Nyquist (Aliasing)

- **Nyquist Frequency:** sampling a signal with a sample rate f_{sample}, aliasing occurs if the signal has frequencys higher than $\frac{1}{2} f_{sample}$

$$f_{Nyquist} = \frac{1}{2} f_{sample}$$

The Nyquist Frequency is the highest signal frequency that can be correctly reconstructet (without aliasing) with a sampling rate f_{sample}.

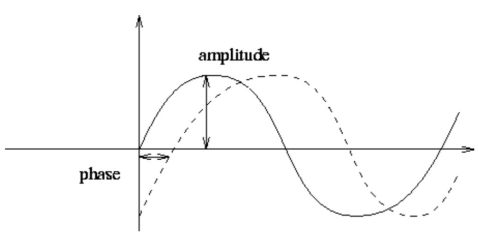
- **Nyquist Rate:** the required sample rate to reconstruct a given signal correctly has to be greater than two times the max. frequency of the signal. (Nyquist-Shannon-Theorem) $f_{n\text{-}rate} > 2 \cdot f_{max}$

Fourier Transform

A periodic, continous function can be expressed as sum of a number of sine and cosine waves.

$$\mathcal{F}(u) = \int_{-\infty}^{\infty} f(x) \cdot e^{-2\pi i u x} \, dx$$

Amplitude and Phase



Amplitude after FT is the real part

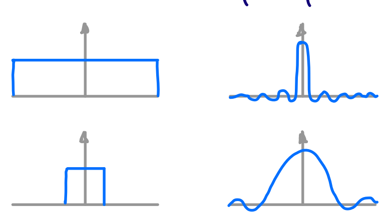
Phase after FT is the imaginary part

→ Phase is important for reconstruction!

Fourier Spectra

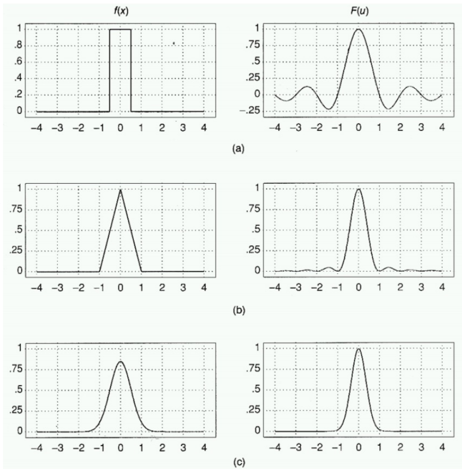
Transform behaviour: wide in spatial domain ↔ narrow in frequency domain

→ and vice versa

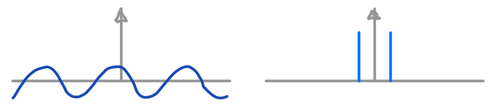


Basic 1D-Transforms

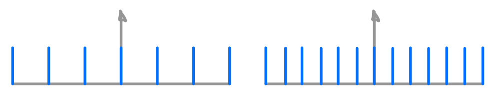
- box → sinc
- triangle → sinc²
- gauss → gauss



- sine → spike

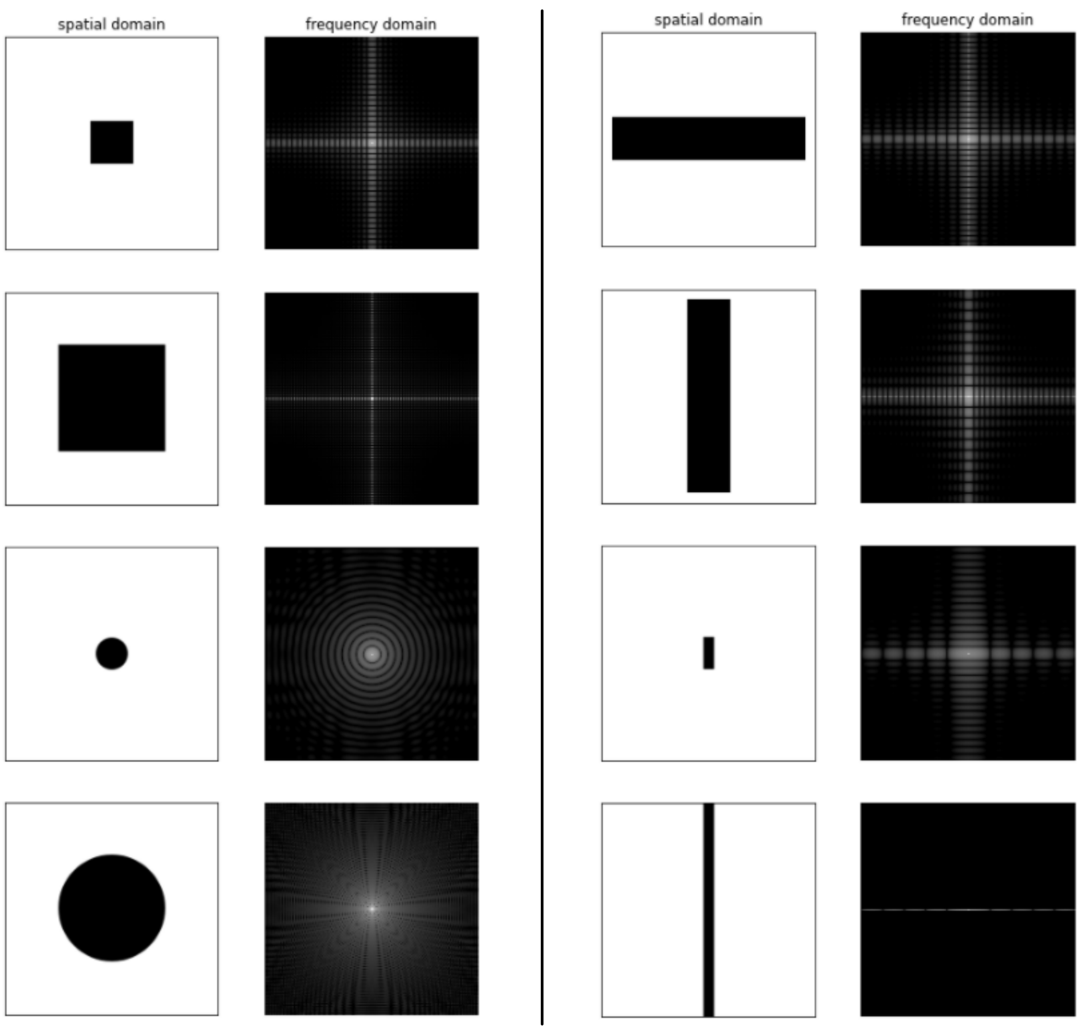


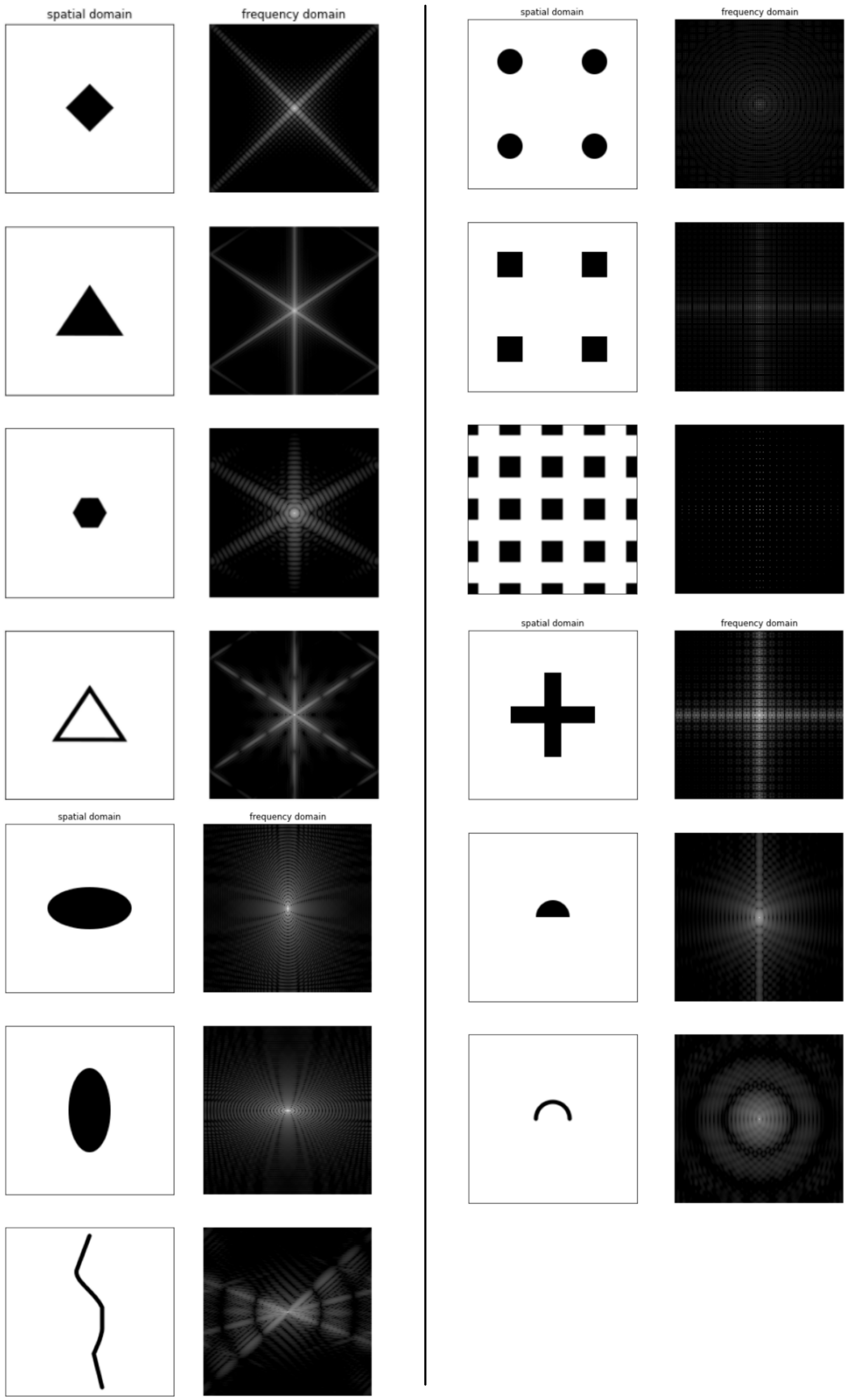
- comp → comp



Basic 2D-Transforms

an edge in spatial domain is an orthogonal line in frequency domain




spatial domain
frequency domain
spatial domain
frequency domain
spatial domain
frequency domain
spatial domain
frequency domain

Filtering (Image Processing)

Low-Pass-Filter

frequencies higher than a threshold are ignored

→ multiplication with box function in frequency

→ blurres image (Gauss-Filter)

High-Pass-Filter

frequencies lower than a threshold are ignored

→ amplifies edges in images (Laplace Filter, Sobel)

Band-Pass-Filter

intermediate frequencies are extracted

Median Filter (Non-Linear)

selects value at position $\frac{n}{2}$

reduces noise by preserving edges

Bilateral Filter

convolution with two gaussians (spacial and value distance)

→ smoothes surfaces but preserves edges

Wavelet Transform

disadvantages of FT:
- single spot in spatial domain influences whole frequency spectrum
- loss of spatial information

Wavelet Representation as linear combination of basis function with varying frequency

Wavelet representation reduces need of detail coefficients (40-25%) to represent image

With higher level details are more sparse → removing those leads to slight smoothness

Transformation via downsampling (like Mip Mapping)

A-Trous-Wavelets

- combine idea of wavelets with bilateral filter
- create detail levels by increasing filter mask at each iteration, filling zero holes
- store detail levels and smoothed image of highest level
- for reconstruction weight each level different to remove noise or smooth surfaces