Staff Planning Considering Switching with Multi-Objective and Stochastic Demand

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2021/6/25

Abstract

This is a simple and basic tutorial on how to adjust a staff planning model with ϵ-constraint method and 2-stage method when staffs may change their job position during working schedule.

Key words operation research, planning, stochastic programming, multi-objective decision, ϵ-constraint method, 2-stage method

1 Basic Model

We start from a model that only consider how to minimize cost. The decision here is to decide the amount or workers on each job position in each time period. The model assume there are 2 type of skills related to two kinds of job. To do a job, one must have the relative skill; some workers may have two skills, which allows them to switch between two jobs. All the demand must be satisfy by either full-time worker or outsourcing.

Here is the model we start from. Here we assume all the information is known (including the scenario of demand).

Set	Meaning
I	Schedules of when to work and when to leave
J	Set of jobs
T	Time period in a day, $T = 0, ..., 23$
S	Scenario of different demand.

Table 1: Set

Parameter	Meaning
A_{it}	Schedule i include period t or not (binary)
D_{sjt}	Demand of worker needed on job j at period t
E_j	Number of workers who have the skill for job j
L_{jt}	Number of outsourcing workers who have the skill for job j on period t
B	Number of workers who have both skills, $B < E_j$
C_t	Cost of hiring a worker to work at period t
F	Cost of letting a worker change jobs at the middle of a day
G_{jt}	Cost of satisfied an unit of demand by outsourcing
P_s	Probability of scenario s

Table 2: Parameter

Variable	Meaning
x_i	Number of worker work on schedule i (integer)
y_{sjt}	Number of worker work on job j on period t in scenario s (integer)
z_{sjt}	Difference of number of workers on job j at the start of period t in scenario s (integer)
l_{sjt}	Number of outsourcing on job j, period t in scenario s

Table 3: Decision Variables

Basic model

$$\min \quad \sum_{s\in S} P_s \left(\sum_{j\in J}\sum_{t\in T} y_{sjt}C_t + \sum_{t\in T}\sum_{j\in J} l_{sjt}G_{jt} + \sum_{t\in T}\sum_{j\in J} z_{sjt}F \right) \tag{1}$$

$$\text{s.t.} \quad \sum_{j\in J} y_{sjt} = \sum_{i\in I} x_i A_{it} \qquad \forall t\in T,\ s\in S \tag{2}$$

$$y_{sjt} + l_{sjt} \geq D_{sjt} \qquad \forall j\in J,\ t\in T,\ s\in S \tag{3}$$

$$\sum_{i\in I} x_i \leq \sum_{j\in J} E_j - B \tag{4}$$

$$y_{sjt} \leq E_j \qquad \forall j\in J,\ t\in T,\ s\in S \tag{5}$$

$$l_{sjt} \leq L_{jt} \qquad \forall j\in J,\ t\in T,\ s\in S \tag{6}$$

$$z_{sjt} = \mid y_{sjt} - y_{s,j,t-1} \mid \qquad \forall j\in J,\ t\in \{1,...23\},\ s\in S \tag{7}$$

$$x_i, y_{sjt}, z_{st}, r_{sjt} \geq 0 \qquad \forall i,j,t,s \tag{8}$$

2 ϵ-constraint Model

Now let's consider one more objective function – maximize the total extra manpower, r. By getting more manpower, the worker could be less pressured at work, which make adding the objective function reasonable.

Hence we get a new objective function

$$\max \quad \sum_{s\in S} P_s \left(\sum_{j\in J} \sum_{t\in T} r_{sjt} \right) \tag{9}$$

and also two new constraints to define r and $\overline{r}$

$$r_{sjt} = y_{sjt} + l_{sjt} - D_{sjt} \qquad \forall j \in J,\ t \in T,\ s \in S \tag{10}$$

$$\overline{r} = \sum_{s\in S} \sum_{j\in J} \sum_{t\in T} r_{sjt} \ / \ (S \times J \times T) \tag{11}$$

$$r_{sjt} \geq 0 \qquad \forall j, t, s \tag{12}$$

Here, $(S \times J \times T)$ is a constant equals to the number of variable r_{sjt}.

Since there are two objective function now, optimal solver can not solve it. To let the model become one objective, a naive way it using weighted-sum. But here we will introduce another method – ϵ-constraint method.

ϵ-constraint method combines multiple objective functions by calculate a series of solution and find the efficient frontier.

In a model of ϵ-constraint method, there is only one objective function while the other objective functions being changed to constraints. Each constraint should force an objective value of a particular objective function be lower/higher than a value ϵ. With different value of ϵ, we are able to get a series of models which comes up with a series of optimal solutions.

For the model, the only change is the objective function

$$\max \quad \sum_{s\in S} P_s \left(\sum_{j\in J} \sum_{t\in T} r_{sjt} \right) \tag{13}$$

is change into constraint

$$\text{s.t.} \quad \sum_{j\in J} \sum_{t\in T} r_{sjt} \geq \epsilon_s \quad \forall s \in S \tag{14}$$

Then how to decide the value of ϵ_s? For a two-objective model, you should do the following:

1. Solve the model with two objective function alone (consider only one a time), and get two optimal solutions.
2. Put each of the solution you get in step 1 into another objective function, and get the objective value. Now you should have 4 objective value:
   (a) Optimal objective value of objective function 1
   (b) Optimal objective value of objective function 2
   (c) Objective value of objective function 1, using solution of (b)
   (d) Objective value of objective function 2, using solution of (a)
3. Decide how many pieces you want to cut. Here we assume it is n.
4. For objective function 2, set $\epsilon_s 0$ as the lowest objective value get by objective function 2 (which is (d) in step 2) and set $\epsilon_s n$ as the highest objective value get by objective function 2 (which is (b) in step 2). [1]
5. Equally divided $\epsilon_s 0$ $\epsilon_s n$ into n part. Those $n+1$ values are the ϵ set. (Their value should all between $\epsilon_s 0$ $\epsilon_s n$.)

By using the ϵ into model, we can get a series of solutions. Check the feasibility of all solutions and draw the plot to find efficiency frontier.

3 2-stage Model

Now let's go ahead to deal with stochastic demand. Here we are using 2-stage method, which means in Stage 1 we do not know the demand but need to do some decision, and in Stage 2 we already know it and can make decision base on what we have done in Stage 1.

The Stage 1 model here is for decide how many worker to hire and when should them show up.

[1] If your second objective function is to minimize, exchange the lowest and highest value.

Stage 1.

π_s^* is known from stage 2.

$$\min \quad \sum_{i\in I}\sum_{t\in T} x_i A_{it} C_t + \mathbb{E}(\pi_s \mid s \in S) \tag{15}$$

$$\text{s.t.} \quad x_i \geq 0 \qquad \forall i \in I \tag{16}$$

Here, the expected value is $\mathbb{E}(\pi_s \mid s \in S) = \sum_{s\in S} P_s \pi_s$.

Stage 2 model is to decide how many workers should work on which job position in which period, and how many outsourcing to use.

Stage 2.

Assume that x_i is set and the scenario is known, the decision in this stage is to decide y_{sjt}.

$$\min \quad \pi_s = \sum_{t\in T}\sum_{j\in J} z_{sjt} F + \sum_{t\in T}\sum_{j\in J} l_{sjt} G_{jt} \tag{17}$$

$$\text{s.t.} \quad \sum_{j\in J}\sum_{t\in T} r_{sjt} \geq \epsilon_1 \tag{18}$$

$$\sum_{j\in J} y_{sjt} = \sum_{i\in I} x_i A_{it} \qquad \forall t \in T \tag{19}$$

$$y_{sjt} + l_{sjt} \geq D_{sjt} \qquad \forall j \in J,\ t \in T \tag{20}$$

$$y_{sjt} \leq E_j \qquad \forall j \in J,\ t \in T \tag{21}$$

$$l_{sjt} \leq L_{jt} \qquad \forall j \in J,\ t \in T \tag{22}$$

$$r_{sjt} = y_{sjt} + l_{sjt} - D_{sjt} \qquad \forall j \in J,\ t \in T \tag{23}$$

$$z_{sjt} =\mid y_{sjt} - y_{s,j,t-1} \mid \qquad \forall j \in J,\ t \in \{1, ...23\} \tag{24}$$

$$x_i, y_{sjt}, z_{st}, r_{sjt}, l_{sjt} \geq 0 \qquad \forall i, j, t \tag{25}$$

We may replace Constraint (24) by Constraints (26) and (27) to make this constraint become linear.

$$z_{sjt} \geq y_{sjt} - y_{s,j,t-1} \qquad \forall j \in J,\ t \in \{1, ...23\} \tag{26}$$

$$z_{sjt} \geq y_{s,j,t-1} - y_{sjt} \qquad \forall j \in J,\ t \in \{1, ..., 23\} \tag{27}$$

That's all in this tutorial, thanks for reading!