Computer Graphics (Graphische Datenverarbeitung)

- BRDFs -

WS 2021/2022

Corona

- Regular random lookup of the 3G certificates

- Contact tracing: We need to know who is in the class room
  - New ILIAS group for every lecture slot
  - Register via ILIAS or this QR code (only if you are present in this room)

Overview

- Last time
  - Radiance
  - Light sources
  - Rendering Equation & Formal Solutions

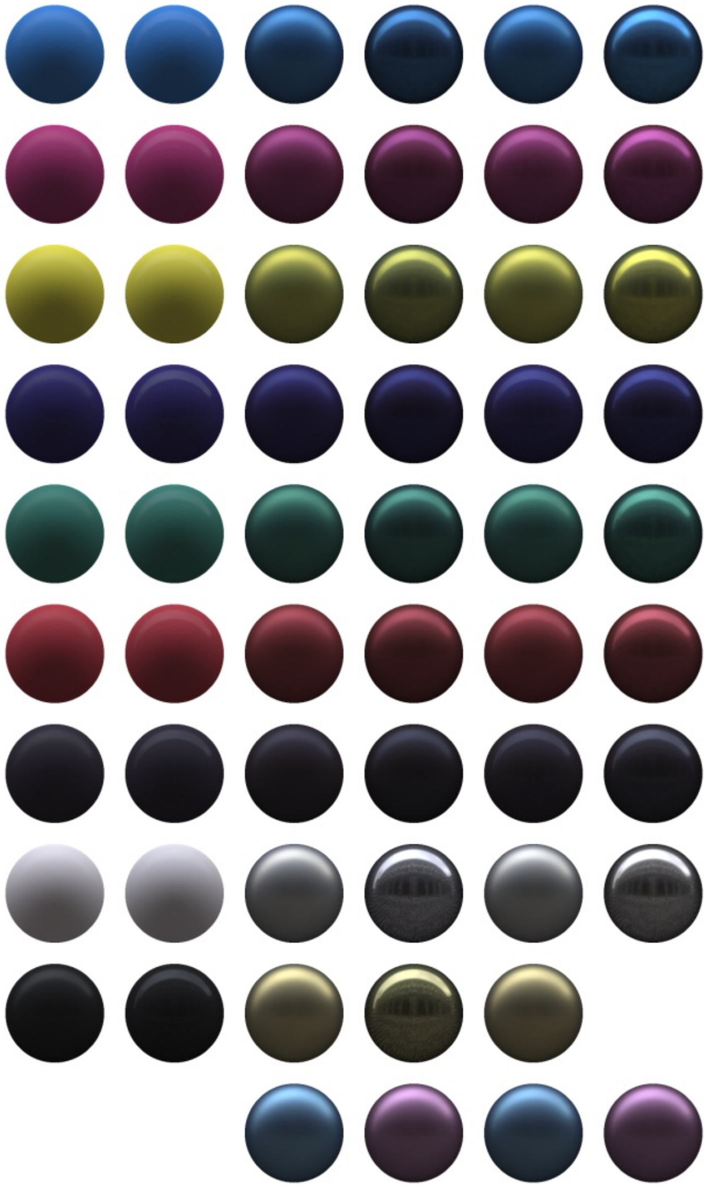
- Today
  - Bidirectional Reflectance Distribution Function (BRDF)
  - Reflection models
  - Shading
  - Spatially varying reflection properties

- Next lectures
  - Texturing
  - Filtering
  - Anti aliasing

BRDFs

Some Example of BRDFs

- Surface appearance varies in
  - Color / absorption
  - Specular roughness
  - Specular intensity, color
  - ….



Reflection Equation - Reflectance

- Reflection equation

$$L_o(\underline{x}, \underline{\omega}_o) = \int_{\Omega_+} f_r(\underline{\omega}_i, \underline{x}, \underline{\omega}_o)\, L_i(\underline{x}, \underline{\omega}_i) \cos\theta_i \; d\underline{\omega}_i$$

- BRDF
  - Ratio of reflected radiance to incident irradiance

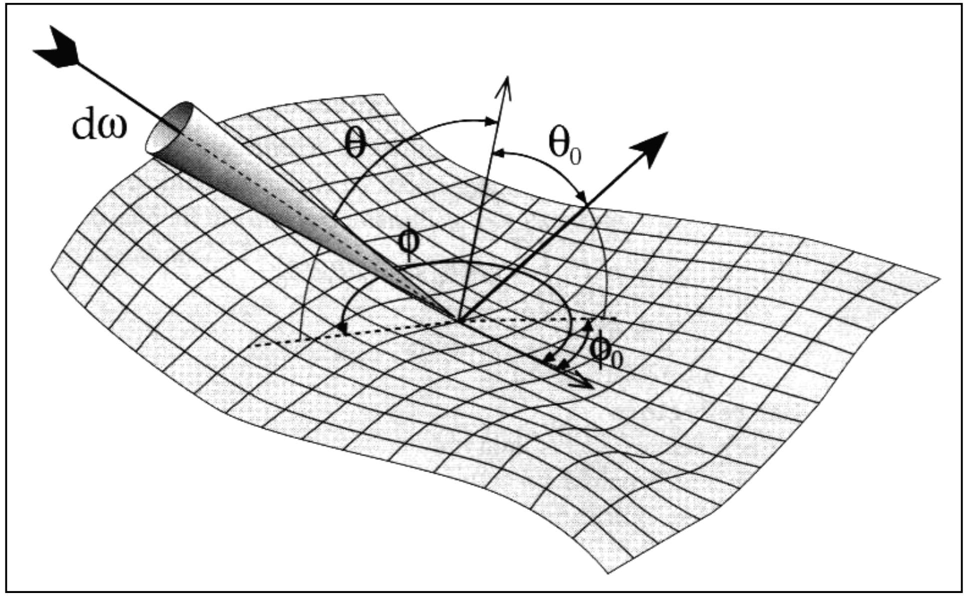
$$f_r(\omega_o, x, \omega_i) = \frac{dL_o(x, \omega_o)}{dE_i(x, \omega_i)} \qquad [1/sr]$$

Bidirectional Reflectance Distribution Function

- BRDF describes surface reflection for light incident from direction (θ_i,φ_i) observed from direction (θ_o,φ_o)
- Bidirectional
  - Depends on two directions and position (6-D function)
- Distribution function
  - Can be infinite, locally
- Unit [1/sr]

$$f_r(\underline{\omega}_o, \underline{x}, \underline{\omega}_i) = \frac{dL_o(\underline{x}, \underline{\omega}_o)}{dE_i(\underline{x}, \underline{\omega}_i)}$$

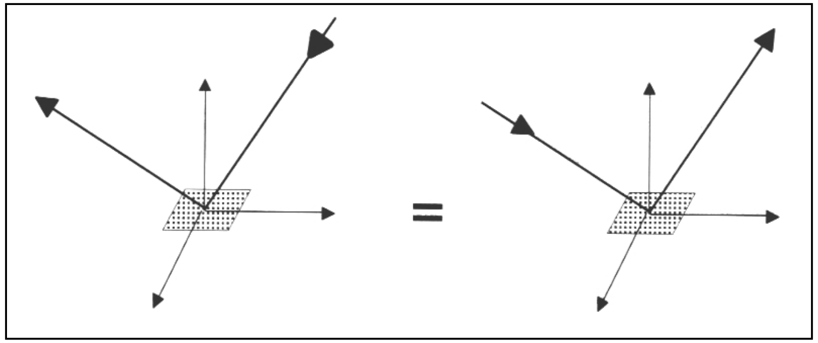
$$= \frac{dL_o(\underline{x}, \underline{\omega}_o)}{dL_i(\underline{x}, \underline{\omega}_i)\cos\theta_i \, d\underline{\omega}_i}$$



BRDF Properties

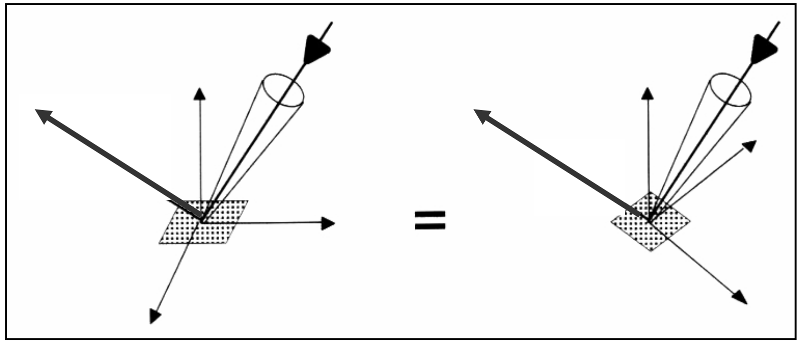
- Helmholtz reciprocity principle
  - BRDF remains unchanged if incident and reflected directions are interchanged

$$f_r(\omega_o, \omega_i) = f_r(\omega_i, \omega_o)$$



- Smooth surface: isotropic BRDF
  - reflectivity independent of rotation around surface normal
  - BRDF has only 3 instead of 4 directional degrees of freedom

$$f_r(\underline{x}, \theta_i, \theta_o, \varphi_o - \varphi_i)$$



BRDF Properties

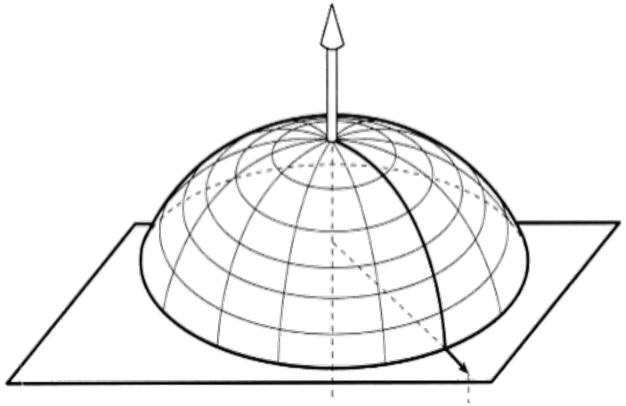
- Characteristics
  - BRDF units [sr^{-1}]
  - Range of values:
    - From 0 (absorption) to ∞ (reflection, δ -function)
  - Energy conservation law
    - No self-emission
    - Possible absorption

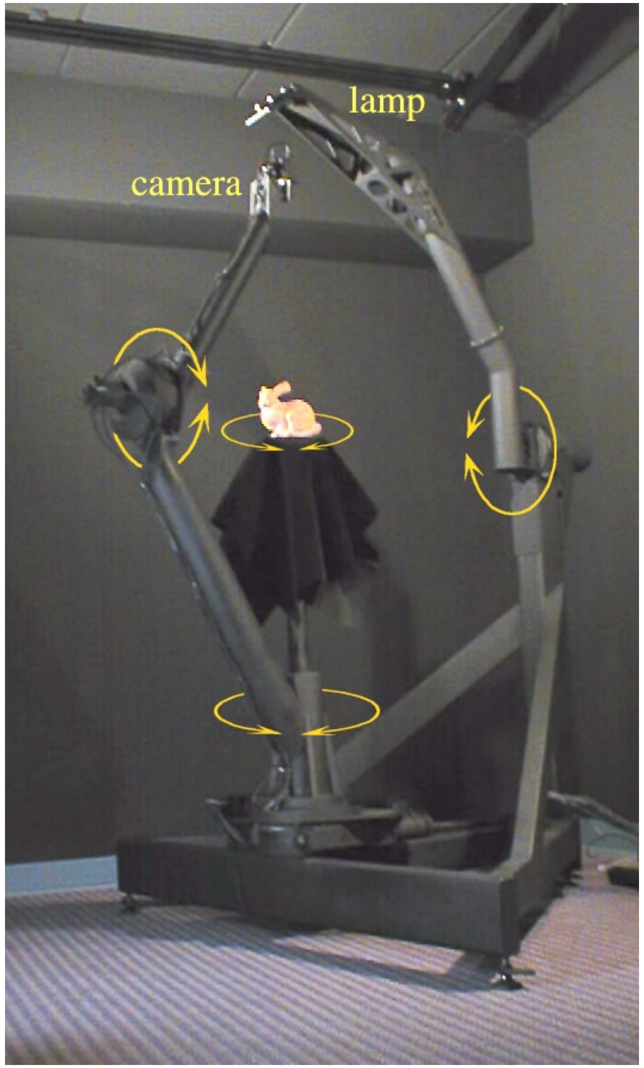
$$\int_{\Omega} f_r(\underline{\omega}_o, \underline{x}, \underline{\omega}_i) \cos\theta_o d\omega_o \leq 1 \quad \forall\, \theta, \varphi$$

  - Reflection only at the point of entry ($x_i = x_o$) – opaque surfaces
    - No subsurface scattering
    - No refraction or transmission

BRDF Measurement

- Gonio-Reflectometer
- BRDF measurement
  - point light source position (θ, φ)
  - light detector position (θ_o, φ_o)
- 4 directional degrees of freedom
- BRDF representation
  - m incident direction samples (θ, φ)
  - n outgoing direction samples (θ_o, φ_o)
  - $m*n$ reflectance values (large!!!)

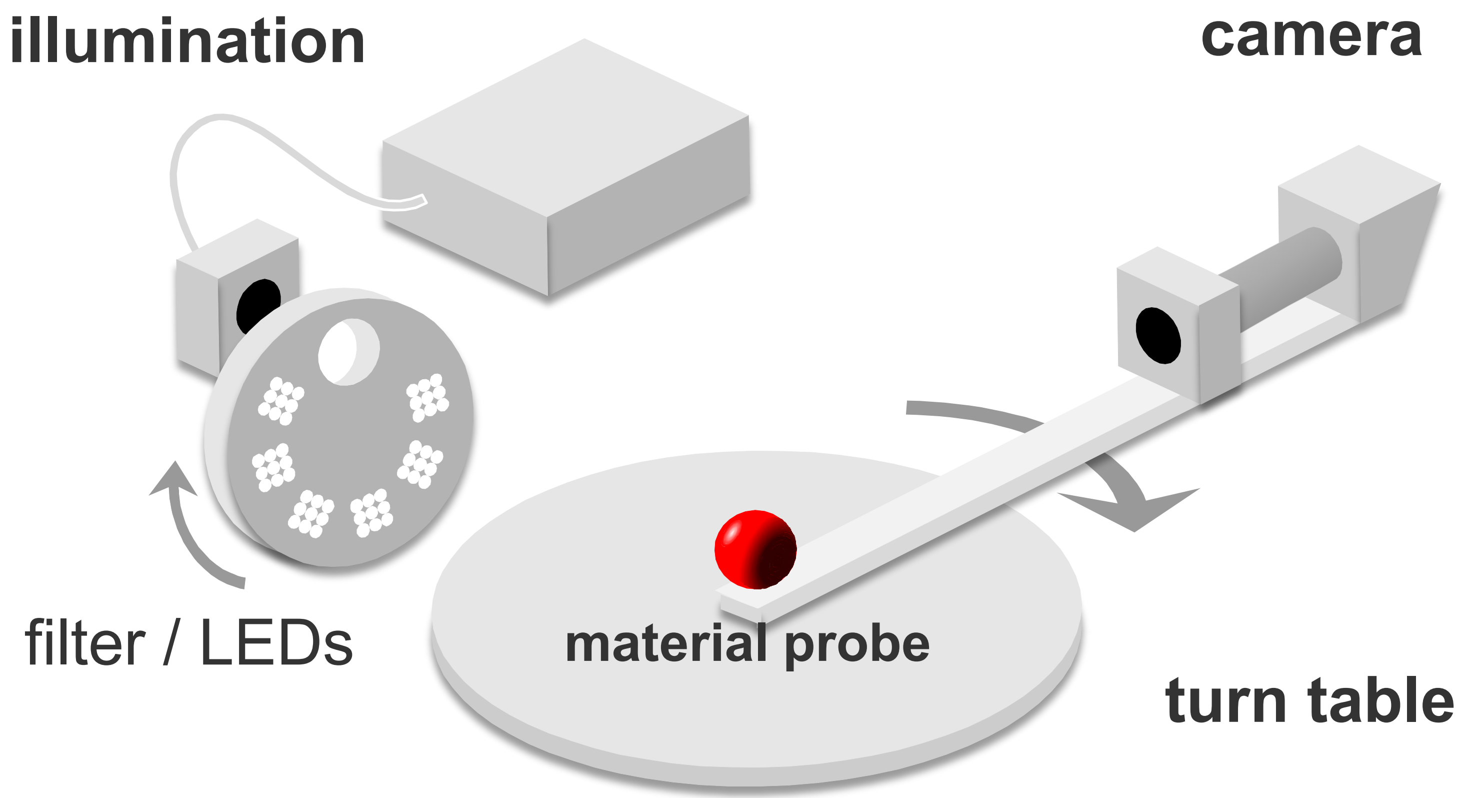




Stanford light gantry

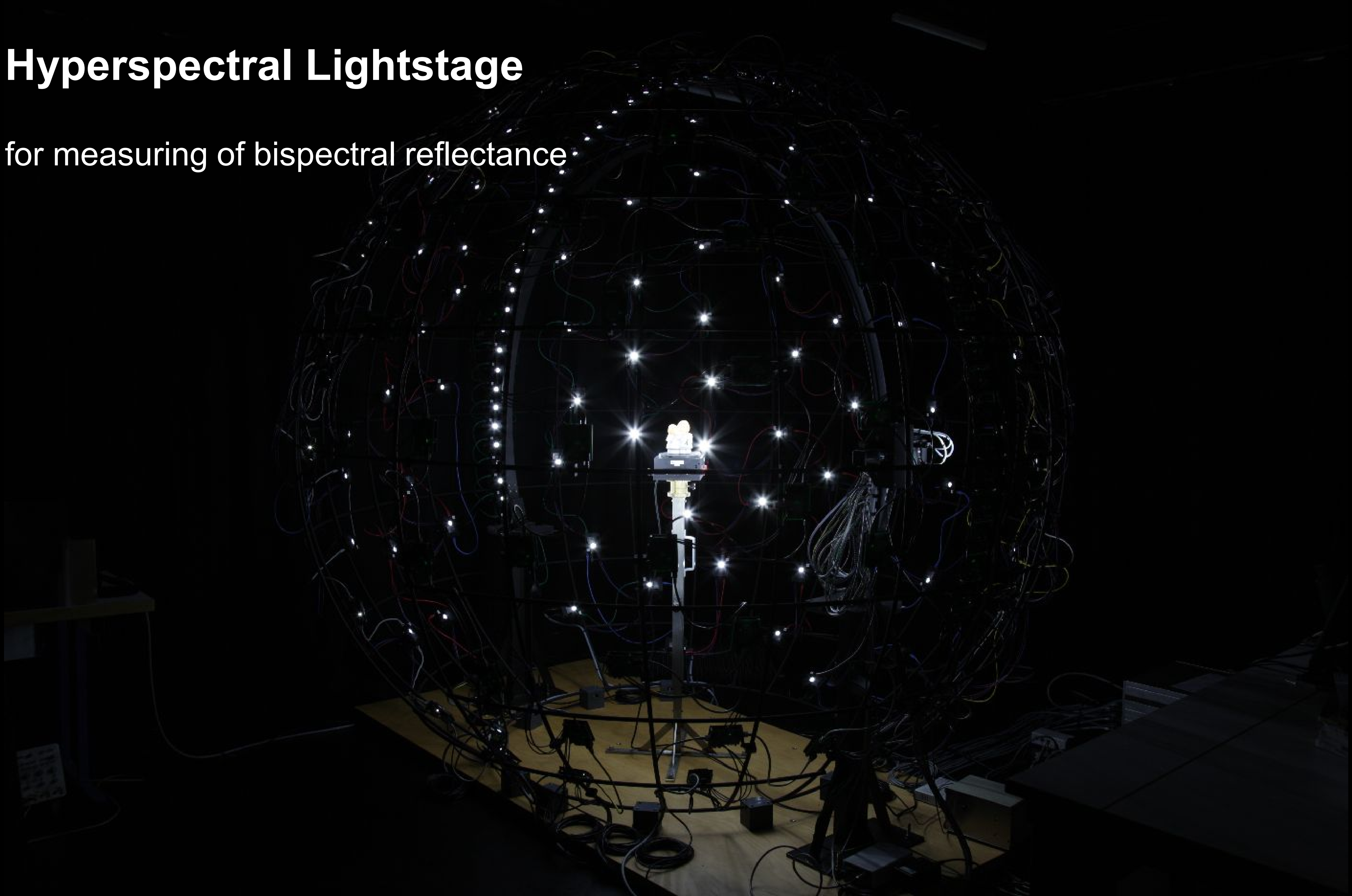
BRDF Measurement

Assumption: homogeneous material



Hyperspectral Lightstage

for measuring of bispectral reflectance

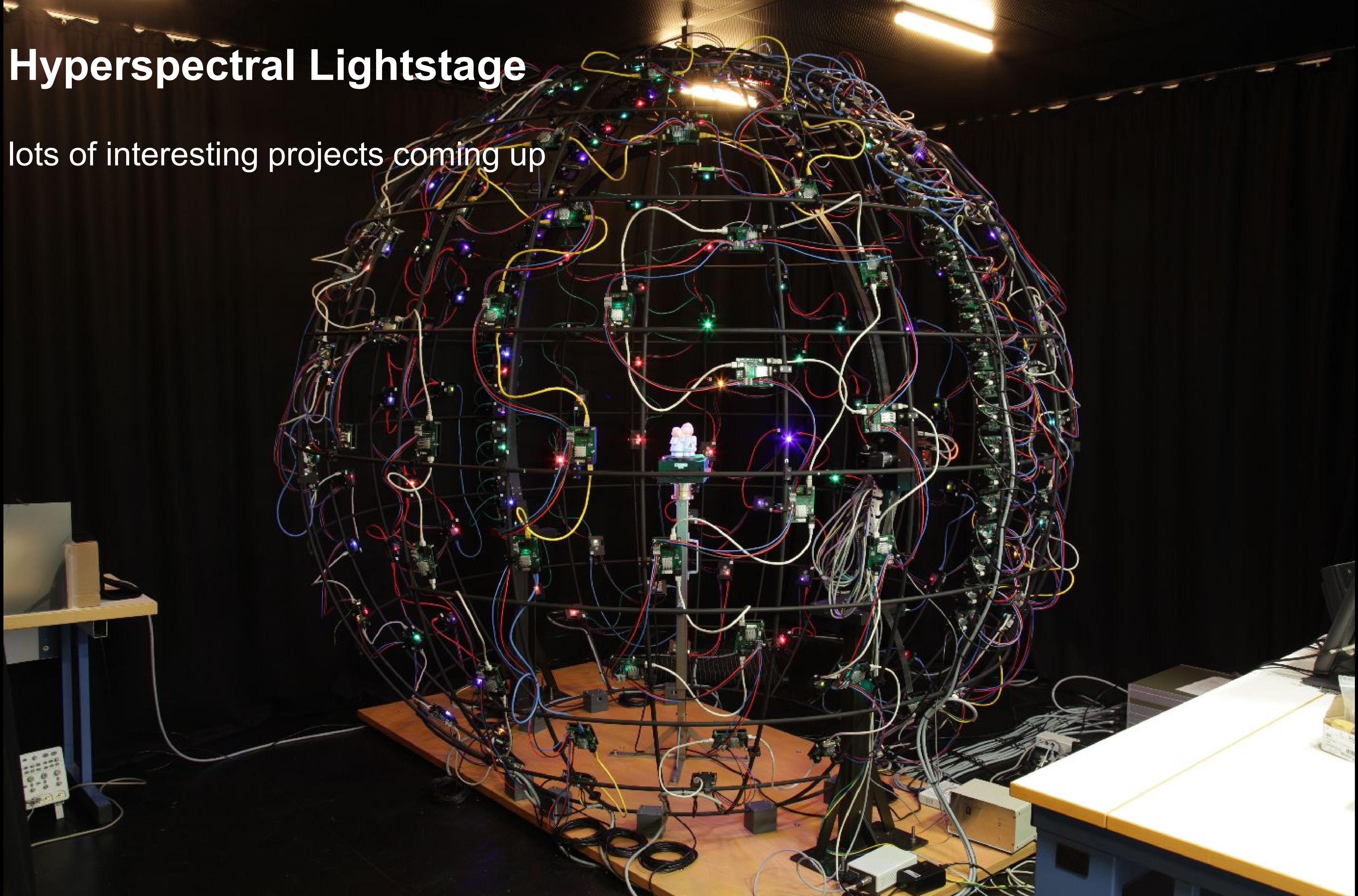


Hyperspectral Lightstage

controlled spectrum from each direction

Hyperspectral Lightstage

lots of interesting projects coming up



Compressive Higher-order Sparse and Low Rank Acquisition with a Hyperspectral Light Stage

paper id: 0226

(this video has audio)

Tabulated BRDF

- **4D Table parameterized by** $(\theta_i, \phi_i, \theta_o, \phi_o)$

$\theta_i \rightarrow$; $\phi_i \downarrow$

0.0005	0.001	0.002	0.0025	0.0025
0.0008	0.0012	0.0025	0.0028	0.003
0.0009	0.0012	0.003	0.004	0.005
0.00095	0.0011	0.004	0.0032	0.004
0.00089	0.001	0.003	0.003	0.003

Tabulated BRDF

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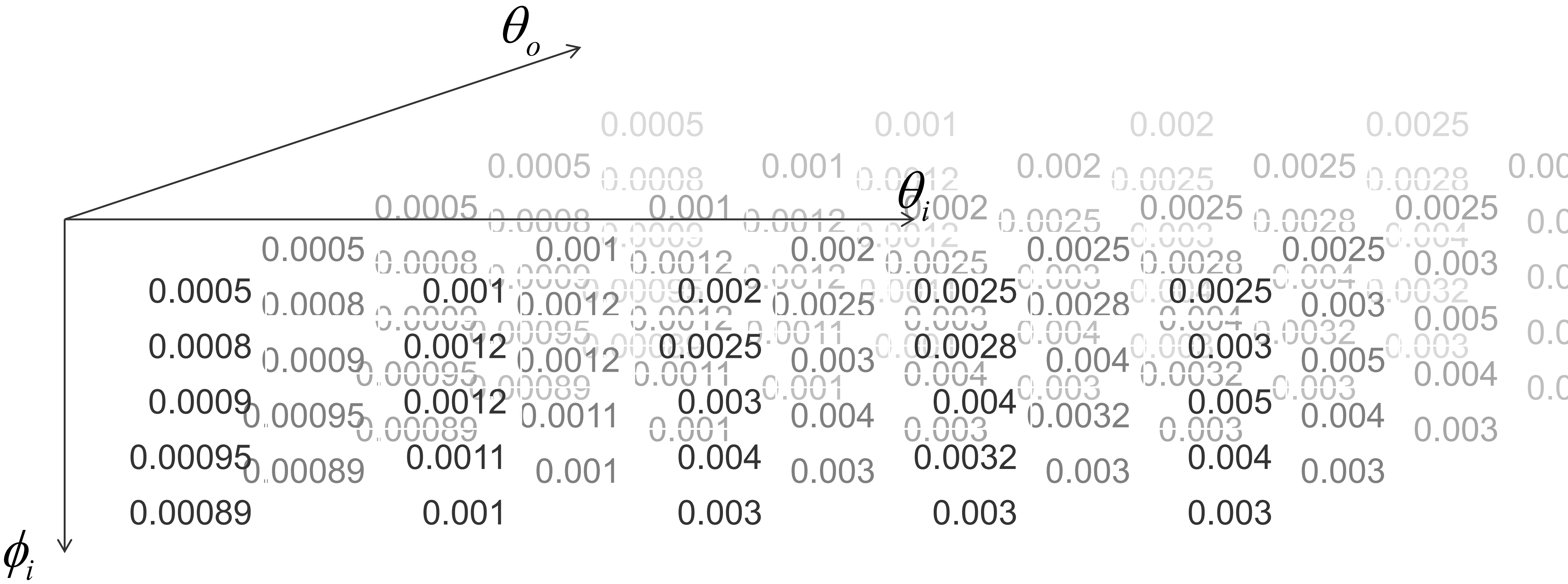

Tabulated BRDF

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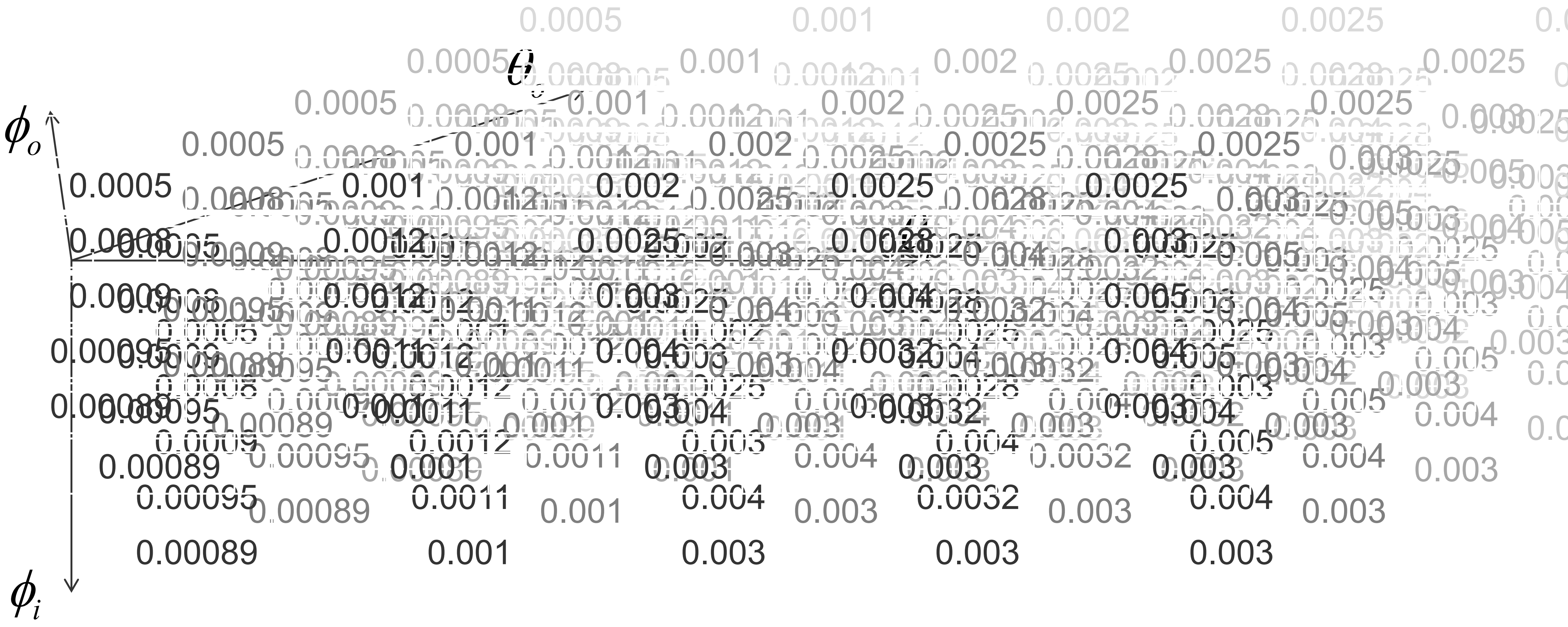
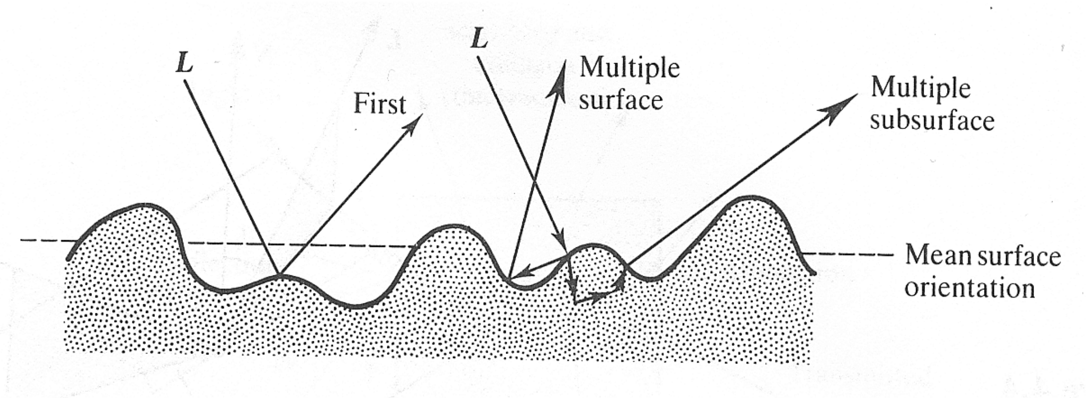


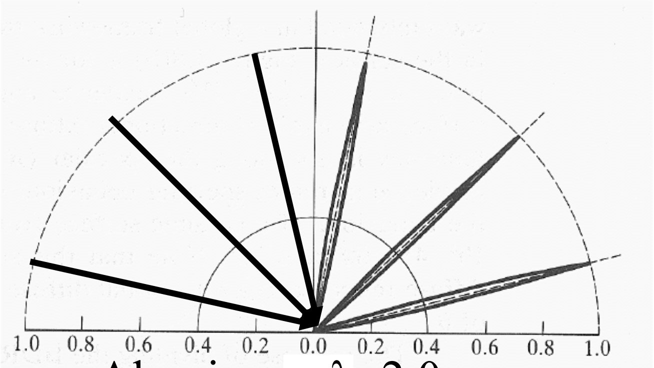
Rendering from Measured BRDF

- Linearity, superposition principle
  - Complex illumination: integrating light distribution against BRDF
  - Sampled BRDF: superimposed point light sources
- Interpolation
  - Look-up during rendering
  - Sampled BRDF must be filtered
- BRDF Modeling
  - Fit parameterized BRDF model to measured data
  - Continuous function
  - No interpolation
  - Fast evaluation
- Representation in spherical harmonics basis
  - Mathematically elegant filtering, illumination-BRDF integration

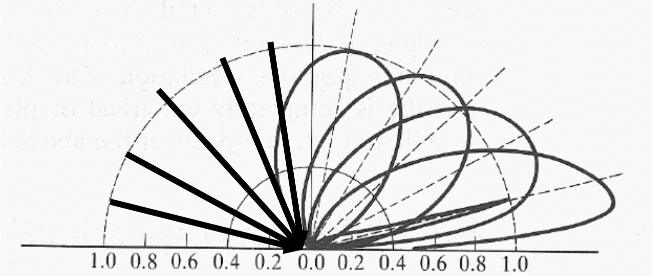
Reflectance

- Reflectance may vary with
  - Illumination angle
  - Viewing angle
  - Wavelength
  - (Polarization, ...)
- Variations due to
  - Absorption
  - Surface micro-geometry
  - Index of refraction / dielectric constant
  - Scattering

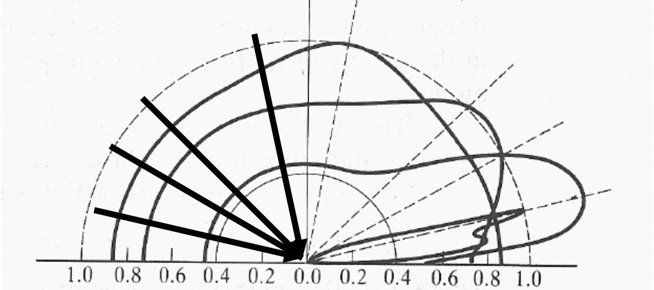




Aluminum; λ=2.0μm



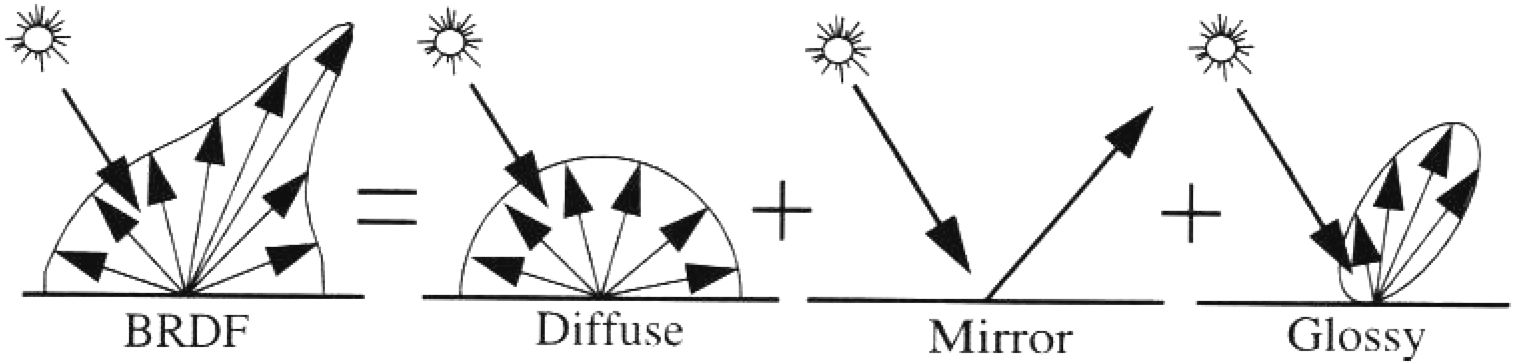
Aluminum; λ=0.5μm



Magnesium; λ=0.5μm

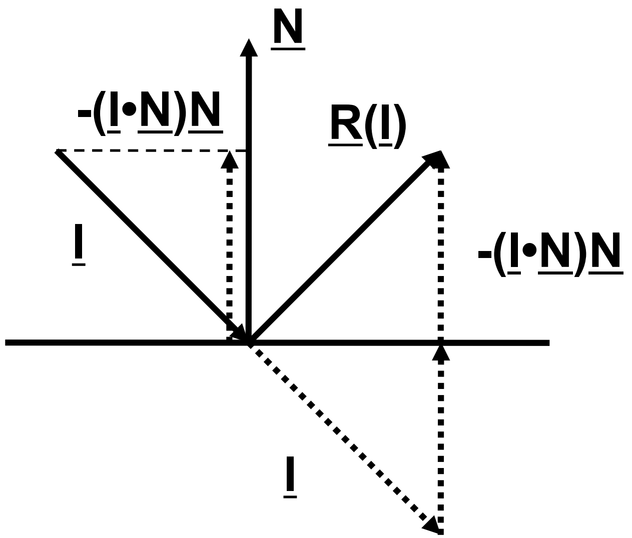
BRDF Modeling

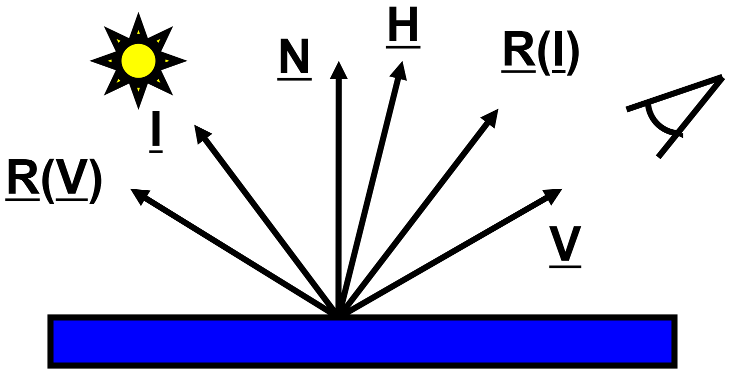
- Phenomenological approach
  - Description of visual surface appearance
- Ideal specular reflection
  - Reflection law
  - Mirror
- Glossy reflection
  - Directional diffuse
  - Shiny surfaces
- Ideal diffuse reflection
  - Lambert's law
  - Matte surfaces

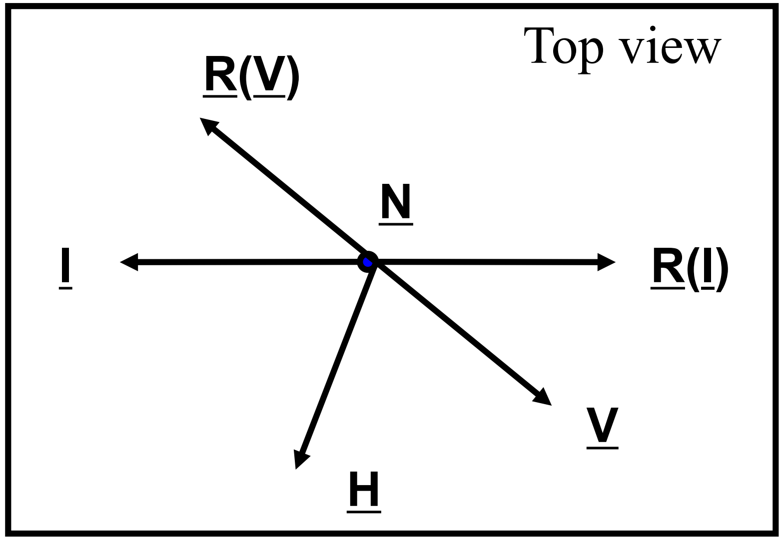


Reflection Geometry

- Direction vectors (normalize):
  - N: surface normal
  - I: vector to the light source
  - V: viewpoint direction vector
  - H: halfway vector
    - H= (I + V) / |I + V|
  - R(I): reflection vector
    - R(I)= I - 2(I•N)N
  - Tangential surface: local plane



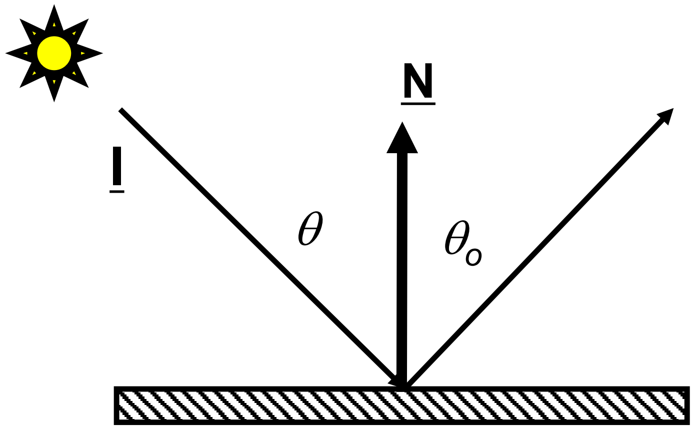




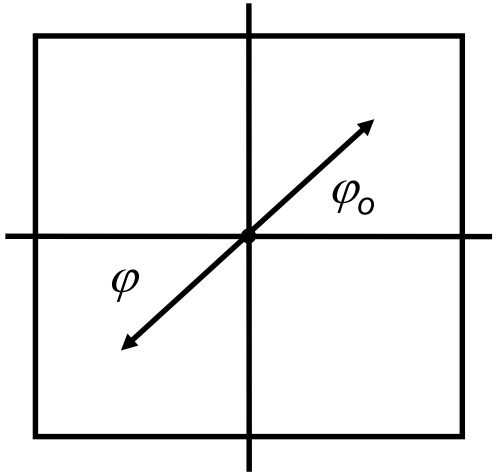
Ideal Specular Reflection

- Angle of reflectance equal to angle of incidence
- Reflected vector in a plane with incident ray and surface normal vector

$$\underline{\mathbf{R}}+(-\underline{\mathbf{I}}) = 2\cos\theta\,\underline{\mathbf{N}} = -2(\underline{\mathbf{I}}\bullet\underline{\mathbf{N}})\,\underline{\mathbf{N}}$$
$$\underline{\mathbf{R}}(\underline{\mathbf{I}}) = \underline{\mathbf{I}} - 2(\underline{\mathbf{I}}\bullet\underline{\mathbf{N}})\,\underline{\mathbf{N}}$$



$\theta = \theta_o$



$\varphi = \varphi_o + 180°$

Ideal Specular Reflection

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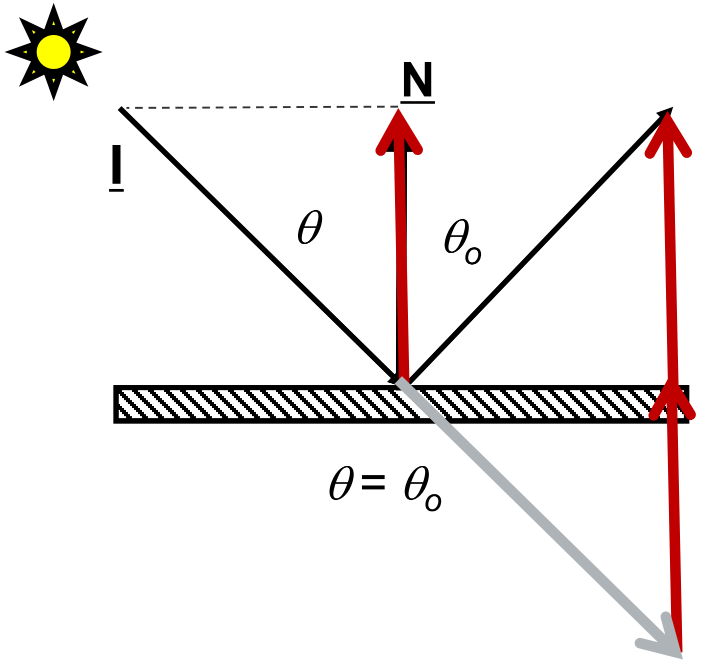

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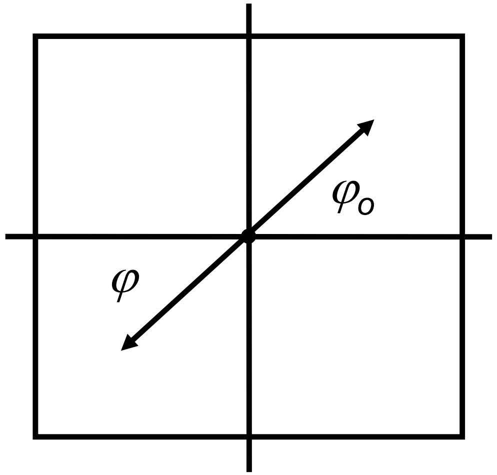


$\varphi = \varphi_o + 180°$

Mirror BRDF

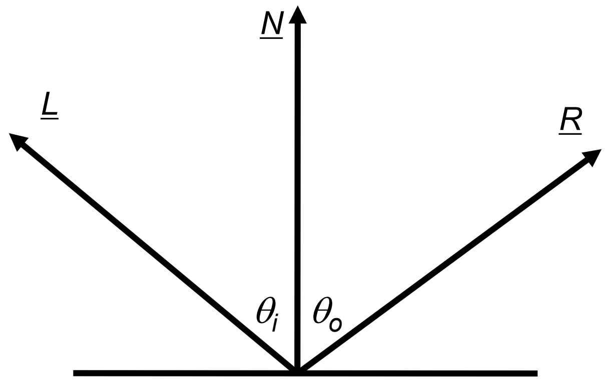
- Dirac Delta function δ(x)
  - δ(x) : zero everywhere except at x=0
  - Unit integral iff integration domain contains zero (zero otherwise)

$$f_{r,m}(\omega_o, x, \omega_i) = \rho_s(\theta_i) \cdot \frac{\delta(\cos\theta_i - \cos\theta_o)}{\cos\theta_i} \cdot \delta(\varphi_i - \varphi_o \pm \pi)$$

$$L_o(x, \omega_o) = \int_{\Omega_+} f_{r,m}(\omega_o, x, \omega_i) L_i(\theta_i, \varphi_i) \cos\theta_i \, d\underline{\omega}_i = \rho_s(\theta_i) L_i(\theta_o, \varphi_o \pm \pi)$$

- Specular reflectance ρ_s
  - Ratio of reflected radiance in specular direction and incoming radiance
  - Dimensionless quantity between 0 and 1

$$\rho_s(\theta_i) = \frac{\Phi_o(\theta_o)}{\Phi_i(\theta_i)}$$



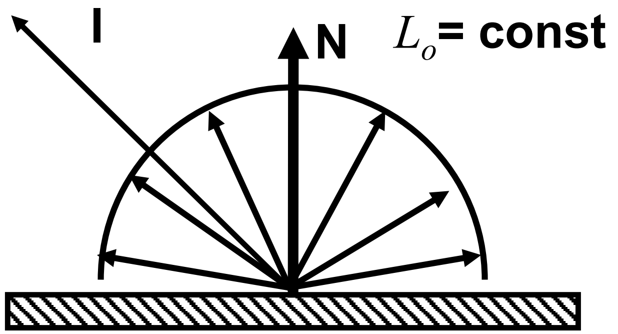
Diffuse Reflection

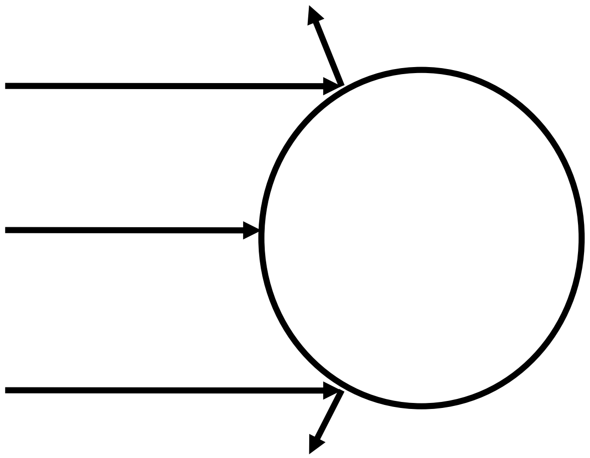
- Light equally likely to be reflected in any output direction (independent of input direction)
- Constant BRDF

$$f_{r,d}(\underline{\omega}_o, \underline{x}, \underline{\omega}_i) = k_d = \text{const}$$

$$L_o(\underline{x}, \underline{\omega}_o) = \int_\Omega k_d L_i(\underline{x}, \underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i = k_d \int_\Omega L_i(\underline{x}, \underline{\omega}_i) \cos\theta_i \, d\underline{\omega}_i = k_d E$$

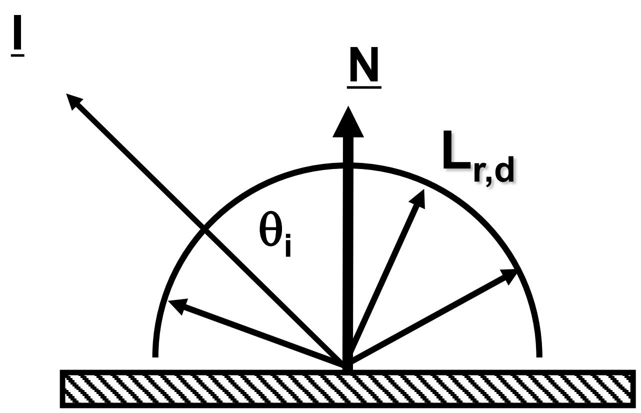
- kd: diffuse coefficient, material property [1/sr]





Lambertian Diffuse Reflection

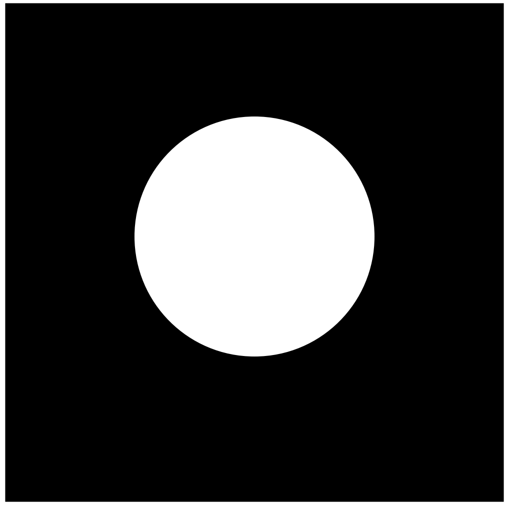
- **Radiosity** $$B = \int_{\Omega} L_o(\underline{x}, \underline{\omega}_o) \cos\theta_o \, d\underline{\omega}_o = L_o \int_{\Omega} \cos\theta_o \, d\underline{\omega}_o = \pi L_o$$
- **Diffuse Reflectance** $$\rho_d = \frac{B}{E} = \pi k_d$$
- **Lambert's Cosine Law** $$B = \rho_d E = \int_{\Omega} \rho_d L_i \cos\theta_i$$
- **For each light source**
  - **$L_{r,d} = k_d L_i \cos\theta_i = k_d L_i (\underline{l} \bullet \underline{N})$**

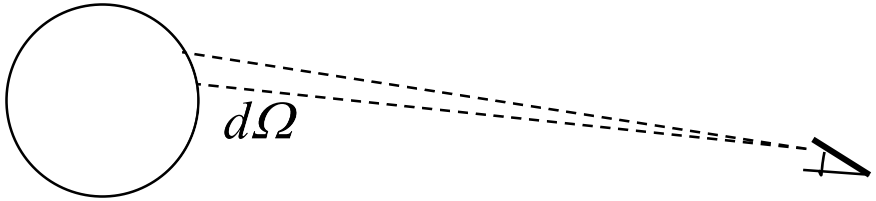


Lambertian Objects

Self-Luminous
spherical Lambertian Light Source

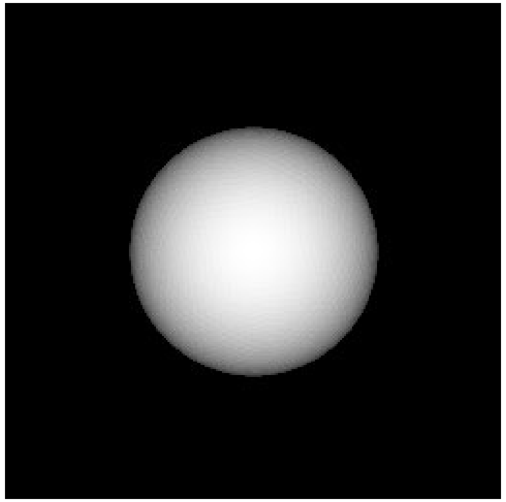
$$\Phi_0 \propto L_0 \cdot d\Omega$$

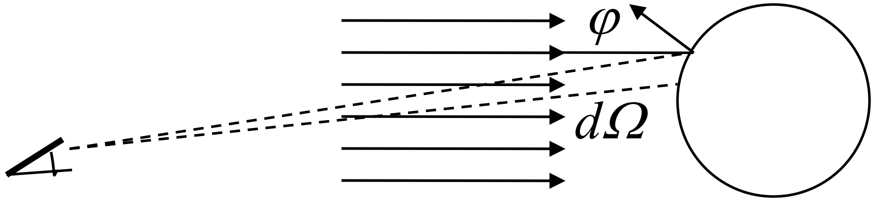




Eye-light illuminated
Spherical Lambertian Reflector

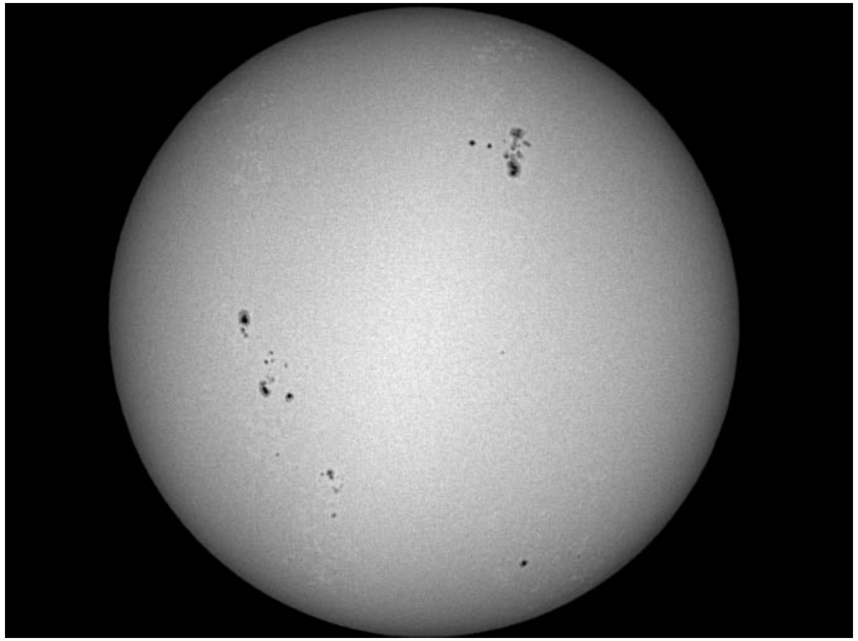
$$\Phi_1 \propto L_0 \cdot \cos\varphi \cdot d\Omega$$





Lambertian Objects II

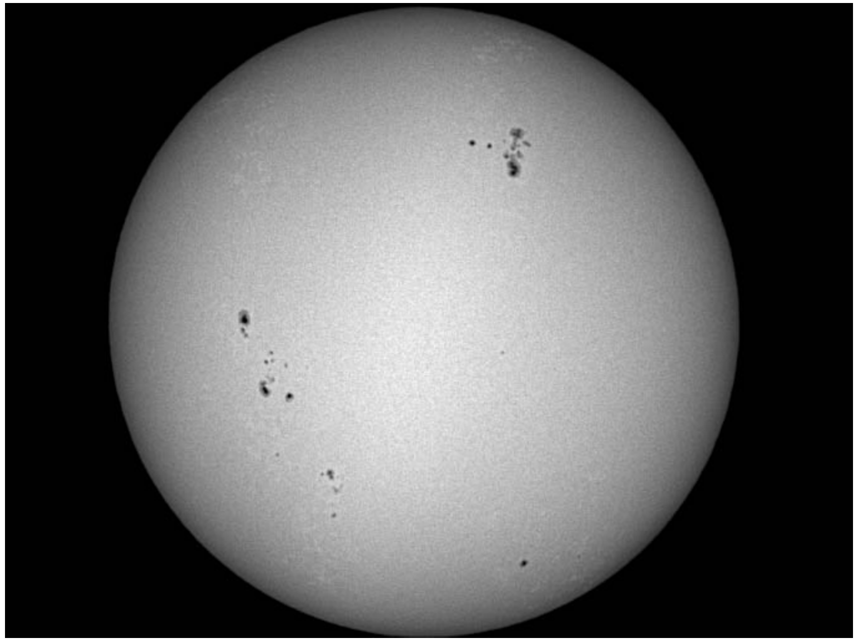
The Sun



The Moon

Lambertian Objects II

The Sun



The Moon

- Absorption in photosphere
- Path length through photosphere longer from the Sun's rim

- Surface covered with fine dust
- Dust on TV visible best from slanted viewing angle

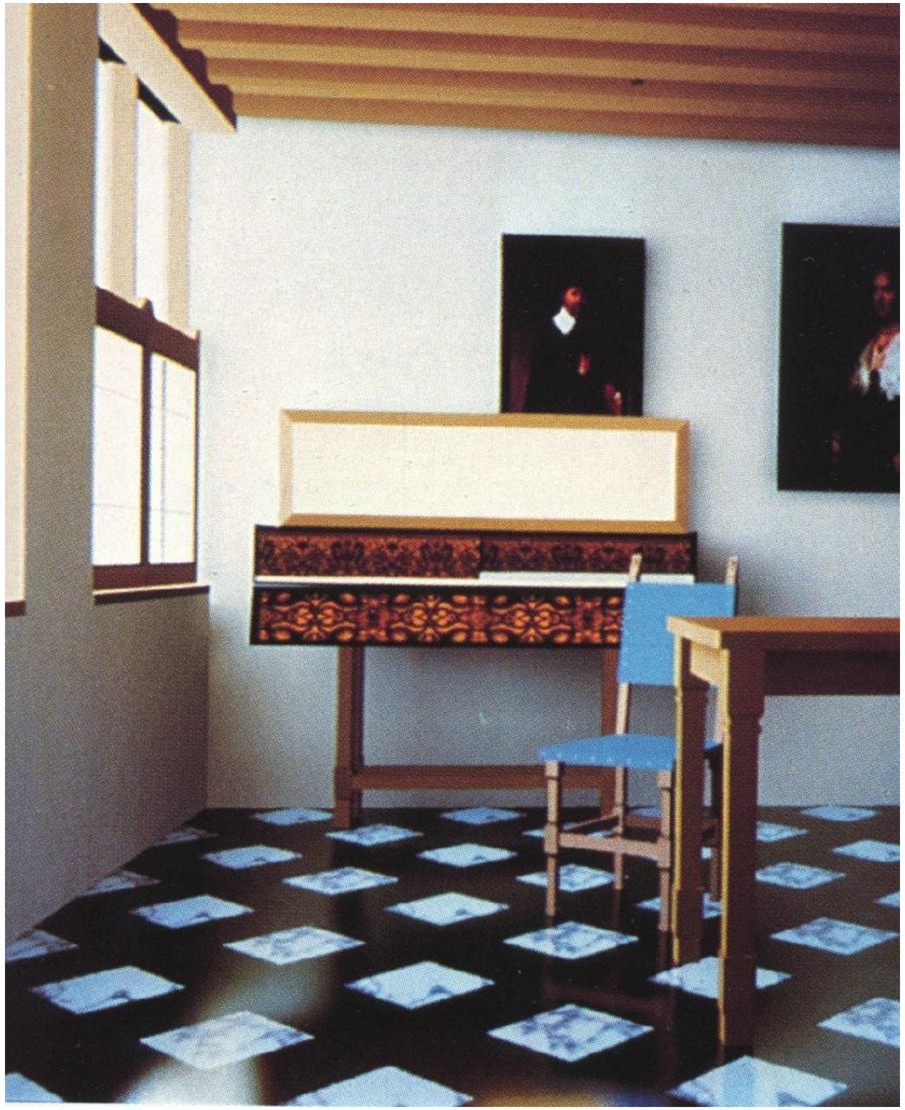
$\Rightarrow$ Neither the Sun nor the Moon are Lambertian

“Diffuse” Reflection

- Theoretical explanation
  - Multiple scattering
- Experimental realization
  - Pressed magnesium oxide powder
  - Almost never valid at high angles of incidence

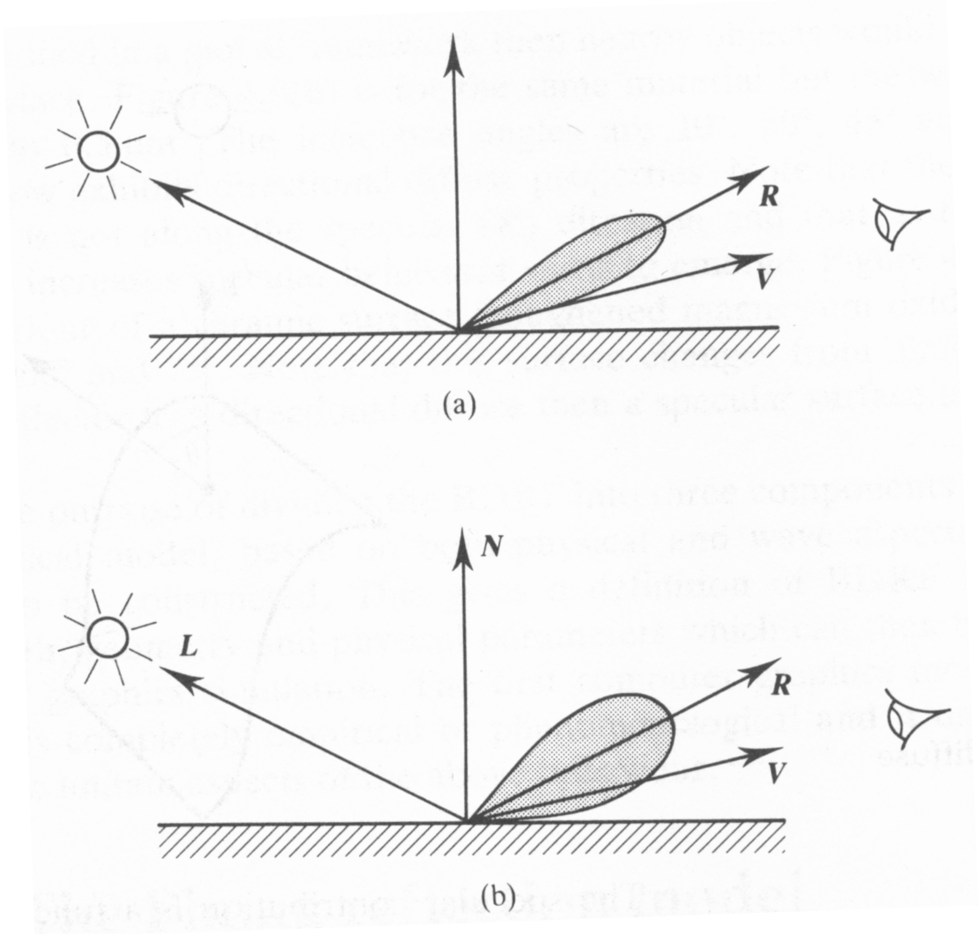
- Paint manufacturers attempt to create ideal diffuse paints

Glossy Reflection



Glossy Reflection

- Due to surface roughness
- Empirical models
  - Phong
  - Blinn-Phong
- Physical models
  - Blinn
  - Cook & Torrance



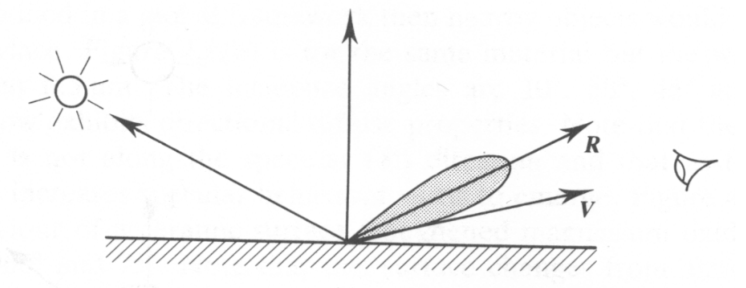
Phong Reflection Model

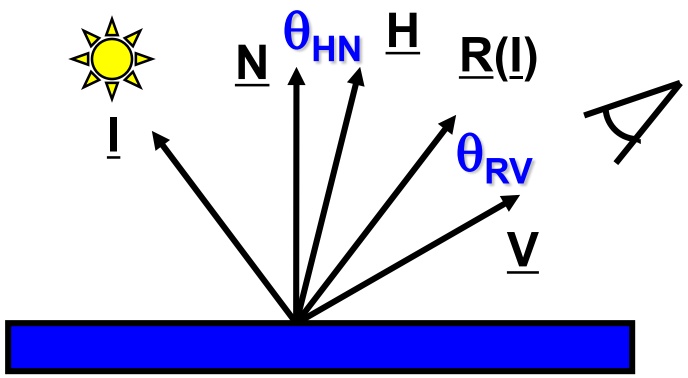
- Cosine power lobe

$$f_r(\omega_o, x, \omega_i) = k_s\left(\underline{R}(\underline{I}) \cdot \underline{V}\right)^{k_e}$$

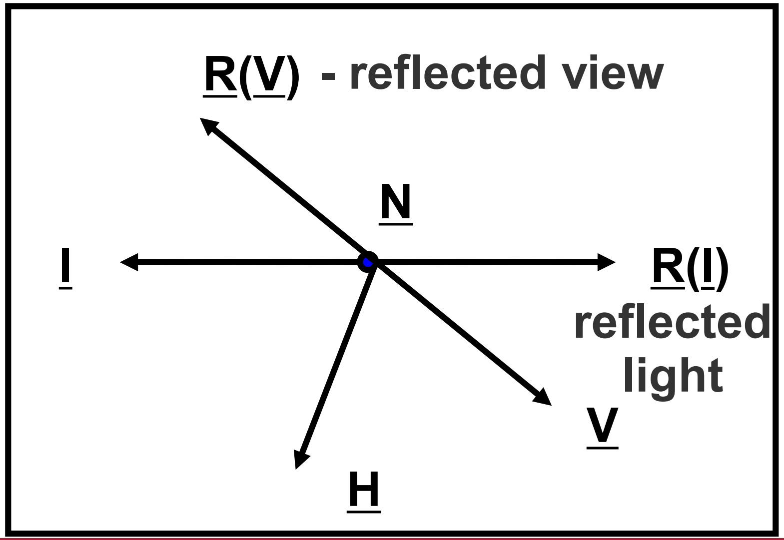
$$L_s = L_i k_s \cos^{k_e} \theta_{RV}$$

- Dot product & power
- Not energy conserving/reciprocal
- Plastic-like appearance





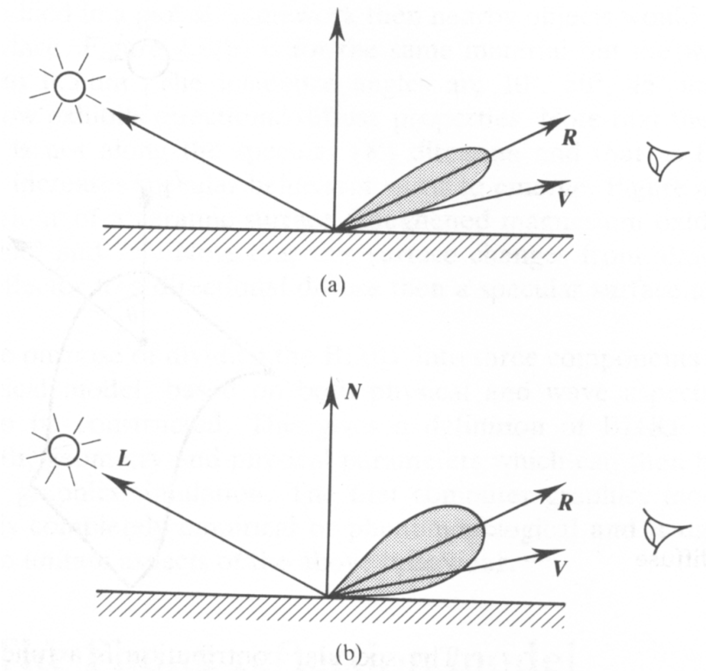
birds eye view at the surface

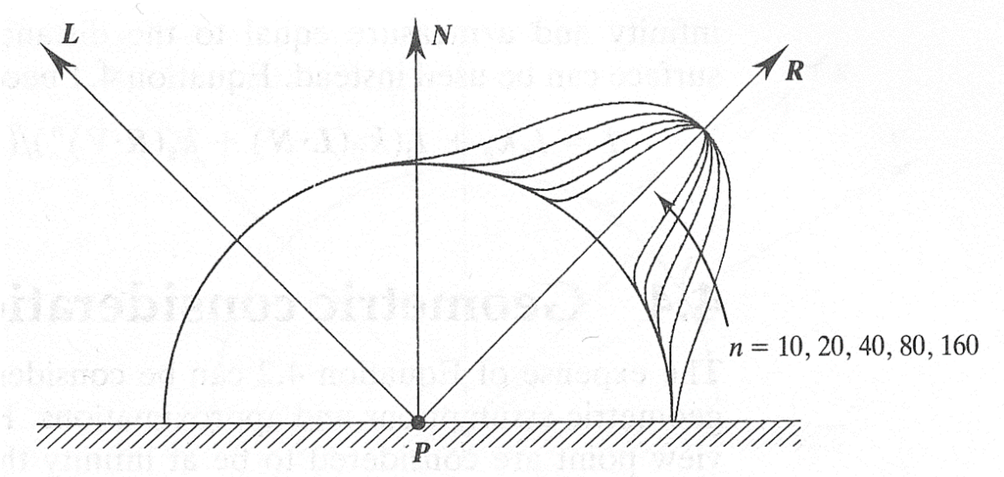


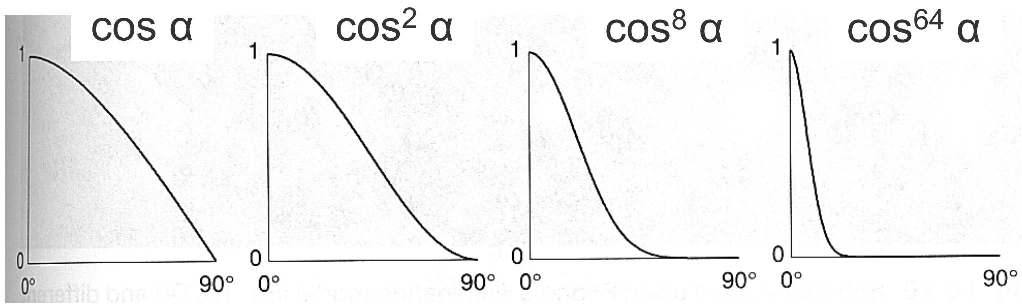
Phong Exponent ke

$$f_r(\omega_o, x, \omega_i) = k_s\left(\underline{R}(\underline{I}) \cdot \underline{V}\right)^{k_e}$$

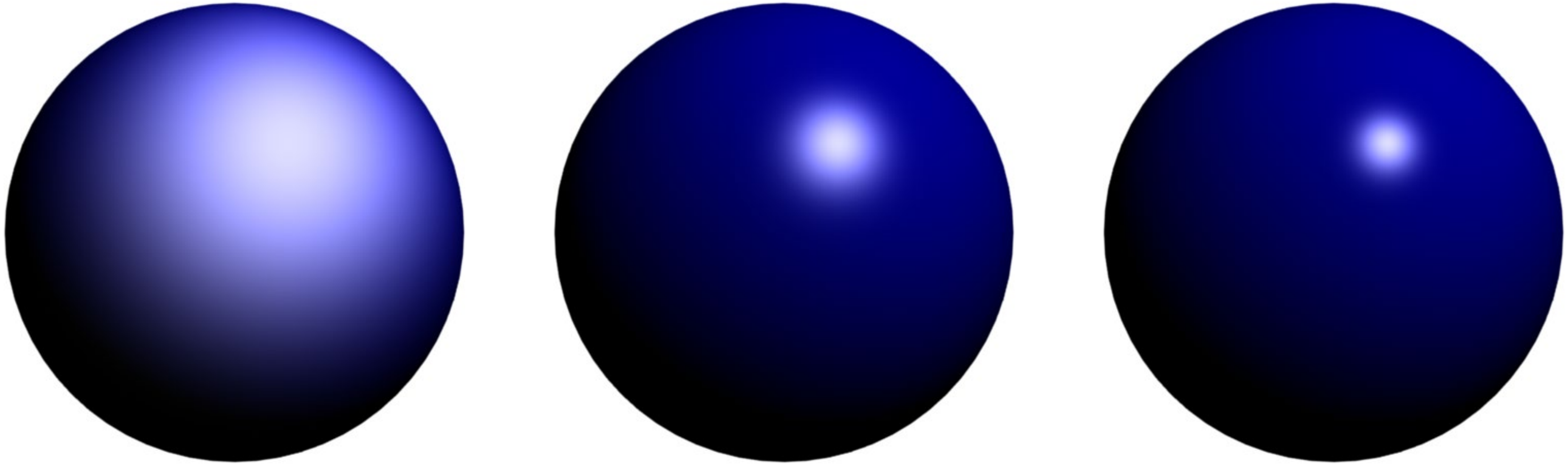
- Determines size of highlight







Phong Lobes



$k = 4$ $\qquad$ $k = 50$ $\qquad$ $k = 120$

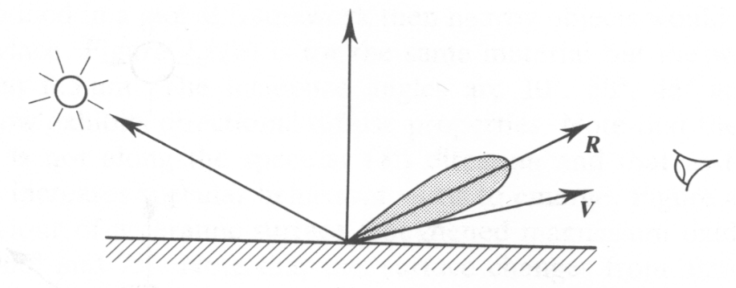
$$f_r(\omega_o, x, \omega_i) = k_s \left(\underline{R}(\underline{I}) \cdot \underline{V} \right)^{k_e}$$

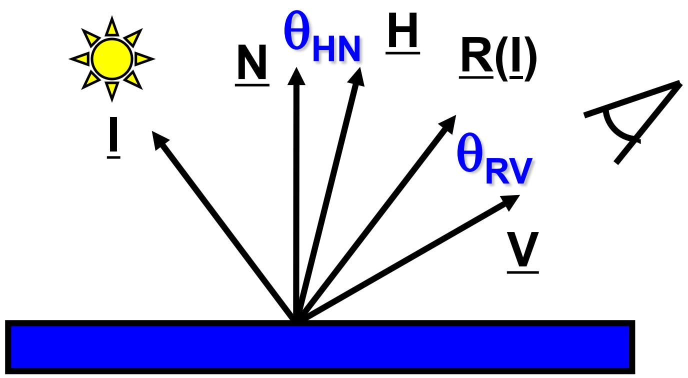
Blinn-Phong Reflection Model

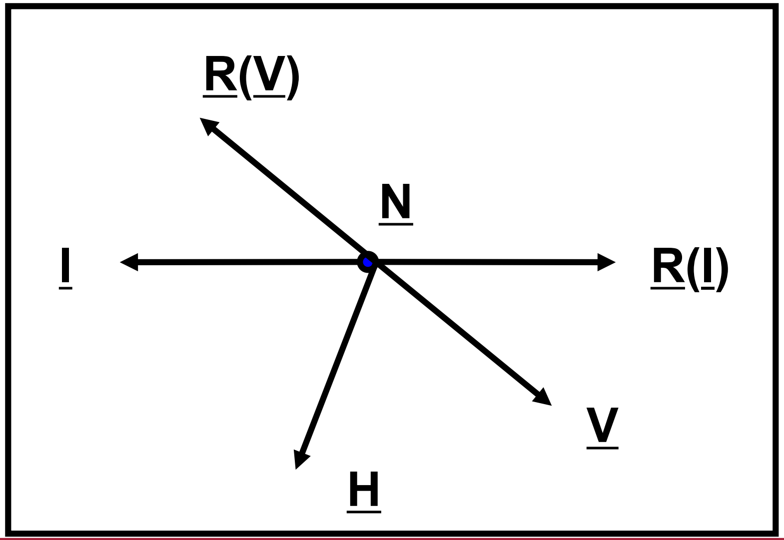
- Blinn-Phong reflection model

$$f_r(\omega_o, x, \omega_i) = k_s (H \cdot N)^{k_e}$$
$$L_s = L_i k_s \cos^{k_e} \theta_{HN}$$
$$\theta_{RV} \Rightarrow \theta_{HN}$$

  - Light source, viewer far away
  - I, R constant: H constant
    - θ_{HN} less expensive to compute







Phong Illumination Model

- Extended light sources: l point light sources

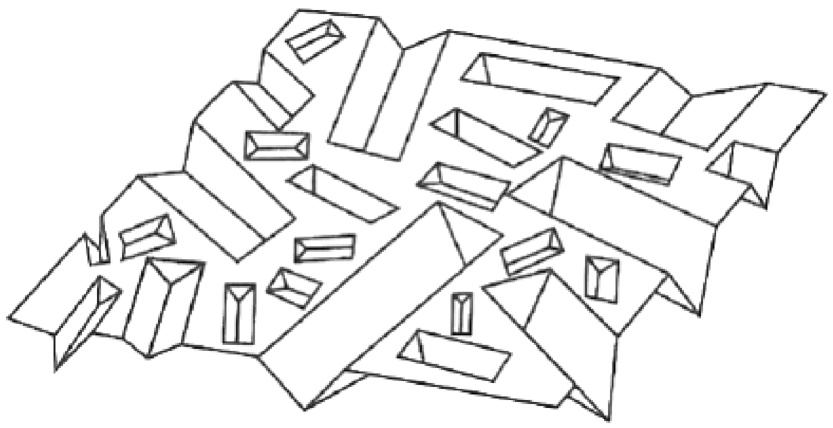
$$\mathrm{L_r} = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (R(I_l) \cdot V)^{k_e} \quad \text{(Phong)}$$

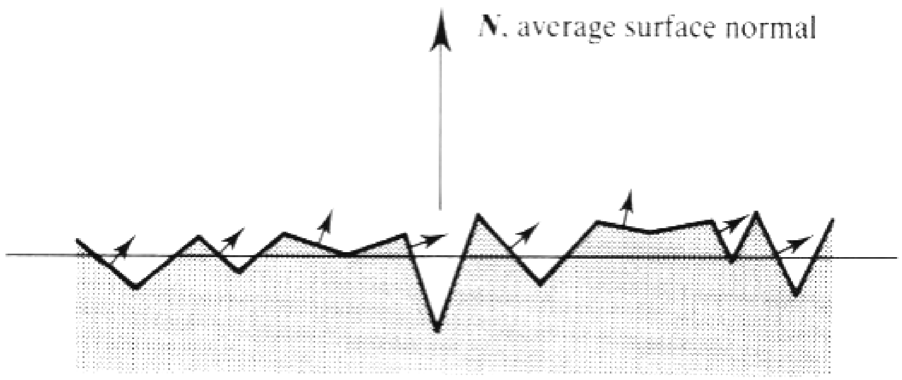
$$\mathrm{L_r} = k_a L_{i,a} + k_d \sum_l L_l (I_l \cdot N) + k_s \sum_l L_l (H_l \cdot N)^{k_e} \quad \text{(Blinn)}$$

- Color of specular reflection equal to light source
- Heuristic model
  - Contradicts physics
  - Purely local illumination
    - Only direct light from the light sources
    - No further reflection on other surfaces
    - Constant ambient term
- Often: light sources & viewer assumed to be far away

Microfacet Model

- Isotropic microfacet collection
- Microfacets assumed as perfectly smooth reflectors
- BRDF
  - Distribution of microfacets
    - Often probabilistic distribution of orientation or V-groove assumption
  - Planar reflection properties, Self-masking, shadowing
  - **Glossy reflection by microfacets oriented such that we see perfect reflection. Intensity ~ number of correctly oriented patches**





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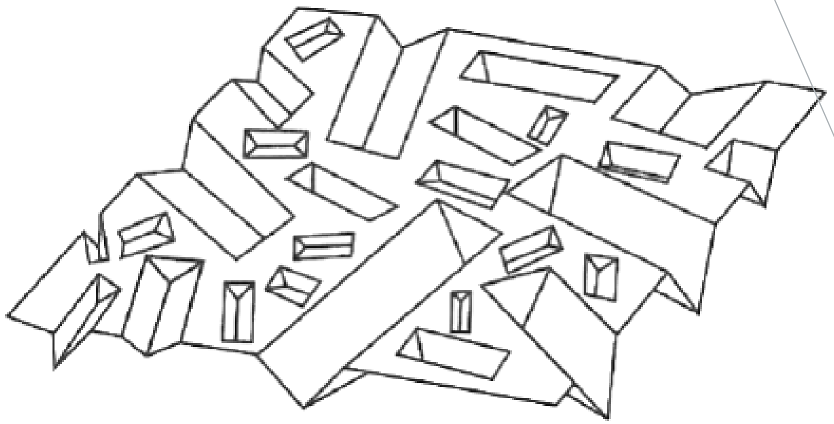

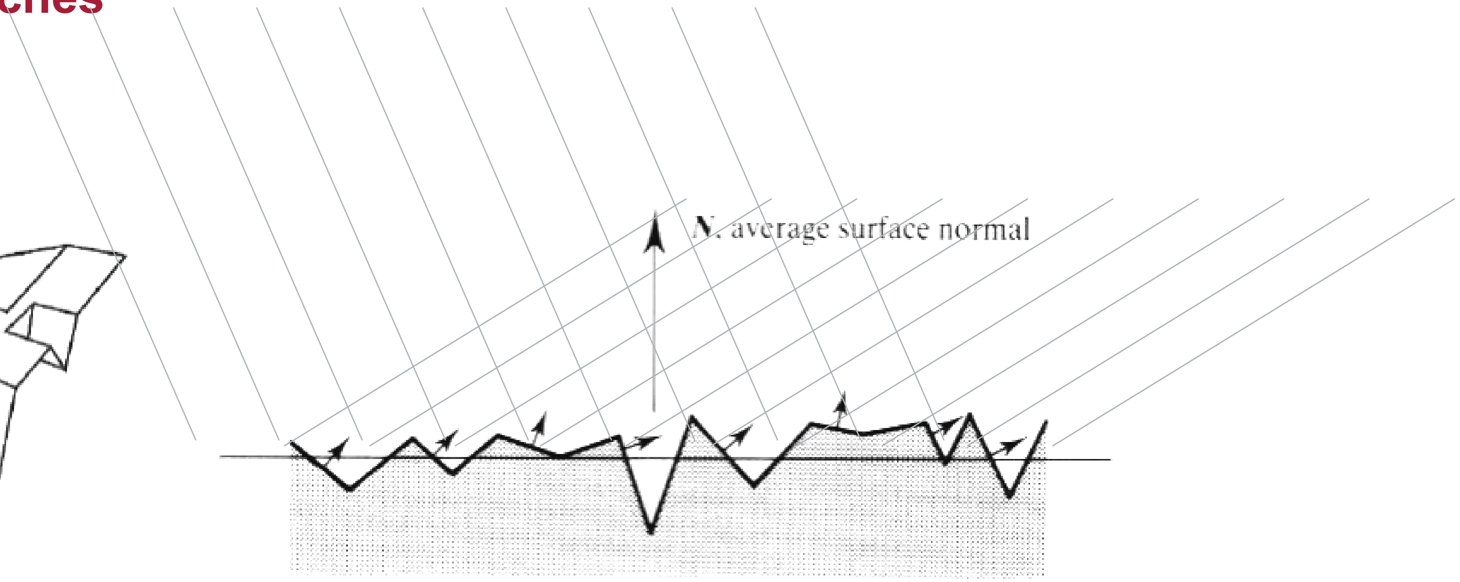

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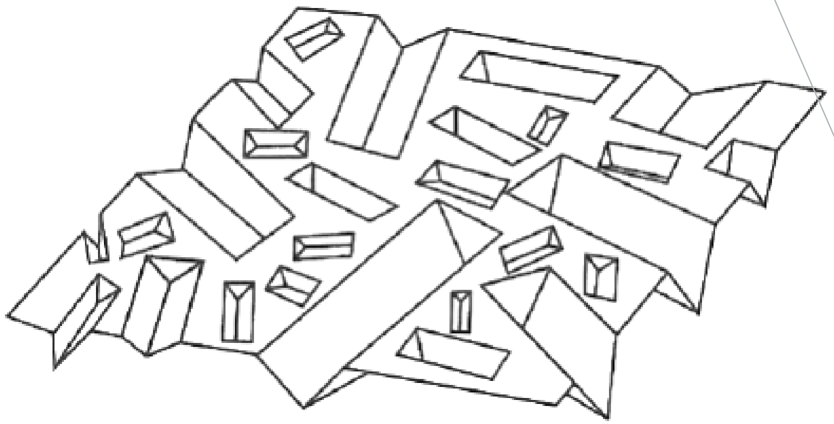

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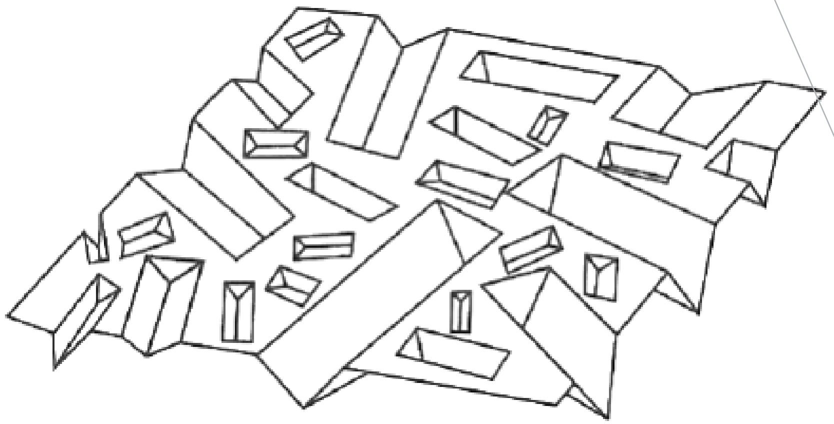
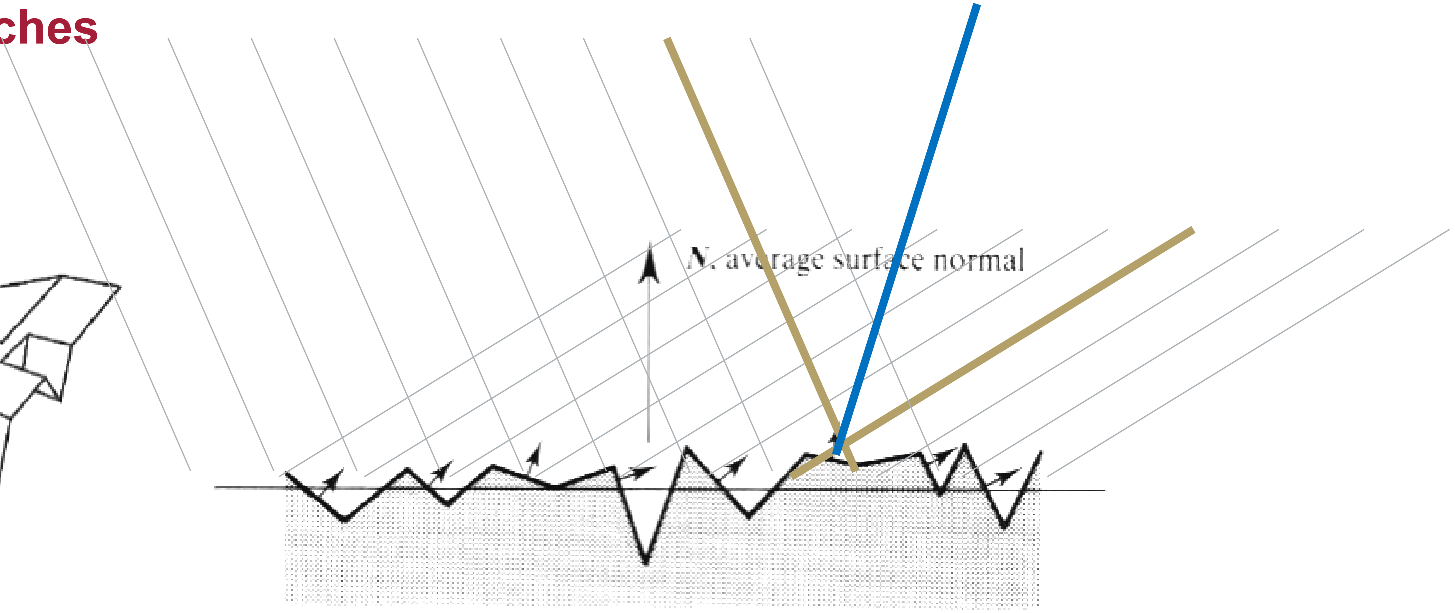


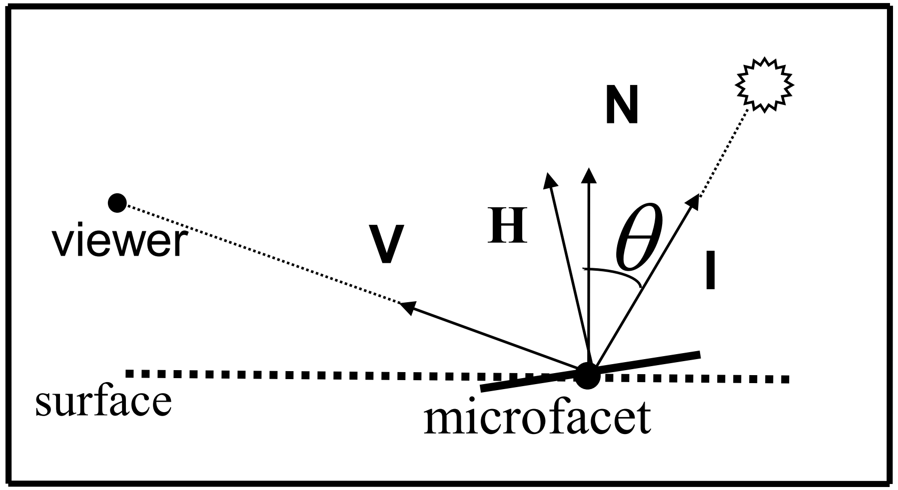


Ward Reflection Model

- BRDF

$$f_r = \frac{\rho_d}{\pi} + \rho_s \frac{1}{\sqrt{(I \cdot N)(V \cdot N)}} \cdot \frac{\exp(-\tan^2 \angle(H,N)/\sigma^2)}{4\pi\sigma^2}$$

  - σ standard deviation (RMS) of surface slope
  - Simple expansion to anisotropic model (σ_x, σ_y)
  - Empirical, not physics-based
  - Inspired by notion of reflecting microfacets
  - Convincing results
  - Good match to measured data



Physics-inspired BRDFs

- Notion of reflecting microfacet
- Specular reflectivity of the form

$$f_r = \frac{D \cdot G \cdot F_\lambda(\lambda, \theta_i)}{\pi \, \underline{N} \cdot \underline{V}}$$

  - D : statistical microfacet distribution
  - G : geometric attenuation, self-shadowing
  - F : Fresnel term, wavelength, angle dependency of reflection along mirror direction
  - N•V : flaring effect at low angle of incidence

- Cook-Torrance model
  - F : wavelength- and angle-dependent reflection
  - Metal surfaces

Cook-Torrance Reflection Model

- Cook-Torrance reflectance model is based on the *microfacet* model. The BRDF is defined as the sum of a diffuse and specular components:

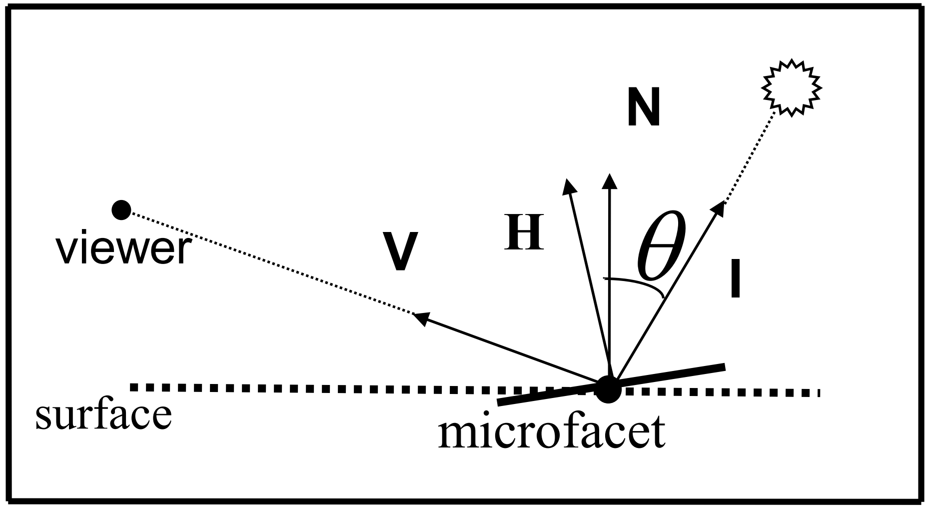
$$f_r = k_d \rho_d + k_s \rho_s; \qquad k_d + k_s \leq 1$$

  where k_s and k_d are the specular and diffuse coefficients.

- Derivation of the specular component ρ_s is based on a physically derived theoretical reflectance model

Cook-Torrance Specular Term

$$\rho_s = \frac{F_\lambda DG}{\pi(\underline{N}\cdot\underline{V})(\underline{N}\cdot\underline{I})}$$

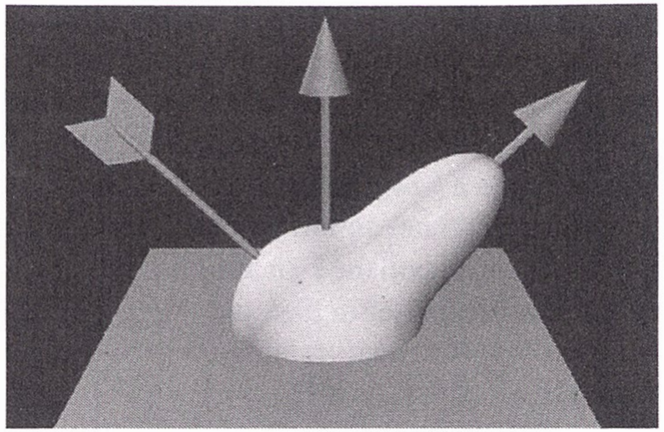


- D : Distribution function of microfacet orientations
- G : Geometrical attenuation factor
  - represents self-masking and shadowing effects of microfacets
- F_λ : Fresnel term
  - computed by Fresnel equation
  - relates incident light to reflected light for each planar microfacet

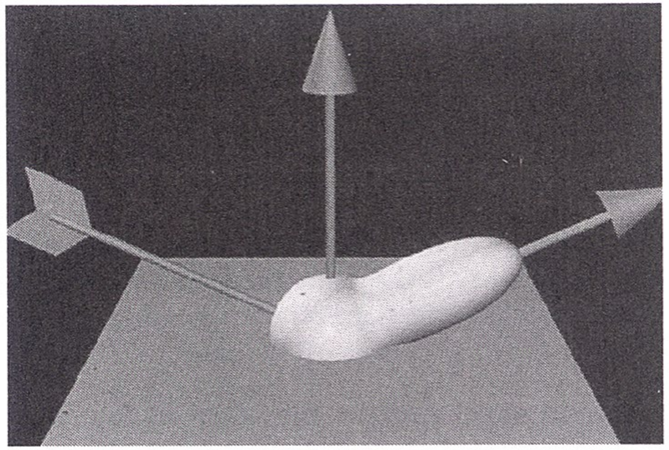
$$F_\lambda \approx (1+(V\cdot N))^\lambda$$

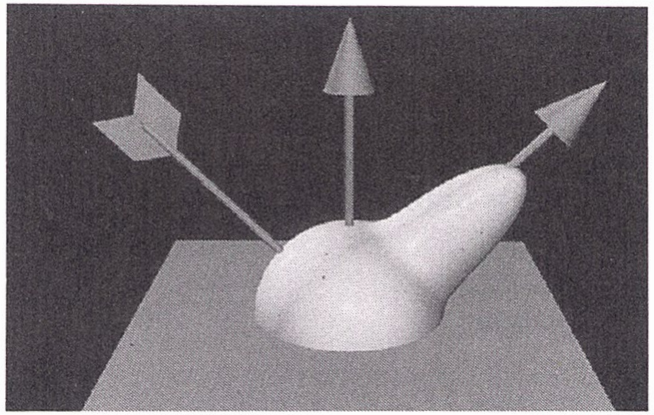
- N·V : Proportional to visible surface area
- N·I : Proportional to illuminated surface area

Comparison Phong vs. Torrance

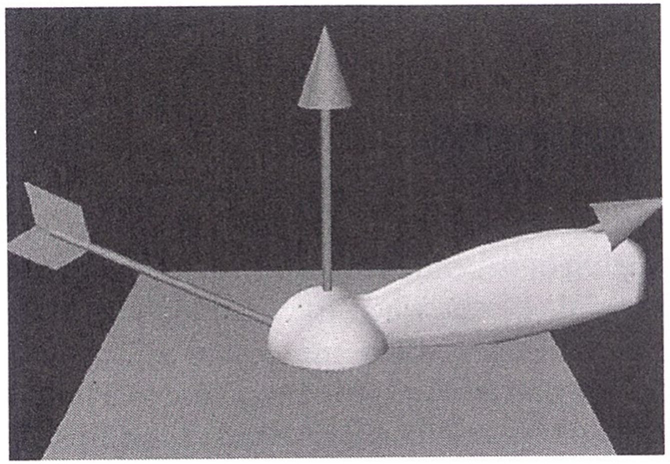


Phong





Torrance

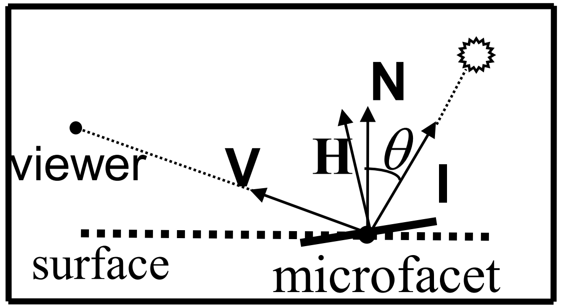


off-specular peak!

Microfacet Distribution Functions

- Isotropic Distributions
  - α : angle to average normal of surface
  - Characterized by half-angle β

$$D(\underline{\omega}) \Rightarrow D(\alpha) \qquad \alpha = \mathbf{N} \cdot \mathbf{H}$$



- Blinn

$$D(\beta) = \frac{1}{2}$$

- Torrance-Sparrow

$$D(\alpha) = \cos^{\frac{\ln 2}{\ln \cos \beta}} \alpha$$

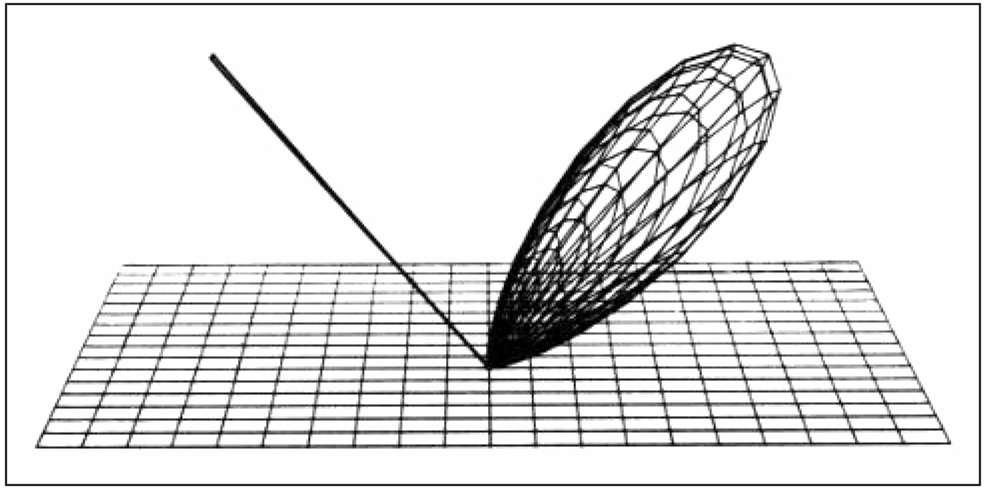
- Beckmann
  - m : average slope of the microfacets
  - Used by Cook-Torrance

$$D(\alpha) = e^{-\left(\frac{\sqrt{2}}{\beta}\alpha\right)^2}$$

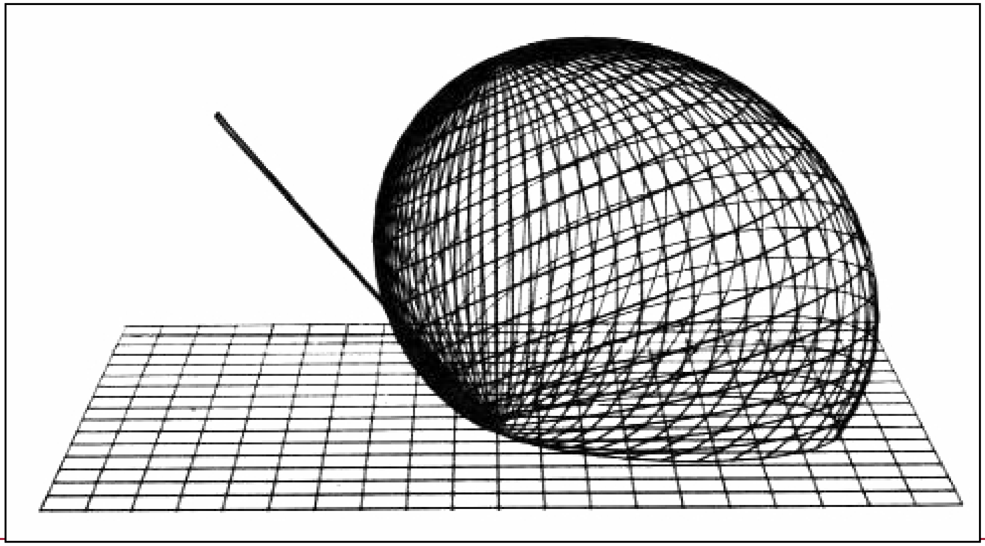
$$D(\alpha) = \frac{1}{4m^2 \cos^4 \alpha} e^{-[\tan \alpha / m]^2}$$

Beckman Microfacet Distribution Function

$m=0.2$



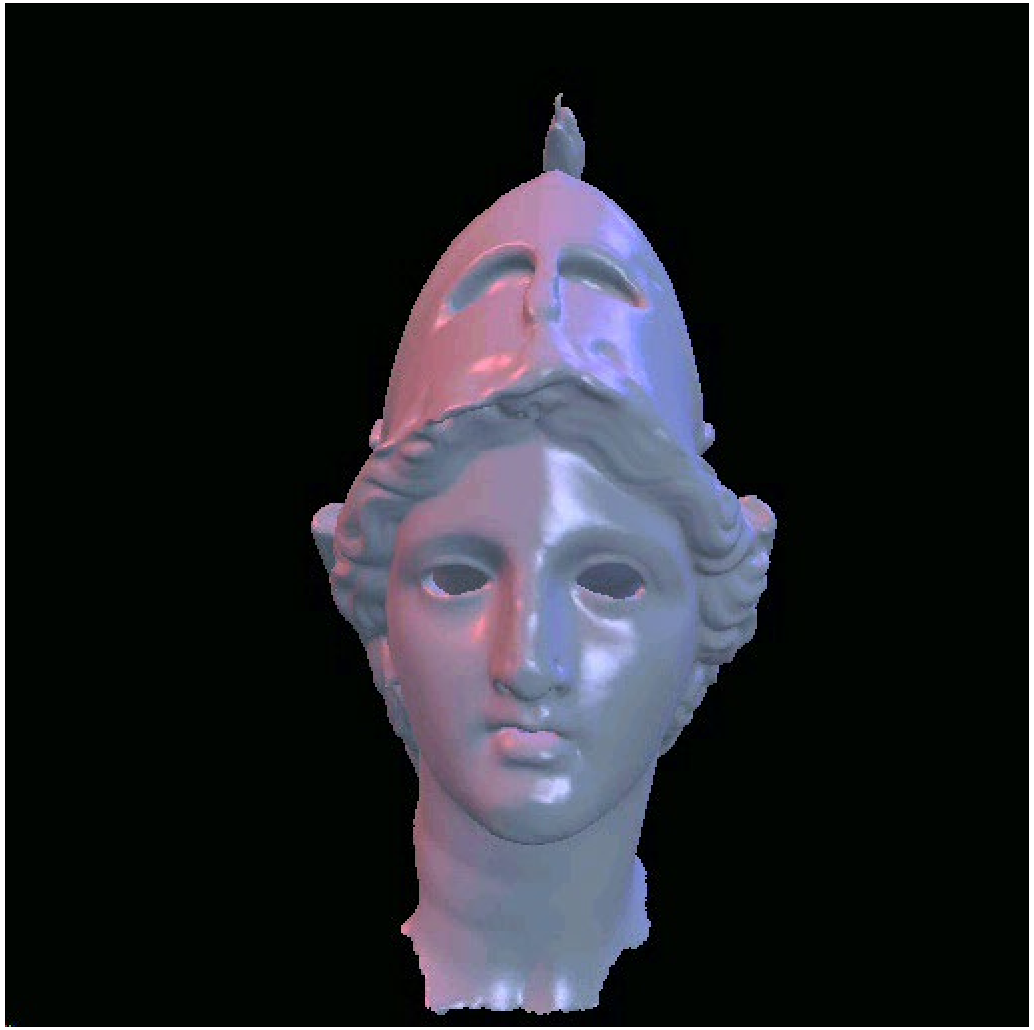
$m=0.6$



Spatially Varying BRDFs

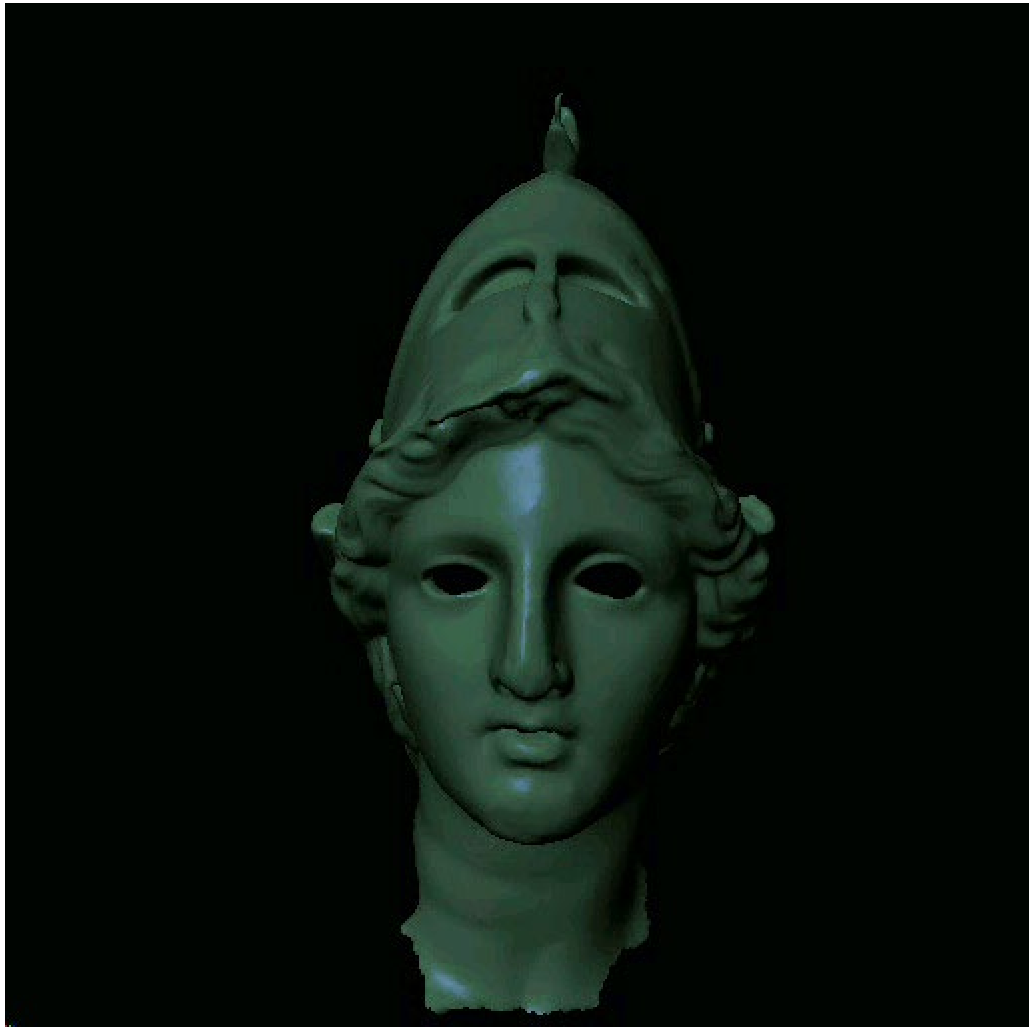
3D Geometry model

- Pure geometry
  - no color



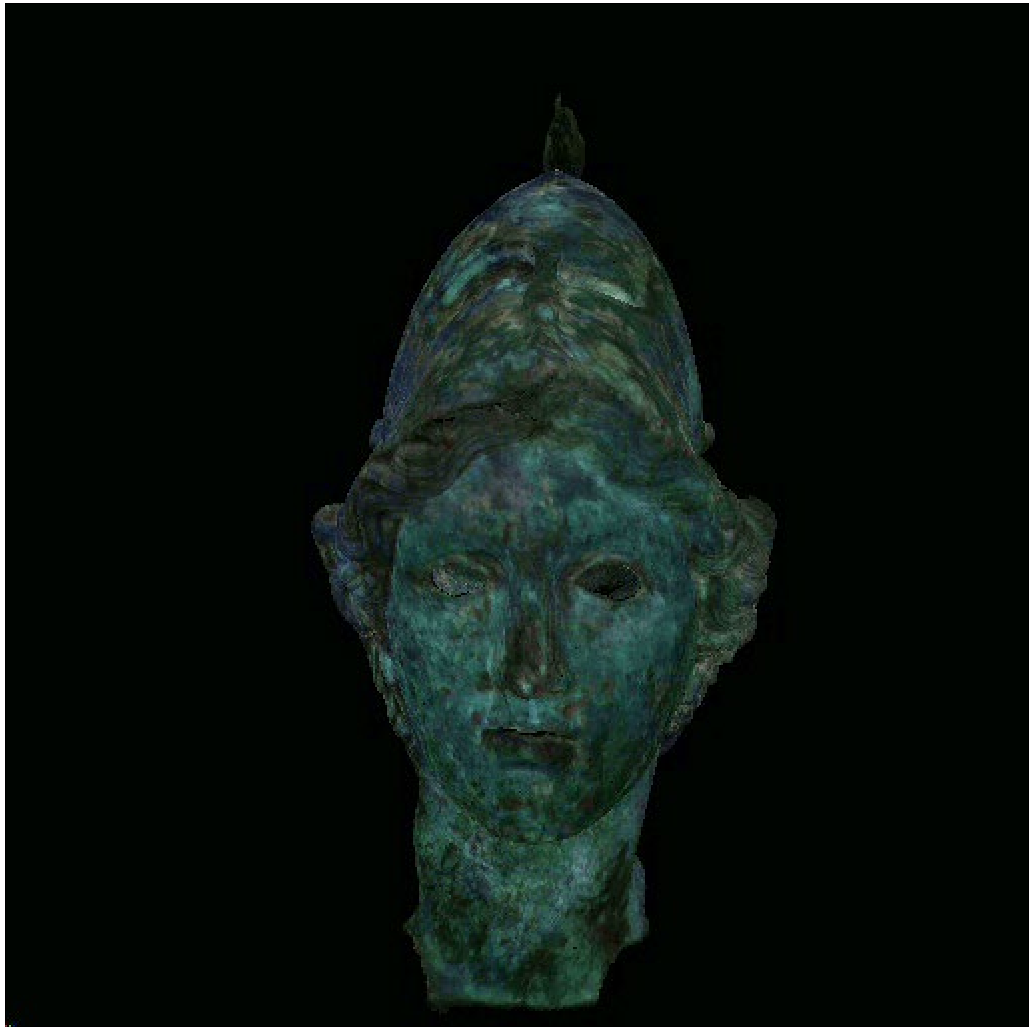
BRDF

- by geometry plus a single BRDF
  - no variation



Geometry plus Texture

- representing diffuse color only - shading is missing
  - no relighting
  - totally flat



Geometry plus spatially varying BRDFs

- realistic object appearance

Questions

- What is a BRDF? How is it defined?
- What are the properties described by it?
- Which phenomena are not covered?
- How can a BRDF be represented?
- How do you measure a BRDF?

Wrap-up

- Today
  - BRDFs
  - Properties
  - Microfacet model

- Next lecture
  - Textures
  - Textures to modify surface properties
  - Texture Parameterization
  - Procedural Shading
  - Texturing Filtering

References

- *An Overview of BRDF Models*, Rosana Montes and Carlos Ureña, Technical Report LSI-2012-001, Dept. Lenguajes y Sistemas Informáticos, University of Granada, Granada, Spain

- *Experimental Analysis of BRDF Models*. Addy Ngan, Frédo Durand, and Wojciech Matusik. Eurographics Symposium on Rendering 2005.

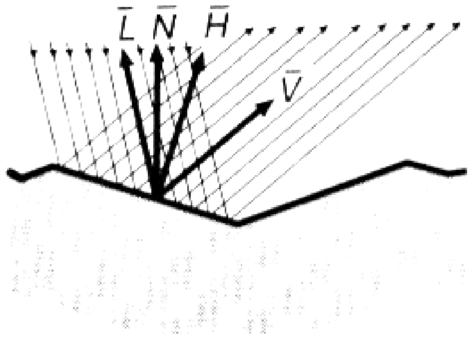
- *Acquisition and A of Bispectral Bidirectional Reflectance and Reradiation Distribution Functions*. Matthias B. Hullin, Johannes Hannika, Boris Ajdin, Jan Kautz, Hans-Peter Seidel, Hendrik P.A. Lensch. ACM Trans. on Graphics. Proceedings of SIGGRAPH 2010.

AppendiX
Geometric Attenuation Factor

Geometric Attenuation Factor

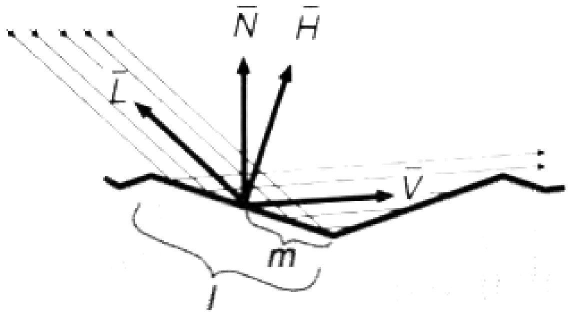
- V-shaped grooves
- Fully illuminated and visible

$$G = 1$$



- Partial masking of reflected light

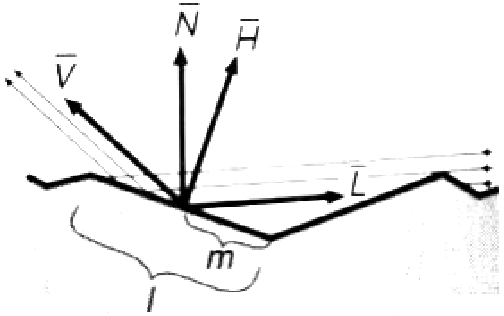
$$G = \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{V})}{(\underline{V} \cdot \underline{H})}$$



- Partial shadowing of incident light

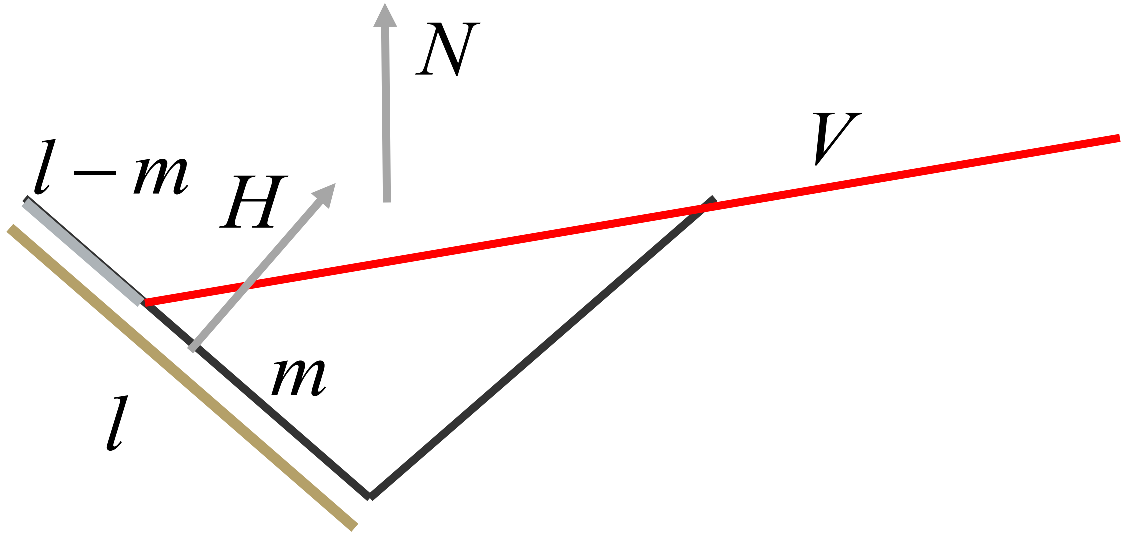
$$G = \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{I})}{(\underline{V} \cdot \underline{H})}$$

$$G = \min\left\{1, \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{V})}{(\underline{V} \cdot \underline{H})}, \frac{2(\underline{N} \cdot \underline{H})(\underline{N} \cdot \underline{I})}{(\underline{V} \cdot \underline{H})}\right\}$$



Geometric Attenuation Factor

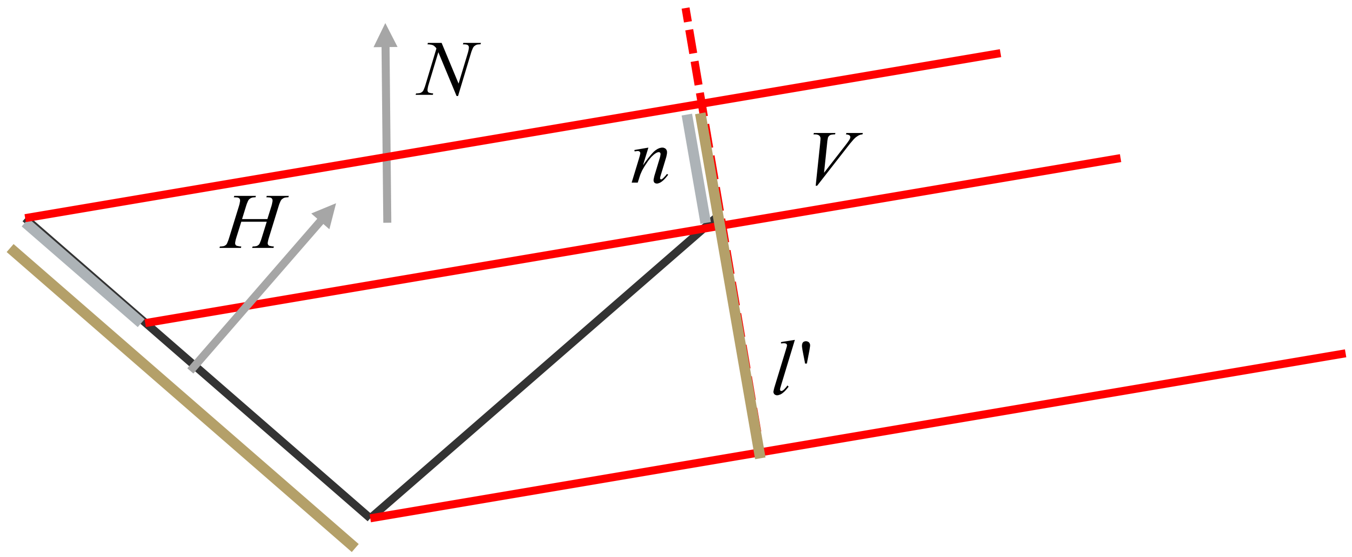
- Area projected into view direction
- Assume entire v-shape is illuminated



$$G = \frac{l-m}{l}$$

Geometric Attenuation Factor

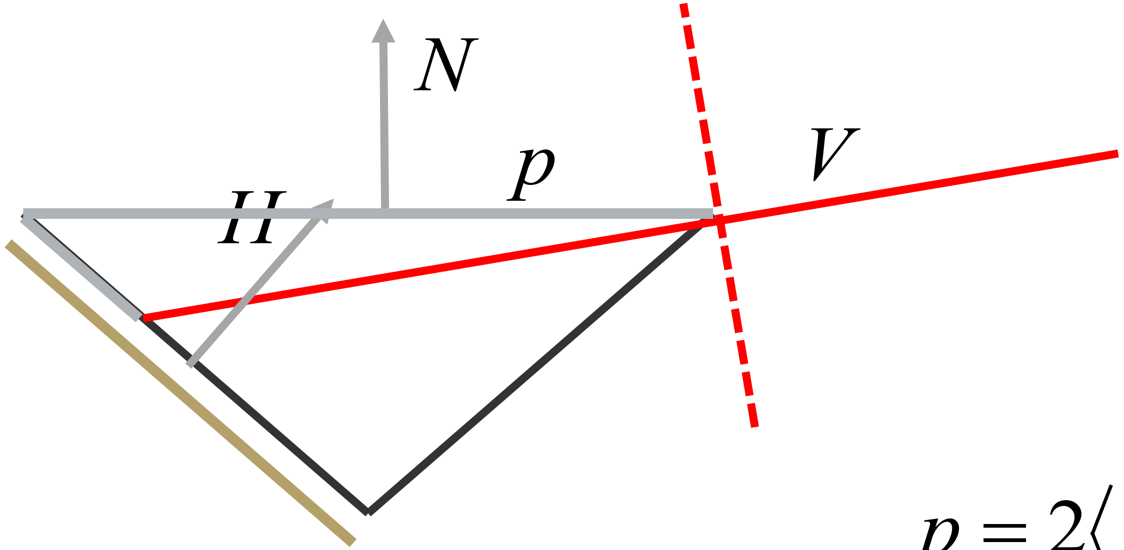
- Area projected into view direction



$$G = \frac{n}{l'} = \frac{n}{\langle H \cdot V \rangle}$$

Geometric Attenuation Factor

- Area projected into view direction

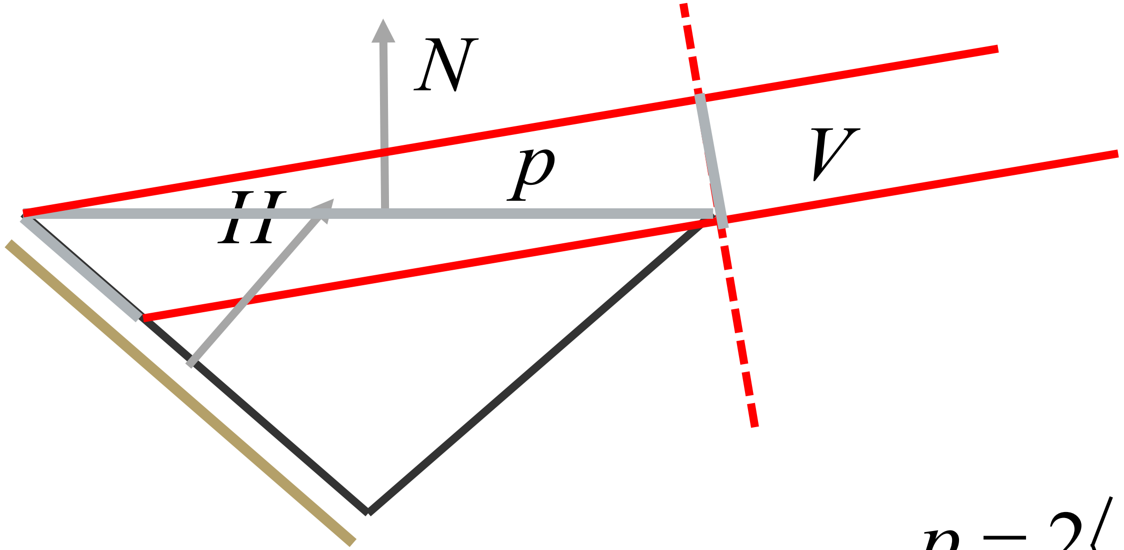


$$p = 2\langle N \cdot H \rangle$$

$$G = \frac{n}{l'} = \frac{n}{\langle H \cdot V \rangle}$$

Geometric Attenuation Factor

- Area projected into view direction



$$p = 2\langle N \cdot H \rangle$$

$$G = \frac{n}{l'} = \frac{p \cdot \langle N \cdot V \rangle}{\langle H \cdot V \rangle} = \frac{2\langle N \cdot H \rangle \cdot \langle N \cdot V \rangle}{\langle H \cdot V \rangle}$$