University of Tübingen
Prof. Dr.-Ing. Hendrik P.A. Lensch
Chair of Computer Graphics
Faezeh Zakeri (faezeh-sadat.zakeri@uni-tuebingen.de)
Zabiha Kehribar (zabiha.kehribar@student.uni-tuebingen.de)

22 February 2022

Graphic Data Processing
Example Exam WS 21/22

⇒ **Please read the instructions on the next page carefully!** ⇐

Approximate time for solving this exam: 60 minutes
(The final exam will be targeted on 120 minutes!)

You may use the first column ("done") to check which exercises you completed. Please leave the last column blank.

exercise	done	max. points	points gained
1		13.5	
2		3	
3		4	
4		5	
5		10	
6		5	
7		7	
8		8	
Sum		**55.5**	

Some instructions which will also be valid for the final exam:

- No tools, books, and lecture notes are allowed for this exam.
- You are allowed to use a ruler and a simple calculator.
- For your answers use the empty space below the exercises and the back sides.
- Be careful to write in a readable way! For things we cannot read we can't give any points.
- Answer only the questions that are posed.
- If you are asked to calculate something, write down every step of your calculation.
- Always write short and precise answers and don't spend too much time on any specific question.
- The amount of points you can get in an exercise roughly corresponds to the relative difficulty.
- The answers can be given either in English or in German.
- Rule for multiple choice: There are no negative points as a penalty for wrong answers.

Good Luck!

1 Spatial Acceleration Structures (13.5 Points)

Ray tracing an unorganized polygon soup is rather slow. Describe three different spatial acceleration structures and explain their benefits and drawbacks with regard to traversal complexity, building time and space requirements. (3-4 sentences for each).

- Grid
  - Easily constructed, trivial insertion
  - Iterate through voxels, pierced by ray (rather costly)
  - Naive, high memory costs (a^k?)
- Octree / Quadtree
  - Similar as grid, but sort of adaptive
  - Theoretically optimal
  - One-time build-up $\in \mathcal{O}(n)$
  - Traversal $\in \mathcal{O}(k \log n)$
  - Split at points
- kd-Tree
  - Construct $\in \mathcal{O}(n(k + \log n))$
  - Traversal $\in \mathcal{O}(n^{1-\frac{1}{k}} + a)$ with (a result size)
  - Memory $\in \mathcal{O}(kn)$
  - Split at hyper-planes
  - Can be balanced with median-split
  - Special form of BSP (is axis aligned)

2 Rendering Equation (3 Points)

Complete the formula of Rendering Equation for surface models:

$$L(x, \omega_o) = \underbrace{L_e(x, \omega_o)}_{\text{emitted radiance}} + \int_\Omega \underbrace{f(x, \omega_o, \omega_i)}_{\text{BRDF}} \cdot \underbrace{L_i(x, \omega_i)}_{\text{incoming radiance}} \cdot \underbrace{\omega_i \cdot n}_{\text{cosine shading}} \cdot \underbrace{d\omega_i}_{\text{integral over } \Omega}$$

Explain briefly the different factors and terms in this equation.

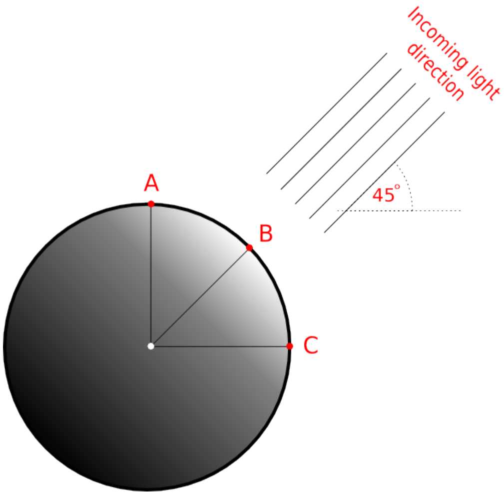
3 BRDF (4 points)

Which of these properties does a BRDF (Bidirectional Reflectance Distribution Function) need to fulfill to be physically valid:

	true	false
conservation of energy	X	
wavelength limited		X
Helmholtz reciprocity	X	
can be negative		X
the unit is $\frac{1}{m^2}$		X
the unit is $\frac{1}{sr}$	X	
the diffuse BRDF is constant	X	
$\int_\Omega f_r(\omega_i, x, \omega) \cos\theta_i \; d\omega_i \leq 1$	X	

4 Shading with a Directional Light Source (5 points)

Compute the irradiance of a directional light source on a sphere for three points A, B and C, with the incoming radiosity (from a surface perpendicular to the light direction) being L_s.



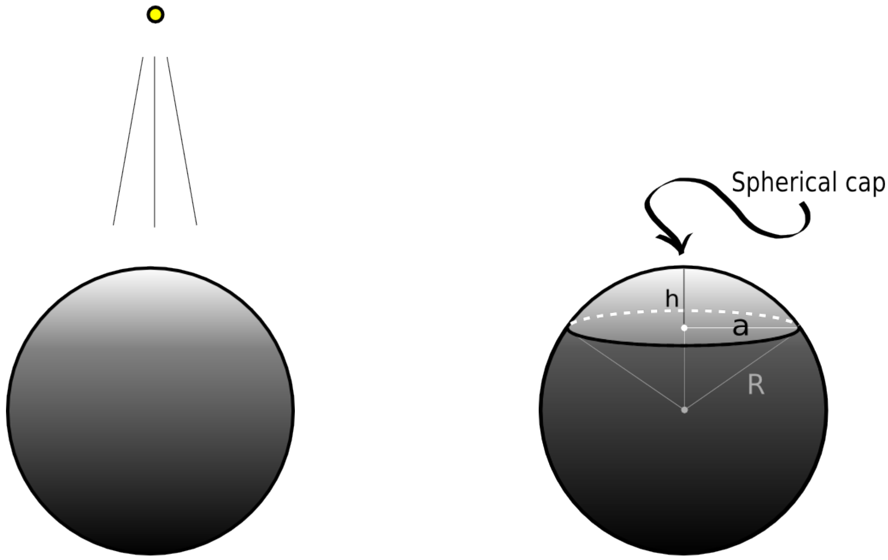
$$L(x, \omega_o) = L_e(x, \omega_o) + \int_\Omega f(x, \omega_o, \omega_i) \cdot L_i(x, \omega_i) \cdot \omega_i \cdot n \cdot d\omega_i$$

$$E_{A,B,C} = \int_\Omega L_s \cdot \cos\omega \cdot d\omega$$

$$E_{A,C} = L_s \cdot \frac{\sqrt{2}}{2}$$

$$E_B = L_s \cdot 1$$

5 Shading with a Point Light Source (10 points)



A point light source illuminates a sphere with radius r, as illustrated in the picture. The distance of the light source to the center of the sphere is d. Given the light source power ϕ_s compute the total radiant power incident on the sphere Note:

- Do not compute any integral.
- Pythagoras says: $a^2 + b^2 = c^2$ for a rectangular triangle.
- The height h of a rectangular triangle can then be computed by $h \cdot c = a \cdot b$.
- The area of a spherical cap is given by $S = \pi(a^2 + h^2)$.

Compute sphere cap around light source:

$L = L_S \frac{A_{cap}}{A_l}$ $\quad A_{cap} = \pi(a^2 + h^2)$

$A_l = 4\pi l^2$

Compute l and derive angles:

$l = \sqrt{d^2 - r^2}$ $\quad \sin\alpha = \frac{r}{d}$

$\sin\alpha = \frac{a}{l}$

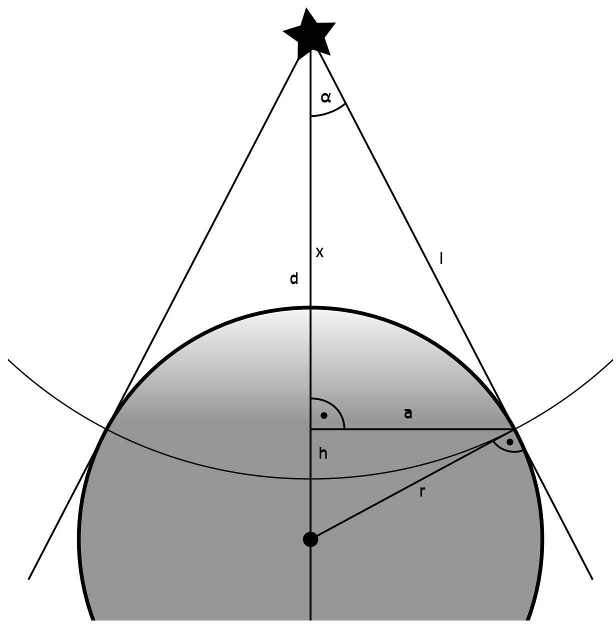
Therefore:

$\frac{r}{d} = \frac{a}{l}$ $\quad a = \frac{rl}{d}$

$x = \sqrt{l^2 - a^2}$ $\quad h = l - x$

All together:

$$
\begin{aligned}
L &= \frac{\cancel{\pi}(a^2 + h^2)}{4\cancel{\pi}l^2} \\
&= \ldots \\
&= L_s \frac{|d|\sqrt{d-r}\sqrt{r+d} + d^2}{2d^2}.
\end{aligned}
$$



6 Sampling theory / Bayer Pattern (5 points)

Most affordable color cameras use a color filter array in front of the image sensor so that each sensor pixel captures only one of the three primary colors. The most popular pattern is the tiling by Bayer (1975), which is depicted below.

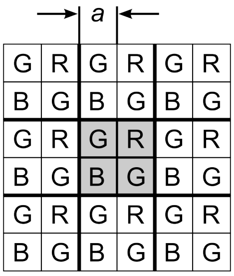


Figure 1: Bayer pattern. We assume square pixels of length a.

Using a sensor with pixel size a and Bayer color filter, what is the highest spatial frequency for each color channel individually that can be captured without aliasing (in every direction)?

A signal with f_{max} can be reconstructed with sample distance $\tau < \frac{1}{2f_{max}}$.

$$2a < \frac{1}{2f_{max}}$$
$$4a < \frac{1}{f_{max}}$$
$$\frac{1}{4a} > f_{max}$$

Is it the same for red, green and blue?

For red and blue yes.
Green is theoretically $\frac{1}{4a} > f_{max}$ but due to per-pixel averaging, the signal will be blurred.

7 Affine Transformations (4+3 points)

a) Affine Invariance: Explain what affine invariance means and give a simple example.

Affine transformations (affinities) preserve collinearity:
Preserve parallel lines, ratios of parallel lines.
Invariance can mean, that a transformation is affine.
Transformations that are affine: Rotation, (Non) Uniform Scaling, Shearing, Translation.

b) Perspective Transformation: Is the perspective projection an affine transformation (explain)?

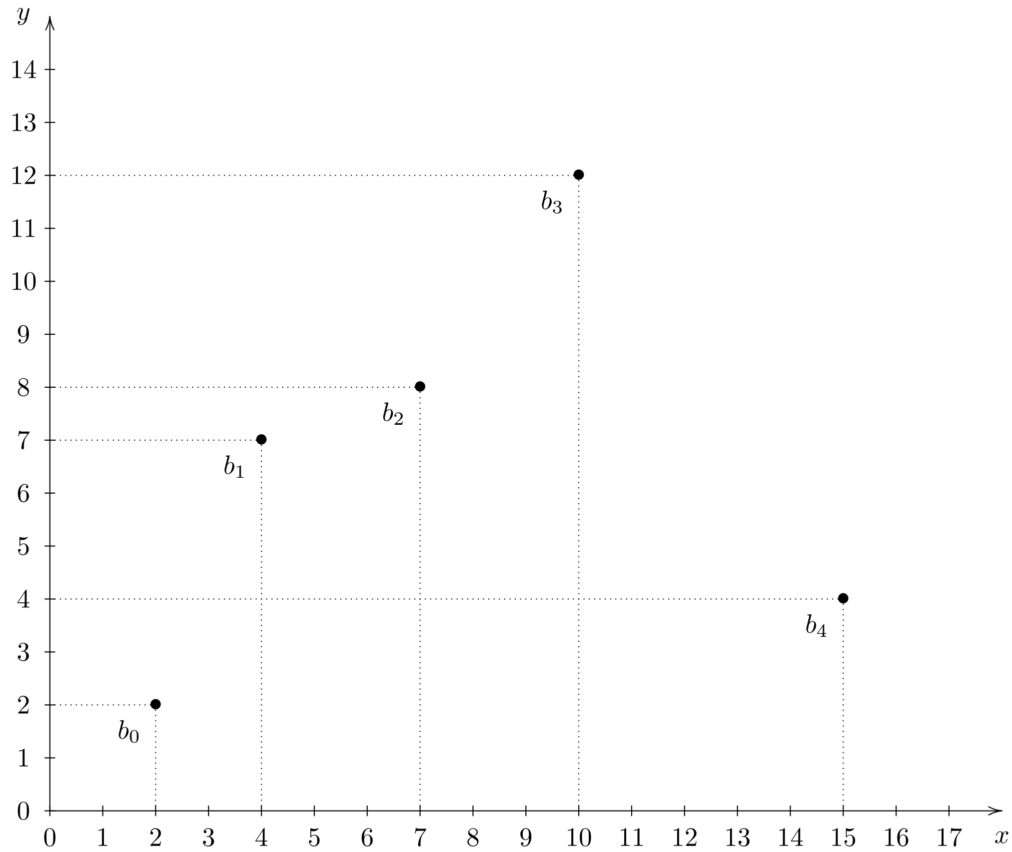
No, see railroad going towards the horizon.

8 Splines (3+5 points)

a) How many times can a spline of degree n be differentiated? Assume spline segments to be polinomials of degree n and the joints between the segments to be as smooth as possible. Explain!

Degree $n \to n-1$-times differentiate-able

b) Given five control points $b_0, \ldots b_4$ of a Bezier curve $P(t)$ with degree 4, $t \in [0,1]$, as depicted below. Apply the *deCasteljau* algorithm graphically and numerically to compute the point $P_{0.5} = P(t = 0.5)$ on the curve.



- $b_0(2,2)$, $b_1(4,7)$, $b_2(7,8)$, $b_3(10,12)$, $b_4(15,4)$

$b_0(2,2)$

$b_{01}(3, 4.5)$

$b_1(4,7)$ $\quad$ $b_{012}(4.25, 6)$

$b_{12}(5.5, 7.5)$ $\quad$ $b_{0123}(5.625, 7.375)$

$b_2(7,8)$ $\quad$ $b_{123}(7, 8.75)$ $\quad$ $b_{12345}(7.1875, 8.125)$

$b_{23}(8.5, 10)$ $\quad$ $b_{1234}(8.75, 8.875)$

$b_3(10,12)$ $\quad$ $b_{234}(10.5, 9)$

$b_{35}(12.5, 8)$

$b_4(15,4)$