Exercise 1

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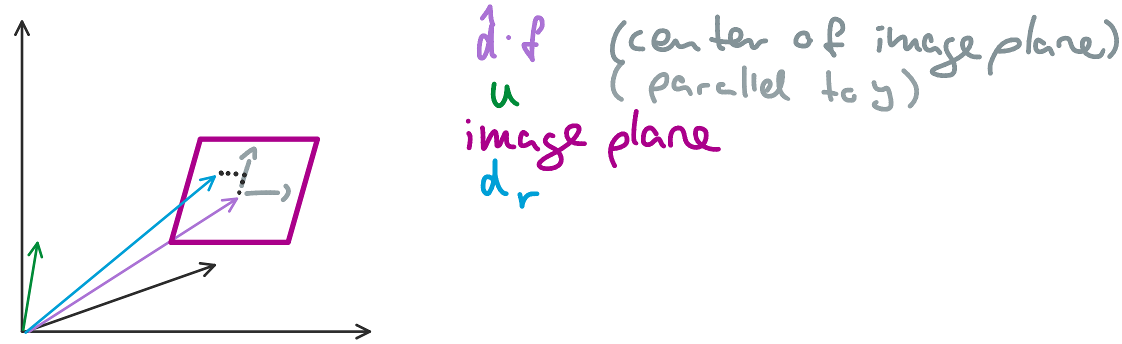
(We worked on most exercises together with Stephan Amann and Amelie Schäfer so solutions might be similar.)

① given: origin o, viewing direction of camera d,
up-vector u, focal length f,
image resolution resX × resY,
x, y coordinates of $d_r$

find: ray direction $d_r$ (without o)




We get to $d_r$ by calculating the center of the image plane and then adding the coordinates according to the x and y vectors of the plane (as well as considering the coordinate system used and the resolution).

The image plane center is calculated by going f length into d direction.
We use a normalized d since we are unsure if it already is (not given).

image plane center = $\hat{d} \cdot f$

The y vector in the image plane is equal in direction to u, thus we use $\hat{u}$ and resY/2 for needed length.
The x vector is calculated with the cross product of u and d.

To calculate the 2D plane in a 3D system we combine as follows while also subtracting 0,5 to get the center of the $d_r$ pixels, which gives $d_r$.

$$d_r = \hat{d} \cdot f + 2 \cdot \left(\frac{x}{resX} - 0{,}5\right) \cdot (\hat{u} \times \hat{d}) \cdot \left(\frac{resX}{2}\right)$$
$$+ 2 \cdot \left(\frac{y}{resY} - 0{,}5\right) \cdot \hat{u} \cdot \left(\frac{resY}{2}\right)$$

The last step is to normalize $d_r$.

$$\hat{d}_r = \frac{1}{|d_r|} d_r$$

② given $r(t) = o + td$, $o = (o_x, o_y, o_z)$, $d = (d_x, d_y, d_z)$ compute $t$

a) given $(p - a) \cdot n = 0$, $a = (a_x, a_y, a_z)$, $n(n_x, n_y, n_z)$

Taking the plane equation and substituting r as point on the plane gives:

$$((o + td) - a) \cdot n = 0$$

Solving for t

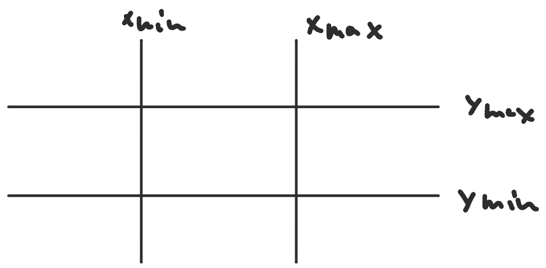
$$o \cdot n + td \cdot n - a \cdot n = 0$$

$$td = a - o$$

$$t = \frac{a - o}{d}$$

b) given $min(x_{min}, y_{min}, z_{min})$, $max = (x_{max}, y_{max}, z_{max})$

For each of the dimensions of the ray $(d_x, d_y, d_z)$ we first check whether the direction is positive or not to then assign them accordingly.



if ( $d_x$ >= 0) {

$$t_{x_{min}} = \frac{x_{min} - O_x}{d_x}$$

$$t_{min} = t_{x_{min}}$$

$$t_{x_{max}} = \frac{x_{max} - O_x}{d_x}$$

$$t_{max} = t_{x_{max}}$$

}

else {

$$t_{x_{min}} = \frac{x_{max} - O_x}{d_x}$$

$$t_{x_{max}} = \frac{x_{min} - O_x}{d_x}$$

}

if ( $d_y$ >= 0) {

$$t_{y_{min}} = \frac{y_{min} - O_y}{d_y}$$

$$t_{y_{max}} = \frac{y_{max} - O_y}{d_y}$$

}

else {

$$t_{y_{min}} = \frac{y_{max} - O_y}{d_y}$$

$$t_{y_{max}} = \frac{y_{min} - O_y}{d_y}$$

}

If the ray is out of the bounds of the box it misses:

if ( $t_{min} > t_{y_{max}}$ ) or ( $t_{y_{min}} > t_{max}$ )

return false

if ( $d_z$ >= 0) {

$$t_{z_{min}} = \frac{z_{min} - O_z}{d_z}$$

$$t_{z_{max}} = \frac{z_{max} - o_z}{d_z}$$

}

else {

$$t_{z_{min}} = \frac{z_{max} - o_z}{d_z}$$

$$t_{z_{max}} = \frac{z_{min} - o_z}{d_z}$$

}

If the ray is out of the bounds of the box it misses:

if( $(t_{min} > t_{z_{max}})$ or $(t_{z_{min}} > t_{max})$)

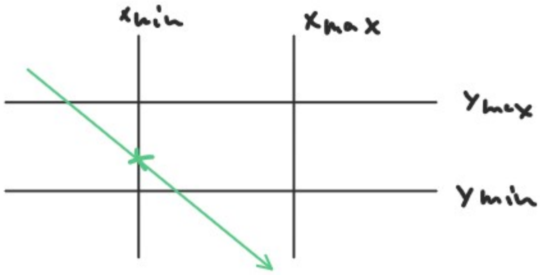
return false

If there is an intersection, we now check for the nearest hit point by comparin the calculated t's with each other as follows.

case 1: $t_{x_{min}} < t_{y_{min}}$ or $t_{x_{min}} < t_{z_{min}}$

intersection point at $r(t_{x_{min}}) = o + t_{x_{min}} d$

example in 2D




case 2: $t_{y_{min}} < t_{x_{min}}$ or $t_{y_{min}} < t_{z_{min}}$

intersection point at $r(t_{y_{min}}) = o + t_{y_{min}} d$

case 3: $t_{z_{min}} < t_{x_{min}}$ or $t_{z_{min}} < t_{y_{min}}$

intersection point at $r(t_{z_{min}}) = o + t_{z_{min}} d$

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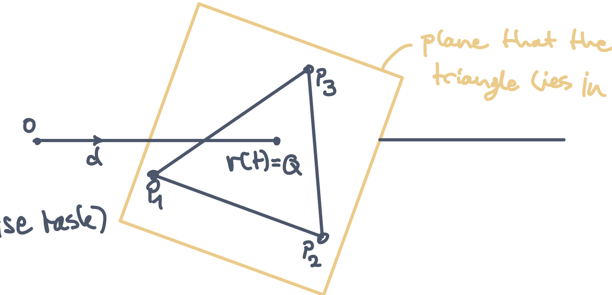
given: $r(t) = O + t\,d$,
$O = (O_x, O_y, O_z)$,
$d = (d_x, d_y, d_z)$

and we know the intersection is inside the triangle!
(given by excercise task)



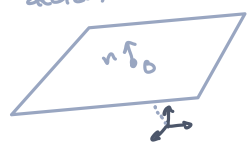

Ray r is cast from Origin O with Direction d, which intersects with the triangle made up with Vertices $P_1, P_2, P_3$ at point Q.

The triangle lies on a plane (square around triangle)

Goal: find parameter t for intersection point Q of ray with triangle

we know the equation for a given plane

sketch:



$$Ax + By + Cz + D = 0 \quad \rightarrow (x,y,z)^T = \text{Vector } v$$

for the coordinates/components of the normal n to the plane we can use A, B, C

$$n = (A, B, C)^T$$

while D portrays the distance from origin (0,0,0) to the plane
and x, y, z are coordinates of any point that lies in the plane

Therefore we can use one of the vertices $P_1$, $P_2$ or $P_3$ as v
(obviously $P_1, P_2$ or $P_3$ lie in plane)
Which gives us (choosing here $P_1$)

$$n \cdot P_1 + D = 0 \iff n \cdot P = D$$

Also lying in plane is our intersection Point $Q = r(t) = O + td$

$\Rightarrow$ to solve for t, we can substitute v with Q

$$Q = O + td, \quad Ax + By + Cz + D = 0$$

$$A \cdot Q_x + B \cdot Q_y + C \cdot Q_z + D = 0$$

$$\iff A \cdot (O_x + t d_x) + B \cdot (O_y + t d_y) + C \cdot (O_z + t d_z) + D = 0$$

$$\iff AO_x + BO_y + CO_z + At d_x + Bt d_y + Ct d_z + D = 0$$

$$\iff t(Ad_x + Bd_y + Cd_z) + AO_x + BO_y + CO_z + D = 0$$

$$\iff t(Ad_x + Bd_y + Cd_z) = -(AO_x + BO_y + CO_z + D)$$

$$\iff t = -\frac{(AO_x + BO_y + CO_z + D)}{Ad_x + Bd_y + Cd_z}$$

$$\iff t = -\frac{n \cdot o + D}{n \cdot d}$$

$\rightarrow$ used different approach in implementation due to difficulties with this one