A Fair and Dynamic Incentive Mechanism for Federated Learning in Internet of Things

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I. Complexity Analysis

We analyze the computational and communication complexity of the proposed FedFD framework by decomposing each component.

1) *Complexity of AW-DPP*. For each client i, computing the non-IID degree L_i in Eq. (2) requires evaluating the KL divergence over Y label categories, resulting in a complexity of $O(Y)$. The dynamic attribute m_i^t in Eq. (3) and Eq. (4) is obtained using an exponential forgetting function over its historical records $\{m_i^0, \ldots, m_i^t\}$, which incurs $O(t)$ cost in the t-th round. Updating the weights w_j^t according to Eq. (13) - Eq. (16) involves only constant-time operations. The exponential mechanism used to perturb the bid b_i in Eq. (9)-Eq. (11), over a discretized candidate bid set of size M, adds another $O(M)$ computational cost. Therefore, the per-client complexity of AW-DPP is $O(Y+t+M)$. With N clients and participation rate q, the total per-round complexity becomes $O\big(qN(K+M+t)\big)$.

2) *Complexity of FS-AQB* . In each round, computing the UCB-based reward $\tilde{r}_i^t$ in Eq. (22), updating the virtual queue Q_i^t in Eq. (24) and Eq. (25) require $O(N)$ time. Selecting the winning set $\mathcal{W}^t$ according to Eq. (25) requires sorting all clients, which dominates the selection cost with $O(N \log N)$.

3) *Complexity of local training and dual privacy protection*. For each selected client $i \in \mathcal{W}^t$, local training over dataset D_i with E epochs incurs a complexity of $O(ED_i)$. Adding Gaussian noise to model parameters in Eq. (**??**), where the parameter dimension is d, requires $O(d)$ operations. Thus, the total computation across the selected clients is $O(\sum_{i\in\mathcal{W}^t}(ED_i+d))$.

4) *Communication complexity*. In each round, the server broadcasts the global model θ^t of size V to the K selected clients and receives their updated local models. The communication overhead is therefore $O(KV)$.

5) *Overall complexity*. Combining the above components, the total per-round computational complexity of FedFD is $O(qN(Y+M+t) + N\log N + \sum_{i\in\mathcal{W}^t}(ED_i+d))$, while the per-round communication complexity is $O(KV)$. The dominant computational components arise from the sorting cost $O(N\log N)$ in FS-AQB and the local update cost $O(ED_i)$ from selected clients, indicating that FedFD achieves scalability while supporting dual privacy protection and fair client selection.

II. Theoretical Analysis

Theorem 1: The incentive mechanism FedFD satisfies γ-incentive compatibility (truthfulness).

Proof: Before proving the γ-truthfulness of FedFD, we first demonstrate the truthfulness of our proposed methods without bid perturbation. In this case, Myerson's theorem [1] shows that FedFD is stimulus compatible when the following conditions are met. 1) The problem of determining the winner in the device selection process is monotonous; 2) In each round, the payment problem is calculated based on the critical value.

For monotonicity problems, assume that device i with bid b_i^t in the t-th round and device i wins in this round. Since we determine the winner based on a greedy selection criterion $\frac{\widetilde{r}_i^{t-1}S_i^t}{b_i^t} - \eta Q_i^t$, even if device i has a lower bid $\widetilde{b}_i^t$ in round t, device i can still win in this round.

For the critical payment problem, we use $\mathcal{W}_{-i}^t$ to represent the winner set when removing client i with b_i^t. Due to the fact that we sorted the values of w in descending order during the selection process, *i.e.*, for all devices, $\frac{\widetilde{r}_1^{t-1}S_1^t}{b_1^t} \geq \cdots \geq \frac{\widetilde{r}_K^{t-1}S_K^t}{b_K^t} \geq \frac{\widetilde{r}_{K+1}^{t-1}S_{K+1}^t}{b_{K+1}^t} \cdots \geq \frac{\widetilde{r}_N^{t-1}S_N^t}{b_N^t}$. we replaced device i with the $K+1$-th device and the bid is b_{K+1}^t. For each winner, the candidate bid is the same. We give the critical payment for device i as $p_i^t = \frac{\widetilde{r}_i^{t-1}S_i^t}{\widetilde{r}_{K+1}^{t-1}S_{K+1}^t} \cdot b_{K+1}^t$, and if $b_i^t \leq p_i^t$, we can obtain $\frac{r_i^t S_i^t}{b_i^t} \geq \frac{r_{K+1}^t S_{K+1}^t}{b_{K+1}^t}$. This means that device i must win before device $K+1$. If $b_i^t > p_i^t$, we can obtain $\frac{r_i^t S_i^t}{b_i^t} < \frac{r_{K+1}^t S_{K+1}^t}{b_{K+1}^t}$. This means that the $K+1$-th device needs to win before the i-th device. Therefore, $p_i^t = \frac{\widetilde{r}_i^{t-1}S_i^t}{\widetilde{r}_{K+1}^{t-1}S_{K+1}^t} \cdot b_{K+1}^t$ is the key payment in this process. Based on the monotonicity and critical payment in Myerson's theorem, we can prove that FedFD is incentive compatible in the absence of bid perturbations.

Next, we demonstrate the γ-truthfulness of FedFD when bids are perturbed. Since we have already verified the truthfulness of FedFD under conditions without bid interference, we fix the device attribute $\mathcal{X}_i$ and evaluate the mechanism under different bid fluctuations (bid interference violates monotonicity). For any device, we use b_i^t and $\hat{b}_i^t$ to represent the truthful and untruthful bid of the device, and their corresponding perturbation bids are ${b'}_i^t$ and $\hat{b'}_i^t$, respectively. After the bid of the device is disturbed, the expected bids for truthful and untruthful bids are $\mathbb{E}[b_i^t|{b'}_i^t]$ and $\mathbb{E}[\hat{b}_i^t|\hat{b'}_i^t]$, respectively. The further payment we have set in the main paper is

$$
{p'}_i^t = \hat{p}_i^t + b_{max} - b_{min} \tag{1}
$$

According to Eq. (1), their corresponding utilities are $\mathbb{E}[u_i^t(b_i^t)] = \mathbb{E}[{p'}_i^t|{b'}_i^t] + b_{max} - b_{min} - \mathbb{E}[b_i^t|{b'}_i^t]$ and

$\mathbb{E}[u_i^t(\hat{b}_i^t)] = \mathbb{E}[\hat{p'_i}^t|\hat{b'_i}^t] + b_{max} - b_{min} - \mathbb{E}[\hat{b}_i^t|\hat{b'_i}^t]$, respectively. When the bidding of the device is fixed, auctioning with perturbed truthful bid and untruthful bid may result in the following four bidding outcomes:

- (b'_{Win}, $\hat{b}'_{Win}$): During the auction process, device i wins with both perturbed truthful and untruthful bids. In this case, we calculate the expected critical payment for device i using the same expected bid and Eq. (1), so we can obtain $\mathbb{E}[b_i^t|{b'_i}^t] = \mathbb{E}[\hat{b}_i^t|\hat{b'_i}^t]$.
- (b'_{Lose}, $\hat{b}'_{Lose}$): During the auction process, device i loses with both perturbed truthful and untruthful bids. Thus $\mathbb{E}[b_i^t|{b'_i}^t] = \mathbb{E}[\hat{b}_i^t|\hat{b'_i}^t] = 0$.
- (b'_{Win}, $\hat{b}'_{Lose}$): During the auction process, device i wins with a perturbed truthful bid, but loses with a perturbed untruthful bid. In this case, $\mathbb{E}[b_i^t|{b'_i}^t] \geq \mathbb{E}[\hat{b}_i^t|\hat{b'_i}^t] = 0$
- (b'_{Lose}, $\hat{b}'_{Win}$): During the auction process, device i loses with a perturbed truthful bid, but wins with a perturbed untruthful bid. In this case, we can obtain $\mathbb{E}[u_i^t(\hat{b}_i^t)] = 0$. Since the $\mathcal{X}_i$ of all devices is the same, the critical payment for the winner who makes untruthful bid is the same as the expected bid of new winner $g^t \in \mathcal{W}^t$, *i.e.*,$\mathbb{E}[{p'_i}^t|{b'_i}^t] = \mathbb{E}[b_g^t|{b'_g}^t]$. In addition, we must guarantee that $\mathbb{E}[b_i^t|{b'_i}^t] > \mathbb{E}[b_g^t|{b'_g}^t]$ to ensure that $\mathbb{E}[b_i^t|{b'_i}^t]$ is not a winning bid. Therefore, we can obtain $0 = \mathbb{E}[u_i^t(\hat{b}_i^t)] \leq \mathbb{E}[u_i^t(\hat{b}_i^t)] = \mathbb{E}[b_g^t|{b'_g}^t] + b_{max} - b_{min} - \mathbb{E}[b_i^t|{b'_i}^t] < \gamma$.

The above analysis proves that $\mathbb{E}[u_i^t(b_i^t)] \geq \mathbb{E}[u_i^t(\hat{b}_i^t)]$ holds in all cases except for (b'_{Lose}, $\hat{b}'_{Win}$). And we also prove in the case of (b'_{Lose}, $\hat{b}'_{Win}$) that the utility gain obtained from an untruthful bid no more than γ. Therefore, we can observe that the reverse auction mechanism in the FedFD satisfies γ-incentive compatibility (truthfulness). ■

Theorem 2: FedFD meets the requirement of individual rationality.

Proof: Based on the training cost of the device, we define the utility function u_i of device i as

$$u_i^t(Q_i^t) = \begin{cases} p_i^t - c_i^t, & x_i^t = 1 \\ 0, & x_i^t = 0, \end{cases} \tag{2}$$

where p_i^t represents the final payment made by the server to device i in round t, and c_i^t represents the cost incurred by device i in the t-th round. If device i isn't selected to participate in the federation, then $p_i^t = 0$ and $c_i^t = 0$.

For any winning device i, we can see from Eq. (1) that we perform further operations on the critical payment $\hat{p}_i^t$. From Eq. (1), it can be seen that ${p'_i}^t \geq b_{max}$, therefore we can obtain $u_i^t = {p'_i}^t - c_i^t \geq b_{max} - c_i^t \geq 0$, which proves that FedFD conforms to individual rationality. ■

Theorem 3: The upper bound of expected regret achieved by FedFD is $O(NK^3 \ln(B + NK^2 \ln(NK^2)))$.

Proof: Firstly, from the perspective of reward payment during the auction process, regret is the difference in total reward between the best strategy and the online selection strategy, as shown in Eq. (3).

$$\nabla_{max} = \sum_{i \in \mathcal{W}^{t*}} r_i - \min_{\mathcal{W}^t \neq \mathcal{W}^{t*}} \sum_{i \in \mathcal{W}^t} r_i \tag{3}$$

The optimal strategy refers to the optimal winner determination and payment plan given with knowledge of the rewards and bids of all devices. Here, we denote the optimal set as W. When analyzing the upper bound of Regret, we assume that every device is truthful. This assumption has been validated in the above.

Ignoring fair selection, we use $\frac{\widetilde{r}_i^{t-1} S_i^t}{\mathbb{E}[b_i^t|{b'_i}^t]}$ as the criterion for device selection. The difference between choices can be expressed as:

$$\Delta_{max} = \sum_{i \in \mathcal{W}^{t*}} \frac{\widetilde{r}_i^{t-1} S_i^t}{\mathbb{E}[b_i^t|{b'_i}^t]} - \min_{\mathcal{W}^t \neq \mathcal{W}^{t*}} \sum_{i \in \mathcal{W}^t} \frac{\widetilde{r}_i^{t-1} S_i^t}{\mathbb{E}[b_i^t|{b'_i}^t]} \tag{4}$$

$$\Delta_{min} = \sum_{i \in \mathcal{W}^{t*}} \frac{\widetilde{r}_i^{t-1} S_i^t}{\mathbb{E}[b_i^t|{b'_i}^t]} - \max_{\mathcal{W}^t \neq \mathcal{W}^{t*}} \sum_{i \in \mathcal{W}^t} \frac{\widetilde{r}_i^{t-1} S_i^t}{\mathbb{E}[b_i^t|{b'_i}^t]} \tag{5}$$

We consider a vector v_i^t to represent the records of the non-optimal set, and use $T(B)$ to represent the calculated time interval. We need to constrain v_i^t and time interval $T(B)$ to analyze the regret upper bound.

When a non-optimal set of devices is selected, the vectors v_i^t will be increased by one. We define a binary variable $\phi_i^t \in \{0, 1\}$ to represent the variation of vector ϕ_i^t in t. Therefore, we have the following inequality:

$$\begin{aligned} v_i^T &= \sum_{t=2}^{T} \mathbb{I}\{\phi_i^t = 1\} = \omega + \sum_{t=2}^{T} \mathbb{I}\{\phi_i^t = 1, v_i^t \geq \omega\} \\ &\leq \omega + \sum_{t=2}^{T} \mathbb{I}\{\sum_{k \in \mathcal{W}^t} \frac{\widetilde{r}_i^{t+1} S_i^t}{\mathbb{E}[b_i^t|{b'_i}^t]} \geq \sum_{k \in \mathcal{W}^{t*}} \frac{\widetilde{r}_i^{t+1} S_i^t}{\mathbb{E}[b_i^t|{b'_i}^t]}, v_i^t \geq \omega\} \\ &\leq \omega + \sum_{t=1}^{T} \sum_{\beta_1^t = \omega}^{t} \cdots \sum_{\beta_1^t = 1}^{t} \mathbb{I}\{\sum_{k=1}^{K} \frac{\widetilde{r}_{i_k}^t S_i^t}{\mathbb{E}[b_{i_k}^t|{b'_{i_k}}^t]} \geq \sum_{k=1}^{K} \frac{\widetilde{r}_{i_k^*}^t S_i^{t*}}{\mathbb{E}[b_{i_k^*}^t|{b'_{i_k^*}}^t]}\} \end{aligned} \tag{6}$$

From Eq. (6), it can be seen that the bound of $\sum_{k=1}^{K} \frac{\widetilde{r}_{i_k}^t S_i^t}{\mathbb{E}[b_{i_k}^t|{b'_{i_k}}^t]} \geq \sum_{k=1}^{K} \frac{\widetilde{r}_{i_k^*}^t S_i^{t*}}{\mathbb{E}[b_{i_k^*}^t|{b'_{i_k^*}}^t]}$ is crucial for the bound of v_i^T. Specifically, for $\sum_{k=1}^{K} \frac{\widetilde{r}_{i_k}^t S_i^t}{\mathbb{E}[b_{i_k}^t|{b'_{i_k}}^t]} \geq \sum_{k=1}^{K} \frac{\widetilde{r}_{i_k^*}^t S_i^{t*}}{\mathbb{E}[b_{i_k^*}^t|{b'_{i_k^*}}^t]}$, at least one of the following formulas will occur:

$$\sum_{k=1}^{K} \frac{\overline{r}_{i_k^*}^t}{\mathbb{E}[b_{i_k^*}^t|{b'_{i_k^*}}^t]} \leq \sum_{k=1}^{K} \frac{r_{i_k^*}^t - \sqrt{\frac{2 \ln t}{\beta_{i_k^*}^t}}}{\mathbb{E}[b_{i_k^*}^t|{b'_{i_k^*}}^t]} \tag{7}$$

$$\sum_{k=1}^{K} \frac{\overline{r}_{i_k}^t}{\mathbb{E}[b_{i_k^*}^t|{b'_{i_k}}^t]} \geq \sum_{k=1}^{K} \frac{r_{i_k}^t + \sqrt{\frac{2 \ln t}{\beta_{i_k}^t}}}{\mathbb{E}[b_{i_k}^t|{b'_{i_k}}^t]} \tag{8}$$

$$\sum_{k=1}^{K} \frac{r_{i_k^*}^t}{\mathbb{E}[b_{i_k^*}^t|{b'_{i_k^*}}^t]} < \sum_{k=1}^{K} \frac{r_{i_k}^t - \sqrt{\frac{2 \ln t}{\beta_{i_k}^t}}}{\mathbb{E}[b_{i_k}^t|{b'_{i_k}}^t]} \tag{9}$$

We use Chernoff Hoeffding bound [2] to illustrate the probability of Eq. (7) and Eq. (8) being bounded.

Lemma 1 (Chernoff Hoeffding Bound): Assuming $x_1, x_2, ..., x_n$ are n independent distributed random variables with values between $[0, 1]$, where $\tilde{\mu} = (\sum_i x_i)/n$ represents the mean in the actual sample and $\bar{\mu}$ represents the mean in the theoretical distribution, for all $u \geq 0$, there exists a probability inequality

$$\mathbb{P}[\bar{\mu} \leq n\tilde{\mu} + u] \leq e^{(-2u^2/n)},\\ \mathbb{P}[\bar{\mu} \geq n\tilde{\mu} + u] \leq e^{(-2u^2/n)} \tag{10}$$

We analyze Eq. (7) and Eq. (8) using the Chernoff Hoeffding inequality, and choose a certain value of ω to ensure that Eq. (9) is impossible. We can obtain $\omega \geq \frac{4K^2(K+1)\ln(T(B))}{C_{min}\Delta_{min}}$.

Therefore, Eq. (6) can be bounded by

$$\begin{aligned}\mathbb{E}[v_i^T] &\leq \lceil \frac{4K^2(K+1)\ln(T(B))}{C_{min}\Delta_{min}} \rceil \\ &+ \sum_{t=1}^{\infty}(t-\omega+1)^K t^K 2Kt^{-2(K+1)} \\ &\leq \frac{4K^2(K+1)\ln(T(B))}{(C_{min}\Delta_{min})^2} + 1 + 2K\sum_{t=1}^{\infty} t^{-2} \\ &\leq \frac{4K^2(K+1)\ln(T(B))}{(C_{min}\Delta_{min})^2} + 1 + \frac{K\pi^2}{3} \\ &= l_1 \ln(T(B)) + l_2\end{aligned} \tag{11}$$

where $l_1 = \frac{4K^2(K+1)}{(C_{min}\Delta_{min})^2}$ and $l_2 = 1 + \frac{K\pi^2}{3}$.

According to our definition of critical payment $\hat{p}_i^t$, the overall payment during the iteration can be expressed as

$$P^{\circ} = \sum_{i\in\mathcal{W}^*} \frac{\widetilde{r}_i^{t-1} S_i^t}{\widetilde{r}_{K+1}^{t-1} S_{K+1}^t} \cdot \mathbb{E}[b_{K+1}^t | {b'_{K+1}}^t] \geq \sum_{i\in\mathcal{W}^*} b_i^t = P^*. \tag{12}$$

According to Eq. (12), we can conclude that the payment of the selected client is always more than its actual bid. In other words, for FedFD, the overall payment of the selected devices is more than KC_{min}. Therefore, we can obtain

$$\begin{aligned}T(B) &\leq T^*(B) + T(\sum_{i\notin\mathcal{W}^*} \beta_i^T(B)C_{max}) \\ &\leq T^*(B) + C_{max}T(\sum_{k=1}^{N} v_i^T) \\ &\leq T^*(B) + \frac{NC_{max}}{KC_{min}}\mathbb{E}[v_i^T] \\ &\leq \frac{B}{P^*} + \frac{NC_{max}}{KC_{min}}(l_1\ln(T(B)) + l_2).\end{aligned} \tag{13}$$

Due to inequality $\ln x \leq x - 1$, we can obtain

$$\ln(T(B)) \leq \frac{KC_{min}}{2NC_{max}l_1}T(B) - 1 + \ln(\frac{2NC_{max}l_1}{KC_{min}}). \tag{14}$$

According to Eq. (13) and Eq. (14), we can obtain

$$\begin{aligned}T(B) &\leq \frac{B}{P^*} + \frac{NC_{max}}{KC_{min}}(l_1\ln(T(B)) + l_2) \\ &\leq \frac{2B}{P^*} + \frac{1}{2}T(B) + \frac{NC_{max}}{KC_{min}}(l_2 - l_1 + l_1\ln\frac{2NC_{max}l_2}{KC_{min}}).\end{aligned} \tag{15}$$

Thus, we derive

$$T(B) \leq \frac{2B}{P^*} + \frac{2NC_{max}}{KC_{min}}(l_2 - l_1 + l_1\ln\frac{2NC_{max}l_2}{KC_{min}}). \tag{16}$$

For convenience, we rephrase Eq. (16) as $T(B) \leq (2B/P^*) + l_3$, which is the upper bound of $T(B)$.

Then we analyzed the lower bound of the time interval $T(B)$. We use B^* to represent the budget for selecting the optimal set, and $\bar{B}$ to represent the remaining budget for selecting the non-optimal set. In addition, we use $\bar{T}(B)$ to represent all rounds of payments made through critical payments during the iteration process. Therefore, we can obtain $\bar{T}(B) \leq T^*(B)$.

$$\begin{aligned}T(B) &= T(B^* + \bar{B}) \geq T^*(B) \\ &\geq \bar{T}(B)(B - C_{max})N(l_1\ln(T(B)) + l_2) \\ &\geq \frac{B - NC_{max}(l_1\ln(\frac{2B}{P^*} + l_3) + l_2))}{P^*} - 1.\end{aligned} \tag{17}$$

We obtained Eq. (17) by taking the logarithm of Eq. (16), and then we can obtain the lower bound of $T(B)$.

Based on the bounds of v and T calculated above, as well as the definition of regret, we can conclude that the regret of auctioning through reward payment is:

$$\begin{aligned}R_a(B) &= \frac{B}{P^*}r^* - T(B)r^* + T(B)r^* - \mathbb{E}[\sum_t^T \sum_i r_i^t] \\ &\leq \frac{B}{P^*}r^* - T(B)r^* + \sum_i^N v_i^T \nabla_{max} \\ &\leq \frac{B}{P^*}r^* - T(B)r^* + \sum_i^N v_i^T \nabla_{max}(l_1\ln(\frac{2B}{P^*} + l_3) + l_2) \\ &\leq (l_1 r^* \frac{NC_{max}}{P^*} + N\nabla_{max}l_1)\ln(\frac{2B}{P^*} + l_3) \\ &+ l_2 r^* \frac{NC_{max}}{P^*} + N\nabla_{max}l_2 + r^*\end{aligned} \tag{18}$$

We use virtual queues in Lyapunov optimization to achieve fairness in the device selection process, and this part of the regret analysis is similar to that in [3]. We define the Lyapunov function as $L(Q^t) = \frac{1}{2}\sum_{i=1}^K {Q_i^t}^2$, where $Q^t = [Q_1^t, \cdots, Q_K^t]$ is its virtual queue in the t-th round. Therefore, we can obtain the drift of the Lyapunov function as

$$\begin{aligned}&L(Q^{t+1}) - L(Q^t) \\ &\leq \frac{1}{2}\sum_{i=1}^K (Q_i^t + h_i^t - x_i^t)^2 - \frac{1}{2}\sum_{i=1}^K {Q_i^t}^2 \\ &\leq \frac{K}{2} + \sum_{i=1}^K h_i^t Q_i^t - \sum_{i=1}^K x_i^t Q_i^t\end{aligned} \tag{19}$$

Let's denote the expected reward of device i as $z_i = \mathbb{E}[r_i^t]$, and denote $\mathcal{W}^*$ as the optimal set selected in round t. Therefore, the regret of algorithm FedFD in fairness of choice can be expressed as $R_f = \frac{1}{T}\sum_t^T \mathbb{E}[z_{\mathcal{W}^*}^t - z_{\mathcal{W}}^t]$.

In addition, we further defined the quantity of choices as follows:

$$\Upsilon^t = z_{\mathcal{W}^*}^t - z_{\mathcal{W}}^t = \sum_i^K z_i {x_i^t}^* - \sum_i^K z_i x_i^t, \tag{20}$$

where x_i^t represents whether device i is selected in the t-th round.

According to the analysis of Lyapunov drift, expected drift and regret can be expressed as

$$\begin{aligned} &\mathbb{E}[L(Q^{t+1}) - L(Q^t) + \eta \Upsilon^t] \\ &\leq \frac{K}{2} + \mathbb{E}[\underbrace{\sum_{i=1}^{K} (Q_i^t + \eta b_i)(x_i^{t^*} - x_i^t)]}_{F^t}. \end{aligned} \tag{21}$$

By Eq. (21), we can obtain $\frac{1}{T\eta}\mathbb{E}[L(Q^T) - L(Q^0)] + \frac{1}{T}\sum_{t=0}^{T}\mathbb{E}[\Upsilon^t] \leq \frac{K}{2\eta} + \frac{1}{T\eta}\sum_{t=0}^{T}[F^t]$. When $L(Q^0) = 0$ and $L(Q^T) \geq 0$, we can get

$$\frac{1}{T}\sum_{t=0}^{T}\mathbb{E}[\Upsilon^t] \leq \frac{K}{2\eta} + \frac{1}{T\eta}\sum_{t=0}^{T}\mathbb{E}[F^t] \tag{22}$$

Based on the selection strategy in the Section V-A, we replace $\frac{\widetilde{r}_i^{t-1} S_i^t}{\mathbb{E}[b_i^t | b_i']}$ with Z, we can obtain $Q_{\mathcal{W}^t} + \eta Z_{\mathcal{W}^t} \geq Q_{\mathcal{W}^{t*}} + \eta Z_{\mathcal{W}^{t*}}$.

$$\begin{aligned} &\sum_{t=0}^{T}\mathbb{E}[F^t] \\ &= \sum_{t=0}^{T}\mathbb{E}[\sum_{i=1}^{K}(Q_i^t + \eta z_i)(x_i^{t^*} - x_i^t)] \\ &\leq \sum_{t=0}^{T}\mathbb{E}[(Q_{\mathcal{W}^{t*}} + \eta z_{\mathcal{W}^{t*}}) - (Q_{\mathcal{W}}^t + \eta z_{\mathcal{W}}^t) \\ &+ (Q_{\mathcal{W}}^t + \eta Z_{\mathcal{W}^t}) - (Q_{\mathcal{W}^{t*}} + \eta Z_{\mathcal{W}^{t*}}) \\ &= \underbrace{\eta \sum_{t=0}^{T}\mathbb{E}[Z_{\mathcal{W}^t} - z_{\mathcal{W}}^t]}_{b_1} + \underbrace{\eta \sum_{t=0}^{T}\mathbb{E}[Z_{\mathcal{W}^{t*}} - z_{\mathcal{W}^{t*}}]}_{b_2}. \end{aligned} \tag{23}$$

Now, in order to obtain R_f, we need to obtain upper bounds for b_1 and b_2. According to Lemma 1 and literature [4], we can conclude that $b_1 < 2\sqrt{6KT\ln T} + (1 + \pi^2/4)K$ and $b_2 < (\pi^2/6)K$.

Finally, we can obtain the regret of FedFD that $R(B) = R_a + R_f$. The regret $R(B)$ is bounded by $O(NK^3 \ln(B + NK^2 \ln(NK^2)))$. ∎

References

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