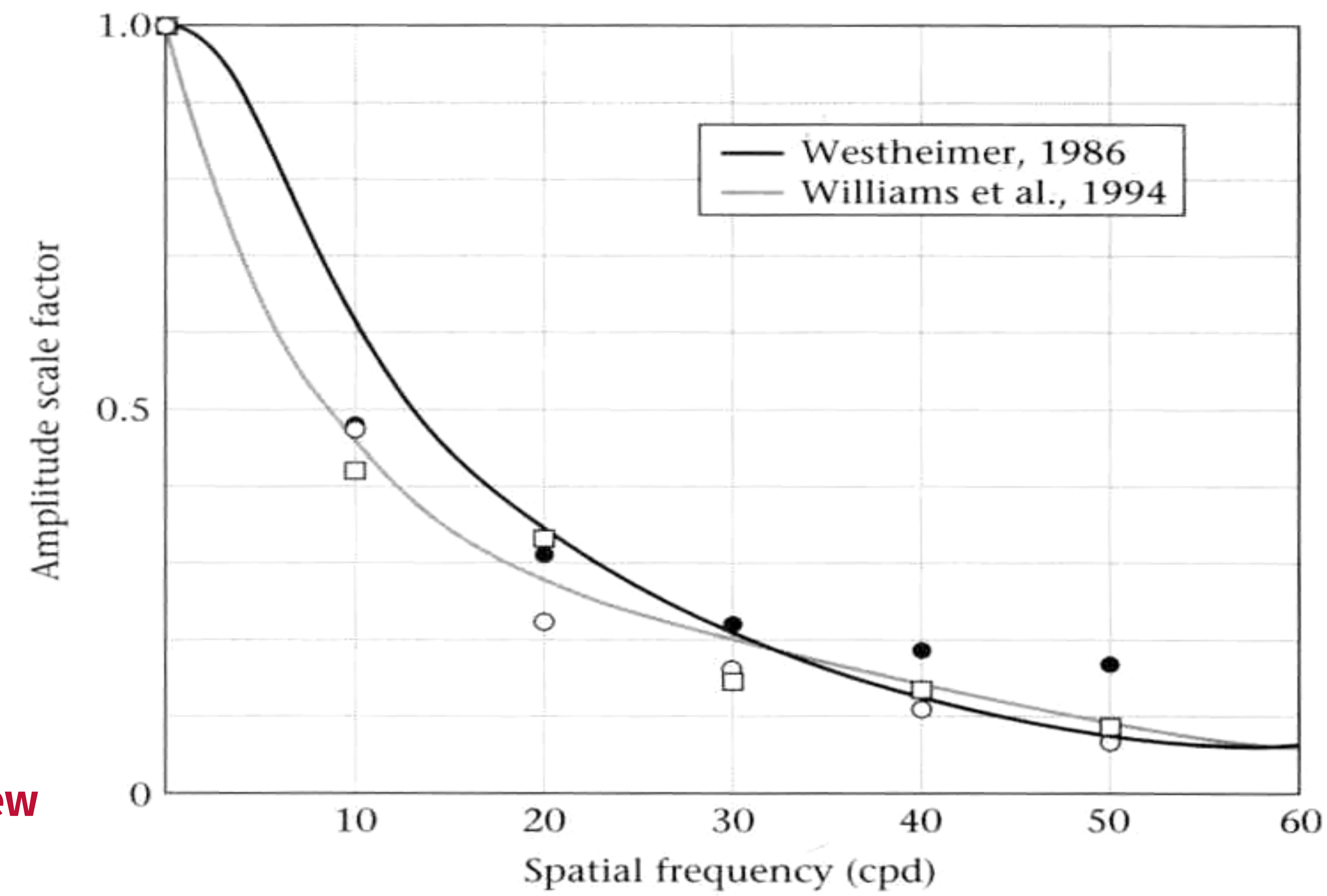
from: http://en.wikipedia.org/wiki/Fourier_transform

From wikipedia (Lucas Barbosa): https://en.wikipedia.org/wiki/File:Fourier_transform_time_and_frequency_domains_(small).gif



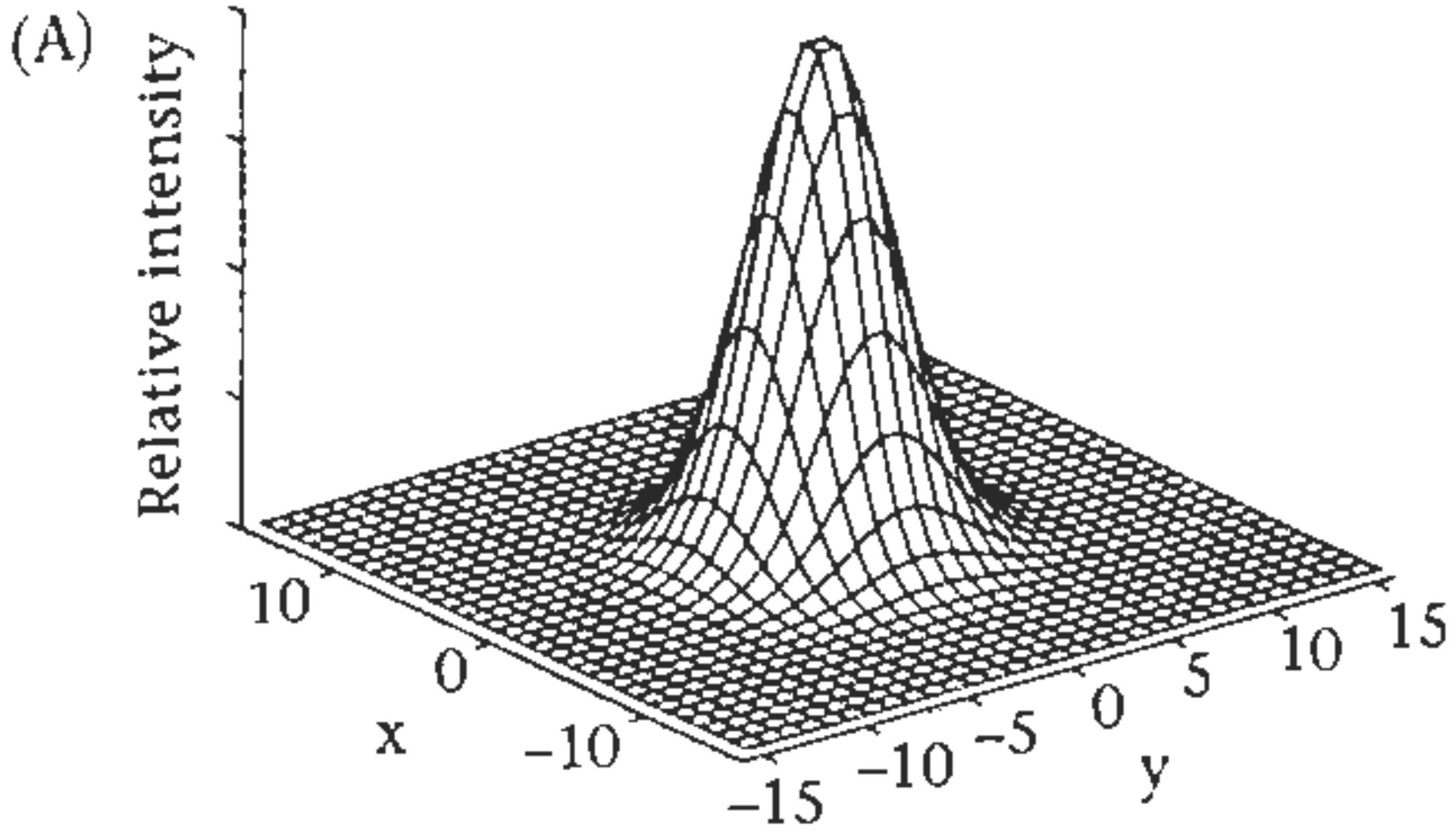
source: http://faculty.salina.k-state.edu/tim/mVision/ch11-freq-domain/fourier_transform.html

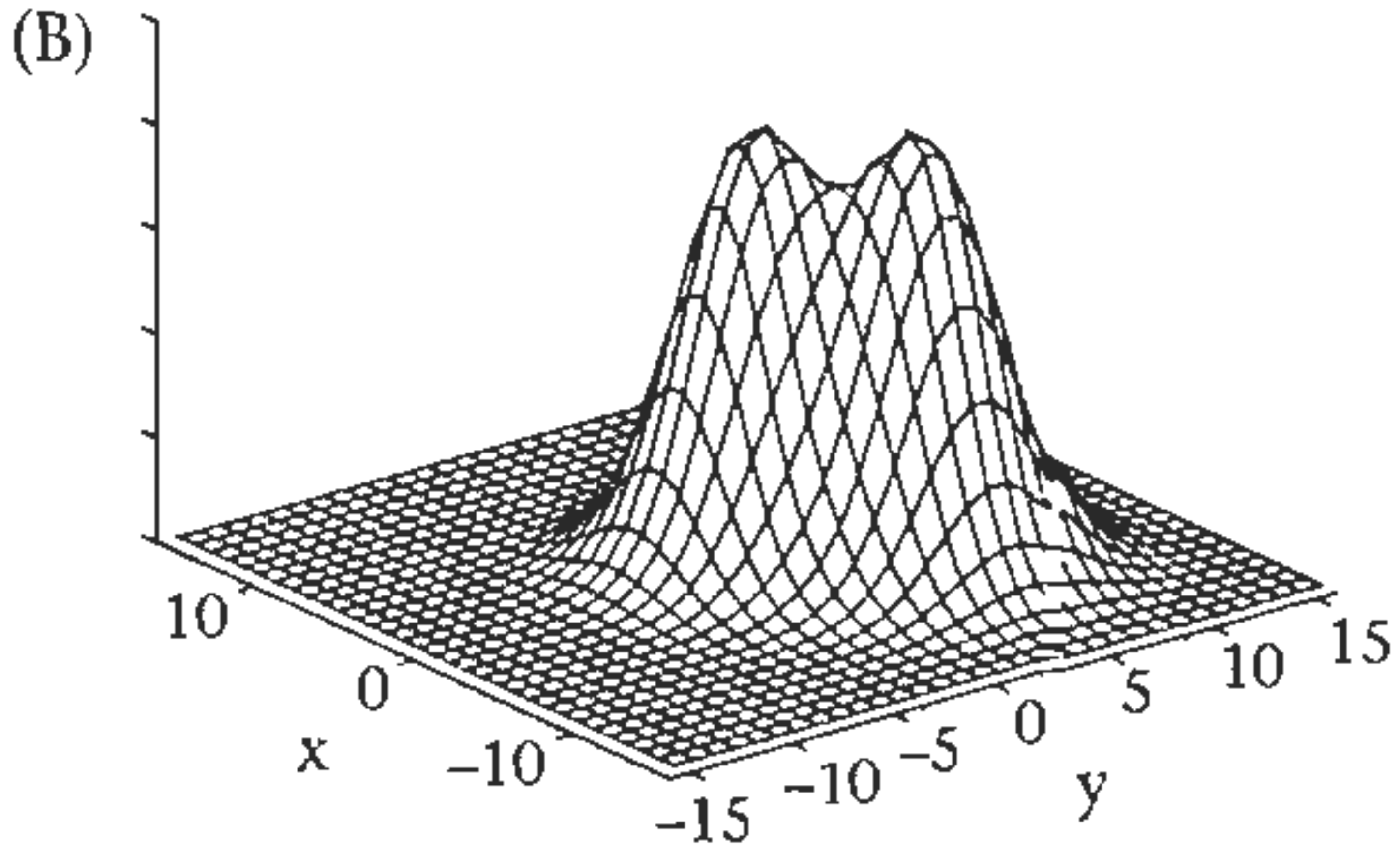
Modulation Transfer Function (MTF)

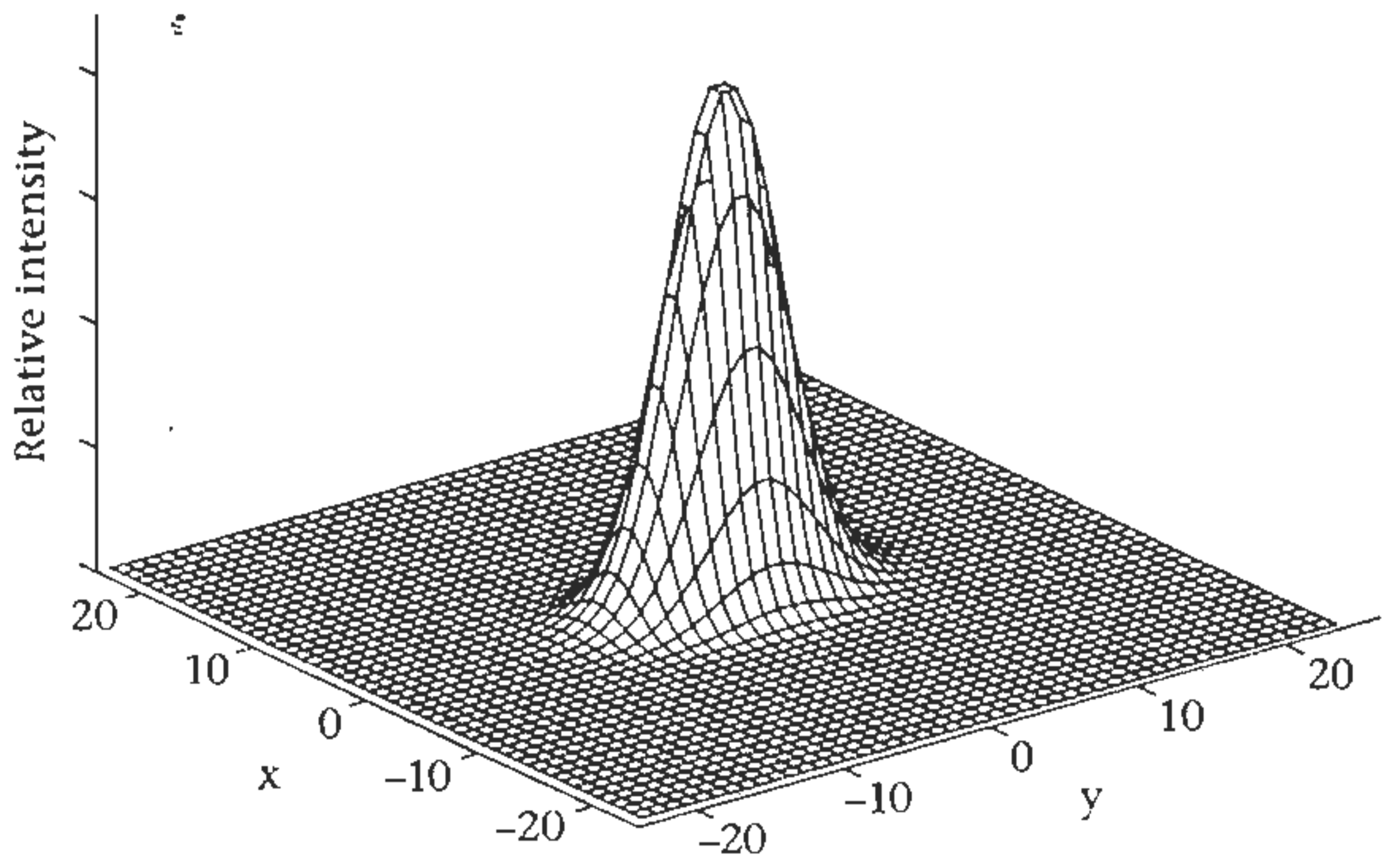


Sine-wave view of a linear system in the Fourier domain

Pointspread function & Astigmatism



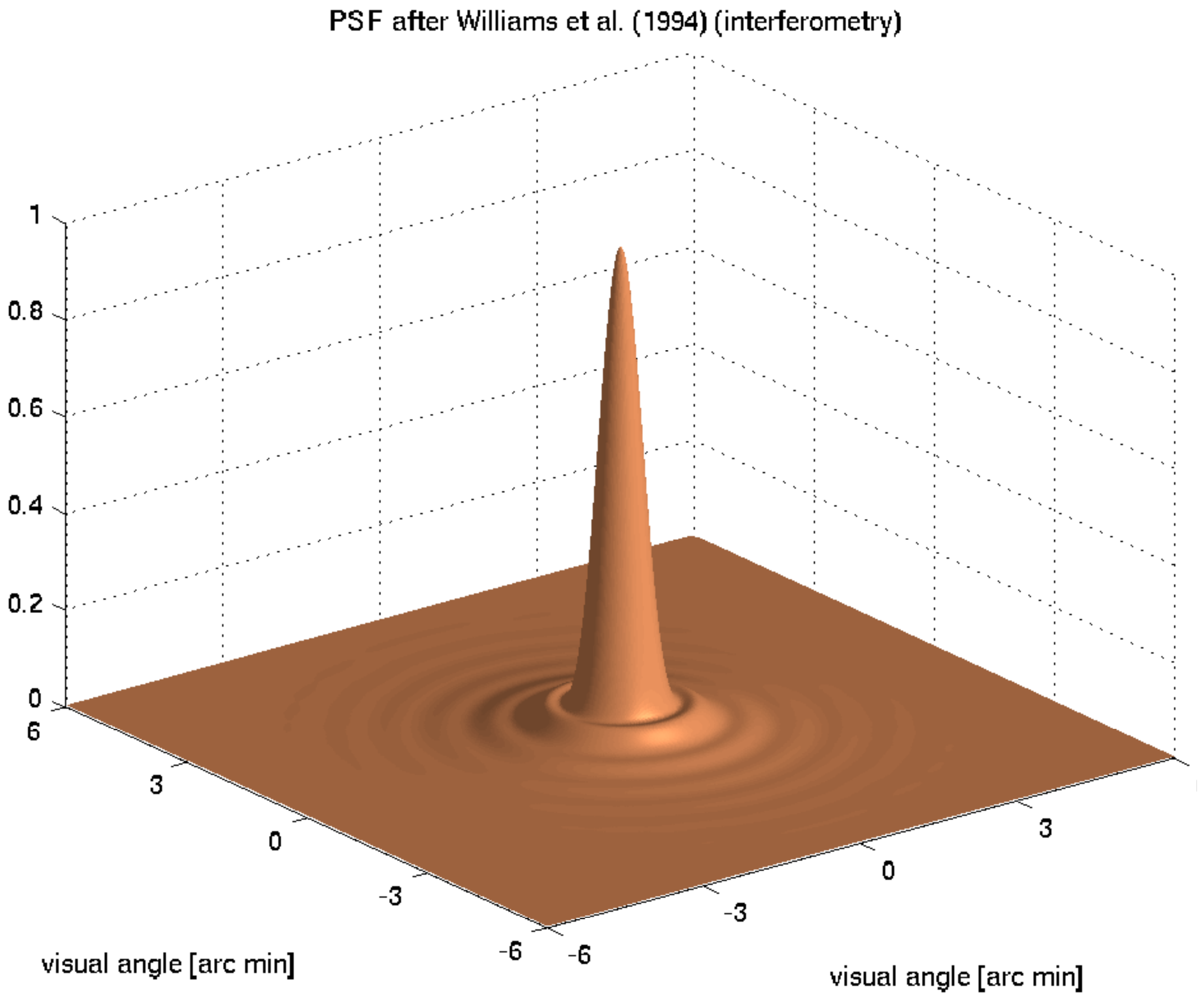


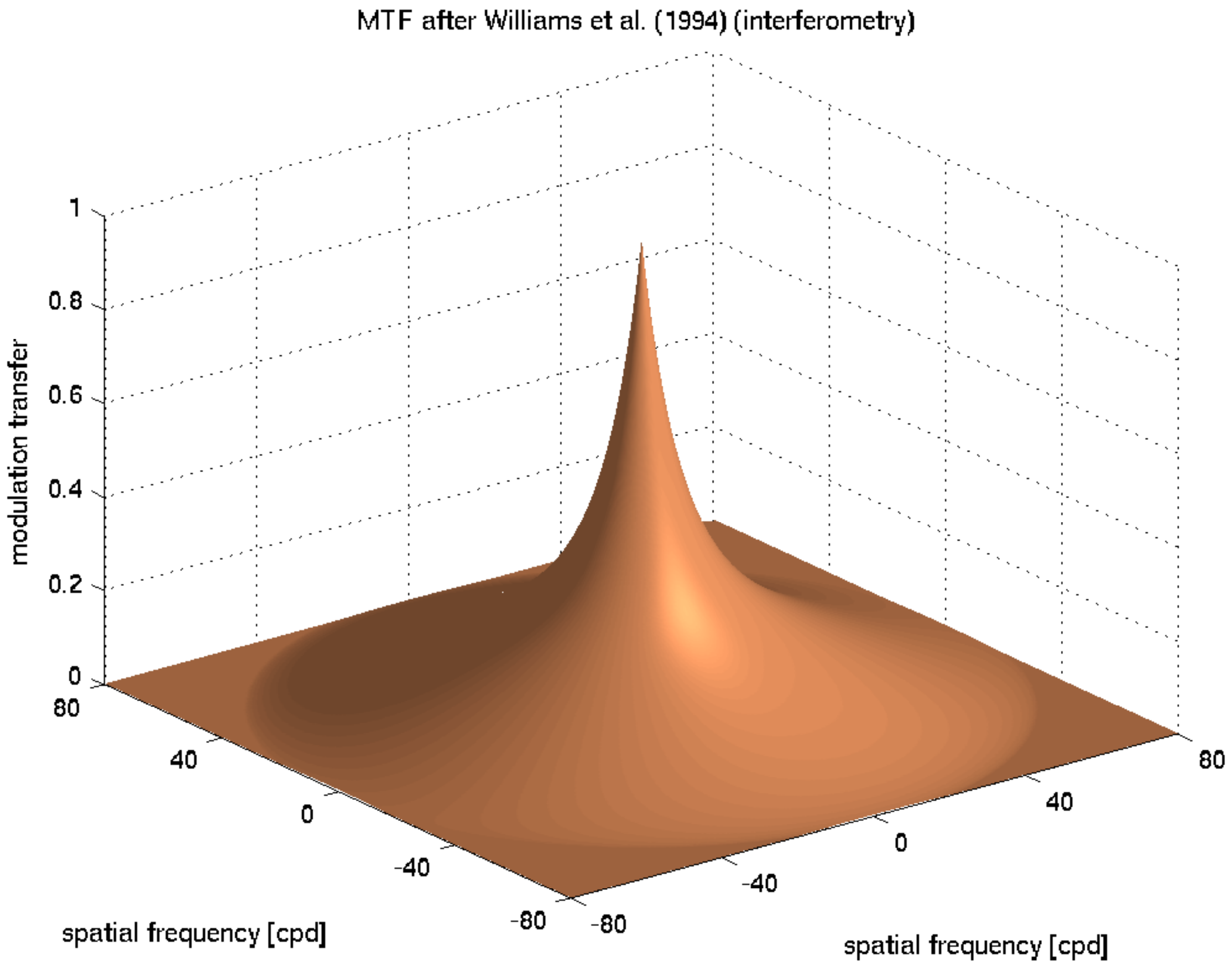


Human MTF

The modulation transfer function (MTF) is simply the Fourier transform of the LSF or the PSF (in 1-D or 2-D). It shows the ratio of the output-to-input amplitude at a given frequency (no phase information)

Because application of the Fourier transform simply represent exactly the same information in a different way, the MTF and LSF/PSF are two ways of viewing the same thing: the optical quality of the eye





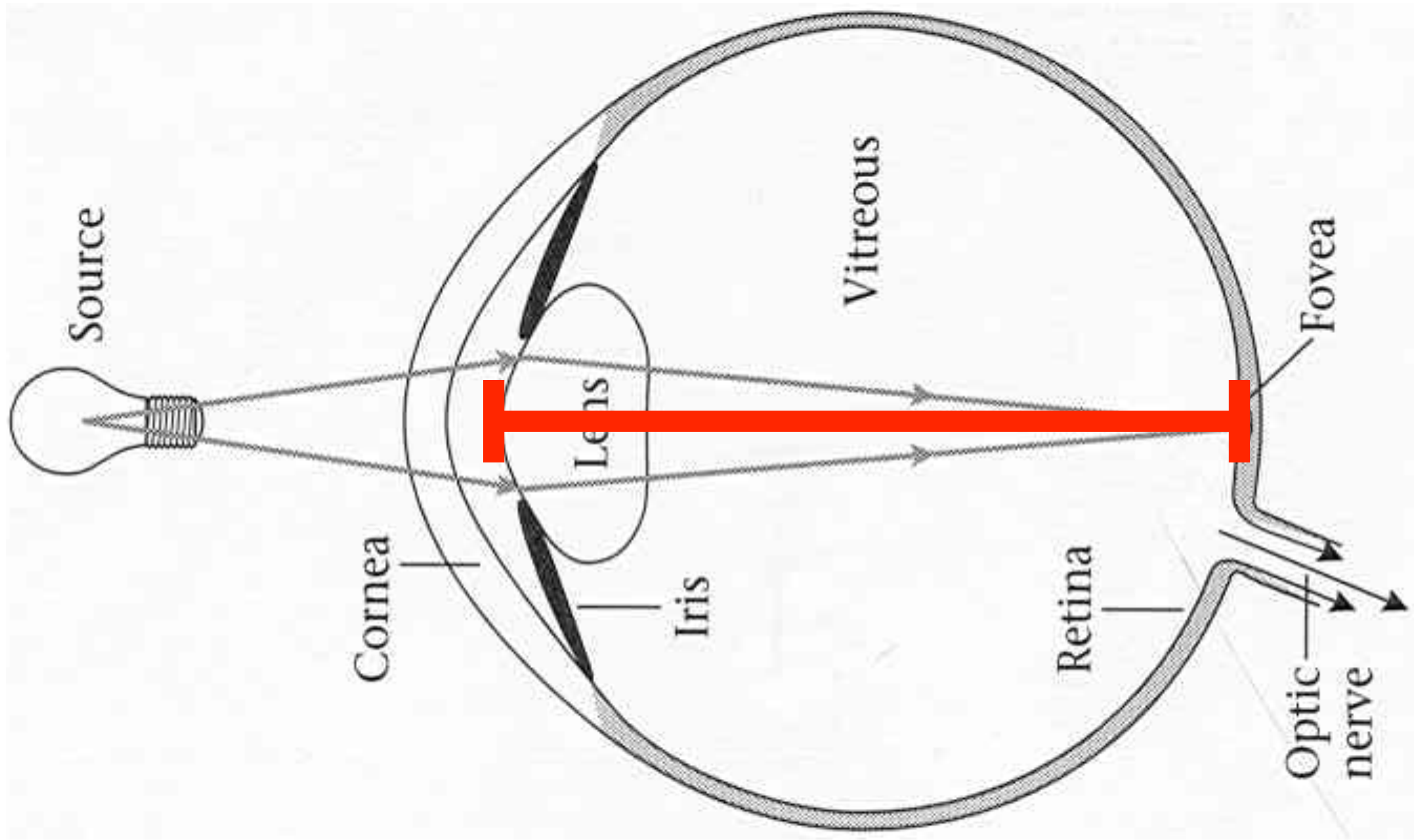
Depth of field & accommodation

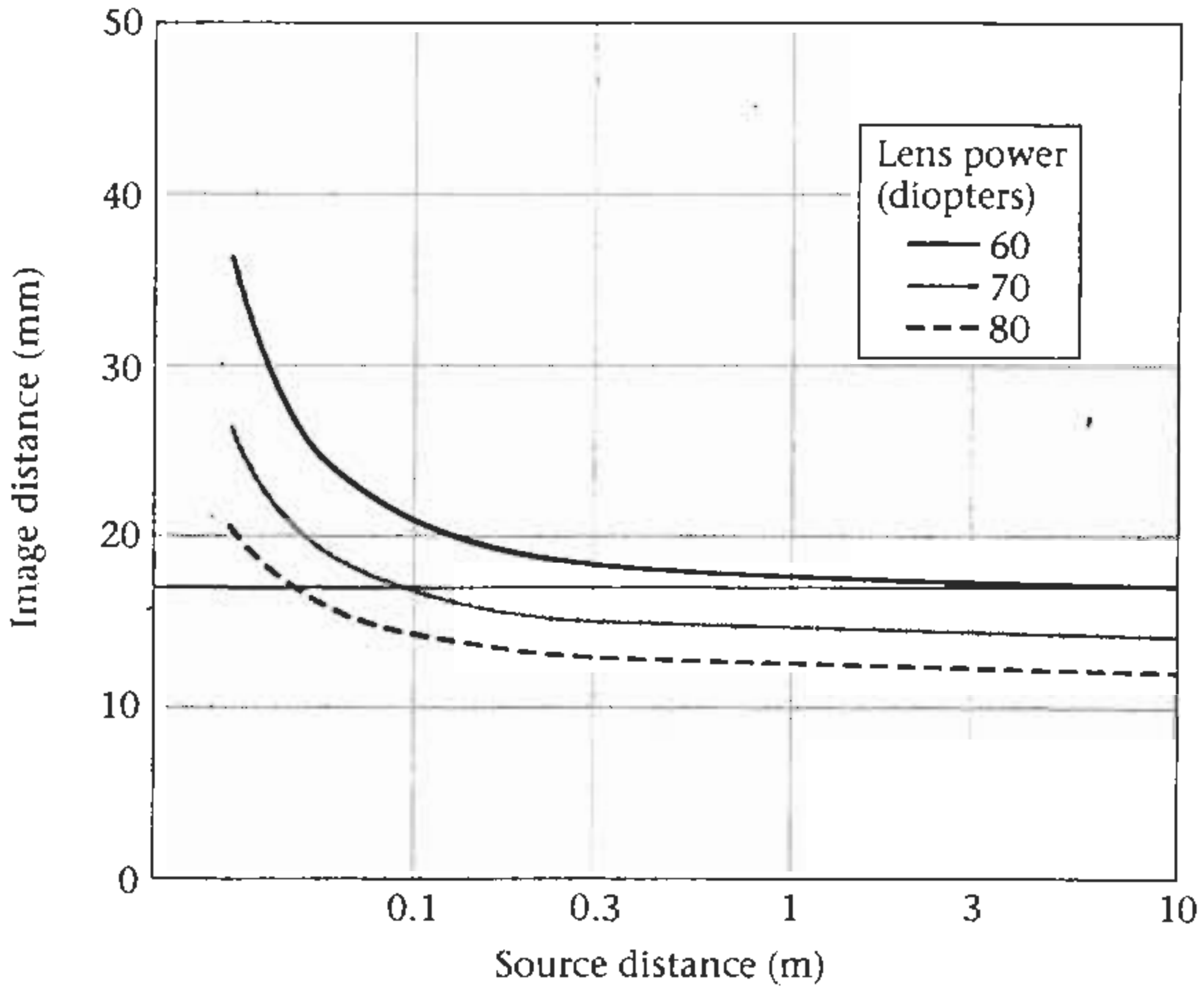
The optical power of a lens is a measure of how strongly a lens bends the incoming light (refraction)

Measured in **diopters**, which is the reciprocal of **focal length** in meters

The average human eye focusing a source at optical infinity onto the retina has a distance from mid-cornea to retina of 0.017 m.

1/0.017 = 58.8 diopters.

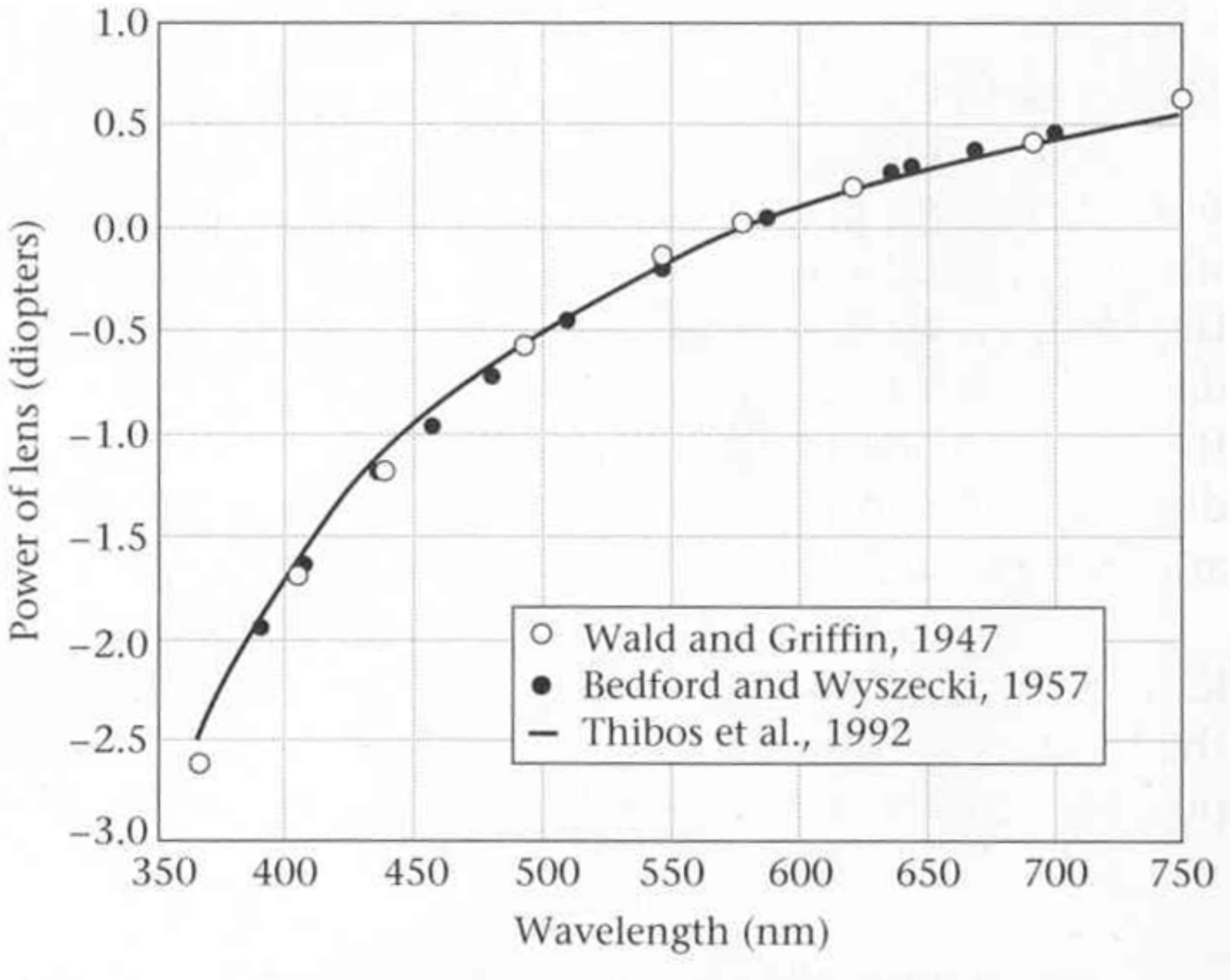




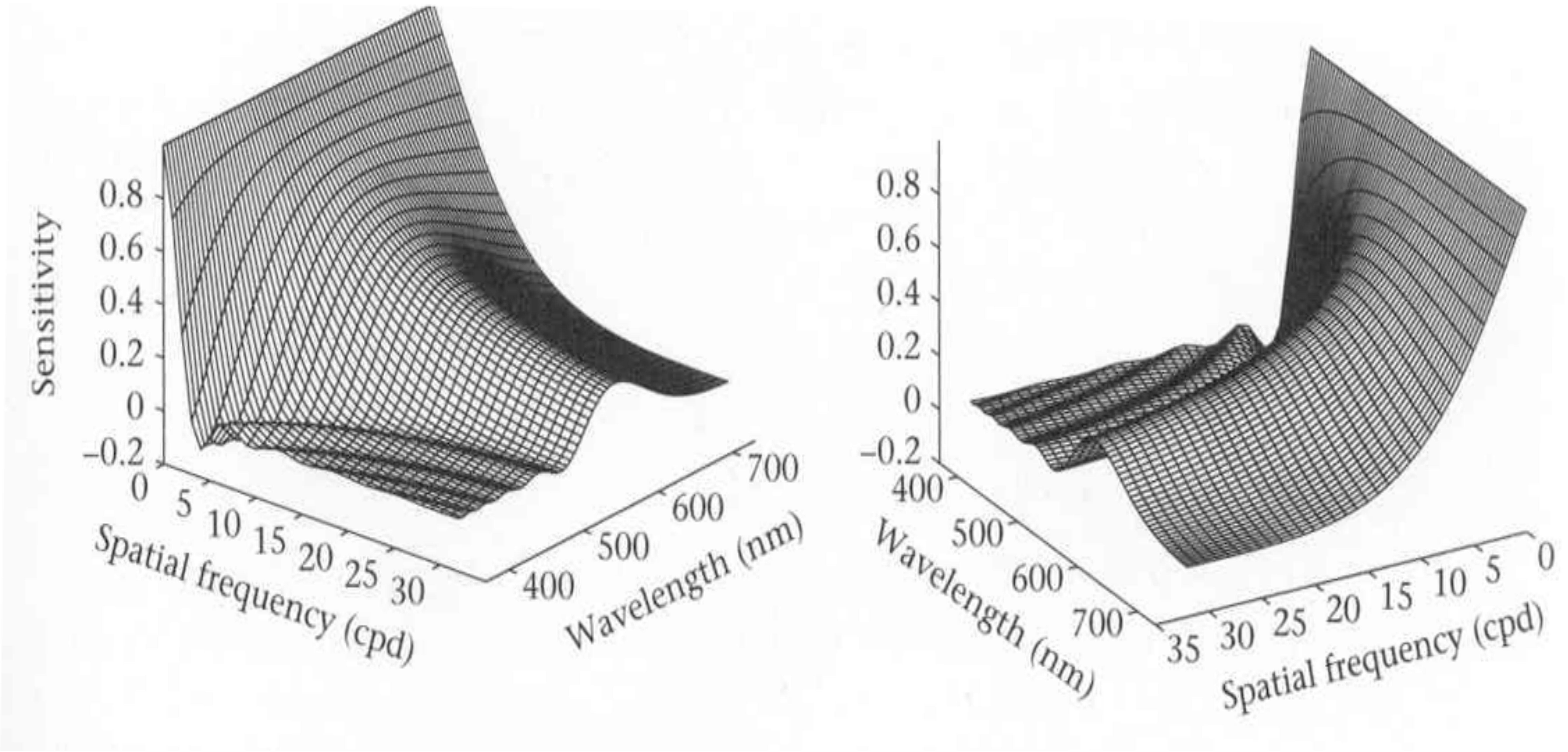
Chromatic Aberration of the Human Eye

Final source of image degradation: chromatic aberration.

Cornea and lens refract the light to focus on the retina. Refraction, however, is a function of the wavelength of light. Only one wavelength can be in focus at a time



Human MTF including the effects of chromatic aberration



Questions

Why are linear systems so important?

If optical systems were not linear—what would the implication be for glasses and contact lenses?

The End

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