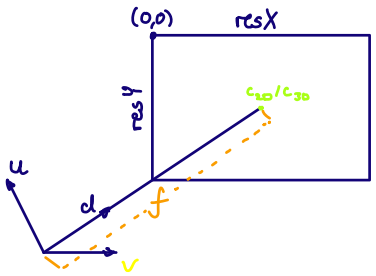
Perspective Camera Model

→ derive equation for camera ray through a pixel (x,y)

We assume the position $(0,0)$ on the image plane is in the upper left corner and the metric of the scene is pixel.



Given variables:

o = camera origin

$\vec{d}$ = viewing direction

$\vec{u}$ = up-vector (camera orientation)

$\vec{d} \perp \vec{u}$

f = focal length (distance camera to plane)

resX, resY = dimensions of the image plane

Steps:

1. compute perpendicular vector v

$$\vec{v} = \vec{u} \times \vec{d}$$

2. get center of the image plane in 2D and 3D space

$$c_{2D} = \begin{pmatrix} resX/2 \\ resY/2 \end{pmatrix}$$

$$c_{3D} = o + f \cdot \vec{d}$$

3. position of pixel (x,y) relative to c_{2D}

$$r = c_{2D} - \begin{pmatrix} x \\ y \end{pmatrix}$$

▽ if the ray has to go through the center of the pixel, first do $\begin{pmatrix} x + \frac{y}{2} \\ y + \frac{y}{2} \end{pmatrix}$ ▽

4. calculate world coordinates of the pixel by shifting c_{3D} in u and v

$$i = c_{3D} + r_x \cdot \vec{v} + r_y \cdot \vec{u}$$

5. direction vector of the ray

$$\vec{rd} = i - o$$

As one Formula:

$$\vec{rd} = \left[(o + f\vec{d}) + \left(\frac{resX}{2} - x \right) \cdot (\vec{u} \times \vec{d}) + \left(\frac{resY}{2} - y \right) \cdot \vec{u} \right] - o$$

→ if center of pixel is expected, mind to add $\frac{y}{2}$ to x and y !

$$V_S(x,y) = (f \cdot d + o) + \left(\frac{resX}{2} - x\right) \cdot (u \times w) + \left(\frac{resY}{2} - y\right) \cdot (u)$$