

# Calculus

## HW1: Expressing Functions and Exploring Mathematical Models

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# Tasks

- Basic concepts (33%)
- Alternative expression for functions (20%)
- Mathematical model for physics (27%)
- Report (20%)

# Basic concepts

## Q1 (2%)

Is it true that  $f = g$ ? Please explain why.

①  $f(x) = x + \sqrt{2-x}$  and  $g(u) = u + \sqrt{2-u}$

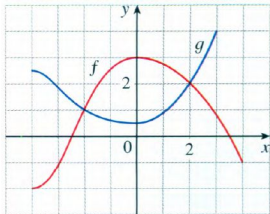
②  $f(x) = \frac{x^2-x}{x-1}$  and  $g(x) = x$

## Grading criteria

- Whether the true/false answers are correct
- Level of justification

## Q2 (8%)

The graphs of  $f$  and  $g$  are given.



- 1 State the values of  $f(-4)$  and  $g(3)$ .
- 2 Which is larger,  $f(-3)$  or  $g(-3)$ ?
- 3 For what values of  $x$  is  $f(x) = g(x)$ ?
- 4 On what interval(s) is  $f(x) \leq g(x)$ ?
- 5 State the solution(s) of the equation  $f(x) = -1$ .
- 6 On what interval(s) is  $g$  decreasing?
- 7 State the domain and range of  $f$ .
- 8 State the domain and range of  $g$ .

## Grading criteria

- Whether the answers are correct

### Q3 (2%)

Determine whether the equation, table, or graph defines  $y$  as a function of  $x$ .

1  $3x^2 - 2y = 5$

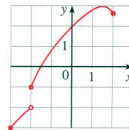
2  $x^2 + (y - 3)^2 = 5$

$x$ Height (cm)	$y$ Shoe size
180	12
150	8
150	7
160	9
175	10

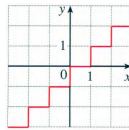
(a)

$x$ Year	$y$ Tuition cost (\$)
2016	10,900
2017	11,000
2018	11,200
2019	11,200
2020	11,300

(b)



(a)



(b)

### Grading Criteria

- Whether the answers are correct.

#### Q4 (8%)

Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential function, or logarithmic function. Explain why.

1  $f(x) = x^3 + 3x^2$

2  $r(t) = t^{\sqrt{3}}$

3  $v(t) = 8^t$

4  $g(u) = \log_{10} u$

5  $f(t) = \frac{3t^2+2}{t}$

6  $y = \frac{1}{x^2}$

7  $y = \frac{x}{x^2+1}$

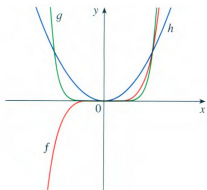
8  $g(t) = \cos(t)^2 - \sin(t)$

#### Grading criteria

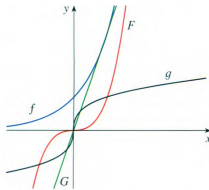
- Whether the answers are correct
- Level of justification

### Q5 (4%)

Match each equation with its graph. Explain your choices. (Do not explain by comparing through direct drawing of graphs using a computer or manually.)



- ①  $y = x^2$
- ②  $y = x^5$
- ③  $y = x^8$



- ①  $y = 3x$
- ②  $y = 3^x$
- ③  $y = x^3$
- ④  $y = \sqrt[3]{x}$

### Grading criteria

- Whether the answers are correct
- Level of justification

### Q6 (3%)

What do all members of the family of linear functions  $f(x) = 1 + m(x + 3)$  have in common? Sketch several members of the family.

### Q7 (3%)

Sketch several members of the family of polynomials  $P(x) = x^3 - cx^2$ . How does the graph change when  $c$  changes?

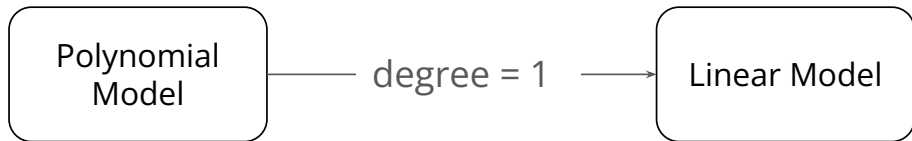
### Grading criteria

- Rationality
- Level of detail

### Q8 (3%)

Draw a **single** mind map to illustrate the relation between mentioned mathematical models (linear, polynomials, power, rational, algebraic, trigonometric, exponential, logarithmic)

(Part of the mind map may be like follows)



### Grading Criteria

- The completeness of course coverage
- The clarity of the mind-map and its associated description

# Alternative expression for functions

## Q1 (3%)

You place a frozen pie in an oven and bake it for an hour. Then you take it out and let it cool. Describe how the temperature of the pie changes as time passes. Then sketch a rough graph of the temperature of the pie as a function of time.

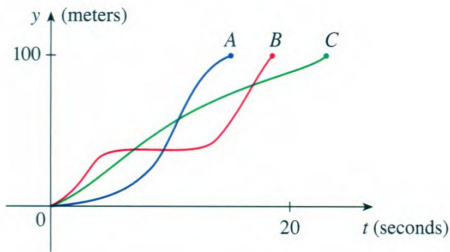
- 1 **Verbally:** Describe how the temperature of the pie changes as time passes.
- 2 **Visually:** Sketch a rough graph of the temperature of the pie as a function of time.

## Grading criteria

- Rationality
- Level of detail in description
- Consistency between the graph and the description

## Q2 (3%)

Three runners compete in a 100-meter race. The graph depicts the distance run as a function of time for each runner.



- 1 Describe in words what the graph tells you about this race.
- 2 Who won the race? Did each runner finish the race?
- 3 What are the ranges of the three functions?

## Grading criteria

- Rationality
- Level of detail in description
- Consistency between the graph and the description

### Q3 (3%)

Researchers measured the blood alcohol concentration (BAC) of eight adult male subjects after rapid consumption of 30 mL of ethanol (corresponding to two standard alcoholic drinks). The table shows the data they obtained by averaging the BAC (in  $g/dL$ ) of the eight men.

$t$ (hours)	BAC	$t$ (hours)	BAC
0	0	1.75	0.022
0.2	0.025	2.0	0.018
0.5	0.041	2.25	0.015
0.75	0.040	2.5	0.012
1.0	0.033	3.0	0.007
1.25	0.029	3.5	0.003
1.5	0.024	4.0	0.001

- 1 Visually:** Use the readings to sketch a graph of the BAC as a function of  $t$ .
- 2 Verbally:** Use your graph to describe how the effect of alcohol varied with time.

### Grading criteria

- Preciseness of the graph from the table.
- Level of detail in description
- Consistency between the graph and the description

#### Q4 (3%)

Express the function  $h(x) = \sqrt{4 - x^2}$ .

- 1 Find the domain and the range.
- 2 Sketch the graph.
- 3 Describe the graph.

#### Grading criteria

- Whether the domain and range are correct
- Preciseness of the graph from the formula
- Level of detail in description
- Consistency between the graph and the description

### Q5 (8%)

A Norman window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 10 m, express the area  $A$  of the window as a function of the width  $x$  of the window.



- ① The formula.
- ② A table.
- ③ The graph.
- ④ A description.

### Grading criteria

- Whether the formula is correct
- Level of importance of the data in the table
- Preciseness of the graph
- Level of detail in the description

### Q6 (3%)

Describe the differences in understanding a function from different forms of expression (table, description, formula, and graph).

- 1 Which transform is more difficult for you, and describe the challenges.

### Grading criteria

- Level of detail and depth in your description

# Mathematical model for physics

## Q1 (7%)

How to install python, numpy, and matplotlib in your computer?

- 1 List the steps.
- 2 What are the challenges or issues?
- 3 How do you overcome the challenges.

## Grading criteria

- Level of detail
- Clarity

## Q2 (20%)

The table shows the mean (average) distances  $d$  of the planets from the sun (taking the unit of measurement to be the distance from the earth to the sun) and their periods  $T$  (time of revolution in years).

Planet	$d$	$T$
Mercury	0.387	0.241
Venus	0.723	0.615
Earth	1.000	1.000
Mars	1.523	1.881
Jupiter	5.203	11.861
Saturn	9.541	29.457
Uranus	19.190	84.008
Neptune	30.086	164.784

Implement **kepler\_law.py** and **test\_kepler\_law.py** to answer:

- 1 Fit a power model to the data.
- 2 Does the model corroborate *Kepler's Third Law*:

## Kepler's Third Law

The square of the period of revolution of a planet is proportional to the cube of its mean distance from the sun.

(Continuing on Q2: tracing on **test\_kepler\_law.py**)

```
import numpy as np
import matplotlib.pyplot as plt
from kepler_law import *
```

(See following slides...)

```
def main():
    # Example data: [Mercury, Venus, Earth, Mars, Jupiter]
    # TODO: Replace these data with real data in the lecture slide
    distances = [0.39, 0.72, 1.00, 1.52, 5.20] # In astronomical units
                (AU)
    periods =    [0.39, 0.72, 1.00, 1.52, 5.20] # In Earth years

    (See the next slide...)

if __name__ == "__main__":
    main()
```

- Import 3rd party libraries and our own **kepler\_law.py**
- TODO: Fill the real data.

## Grading Criteria

- Correctness.

(Continuing: tracing on **test.kepler.law.py**)

```
# Fit power model
a, b = period_vs_distance_fit(distances, periods)
print(f"Fitted power model: T = {a:.4f} * d^{b:.4f}")

# Draw plot and save image (TODO: Report the figure)
draw_period_vs_distance(distances, periods, a, b)
print("Plot saved as 'period-vs-distance.png'")

# Calculate MSE
mse = calculate_model_mse(distances, periods, a, b)
print(f"Mean Squared Error (MSE): {mse:.6f}")

# Test Kepler's Law (TODO: Report the errors)
# An example:
diff = kepler_law_test(period1=1.00, distance1=1.00, period2=1.88,
                       distance2=1.88)
print(f"Kepler Law Ratio Difference (Earth vs XXXX): {diff:.6f}")
```

- The main workflow.
- TODO: Report the figure.
- TODO: Use **kepler\_law\_test** to explain whether the model corroborate to Kepler's Law

## Grading Criteria

- Experiment richness.
- Rationality of the description.

```
def draw_period_vs_distance(distances, periods, a, b):  
    # Draw original data and fitted curve, save as image file  
    # Scatter plot of original data  
    plt.scatter(distances, periods, color='blue', label='Observed Data'  
                )  
  
    # Generate smooth curve using fitted model  
    d_range = np.linspace(min(distances), max(distances), 100)  
    T_fit = a * d_range**b  
  
    # Plot fitted curve  
    plt.plot(d_range, T_fit, color='red', label=f'Fitted Model: T = {a  
        :.2f} * d^{b:.2f}')
```

plt.xlabel('Distance (AU)')

plt.ylabel('Period (years)')

plt.title('Period vs. Distance')

plt.legend()

plt.grid(True)

- Draw data points and the fitted curve.
- BONUS: Explain the meaning of each code line.

## Grading Criteria

- Level of detail in the description

(Continuing: tracing on **test\_kepler.law.py**)

```
def calculate_model_mse(distances, periods, a, b):  
    mse = 0  
    # TODO: Calculate the mean squared error between model and actual  
    data  
    return mse  
  
def kepler_law_test(period1, distance1, period2, distance2):  
    diff = 0  
    # TODO: Design and implement an error to compare (period1,  
    distance1) and (period2, distance2)  
    # according to the Kepler's Third Law (period^2 is proportional to  
    distance^3)  
    return diff
```

- Evaluation tool functions.
- TODO: Implement the MSE and the error for validating the Kepler's Law.

### Grading Criteria

- Correctness of implementation
- Rationality of the Kepler's Law error design

(Continuing: tracing on **kepler\_law.py**)

```
import numpy as np

def period_vs_distance_fit(distances, periods):
    a = 1.0
    b = 1.0
    # TODO: Fit the power function  $T = a * d^b$ 
    # Hints:
    # 1. Apply logarithm to convert to linear model:  $\log(T) = \log(a) + b * \log(d)$ 
    # 2. Use numpy.log and numpy.polyfit (with degree 1)

    return a, b # Return parameters of the power model
```

- TODO: Fit data with power model formulation.

## Grading Criteria

- Correctness

# Report

## Content:

- Answers to all of the above task questions (including both required and bonus questions) (3%)
- A description of how your team divided and collaborated on the work (10%)
- A record of working time and the tasks completed (3%)
- Reflections on what you learned (4%)

## Grading Criteria

- Clarity of descriptions and figures
- Appropriateness of formatting/layout
- Consistency of writing style
- Contribution of each member

Write members' names on the report. If one's name is not on the report, it will be regarded that this one has no contribution.

## Division and Collaboration

Explain your roles on the work:

- **Implementer:** The one who answer a question or write a piece of program.
  - **Reviewer:** The one who examine an answer or inspect/test a piece of program.
- 
- The implementer and reviewer cannot be the same person.
  - The roles of implementer or reviewer for different problems or code segments can be taken by different ones by exchanging.
  - Each team member must describe their key responsibilities for the task(s), any challenges encountered, any unresolved issues.
  - Meeting Records (online or in-person):
    - Clarification of assignment requirements
    - Progress planning
    - Consolidation of questions
    - Discussion of role assignments
  - Bonus: an implementer overcomes weakness with certain task with help of reviewers.

## Grading Criteria

- Completeness in addressing the assignment requirements
- Clarity in each role's description
- Completeness and clarity of the meeting records
- Reasonableness of the progress planning

## Working Time

Provide a table to show each member's working time on the associated working items.

(A proper format could be as follows)

Member name 1	Working hours 1	working item 1
Member name 2	Working hours 2	working item 2

- Bonus: analyze why the working hours is high or low by self.

## Grading Criteria

- Clarity
- Level of detail
- Completeness
- Reasonableness

## Reflection on what you learned

Retrospective shared from each member.

- What knowledges of this topics are gained
- Personal learning or working style
- Group learning or working style
- Personal or group challenges for this project

## Grading Criteria

- Level of detail
- Clarity
- Reasonableness

# Submission Guidelines

- Put all the codes into a folder *codes*.
- Named the report with *report.pdf*
- Put **codes** and *report.pdf* into a folder **hw1**.
- Compress **hw1** and specify the file format as hw1.zip.
- Upload hw1.zip onto Digital Platform 3.
- Deadline: 12:00 a.m. on October 6th.

Unsatisfied submission will cause 5 point subtraction on the HW score.

# The End

Questions? Comments?

# Origin of The Latex Template

The latex template is downloaded from:  
<https://www.LaTeXTemplates.com>

The Author: Vel (vel@latextemplates.com)