Rendering Equation

$$L(x,\omega_o) = L_e(x,\omega_o) + \int_\Omega f_r(\omega_i, x, \omega_o)\, L_i(x,\omega_i)\, (\omega_i \cdot n)\, d\omega_i$$

x is the surface point we hit

ω_o is the vector from x to the source of the ray (e.g. camera)

→ $L(x,\omega_o)$ is the color we get from this ray (basically pixel color)

→ $L(x,\omega_o)$ is the visible surface radiance

$L_e(x,\omega_o)$ is the emitted radiance of point x in direction ω_o

→ if x is not a light source, $L_e(x,\omega_o) = 0$

$\int_\Omega \dots d\omega_i$ means that we shoot rays from x in all directions over hemisphere

ω_i is the vector from x in one of those directions

$L_i(x,\omega_i)$ describes the incoming radiance at point x from direction ω_i

→ this is a recursive call of the rendering equation

n is the surface normal at point x

$\omega_i \cdot n$ is the dot product of the surface normal and the incoming ray

→ this is the cosine shading angle $\cos\theta_i$

→ it weights the reflectance depending on incoming angle

$f_r(\omega_i, x, \omega_o)$ is the BRDF and describes the reflectance properties of the material at point x

→ it describes how much of the incoming light from ω_i is reflected in the direction ω_o

So what is described by the rendering Equation?

The radiance of a point x on a surface in an outgoing direction ω_o.

How does it describe global Light transport?

Every point that is illuminated and reflects light indirectly illuminates all other surface points in the scene. By evaluating the whole rendering equation this simulates global light transport.

What can the rendering equation express?

basically the rendering equation only expresses reflectance on opaque surfaces

- specular, diffuse and glossy reflectance
- polarisation (more complex but possible)
- reflection into incoming direction (principle that outgoing light at point x somewhere is incoming light at point y) not sure about this!

Radiometric Quantities

radiance: power (flux) traveling from some point x into direction ω_o

$$L(x,\omega) = \frac{d^2\Phi}{dA \cos\theta \, d\omega} \left[\frac{W}{m^2\,sr}\right]$$

$d^2\Phi$ = differential power

$dA \cos\theta$ = differential area

$d\omega$ = solid angle

irradiance: total power per unit area incident at point x

$$E = \int_{\Omega^+} L_i(x,\omega) \cos\theta \, d\omega \left[\frac{W}{m^2}\right]$$

L_i = incoming radiance

radiocity: outgoing total power per unit area of point x

$$B = \int_{\Omega^+} L_o(x,\omega) \cos\theta \, d\omega \left[\frac{W}{m^2}\right]$$

L_o = outgoing radiance

=> Light Transport in a scene provides dynamic energy equilibrium

-> emitted photons = absorbed photons + escaping photons

Why is radiance important to ray tracing?

radiance is the quantity we need to propagate along the ray

Simplification of the Rendering Equation to approximate global light transport

-> Assumption: no directional dependence of reflection points

- every surface is diffuse Lambertian and scatters light equally into each direction

-> Simplification:

- replace radiance by radiocity $B(x)$
- reflectance $\rho(x)$ instead of BRDF

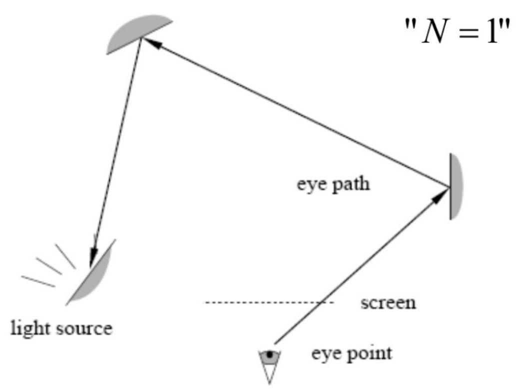
=> Radiosity Equation: $B(x) = B_e + \rho(x) E(x)$

Path Tracing

Monte Carlo Approximation

estimating the integral of the rendering equation by shooting uniformly distributed random sample rays (discretising the integral)

Simple Path Tracing



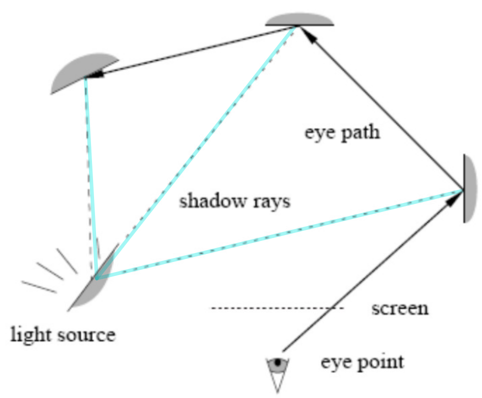
start at camera and shoot towards the scene

at intersection point shoot randomly distributed rays into the scene; for each do the same recursively

-> worst case: no ray ever hits a light source

= black image

Improvement: next event estimate

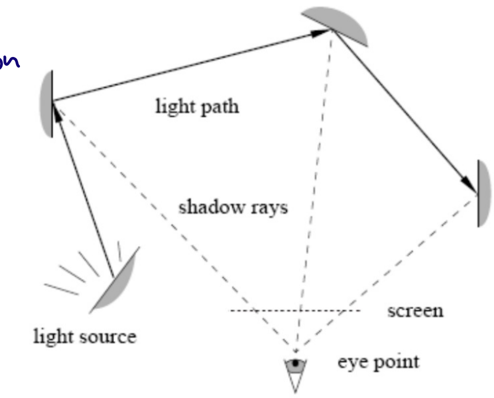


at each intersection first shoot rays towards light sources to evaluate if the point is illuminated

Light Tracing

start at the Light source into random direction

→ next event estimation is also suitable here



Path Tracing vs. Light Tracing

Performance depends on image size vs. scene size, nature of Light transport paths, number of Light sources

BRDF (Bidirectional Reflectance Distribution Function)

tells how much Light will be reflected at a surface point

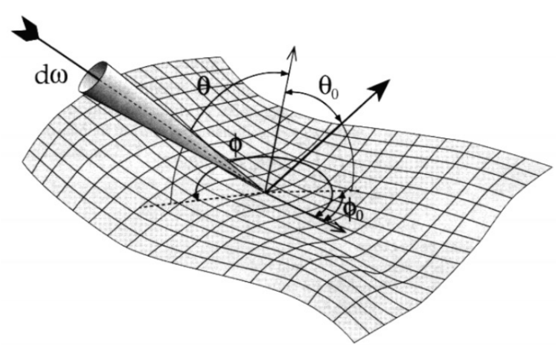
The BRDF defines

- color
- absorption
- specular roughness
- specular intensity / color

BRDF Properties

$$f_r(\omega_i, x, \omega_o) = \frac{dL_o(x,\omega_o)}{dE_i(x,\omega_i)}$$

$$= \frac{dL_o(x,\omega_o)}{dL_i(x,\omega_i)\cos\theta\, d\omega_i} \left[\frac{1}{sr}\right]$$



- bidirectional
- surface reflection for Light incident from direction (θ_i, φ_i) observed from direction (θ_o, φ_o)
- isotropic: no surface orientation we have to account for
- Helmholz reciprocity principle: $f_r(\omega_i, \omega_o) = f_r(\omega_o, \omega_i)$
- value from 0 to ∞ → 0 = total absorption
  → ∞ = perfect mirror

physically valid

- energy conservation law: $\int_{\Omega^+} f_r(\omega_i, x, \omega_o) \cos\theta \, d\omega_i \leq 1$
- NO subsurface scattering, transmission, refraction

  → only reflection at point x

Reflectance

varies with

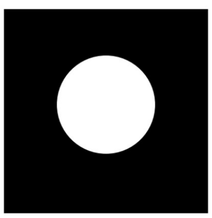
- illumination angle
- viewing angle
- wavelength
- polarisation

caused by

- absorption
- surface micro geometry
- refraction index
- scattering

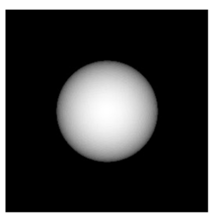
BRDF models combination of diffuse + mirror + glossy reflection

Lambertian light source vs. reflectance



diffuse light source

→ equally bright at each point

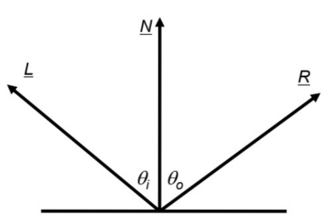


diffuse object (illuminated)

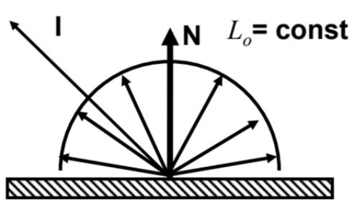
→ falloff where surface normal doesn't point directly towards the light / camera (because of cos θ)

Simple BRDF Models

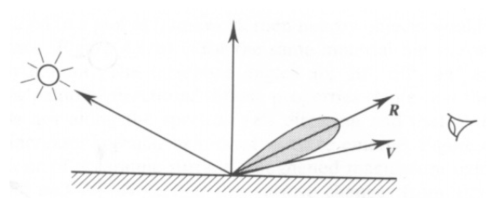
Mirror BRDF



Diffuse Reflection BRDF



(glossy) Phong Reflection BRDF



Further BRDF Models

- Blinn - Phong - Model
- Microfacet Model → Ward - Model, Cook - Torrance - Model, Beckman - Model

BRDF covered Phenomena

- opaque reflection (diffuse, glossy, specular)
- wavelength dependent
- polarisation (very complex)

→ abilities of the rendering equation depend on this

NOT covered phenomena

- flouroscence
- transparency
- subsurface scattering

BRDF covers opaque surfaces only

→ BSDF (Bidrectional Scatter Distribution Function) for glass

↳ $\cos\Theta$ in RE not suitable for fraction

→ subsurface scattering needs $\int_{\Omega} \int_{S}$

↳ more complex

Ω incoming rays

S outgoing rays