Q: $J = \sum_{i=1}^{10} (y_i - ax_i + b))^2$

prof: $J = \sum_{i=1}^{10} (y_i - ax_i + b))^2$

(1) 對 a、b 作一階偏導數

$$\begin{cases} \partial J / \partial a = 2 \sum_{i=1}^{10} (y_i - b - ax_i)(-x_i) = 0 \\ \partial J / \partial b = 2 \sum_{i=1}^{10} (y_i - b - ax_i)(-1) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \partial J / \partial a = \sum_{i=1}^{10} (y_i - b - ax_i)(-x_i) = 0 \\ \partial J / \partial b = \sum_{i=1}^{10} (y_i - b - ax_i)(-1) = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \sum_{i=1}^{10} x_i y_i - b x_i - a x_i^2 = 0 \\ \sum_{i=1}^{10} y_i - b - a x_i = 0 \end{cases}$$

$$\Rightarrow \begin{cases} b \sum_{i=1}^{10} x_i + a \sum_{i=1}^{10} x_i^2 = \sum_{i=1}^{10} x_i y_i \\ 10b + a \sum_{i=1}^{10} x_i = \sum_{i=1}^{10} y_i \end{cases}$$

(2) 利用 crame rule 求解

① $$a = \frac{\begin{vmatrix} \sum_{i=1}^{10} x_i & \sum_{i=1}^{10} x_i y_i \\ 10 & \sum_{i=1}^{10} y_i \end{vmatrix}}{\begin{vmatrix} \sum_{i=1}^{10} x_i & \sum_{i=1}^{10} x_i^2 \\ 10 & \sum_{i=1}^{10} x_i \end{vmatrix}} = \frac{\sum_{i=1}^{10} x_i \sum_{i=1}^{10} y_i - 10 \sum_{i=1}^{10} x_i y_i}{\left(\sum_{i=1}^{10} x_i\right)^2 - 10 \sum_{i=1}^{10} x_i^2} = \frac{-\left(10 \sum_{i=1}^{10} x_i y_i - \sum_{i=1}^{10} x_i \sum_{i=1}^{10} y_i\right)}{-\left(10 \sum_{i=1}^{10} x_i^2 - \left(\sum_{i=1}^{10} x_i\right)^2\right)}$$

$$\xrightarrow{\text{同除}10} \frac{\sum_{i=1}^{10} x_i y_i - \left(\frac{\sum_{i=1}^{10} x_i}{10}\right)\left(\sum_{i=1}^{10} y_i\right)}{\sum_{i=1}^{10} x_i^2 - \left(\sum_{i=1}^{10} x_i\right)^2 / 10} \quad Q'$$

$\because \bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \Rightarrow n\bar{x} = \sum_{i=1}^{n} x_i$，$n\bar{y} = \sum_{i=1}^{n} y_i$ 反之亦然

$$\Longrightarrow \frac{\sum_{i=1}^{10} x_i y_i - 10 \bar{x}\bar{y}}{\sum_{i=1}^{10} x_i^2 - 10\bar{x}^2} = \frac{\sum_{i=1}^{10} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{10} (x_i - \bar{x})^2} = S_{xy} / S_{xx} \quad \#$$

② $$b = \frac{\begin{vmatrix} \sum_{i=1}^{10} x_i^2 & \sum_{i=1}^{10} x_i y_i \\ \sum_{i=1}^{10} x_i & \sum_{i=1}^{10} y_i \end{vmatrix}}{\begin{vmatrix} \sum_{i=1}^{10} x_i & \sum_{i=1}^{10} x_i^2 \\ 10 & \sum_{i=1}^{10} x_i \end{vmatrix}} = \frac{\sum_{i=1}^{10} x_i^2 \sum_{i=1}^{10} y_i - \sum_{i=1}^{10} x_i \sum_{i=1}^{10} x_i y_i}{\left(\sum_{i=1}^{10} x_i\right)^2 - 10 \sum_{i=1}^{10} x_i^2} =$$