Latent Variable Selection with Convex Optimization

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Abstract

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Main Body Sections (TBD)

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Derivations:
Likelihood, assuming normal distribution:

$$L(\theta;x) = p_\theta(X=x|\theta)$$

$$L(\theta;x) = \frac{1}{(2\pi)^{\frac{D}{2}}(det\Sigma)^{\frac{1}{2}}} exp(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu))$$

Log likelihood:

$$l(\theta;x) = logL(\theta;x)$$

$$-log((\frac{1}{2\pi})^{\frac{D}{2}}) - log(det\Sigma)^{\frac{1}{2}} - \frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\Big|_{\mu=\theta_\mu,\Sigma=\theta_\Sigma}$$

Optimize over θ:

$$\max_\theta \ logL(\theta;x)$$

$$\max_\theta \ -\frac{1}{2}log(det\Sigma) - \frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)$$

Optimal solution:

$$\operatorname*{argmax}_\theta \ -log(det\Sigma) - (x-\mu)^T\Sigma^{-1}(x-\mu)$$

$$let\ z = x - \mu$$

$$let\ \Sigma_s = zz^T$$

$$let\ K = \Sigma^{-1}$$

$$(x-\mu)^T\Sigma^{-1}(x-\mu) = tr(z^T\Sigma z) = tr(zz^T\Sigma^{-1}) = tr(\Sigma_s K)$$

$$\operatorname*{argmax}_\theta \ log(detK) - tr(\Sigma_s K)$$

For K positive definite, the above problem can be cast as a convex optimzation.

$$let\ K = S - L$$

$$let\ S = X_O\ given\ X_H$$

$$let\ L\ be\ marginalization\ over\ X_H$$

$$\max_{S,L} \ l(\theta = S-L;\Sigma_s) = \max_{S,L} logdet(S-L) - tr(\Sigma_s(S-L))$$

$$s.t.\ S - L \succ 0, L \succeq 0$$

Conclusion

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