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Graphic Data Processing
Assignment 9

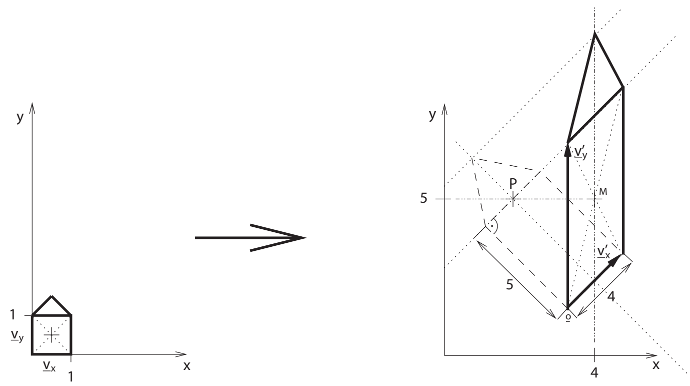
Submission deadline for the exercises: 26. January 2022 6.00 am

Written Solutions

Written solutions have to be submitted digitally as one PDF file via Ilias.

9.1 Transformations (25 Points)

In the picture below the left house should be transformed into the house on the right. The point M is at $(4, 5)$ and lines that look to be parallel are parallel. Please specify the complete transformation matrix as a sequence of primitive transformations (there's no need to calculate the final matrix). Do not guess any numbers.



9.2 Affine Spaces (10 + 5 = 15 Points)

Definition of an affine space: An affine space consists of a set of points P, an associated vector space V and an operation $+ \in P \times V \to P$ that fulfills the following axioms:

(1) `for` $p \in P$ `and` $v, w \in V : (p + v) + w = p + (v + w)$

(2) `for` $p, q \in P$ `there exists a unique` $v \in V$ `such that:` $p + v = q$

a) Prove that the set of points $A = \{(x, y, z, w) \in R^4 \mid w = 1\}$ is an affine space. What is the associated vector space? You do *not* have to show that the associated vector space is a vector space.

b) What is the difference between a point and a vector in that affine space?

9.3 Rotations (25 Points)

Show that an arbitrary rotation T around the origin in 2D can be represented by a combination of a shearing in y, a scaling in x and y and a shearing in x in this order. You have to derive the shearing and scaling for an arbitrary rotation angle ϕ.

$$T = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}$$

9.4 Transformations (12 + 8 = 20 Points)

a) Show that the inverse of a rotation matrix is its transpose.

b) Describe in words what this 2D transformation matrix does:

$$\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

9.5 Rasterization (5 + 10 = 15 Points)

a) How are polygons rasterized that are not triangles? (describe two Methods roughly)

b) Name two methods for line rasterization and describe one of them in detail.