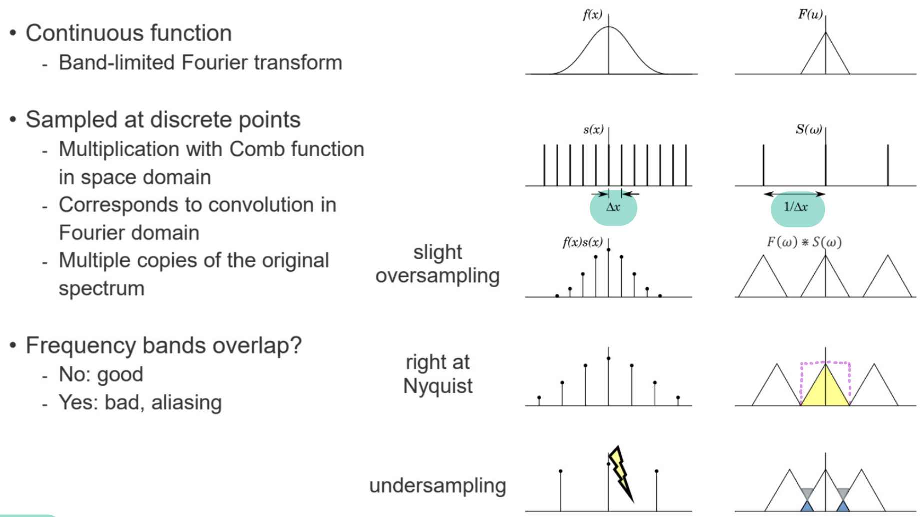
Sampling

- is the multiplication of a signal with a comb-function in the spatial domain
- a continuous (natural) signal is discretised for representation
- the distance of the dirac-pulses in the comp defines the sampling rate



Reconstruction

reconstruct the original frequency band by applying a Low-pass filter

-> box is optimal reconstruction filter but costly, $sinc^2$ as cheaper solution (Artefacts)

[$sinc^2$ is Linear interpolation (triangle function in spatial domain)]

-> distinguish between bad sampling (aliasing) and bad reconstruction

Aliasing Artefacts

- stair cases (e.g. diagonal line)
- Moiré Patterns (eg. aliasing on a checkerboard)

} spatial aliasing

- cart wheels (moving anti-clockwise in movie) } temporal aliasing

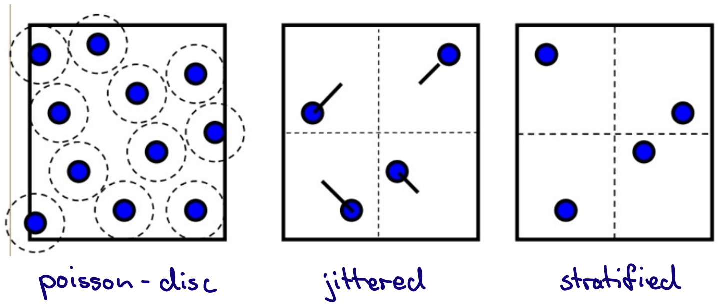
Antialiasing

sources for high frequencies: edges, silhouettes, discontinuities, illumiation

- pre-filtering: apply low-pass filter on original signal (Limit highest frequencies)
- super-sampling: more samples per pixel -> doesn't eliminate aliasing ($f_{nyq.}$ shifted)
  -> irregular supersampling

Sampling Patterns

- regular sampling: N equal distributed samples per pixel
  - -> still aliasing if lines are thinner than a pixel or curvy
- random sampling: N random distributed samples per pixel
  - -> replaces aliasing by noise -> usually destroys image completely
- jittered sampling: random deviation from regular points
  - -> replaces aliasing by soft noise
- stratified sampling: pixel is subdivided into N equal sized areas (grid)
  - -> one sample point is set randomly into each area
  - -> replaces aliasing by soft noise
- Quasi-Monte-Carlo sampling: advanced technique
- poisson disk sampling: random distribution of samples, but minimum distance between points (sampling of human eye)
  - -> replaces aliasing by soft noise (approximates human eye)



=> good sample count per pixel is 4 or 16

Aliasing and Sampling in digital Cameras

- most cameras do low-pass pre-filtering -> (mostly) no aliasing
- optical zoom defines the maximum pixel count the camera can set by sampling
- digital zoom is calculating image bigger without increasing the sample count

Lecture Slides about Sampling and Reconstruction

The *Perfect* Case

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Original function and its band-limited frequency spectrum

Signal sampling:

Mult./conv. with comb

Comb dense enough (sampling≥2*bandlimit)

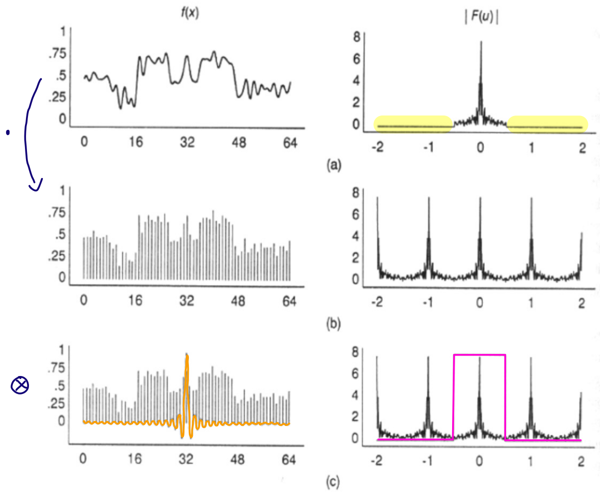
Frequency spectrum is replicated

Bands do not overlap

Correct reconstruction filtering

Fourier: Box (mult.)
Image: sinc (conv.)

Only one copy



result after reconstruction

→ box function

costly to implement

Correct Sampling / *Bad* Reconstruction

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Reconstruction with ideal sinc

Identical signal

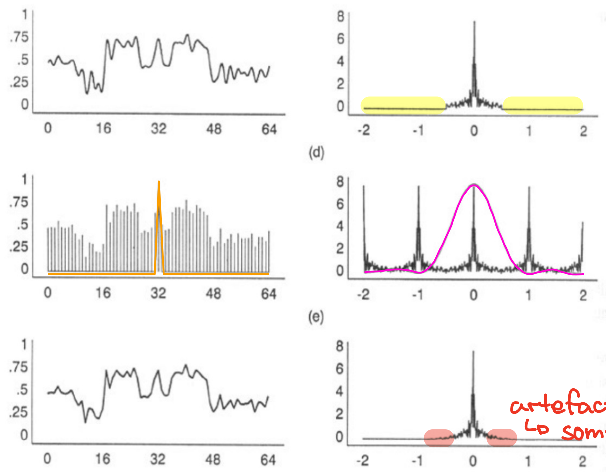
Approximate filtering

Space: tri (conv.)
Fourier: sinc^2 (mult.)

High frequencies are not ignored

→ Aliasing

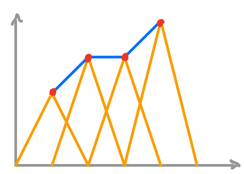
Reconstruction with tri function (= piecewise linear interpolation)



artefacts ↳ some higher frequencies

tri (cap - function) is

Linear interpolation



Sampling with *Too Low Frequency*

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Original function

Sampling below Nyquist:

Comb spaced too far (sampling<2*bandlimit)

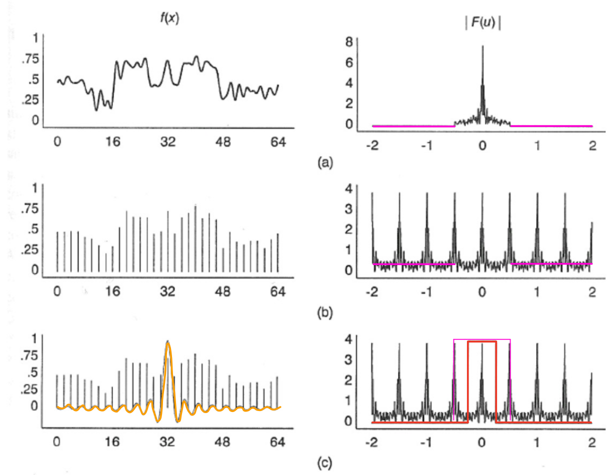
Frequency bands overlap

Correct filtering

Image: sinc (conv.)

Fourier: box (mult.)

Band overlap in frequency domain cannot be corrected - aliasing



→ result will not be similar to input

Sparse Sampling + *Bad* Reconstruction

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Reconstruction with ideal sinc

Reconstruction fails (frequency components wrong due to aliasing !)

Filtering with sinc² function

Reconstruction with tri function (= piecewise linear interpolation)

Even worse reconstruction