Ray Intersection

(aus Vorlesung Ray-Tracing 1) 12.02.

- ray definition $r(t) = o + td$ with $o = \begin{pmatrix} o_x \\ o_y \\ o_z \end{pmatrix}$ origin, $d = \begin{pmatrix} d_x \\ d_y \\ d_z \end{pmatrix}$ direction

- **ray-sphere-intersection:**

  sphere-formula: $r^2 = x^2 + y^2 + z^2$ (if unit circle radius $r = 1$)

  $\rightarrow r^2 = p^2$ (say P is point with coord. x, y, z)

  insert: ~~origin~~ ray r(t) into P

  $r^2 = (o+td)^2$

  $1 = o^2 + 2tdo + t^2d^2 \quad | -1$

  $0 = d^2t^2 + 2dot + o^2 - 1$

  mitternachtsformel: $t_{1,2} = \frac{-2do \pm \sqrt{(2do)^2 - 4d^2(o^2-1)}}{2d^2}$

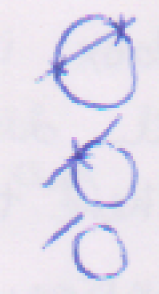
  $= \frac{-2do \pm \sqrt{4d^2o^2 - 4d^2o^2 + 4d^2}}{2d^2} = \frac{-2do \pm 2d}{2d^2}$

  $\rightarrow t_1 = \frac{2d(-o+1)}{2d^2} = \frac{-o+1}{d} \quad \rightarrow t_2 = \frac{2d(-o-1)}{2d^2} = \frac{-o-1}{d}$ //

  three options: 1) two different ts — two intersection points

  2) one t — only one intersect. point

  3) can't solve the equat. — no intersection



  note: if the ts are negative, then the intersections are behind the origin

- **ray-plane-intersection:**

  plane-equation: $0 = p \cdot n - D$ with p point on plane, n plane normal and D distance of plane to origin

  insert ray r(t) into p:

  $0 = (o+td) \cdot n - D$

  $0 = on + tdn - D \quad \rightarrow \quad tdn = -on + D$

  $t = \frac{D - on}{dn}$ //

  to solve: insert given n and d and D, o

- **ray-triangle-intersection:**

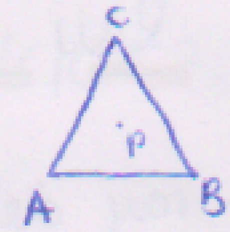
  first: check if ray hits plane on which triangle lies

  generate plane-equation: normal = AB × AC (cross product)

  point: choose one point from triangle

  calculate intersection point: by inserting ray equation into plane equation, solve for t, insert t into r(t)

  when ray and plane parallel: the ray direction and plane normal should be perpendicular (dot-product is 0) then: check if intersection point inside triangle



1) calculate edges:
   edgeAB = $\overline{AB}$, edgeBC = $\overline{BC}$, edge CA = $\overline{CA}$

2) calculate distance from P to first point of edge:
   distAP = P-A , distBP = P-1 , distCP = P-C

3) calculate crossproduct between the two
   cross0 = edgeAB x distAP
   cross1 = edgeBC x distBP
   cross2 = edgeCA x distCP

4) check if all crossproducts · normal are positive, then the point lies in the triangle

inside-outside-test

- ray-box-intersection

first: find intersection points with each plane (3D) or line (2D)

definition: a box is defined by its min. and max value in each dimension

now: check where the ray intersects with the line of the min and max value

x-dimension: $o_x + t d_x = min_x \rightarrow t_{xmin} = \frac{min_x - o_x}{d_x}$

$o_x + t d_x = max_x \rightarrow t_{max} = \frac{max_x - o_x}{d_x}$

repeat for y and z-dimension

but: we now only know the intersection with these planes, not if the intersection points lie on the cube

then:

find biggest tmin: $t_{min} = max(t_{xmin}, t_{ymin}, t_{zmin})$
find smallest tmax: $t_{max} = min(t_{xmax}, t_{ymax}, t_{zmax})$

but check when comparing: if ($t_{min} > t_{ymax}$ or $t_{max} < t_{ymin}$) return false

- ray-cylinder-intersection

cylinder-equation: $x^2 + y^2 = 1$ (infinite)

insert ray: $r(t) = o + td$

$$(o_x + td_x)^2 + (o_y + td_y)^2 = 1$$

$$0 = (o_x^2 + o_y^2) + 2t(d_x + d_y) + t^2(d_x^2 + d_y^2) - 1$$

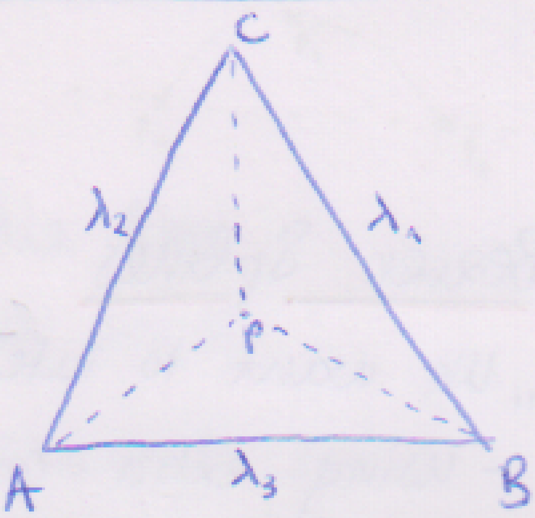
$$= \underbrace{(d_x^2 + d_y^2)}_{a} t^2 + \underbrace{2(d_x + d_y)}_{b} t + \underbrace{(o_x^2 + o_y^2 - 1)}_{c}$$

Solve with mitternachtsformel after inserting d and o

choose nearest t

> ! After finding the correct t, you need to insert it into ray-equation. Then you have the intersection point.

Barycentric Coordinates



- coordinate system to describe a point in an triangle $P = \lambda_1 A + \lambda_2 B + \lambda_3 C$
- important: $\lambda_1 + \lambda_2 + \lambda_3 \overset{!}{=} 1$
- use to check inside triangle: iff $\lambda_{1,2,3} \geq 0$, point is in triangle
- calculation: $\lambda_2 = \dfrac{\vec{AC}_x \vec{AP}_y - \vec{AC}_y \vec{AP}_x}{\vec{AC}_x \vec{AB}_y - \vec{AC}_y AB_x}$ (section on the AC line)

$$\lambda_3 = \frac{AP_x AB_y - AP_y AB_x}{AC_x AB_y - AC_y AB_x} \quad \text{(section on the AB line)}$$

$$\lambda_1 = 1 - \lambda_2 - \lambda_3 \quad \text{(section on the BC line)}$$