EBERHARD KARLS
UNIVERSITÄT
TÜBINGEN

FACULTY OF
SCIENCE
Computer Graphics

Computer Graphics (Graphische Datenverarbeitung)

- Sampling & Antialiasing -

WS 2021/2022

Corona

- Regular random lookup of the 3G certificates

- Contact tracing: We need to know who is in the class room
  - New ILIAS group for every lecture slot
  - Register via ILIAS or this QR code (only if you are present in this room)

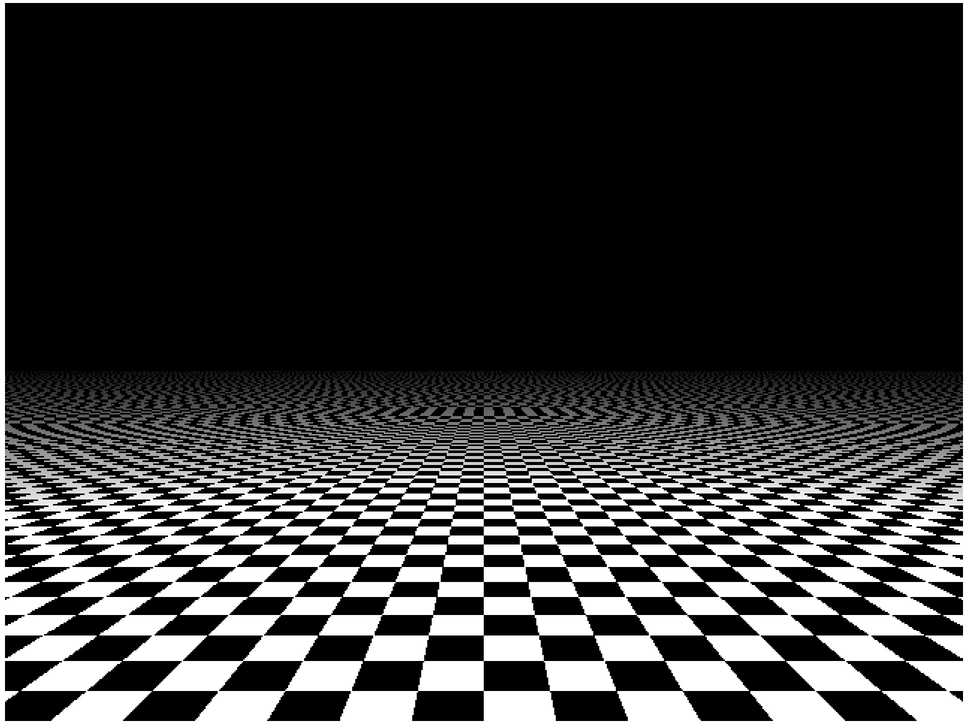
Overview

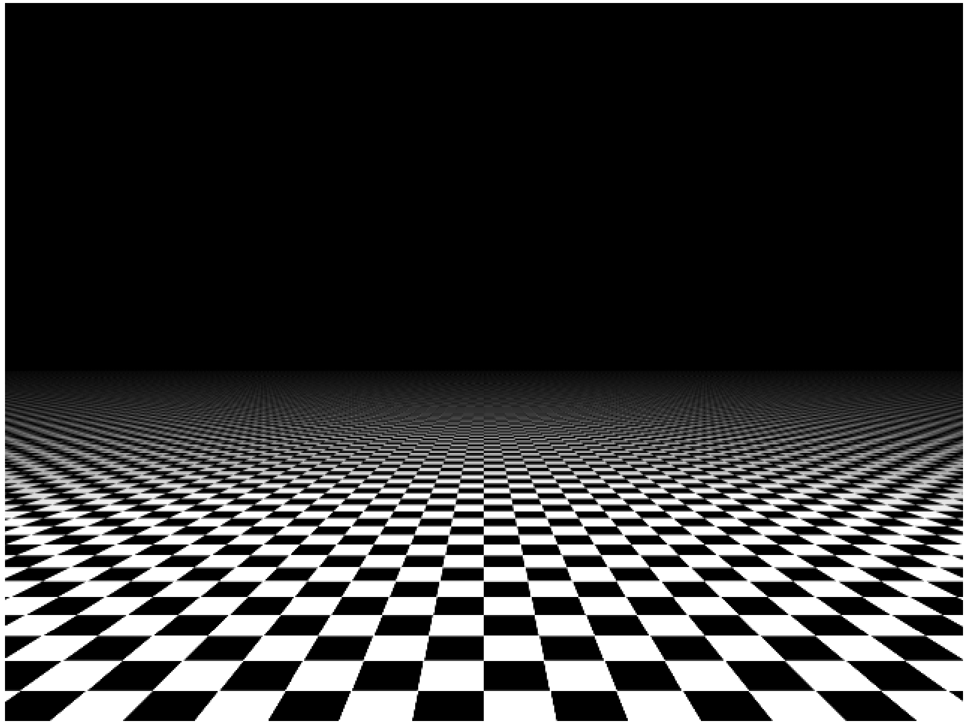
- Last lecture
  - Fourier Transform
  - Filtering

- Now
  - Signal Processing
  - Sampling
  - Anti-aliasing & supersampling

Aliasing

- Ray tracing
  - Textured plane with one ray for each pixel (say, at pixel center)
    - No texture filtering: equivalent to modeling with b/w tiles
  - Checkerboard period becomes smaller than two pixels
    - At the Nyquist limit
  - Hits textured plane at only one point, black or white by chance





Discrete Fourier Transform

- N Equally-spaced function samples f_i
  - Function values known only at discrete points
    - Physical measurements
    - Pixel positions in an image !
- Fourier Analysis

$$a_k = 1/N \sum_i \cos(2\pi\, k\, i / N)\ f_i, \quad b_k = 1/N \sum_i \sin(2\pi\, k\, i / N)\ f_i$$

  - Sum over all measurement points N
  - k=0,1,2, …, ? Highest possible frequency ?

$\Rightarrow$**Nyquist frequency**

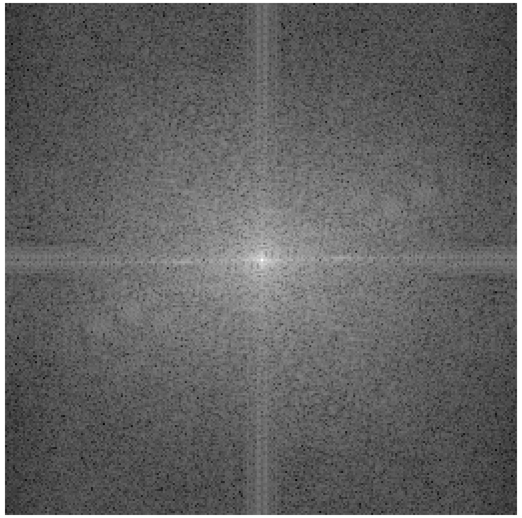
- Sampling rate N_i
- 2 samples / period $\Leftrightarrow$ 0.5 cycles per pixel

$\Rightarrow k \leq N / 2$

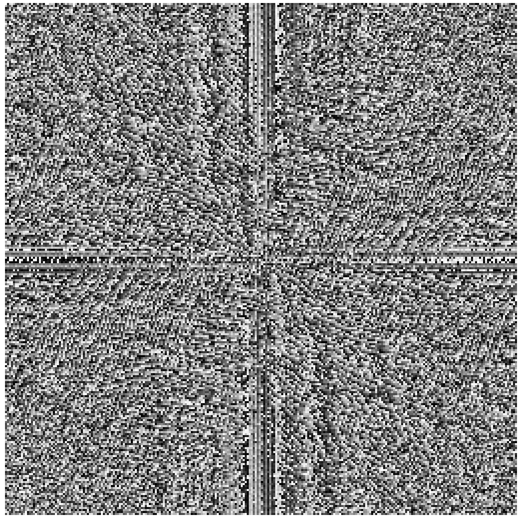
An Example

f(x)

Fourier transformed

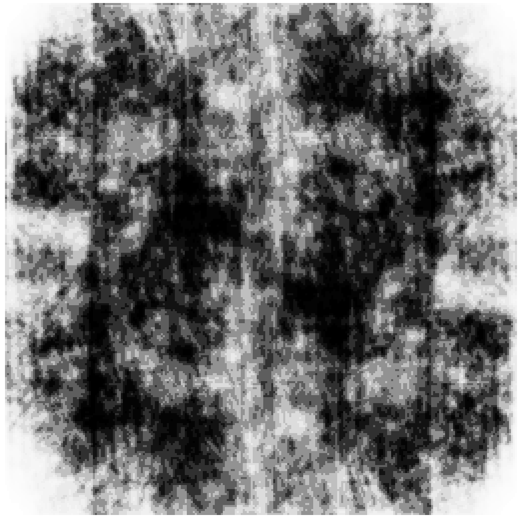


Amplitude



Phase

reconstructed



ignoring Phase

using Phase+Amplitude

Spatial vs. Frequency Domain

- Important basis functions
  - Box ↔ sinc

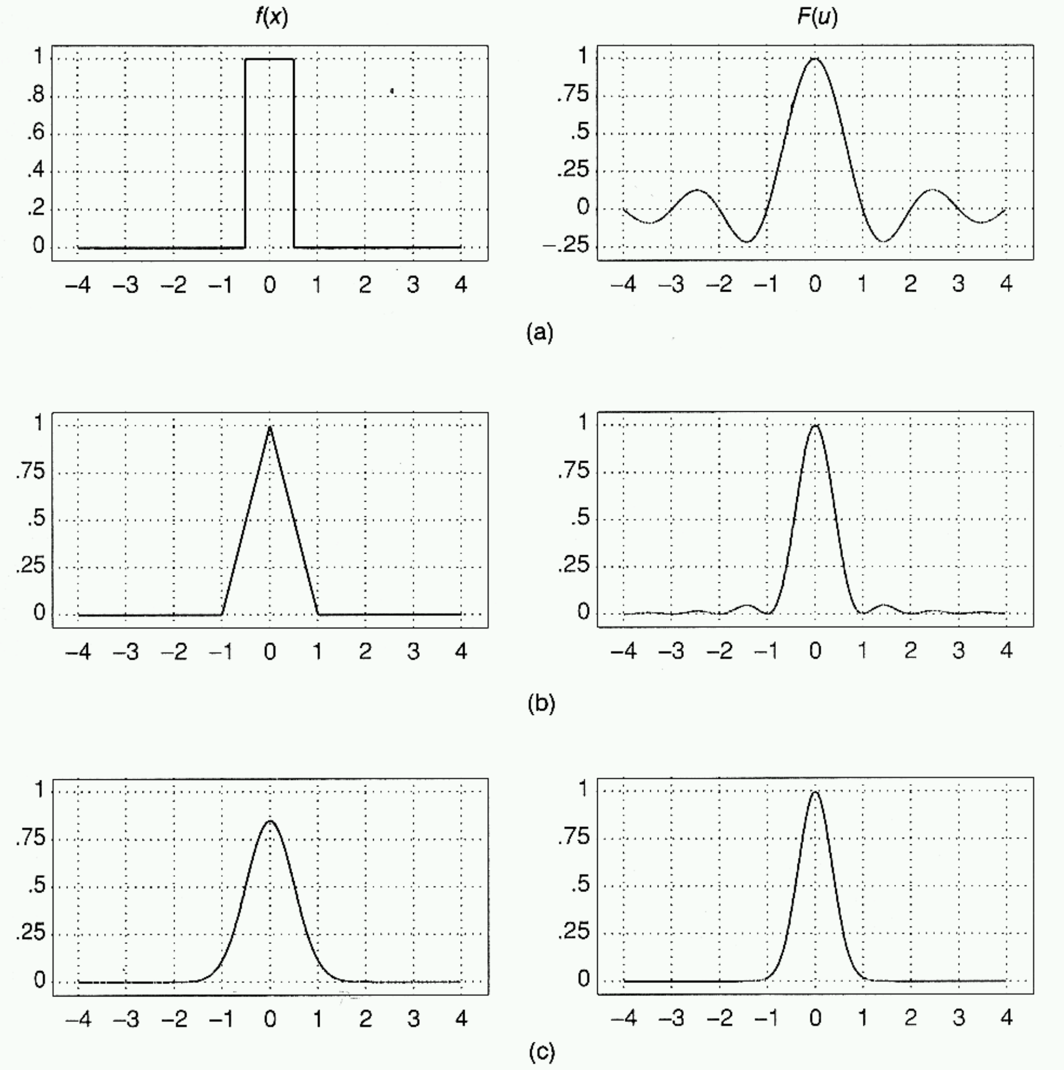
$$\mathrm{sinc}(x) = \frac{\sin(x\pi)}{x\pi}$$

$$\mathrm{sinc}(0) = 1$$

$$\int \mathrm{sinc}(x)\,dx = 1$$

    - Wide box ↔ small sinc
    - Negative values
    - Infinite support

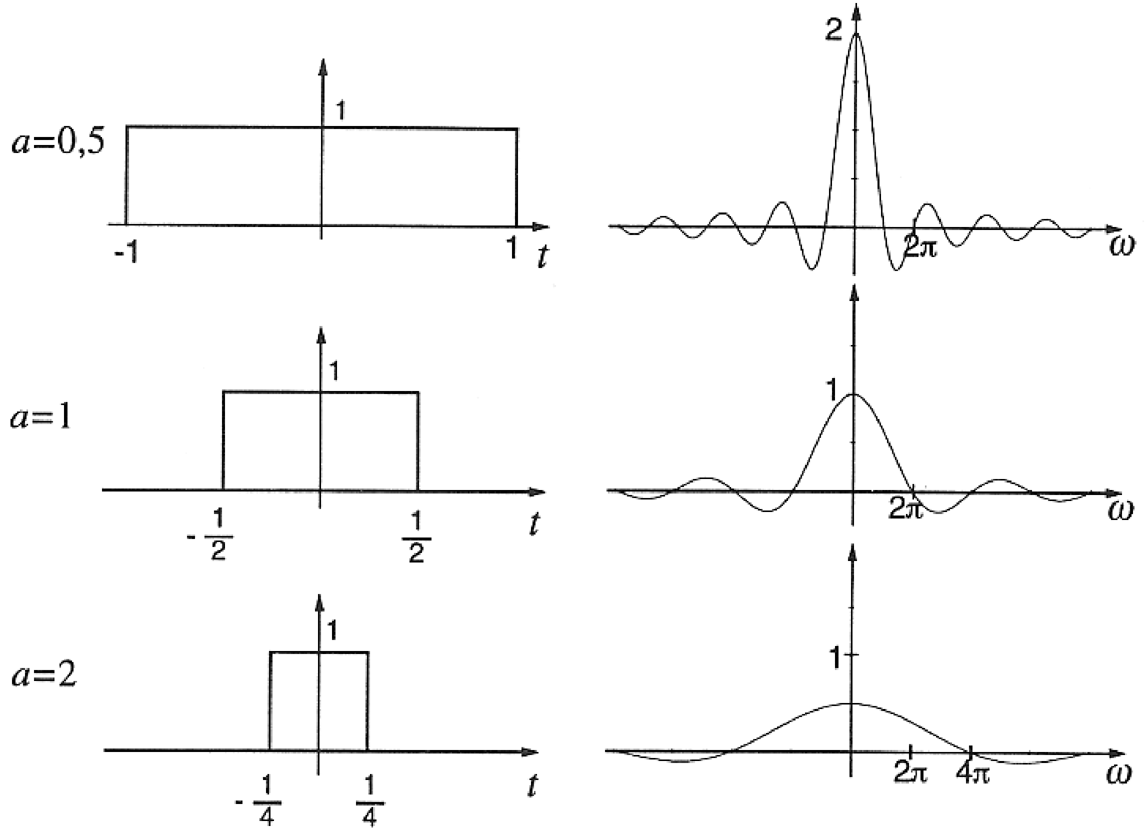
  - Triangle ↔ sinc2
  - Gauss ↔ Gauss



Spatial vs. Frequency Domain

- Transform behavior
- Example: **box function**
  - Fourier transform: sinc
  - Wide box: narrow sinc
  - Narrow box: wide sinc

$$\mathrm{rect}(at) \quad \circ\!\!-\!\!\bullet \quad \frac{1}{|a|}\,\mathrm{si}\left(\frac{\omega}{2a}\right).$$



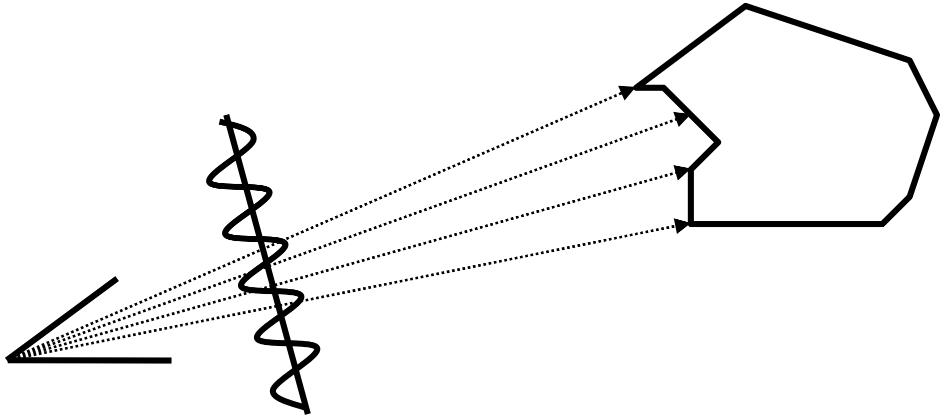
What you should learn today

- What is sampling, aliasing?
- How does the Nyquist-Frequency come into play?
- The difference between sampling and reconstruction
- How to fight aliasing – by anti-aliasing!

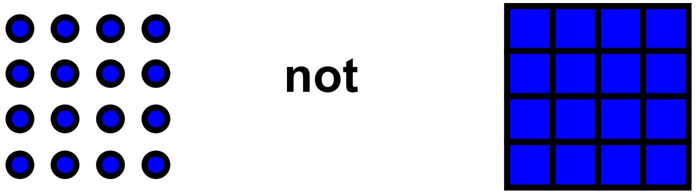
Sampling

The Digital Dilemma

- Nature: continuous signal (2D/3D/4D with time)
  - Defined at every point
- **Acquisition: sampling**
  - Rays, pixel/texel, spectral values, frames, ...



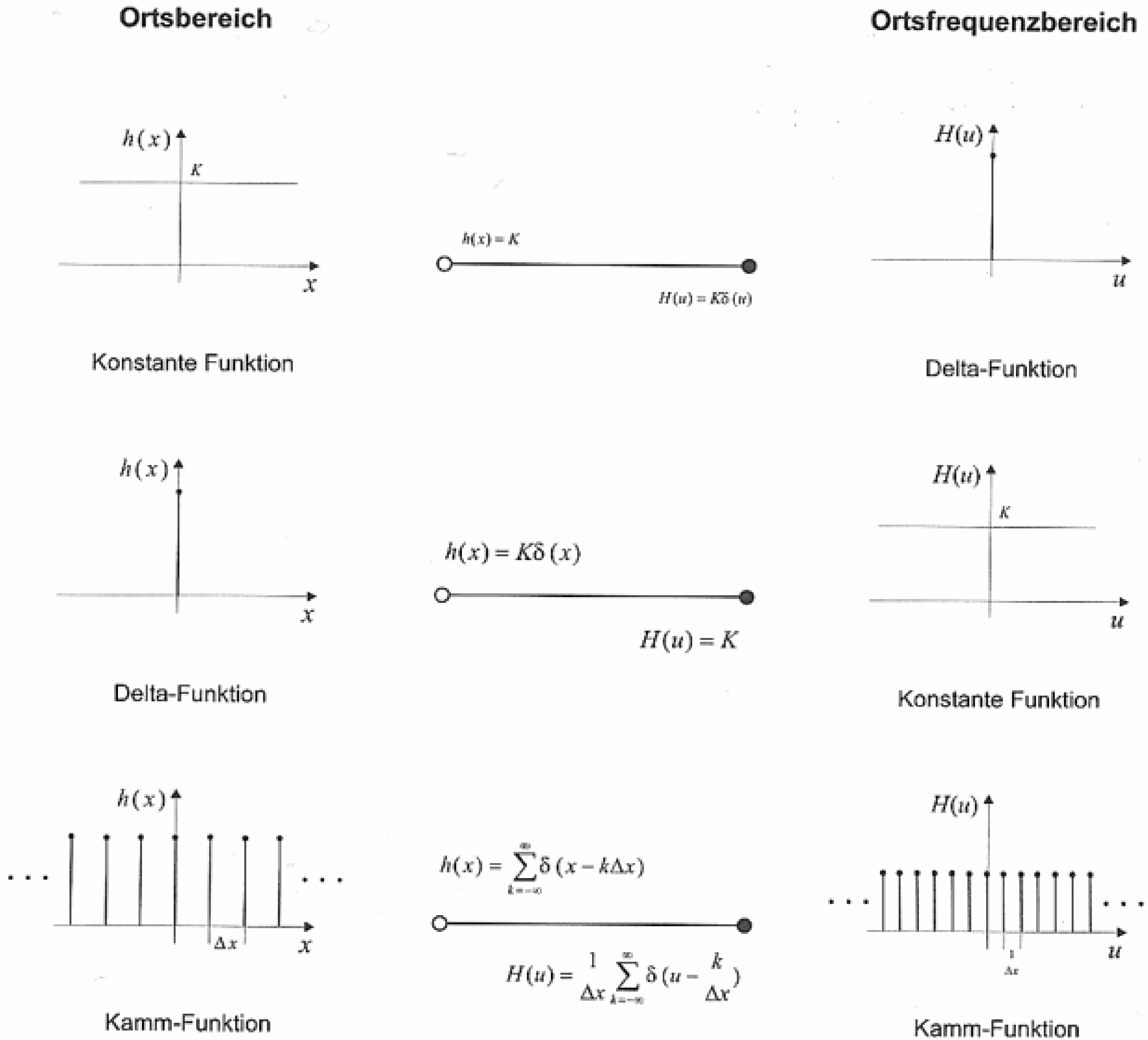
- Representation: discrete data
  - Discrete points, discretized values
- **Reconstruction: filtering**
  - Mimic continuous signal



- Display and perception: faithful
  - Hopefully similar to the original signal, no artifacts

Sampling

- Constant & ∂-Function
  - flash



- Comb/Shah

Sampling

- Constant & δ-Function
  - Duality

$$f(x) = K$$

$$F(\omega) = K\delta(\omega)$$

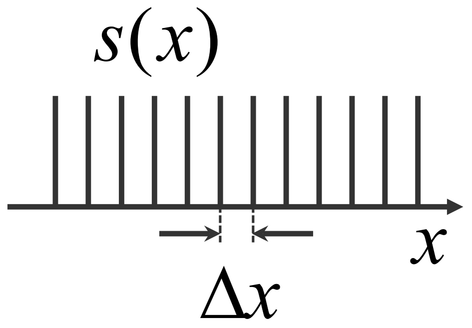
  - And vice versa

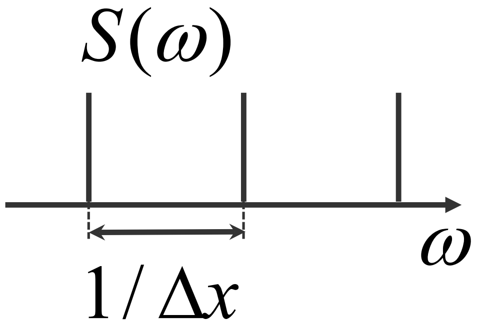
- Comb function
  - Duality: The dual of a comb function is again a comb function
    - Inverse wave length, amplitude scales with inverse wave length

- ß

$$f(x) = \sum_{k=-\infty}^{\infty} \delta(x - k\Delta x)$$

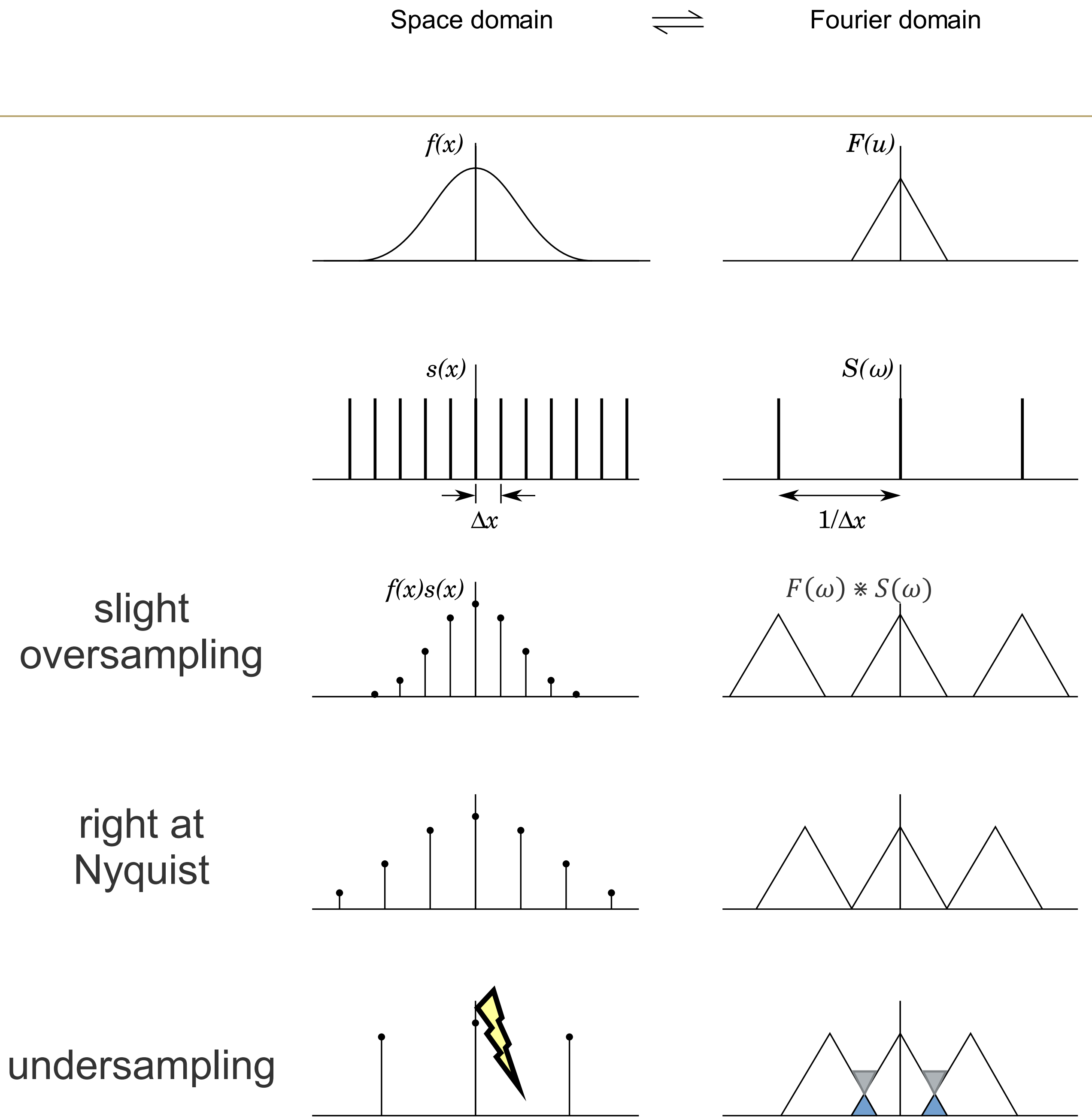
$$F(\omega) = \frac{1}{\Delta x} \sum_{k=-\infty}^{\infty} \delta(\omega - k\frac{1}{\Delta x})$$



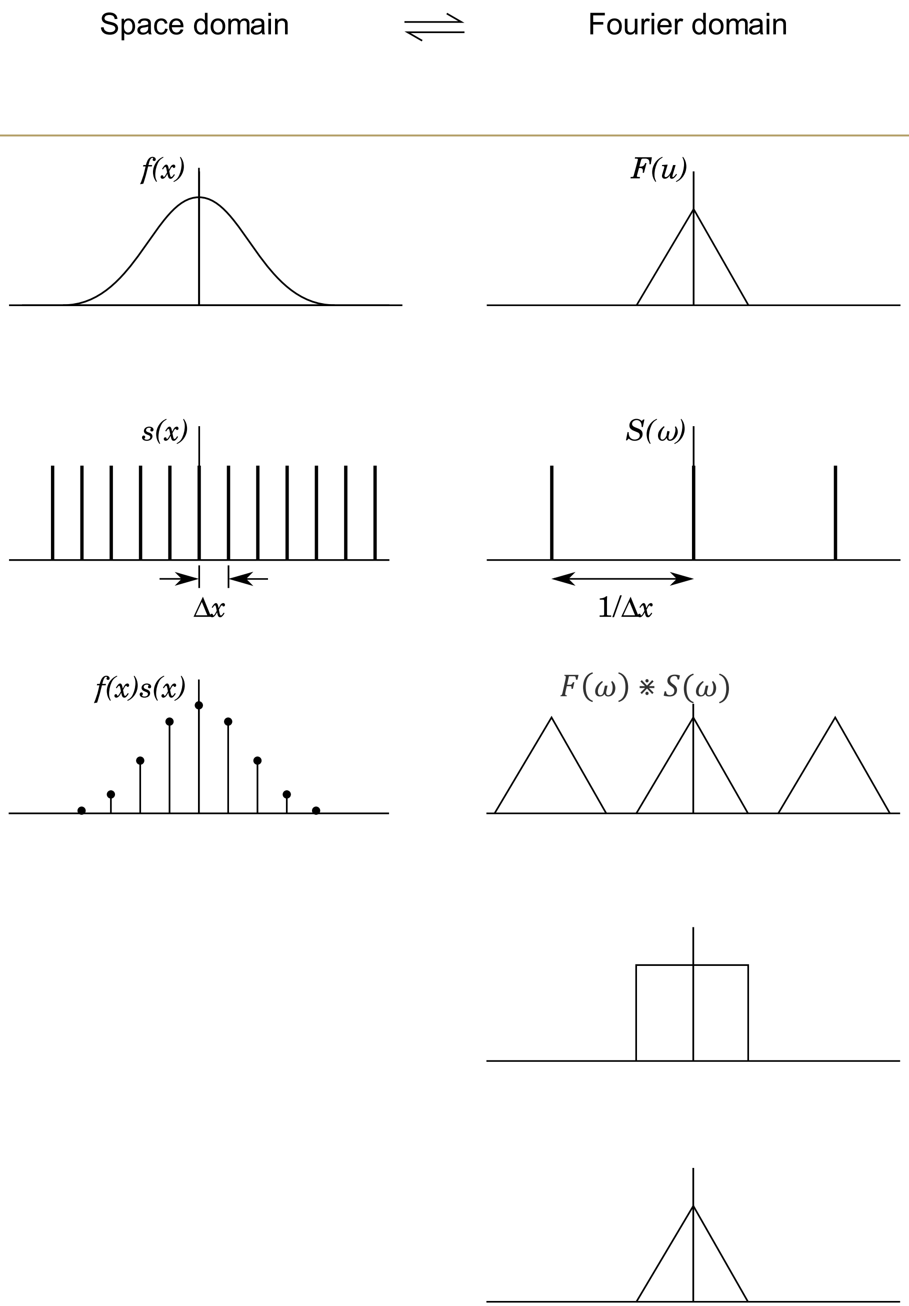


Sampling

- Continuous function
  - Band-limited Fourier transform
- Sampled at discrete points
  - Multiplication with Comb function in space domain
  - Corresponds to convolution in Fourier domain
  - Multiple copies of the original spectrum
- Frequency bands overlap?
  - No: good
  - Yes: bad, aliasing



Reconstruction



- Only original frequency band desired

- Filtering
  - In Fourier domain: multiplication with windowing function around origin
  - In spatial domain: convolution with Fourier transform of windowing function

- Optimal filtering function
  - Box function in Fourier domain
  - Corresponds to sinc in space domain
    - Unlimited region of support
  - Spatial domain only allows for approximations (due to finite support)

Reconstruction Filter

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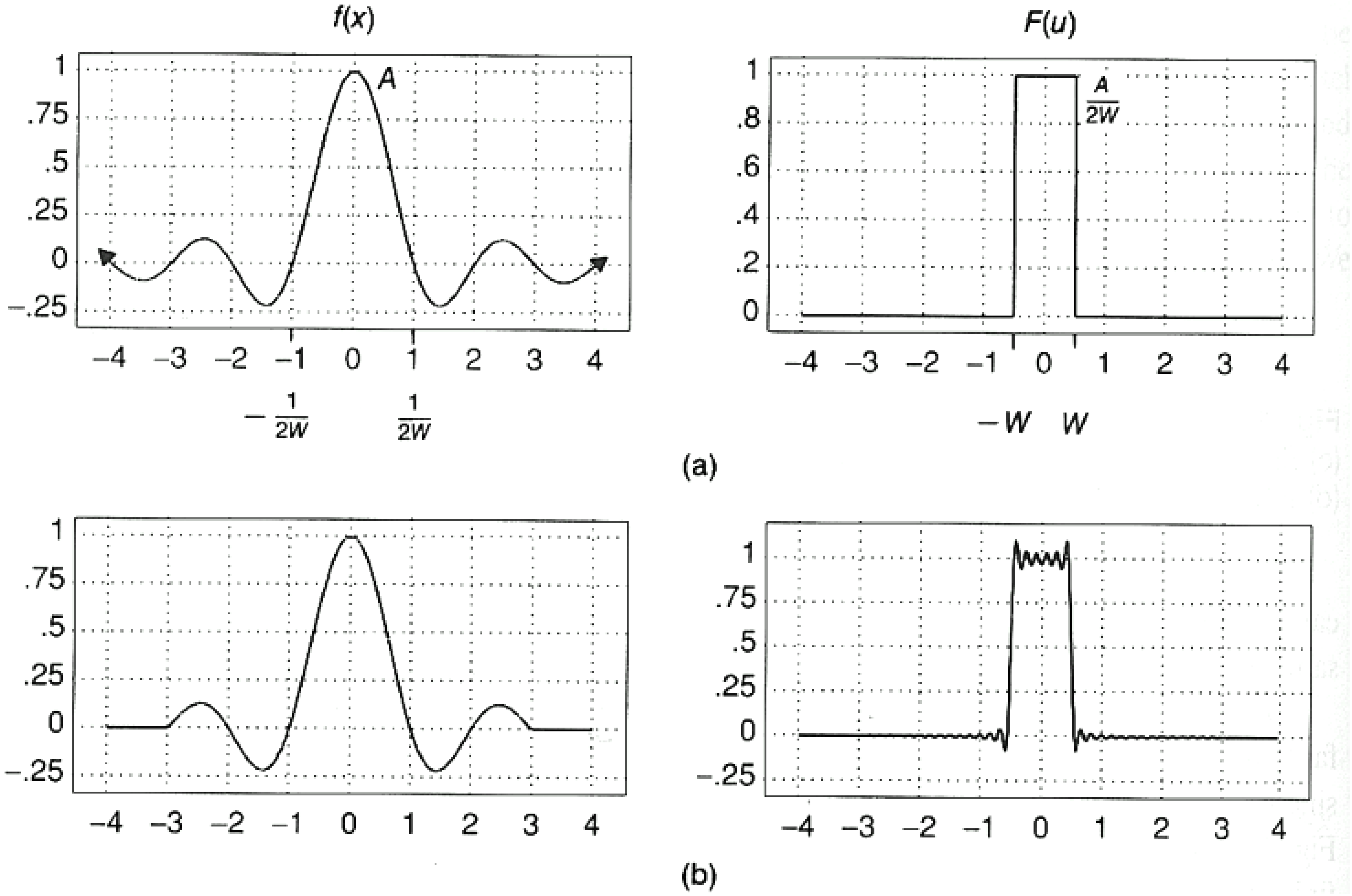
- Cutting off the support is not a good solution



The *Perfect* Case

Original function and its band-limited frequency spectrum

Signal sampling:

Mult./conv. with comb

Comb dense enough (sampling≥2*bandlimit)

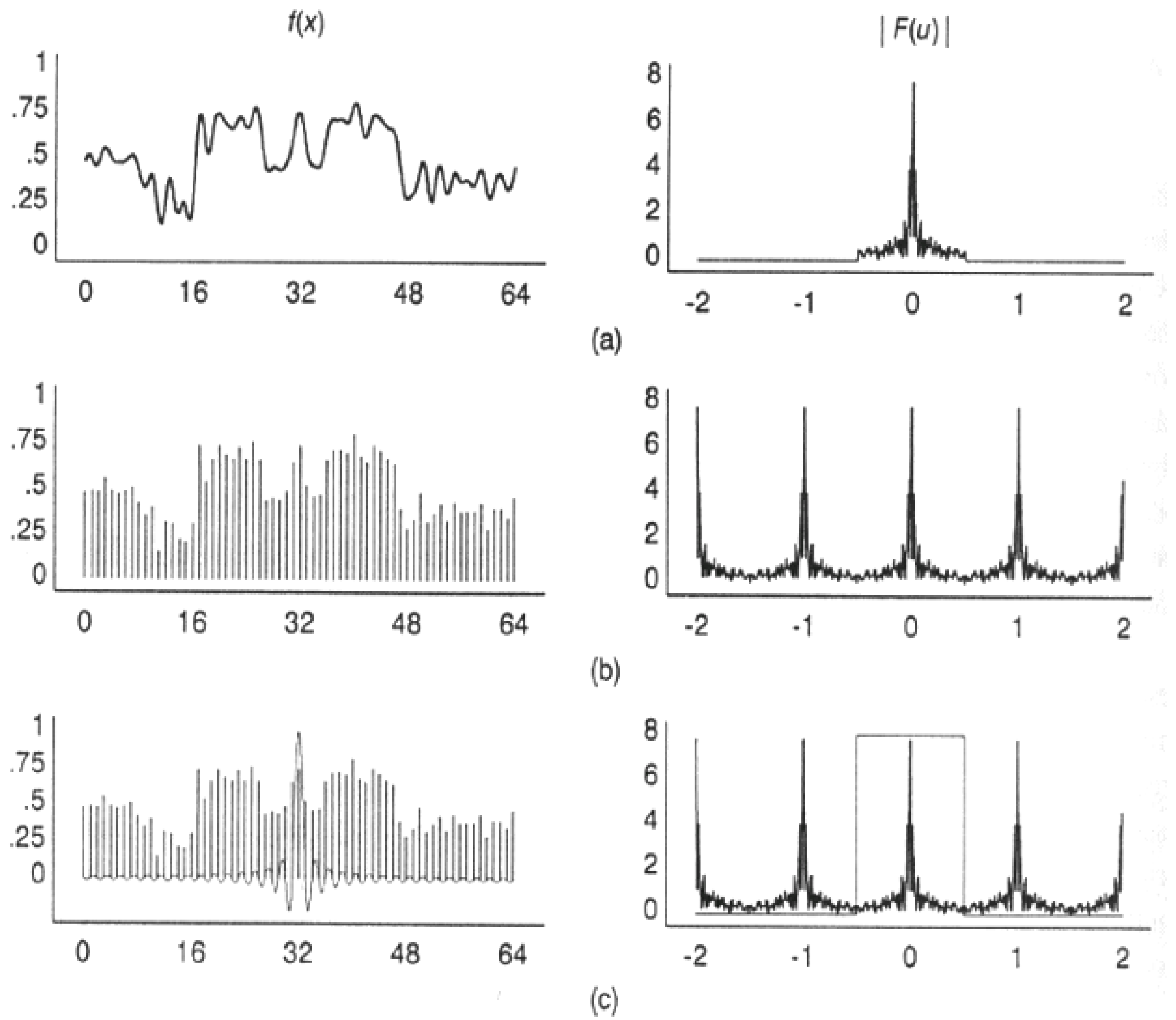
Frequency spectrum is replicated

Bands do not overlap

Correct reconstruction filtering

Fourier: Box (mult.)
Image: sinc (conv.)

Only one copy



Correct Sampling / *Bad* Reconstruction

Reconstruction with ideal sinc

Identical signal

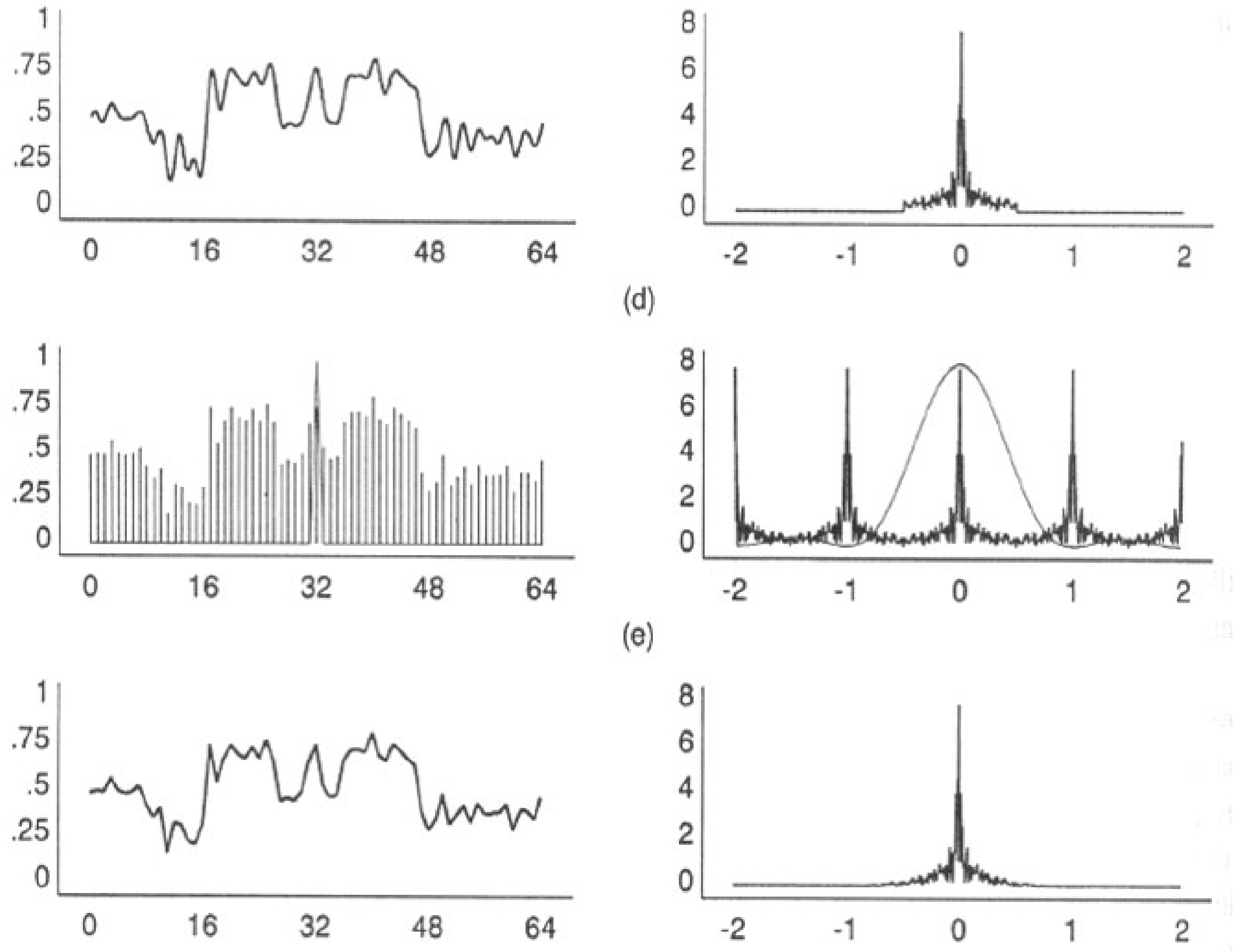
Approximate filtering

Space: tri (conv.)
Fourier: sinc^2 (mult.)

High frequencies are not ignored

→ Aliasing

Reconstruction with tri function (= piecewise linear interpolation)



Sampling with *Too Low Frequency*

Original function

Sampling below Nyquist:

Comb spaced too far (sampling<2*bandlimit)

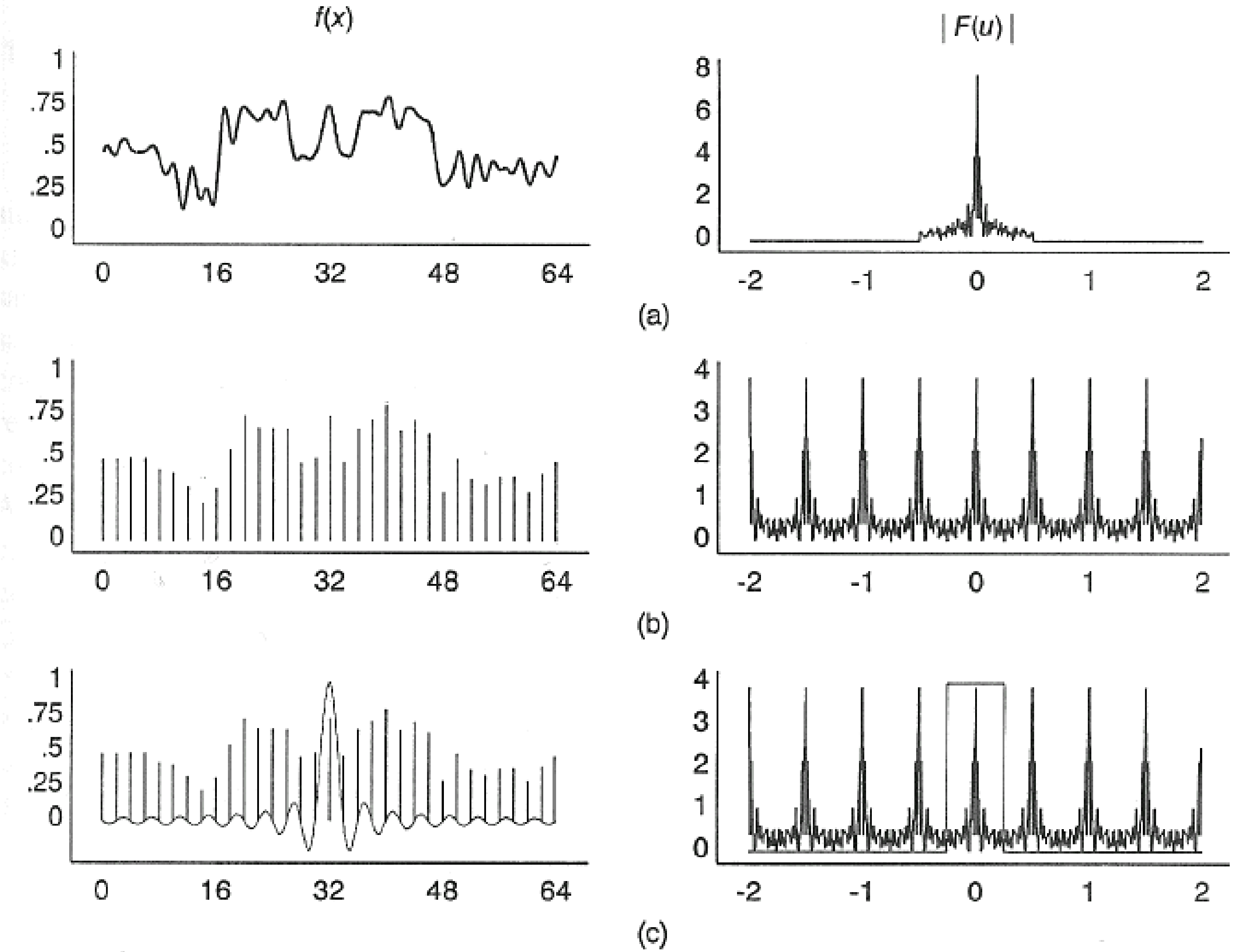
Frequency bands overlap

Correct filtering

Image: sinc (conv.)

Fourier: box (mult.)

Band overlap in frequency domain cannot be corrected - aliasing



Sparse Sampling + *Bad* Reconstruction

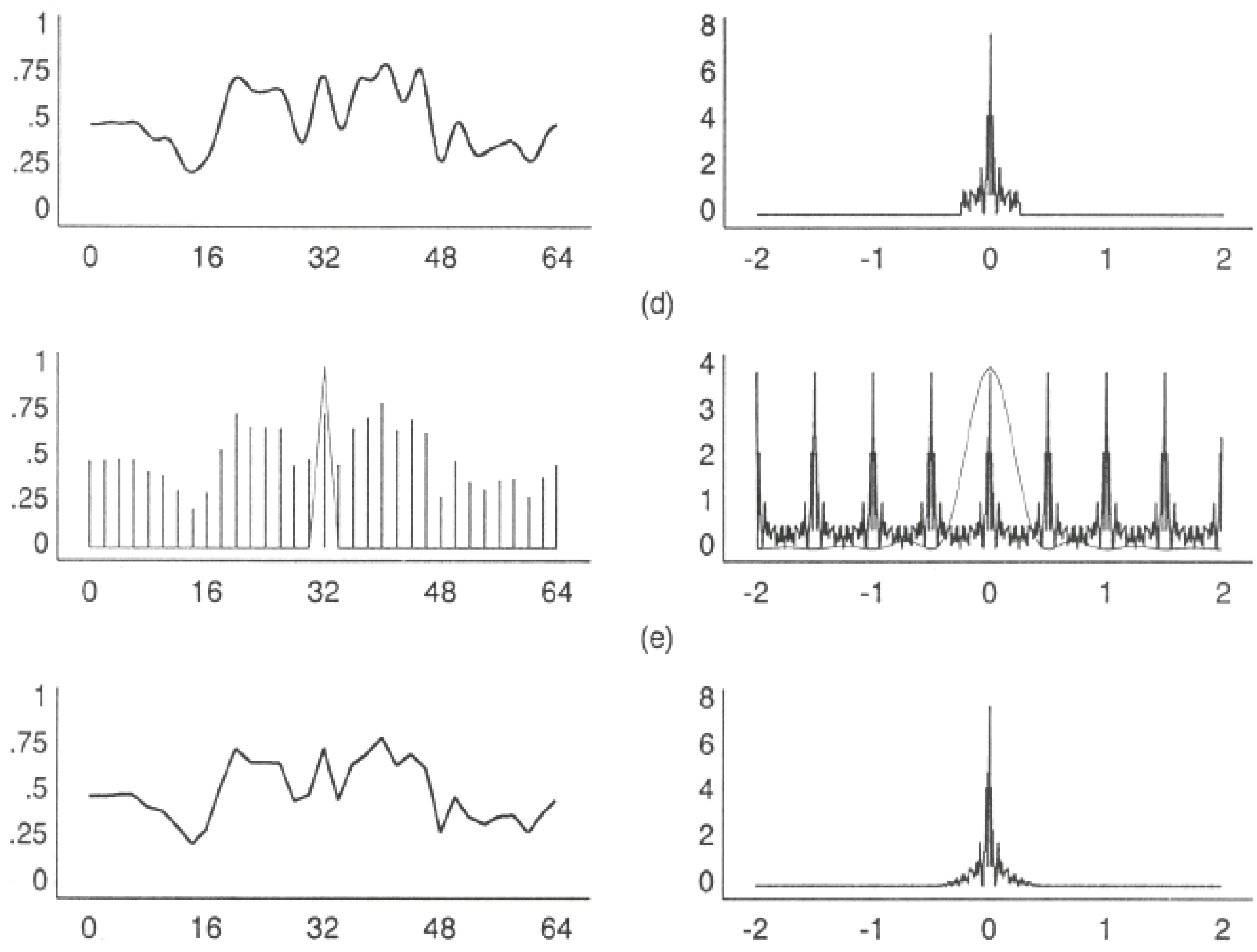
Reconstruction with ideal sinc

Reconstruction fails (frequency components wrong due to aliasing !)

Filtering with sinc^2 function

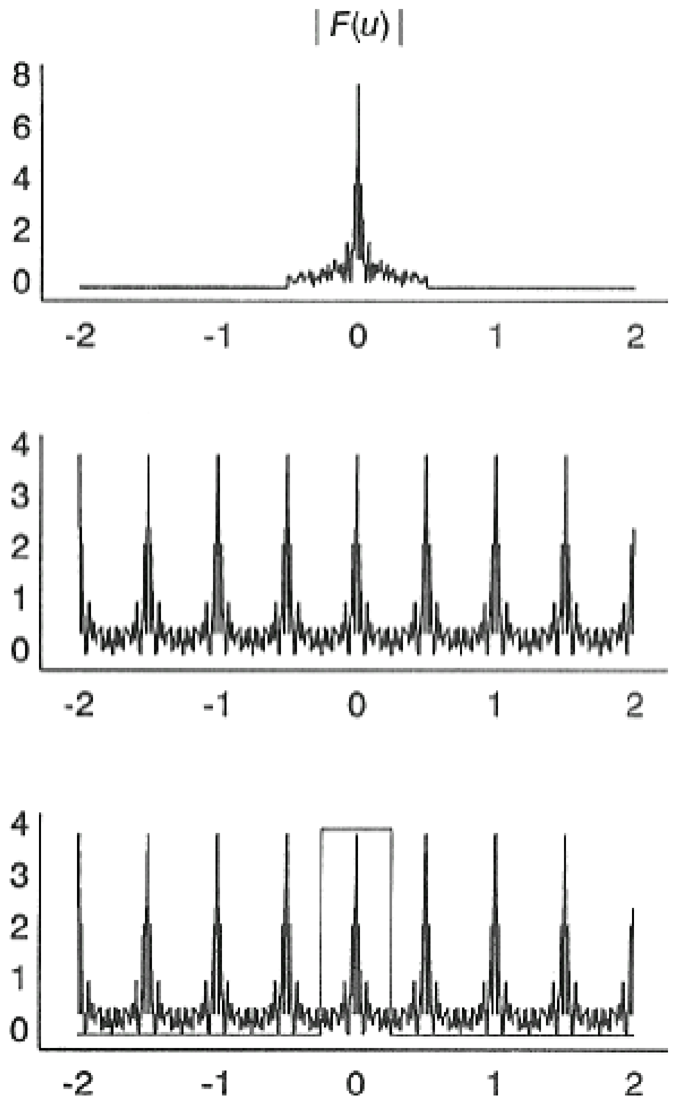
Reconstruction with tri function (= piecewise linear interpolation)

Even worse reconstruction



Aliasing

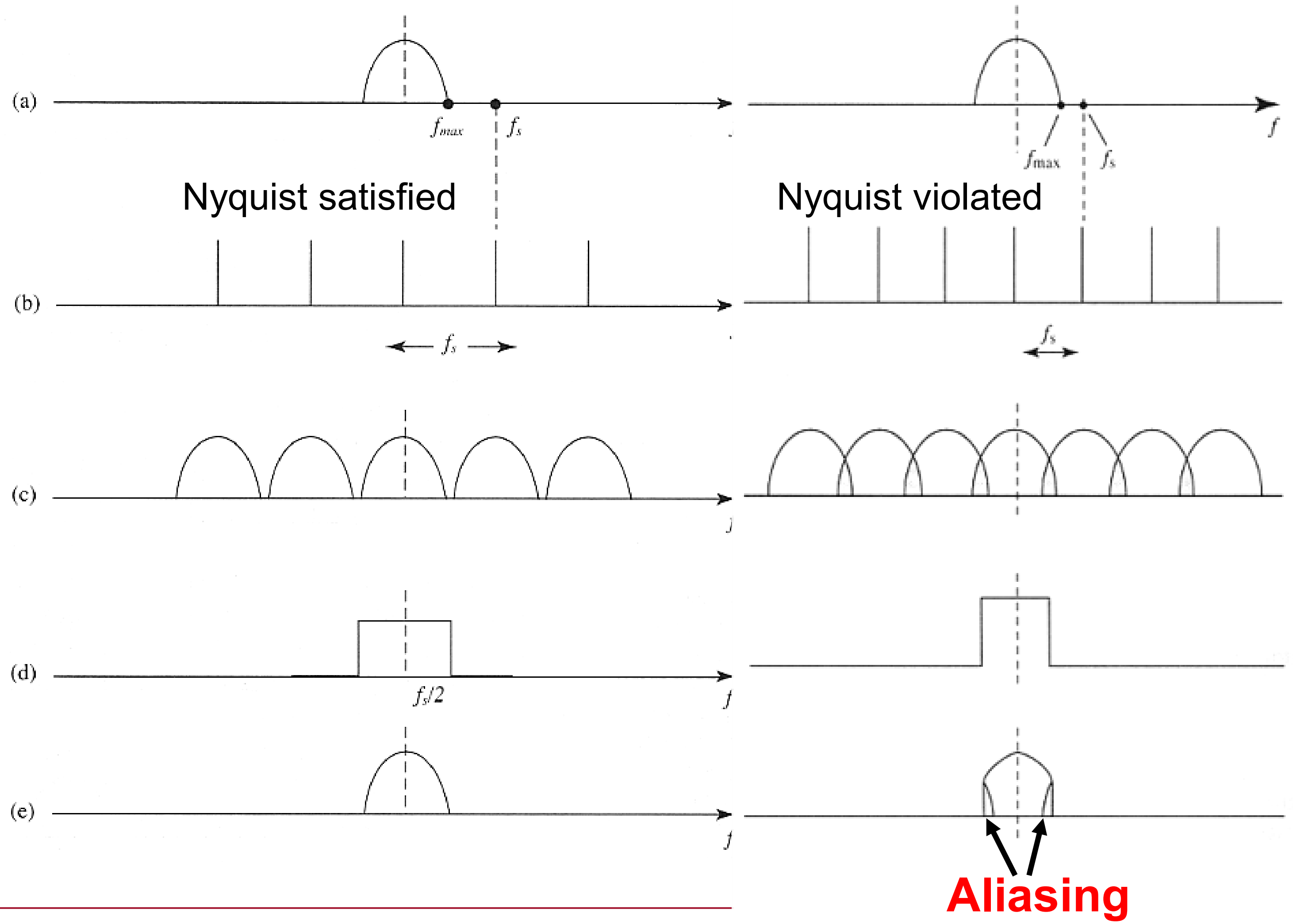
- Overlap between replicated copies in frequency spectrum
- High frequency components from the replicated copies are treated like low frequencies during the reconstruction process



Other examples of Aliasing?

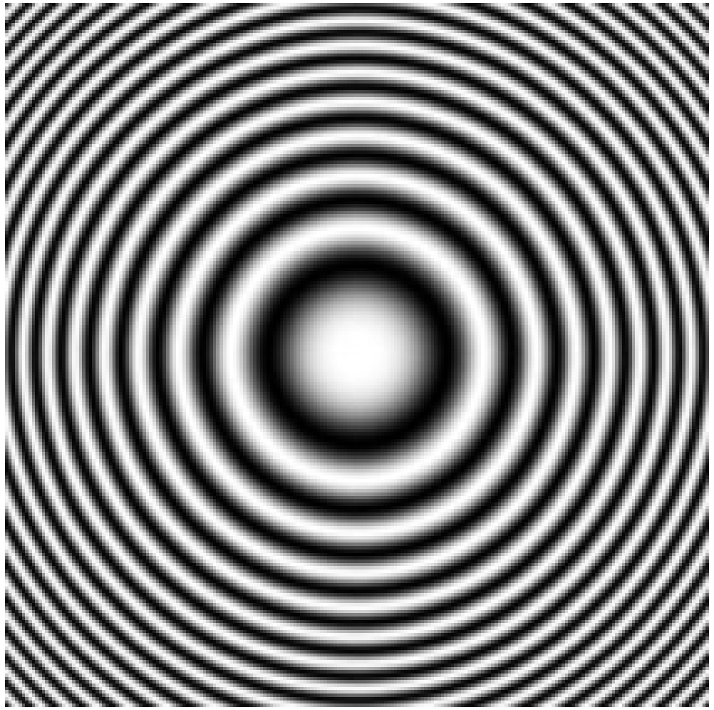
Aliasing

- In Fourier space
- Original spectrum
- Sampling comb
- Resulting spectrum
- Reconstruction Filter
- Reconstructed spectrum

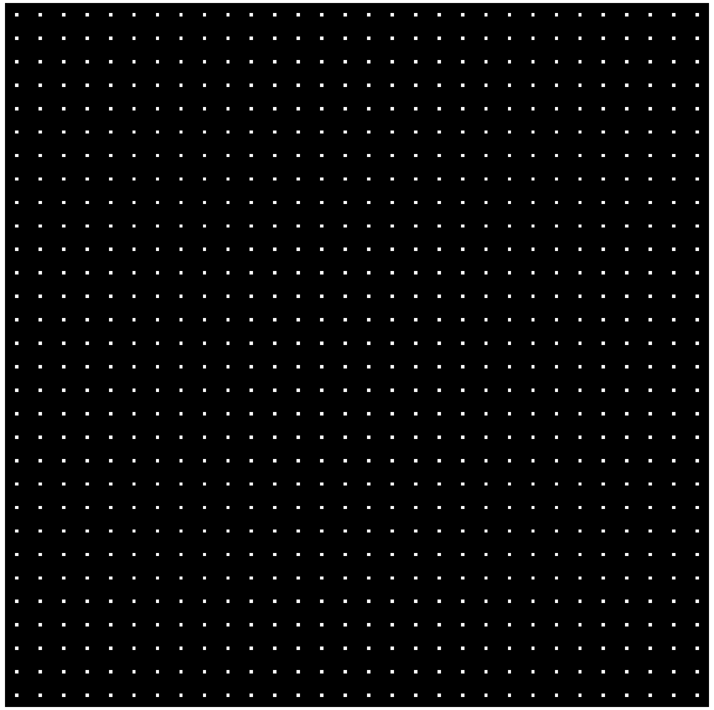


Aliasing 2D

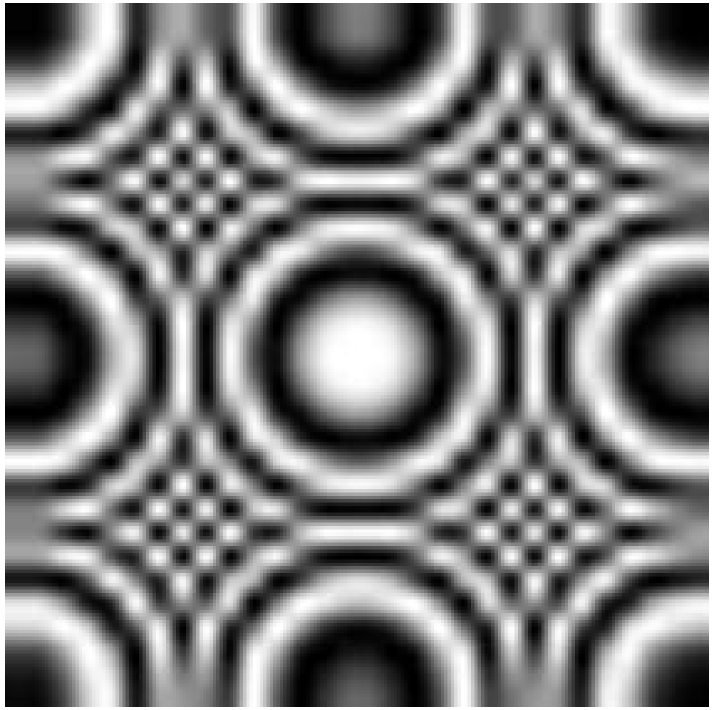
[wikipedia]



original image



sampled at these location

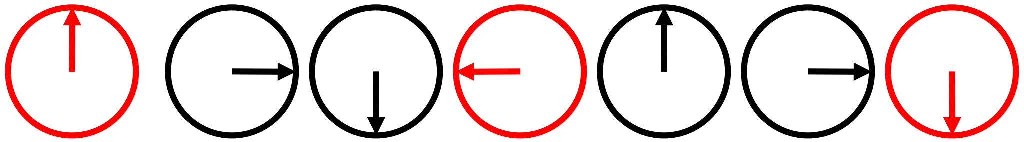


yields this reconstruction.

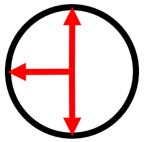
Sampling Artifacts

- Spatial aliasing:
  - Stair cases, Moiré patterns, etc.
- Solutions:
  - Increasing the sampling rate
    - Ok, but infinite frequencies at sharp edges
  - Post-filtering (after reconstruction)
    - Does not work - only leads to blurred stair cases
  - Pre-filtering (Blurring) of sharp geometry features
    - Slowly make geometry transparent at the edges
    - Correct solution in principal
    - Analytic low-pass filtering hard to implement
    - Super-sampling

Sampling Artifacts



- Temporal Aliasing
  - Cart wheels, ...
- Solutions
  - Increasing the frame rate
    - OK
  - Pre-filtering (Motion Blur)
    - Yes, possible for simple geometry (e.g., Cartoons)
    - Problems with texture, etc.
  - Post-filtering (Averaging several frames)
    - Does not work!!!! – only multiple detail
- Important
  - Distinction between **aliasing errors** and **reconstruction errors**

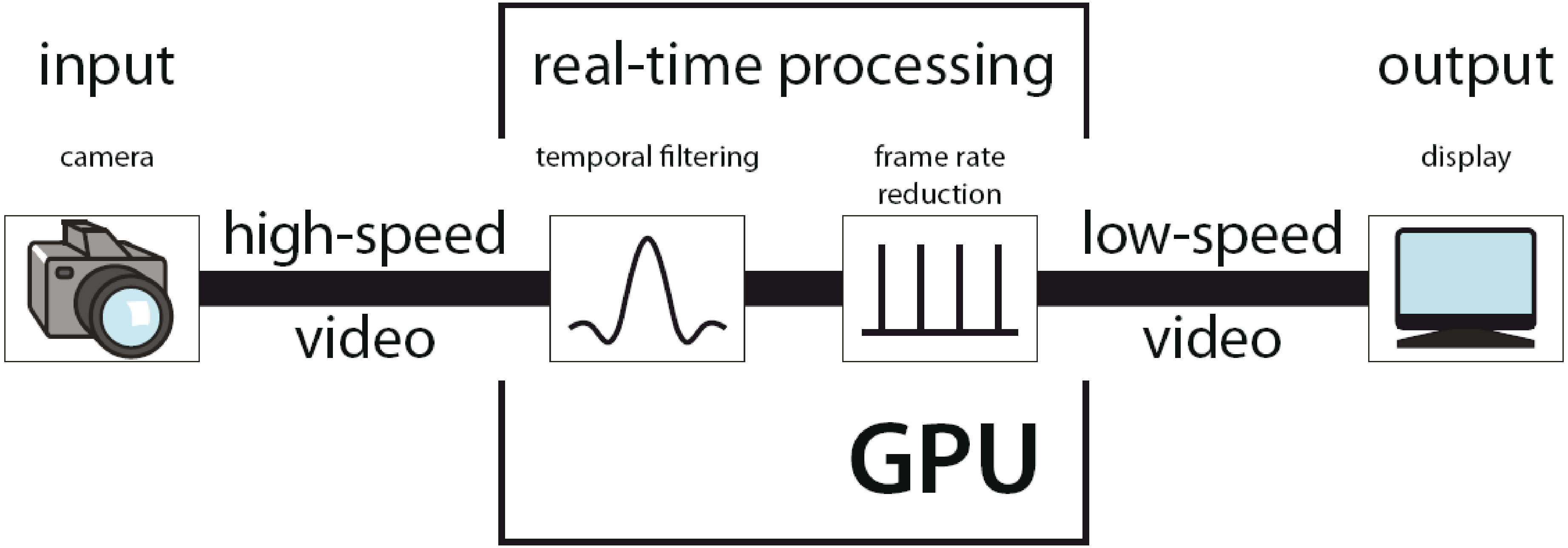


Aliasing

- It all comes from sampling at discrete points
  - Multiplied with comb function, no smoothly weighted filters
  - Comb function: repeats frequency spectrum
- Or, from using non band limited primitives
  - Hard edges → infinitely high frequencies
- In reality, integration over finite region necessary
  - E.g., finite CCD pixel size, anti-aliasing filter
- Computer: Analytic integration often not possible
  - No analytic description of radiance or visible geometry available
- Only way: numerical integration
  - Estimate integral by taking multiple point samples, average
    - Leads to aliasing
  - Computationally expensive
  - Approximate

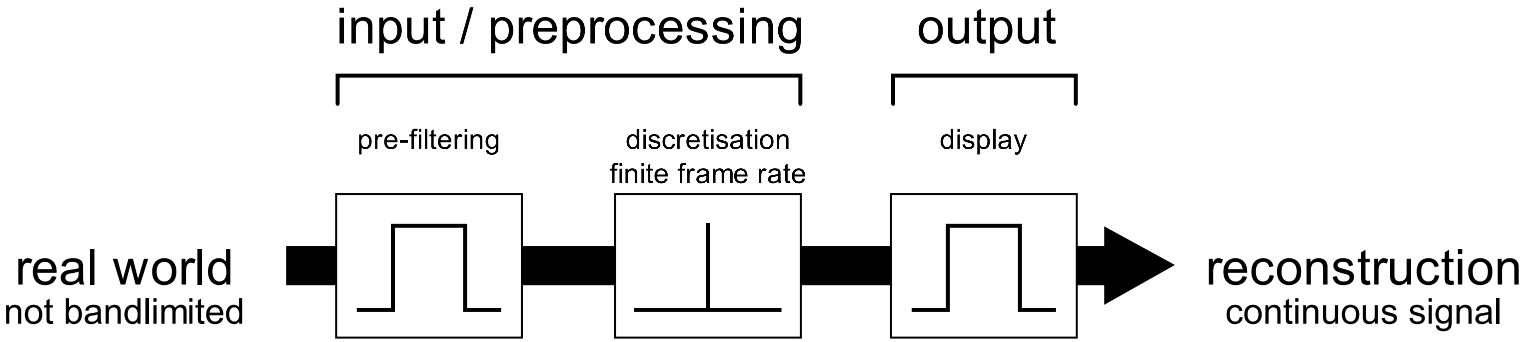
Aliasing in the Temporal Domain

A Shaped Temporal Filtering Camera

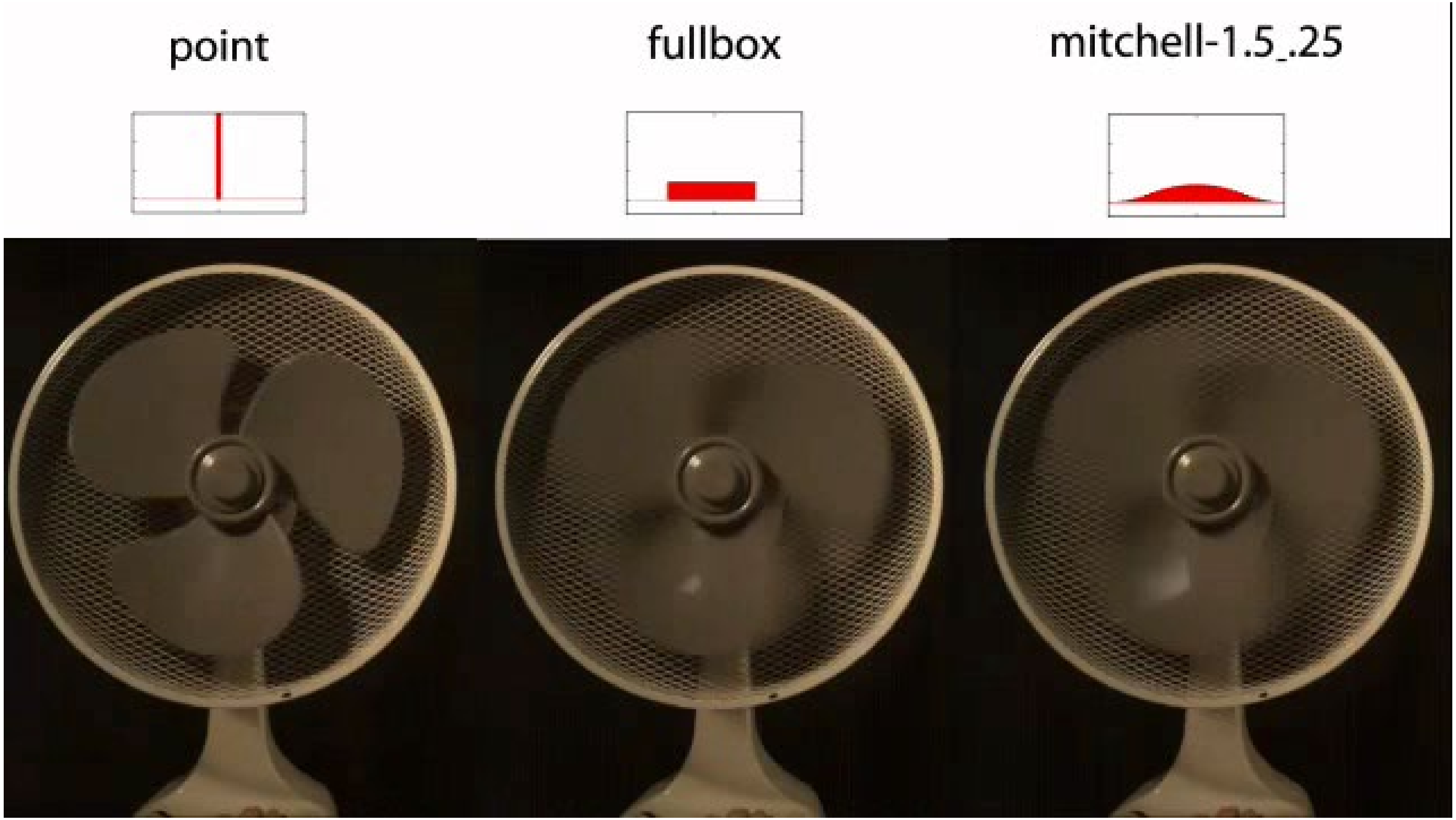


[Fuchs, Chen, Wang, Raskar, Seidel, Lensch – VMV 2009, C&G 2010]

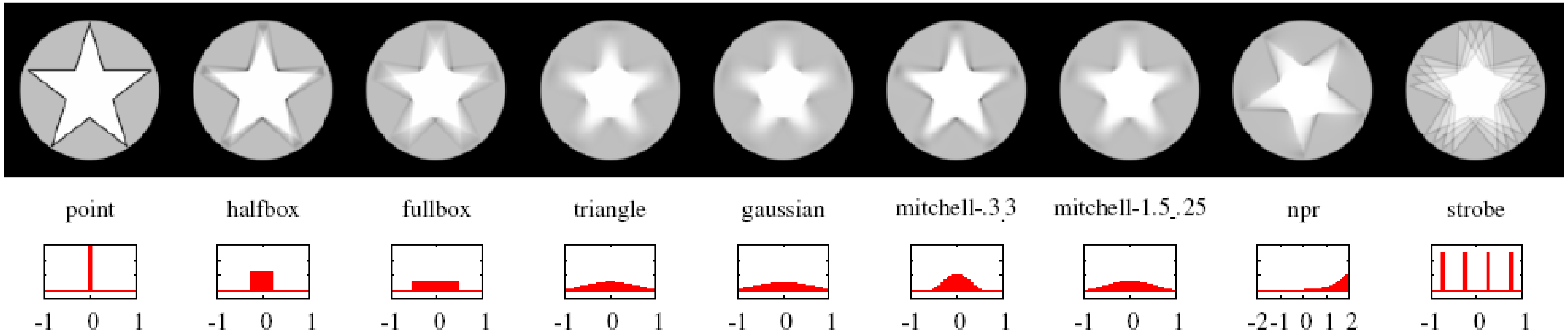
Aliasing in Standard Video Cameras



Aliasing and Prefiltering

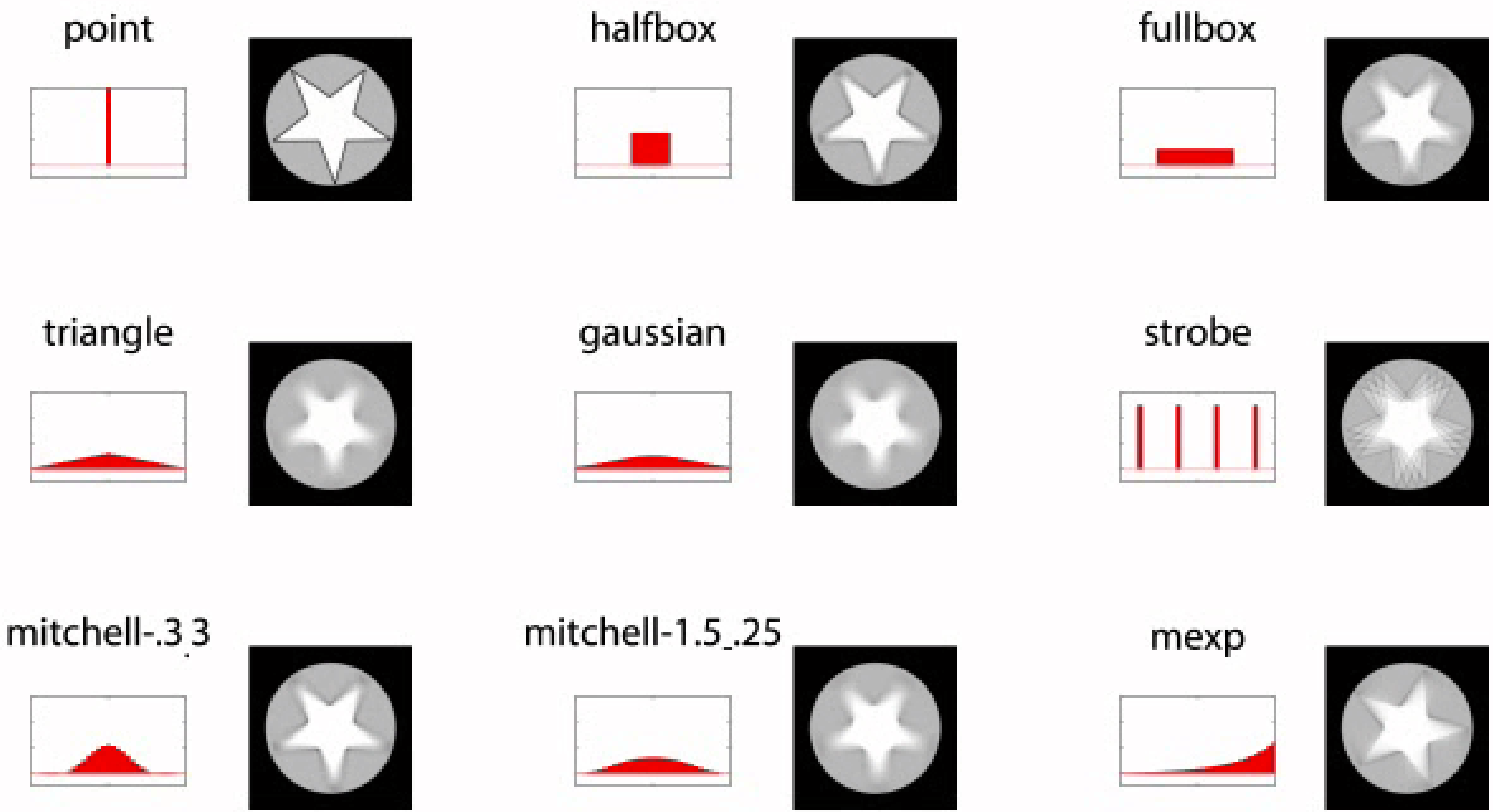


Prefiltering using various Kernels

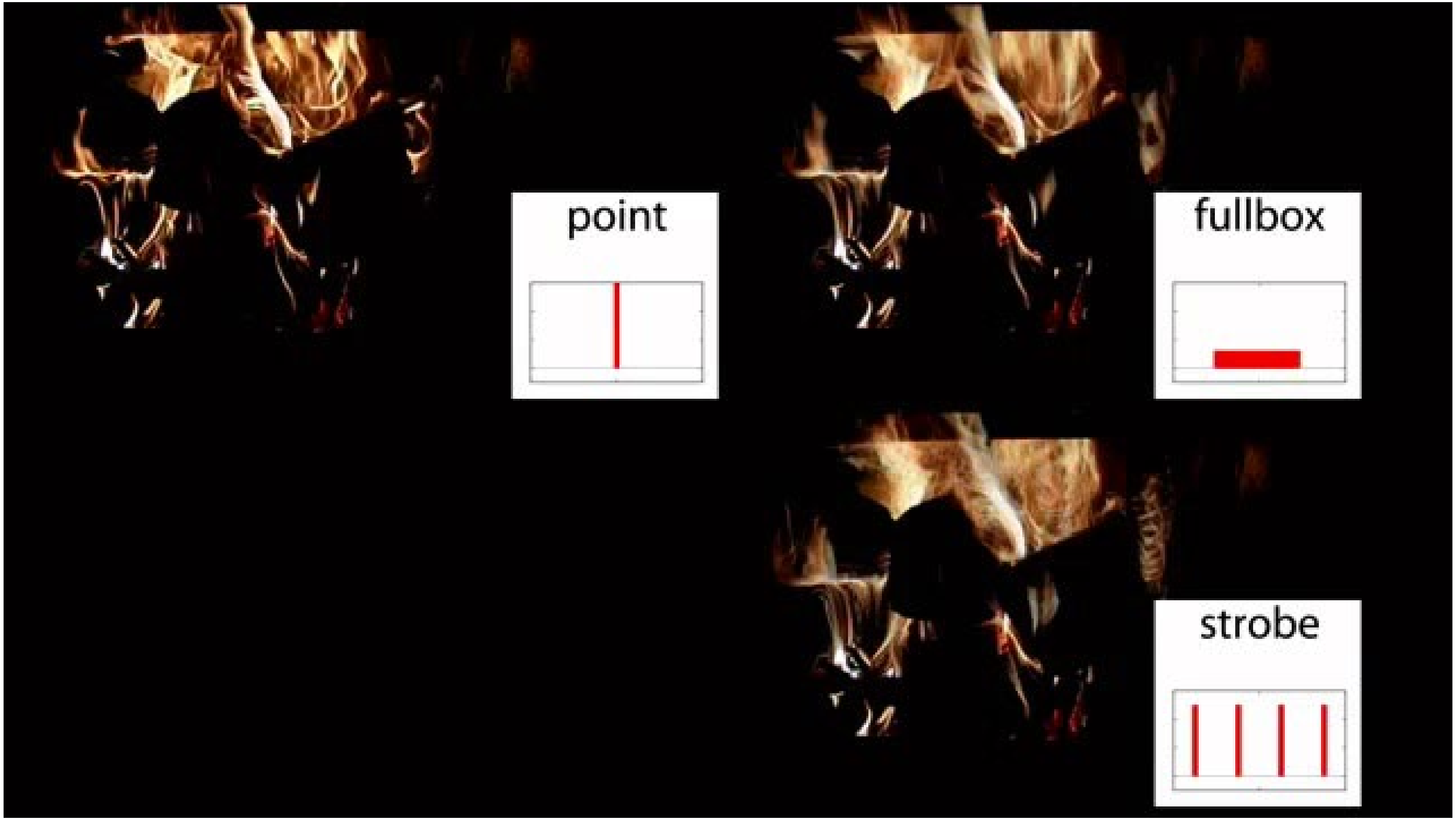


- Each filter results in a different temporal appearance
- Corresponding to a temporal bokeh
- Smoothing might not be the only thing intended

Prefiltering using various Kernels

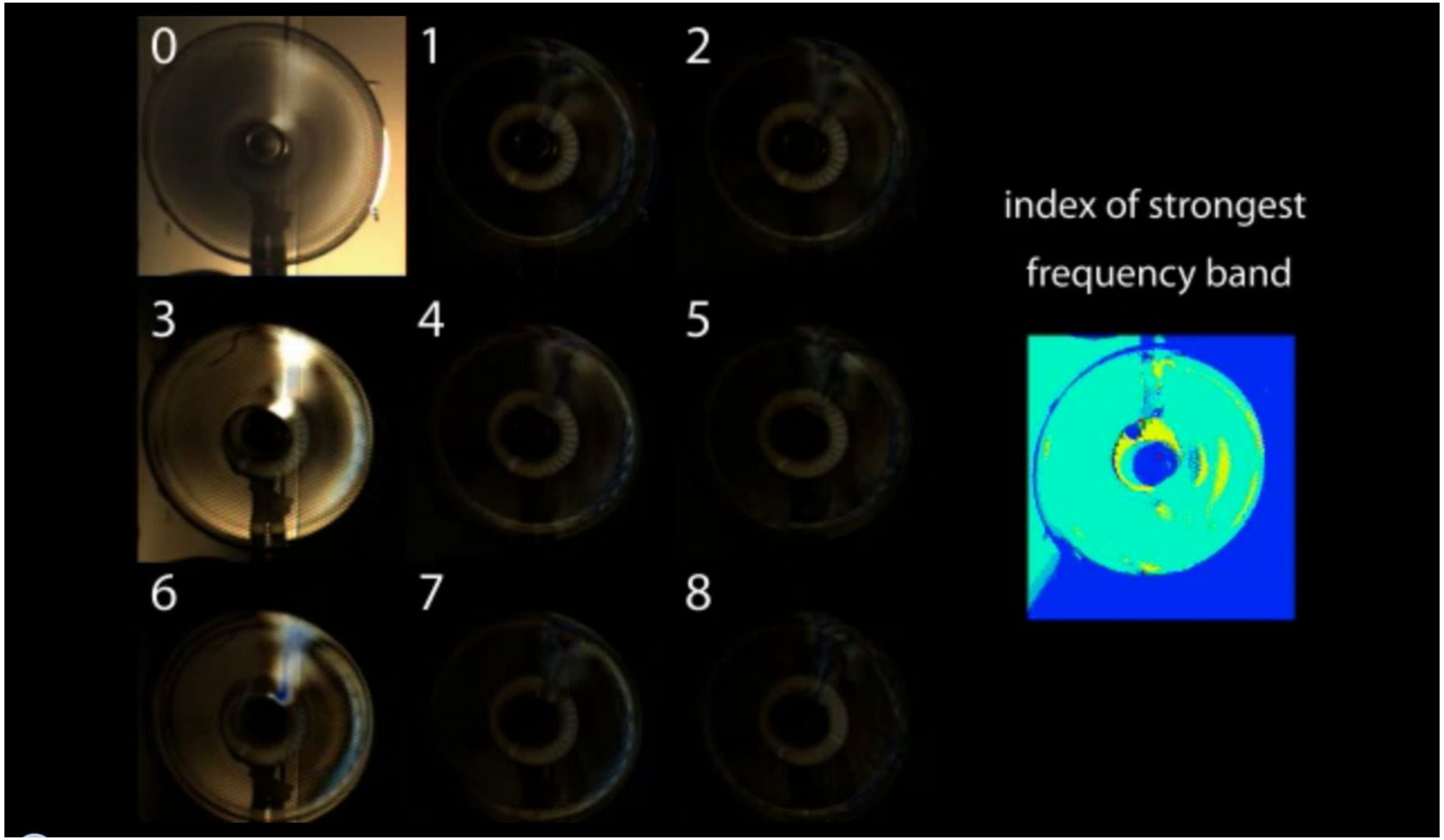


The Temporal Bokeh



Fourier Transform Camera

- Calculate a temporal Fourier transform on the fly
- 250x256@420Hz



Fourier Transform Camera

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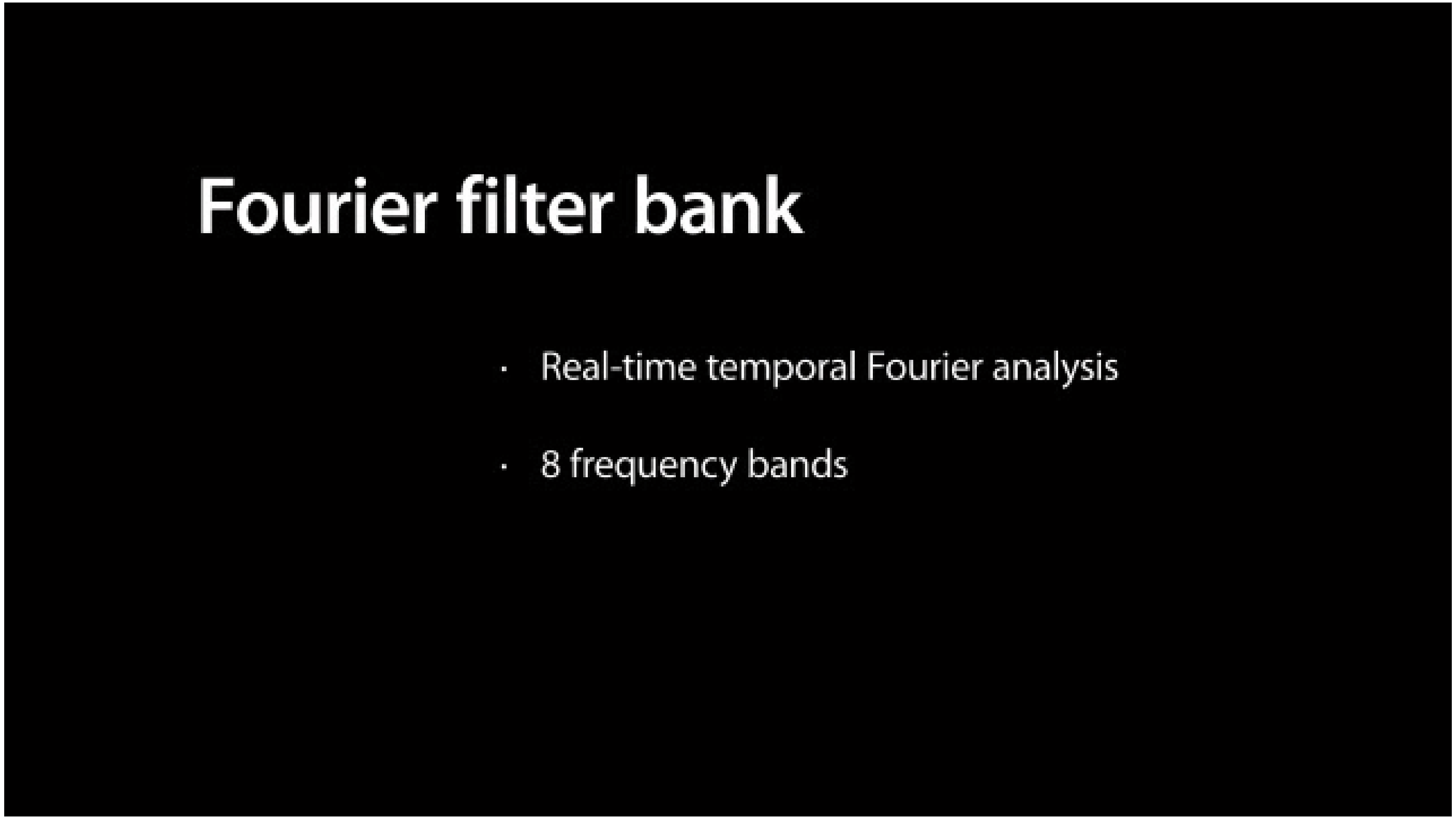
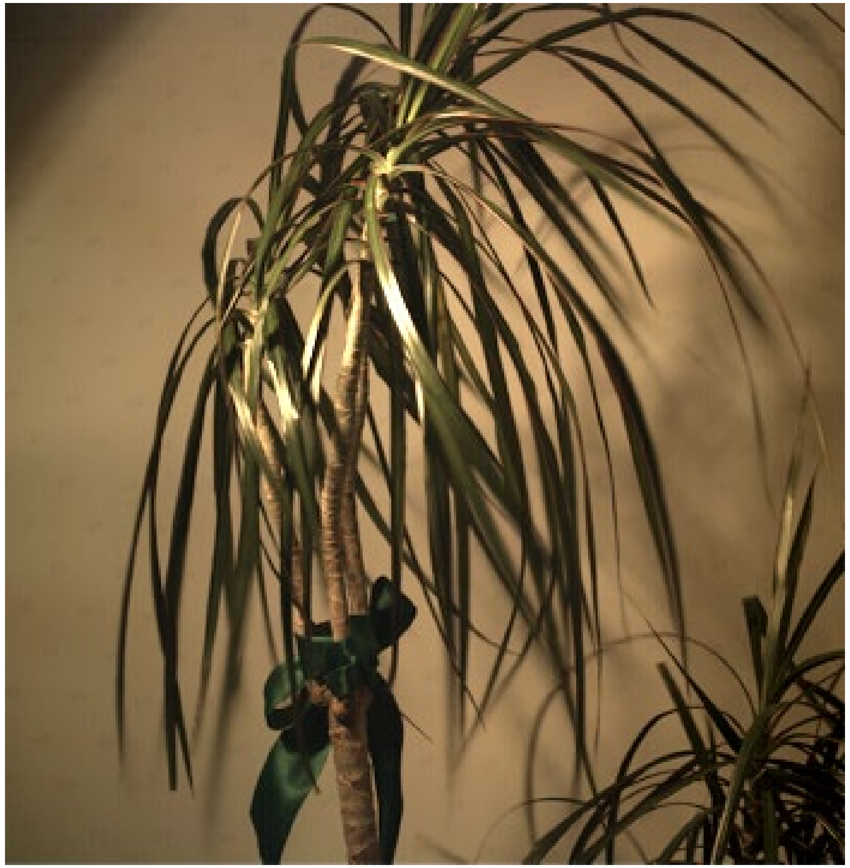


Motion Intensifier



Motion Intensifier

AntiAliasing

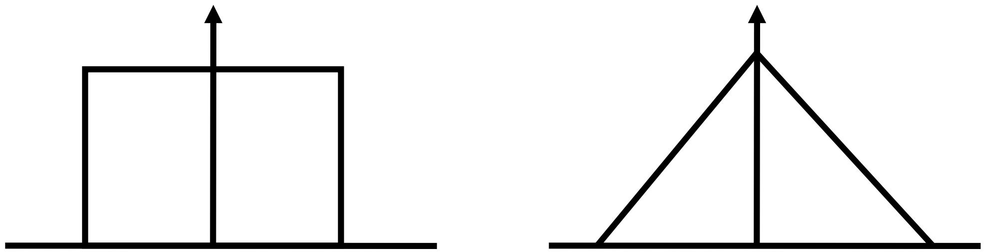
How to avoid aliasing artifacts?

Sources of High Frequencies

- Geometry
  - Edges, vertices, sharp boundaries
  - Silhouettes (view dependent)
  - …
- Texture
  - E.g., checkerboard pattern, other discontinuities, …
- Illumination
  - Shadows, lighting effects, projections, …

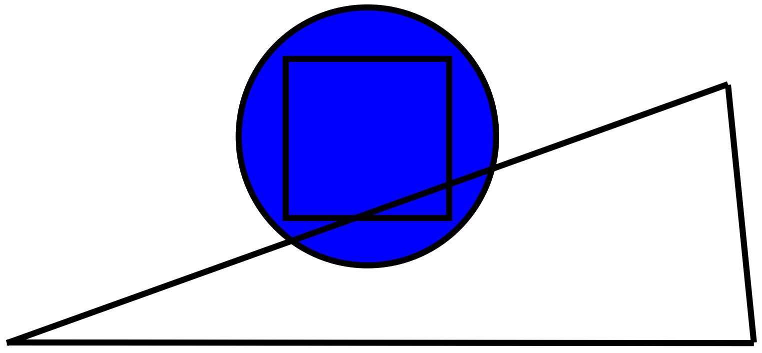
  → Analytic filtering almost impossible

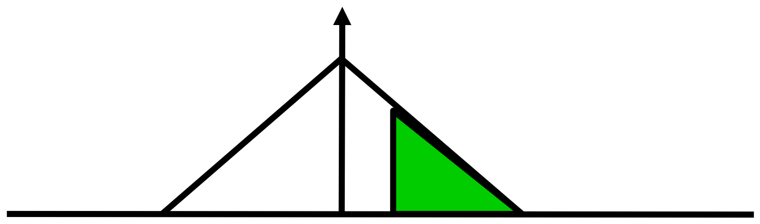
  - Even with the most simple filters



Comparison

- Analytic low-pass filtering
  - Ideally eliminates aliasing completely
  - Hard to implement
    - Weighted or unweighted area sampling
    - Compute distance from pixel to a line
    - Filter values can be stored in look-up tables
      - Possibly taking into account slope
      - Distance correction
      - Non rotationally symmetric filters
    - Does not work at corners
- Over-/Super-sampling
  - Very easy to implement
  - Does not eliminate aliasing completely
    - Sharp edges contain infinitely high frequencies

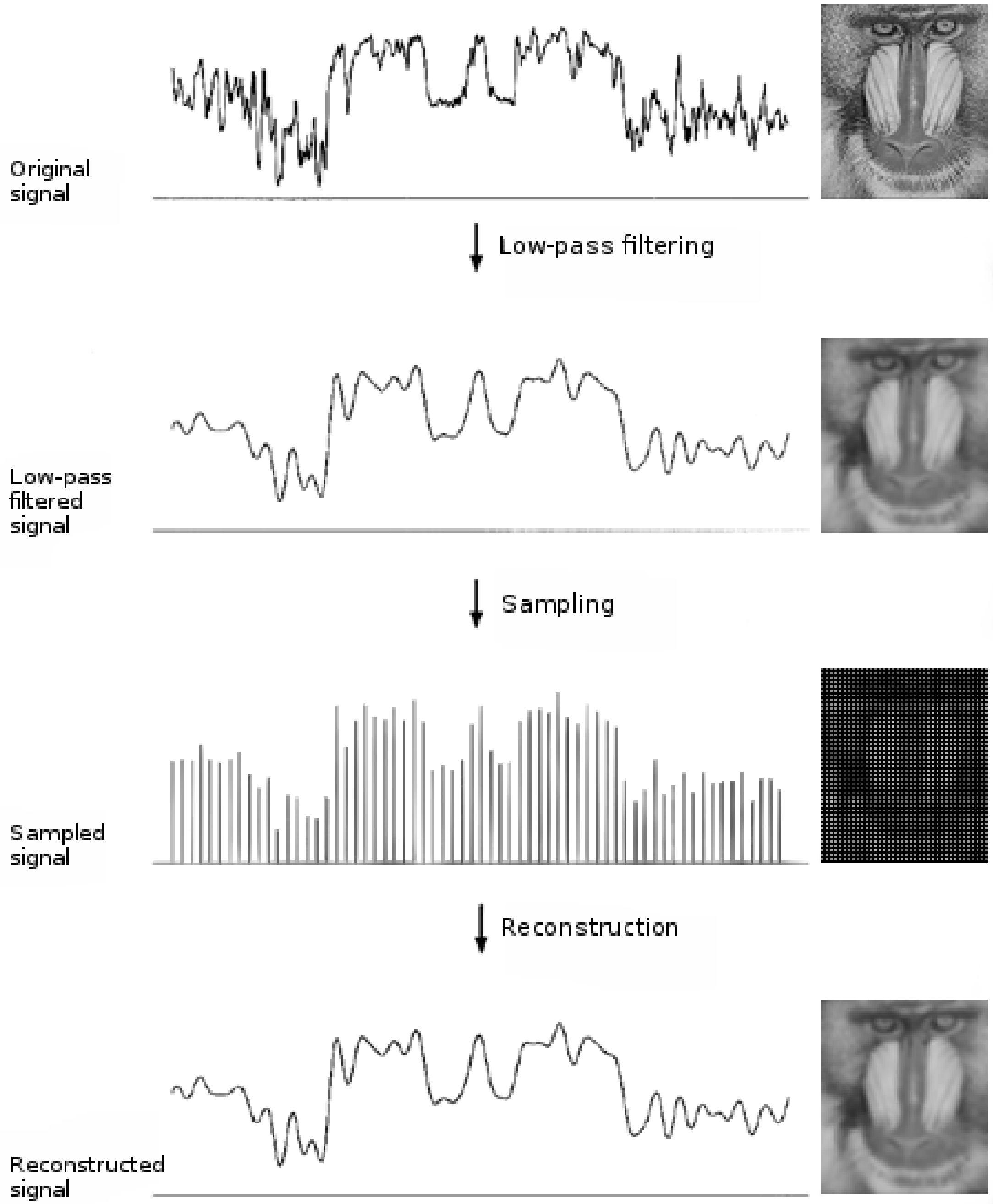




Anti-aliasing by Pre-Filtering

- Filtering before sampling
  - Band-limiting signal
  - Analog/analytic or
  - Reduce Nyquist frequency for chosen sampling-rate
- Ideal reconstruction
  - Convolution with sinc
- Practical reconstruction
  - Convolution with
    - Box filter, Bartlett (Tent)

  → Reconstruction error

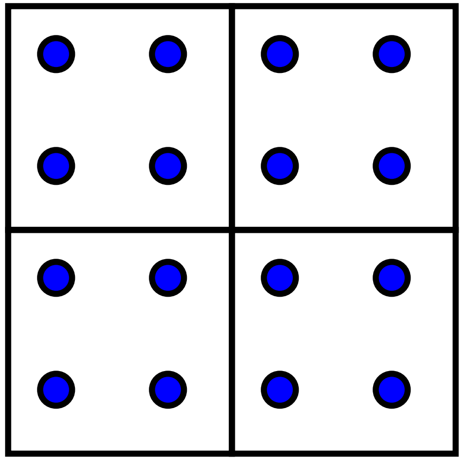


Super-Sampling in Practice

Regular super-sampling

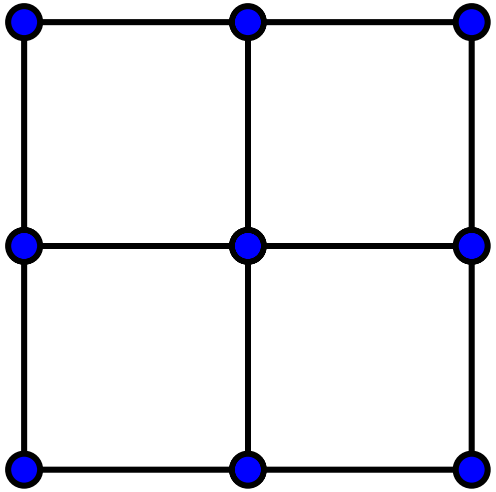
- Averaging of N samples per pixel on a grid
- N:
  - 4 quite good
  - 16 almost always sufficient
- Samples
  - Rays, z-buffer, reflection, motion, ...
- Filter Weights
  - Box filter
  - Others: B-spline, Pyramid (Bartlett), Hexagonal, ...
- Regular super-sampling
  - Nyquist frequency for aliasing only shifted

  → Irregular sampling patterns

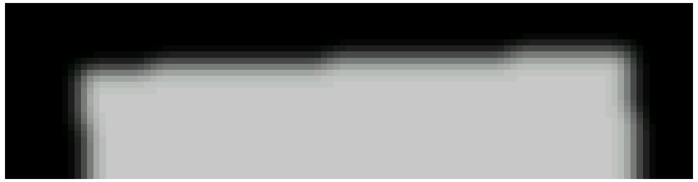


Super-Sampling Caveats

- Popular mistake
  - Sampling at the corners of every pixel
  - Pixel color by averaging
  - Free super-sampling ???
- Problem
  - Wrong reconstruction filter !!!
  - Same sampling frequency,
    but post-filtering with a hat function
  - Blurring: Loss of information
- Post-Reconstruction Blur



1x1 Sampling, 3x3 Blur



1x1 Sampling, 7x7 Blur

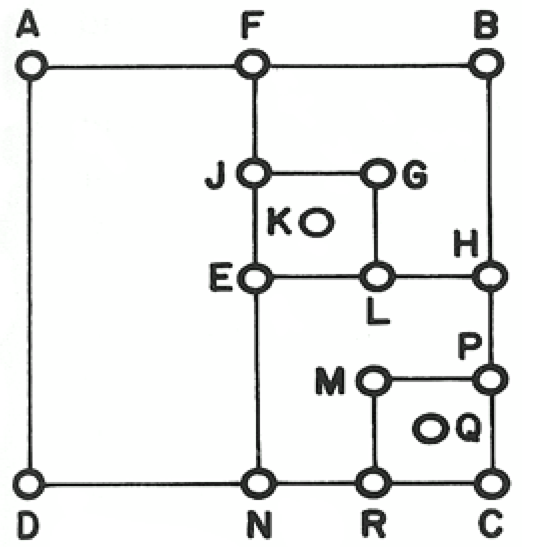
- **„Super-sampling“ does not come for free**

Adaptive Super-Sampling

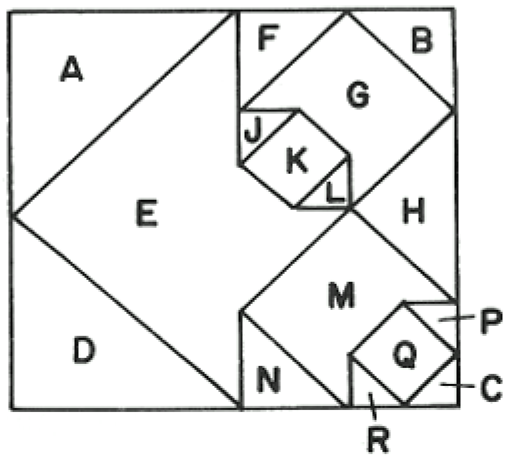
- Adaptive super-sampling
  - Idea: locally adapt sampling density
    - Slowly varying signal: low sampling rate
    - Strong changes: high sampling rate
  - Adapt sampling density locally
  - Decision criterion needed
    - Differences of pixel values
    - Contrast (relative difference)
      - $|A-B| / |A|+|B|$

Adaptive Super-Sampling

- Algorithm
  - Sampling at corners and mid points
  - Recursive subdivision of each quadrant
  - Decision criterion
    - Differences, contrast, object-IDs, ray trees, ...
  - Filtering with weighted averaging
    - ¼ from each quadrant
    - Quadrant: ½ (midpoint + corner)
      - Recursion



$$\frac{1}{4}\left(\frac{A+E}{2}+\frac{D+E}{2}+\frac{1}{4}\left[\frac{F+G}{2}+\frac{B+G}{2}+\frac{H+G}{2}+\frac{1}{4}\left\{\frac{J+K}{2}+\frac{G+K}{2}+\frac{L+K}{2}+\frac{E+K}{2}\right\}\right]\right.$$
$$\left.+\frac{1}{4}\left[\frac{E+M}{2}+\frac{H+M}{2}+\frac{N+M}{2}+\frac{1}{4}\left\{\frac{M+Q}{2}+\frac{P+Q}{2}+\frac{C+Q}{2}+\frac{R+Q}{2}\right\}\right]\right)$$

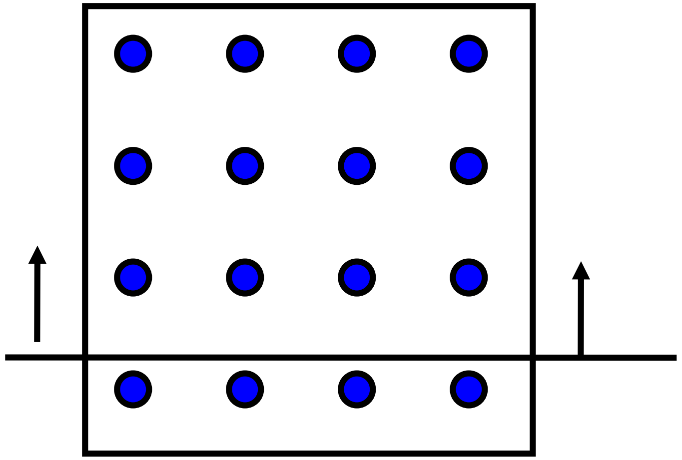


- Extension
  - Jittering of sample points

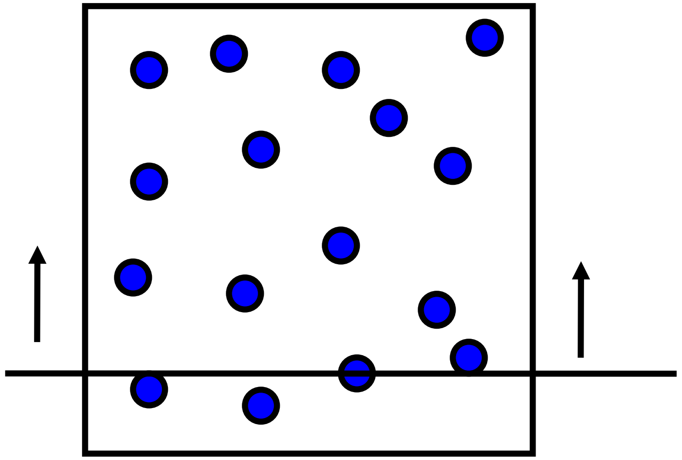
Is there Aliasing in the Human Eye?

Super-Sampling in Practice

- Problems with regular super-sampling
  - Expensive: 4-fold to 16-fold effort
  - Non-adaptive: Same effort everywhere
  - Too regular: Apparent reduction of number of levels
- Introduce irregular sampling pattern



0 → 4/16 → 8/16 → 12/16 → 16/16

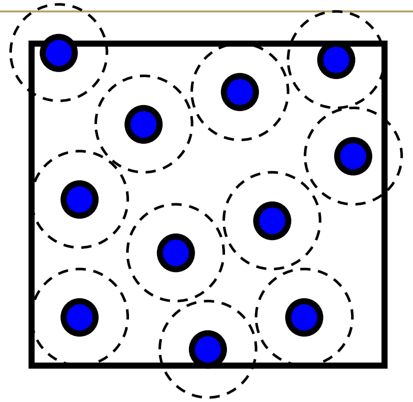


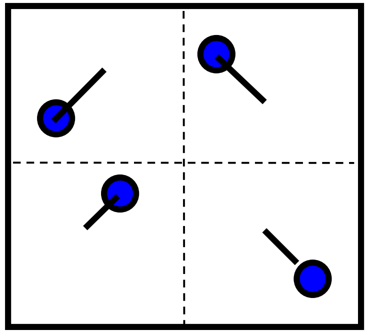
Better, but noisy

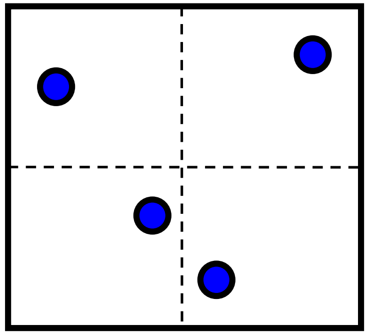
- Stochastic super-sampling
  - Or analytic computation of pixel coverage and pixel mask

Stochastic Sampling

- Requirements
  - Even distribution
  - Little correlation between samples
  - Incremental generation
- Generation of samples
  - Poisson-disk sampling
    - Fixes a minimum distance between samples
    - Random generation of samples
      - Rejection, if too close to other samples
  - Jittered sampling
    - Random perturbation from regular positions
  - Stratified Sampling
    - Subdivision into areas with one random sample each
    - Improves even distribution
  - Quasi-random numbers (Quasi-Monte Carlo)
    - E.g. Halton Sequence
    - Advanced feature

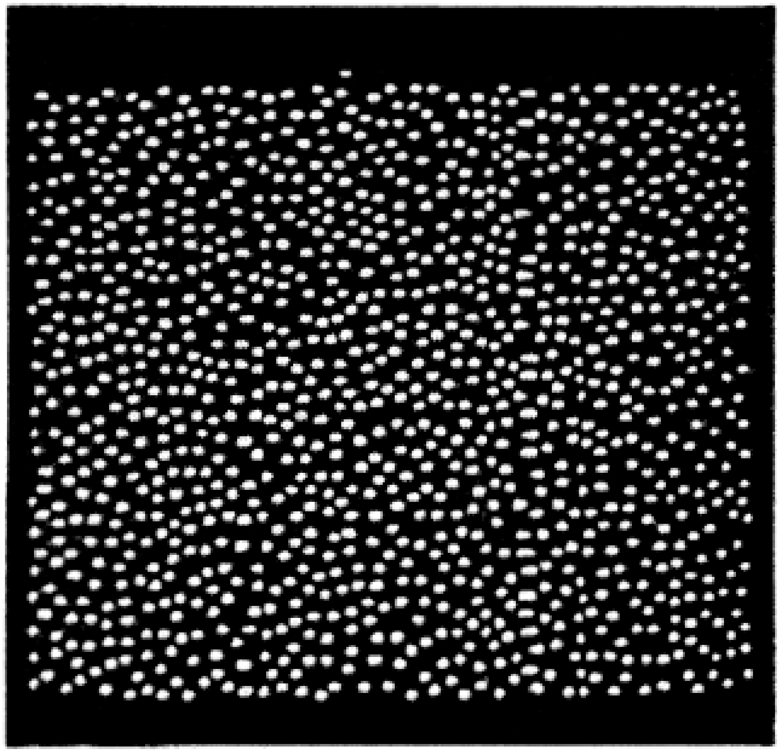






Poisson-Disk Sample Distribution

- Motivation
  - Distribution of the optical receptors on the retina (here: ape)



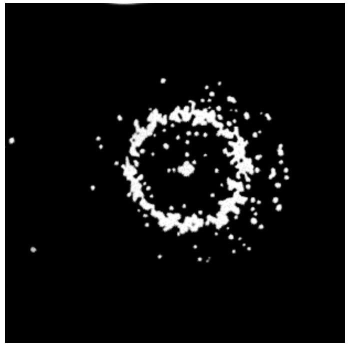
Distribution of the receptors

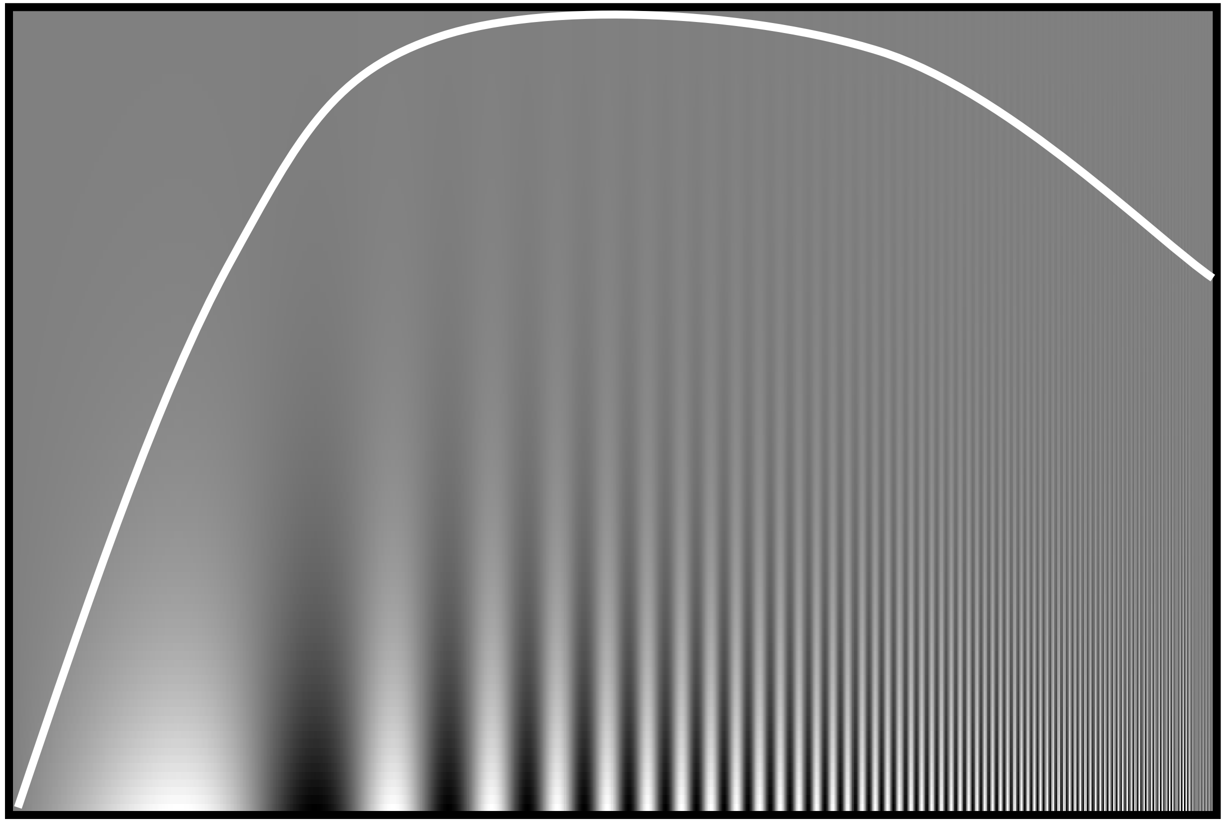
Fourier analysis

© Andrew Glassner, Intro to Raytracing

HVS: Poisson Disk Experiment

- Human Perception
  - Very sensitive to regular structures
  - Insensitive against (high frequency) noise

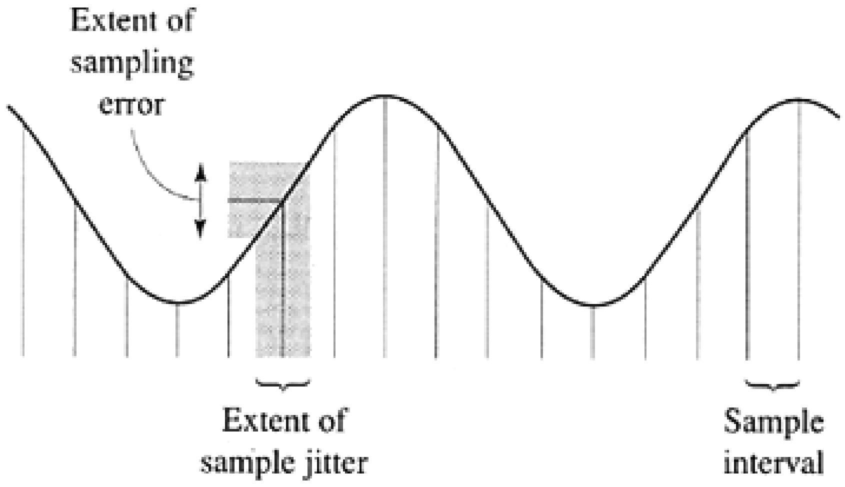




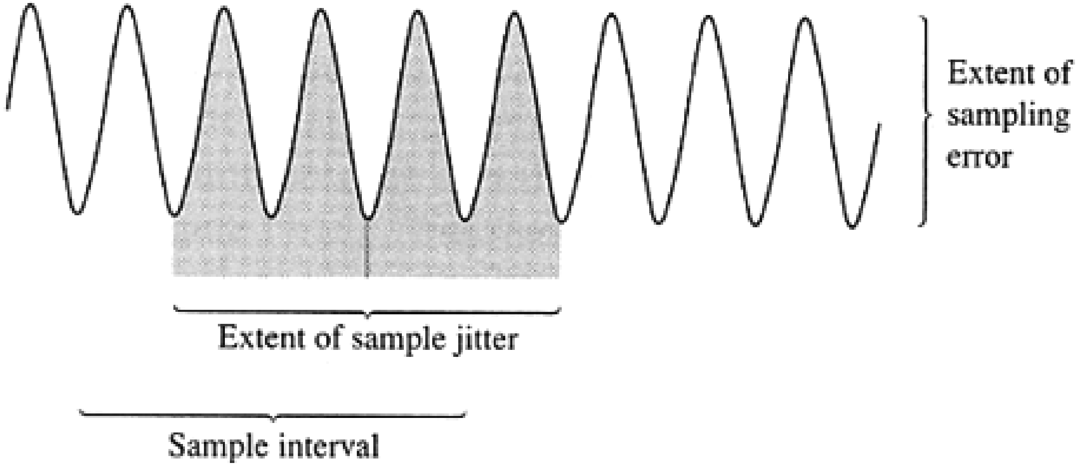
Campbell-Robson contrast sensitivity chart

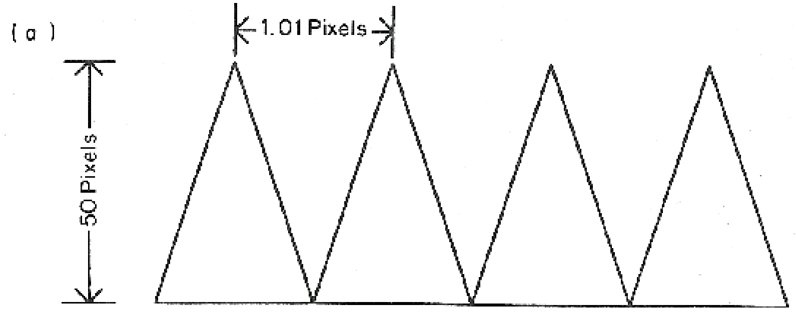
Stochastic Sampling

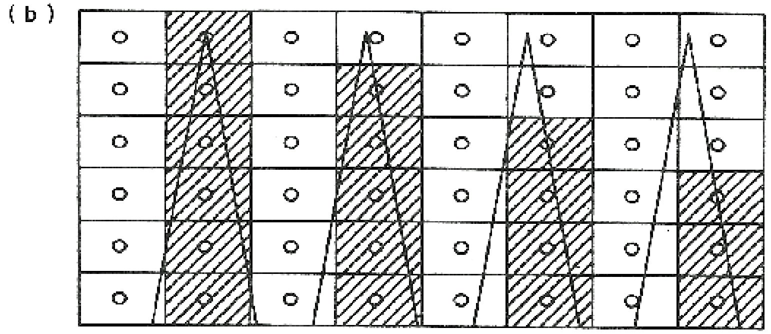
- Stochastic Sampling
  - Transforms energy in high frequency bands into noise
  - Low variation in sample domain
    - Closely reconstructs target value
  - High variation
    - Reconstructs average value



(a)



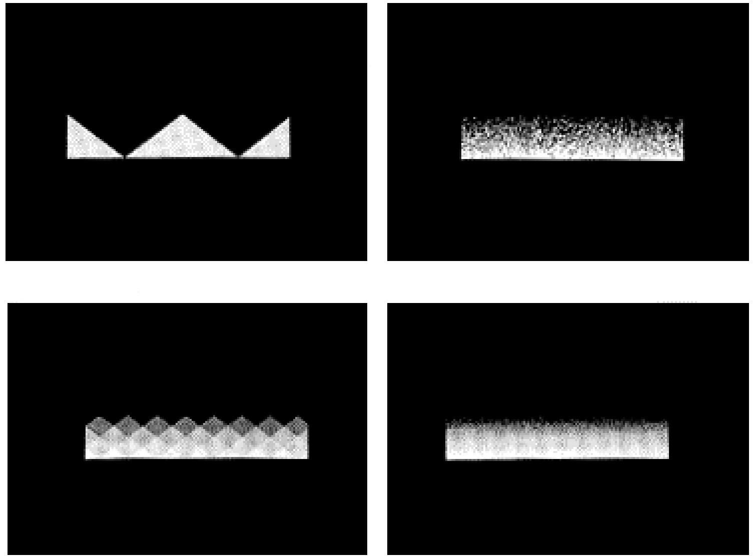


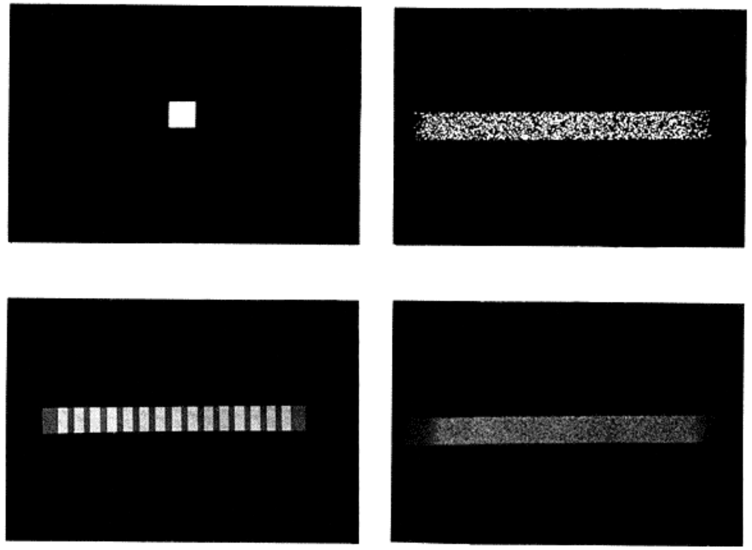


Examples

Triangle comb and rectangular wave (Width: 1.01 pix, Heigth: 50 pix):
1 sample, no jittering
1 sample, jittering
16 samples, no jittering
16 samples, jittering

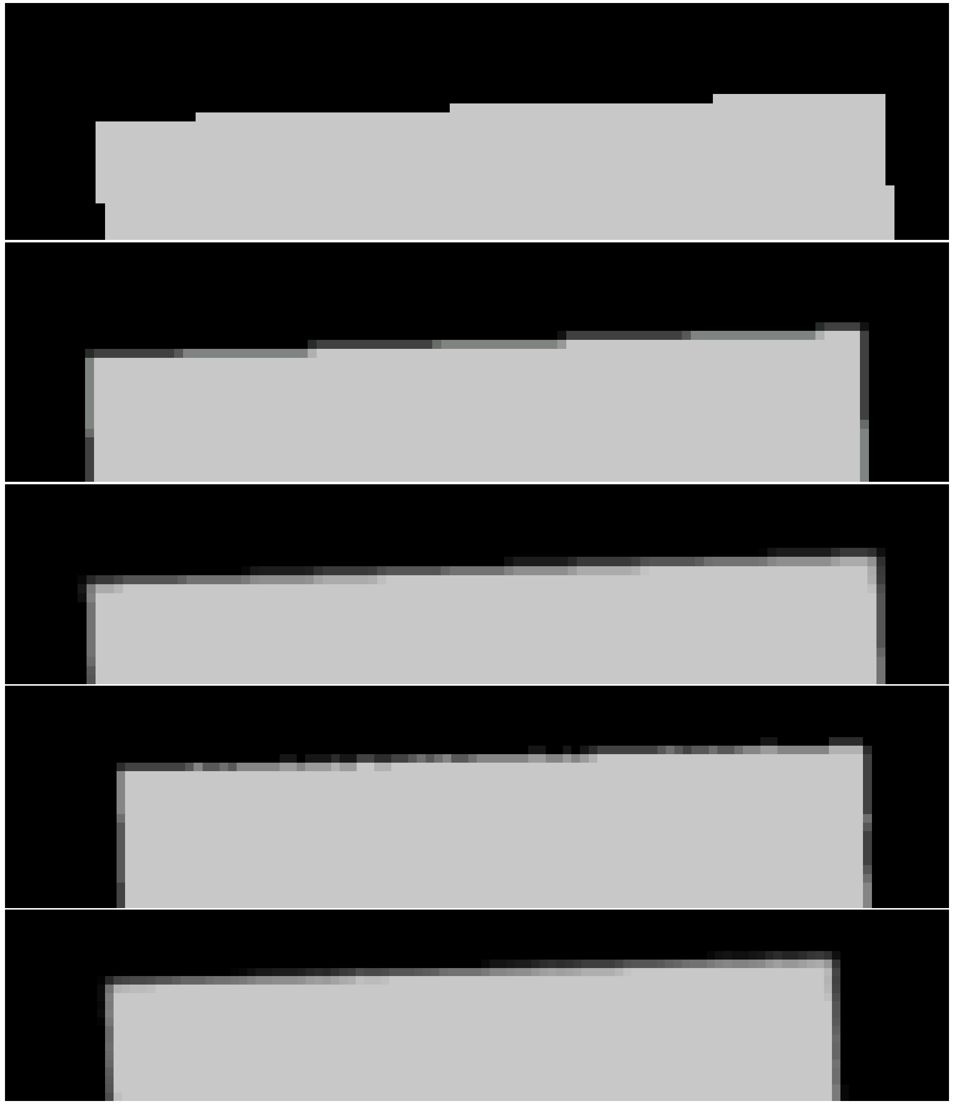
Motion Blur:
1 sample, no jittering
1 sample, jittering
16 samples, no jittering
16 samples, jittering





Comparison

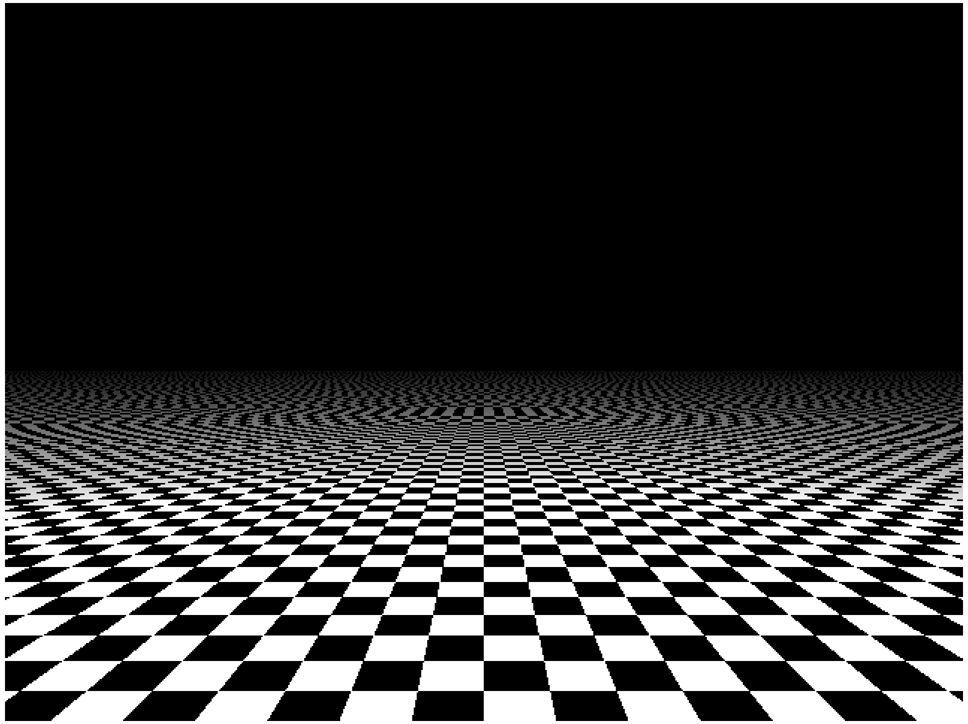
- Regular, 1x1
- Regular 3x3
- Regular, 7x7
- Jittered, 3x3
- Jittered, 7x7

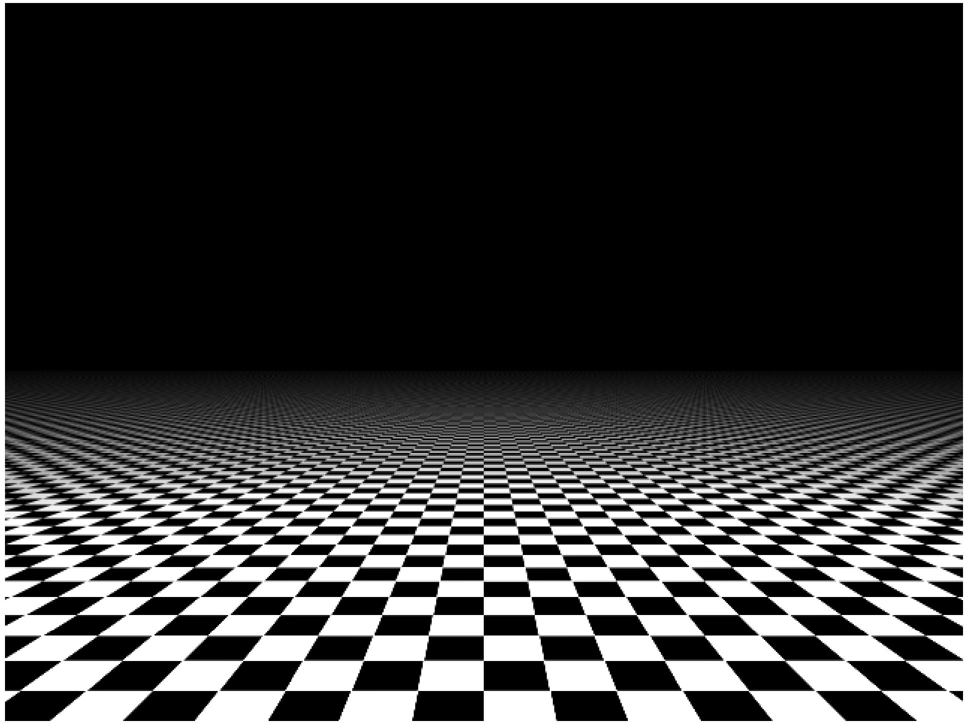


Wrap-UP

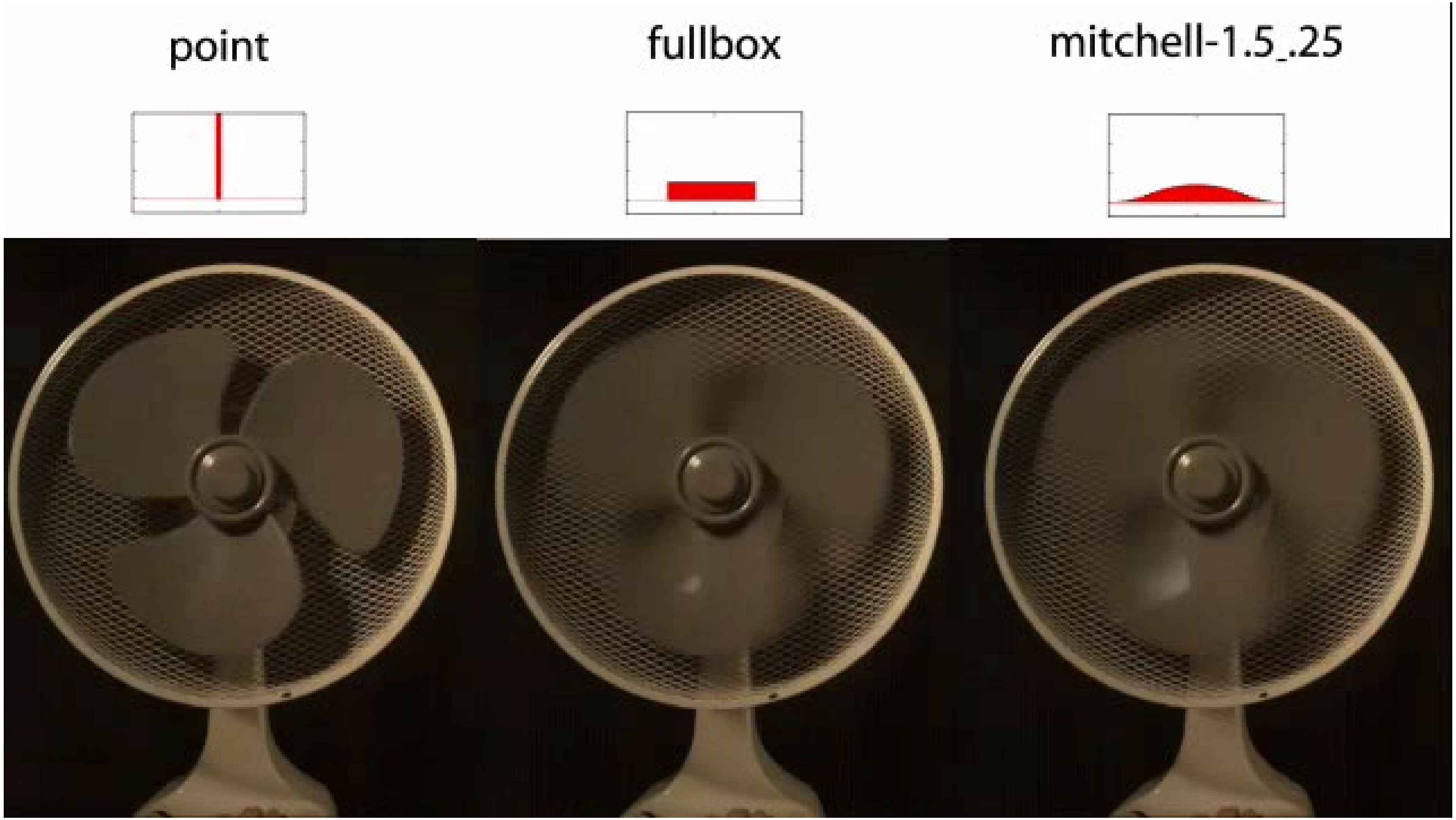
Aliasing

- Ray tracing
  - Textured plane with one ray for each pixel (say, at pixel center)
    - No texture filtering: equivalent to modeling with b/w tiles
  - Checkerboard period becomes smaller than two pixels
    - At the Nyquist limit
  - Hits textured plane at only one point, black or white by chance



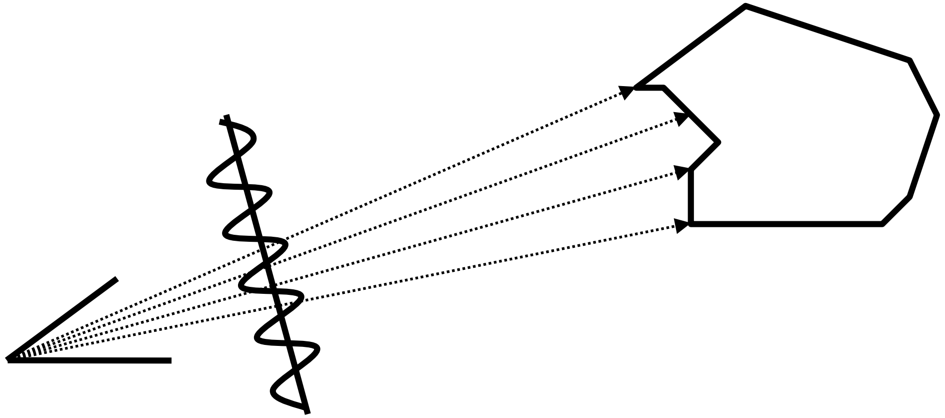


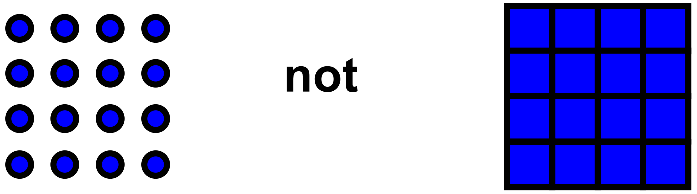
Aliasing and Prefiltering



The Digital Dilemma

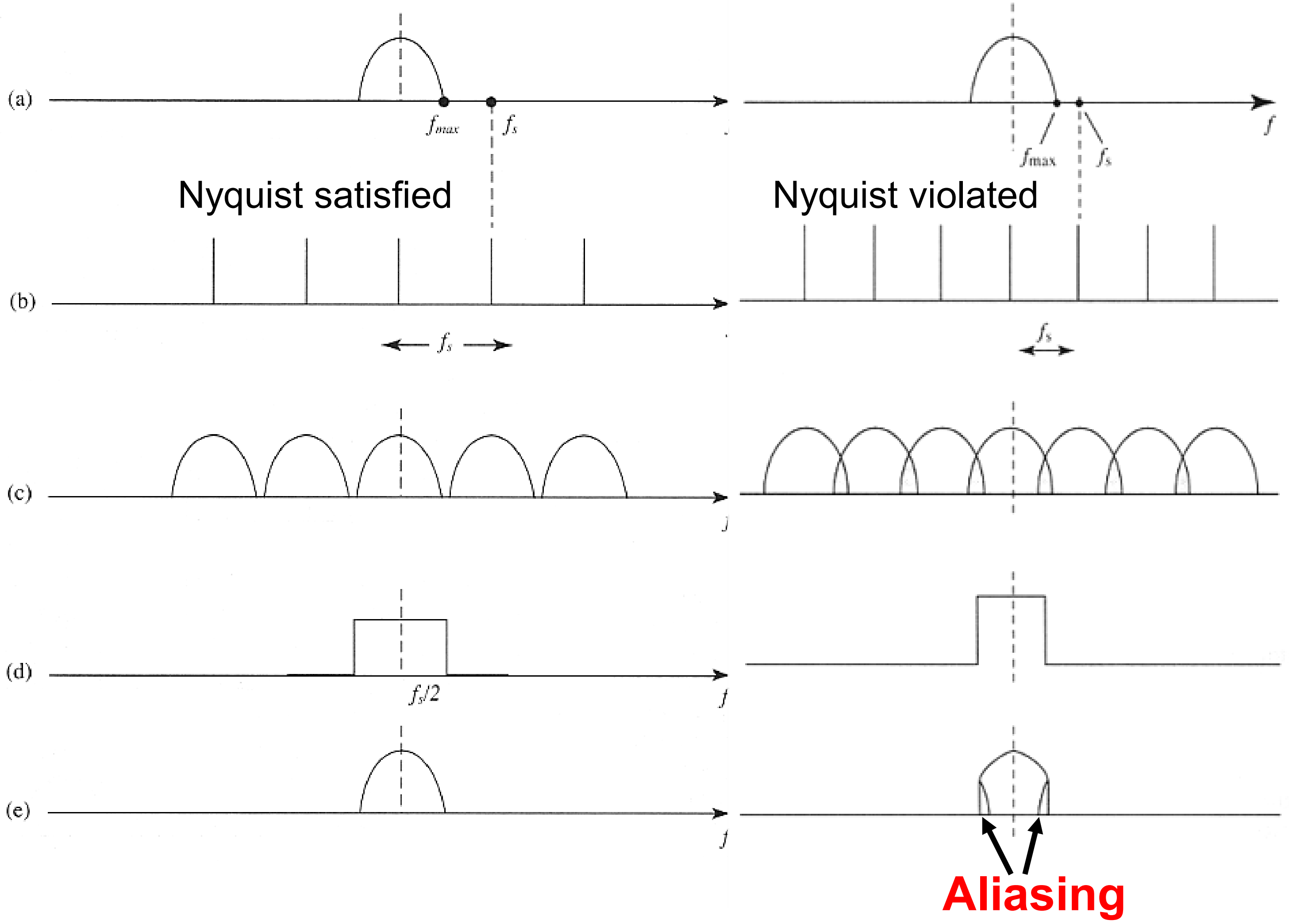
- Nature: continuous signal (2D/3D/4D with time)
  - Defined at every point
- **Acquisition: sampling**
  - Rays, pixel/texel, spectral values, frames, ...
- Representation: discrete data
  - Discrete points, discretized values
- **Reconstruction: filtering**
  - Mimic continuous signal
- Display and perception: faithful
  - Hopefully similar to the original signal, no artifacts





Aliasing

- In Fourier space
- Original spectrum
- Sampling comb
- Resulting spectrum
- Reconstruction Filter
- Reconstructed spectrum

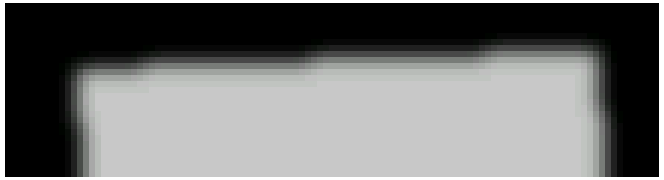


Anti-Aliasing

- Sampling Patterns
  - Super-sampling (slow), ok but infinite frequencies at sharp edges
  - Stochastic sampling

1 sample per pixel

- Post-filtering (after reconstruction)
  - Does not work!



1 sample, 7x7 blur

- Pre-filtering (blurring)
  - Correct solution in principal
  - Analytic low-pass filtering hard to implement

Questions

- Why do we hardly see aliasing in digital photo cameras?
- 10x zoom (3x optical) – what does this mean?

Overview

- Last lectures
  - Artifacts in ray traced images
  - Signal processing
  - Fourier Transform

- Today
  - Sampling, Reconstruction
  - Anti-aliasing & Super-Sampling

- Next
  - Image filters
  - The human visual system

Homework

- Where do you observe sampling, aliasing and anti-aliasing in action at home?
  - Find one example each.