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$$E(\hat{\theta}_1) = E\left(\frac{\sum (X_i - \bar{X})^2}{n}\right)$$

$$= \frac{1}{n} E(\sum X_i^2 - n\bar{X}^2)$$

$$= \frac{1}{n} (n\sigma^2 + \cancel{n\mu^2} - \sigma^2 - \cancel{n\mu^2}) = \frac{n-1}{n} \sigma^2 \rightarrow \text{偏誤估計量}$$

$$E(\hat{\theta}_2) = E\left(\frac{\sum (X_i - \bar{X})^2}{n-1}\right) = \frac{1}{n-1} E(\sum X_i^2 - n\bar{X}^2)$$

$$= \frac{1}{n-1} (n\sigma^2 + \cancel{n\mu^2} - \sigma^2 - \cancel{n\mu^2}) = \sigma^2 \rightarrow \text{不偏估計量}$$

$$E(X_i^2) = \sigma^2 + \mu^2$$

$$E(\bar{X}^2) = \frac{\sigma^2}{n} + \mu^2$$