Computational Neuroscience MSc coursework

Preliminaries

This coursework will consider spike-timing-dependent plasticity (STDP). Throughout, you will simulate a single postsynaptic neuron, receiving input from one or more populations of presynaptic neurons. The strengths of the synapses in the model will evolve according to STDP rules.

Your coursework submission should be ≤ 3 pages of A4 total (including figures), with font size no less than 10 pt, single line spacing, and margins no less than 2 cm. Submit the report in PDF format to SAFE. All plots should have clear x- and y-axis labels, and units. For each missing label or legend, 1% of your grade will be subtracted.

To help you decide which results to include in your report, I will encode meaning in my text colour and formatting. If a sentence is coloured magenta, I expect to see a written response. *For sentences formatted with italics I expect to see a plot.* These sentences will indicate the ideal set of minimal items to include in your report, but please also write or plot additional information as appropriate to support your answers to the questions.

Setting up the basic model

Write a simulation (using e.g. Python or MATLAB) of a single leaky integrate-and-fire neuron with resting potential $V_{rest} = -65$ mV, spike threshold $V_{th} = -50$ mV, reset voltage $V_{reset} = V_{rest} = -65$ mV, a passive membrane leak conductance that gives an input resistance of $R_m = 100$ MΩ, and membrane capacitance $C = 0.1$ nF (check yourself that the membrane time constant works out at $\tau_m = 10$ ms; no need to show evidence in your report). Add $N = 40$ incoming synapses (just model the synaptic responses to spike trains, do not explictly simulate the somatic voltages of these presynaptic neurons). Model all synapses as a conductance-based, following a single-exponential timecourse with decay time constant $\tau_{syn} = 2$ ms, and the initial peak conductances chosen from the uniform distribution $0 \leq \bar{g}_{syn} \leq 2$ nanoSiemens. The simulation should run for at least 200 or 300 seconds biological time, with a simulation timestep $\delta t = 1$ milliseconds. Use the forward Euler method to solve the system of ODEs.

Simulate each input spike train as an independent homogeneous Poisson process with the same average firing rate $\langle r \rangle$. A homogeneous Poisson process implies that average firing rate $\langle r \rangle$ is fixed in time, but that the spikes themselves arrive randomly. Initially set $\langle r \rangle = 10$ Hz. There are multiple ways to simulate Poisson spike trains. A simple (but inefficient) method is to draw a random number on the unit interval for each synapse at every timestep, then if that number is less than $\langle r \rangle \delta t$, assume a spike has occured at that synapse. This method is valid as long as $\langle r \rangle \delta t \ll 1$.

Simulate STDP using the following basic rule. First, let's denote a presynaptic spike time as t_{pre}, postsynaptic spike time as t_{post}, and the difference in pre-post spike times as $\Delta t = t_{post} - t_{pre}$ (so that pre-before-post timings are positive, while post-before-pre timings are negative). Now the rule for changing the synaptic condutance $\bar{g}_{syn} \to \bar{g}_{syn} + f(\Delta t)$, with

$$f(\Delta t) = \begin{cases} A_+ \exp(-|\Delta t|/\tau_+) & \text{if} \quad \Delta t > 0 \\ -A_- \exp(-|\Delta t|/\tau_-) & \text{if} \quad \Delta t \leq 0 \end{cases}.$$

Initially set the four STDP model parameters as: $A_+ = 0.1$ nS , $A_- = 0.12$ nS, $\tau_+ = 20$ ms, $\tau_- = 20$ ms. The synaptic strength updates should follow the 'nearest neighbour' principle: only include the most recent pre and post spikes in your weight update calculations. Finally, impose hard limits on the synaptic strengths during the entire simulations: if an update would make a synaptic weight negative, set it to zero; if an update would make the synaptic weight greater than 2 nS, cap it at exactly 2 nS instead.

Include a flag in your code that lets you switch the simulation mode from having STDP 'on' or 'off'. STDP 'on' mode means that every spike (pre and post) triggers changes in the maximum conductances of the activated synapses according to the above rule, while STDP 'off' mode means that synaptic strengths are fixed and remain stuck at the same values throughout the simulation. One easy way to turn off plasticity is to set the A_+ and A_- parameters equal to zero.

Finally, because these simulations are stochastic, any numbers that you measure will likely change a little bit each time you run the simulation. Hence any values you include in your report should come from averages over multiple realisations (repeats) of the simulations.

Questions

1. (25% of grade) Initially all the synapses have the same strength, at the maximal conductance (2 nS). However after the simulation has run for a while, if STDP is switched on the synapses will change their strengths. Simulate the case where input firing rates $\langle r \rangle = 15$ Hz. What qualitative shape does the synaptic strength distribution converge towards after 200 seconds of simulation time? *Plot a histogram of the steady-state synaptic weights after one run of the simulation. Also plot the average firing rate of the postsynaptic neuron as a function of time across the entire 200 or 300 second simulation (taking 1-second time bins).* Now repeat the same simulation with STDP turned 'off', but with all synaptic strengths set

equal to the mean value of the synaptic strengths you measured in the STDP 'on' simulations. Report the steady-state firing rate (as averaged over the last 10 seconds of the simulation) for both the STDP 'on' and 'off' simulation modes. To get reliable estimates for the firing rate, you may need to average over multiple realisations of the simulations. [Plotting the strength of each synapse as a function of time during the simulation can help you figure out what is going on.]

2. (25% of grade) Vary the input firing rates from 10 Hz up to 20 Hz, both for a set of simulations with STDP switched on and for a set of simulations with STDP switched off. How does the steady-state output firing rate depend on the input firing rates in both cases? *Plot the mean output firing rate as a function of the input firing rates. Plot the steady-state synaptic strength distribution for* $\langle r \rangle = 10$ *Hz and* $\langle r \rangle = 20$ *Hz.* Can you give an explanation for what is happening?

3. (20% of grade) Up to now the input spike trains have been independent, and hence uncorrelated with each other. Now we will add correlations. Do this by making the firing rates of the input spike trains vary as a sinusoidal function of time:

$$\langle r \rangle(t) = \langle r \rangle_0 + B \sin(2\pi f t)$$

where $\langle r \rangle_0$ is the average firing rate. Now since the input firing rates are co-varying in time, the input spike trains will be temporally correlated. The degree of correlation depends on the parameter B, while the temporal extent of correlations depends on f. Fix $f = 10$ Hz, and $\langle r \rangle_0 = 15$ Hz. Vary the degree of correlations by varying B from 0 to 15 Hz. How does the degree of correlation affect the steady-state synaptic weights? *Plot the mean and standard deviation of the steady-state synaptic strengths as a function of* B*. Plot example histograms of the steady-state synaptic strengths for* $B = 0$ *and* $B = 15$ *Hz.*

4. (15% of grade) Measure the temporal cross correlations functions $\rho_i(\Delta t)$ between a few example presynaptic neurons and the postsynaptic neuron (where the subscript i indexes the presynaptic neuron). What does it look like? *Plot the cross-correlation function (cross-correlogram) both for the STDP 'off' and 'on' cases.* Report the time integral of the product of the crosscorrelation with the plasticity function $f(\Delta t)$, both for the STDP 'on' and 'off' cases. Can you explain what is happening?

5. (15% of grade) Now split the 40 input synapses into two equal sized groups ($N_1 = 20$, $N_2 = 20$). Make the first input group's spike trains correlated as before ($B_1 > 0$), but keep the second input group's synapses uncorrelated ($B_2 = 0$). Vary the degree of correlations by varying the sinusoid amplitude parameter B. *Plot the mean steady–state synaptic strength of each group as a function of* B. What do the synaptic strength distributions of the correlated vs uncorrelated group look like? *Plot example histograms of steady-state synaptic strengths for each group of synapses from simulations where* $\langle r \rangle_0 = 15$ *Hz* for both groups of input synapses. How do these simulation results compare with the results from question 3?