Tina Truong
Tanja Huber

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Exercise Nr. 6

(We worked on most exercises together with Stephan Amann and Amelie Schäfer so solutions might be similar.)

6.1

$$
\begin{aligned}
\mathcal{F}\left[f \otimes g\right] &= \mathcal{F}[f(t) \otimes g(t)] \\
&= \mathcal{F}\left[\int_{-\infty}^{\infty} f(\tau) \cdot g(t-\tau) d\tau\right] \\
&= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} f(\tau) \cdot (t-\tau) d\tau\right] \cdot e^{-2\pi ikt} dt \\
&= \int_{-\infty}^{\infty} f(\tau) \left[\int_{-\infty}^{\infty} g(t-\tau) \cdot e^{-2\pi ikt} dt\right] d\tau \\
&\text{Shift Theorem:} \\
&\mathcal{F}[g(t-\tau)] = e^{-2\pi ik\tau} \mathcal{G}(k) \\
&= \int_{-\infty}^{\infty} f(\tau) e^{-2\pi ik\tau} \cdot \mathcal{G}(k) d\tau \\
&= \mathcal{G}(k) \int_{-\infty}^{\infty} f(\tau) \cdot e^{-2\pi ik\tau} d\tau \\
&= \mathcal{G}(k) \cdot \mathcal{F}(k) \\
&= \mathcal{F}[f] \cdot \mathcal{F}[g]
\end{aligned}
$$

6.2

$$
\begin{aligned}
F(B(x)) &= \underbrace{\int_{-\infty}^{-1} B(x) * e^{-2\pi ikx} dx}_{0} + \int_{-1}^{1} B(x) * e^{-2\pi ikx} dx + \underbrace{\int_{-1}^{\infty} B(x) * e^{-2\pi ikx} dx}_{0} \\
&= \int_{-1}^{1} \underbrace{B(x)}_{\text{is 1 for all -1 x 1}} * e^{-2\pi ikx} dx \\
&= \int_{-1}^{1} e^{-2\pi ikx} dx \\
&= \int_{-1}^{1} cos(-2\pi kx) + i \sin(-2\pi kx) dx \\
&= \int_{-1}^{1} cos(-2\pi kx) dx + \int_{-1}^{1} i \sin(-2\pi kx) dx \\
&= \left[\frac{sin(-2\pi kx)}{-2\pi k}\right]_{-1}^{1} + i \left[\frac{-cos(-2\pi kx)}{-2\pi k}\right]_{-1}^{1} \\
&= \frac{sin(-2\pi k)}{-2\pi k} - \frac{sin(2\pi k)}{-2\pi k} + \underbrace{i\left(\frac{-cos(-2\pi k)}{-2\pi k} - \frac{-cos(2\pi k)}{-2\pi k}\right)}_{\text{is 0 since cos is symmetrical}} \\
&= -\frac{sin(-2\pi k) - sin(2\pi k)}{2\pi k} \\
&= -\frac{2sin(\frac{-2\pi k - 2\pi k}{2}) cos(\frac{-2\pi k + 2\pi k}{2})}{2\pi k} \\
&= -\frac{2sin(-2\pi k) cos(0)}{2\pi k} \\
&= -\frac{2sin(-2\pi k)}{2\pi k} \\
&= -\frac{sin(-2\pi k)}{\pi k} \\
&= \frac{sin(2\pi k)}{\pi k}
\end{aligned}
$$

is a $sinc(x) = \dfrac{sin(\pi x)}{\pi x}$ type function