EBERHARD KARLS UNIVERSITÄT TÜBINGEN

FACULTY OF SCIENCE

Computer Graphics

Computer Graphics (Graphische Datenverarbeitung)

- Highlights-

Hendrik Lensch

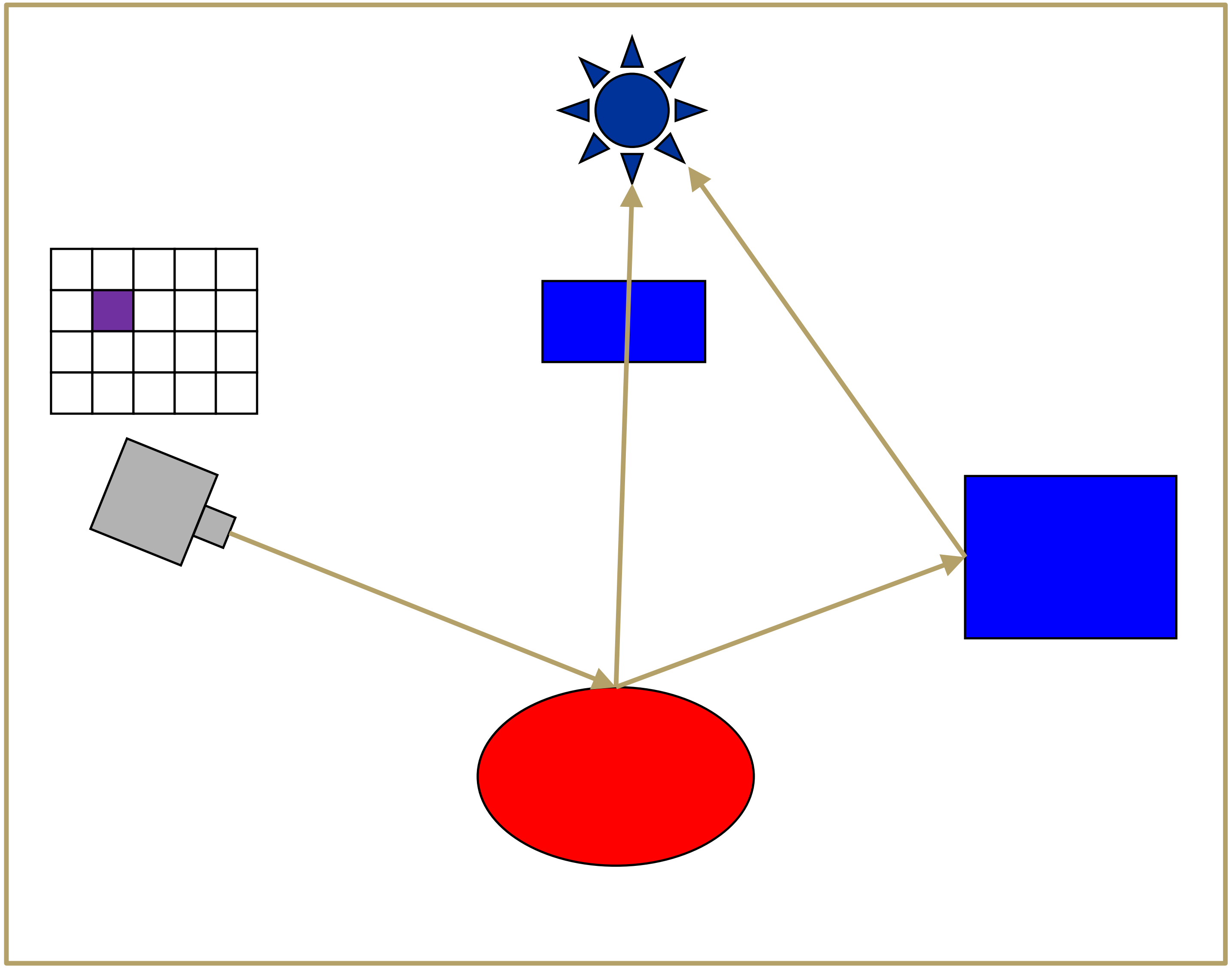
WS 2020/2021

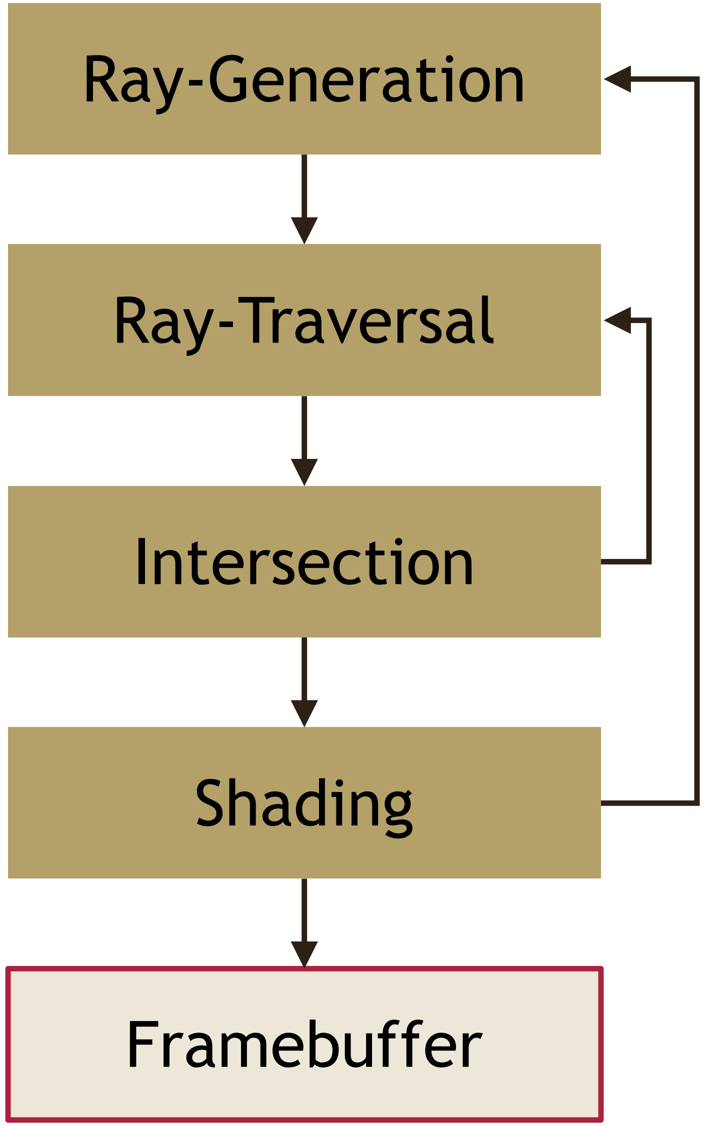
Zur Klausur

- Termin: Dienstag, 23.02.2021, 15:30-18:00 Uhr
- Raum: N10

- Hilfsmittel:
  - Lineal
  - Nicht programmierbarer Taschenrechner

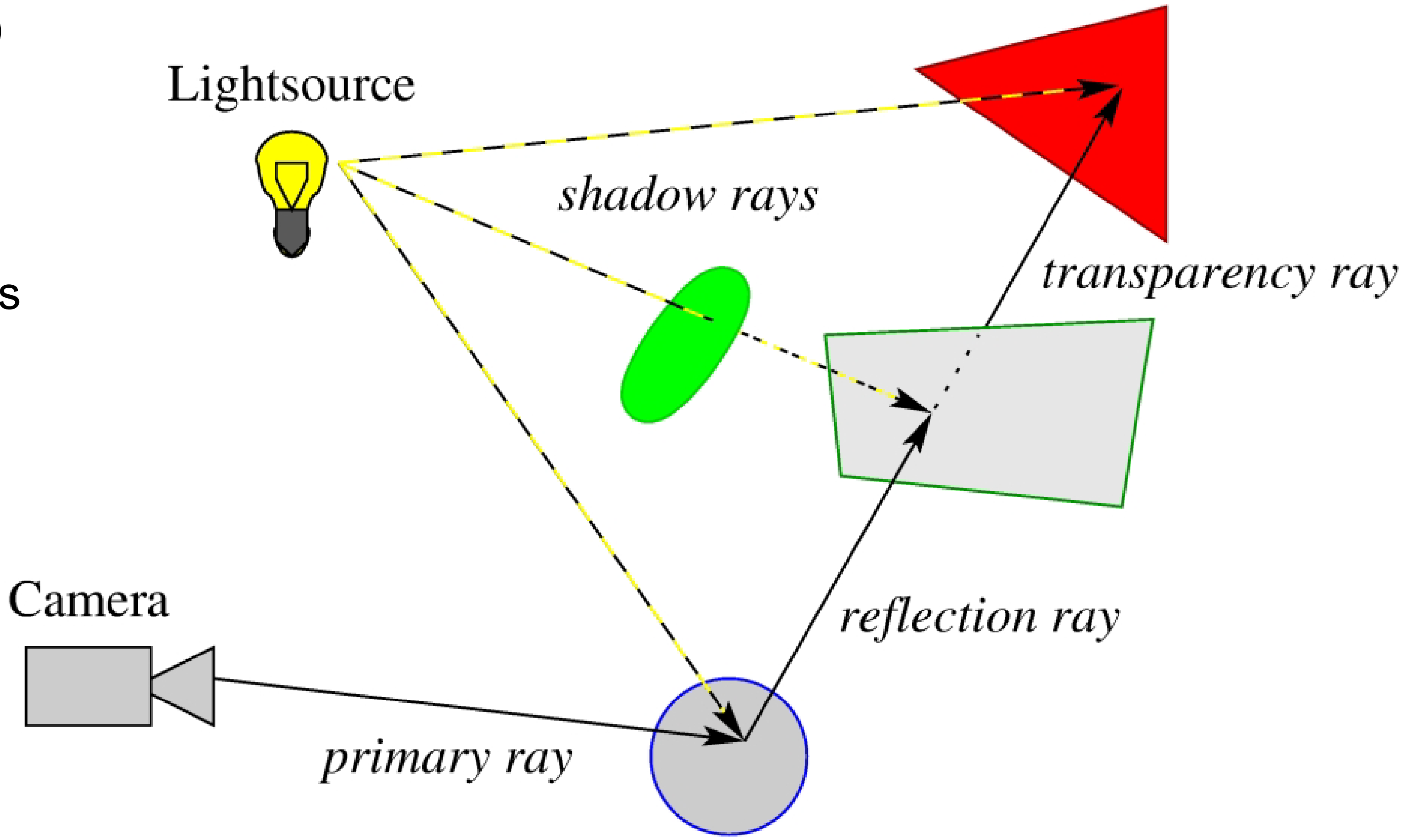
Ray Tracing Pipeline





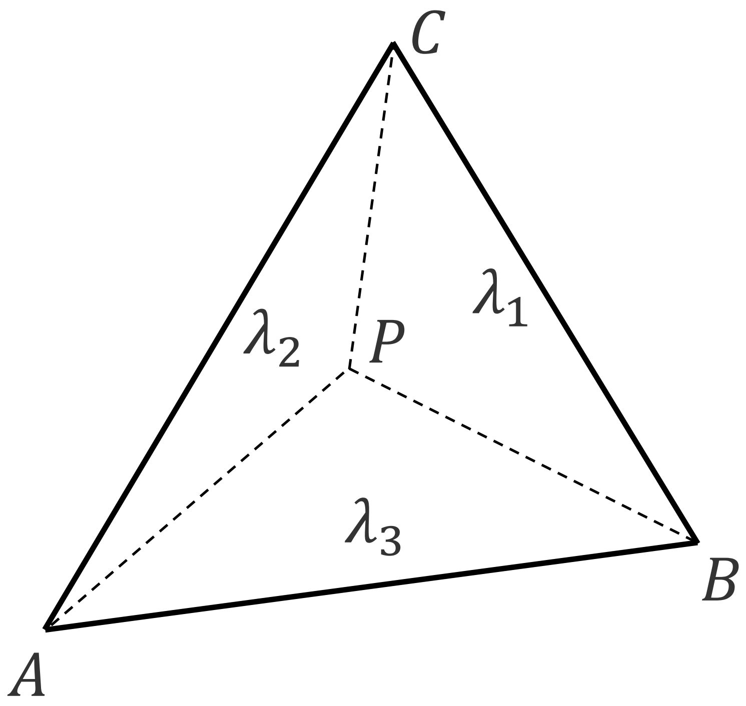
Ray Tracing

- Global effects
- Parallel (as nature)
- Fully automatic
- Demand driven
- Per pixel operations
- Highly efficient



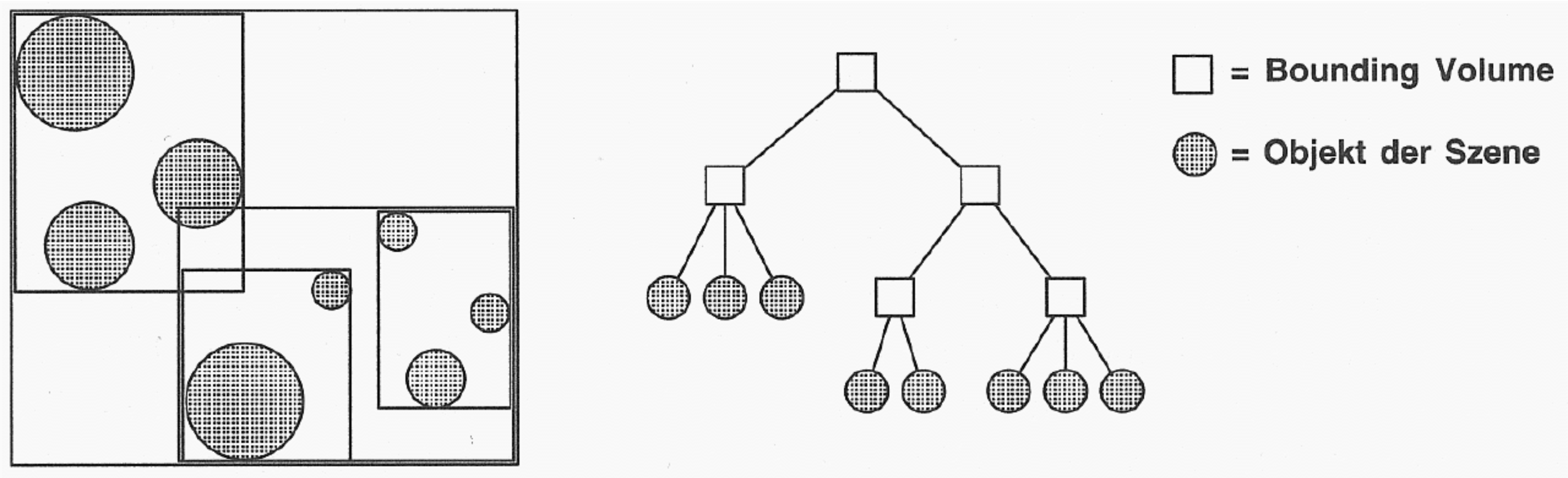
Intersection Ray – Triangle

- Barycentric coordinates
  - Non-degenerate triangle ABC
  - Ratio of signed areas: $\lambda_1 = \Delta(BCP)/\Delta(ABC)$
    $\lambda_2 = \Delta(CAP)/\Delta(ABC)$
    $\lambda_3 = \Delta(ABP)/\Delta(ABC)$
  - Every point P in the plane can be described using
    - $P = \lambda_1 A + \lambda_2 B + \lambda_3 C$
  - $\boldsymbol{\lambda_1 + \lambda_2 + \lambda_3 = 1}$
    - For fixed λ_3, P may move parallel to AB
    - For $\lambda_1 + \lambda_2 = 1$ and $0 < \lambda_3 < 1$
      - $P = (1 - \lambda_3)(\lambda_1 A + \lambda_2 B) + \lambda_3 C$
    - P moves between C and AB
- Point is in triangle, iff all $0 \leq \lambda_i$



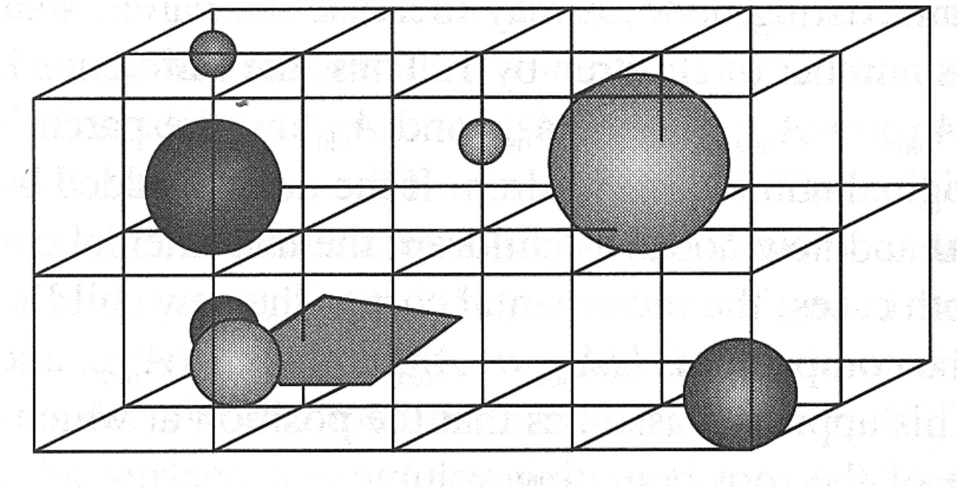
Bounding Volume Hierarchies

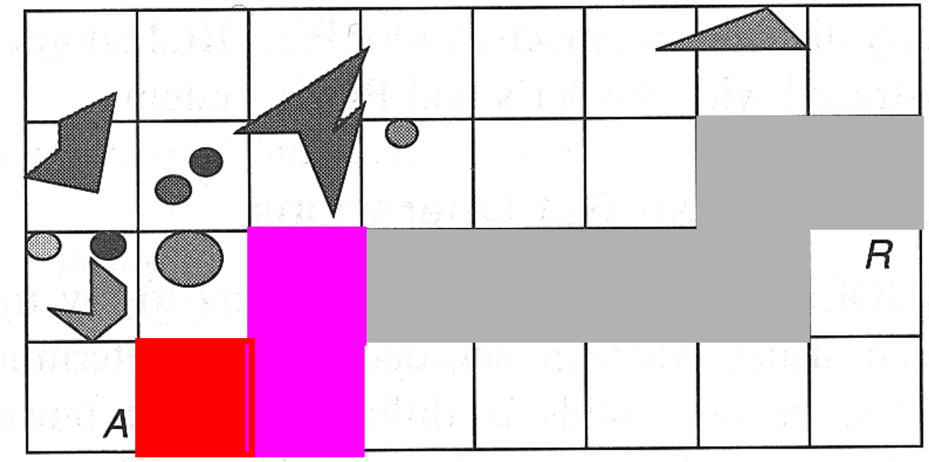
- Idea:
  - Organize bounding volumes hierarchically into new BVs
- Advantages:
  - Very good adaptivity
  - Efficient traversal O(log N)
  - Often used in ray tracing systems
- Problems
  - How to arrange BVs?



Grid

- Grid
  - Partitioning with equal, fixed sized „voxels“
- Building a grid structure
  - Partition the bounding box (bb)
  - Resolution: often $\sqrt[3]{n}$
  - Inserting objects
    - Trivial: insert into all voxels overlapping objects bounding box
    - Easily optimized
- Traversal
  - Iterate through all voxels in order as pierced by the ray
  - Compute intersection with objects in each voxel
  - Stop if intersection found in current voxel

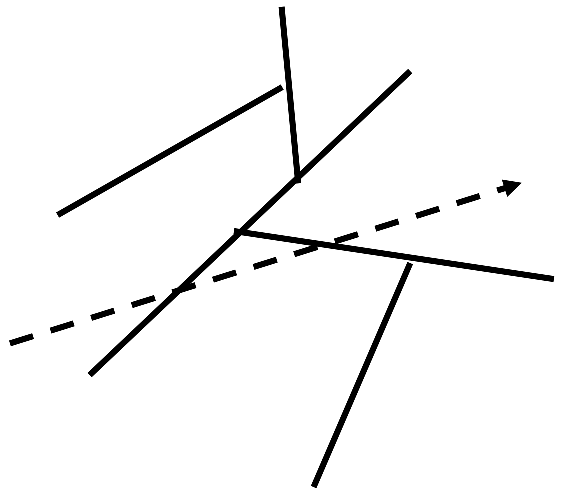


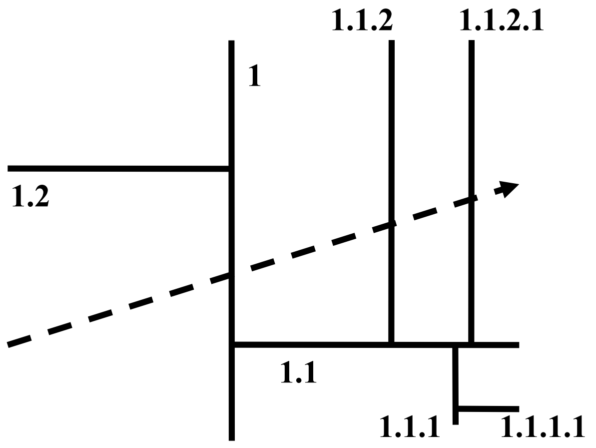


BSP- and kD-Trees

- Recursive space partitioning with half-spaces
- Binary Space Partition (BSP):
  - Recursively split space into halves
  - Splitting with half-spaces in arbitrary position
    - Often defined by existing polygons
  - Often used for visibility in games (→ Doom)
    - Traverse binary tree from front to back
- kD-Tree
  - Special case of BSP
    - Splitting with axis-aligned half-spaces
  - Defined recursively through nodes with
    - Axis-flag
    - Split location (1D)
    - Child pointer(s)

  - See following slides for details



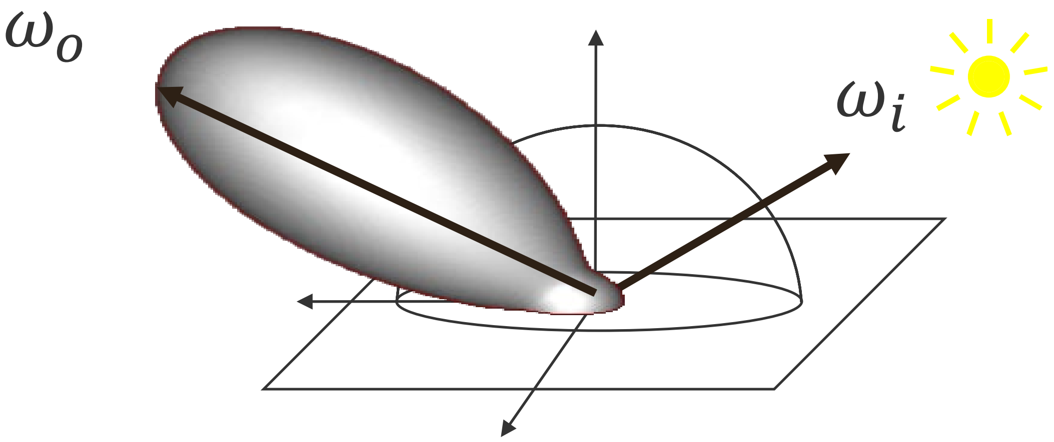


BRDF – 4D

(bidirectional reflectance distribution function)

$$f_r(\omega_i \to \omega_o) = \frac{dL(\omega_i)}{dE(\omega_o)}$$

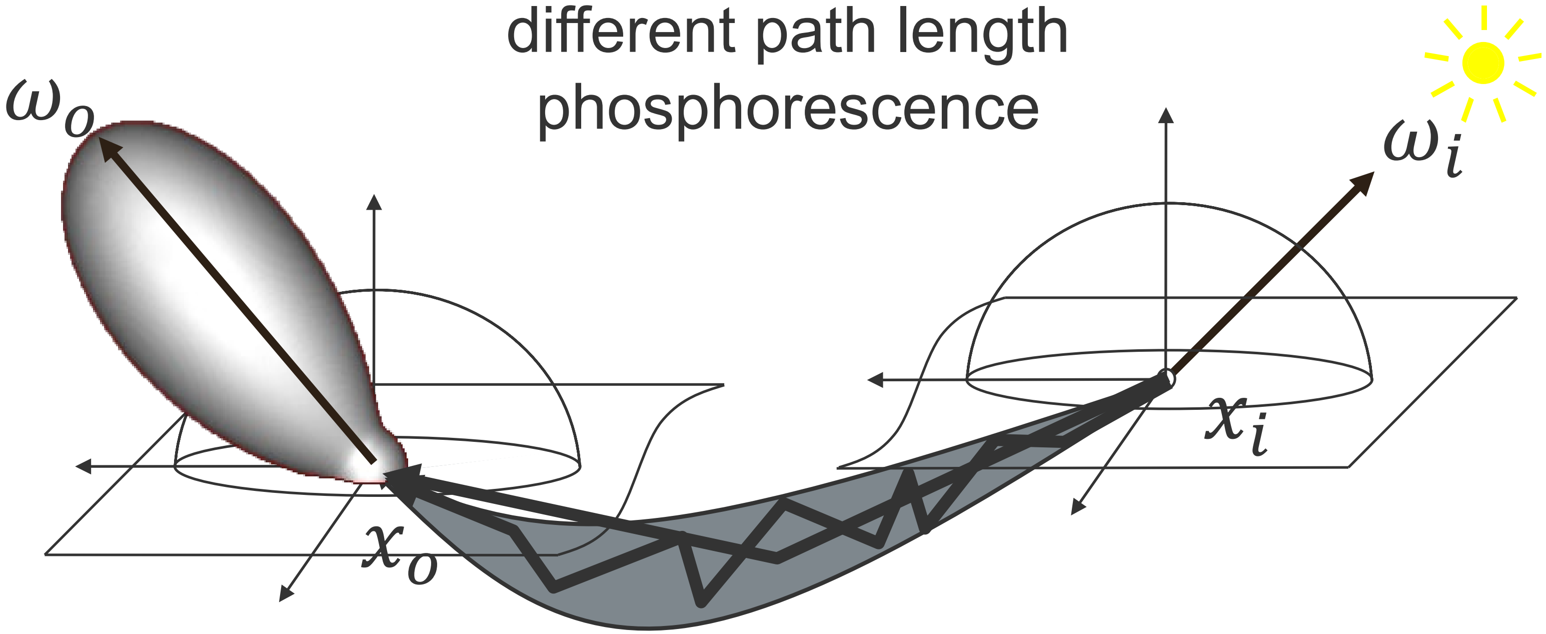
ratio of reflected radiance to incident irradiance



Generalization – 12D

$$f_r\Big((x_i, \omega_i, t_i, \lambda_i) \rightarrow (x_o, \omega_o, t_o, \lambda_o)\Big)$$

different path length
phosphorescence



Phong Illumination Model

- Extended light sources: l point light sources

$$\mathrm{L_r} = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (R(I_l) \cdot V)^{k_e} \quad \text{(P} \qquad \text{ł}$$

$$\mathrm{L_r} = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (H_l \cdot N)^{k_e} \quad \text{(} \quad \text{B}$$

- Color of specular reflection equal to light source
- Heuristic model
  - Contradicts physics
  - Purely local illumination
    - Only direct light from the light sources
    - No further reflection on other surfaces
    - Constant ambient term
- Often: light sources & viewer assumed to be far away

Radiometry

- Definition:
  - Radiometry is the science of measuring radiant energy transfers. Radiometric quantities have physical meaning and can be directly measured using proper equipment such as spectral photometers.

- Radiometric Quantities

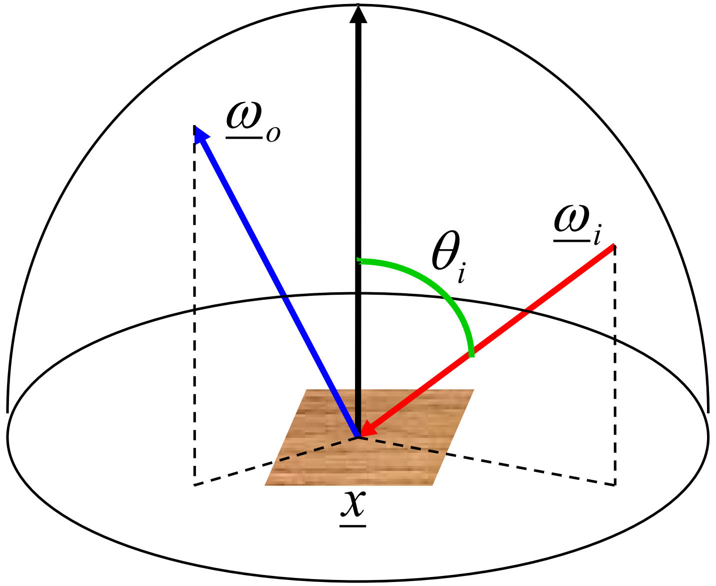
- energy	[watt second]	$n \cdot h\lambda$ (Photon Energy)
- radiant power (total flux)	[watt]	Φ
- radiance	[watt/(m^2 sr)]	L
- irradiance	[watt/m^2]	E
- radiosity	[watt/m^2]	B
- intensity	[watt/sr]	I

(Surface) Rendering Equation

- In Physics: Radiative Transport Equation
- Expresses energy equilibrium in scene

$$L(\underline{x},\underline{\omega}_o) = L_e(\underline{x},\underline{\omega}_o) + \int_{\Omega} f_r(\underline{\omega}_i,\underline{x},\underline{\omega}_o)\, L_i(\underline{x},\underline{\omega}_i)\, \mathrm{c} \quad \theta_i \, d\underline{\omega}_i$$

- total radiance = emitted radiance + reflected radiance
- First term: emissivity of the surface
  - non-zero only for light sources
- Second term: reflected radiance
  - integral over all possible incoming directions of irradiance times angle-dependent surface reflection function
- Fredholm integral equation of 2nd kind
  - unknown radiance appears on lhs and inside the integral
  - Numerical methods necessary to compute approximate solution



Path Tracing

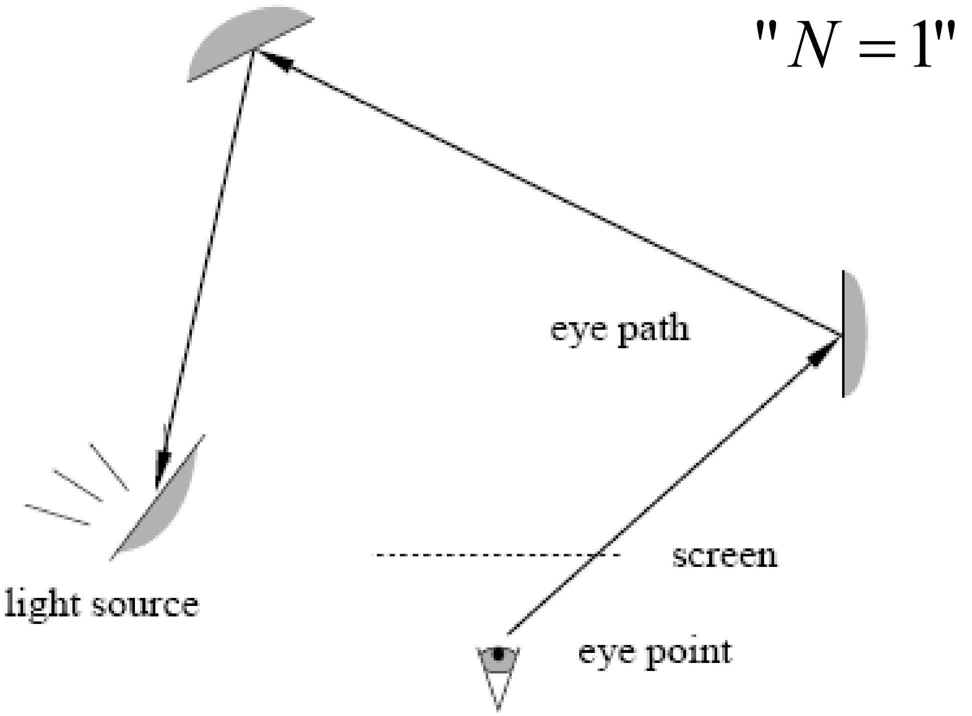
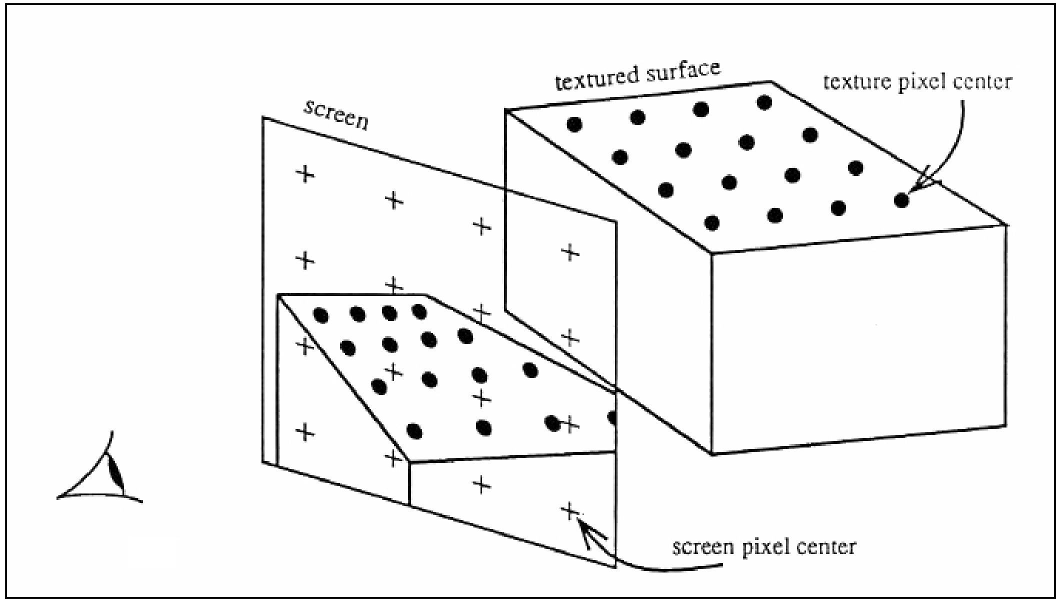
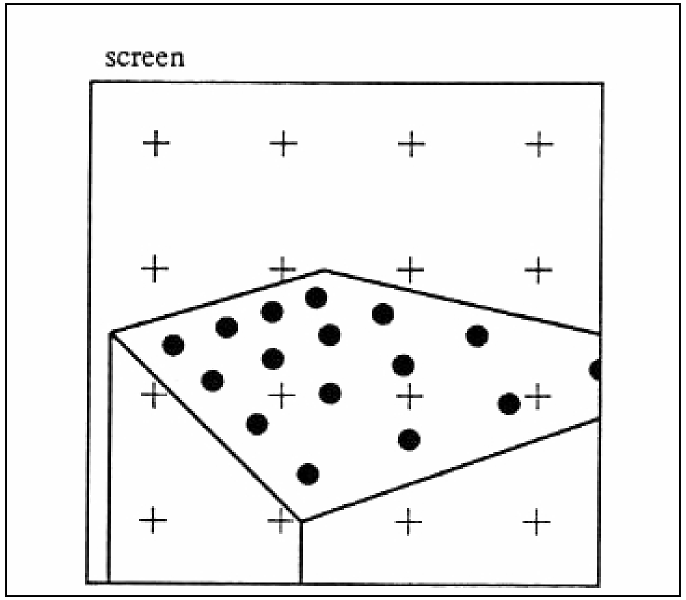


Figure 4.2: Schematic overview of the path tracing algorithm. The radiant flux through a pixel has to be estimated. The tracing of a primary ray from the virtual eye point through a pixel corresponds to sampling the expression for the flux. The subsequent random walk through the scene corresponds to recursively estimating the radiance values. Each time a light source is hit a contribution is added to the estimate.

2D Texturing

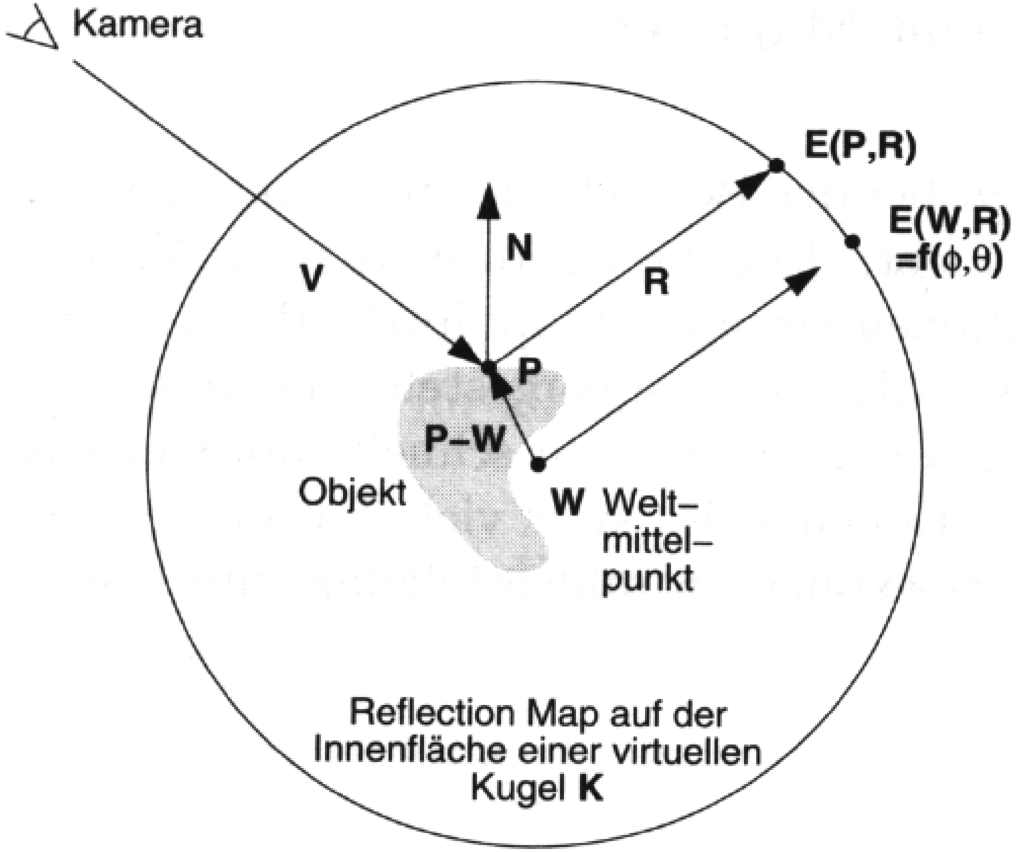




- 2D texture mapped onto object
- Object projected onto 2D screen
- 2D→2D: warping operation
- Uniform sampling ?
- Hole-filling/blending ?

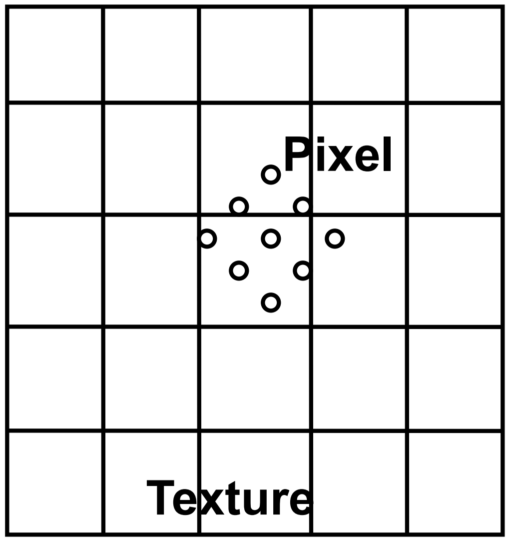
Reflection Map Rendering

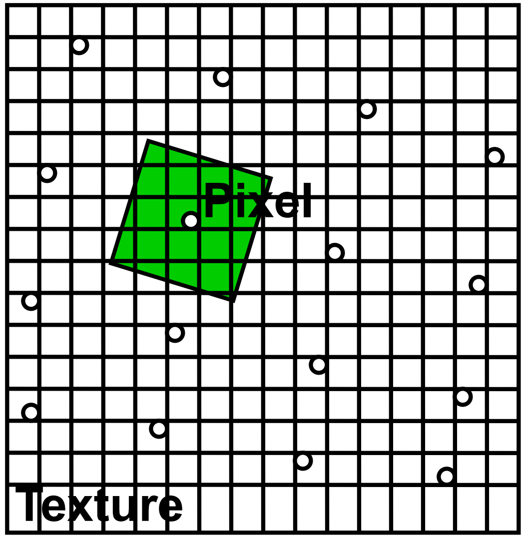
- Spherical parameterization
- O-mapping using reflected view ray intersection



Filtering

- Magnification
  - Map few texels onto many pixels
  - Nearest:
    - Take the nearest texel
  - Bilinear interpolation:
    - Interpolation between 4 nearest texels
    - Need fractional accuracy of coordinates
- Minification
  - Map many texels to one pixel
    - Aliasing:
      - Reconstructing high-frequency signals with low level frequency sampling
  - Filtering
    - Averaging over (many) associated texels
    - Computationally expensive

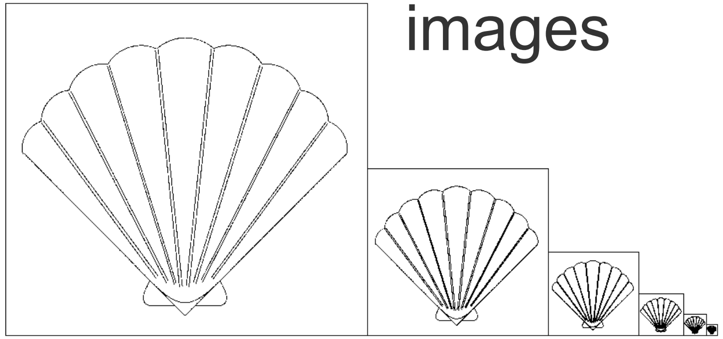


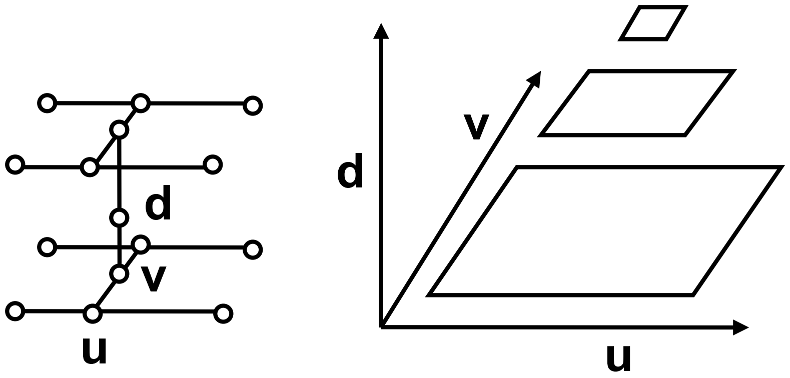


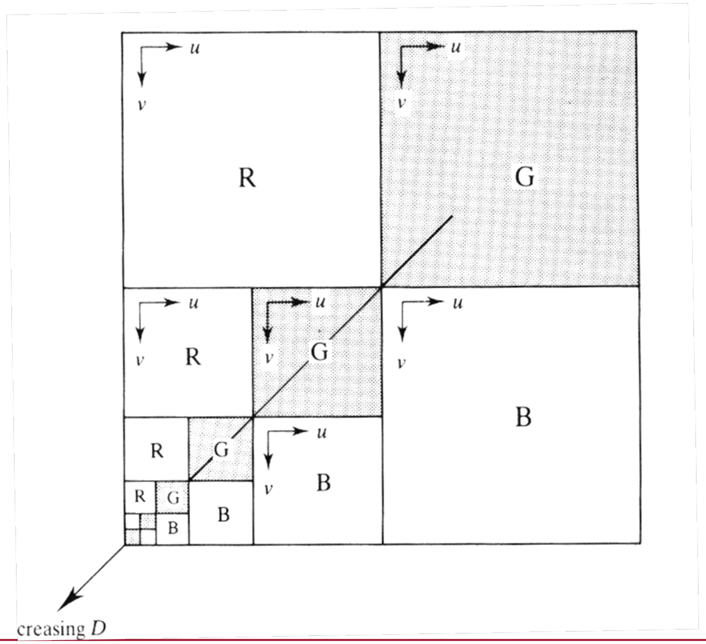
MipMapping II

- Multum In Parvo (MIP): much in little
- Hierarchical resolution pyramid
  - Repeated averaging over 2x2 texels
- Rectangular arrangement (RGB)
- Reconstruction
  - Tri-linear interpolation of 8 nearest texels

original - prefiltered images



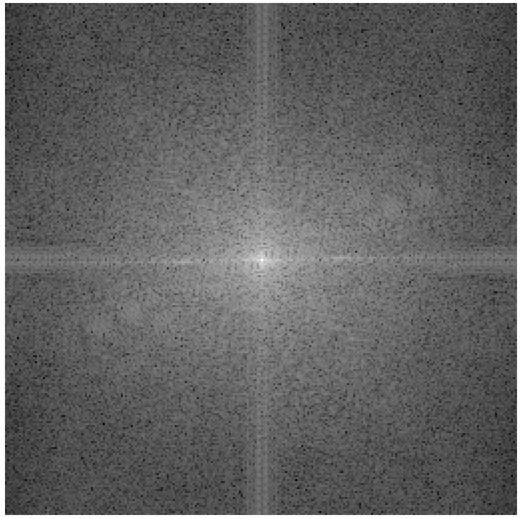




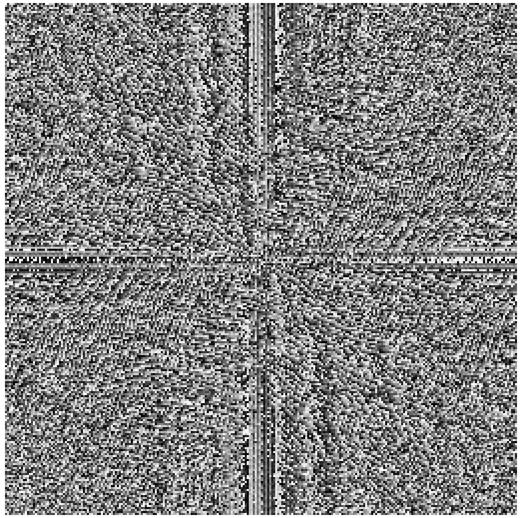
An Example

f(x)

Fourier transformed

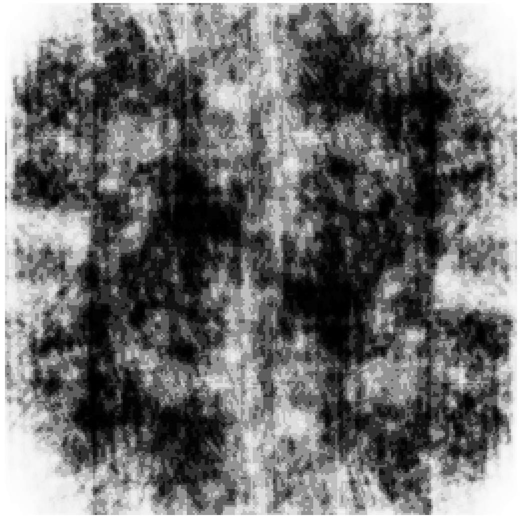


Amplitude



Phase

reconstructed



ignoring Phase

using Phase+Amplitude

Spatial vs. Frequency Domain

- Important basis functions
  - Box ↔ sinc

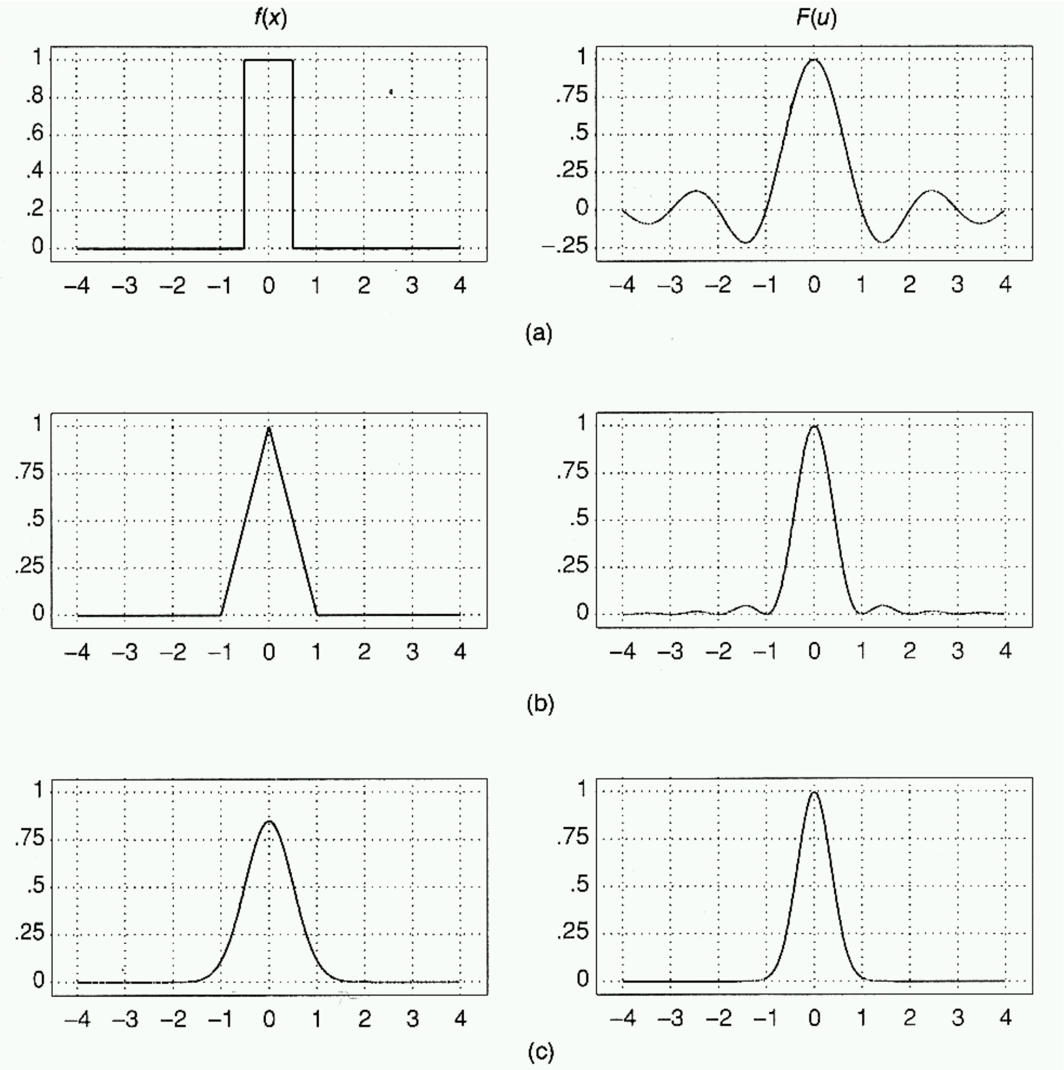
$$\mathrm{sinc}(x) = \frac{\sin(x\pi)}{x\pi}$$

$$\mathrm{sinc}(0) = 1$$

$$\int \mathrm{sinc}(x)\,dx = 1$$

    - Wide box ↔ small sinc
    - Negative values
    - Infinite support

  - Triangle ↔ sinc2
  - Gauss ↔ Gauss



Sparse Sampling + *Bad* Reconstruction

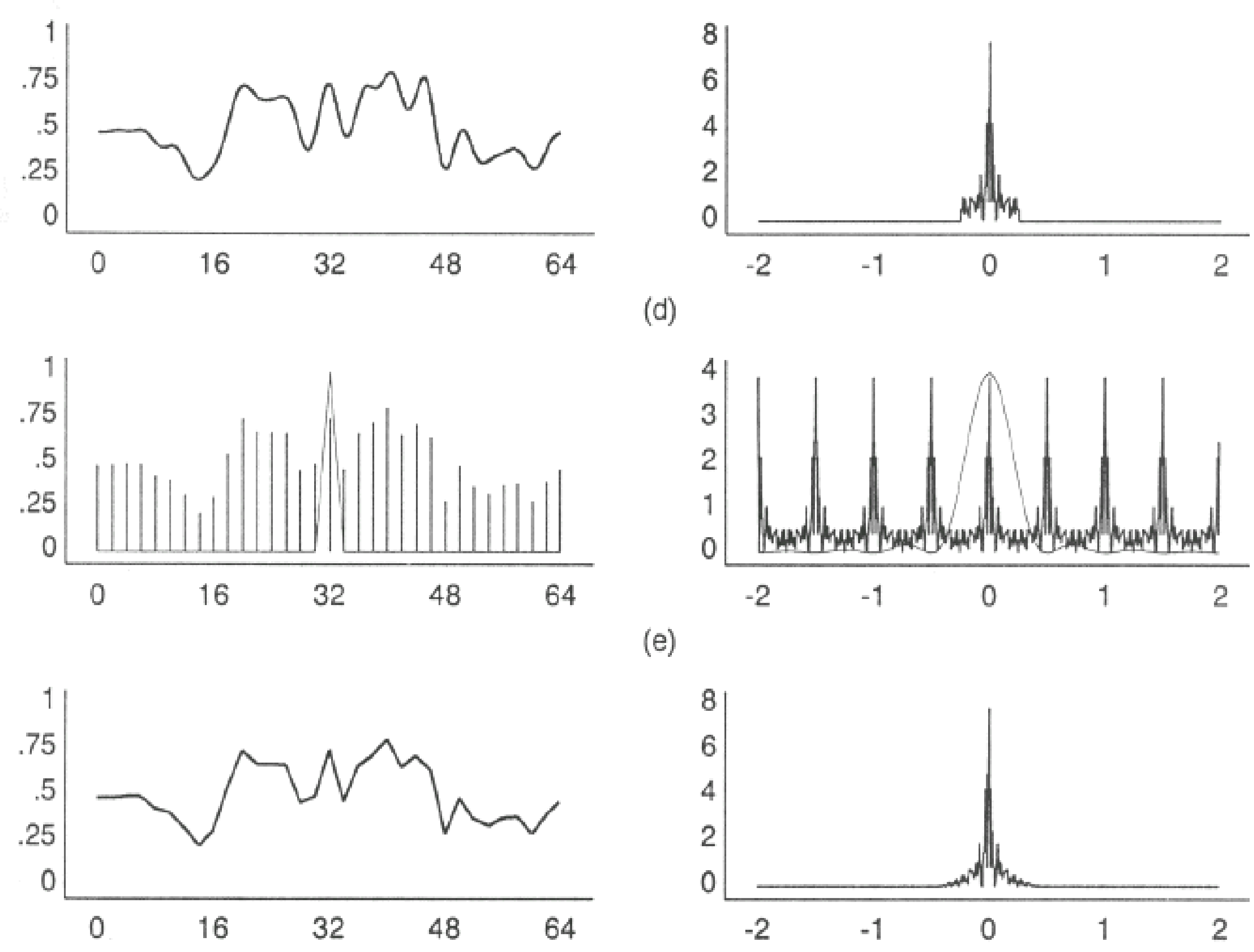
Reconstruction with ideal sinc

Reconstruction fails (frequency components wrong due to aliasing !)

Filtering with sinc^2 function

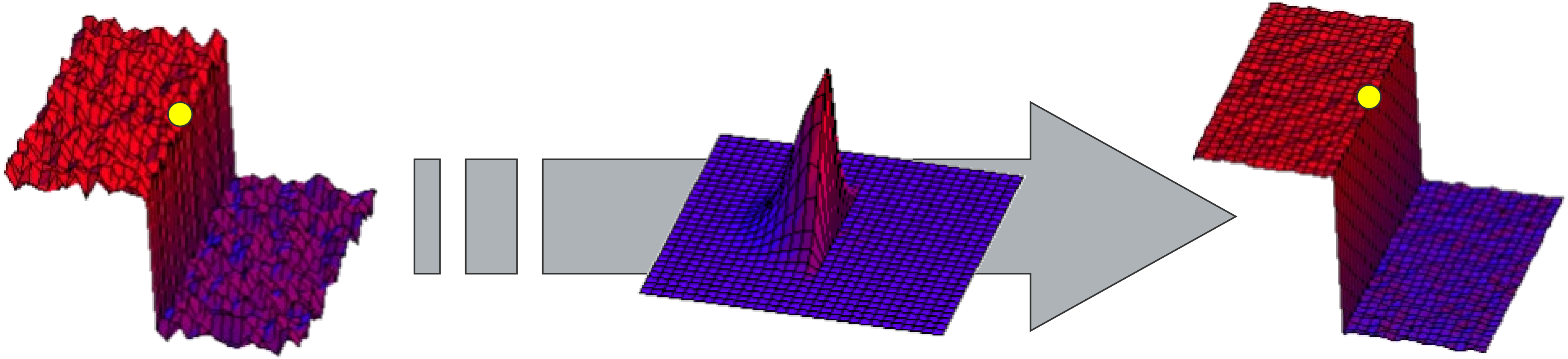
Reconstruction with tri function (= piecewise linear interpolation)

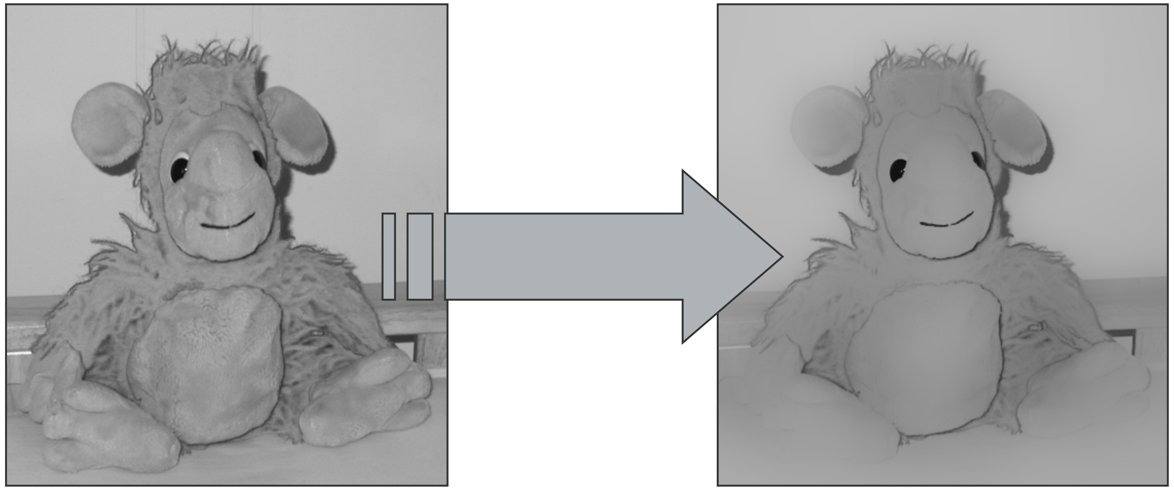
Even worse reconstruction



Large-scale Layer

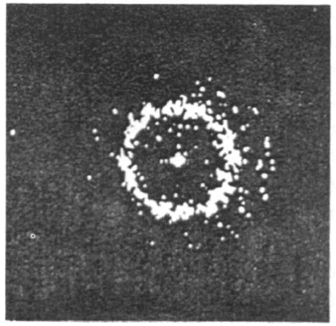
- Bilateral filter – edge preserving filter
- Smith and Brady 1997; Tomasi and Manducci 1998; Durand et al. 2002

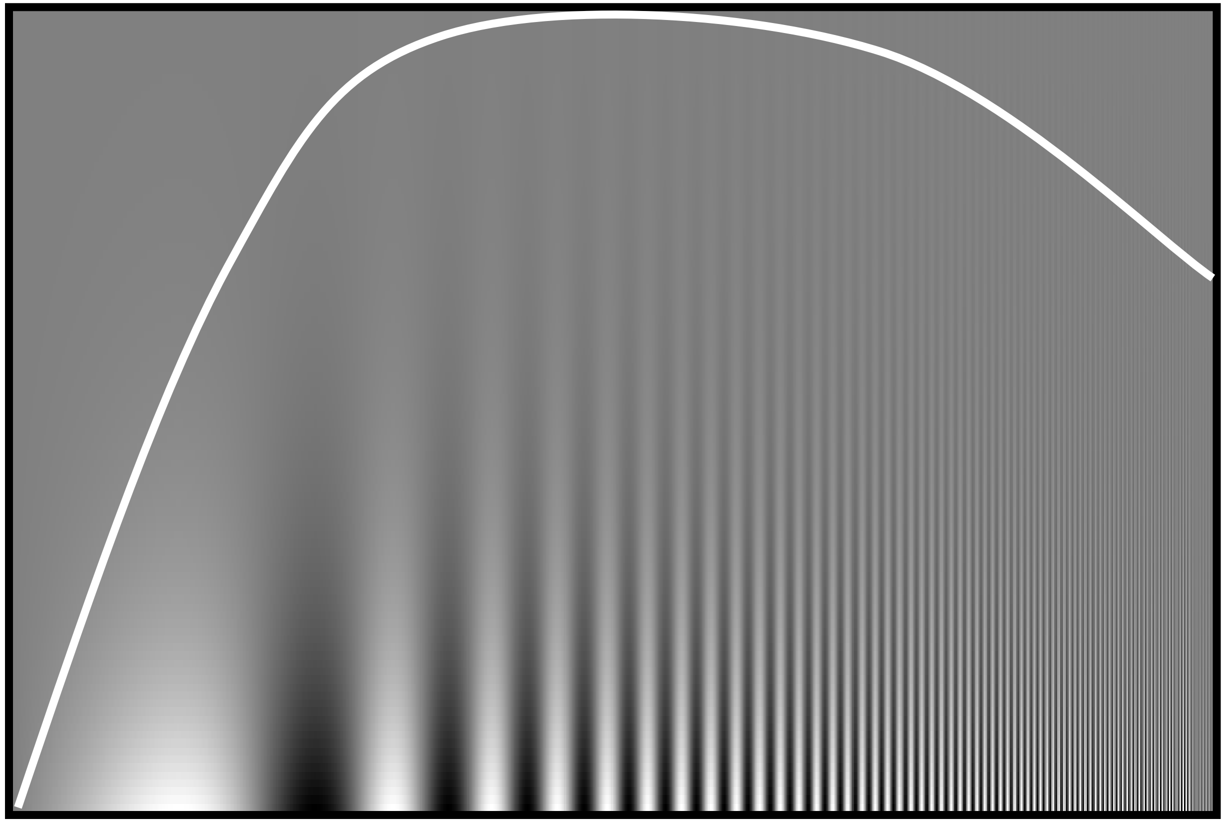




HVS: Poisson Disk Experiment

- Human Perception
  - Very sensitive to regular structures
  - Insensitive against (high frequency) noise

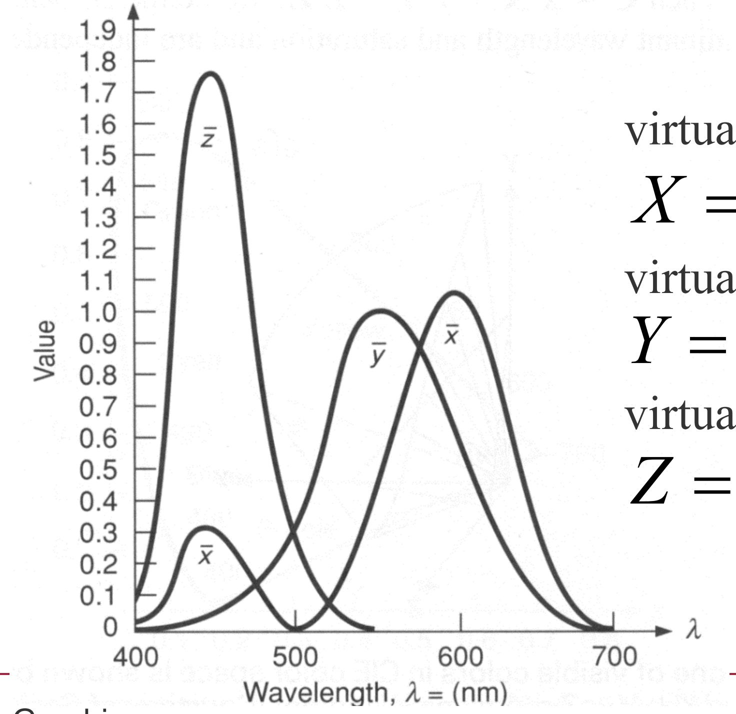




Campbell-Robson contrast sensitivity chart

Standard Color Space CIE-XYZ

- Standardized imaginary primaries CIE XYZ (1931)
  - Imaginary primaries more saturated than monochromatic lights
    - Could match all physically realizable color stimuli
  - Y is roughly equivalent to luminance
    - Shape similar to luminous efficiency curve
  - Monochromatic spectral colors form a curve in 3D XYZ-space
  - Matching curves for virtual CIE XYZ primaries



virtual red

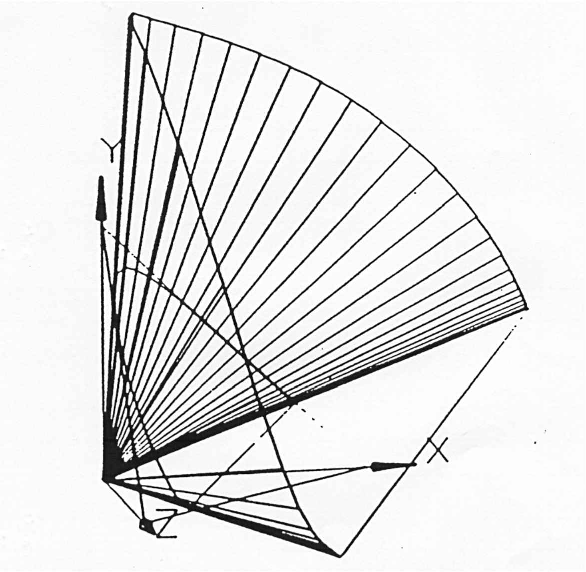
$$X = K_m \int L(\lambda)\bar{x}(\lambda)d\lambda,$$

virtual green

$$Y = K_m \int L(\lambda)\bar{y}(\lambda)d\lambda,$$

virtual blue

$$Z = K_m \int L(\lambda)\bar{z}(\lambda)d\lambda$$



Basic Transformations

- Rotation around major axis

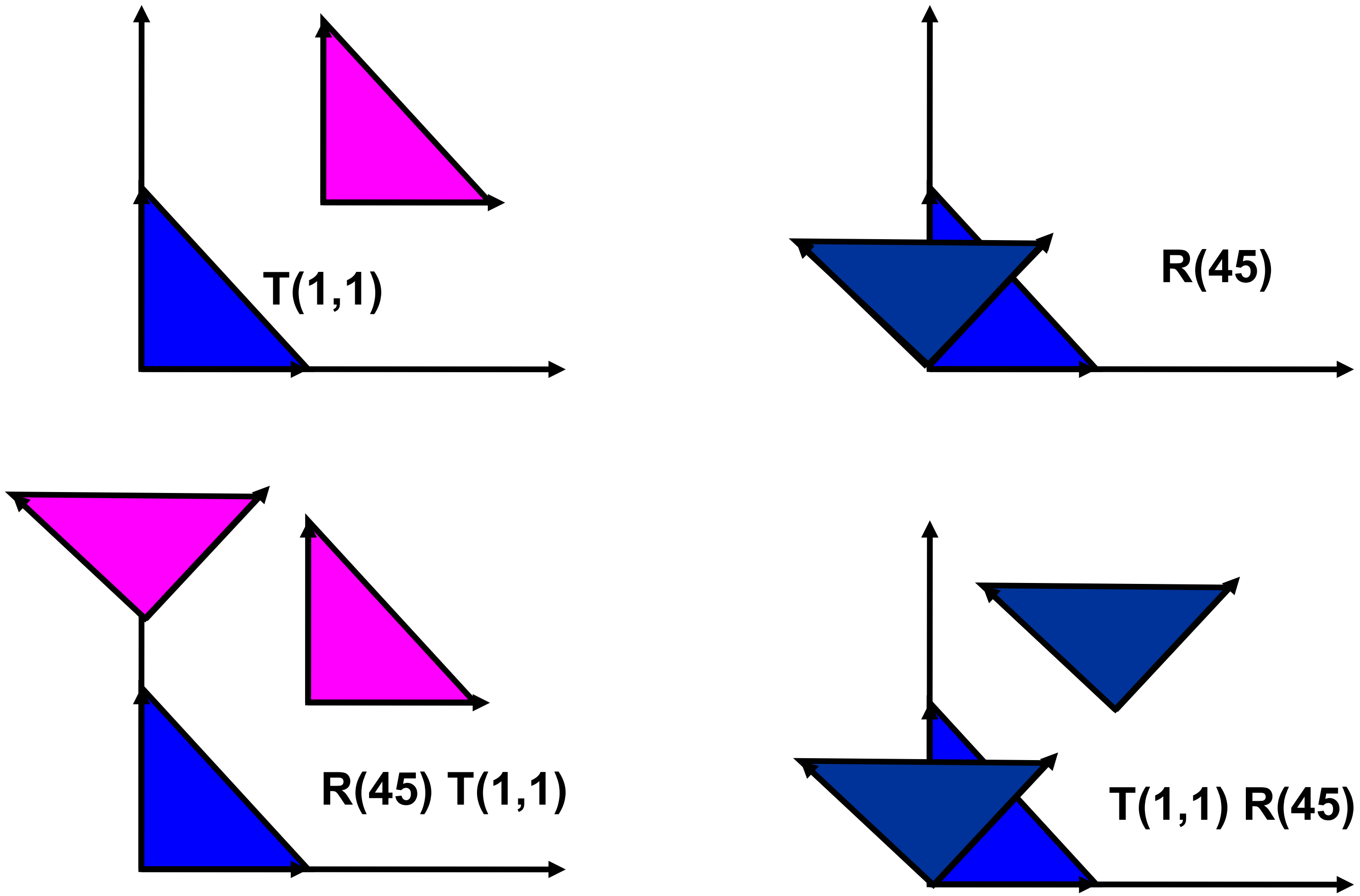
$$R_x(\theta)=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos\theta & -\sin\theta & 0\\ 0 & \sin\theta & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{pmatrix}\qquad R_y(\theta)=\begin{pmatrix}\cos\theta & 0 & \sin\theta & 0\\ 0 & 1 & 0 & 0\\ -\sin\theta & 0 & \cos\theta & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$R_z(\theta)=\begin{pmatrix}\cos\theta & -\sin\theta & 0 & 0\\ \sin\theta & \cos\theta & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

  - Assumes a right handed coordinate system

Concatenation of Transformations

- In general, transformations do not commute



Camera Transformation

- Complete Transformation
  - Perspective Projection

$$K = T_{raster} S_{raster} \quad P_{parallel} \quad P_{persp} S_{far} S_{xy} H \quad RT$$

  - Orthographic Projection

$$K = T_{raster} S_{raster} \quad P_{parallel} \quad S_{xyz} T_{near} H \quad RT$$

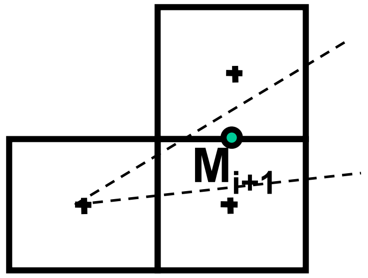
- Other representations
  - Different camera parameters as input
  - Different canonical viewing frustum
  - Different normalized coordinates
    - $[-1 .. 1]^3$ versus $[0 .. 1]^3$ versus ...
  - ...

→ ***Different transformation matrices***

Lines: Bresenham

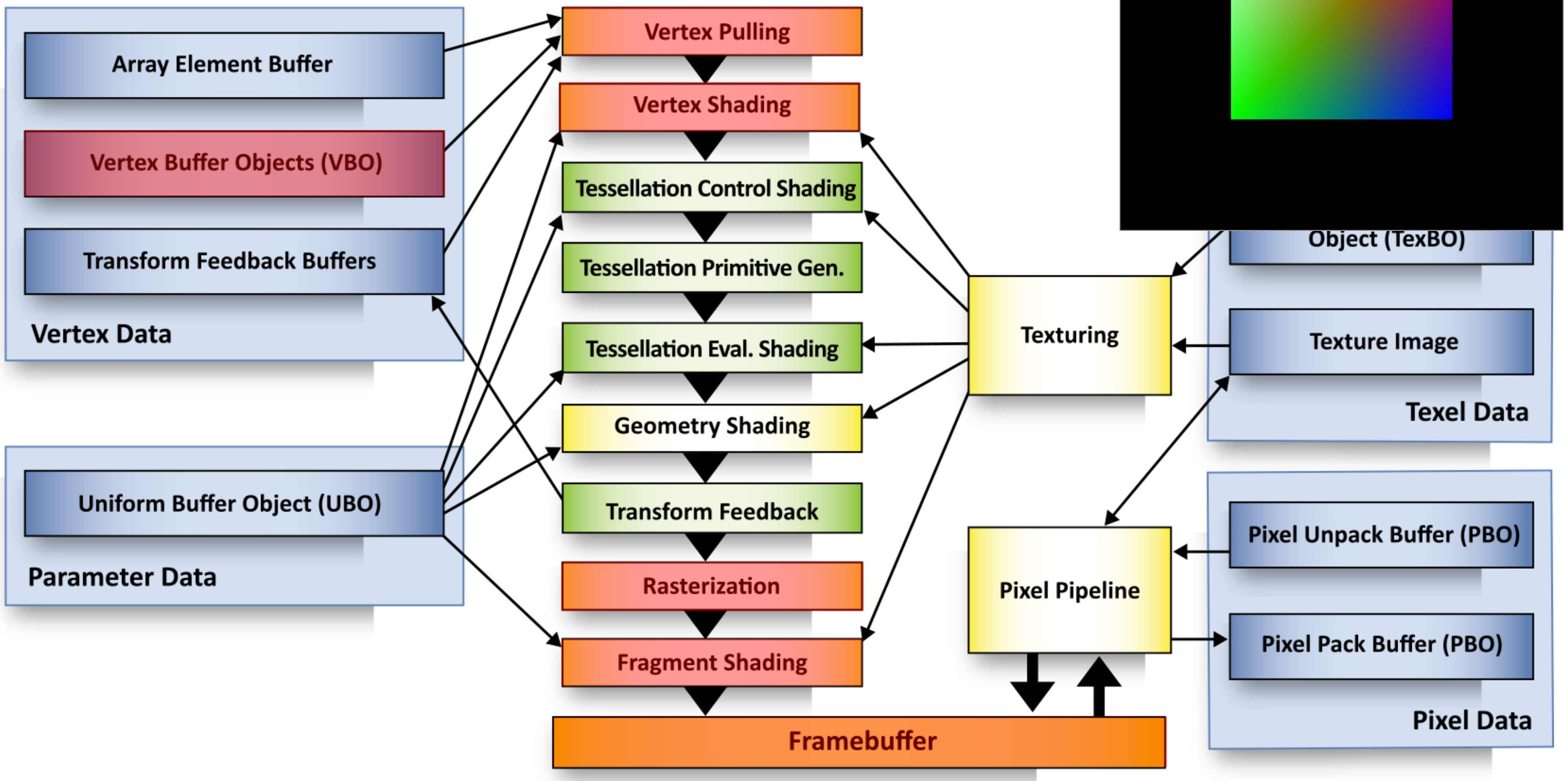
- Decision variable (the midpoint formulation)
  - Measures the vertical distance of midpoint from line:

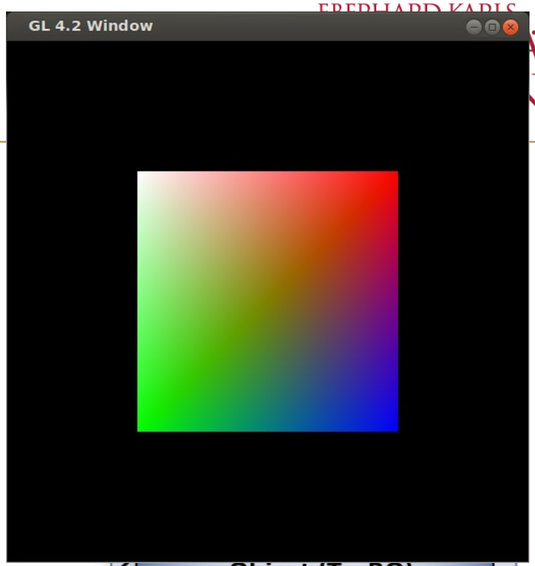
    $d_{i+1} = F(M_{i+1})=F(x_i+1, y_i+1/2) = a(x_i+1) + b(y_i+1/2) + c$



- Preparations for the next pixel
  - if ($d_i \leq 0$)
    - $d_{i+1}= d_i + a = d_i + dy$ // incremental calculation
  - else
    - $d_{i+1}= d_i + a + b = d_i + dy – dx$
    - $y= y + 1$
  - $x = x +1$

Colored Quad





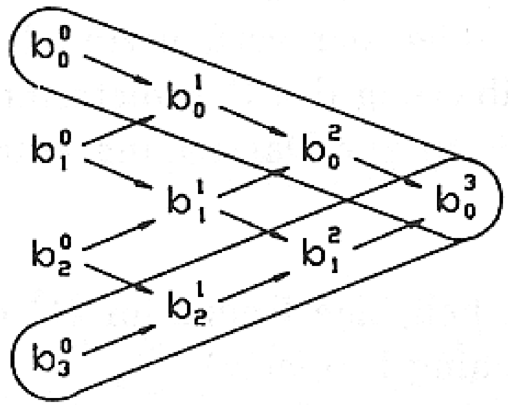
DeCasteljau Algorithm

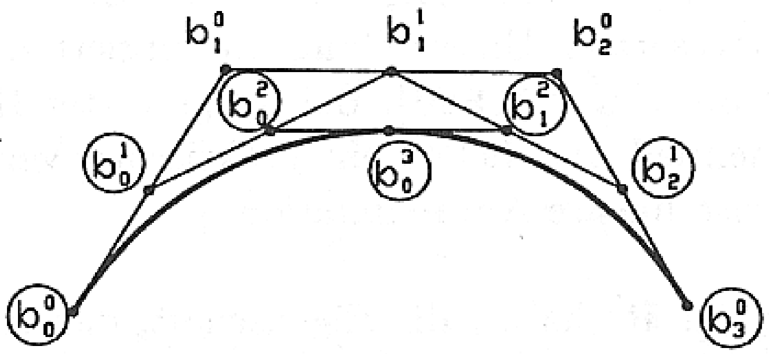
- DeCasteljau-Algorithm:
  - Recursive degree reduction of the Bezier curve by using the recursion formula for the Bernstein polynomials

$$P(t) = \sum_{i=0}^{n} b_i^0 B_i^n(t) = \sum_{i=0}^{n-1} b_i^1 B_i^{n-1}(t) = \cdots = b_i^n(t) \cdot 1$$

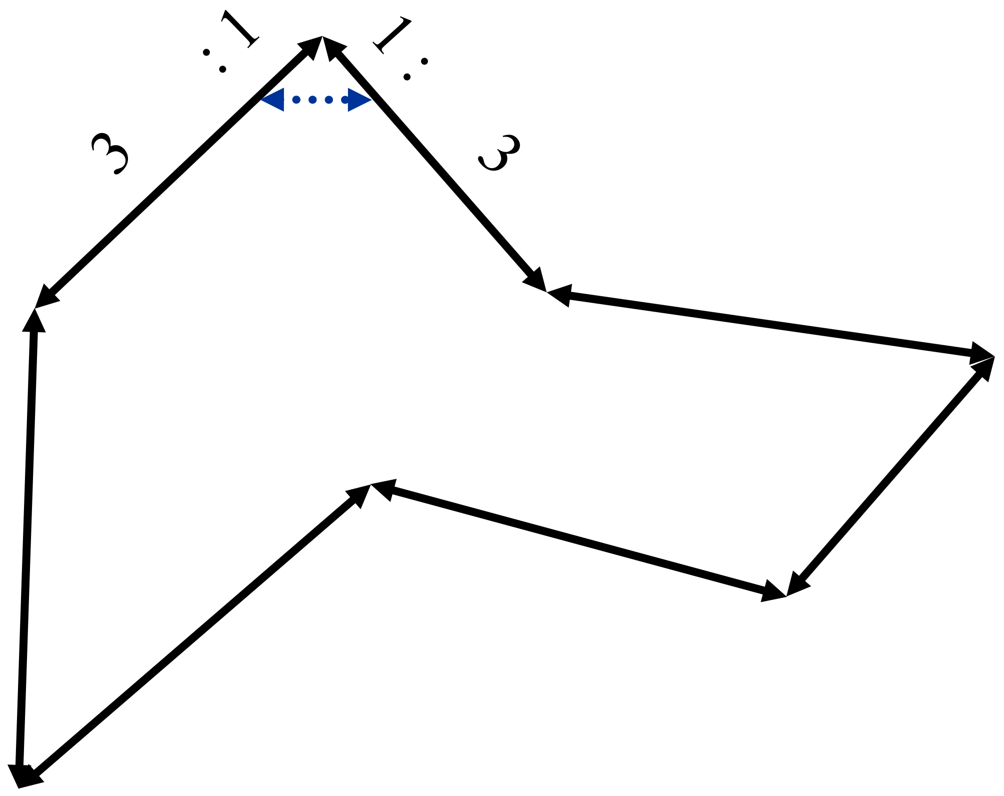
- Example:
  - t= 0.5

$$b_i^k(t) = \mathrm{t} b_i^{k-1}(t) + (1 - \mathrm{t}) b_i^{k-1}(t)$$

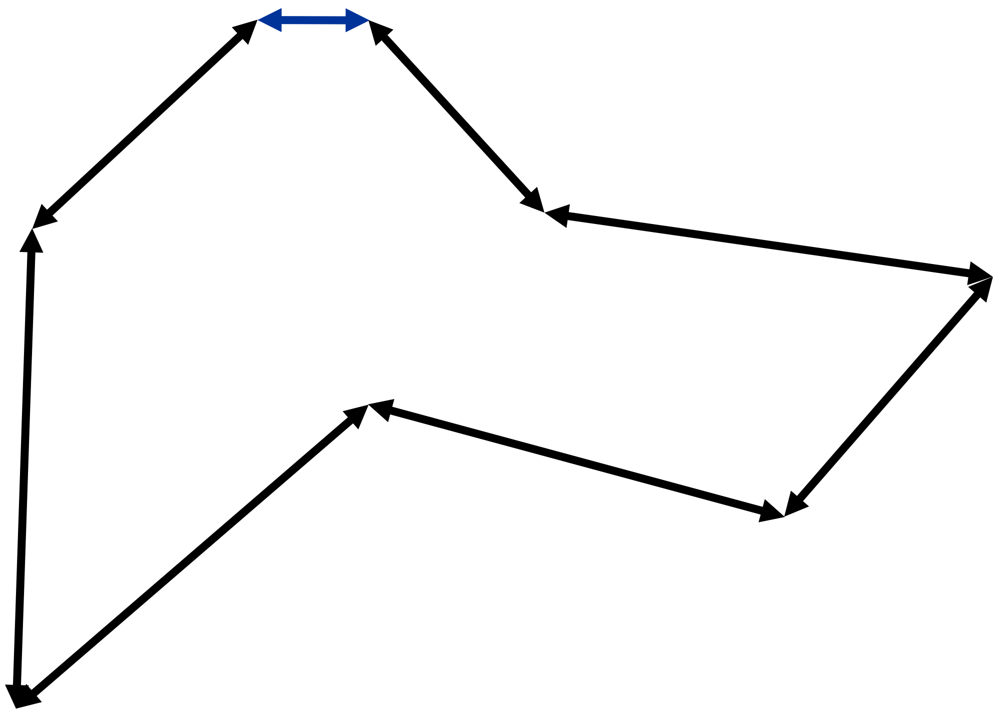




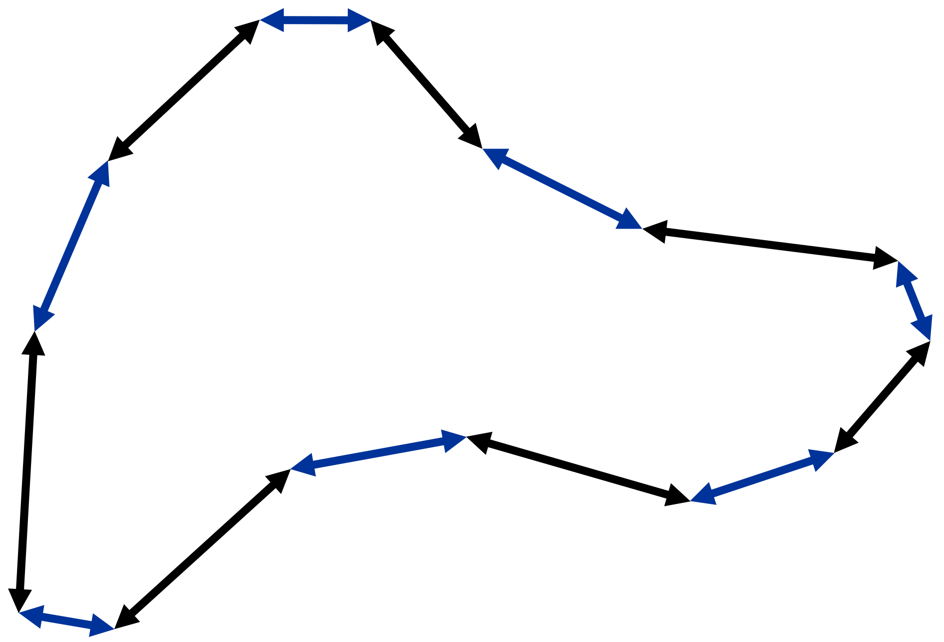
Corner Cutting



Corner Cutting

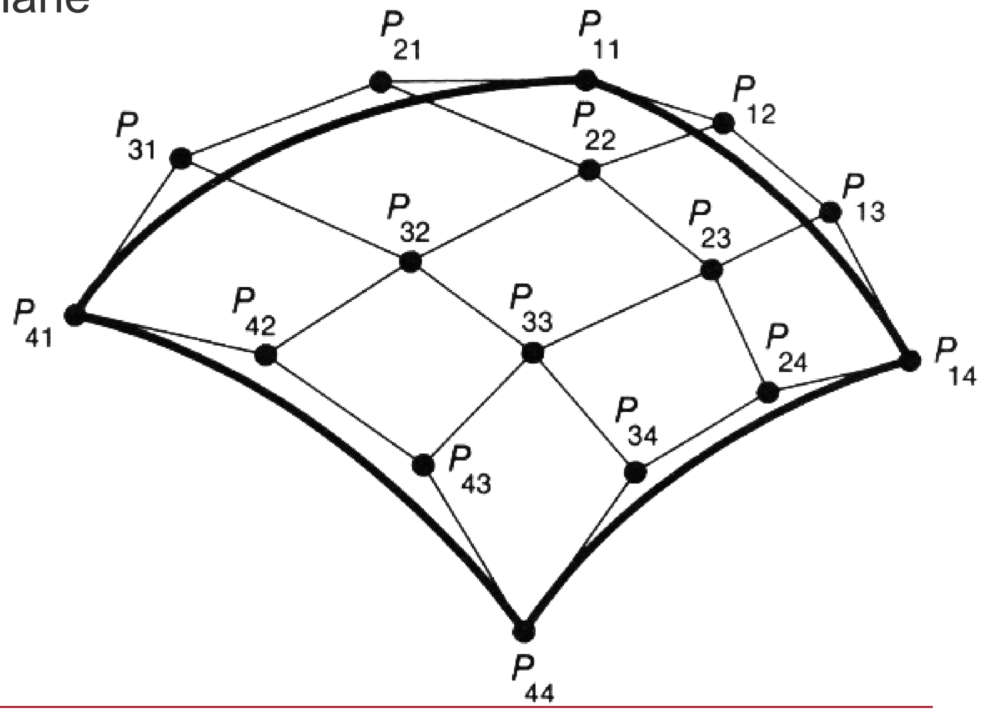


Corner Cutting



Tensor Product Surfaces

- Properties Derived Directly From Curves
- Bézier Surface:
  - Surface interpolates corner vertices of mesh
  - Vertices at edges of mesh define boundary curves
  - Convex hull property holds
  - Simple computation of derivatives
  - Direct neighbors of corners vertices define tangent plane
- Similar for Other Basis Functions



Lumigraph – Depth-corrected Rendering

What color has ray (s,u) ?

- Closest recorded ray

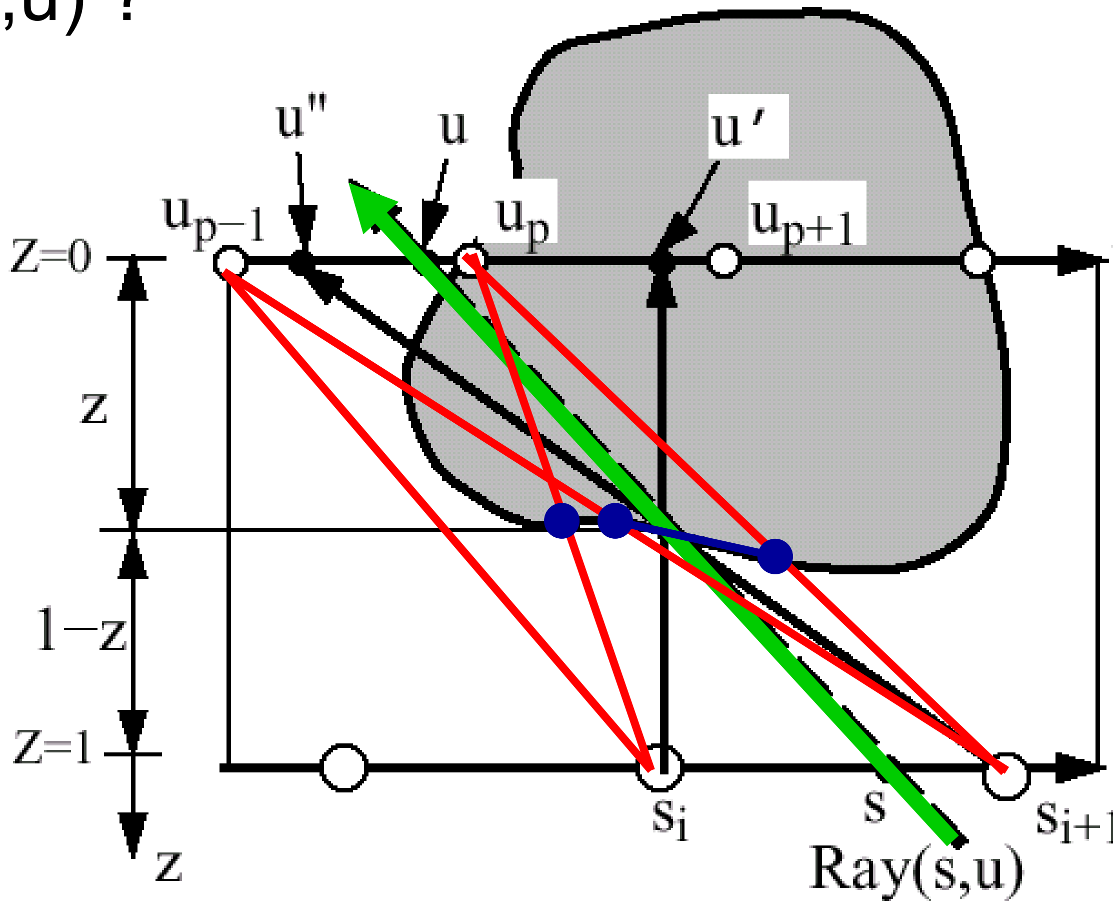
⇒ Wrong surface point

- Neighboring rays

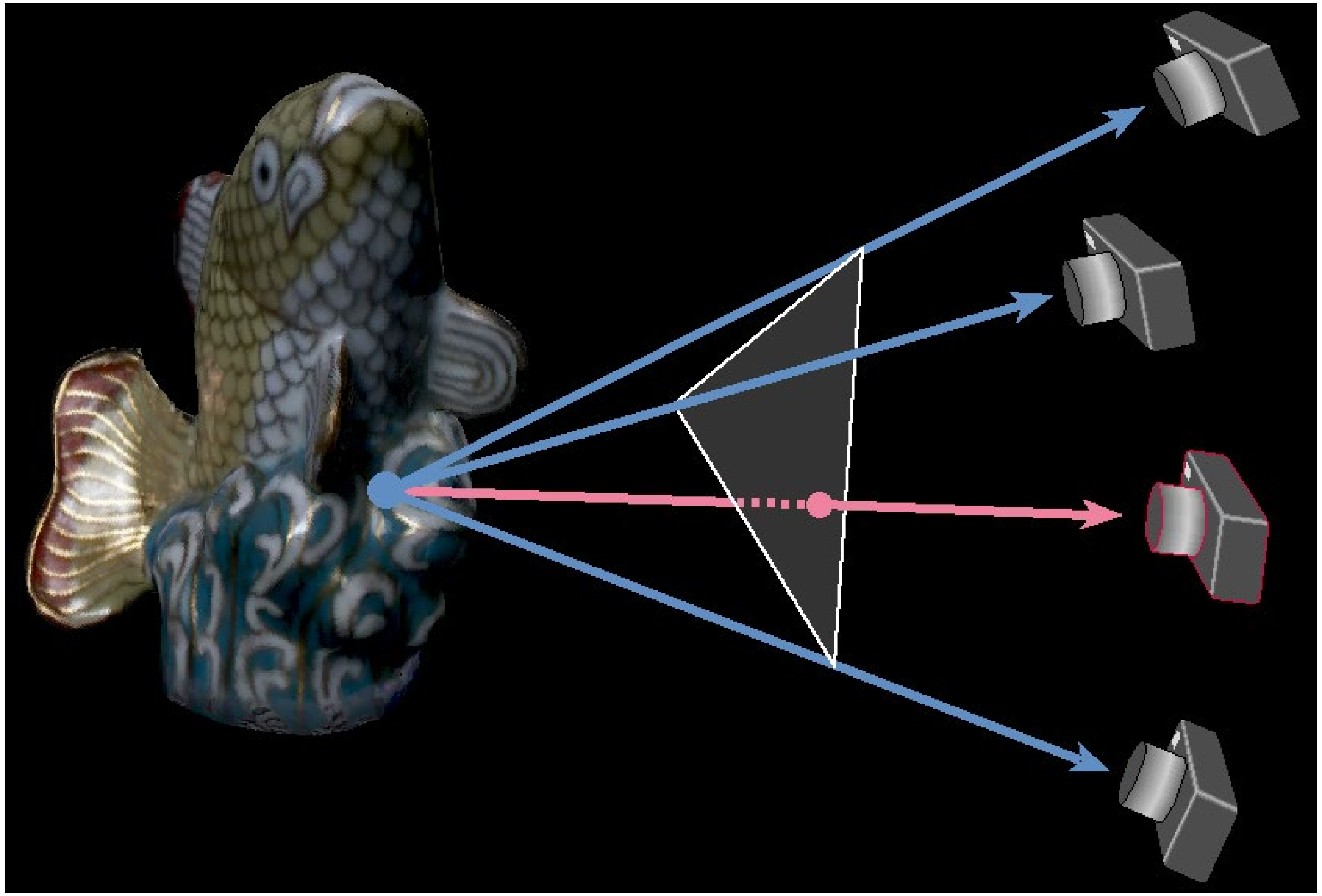
⇒ find surface point closest to ray (s,u)

- Fit planar surface

⇒ interpolate between closest rays



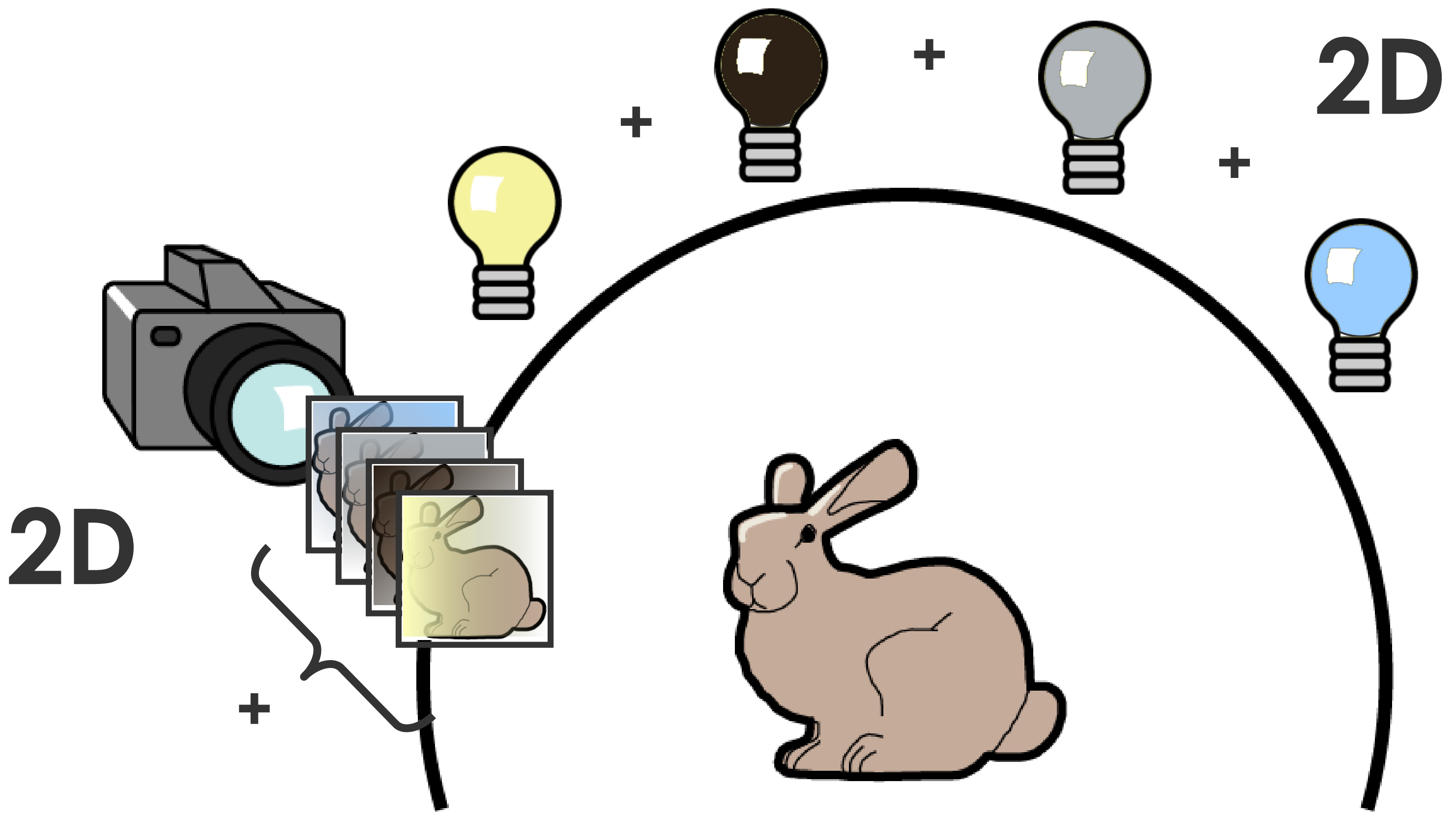
SLF vs. View-dependent Texture Mapping



Debevec *et al.* 1996, 1998
Pulli *et al.* 1997

4D-Reflectance Field

- [Debevec2000]



distant light sources only

Reflectance Field

- Same Scene – Different virtual illumination

Next Term

- Computational Photography (Master) (BSc fine as well)

- Forschungspraktikum (Master – individuell)

- Proseminar
- Bachelor- /Masterarbeiten
- Hiwi

- Massively Parallel Computing (Master)
  - Block lecture (10.-16.03.) (V+Ü)

- ~~Praktikum Computerspiele / Visual Effects~~