

**Latent Variable Selection with Convex Optimization**

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Once the structure is recovered, a model can be trained using one of existing techniques such as loopy belief propagation, mean field, evidence lower bound, etc.

Citation test only [1].

**ABSTRACT**

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**MAIN BODY SECTIONS (TBD)**

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Derivations:

Likelihood, assuming normal distribution:

$$L(\theta; x) = p_\theta(X = x|\theta)$$

$$L(\theta; x) = \frac{1}{(2\pi)^{\frac{D}{2}} (\det \Sigma)^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x - \mu)^T \Sigma^{-1}(x - \mu)\right)$$

Log likelihood:

$$l(\theta; x) = \log L(\theta; x)$$

$$- \log\left(\left(\frac{1}{2\pi}\right)^{\frac{D}{2}}\right) - \log(\det \Sigma)^{\frac{1}{2}} - \frac{1}{2}(x - \mu)^T \Sigma^{-1}(x - \mu) \Big|_{\mu=\theta_\mu, \Sigma=\theta_\Sigma}$$

Optimize over  $\theta$ :

$$\max_{\theta} \log L(\theta; x)$$

$$\max_{\theta} -\frac{1}{2} \log(\det \Sigma) - \frac{1}{2}(x - \mu)^T \Sigma^{-1}(x - \mu)$$

Optimal solution:

$$\operatorname{argmax}_{\theta} -\log(\det \Sigma) - (x - \mu)^T \Sigma^{-1}(x - \mu)$$

$$\text{let } z = x - \mu$$

$$\text{let } \Sigma_s = zz^T$$

$$\text{let } K = \Sigma^{-1}$$

$$(x - \mu)^T \Sigma^{-1}(x - \mu) = \operatorname{tr}(z^T \Sigma z) = \operatorname{tr}(zz^T \Sigma^{-1}) = \operatorname{tr}(\Sigma_s K)$$

$$\operatorname{argmax}_{\theta} \log(\det K) - \operatorname{tr}(\Sigma_s K)$$

For  $K$  positive definite, the above problem can be cast as a convex optimization.

$$\text{let } K = S - L$$

$$\text{let } S = X_O \text{ given } X_H$$

$$\text{let } L \text{ be marginalization over } X_H$$

$$\max_{S, L} l(\theta = S - L; \Sigma_s) = \max_{S, L} \log \det(S - L) - \operatorname{tr}(\Sigma_s(S - L))$$

$$\text{s.t. } S - L \succ 0, L \succeq 0$$

## CONCLUSION

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## REFERENCES

- [1] V Chandrasekaran, P.A. Parrilo, and A.S. Willsky. “Latent Variable Graphical Model Selection via Convex Optimization”. In: *The Annals of Statistics*, Vol. 40, No. 4, 19351967 (2012).