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FACULTY OF
SCIENCE
Computer Graphics

Computer Graphics (Graphische Datenverarbeitung)

- Ray Tracing I -

WS 2021/22

Hendrik Lensch

Overview

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- Last Lecture
  - Introduction
  - Math Primer
- Today
  - Ray Tracing I
    - Background
    - Basic ray tracing
    - What is possible
    - Recursive ray tracing algorithm
- Next Lecture
  - Ray Tracing II: Spatial indices

Background

What do we want, what do we need?

Photorealistic Rendering

- Long standing goal in computer graphics
- Ingredients:
  - Camera model
  - Scene model
  - Illumination model
  - Rendering algorithm

Camera Model

- Perspective

Camera Model

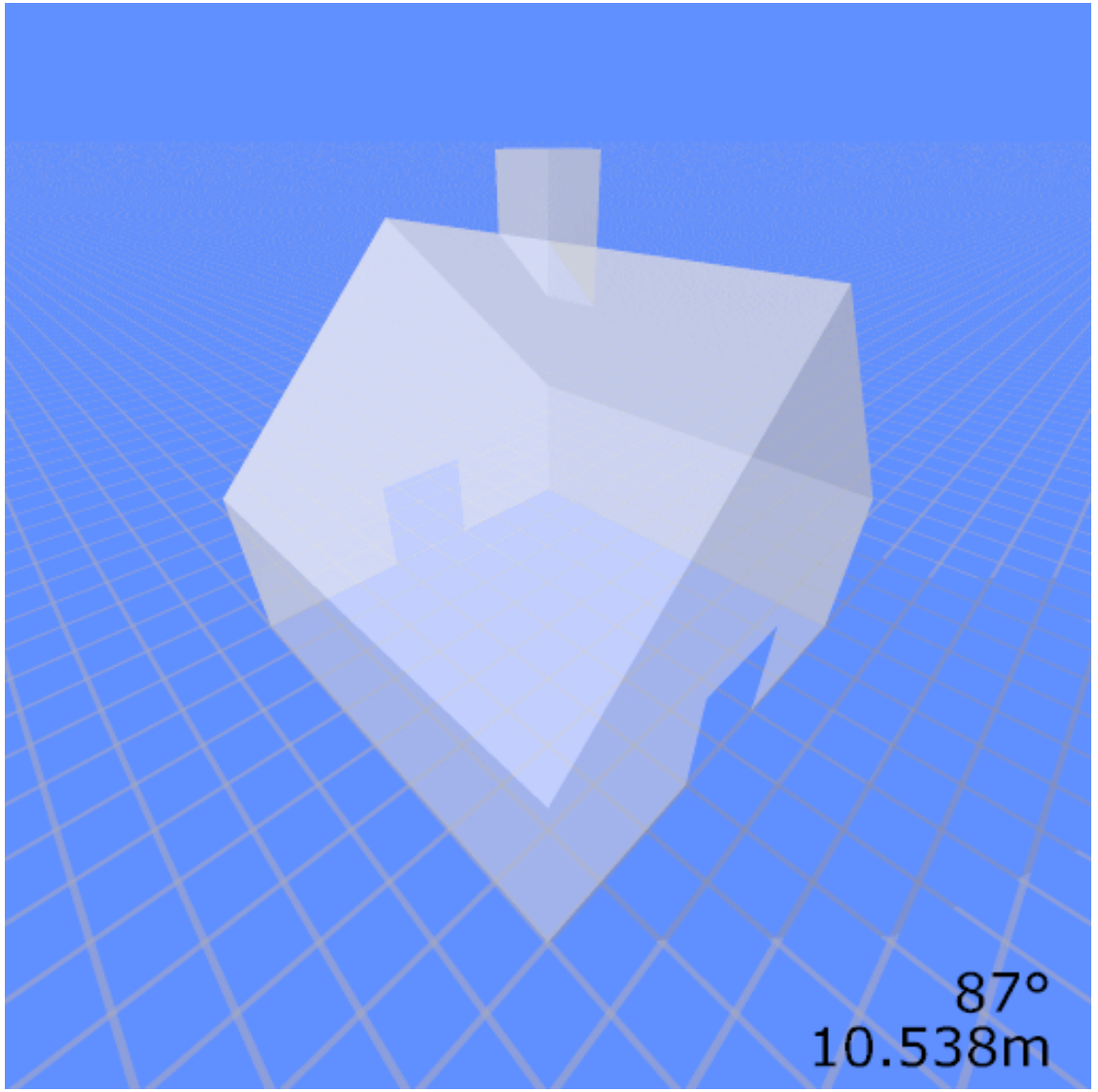
- Perspective

Camera Model

- Perspective

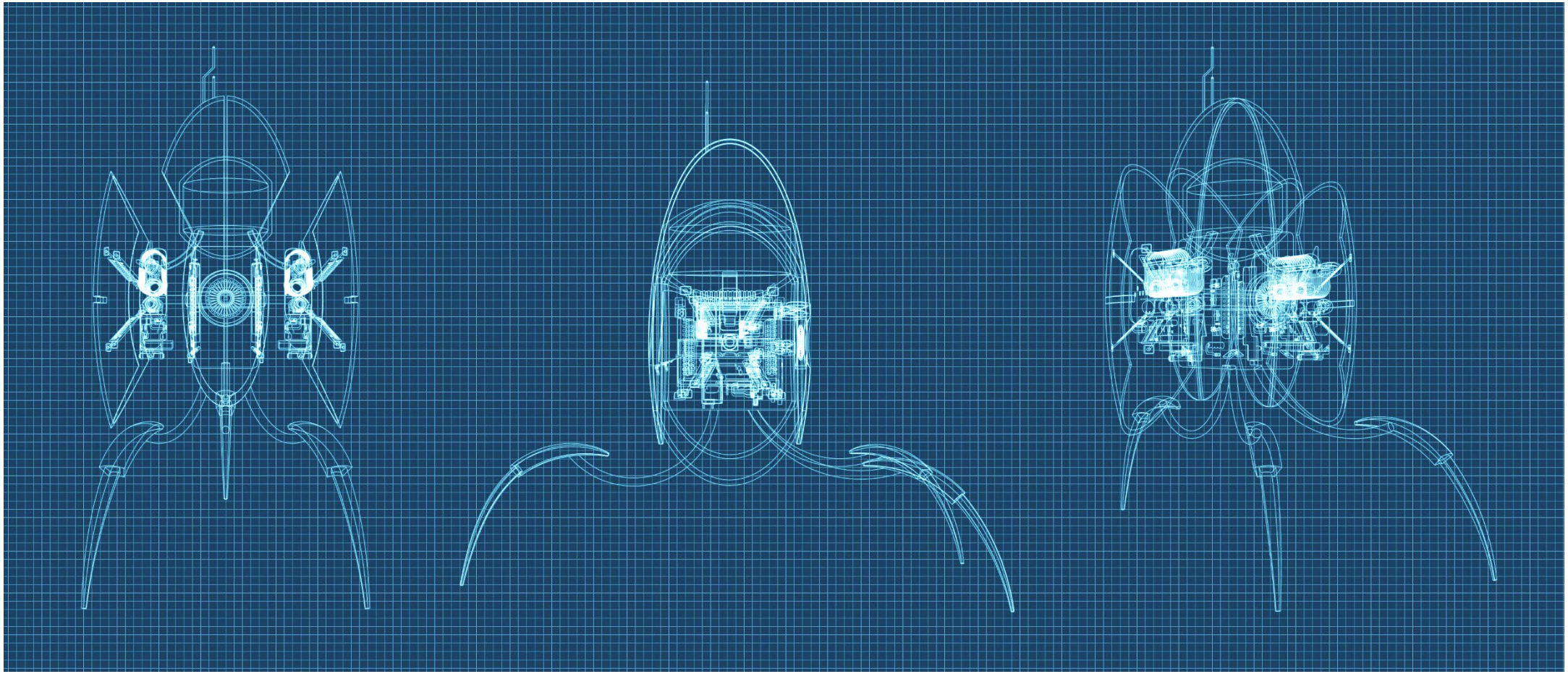
Camera Model

- Perspective projection
- Orthographic projection



Scene Model

- 3D geometry



Scene Model

- 3D geometry
- Color
- Texture
- Material

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- 3D geometry
- Color
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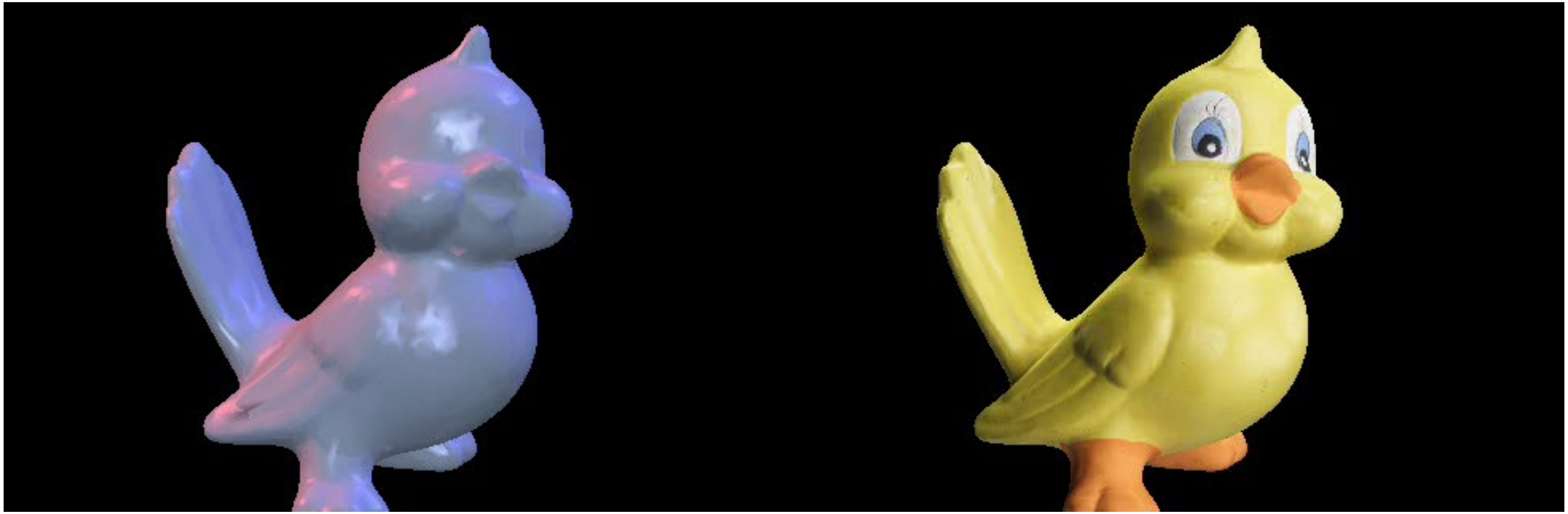
- Material

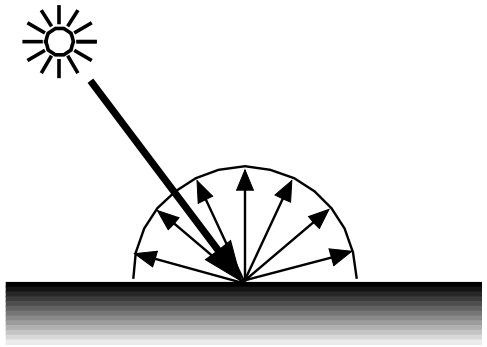


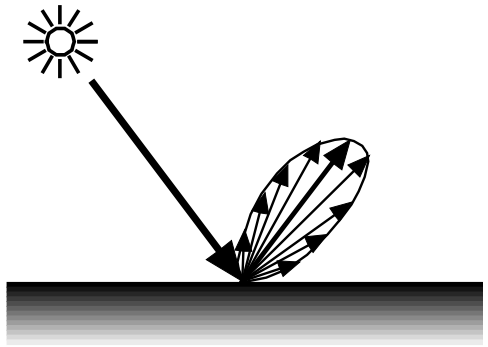
Illumination Model: Local Reflections

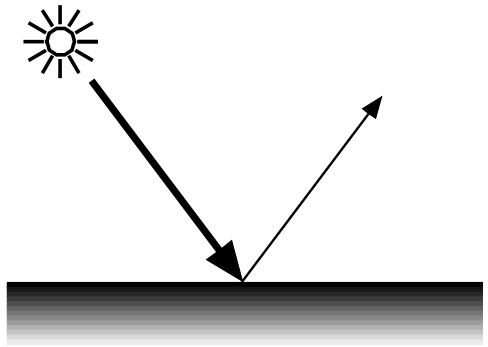
diffuse

glossy

mirror







Local vs. Global Light Transport

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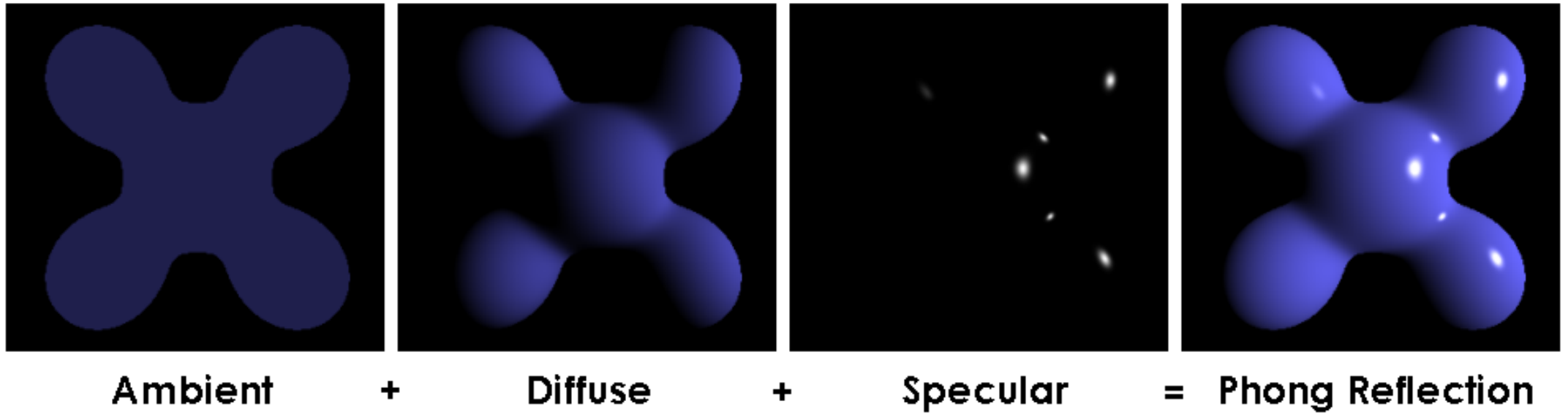
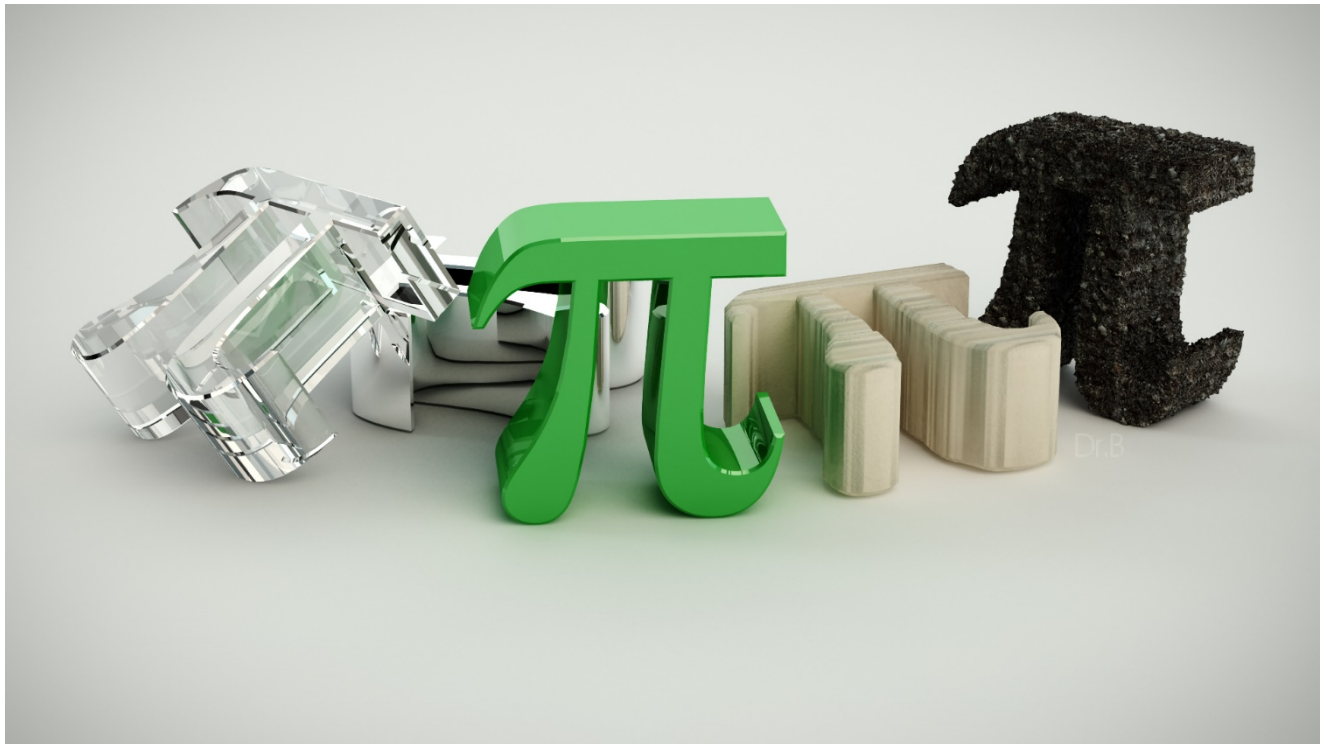
- Local Illumination Models
  - e.g. Phong
  - Light reflects on exactly one surface between light source and eye
  - Indirect light might be included with a constant term



Local vs. Global Light Transport

- Local Illumination Models
  - e.g. Phong
  - Light reflects on exactly one surface between light source and eye
  - Indirect light might be included with a constant term

- Global Illumination Models
  - e.g. Ray Tracing or Radiosity
  - Simulate and measure light transportation through the scene
  - Light interacts between objects and other objects or the environment



Global Illumination Effects

- Light scatters multiple times
  - Shadows
  - (Inter-) Reflections
  - Refractions
  - Caustics
  - …

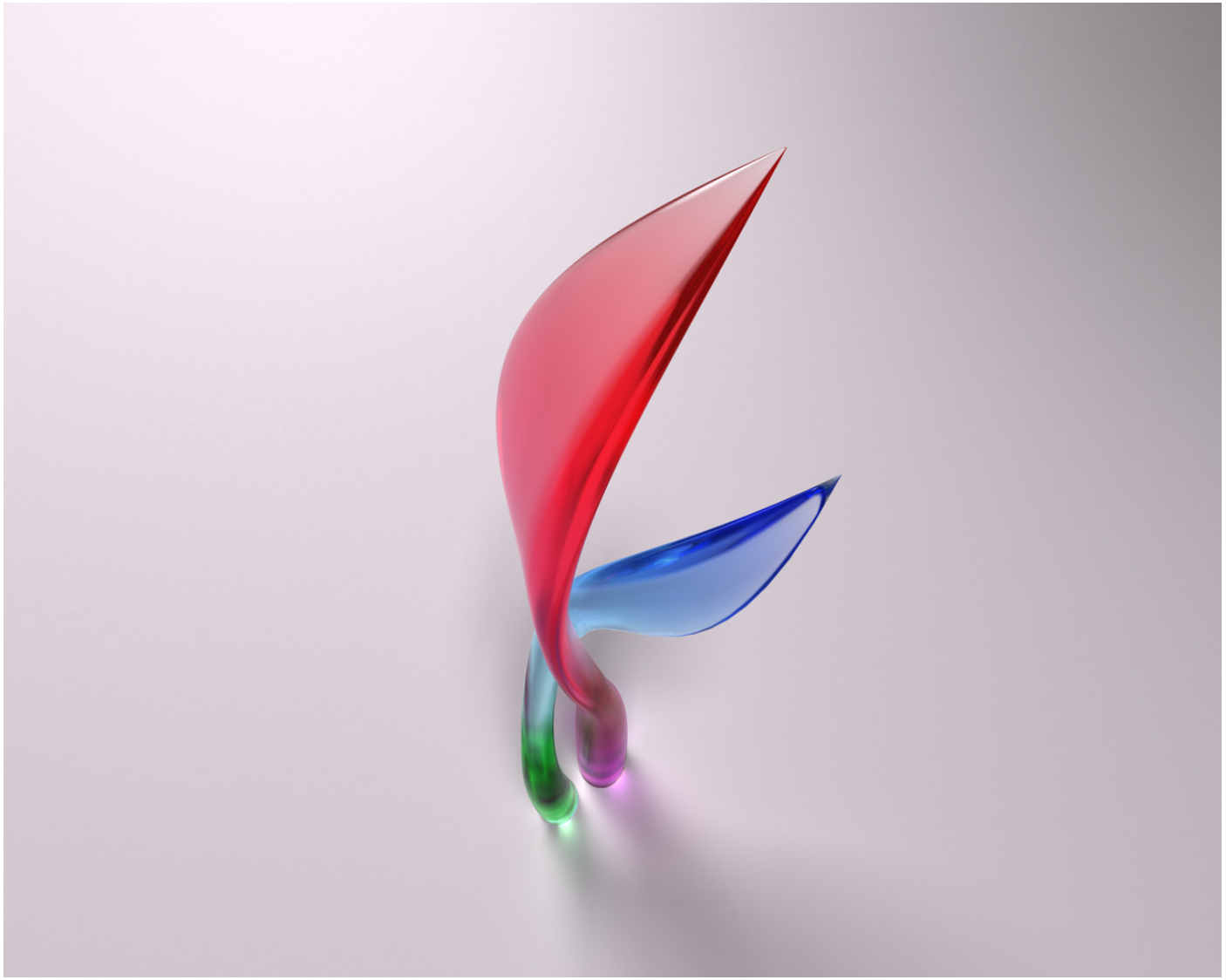


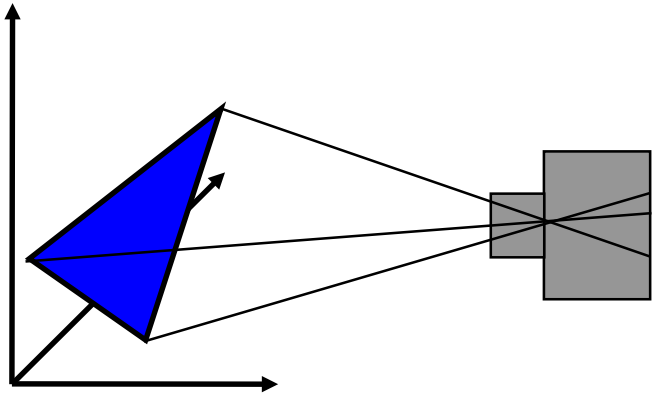
Image Formation

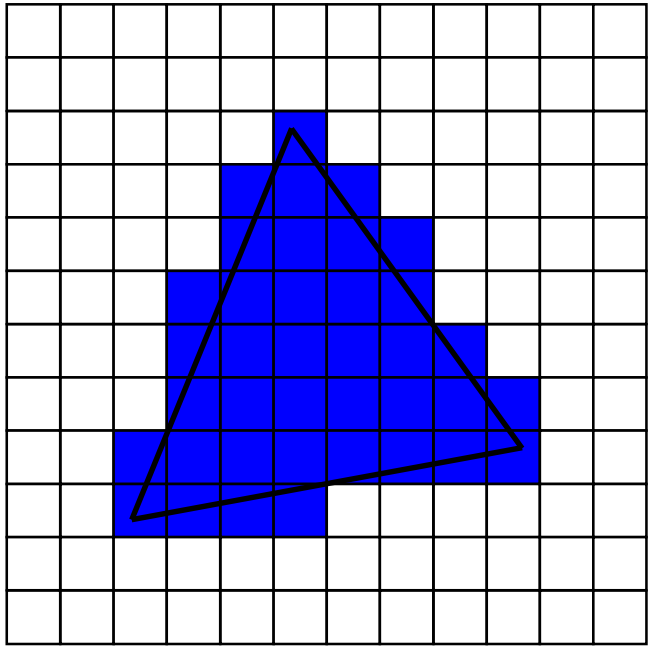
Why do images look like they do?

Image Formation: Rasterization

- Primitive operation of all interactive graphics, so far…
  - Scan convert a single triangle at a time

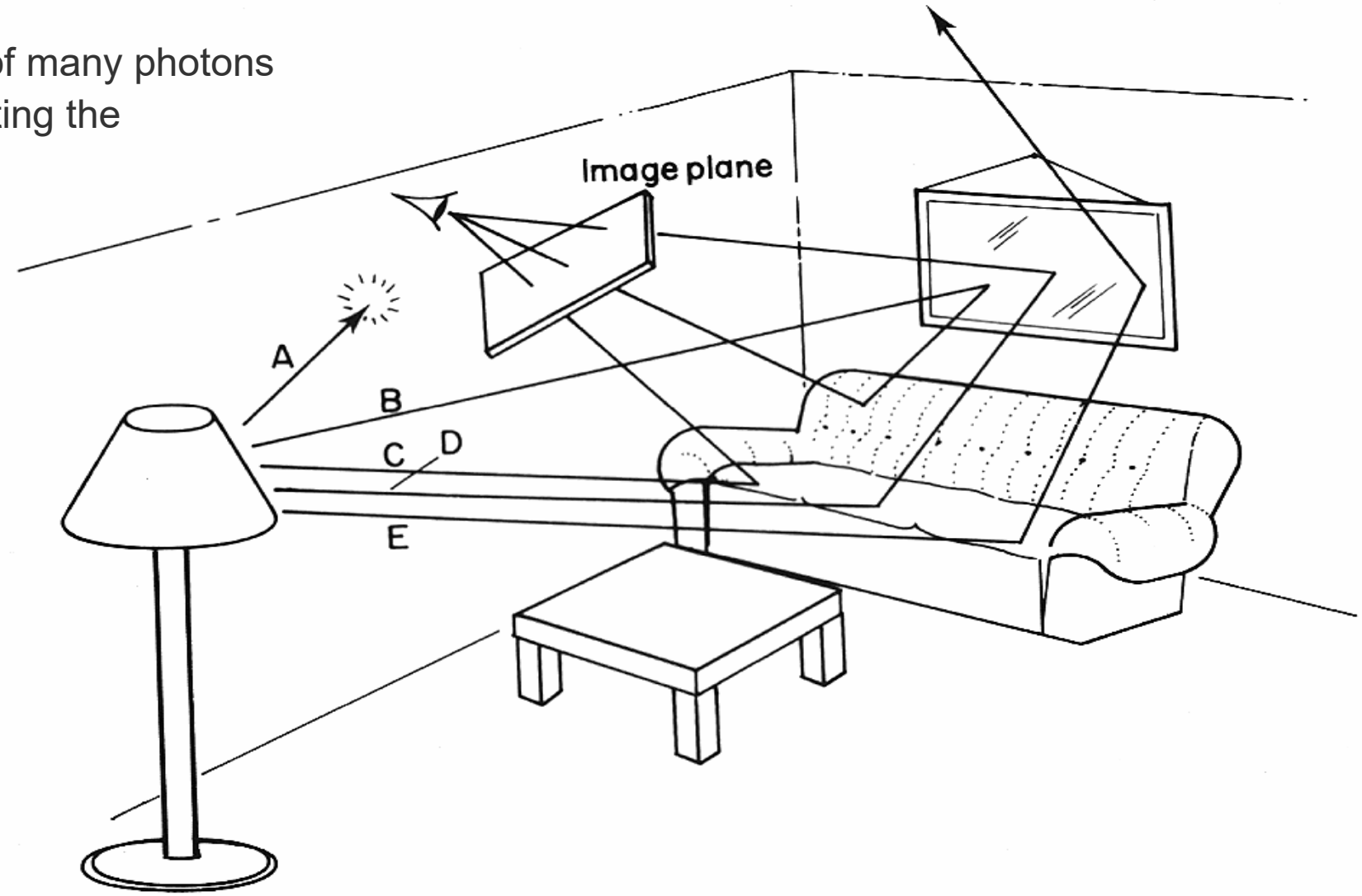
- Sequentially processes every triangle individually
  - Easy to account for direct reflection – local effects
  - But most effects need access to the world:
    - Shadows, Reflection, Global Illumination





Tracing the Paths of Light

- Nature:
  - Follow the path of many photons
  - Record those hitting the film in a camera



Light Transport

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- Light Distribution in a Scene
  - Dynamic equilibrium
  - Newly created, scattered, and absorbed photons

- Forward Light Transport:
  - Start at the light sources
  - Shoot photons into scene
  - Reflect at surfaces (according to some reflection model)
  - Wait until they are absorbed or hit the camera (very seldom)
  - Nature: massive parallel processing at the speed of light

- Backward Light Transport:
  - Start at the camera
  - Trace only paths that transport light towards the camera
  - **Ray tracing**

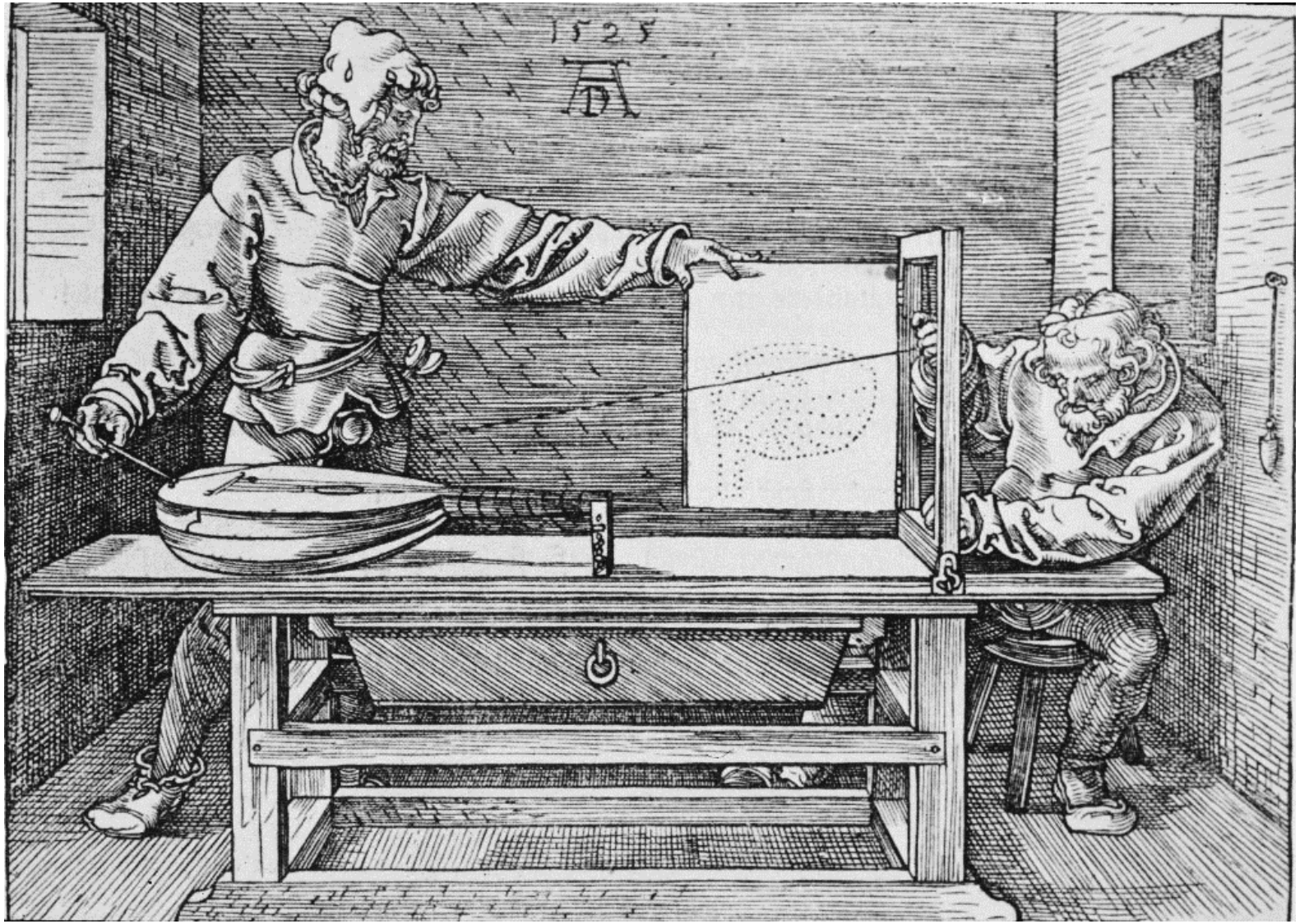
Ingredients

- Surfaces
  - 3D geometry of objects in a scene
- Surface reflectance characteristics
  - Color, absorption, reflection, refraction, subsurface scattering
  - Local property, may vary over surface
  - Mirror, glass, glossy, diffuse, …
- Illumination
  - Position, characteristics of light emitters
  - Repeatedly reflected light → indirect illumination
- Assumption: air/empty space is totally transparent
  - Excludes any scattering effects in participating media volumes
  - Would require solving a much more complex problem
  - In contrast to volume rendering, participating media

Ray Tracing

- The Ray Tracing Algorithm
  - One of the two fundamental rendering algorithms (vs. Rasterization)
- Simple and intuitive
  - Easy to understand and implement
- Powerful and efficient
  - Many optical global effects
    - shadows, reflection, refraction, and others
  - Efficient real-time implementation in SW and HW
- Scalability
  - Can work in parallel and distributed environments
  - Inherent logarithmic scalability with scene size: O(log n) vs. O(n)
  - Output sensitive and demand driven
- Not new
  - Light rays: Empedocles (492-432 BC), Renaissance (Dürer, 1525)
  - Uses in lens design, geometric optics, …

Ray Tracing

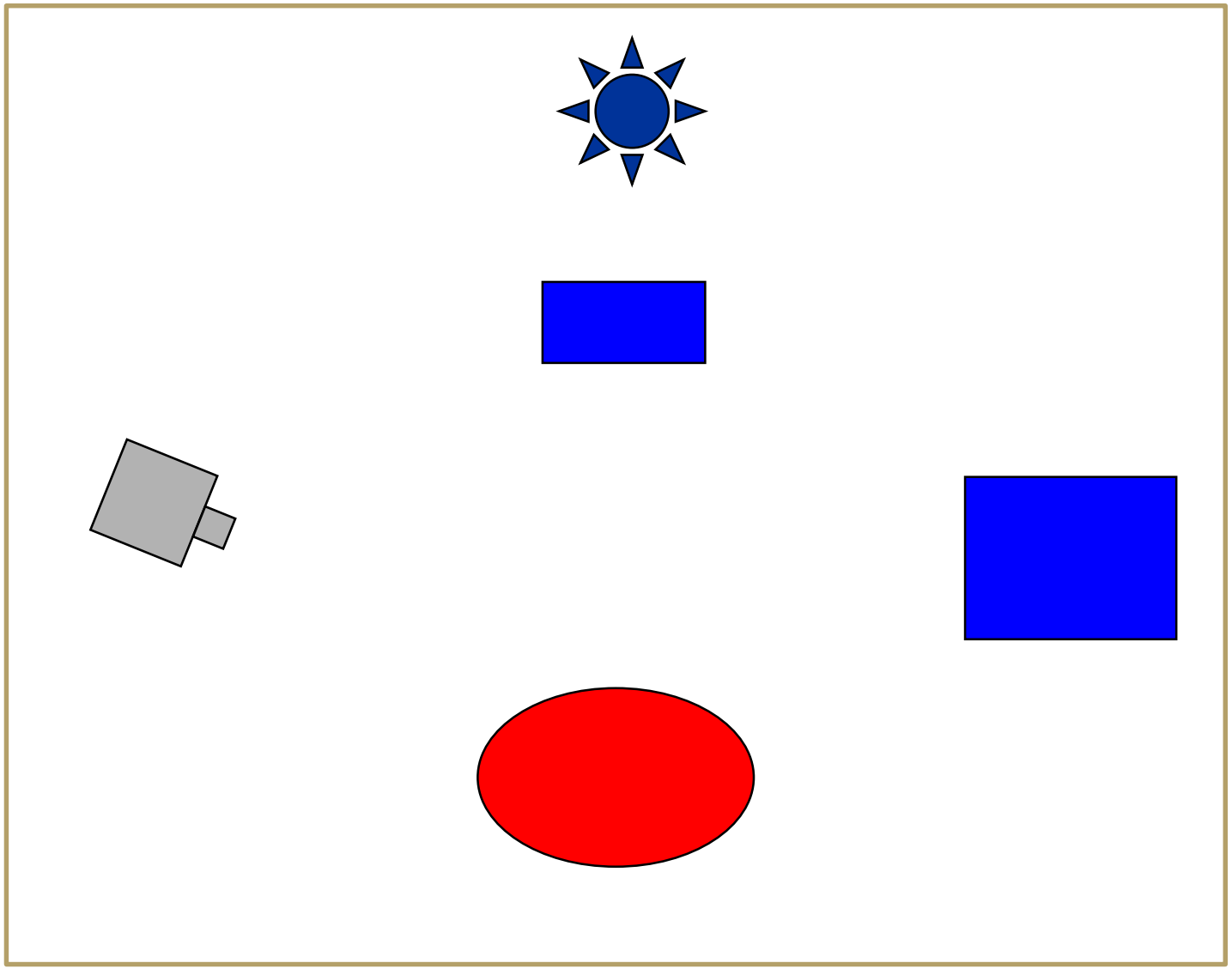


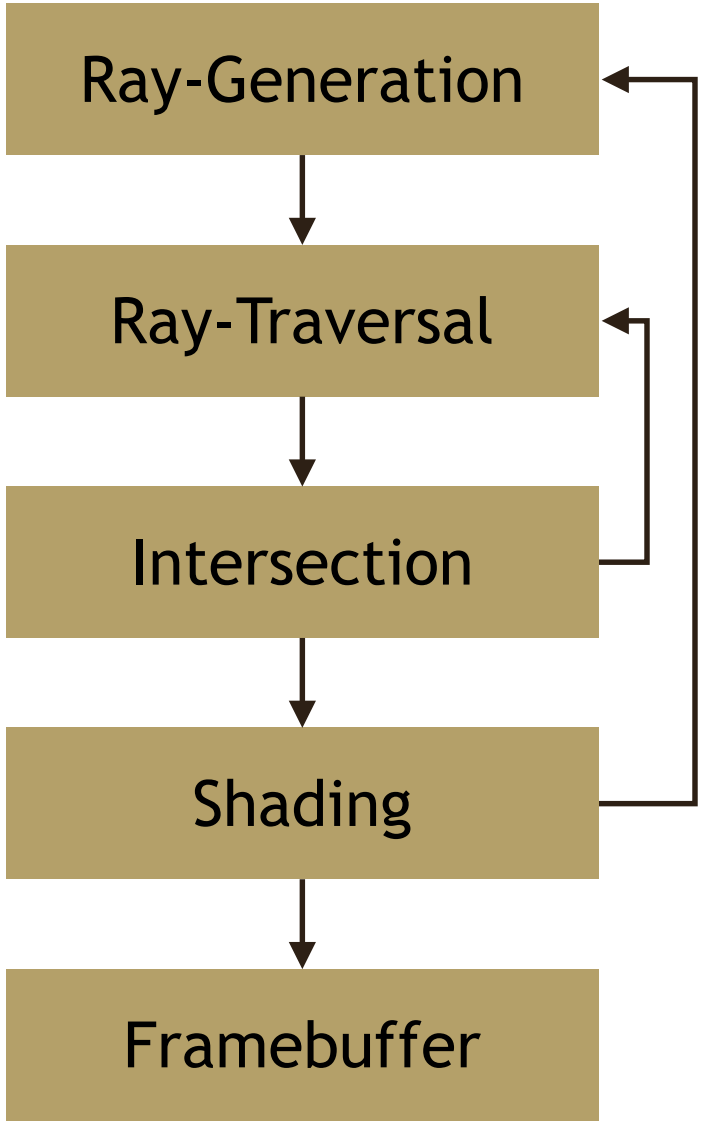
Ray tracing in a woodcut
by Albrecht Dürer, 1525

Global Illumination Effects

- Highly realistic images
  - Ray tracing enables *correct* simulation of light transport

Ray Tracing Pipeline





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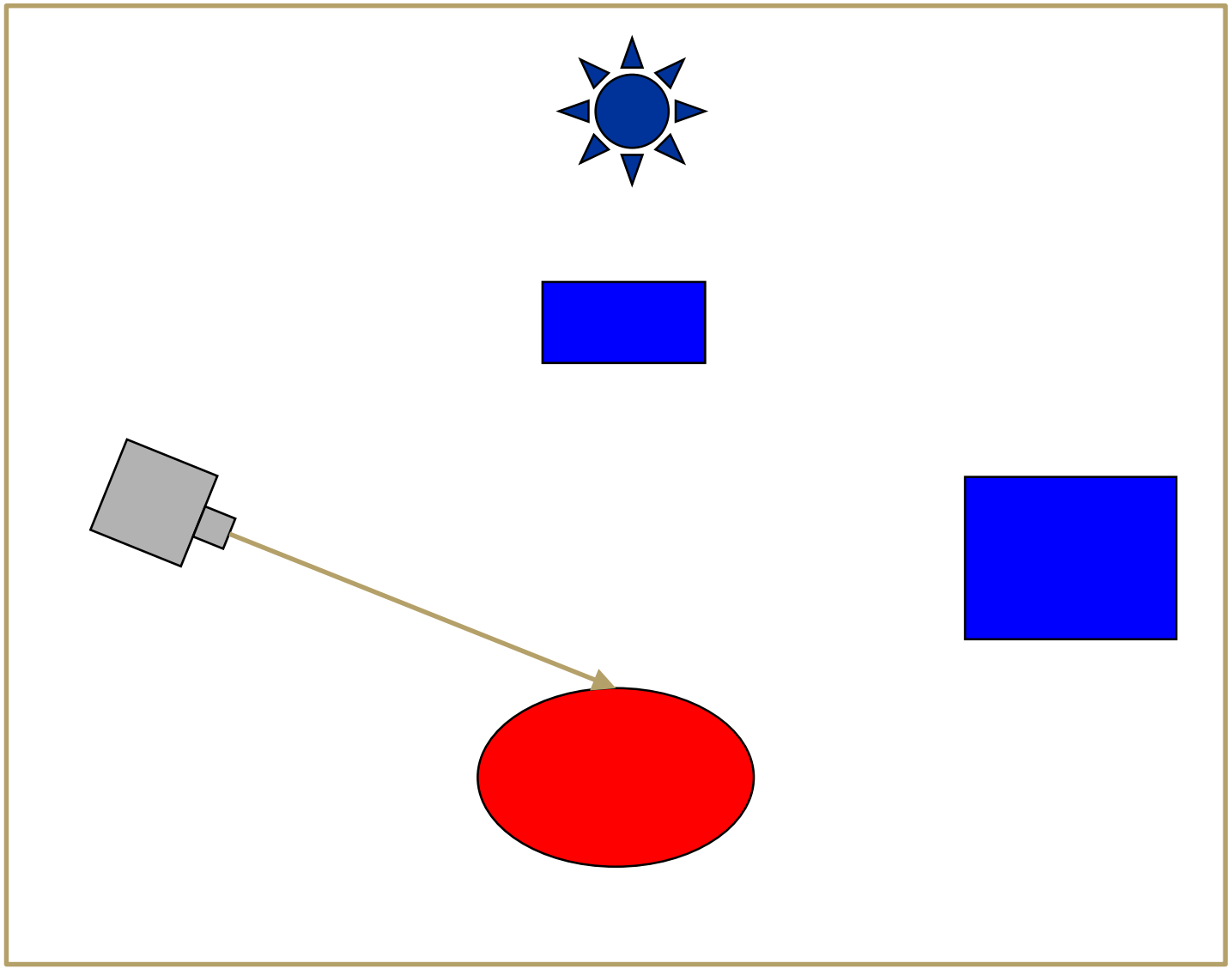

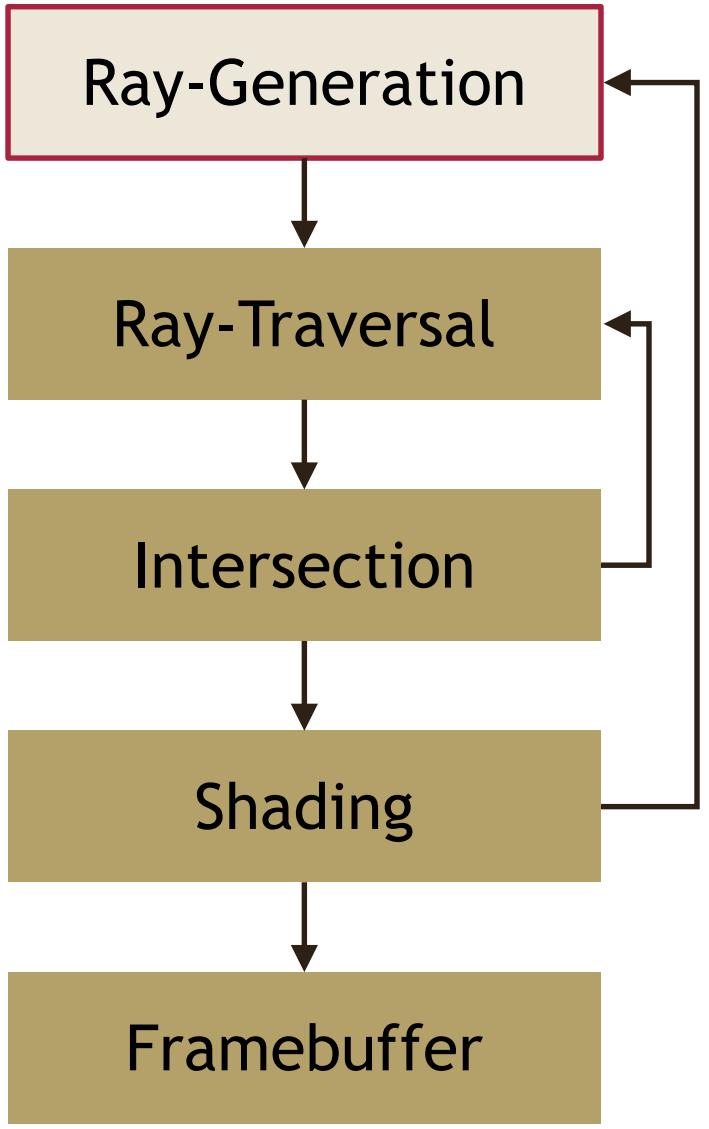

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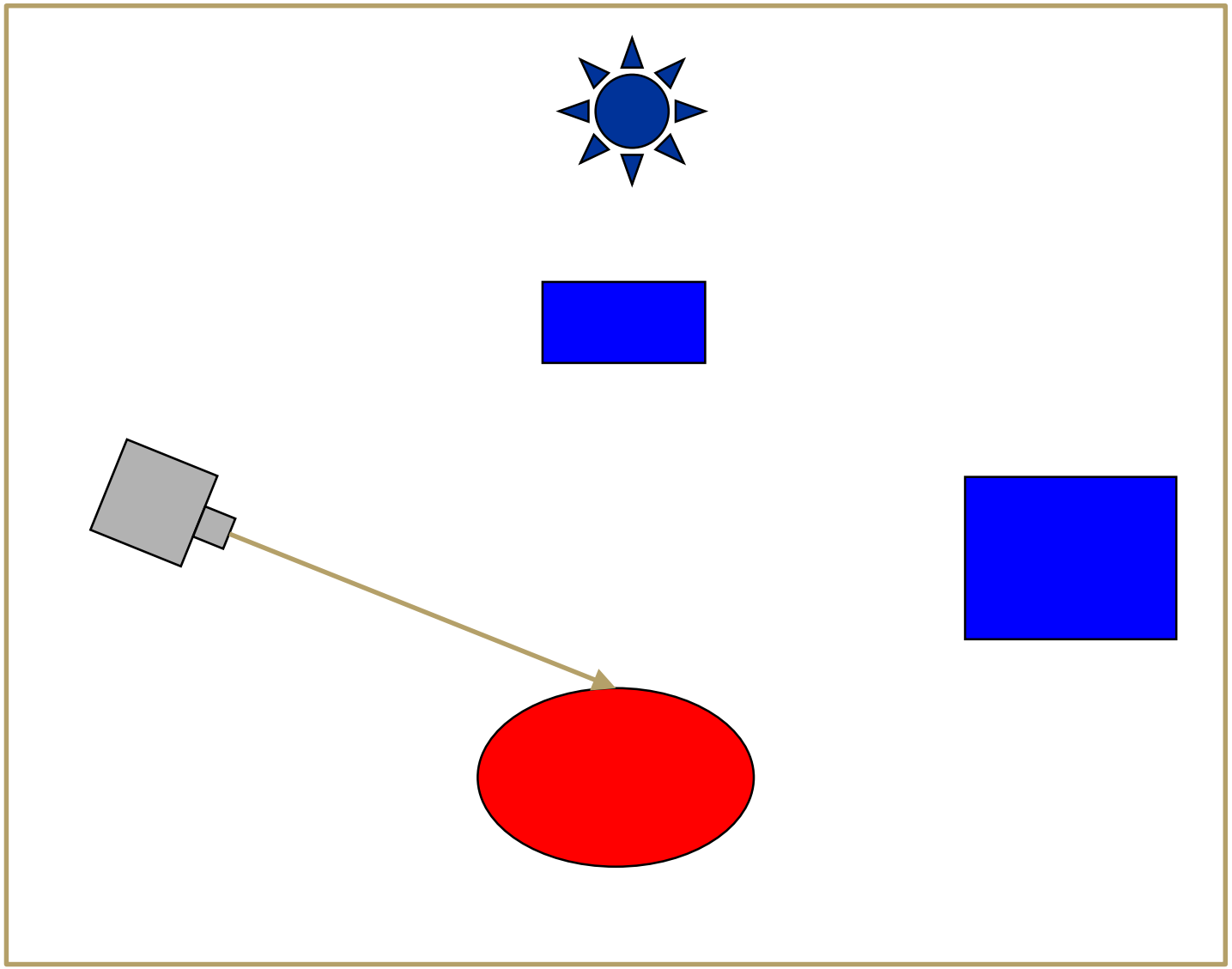

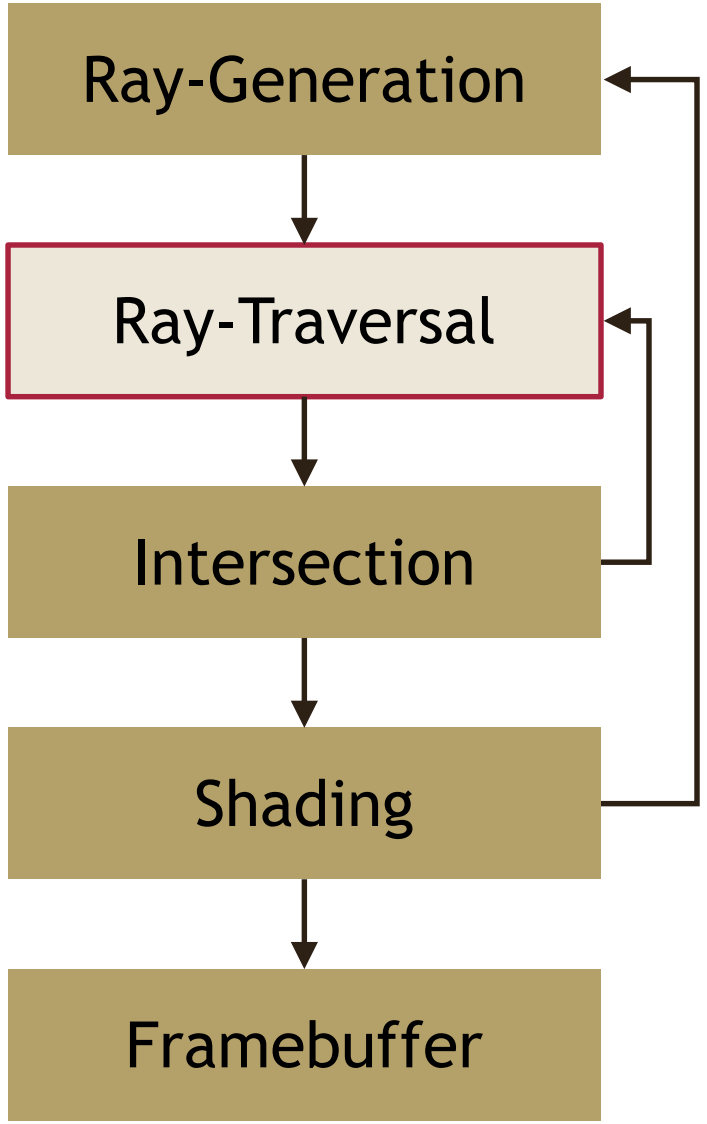

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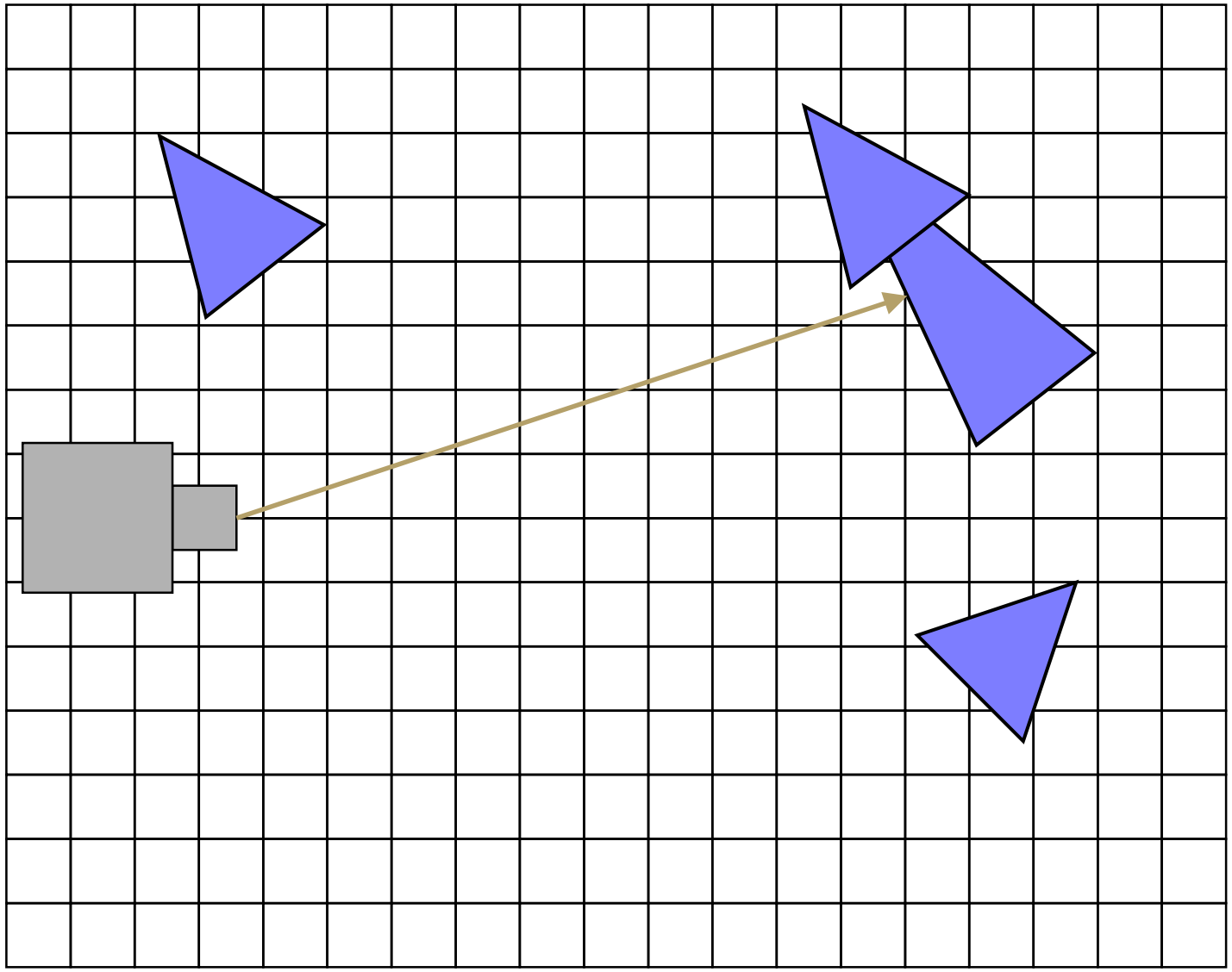

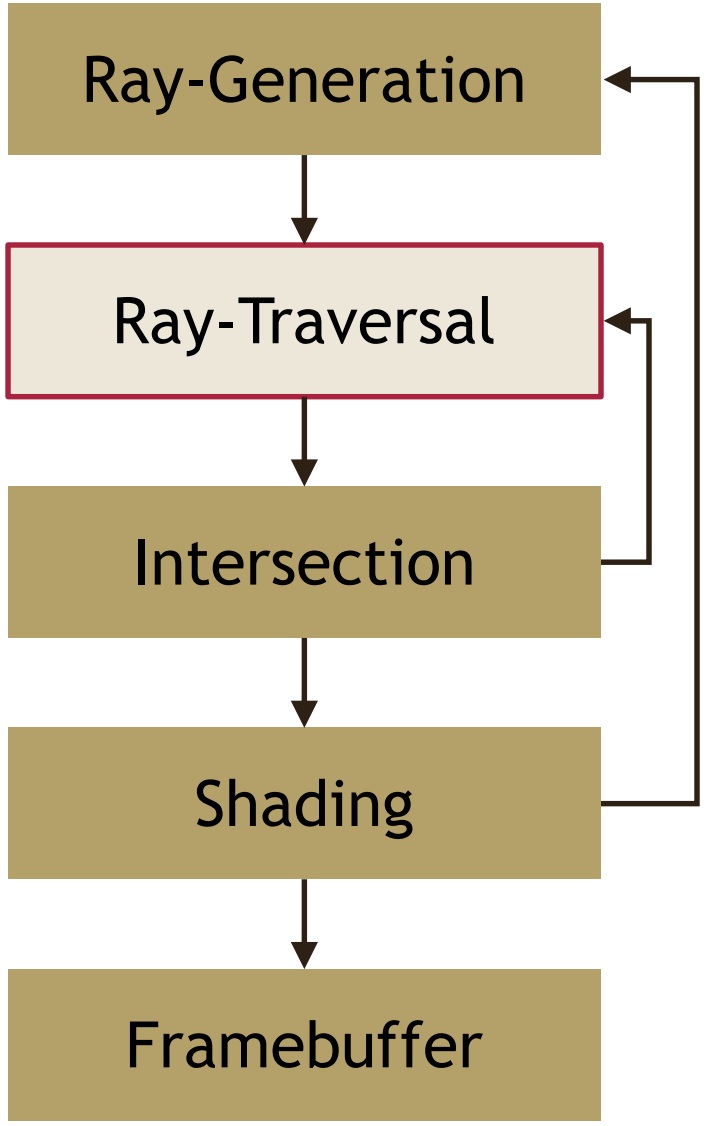

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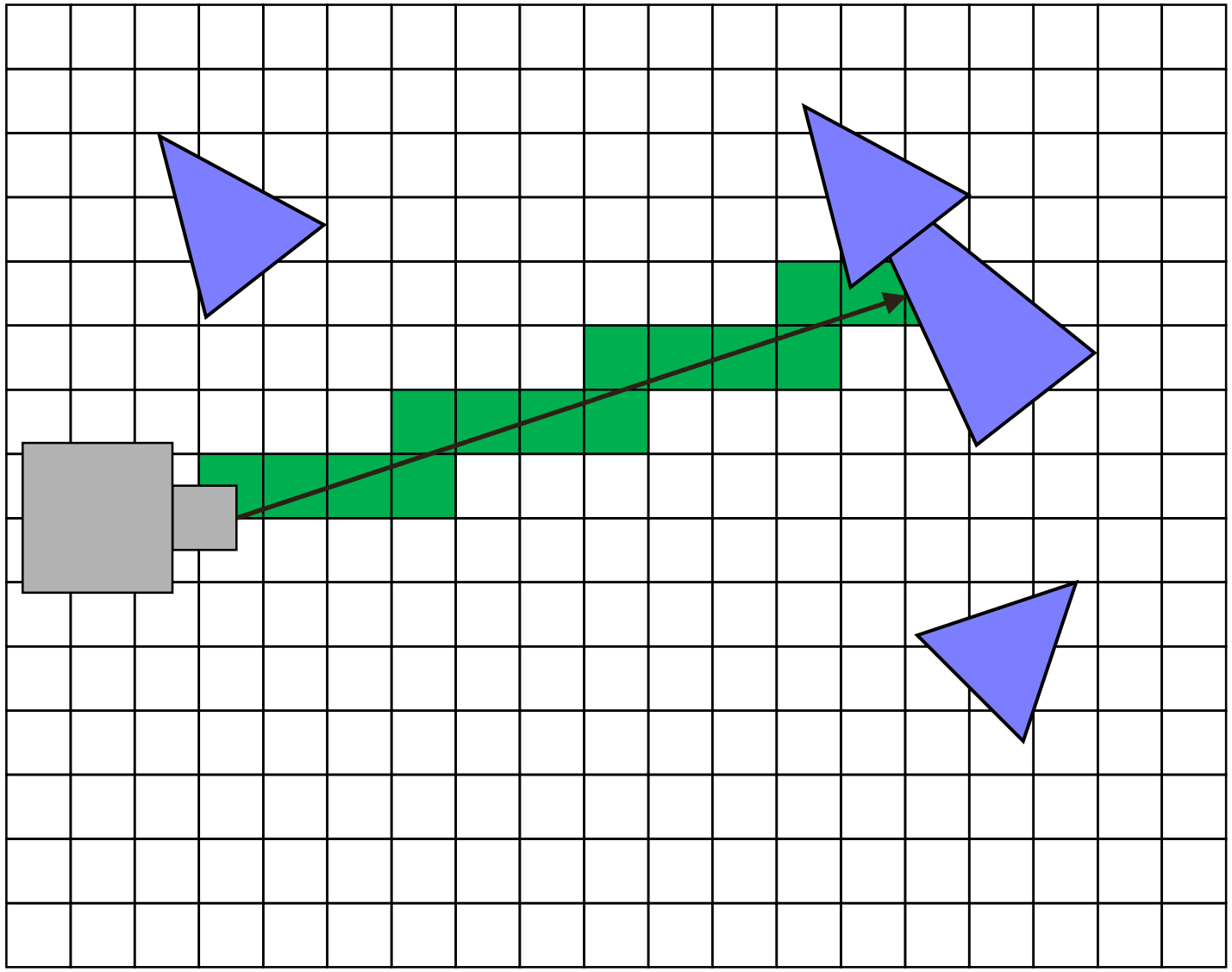

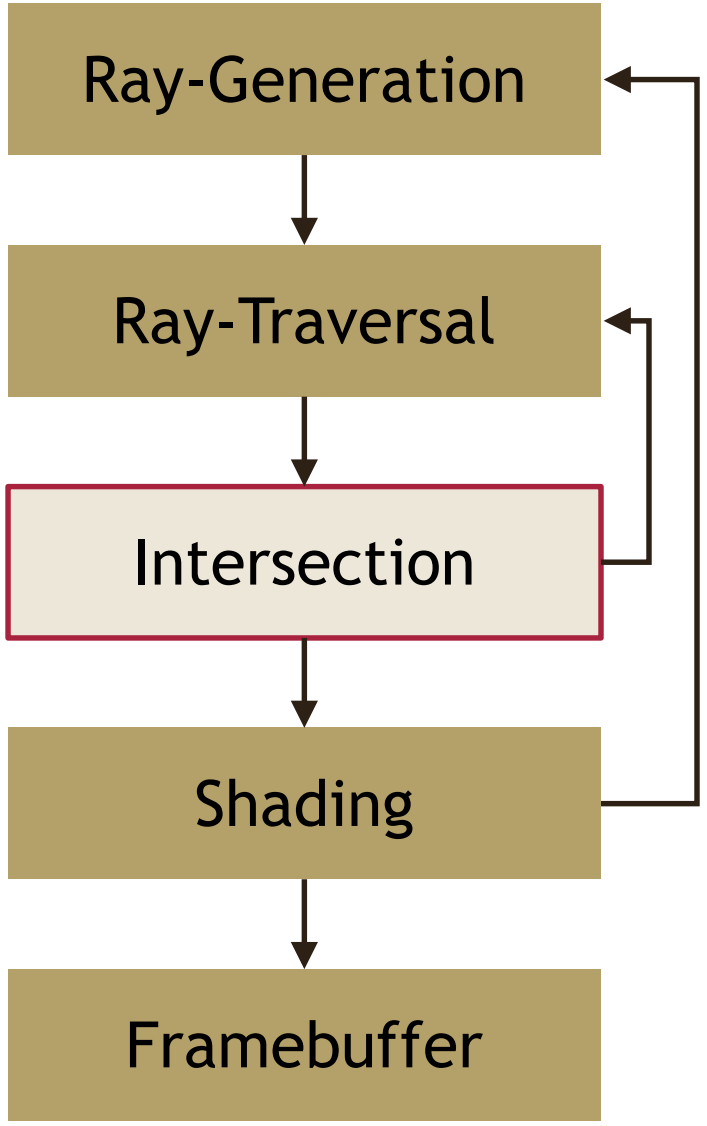

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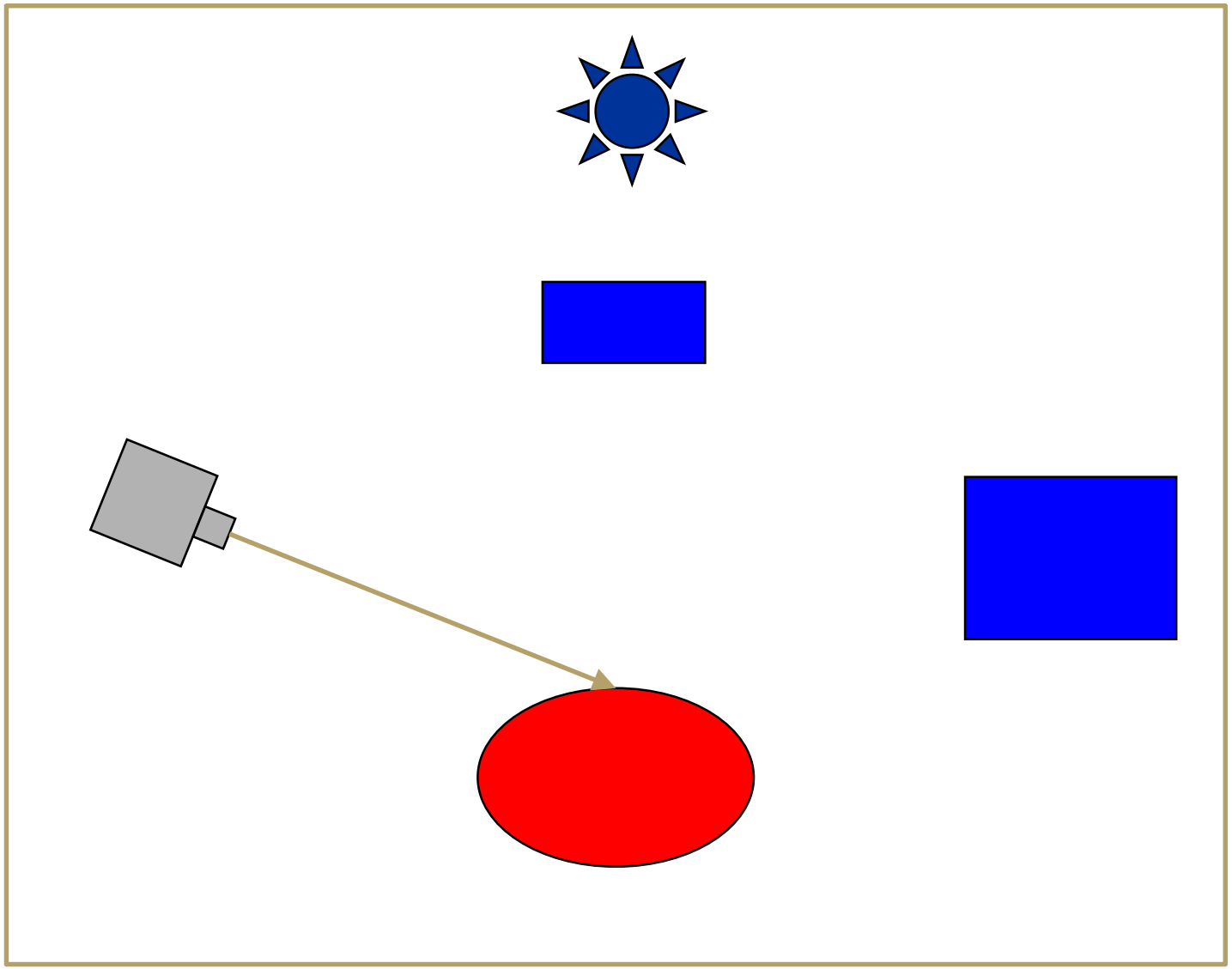

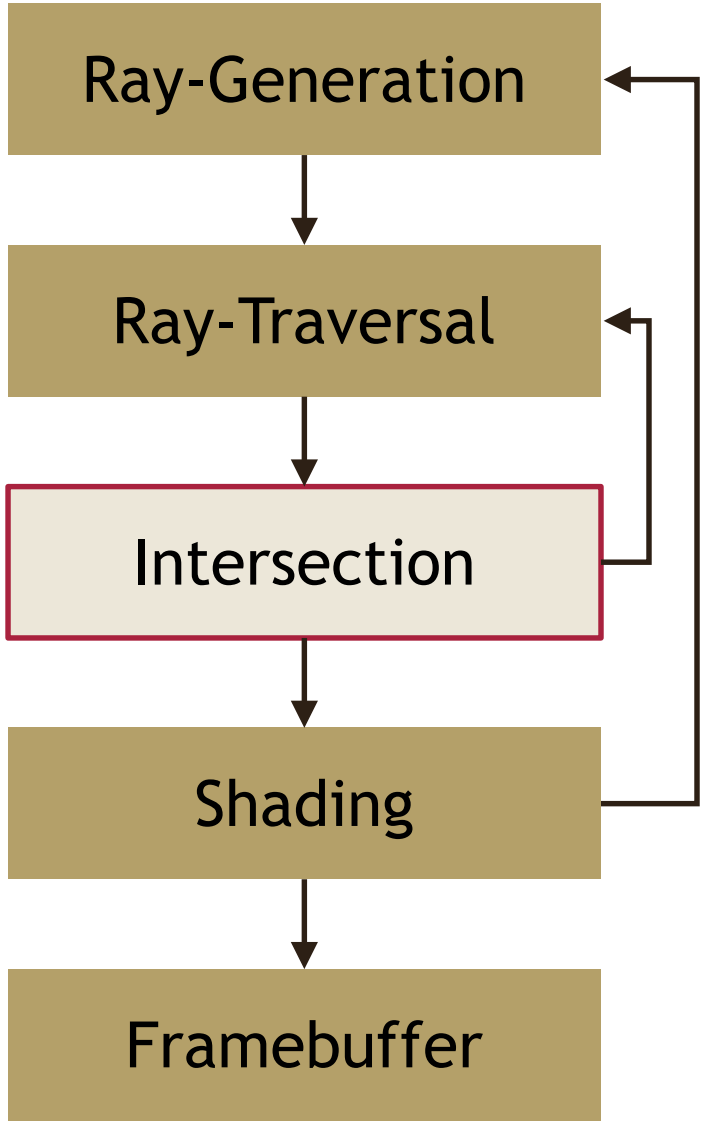

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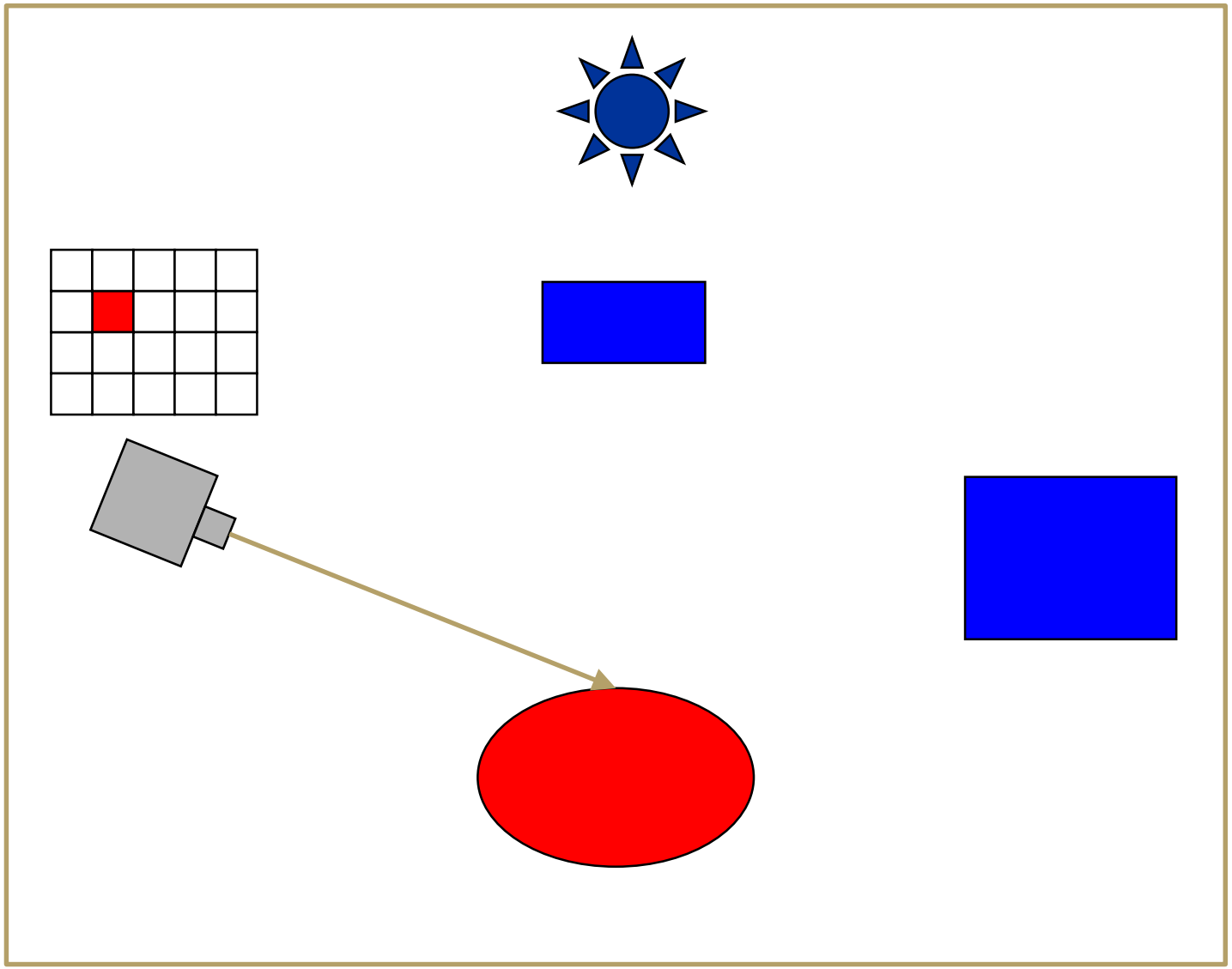

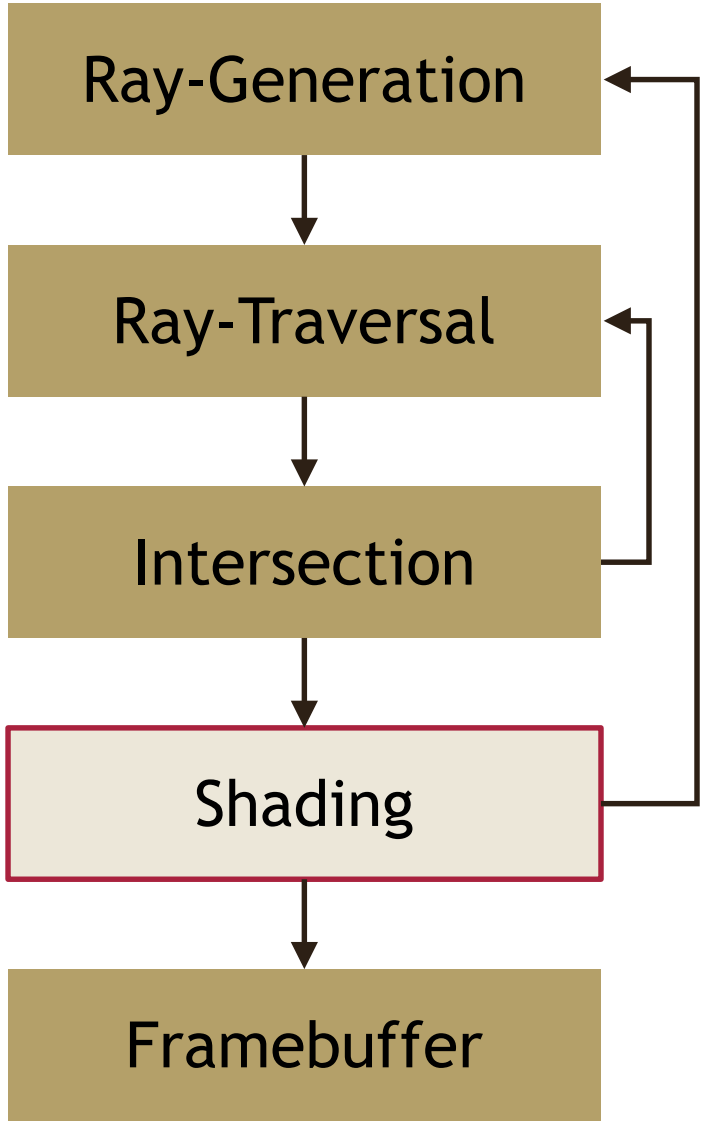

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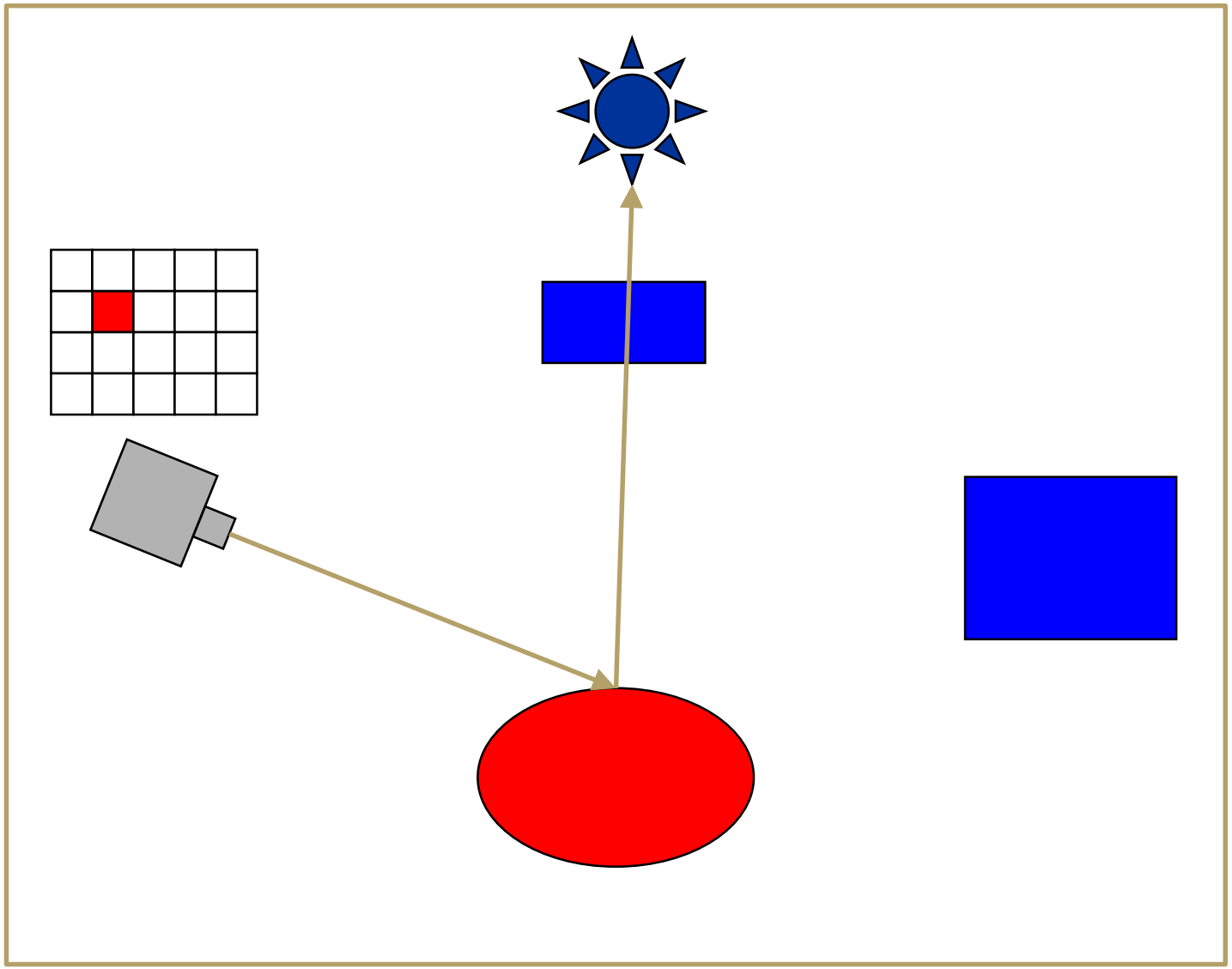

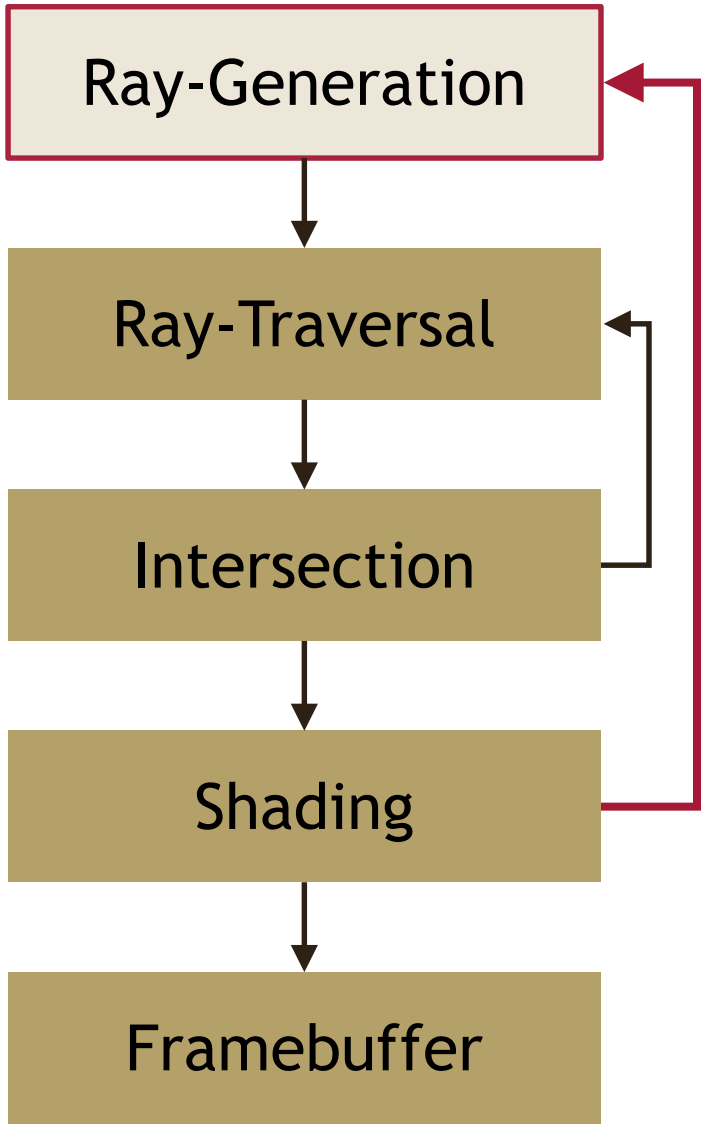

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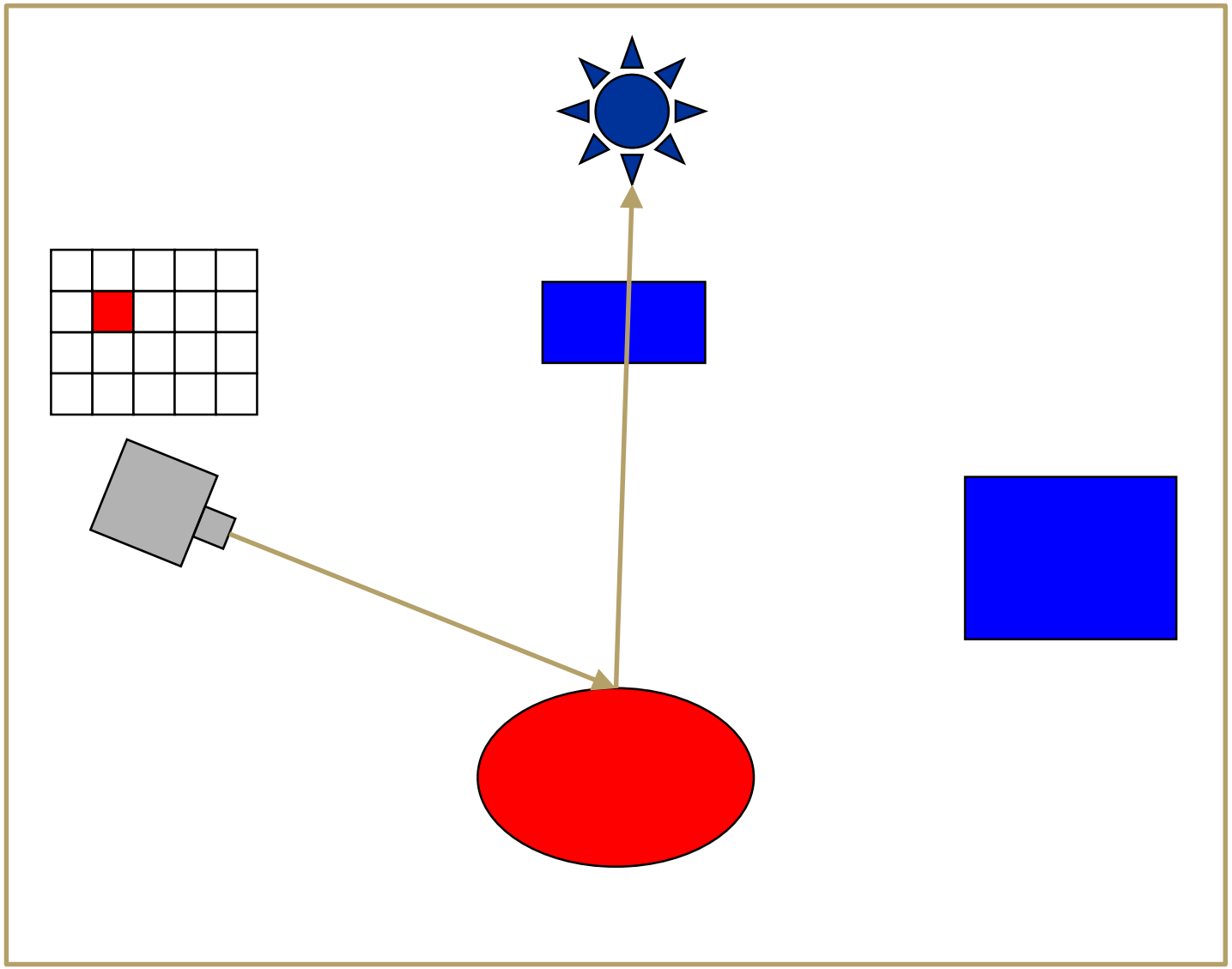

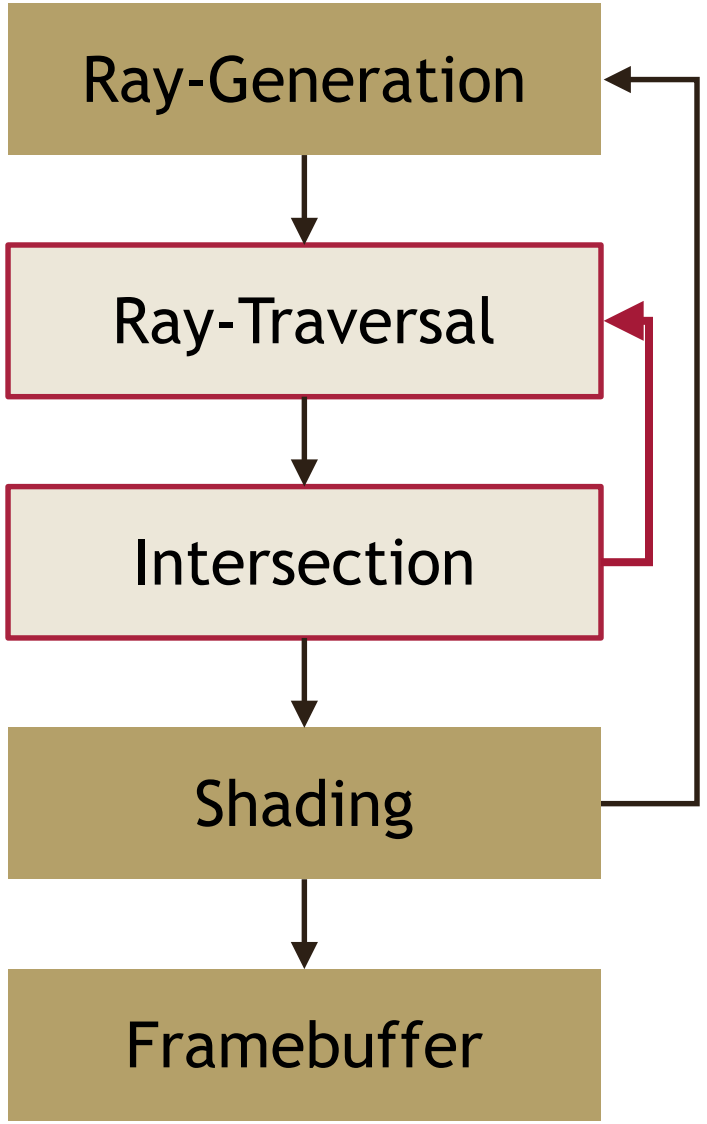

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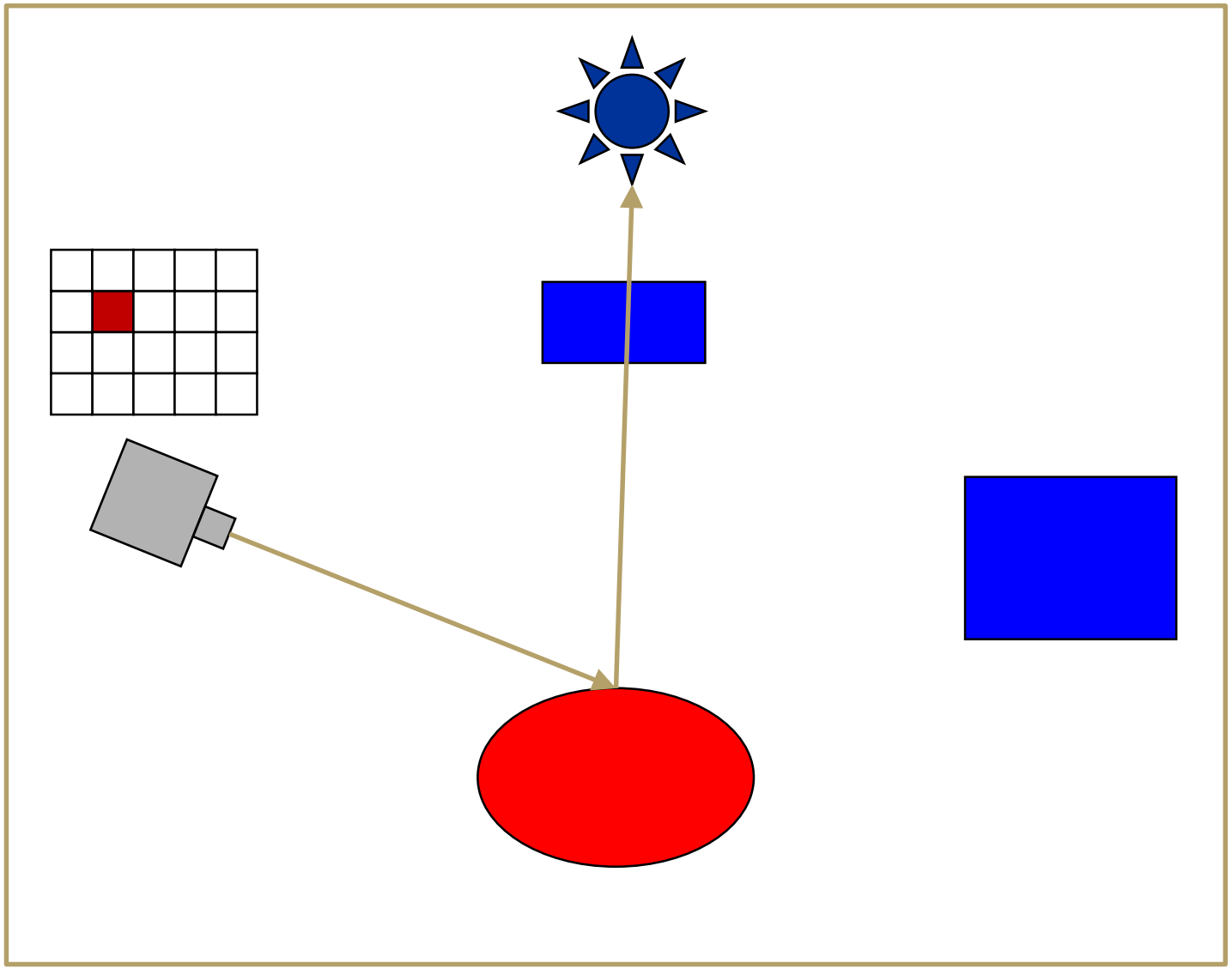

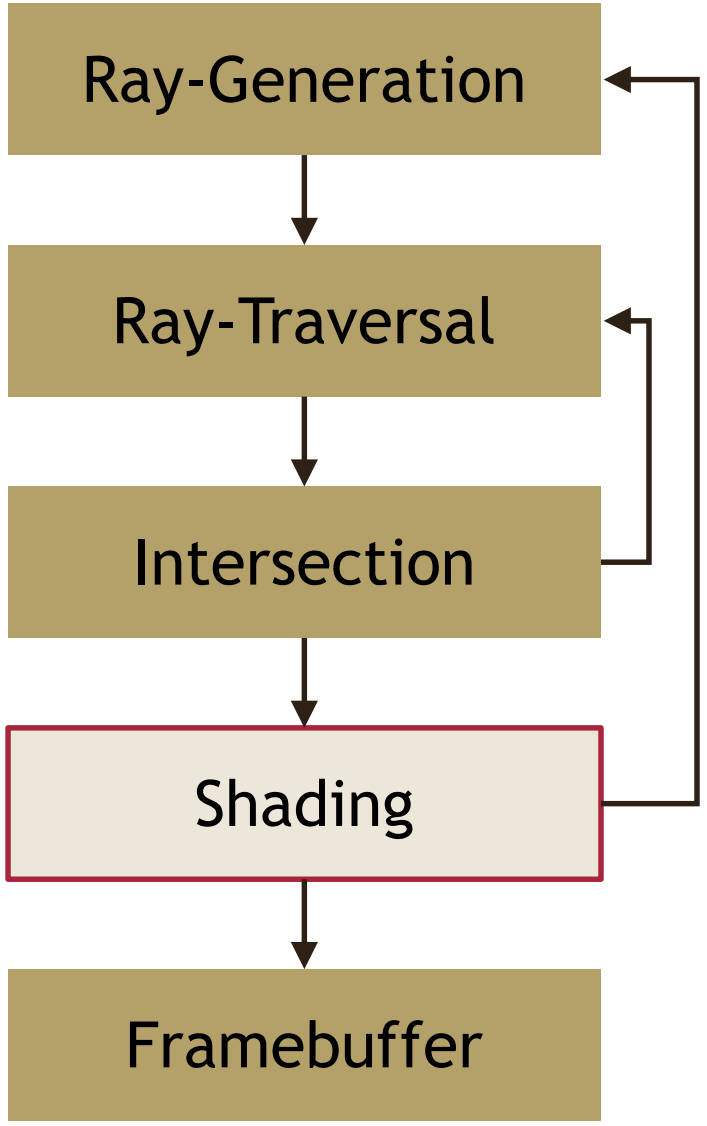

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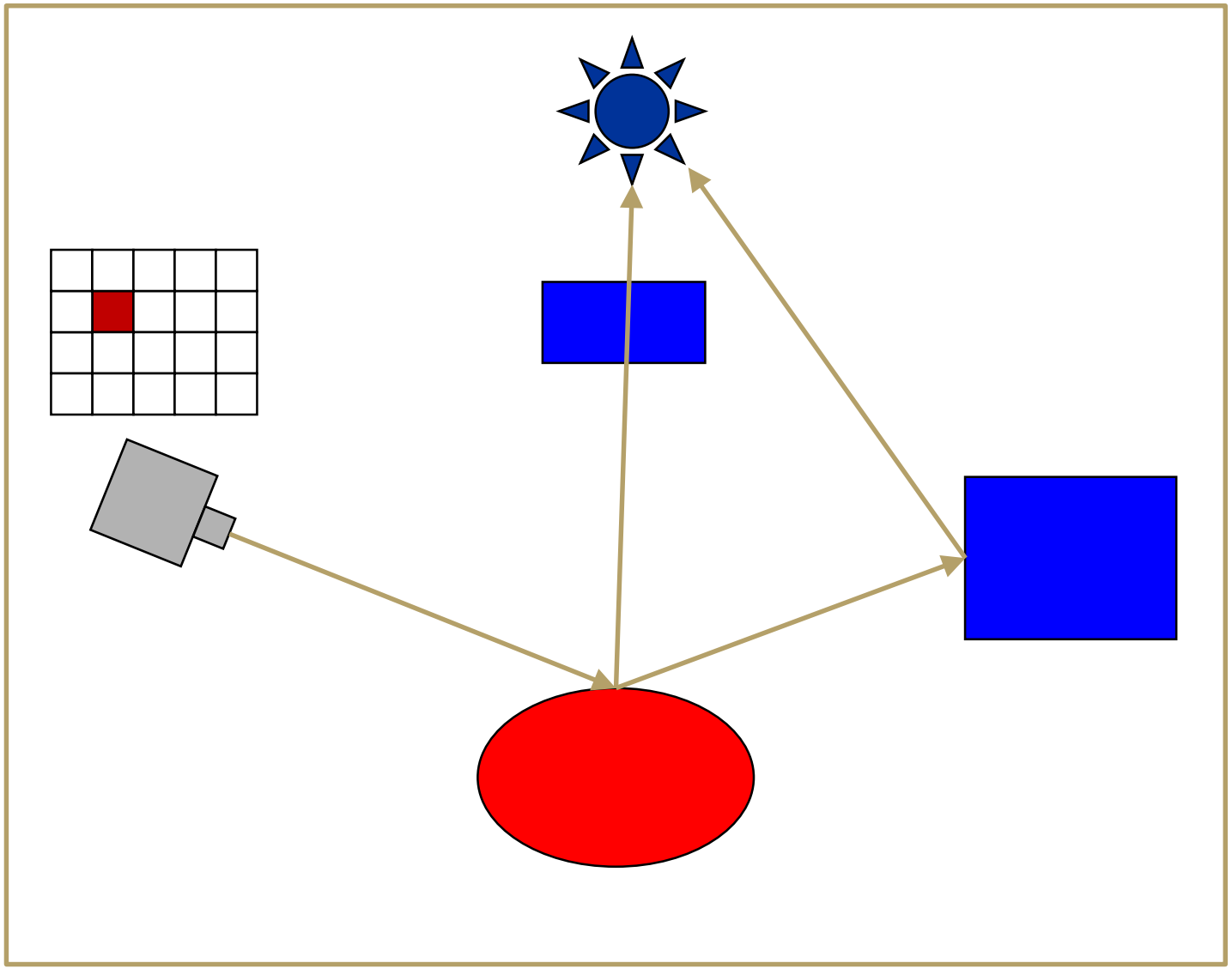

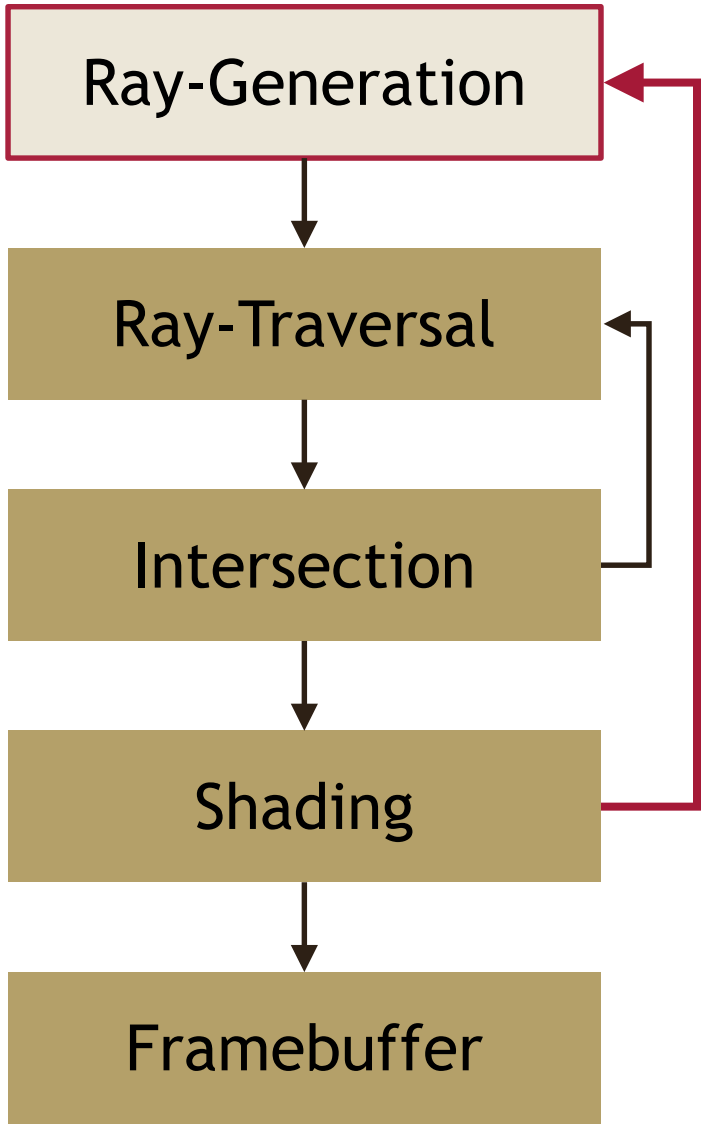

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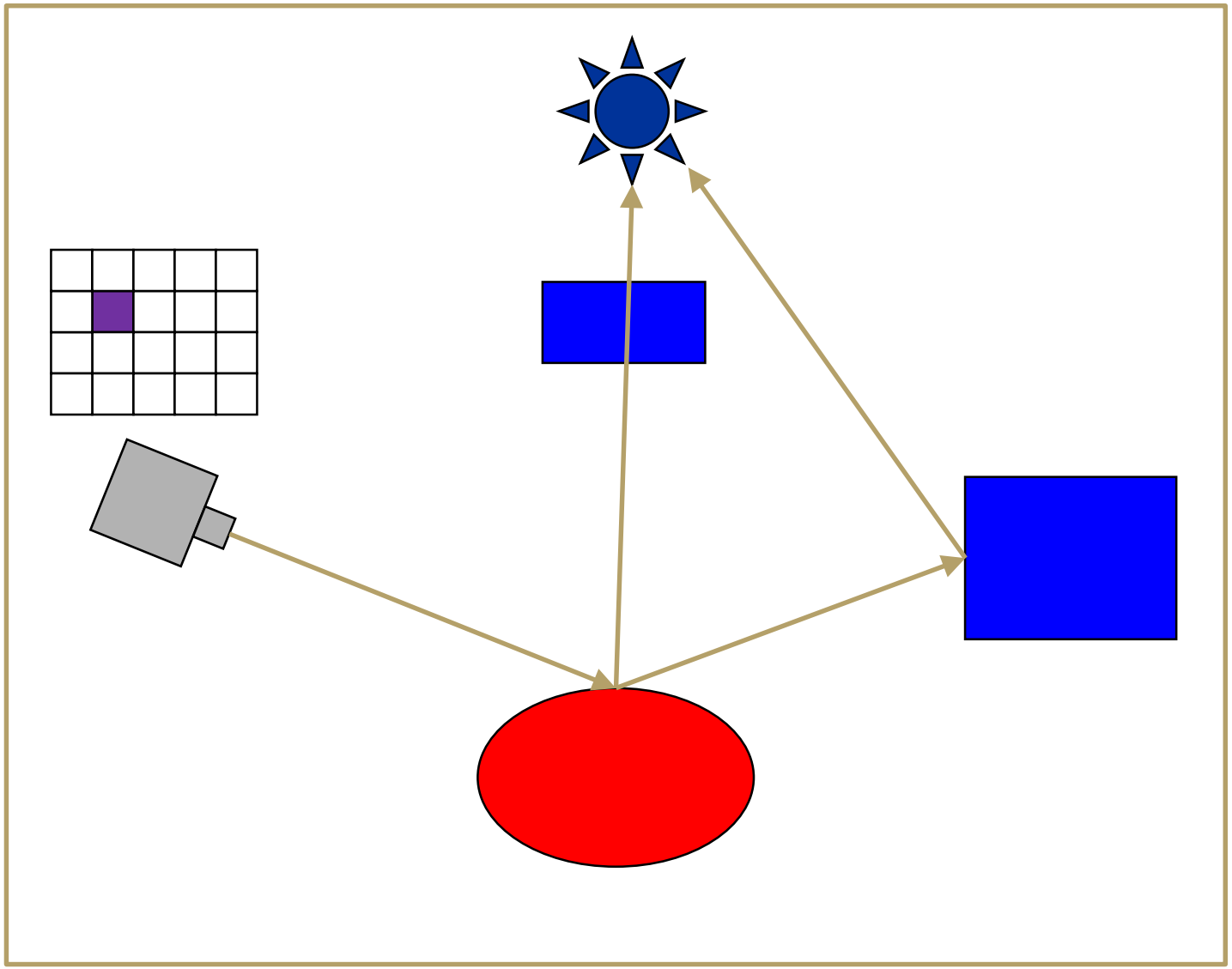

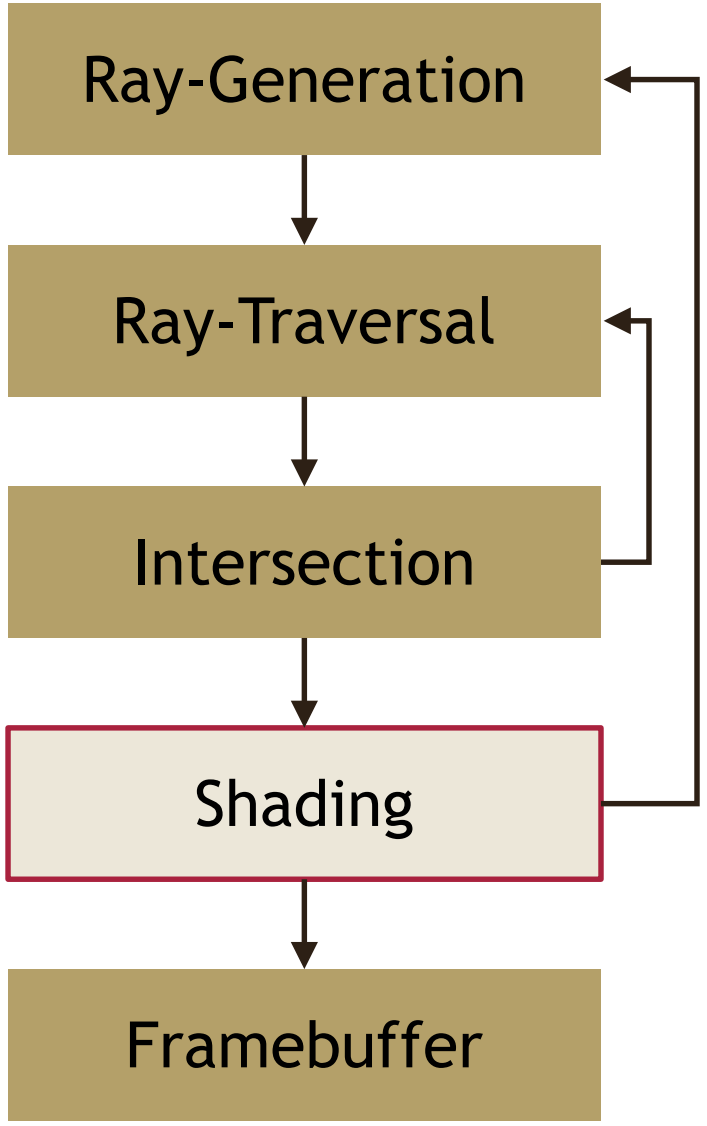

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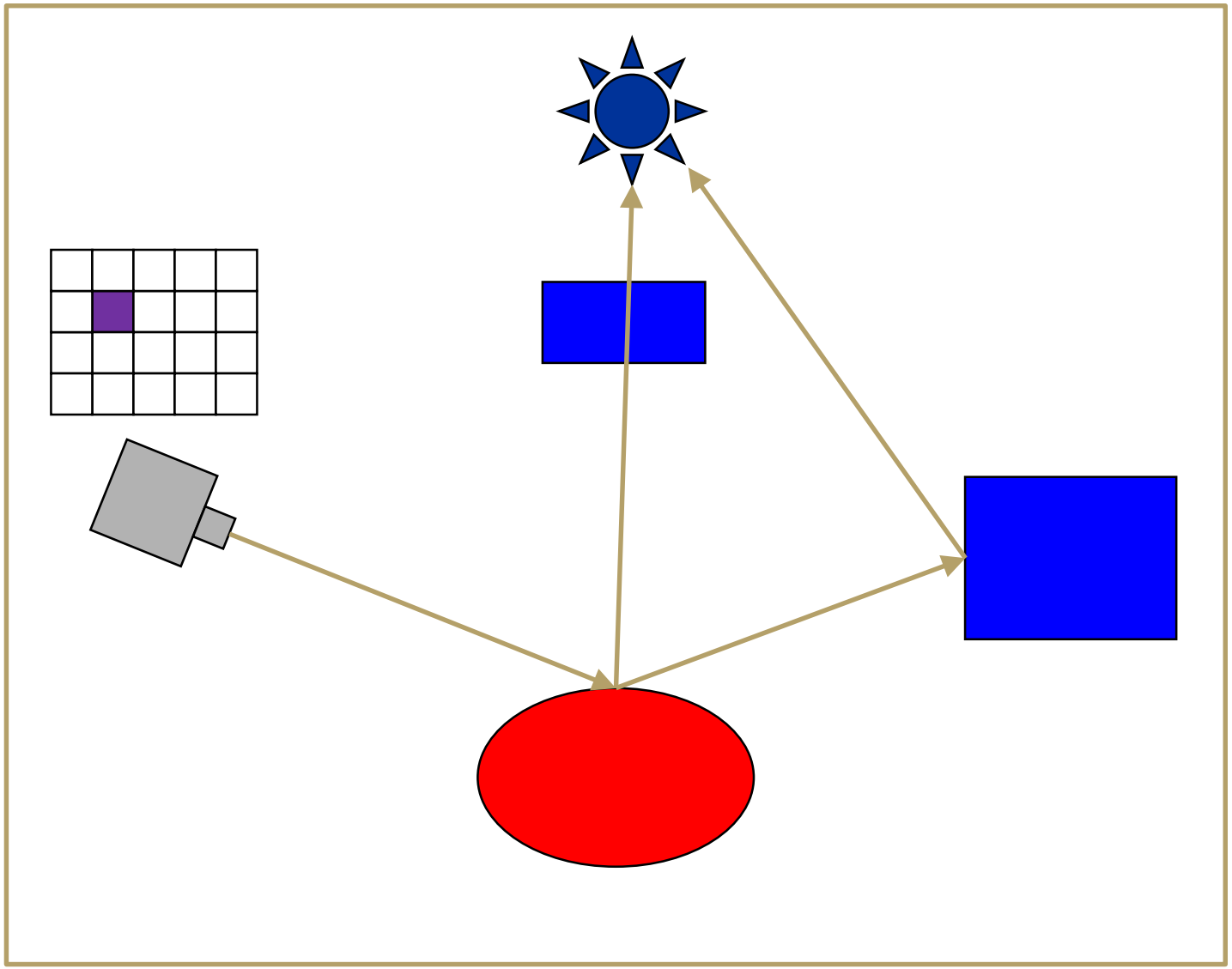
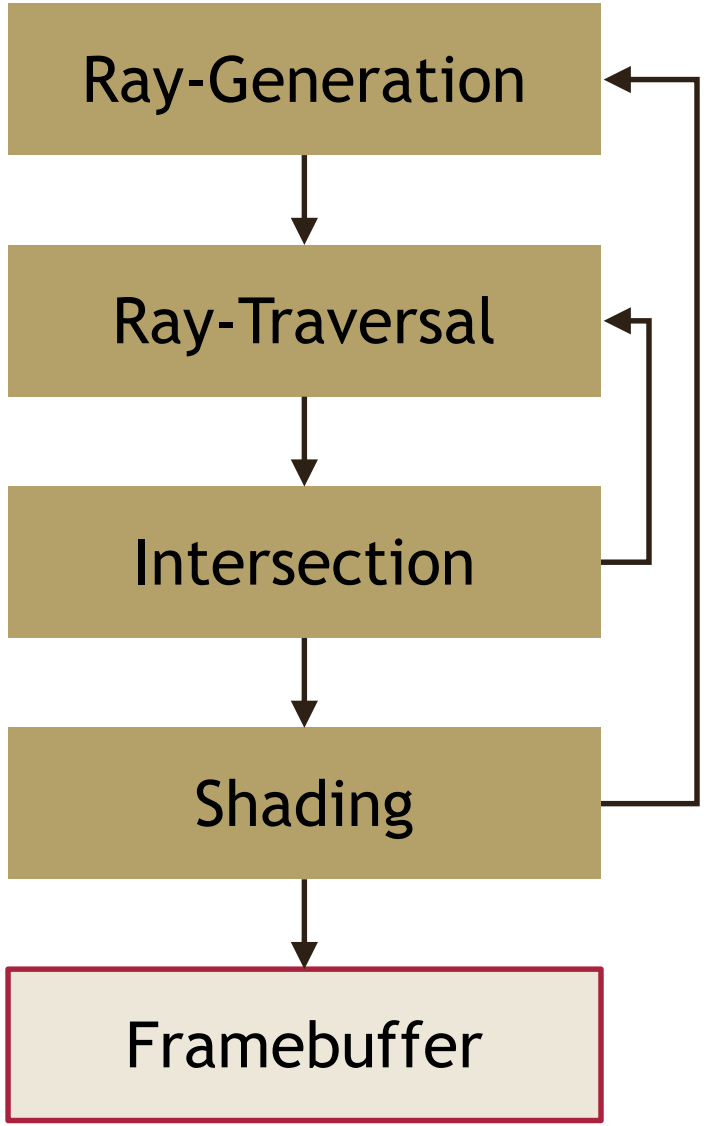
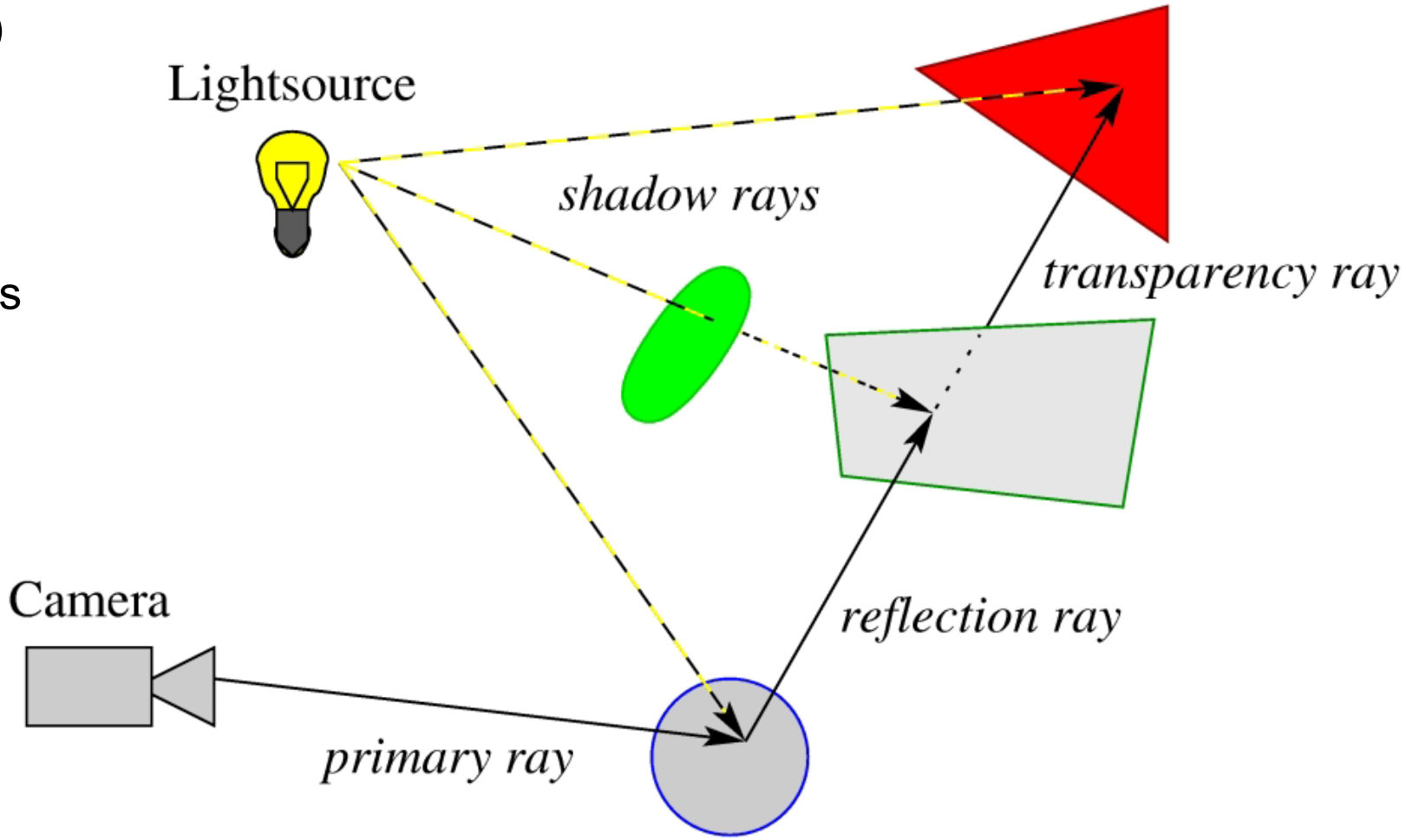




Ray Tracing

- Global effects
- Parallel (as nature)
- Fully automatic
- Demand driven
- Per pixel operations
- Highly efficient



Ray Tracing

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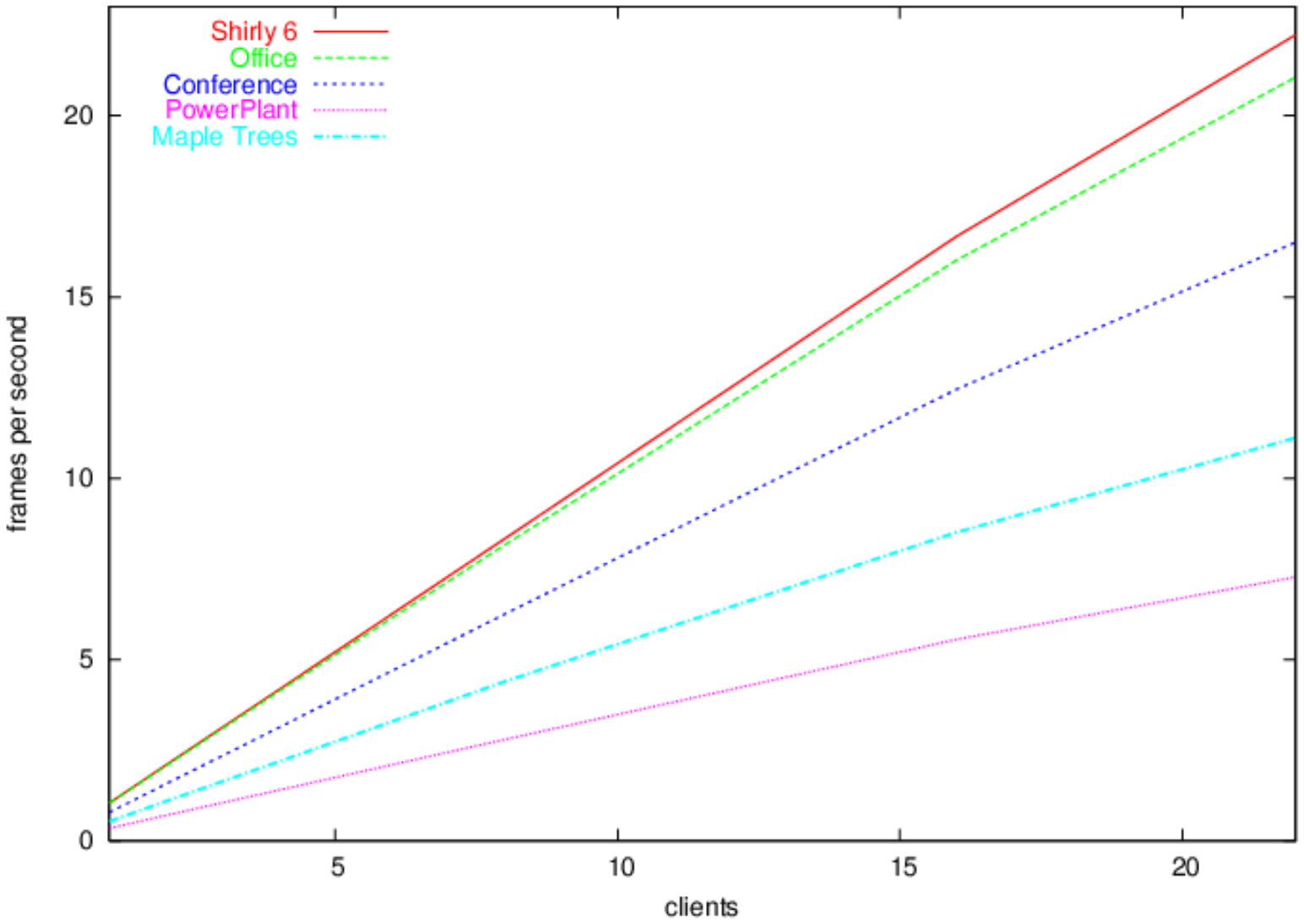
- Offline rendering
  - Only used as an off-line technique
  - Was computationally far too demanding
  - Rendering times of minutes and hours

- Interactive ray tracing (started around 2000)
  - Interactive ray tracing on supercomputers [Parker, U. Utah‘98]
  - Interactive ray tracing on PCs [Wald‘01]
  - Distributed ray tracing on PC clusters [Wald’01]
  - Ray tracing as cloud computing

Scalability

- Highly Scalable
  - Linear in number of
    - Pixels
    - Rays
    - Processors



Enabled by Ray Tracing

- Models Physics of Global Light Transport
  - Dependable, physically-correct visualization

Huge & Realistic 3D Models

Outdoor environment:
~365,000 plants,
~1.5 billion triangles

Rendered in realtime
with skylight illum.
on PC cluster

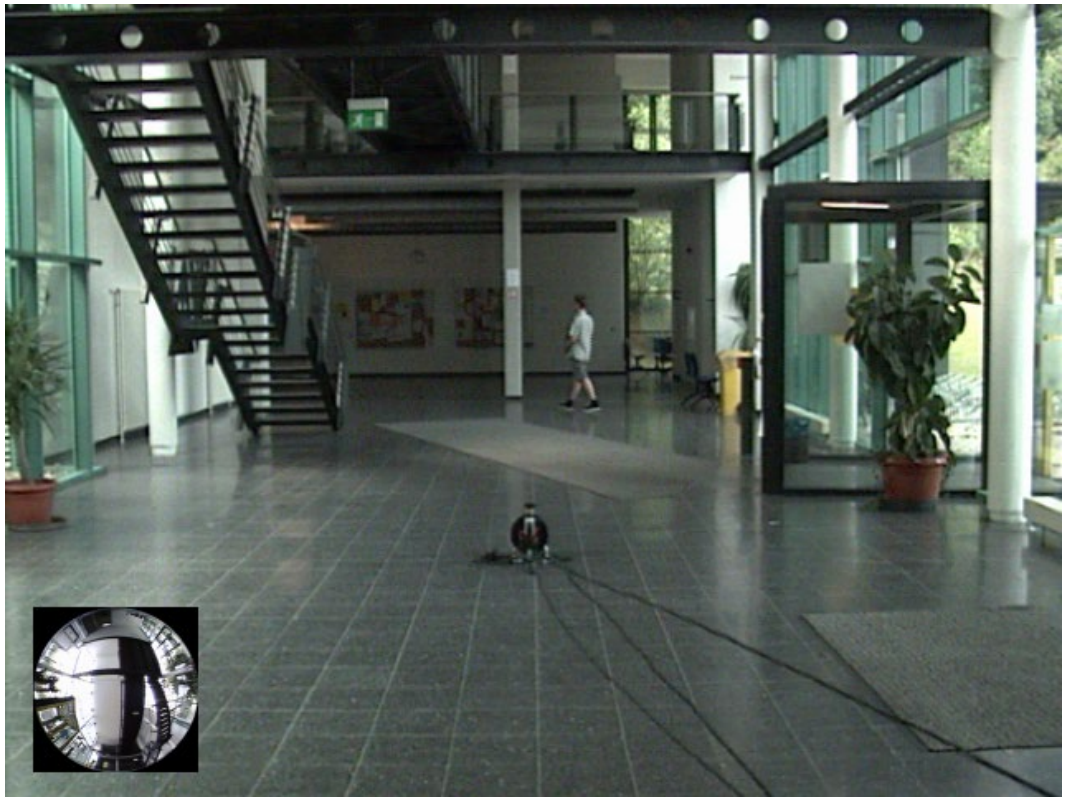
Realtime Lighting Simulation

Measured Materials

- Texture and reflection

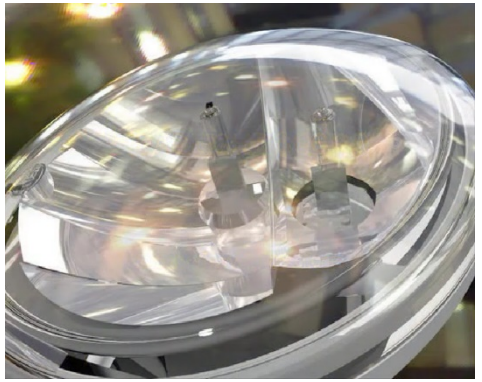
Realistic Visualization: VR/AR

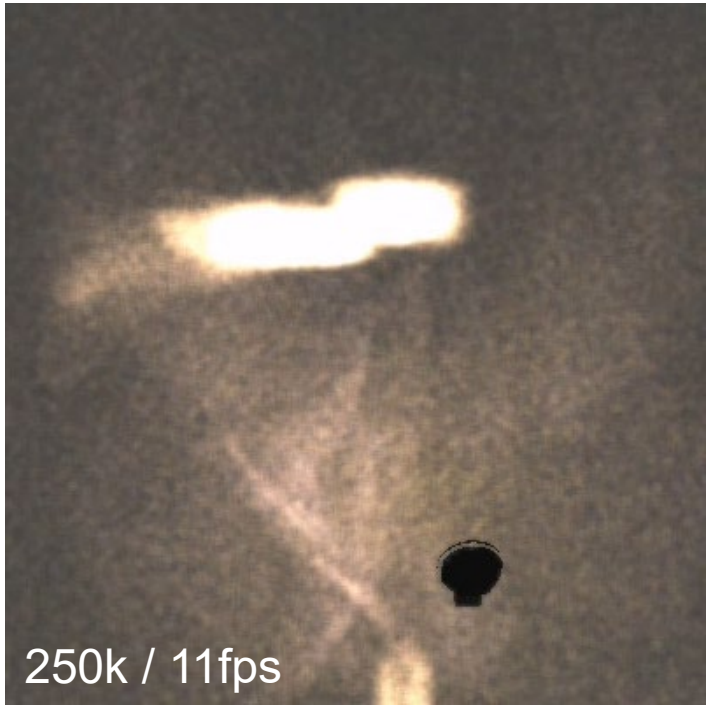
- Lighting measured from environment

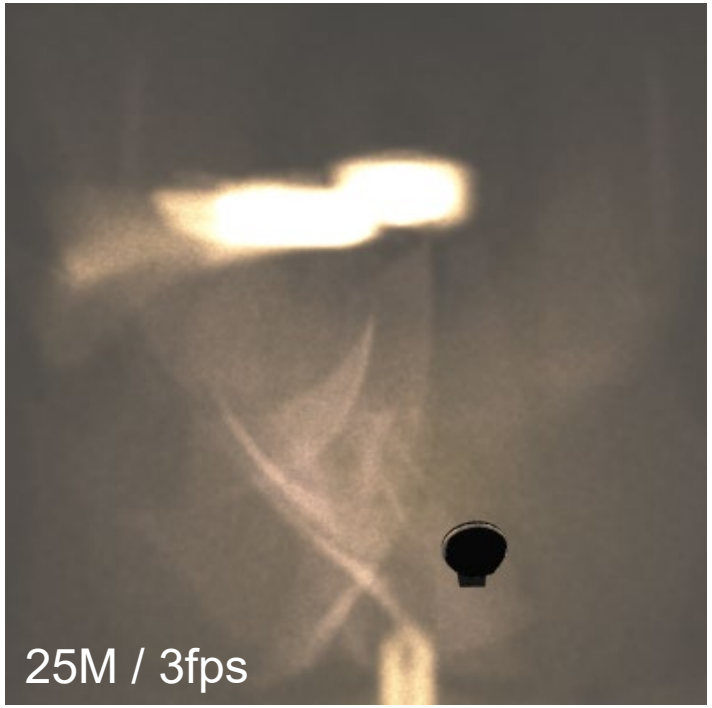


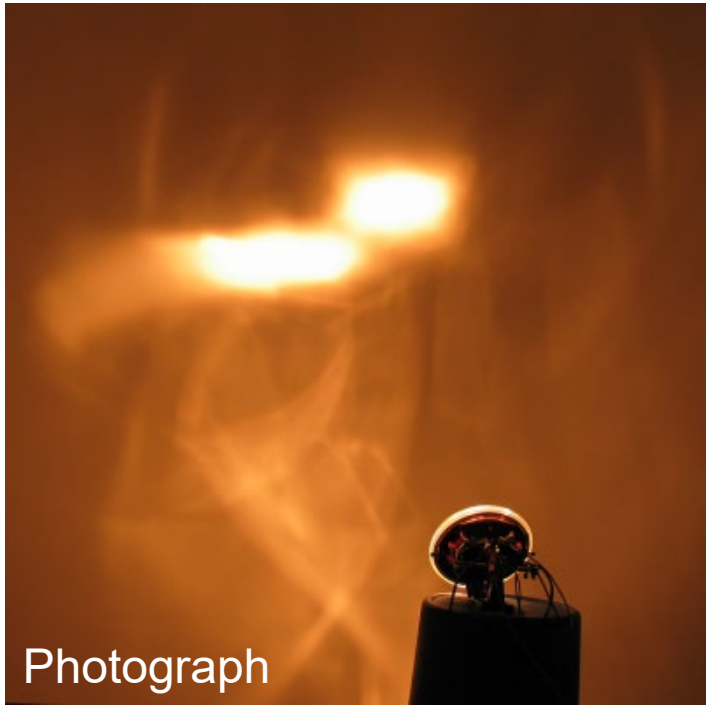
Lighting Simulation

- Complex scattering
- Highly accurate results



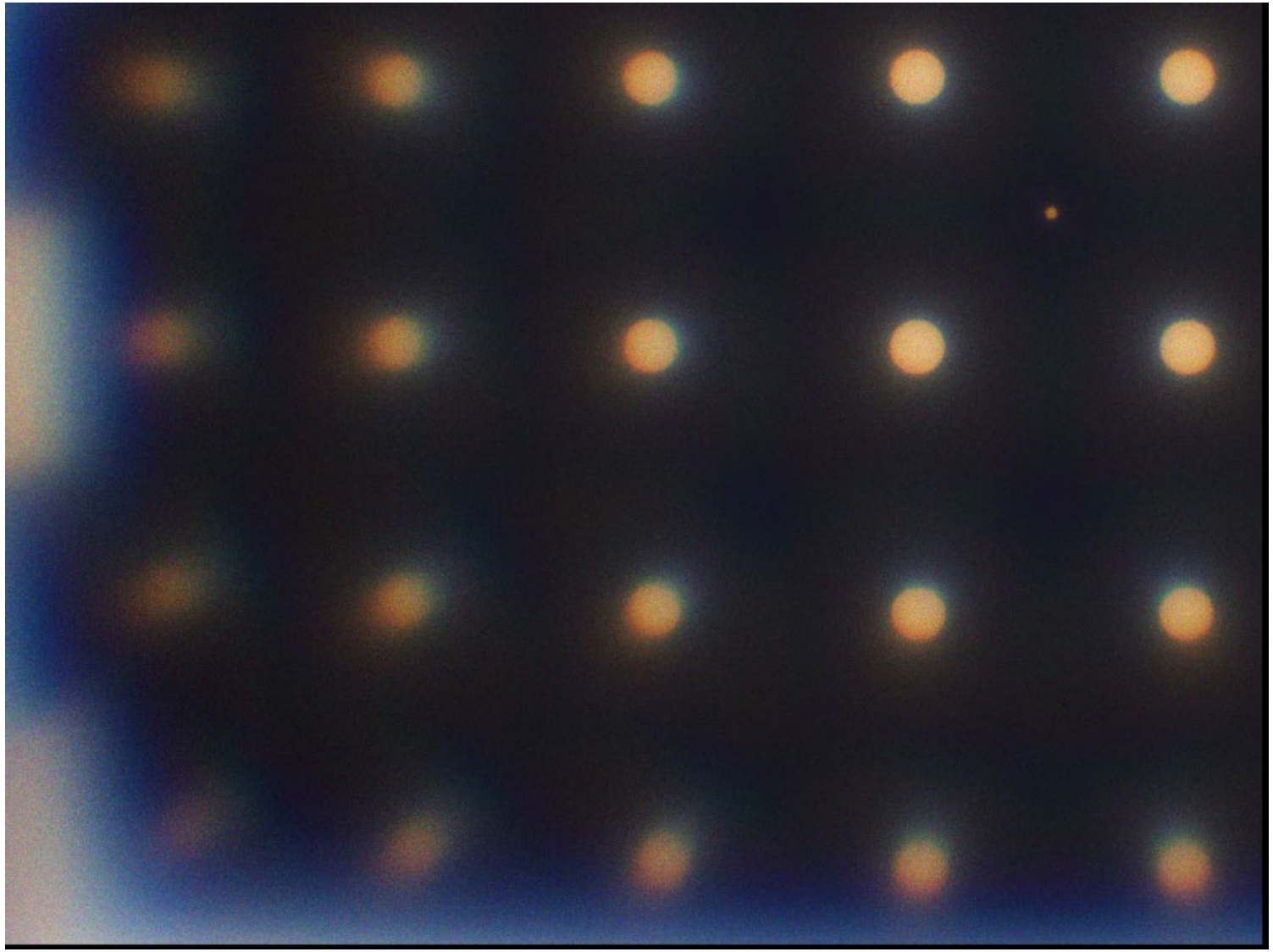






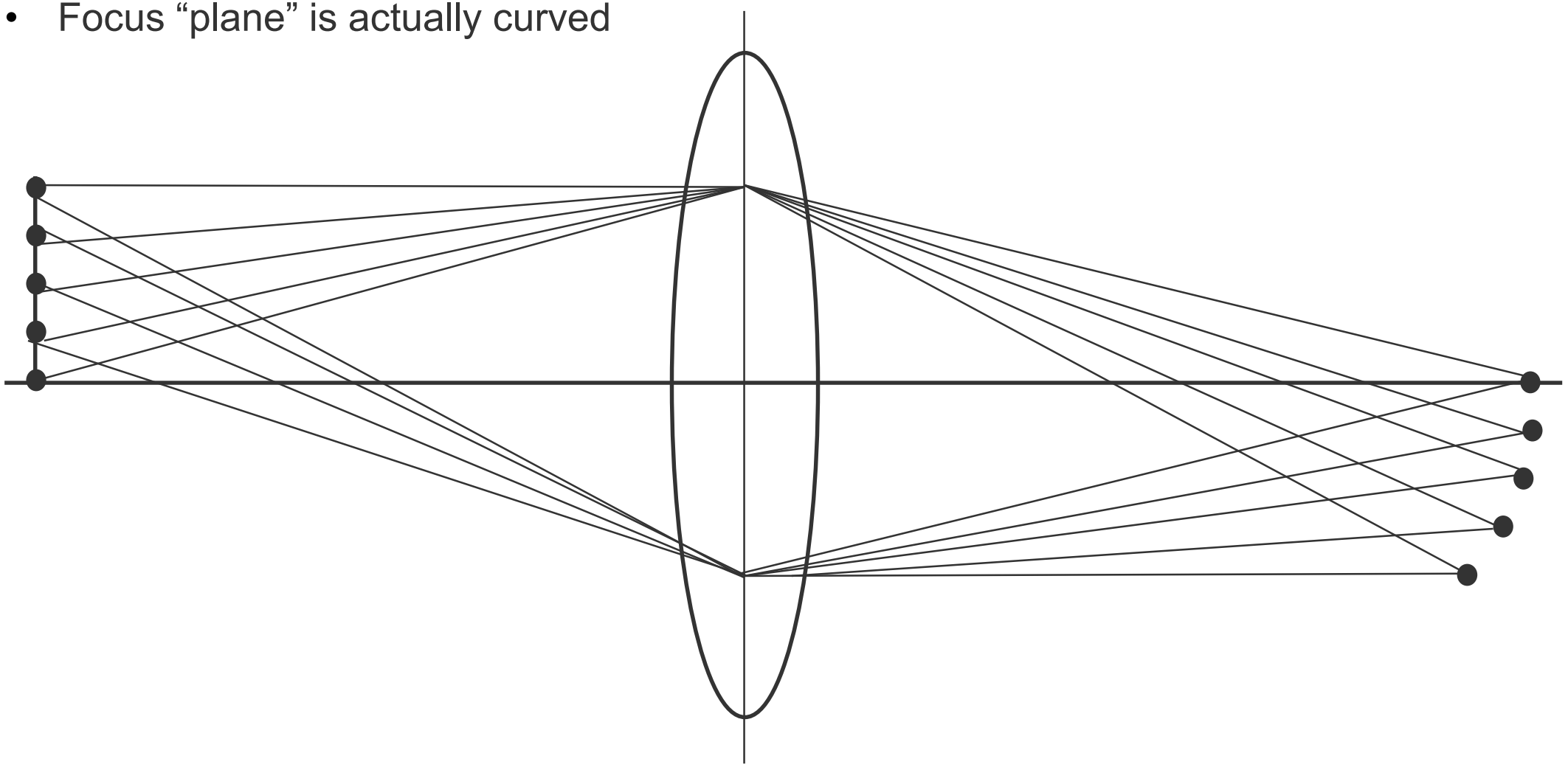
Camera Simulation - Dispersion

- Curvature of field
- Coma
- Chromatic aberration



Camera Simulation - Curvature of Field

- Focus “plane” is actually curved



Camera Simulation – Artificial Artifacts

- Bloom
- Smear

Lens Simulation

- Focus
- Depth of Field

Fundamental Ray Tracing Steps

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- Generation of primary rays
  - Rays from viewpoint along viewing directions into 3D scene
  - (At least) one ray per picture element (pixel)

- Ray tracing
  - Traversal of spatial index structures
  - Intersection of ray with scene geometry

- Shading
  - From intersection, determine “light color” sent along primary ray
  - Determines “pixel color”
  - Needed
    - Local material color and reflection properties
      - Object texture
    - Local illumination of intersection point
      - Can be hard to determine correctly

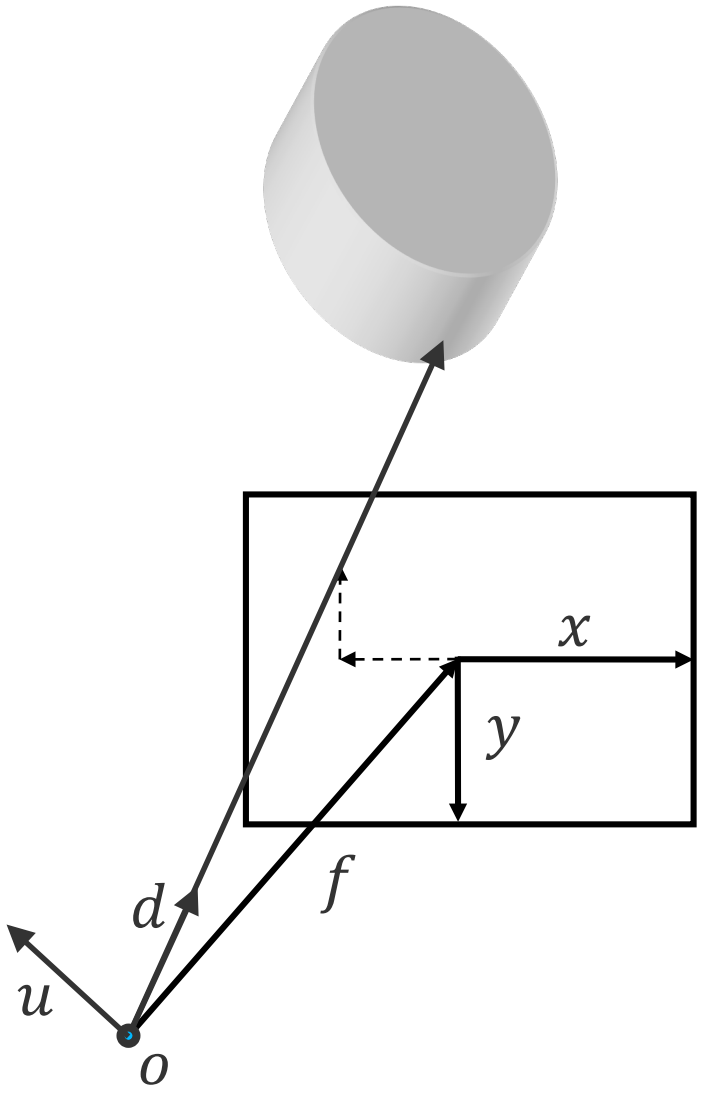
Intersections

Where does the ray hit an object?

Perspective Camera Model

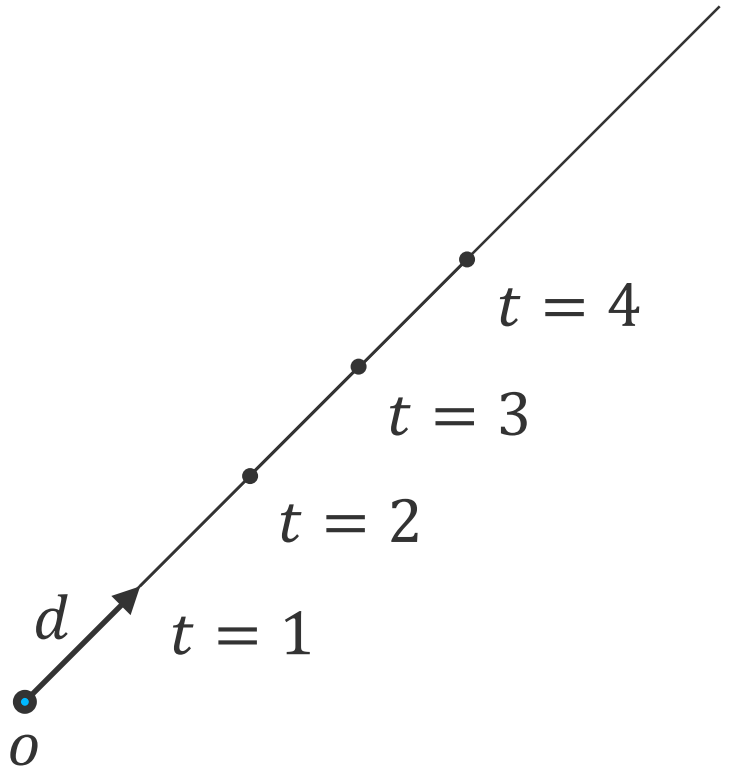
- Definition of the pinhole camera
  - o : Origin (point of view)
  - f : Vector to center of view (focal length)
  - u : Up-vector of camera orientation, in one plane with y vector
  - x, y : Span half the viewing window (frustum) relative to coordinate system (o, f, u)
  - $xres, yres$: Image resolution

```
for j in range(yres):
    for i in range(xres):
        d = f+2*(i/xres-0.5)*x
              +2*(j/yres-0.5)*y
        d = d/norm(d) # normalize
        col = trace(o, d)
        write_pixel(i, j, col)
```



Ray and Object Representation

- Ray in space: $\mathrm{r}(t) = o + td$
  - $o = (o_x, o_y, o_z)^T$
  - $d = (d_x, d_y, d_z)^T$



ray parameterization

Ray and Object Representation

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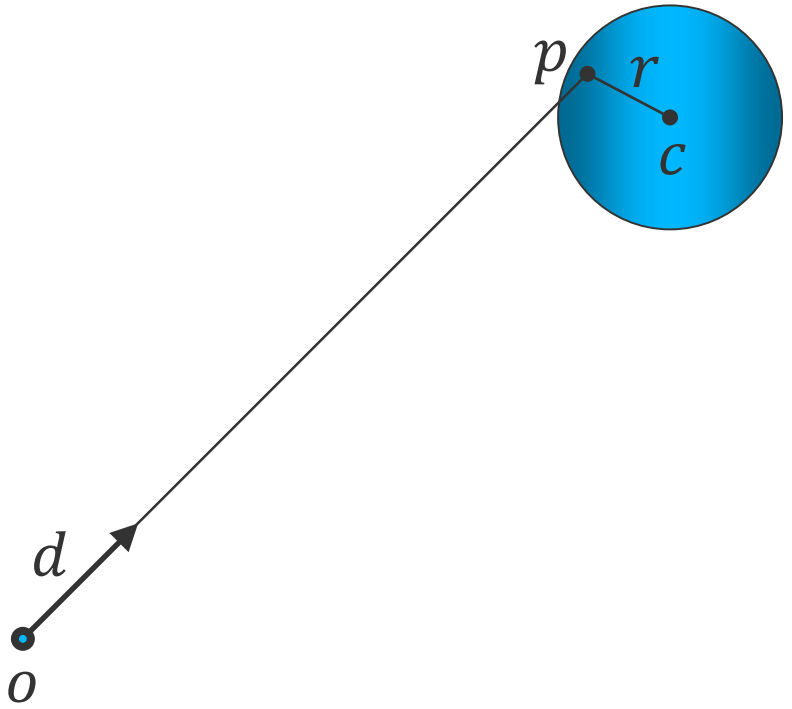
- Scene geometry
  - Sphere: $0 = (p - c)\cdot(p - c) - r^2$
    - c : sphere center
    - r : sphere radius
    - p : any surface point



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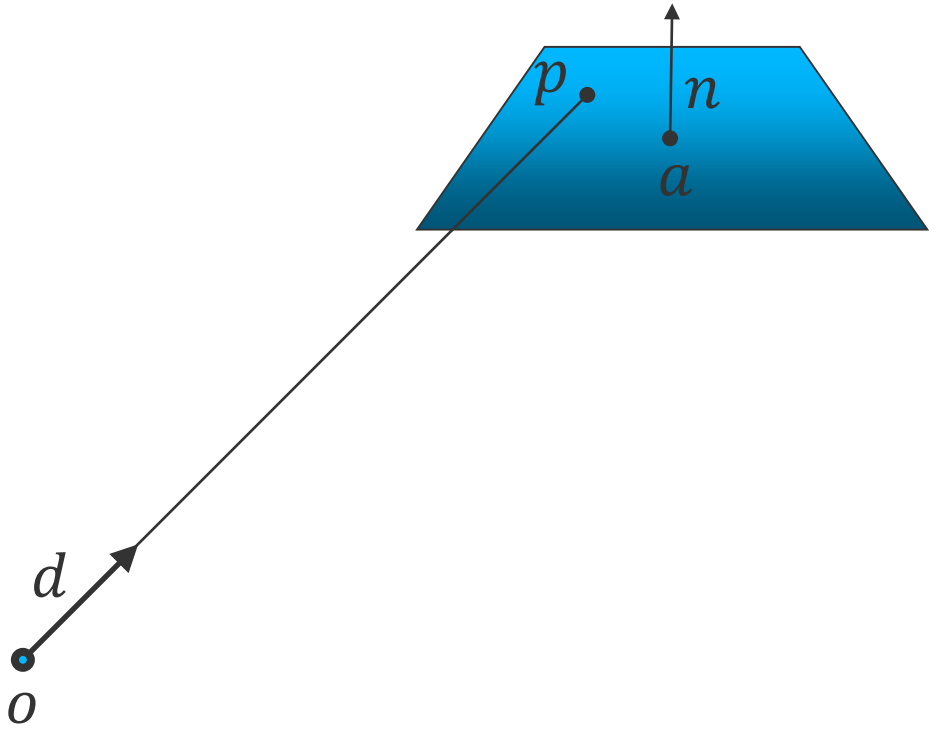
  - Plane: $0 = (p - a)\cdot n$
    - Implicit definition
    - n : surface normal
    - a : one given surface point
    - p : any surface point



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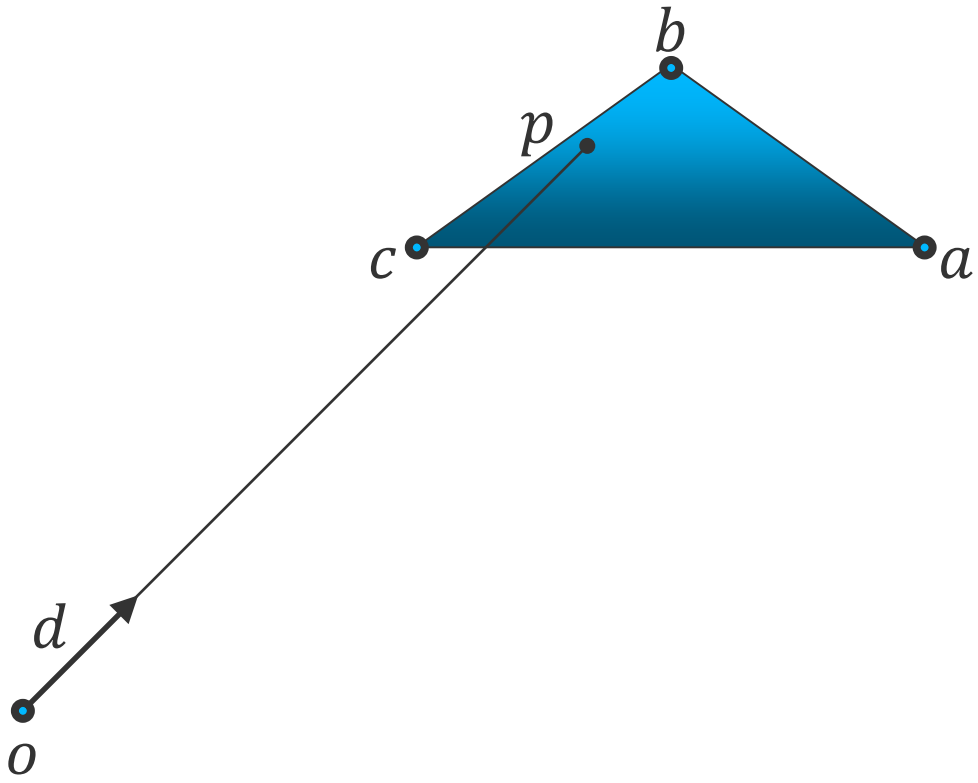
  - Triangle:
    - Plane intersection
      plus barycentric coordinates



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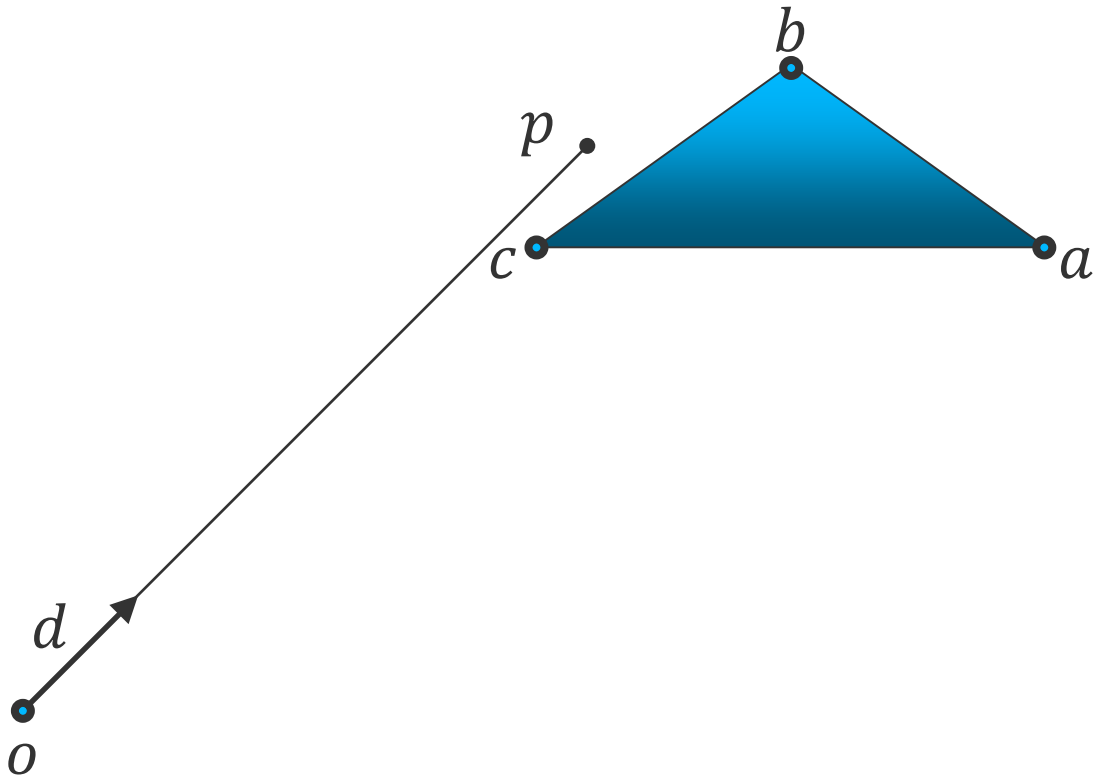
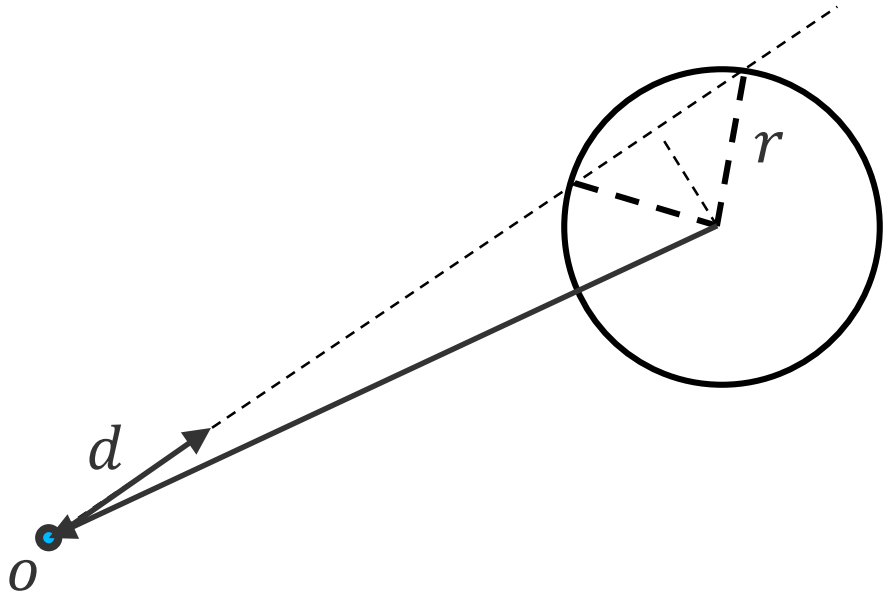


Intersection Ray – Sphere

- Sphere
  - Given a unit sphere at the origin $0 = x^2 + y^2 + z^2 - 1$
  - Given a ray $r(t) = o + td$

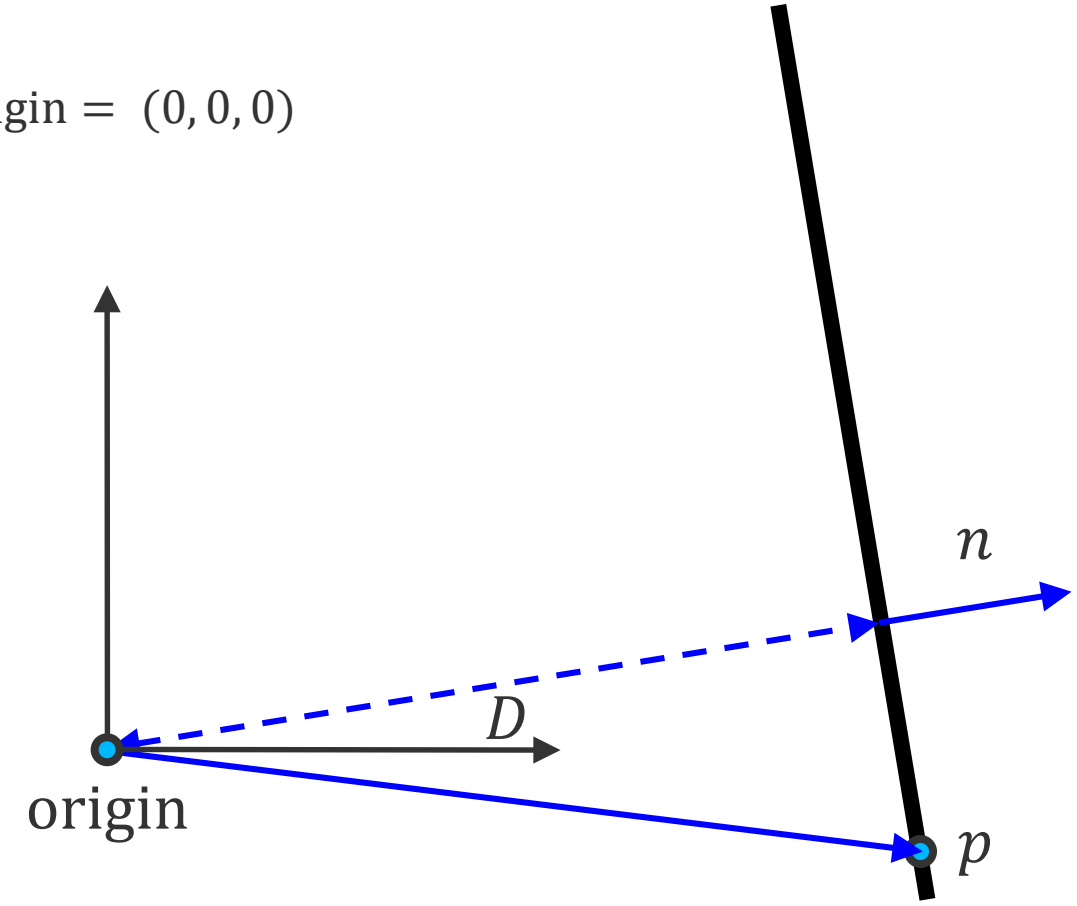
  - Substituting the ray into the equation for the sphere gives
    - $0 = t^2\left(d_x^2 + d_y^2 + d_z^2\right) + 2t\left(o_x d_x + o_y d_y + o_z d_z\right) + \left(o_x^2 + o_y^2 + o_z^2\right) - 1$
    - Easily solvable with standard techniques
    - But beware of numerical imprecision

  - Alternative: Geometric construction
    - Ray and center span a plane
    - Simple 2D construction



Intersection Ray – Plane

- Plane: Implicit representation (Hesse form)
  - Plane equation: $0 = p \cdot n - D$
    - n : Normal vector $|n| = 1$
    - p : Point on the plane
    - D : Normal distance of plane from origin $= (0, 0, 0)$



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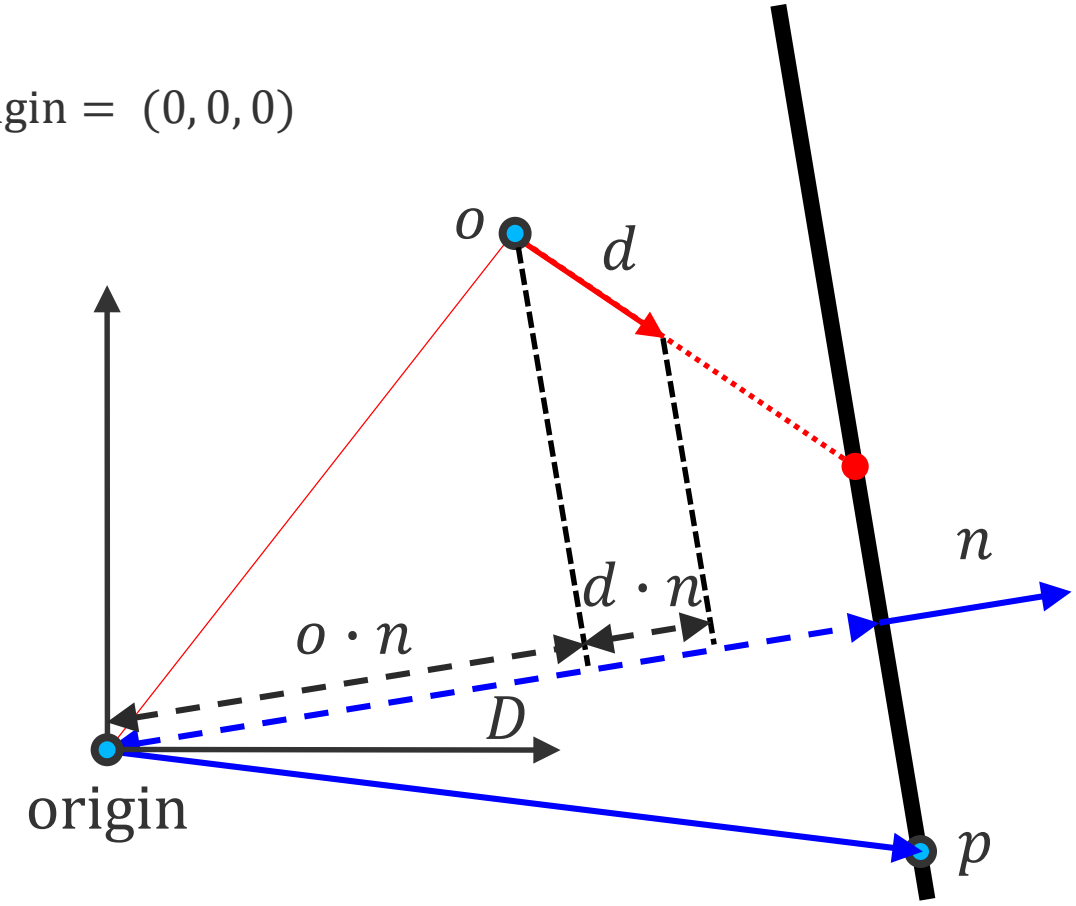
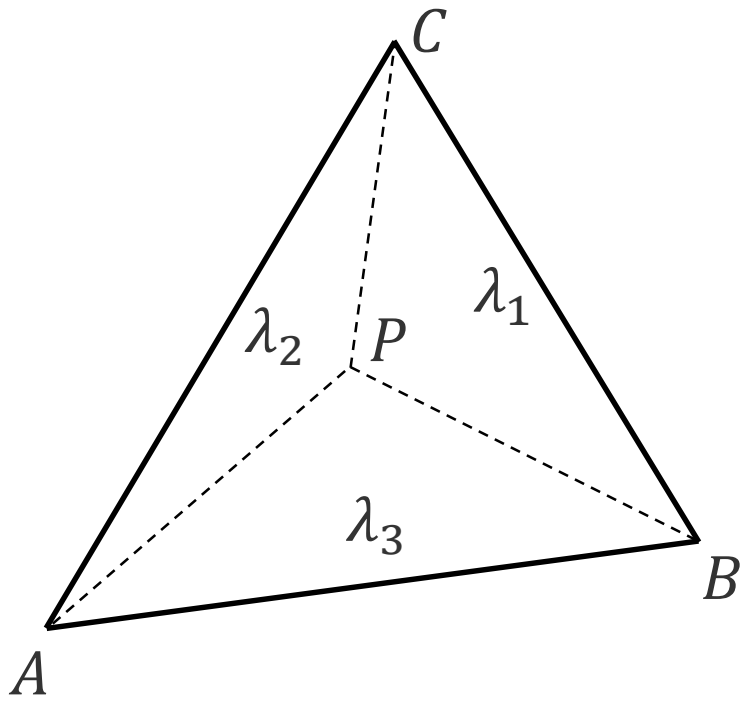
- Two possible approaches
  - Geometric
  - Algebraic
    - Substitute ray $\mathrm{r(t)} = o + td$ for p
    - $(o + td) \cdot n - D = 0$
    - Solving for t gives $t = \frac{D - o \cdot n}{d \cdot n}$



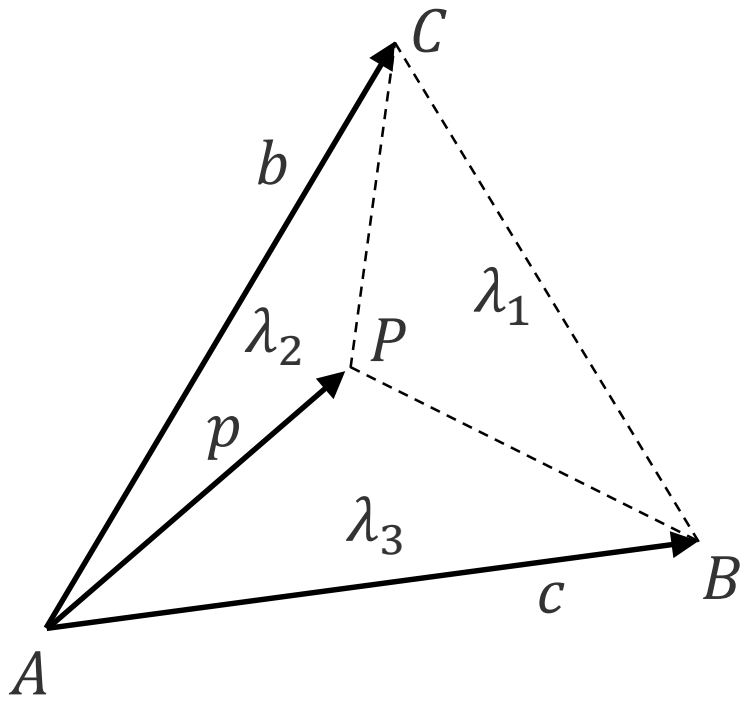
Intersection Ray – Triangle (1)

- Barycentric coordinates
  - Non-degenerate triangle ABC
  - Ratio of signed areas: $\lambda_1 = \Delta(BCP)/\Delta(ABC)$
    $\lambda_2 = \Delta(CAP)/\Delta(ABC)$
    $\lambda_3 = \Delta(ABP)/\Delta(ABC)$
  - Every point P in the plane can be described using
    - $P = \lambda_1 A + \lambda_2 B + \lambda_3 C$
  - $\boldsymbol{\lambda_1 + \lambda_2 + \lambda_3 = 1}$
    - For fixed λ_3, P may move parallel to AB
    - For $\gamma_1 + \gamma_2 = 1$ and $0 < \gamma_3 < 1$
      - $P = (1 - \gamma_3)(\gamma_1 A + \gamma_2 B) + \gamma_3 C$
    - P moves between C and AB
- Point is in triangle, iff all $0 \leq \lambda_i$



Calculation of Barycentric Coordinates 2D

- Hitpoint on 2D Plane P
  - $b = C - A$
  - $c = B - A$
  - $p = P - A$
  - $\lambda_2 = \frac{b_x p_y - b_y p_x}{b_x c_y - b_y c_x}$
  - $\lambda_3 = \frac{p_x c_y - p_y c_x}{b_x c_y - b_y c_x}$
  - $\lambda_1 = 1 - \lambda_2 - \lambda_3$



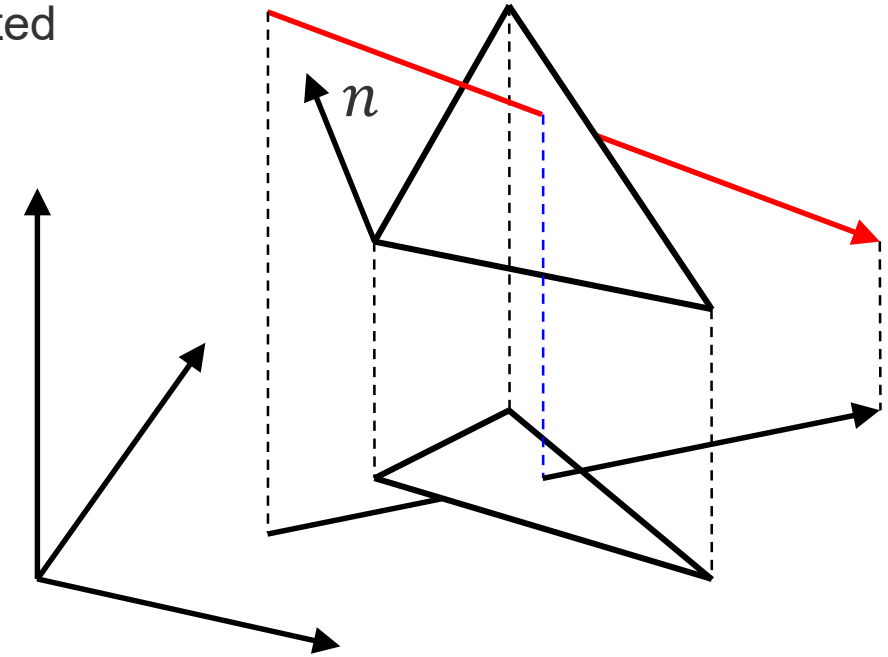
Intersection Ray – Triangle (1)

- Compute intersection with triangle plane

- Given the 3D intersection point
  - Project point into xy, xz, yz coordinate plane
  - Use coordinate plane that is most aligned
    - xy: if n_z is maximal, etc.
  - Coordinate plane and 2D vertices can be pre-computed

- Compute barycentric coordinates in 2D

- Test for positive BCs



Intersection Ray – Triangle (1)

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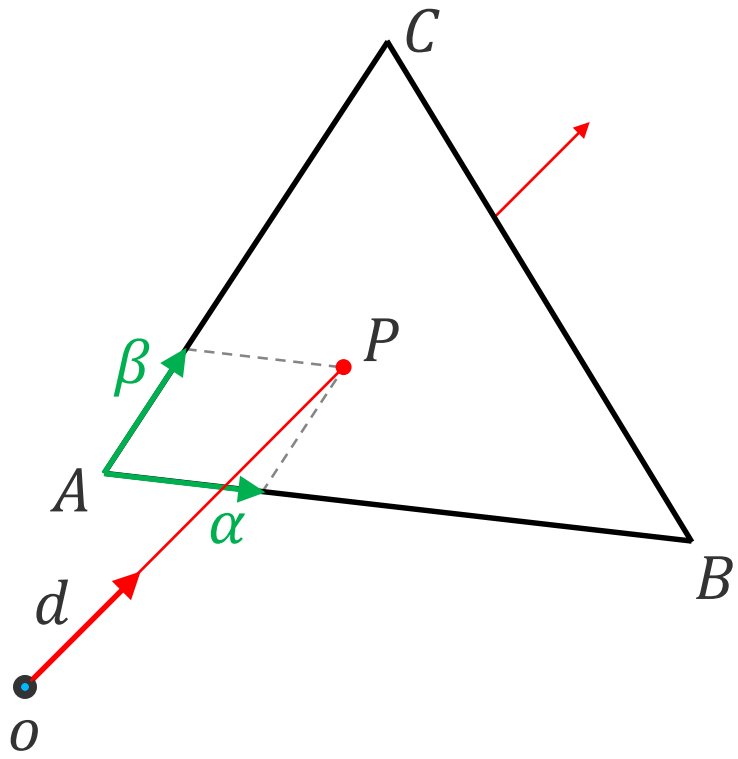
```
# calc edges and normal
b = B-A
c = C-A
n = cross(c,b) # normalize
D = dot(A,n)

# distance test
t_plane = (D-dot(o,n)) / dot(d,n)
if (t_plane < eps or t_plane > t_max):
    return NO_HIT

# determine projection dimensions
if (abs(n[0]) > abs(n[1])):
        #x                                  #z
    k = 0 if (abs(n[0]) > abs(n[2])) else 2
else:
        #y                                  #z
    k = 1 if (abs(n[1]) > abs(n[2])) else 2

u = (k+1) % 3
v = (k+2) % 3
```

```
# calculate hitpoint
p[u] = O[u] + t_plane * d[u] - A[u]
p[v] = O[v] + t_plane * d[v] - A[v]

beta = (b[u]*p[v] - b[v]*p[u]) /
       (b[u] * c[v] - b[v] * c[u])

if beta < 0:
    return NO_HIT

gamma = (c[v]*p[u]-c[u]*p[v] /
        (b[u] * c[v] - b[v] * c[u])

if gamma < 0:
    return NO_HIT

if (beta + gamma) > 1:
    return NO_HIT;

return HIT(t_plane, beta, gamma)
```

Intersection Ray – Triangle (2)

- Check if point is inside triangle parametrically
- Compute α, β
  - $P = \alpha(B - A) + \beta(C - A)$
- Check if point inside triangle
  - $0 \leq \alpha$
  - $0 \leq \beta$
  - $0 \leq \alpha + \beta \leq 1$

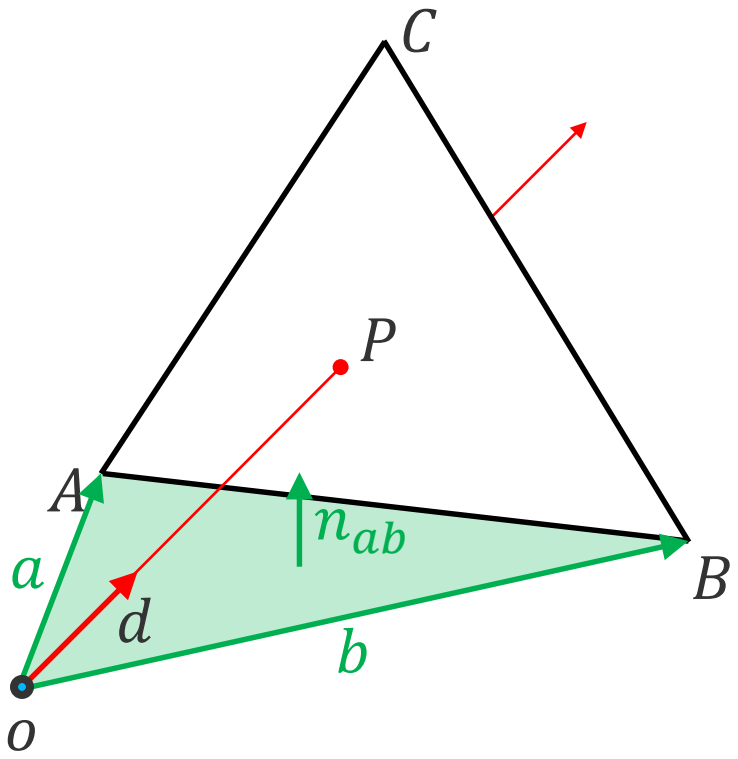


Intersection Ray – Triangle (3)

- Check if point is inside triangle algebraically

- Compute
  - $\langle a, b, c \rangle = \langle A, B, C \rangle - o$
  - $n_{ab} = \frac{a \times b}{|a \times b|}$
  - $n_{bc} = \frac{b \times c}{|b \times c|}$
  - $n_{ca} = \frac{c \times a}{|c \times a|}$

- for $i \in (ab, bc, ca)$:
  - $s_i = o - n_i$
  - if $P \cdot n_i + s_i < 0$:
    - return false



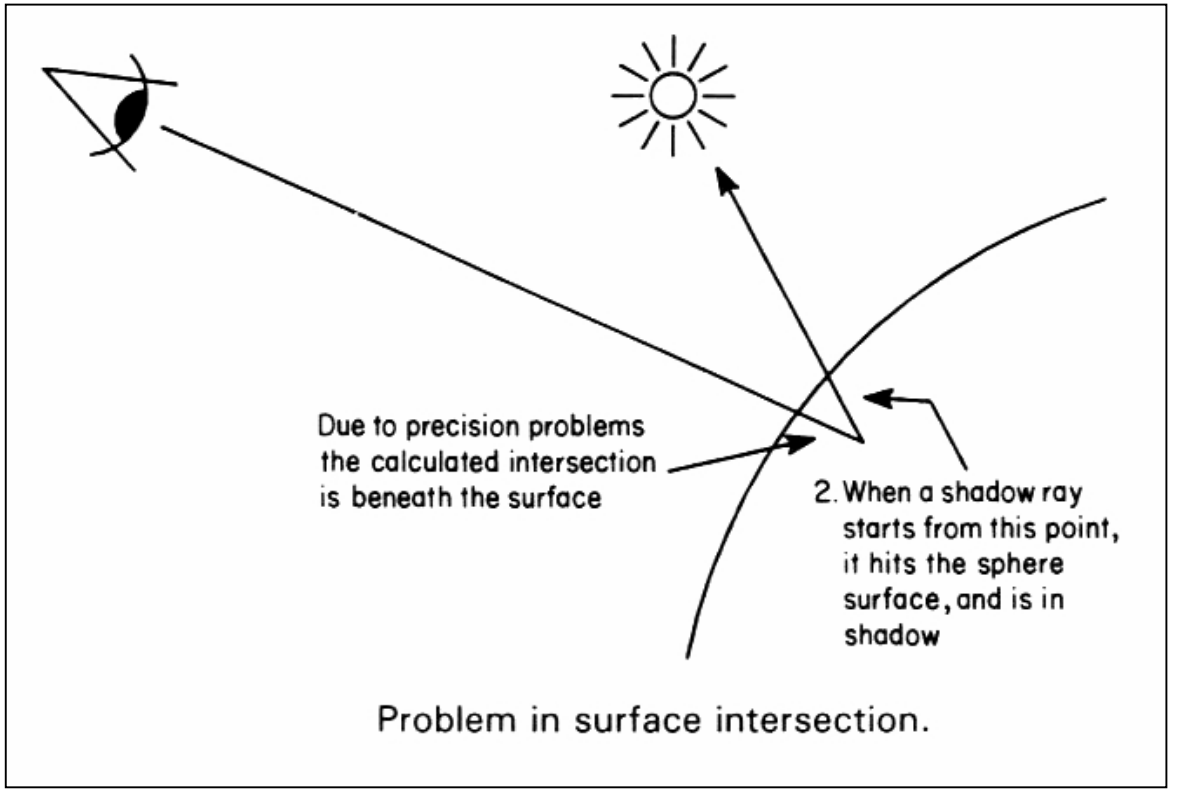
- P is in the triangle if n_{ab}, n_{bc}, n_{ca} all point towards the inside of the triangle

Precision Problems

- Due to floating point arithmetic
- Shadow rays might start just below the surface, reporting the surface itself as occlude
- Solution:
  - check if the value of parameter t is within some tolerance

    $abs(t) < \varepsilon$ with $\varepsilon = 0.000001$

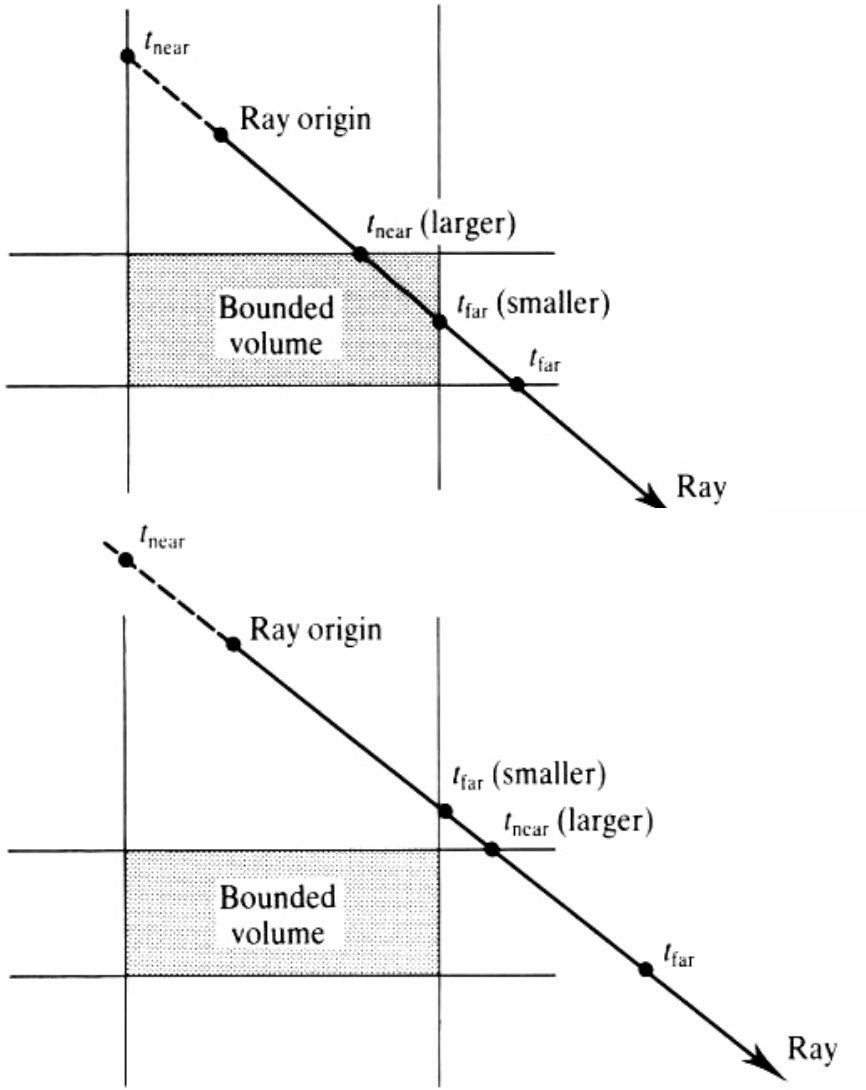
    (Pseudo intersection)



Problem in surface intersection.

Intersection Ray – Box

- Boxes are important for
  - Bounding volumes
  - Hierarchical structures
- Intersection test
  - test pairs of parallel planes in turn
  - calculate intersection distances
    - t_{near} first plane
    - t_{far} second plane
- The ray does not intersect the box if
  - $t_{near} > t_{far}$ for one pair of planes



History of Intersection Algorithms

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- Ray-geometry intersection algorithms
  - Polygons: [Appel ’68]
  - Quadrics, CSG: [Goldstein & Nagel ’71]
  - Recursive Ray Tracing: [Whitted ’79]
  - Tori: [Roth ’82]
  - Bicubic patches: [Whitted ’80, Kajiya ’82]
  - Algebraic surfaces: [Hanrahan ’82]
  - Swept surfaces: [Kajiya ’83, van Wijk ’84]
  - Fractals: [Kajiya ’83]
  - Deformations: [Barr ’86]
  - NURBS: [Stürzlinger ’98]
  - Subdivision surfaces: [Kobbelt et al ’98]
  - …

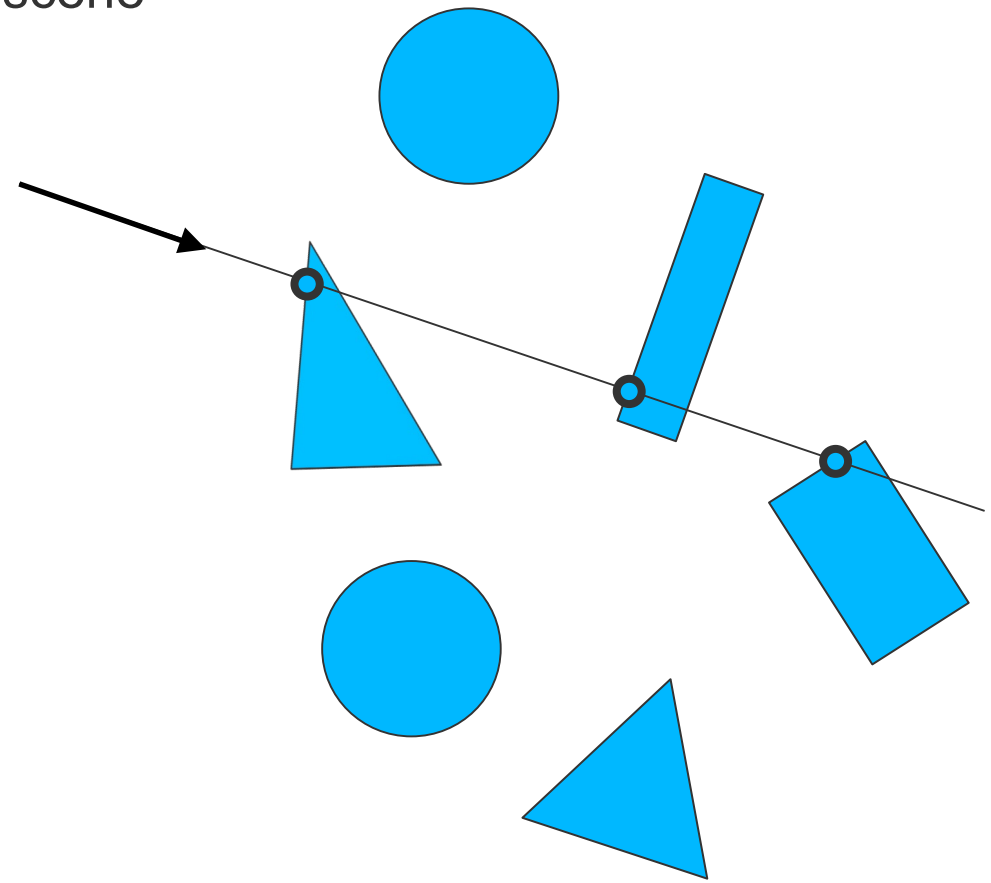
Intersection Ray – Scene

Which object is hit first?

Intersection Ray – Scene

- Find intersection with front-most surface in the scene
- Naïve algorithm:

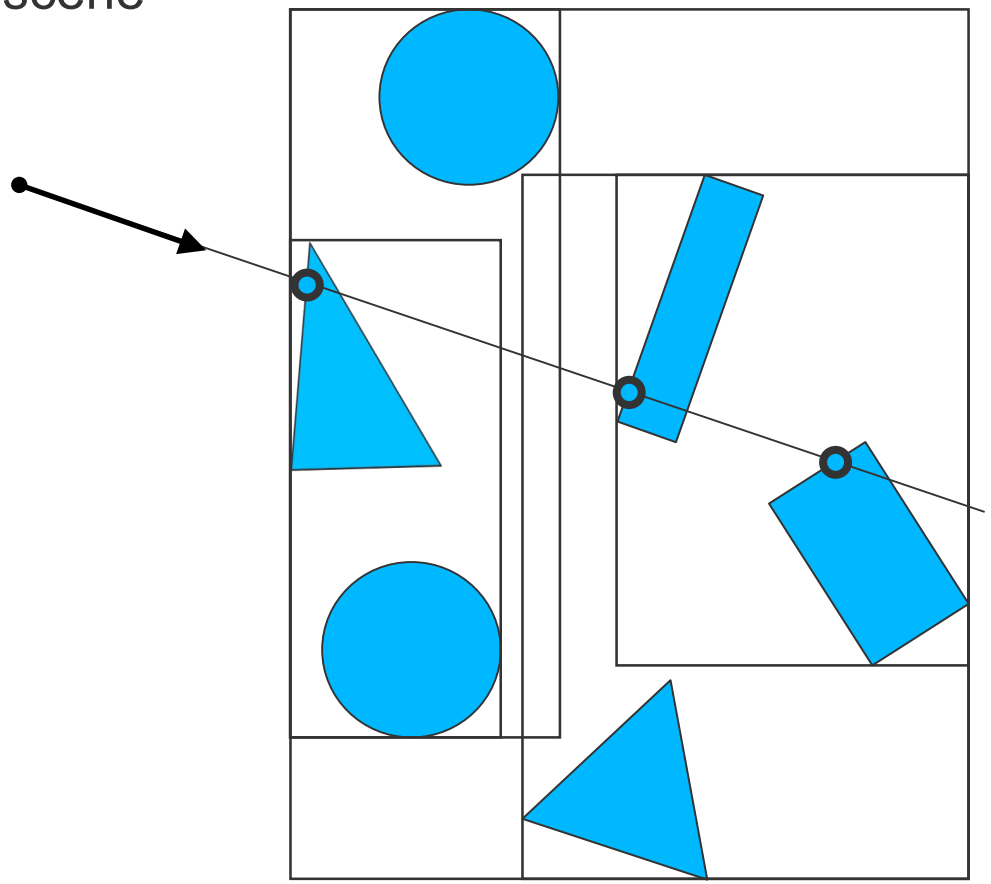
```
def findIntersection(ray, scene):
    min_t = infininty
    min_primitive = None
    for primitive in scene.primitives:
        t = intersect(ray, primitive)
        if t < min_t:
            min_t = t
            min_primitive = primitive
    return min_t, min_primitive
```



- More on acceleration follows later

Intersection Ray – Scene BBox

- Find intersection with front-most surface in the scene
- Cluster multiple primitives in bounding box
- More on acceleration follows later



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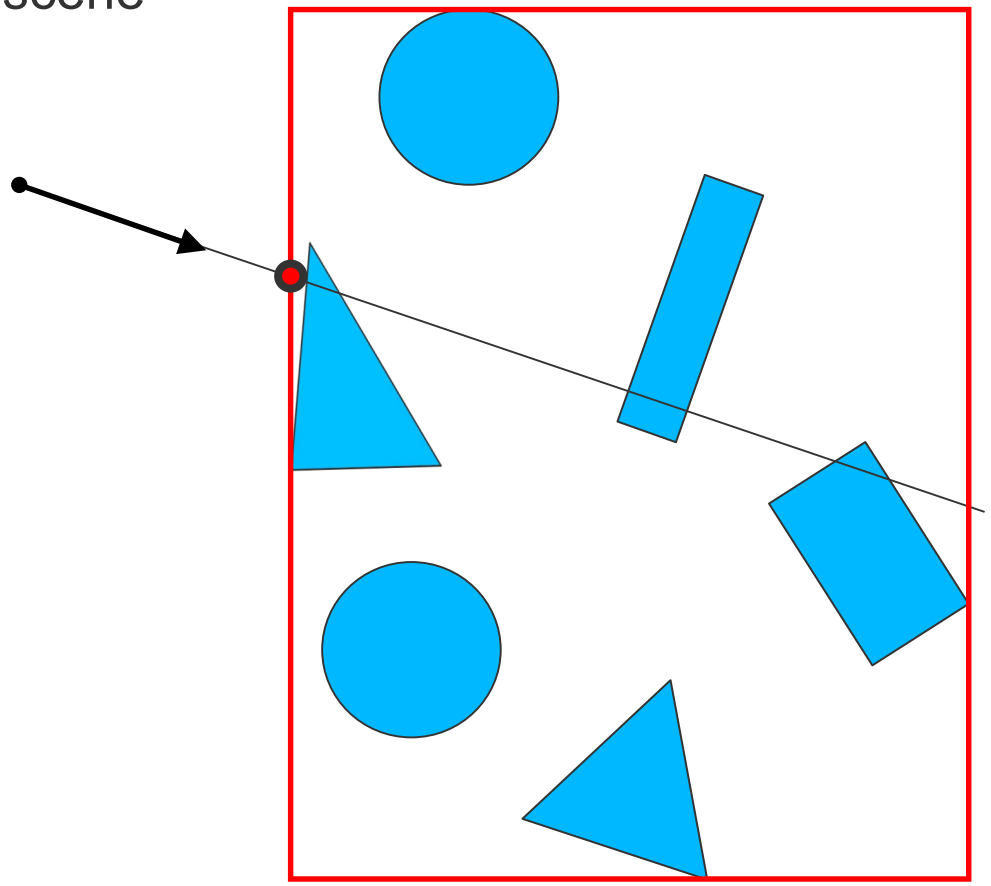

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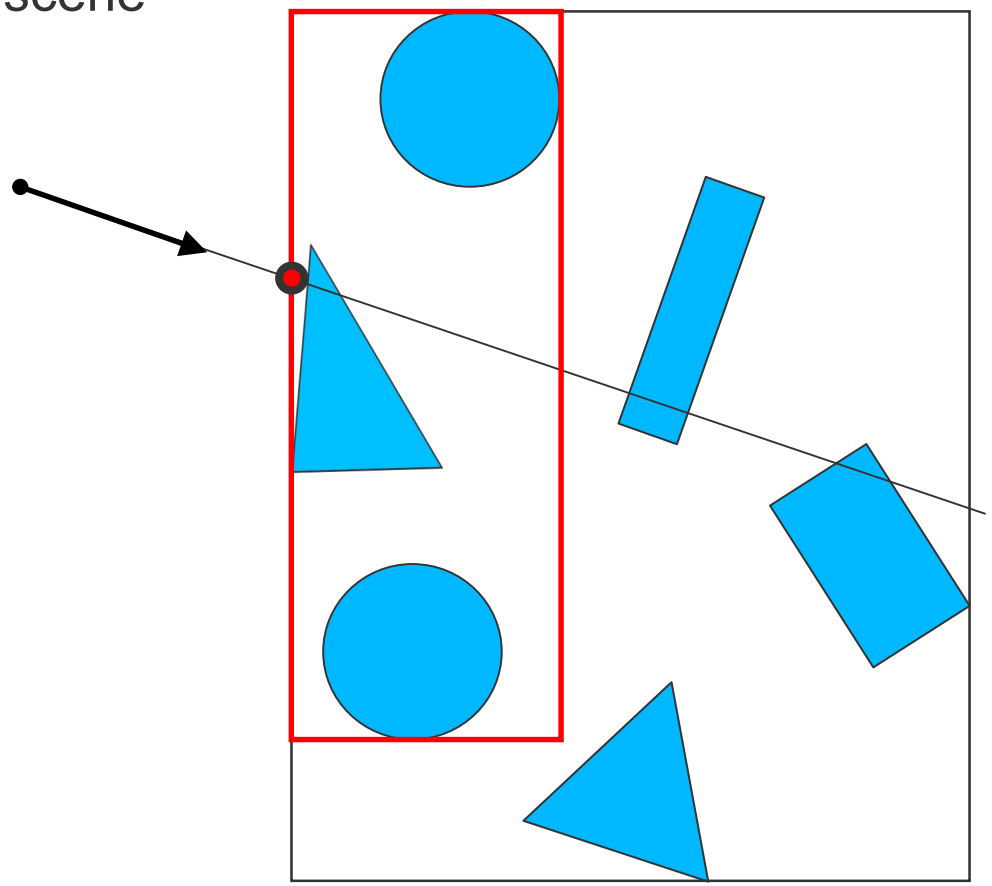

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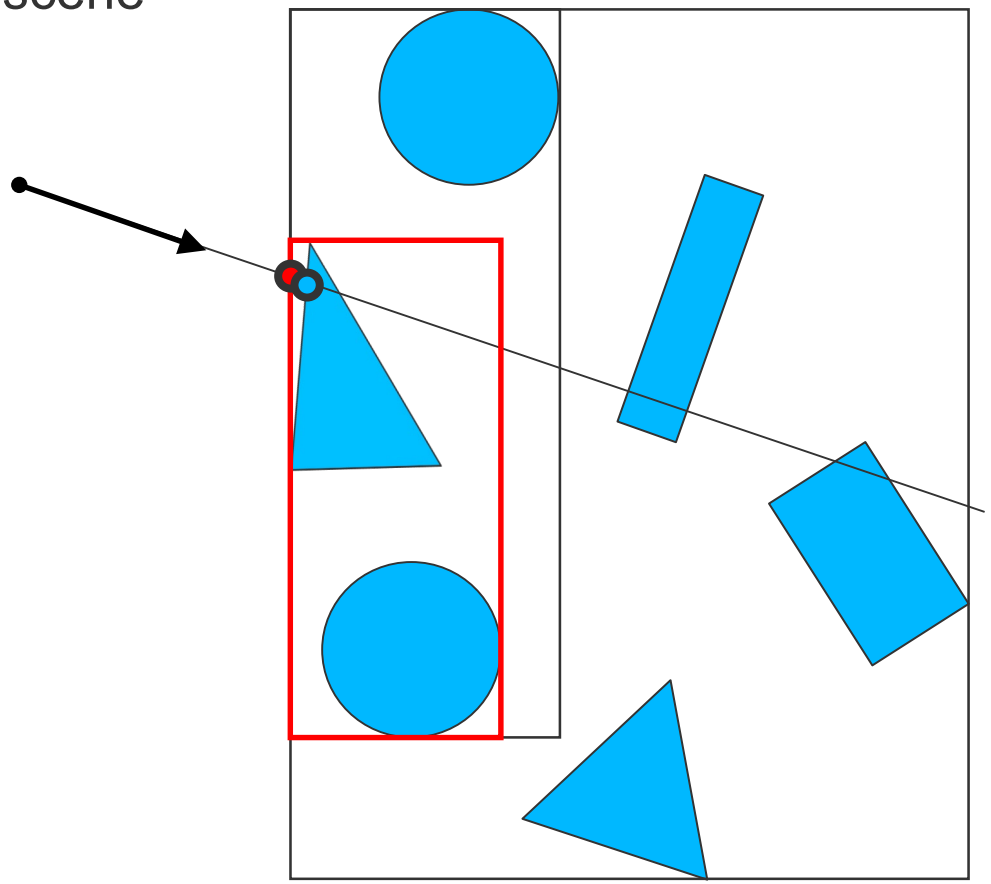

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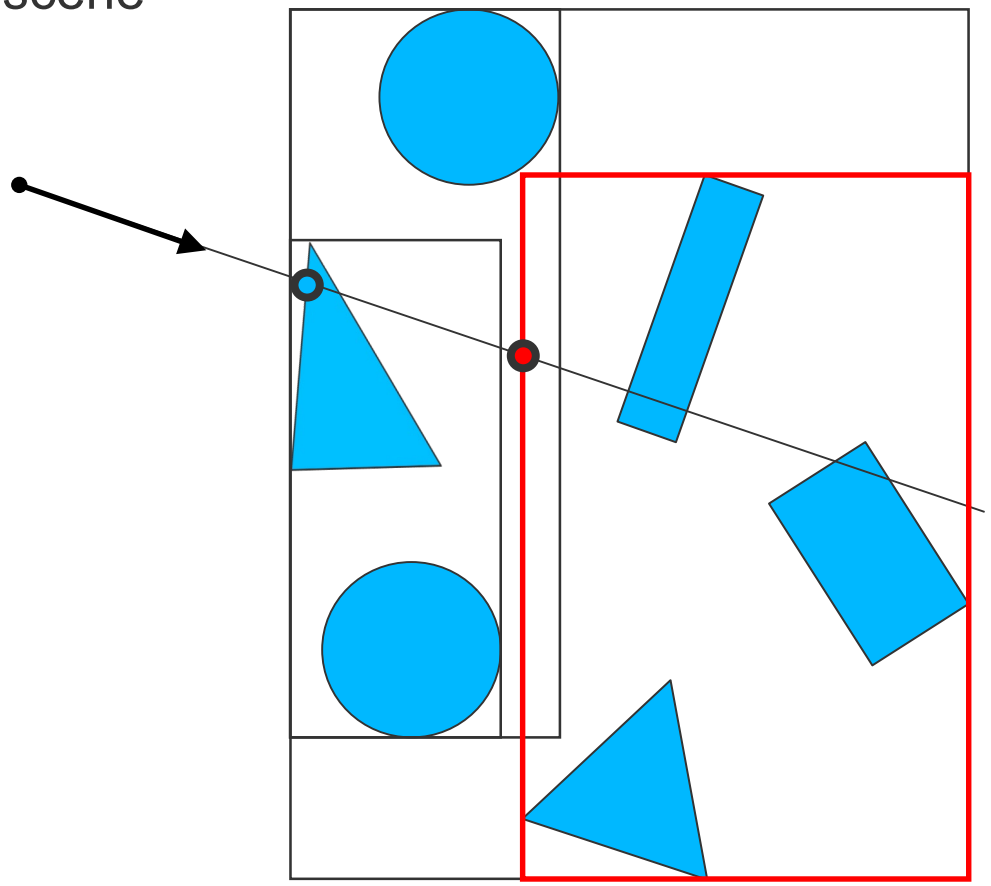

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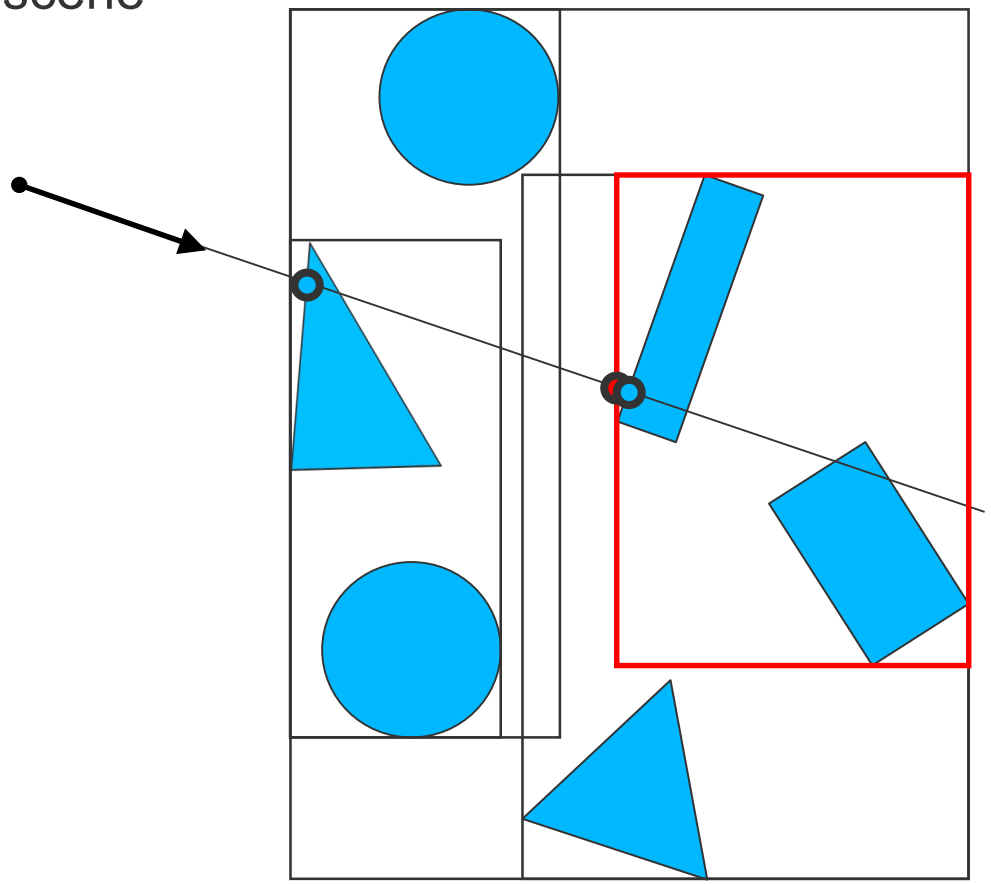
- Cluster multiple primitives in bounding box
- More on acceleration follows later



Shading

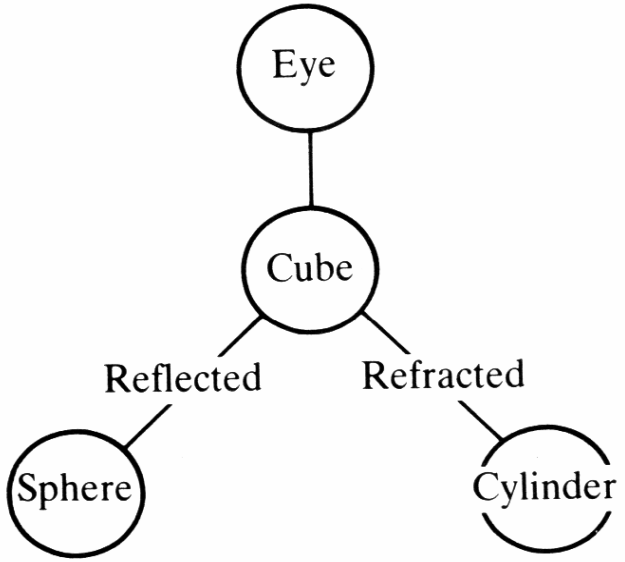
Which *color* do we observe along a ray?

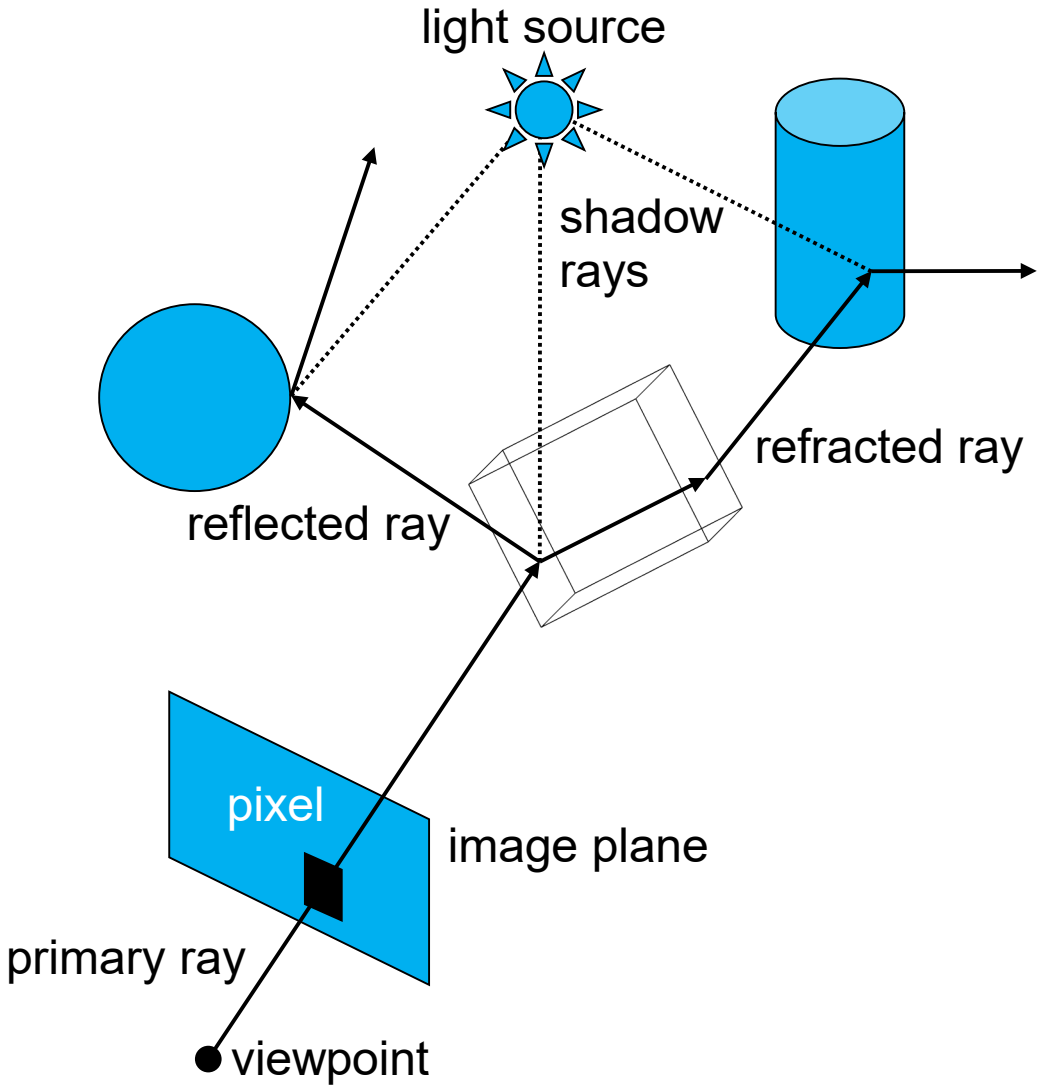
Shading

- Intersection point determines primary ray's *color*
- Diffuse object:
  - Color at intersection point
  - No variation with viewing angle (Lambertian)
  - Must still be illuminated
    - Point light source: shadow ray
    - Scales linearly with received light (Irradiance)
    - No illumination: in shadow = black
- Non-Lambertian Reflectance
  - Appearance depends on illumination and viewing direction
    - Local Bi-directional Reflectance Distribution Function (BRDF)
  - Simple cases:
    - Mirror, glass: secondary rays
- Area light sources and indirect illumination can be difficult

Recursive Ray Tracing

- Search recursively for paths to light sources
  - Interaction of light and material at each intersection point
  - Recursively trace new rays in reflection, refraction, and light direction





Ray Tracing Algorithm


```
def trace(ray, scene):
    hitpoint, material = intersect(ray, scene)
    return shade(ray, hitpoint, material, scene)

def shade(ray, hitpoint, material, scene):
    radiance = 0
    for light in scene.lights:
        rayToLight, distanceToLight = toLight(hitpoint, light)
        if shadowTrace(rayToLight, distanceToLight, scene):
            # add reflected radiance e.g. phong or diffuse
            radiance += reflectedRadiance(material, rayToLight, distanceToLight)
    if material.type == 'mirror':
        radiance += trace(reflect(ray, hitpoint), scene)
    if material.type == 'transparent':
        radiance += trace(refract(ray, hitpoint), scene)
    return radiance

def shadowTrace(ray, dist, scene):
    t, primitive = findIntersection(ray, scene)
    return dist < t
```

Ray Tracing

- Incorporates in a single framework:
  - Hidden surface removal
    - Front to back traversal
    - Early termination once first hit point is found
  - Shadow computation
    - Shadow rays are traced between a point on a surface and a light sources
  - Exact simulation of some light paths
    - Reflection (reflected rays at a mirror surface)
    - Refraction (refracted rays at a transparent surface, Snell's law)

- Limitations
  - Easily gets inefficient for full global illumination computations
    - Many reflections (exponential increase in number of rays)
    - Indirect illumination requires many rays to sample all incoming directions

Ray Tracing: Approximations

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- Usually RGB color model instead of full spectrum
- Finite number of point lights instead of full indirect light
- Approximate material reflectance properties
  - Ambient: constant, non-directional background light
  - Diffuse: light reflected uniformly in all directions,
  - Specular: perfect reflection, refraction
- All are based on purely empirical foundation

Questions

- Write down and explain the principle steps of a recursive ray tracer.

- How do you compute the ray intersections with:
  - Plane
  - Box
  - Triangle
  - Sphere
  - Cylinder

- How do you evaluate the shading for a diffuse surface?

Wrap-Up

- Background
  - Forward light transport vs. backward search in RT
- Ray tracer
  - Ray generation, ray-object intersection, shading
- Ray-geometry intersection calculation
  - Sphere, plane, triangle, box
- Recursive ray tracing algorithm
  - Primary, secondary, shadow rays
- Next lecture
  - Acceleration techniques