

# Vessel Segmentation of Retinal Fundus Images

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## Homework Solution

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### Project Description

The leading causes of irreversible blindness in the United States are mainly age-related macular degeneration, cataract, and glaucoma [1]. Early detection of these diseases is important for patients to optimize their treatment benefits. Among many imaging modalities used for retina like Optical Coherence Tomography (OCT), fundus photography is one of the most commonly adopted methods for ophthalmologists to detect and assess symptoms of eye diseases. The existing algorithms for automatic fundus image analysis can segment and measure the retinal vasculature, which is a major indicator of various diseases [2,3,4]. However, existing methods usually utilize filters on global fundus image to segment retinal vasculature as a whole, which will have insufficient information about local vessel network. Therefore, our project aims at developing a novel machine-learning algorithm to separate and rebuild a vessel starting from specific points generated by user interface.

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### Data

Fundus images are obtained from DRIVE database [5].



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## Methods

- Unit Rectangle
- Vessel Walls Tracing

### ▼ Preprocessing

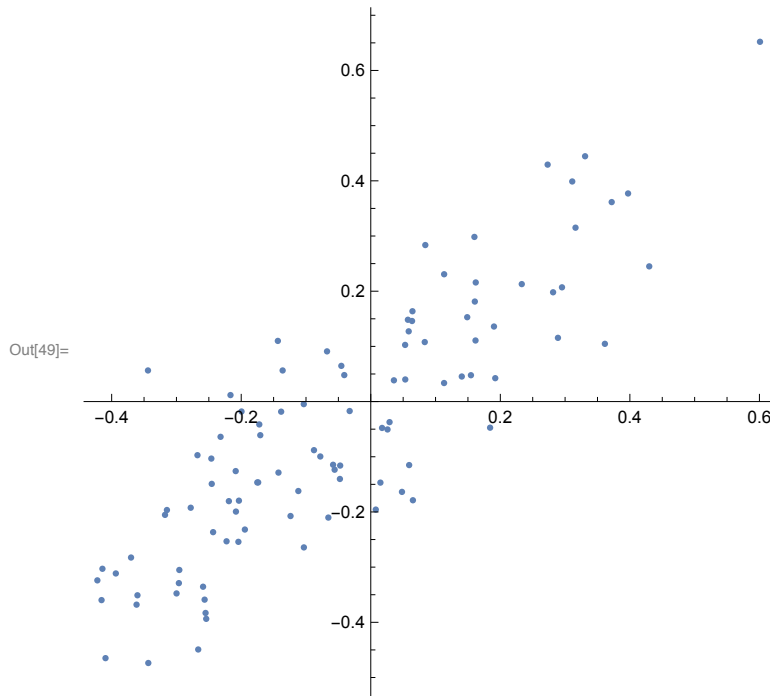
In[48]:= **grad** = **ImageAdjust**[**GradientFilter**[**im1**, 1], {.1, 10}]



In order to improve the running time, we apply gradient filter on the original image, and try to trace the vessel on the white vessel walls with an initial angle.

We generate random angles for each step based on normal distribution.

```
In[49]:= ListPlot[generateRandomSamples[0.25, 0.95, 100], AspectRatio -> 1]
```



We optimize the cost function for every two steps to foresee the growing trend.

Cost Function(  $\theta_1, \theta_2$  ) =

$$-\sum (\text{pixel value}) + \frac{\lambda}{L^2} ((\theta_1 - \theta_{01})^2 + (\theta_2 - \theta_{02})^2) + \frac{\mu}{L^2} (\theta_2 - \theta_1)^2 + \gamma(r - r_0)^2$$

where  $\lambda$  is the curvature constraint,  $\mu$  is the expansion constraint,  $\gamma$  is the diameter constraint,  $r$  is the distance between each pair of pixels, and  $r_0$  is the anticipated vessel diameter.

(Possibly with solving differential equations with respect to each constraint parameters.)

### ▼ Code

```
In[50]:= endpixel[pixel1_, theta_, l_] := Round[pixel1+l*{Cos[theta], Sin[theta]}];

generateRandomSamples[std_, corr_, n_] := Module[
  {dist},
  dist = MultinormalDistribution[
    {0, 0},
    {{std^2, (corr*std)^2}, {(corr*std)^2, std^2}}
  ];
  RandomVariate[dist, n]
];

Clear[findPath]
Options[findPath] = {
  "Length" -> 5,
  "MaxIterations" -> 10,
  "CurvatureConstraint" -> 0,
  "DiameterConstraint" -> 0,
```

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    "ExpansionConstraint" → 0,
    "VesselDiameter" → 4.5,
    "SearchDepth" → 2,
    "SampleNumber" → 100
  };

findPath[image_, {pixel01_, pixel02_}, {theta01_, theta02_}, opts:OptionsPattern[] := Module[
  {steptheta, th1, th2, pixel1, pixel2, l, pos, theta, c, cells, samples, ns, costs, di},
  c = OptionValue["CurvatureConstraint"];
  pos = {{pixel01, pixel02}};
  theta = {{theta01, theta02}};
  l = OptionValue["Length"];
  depth = OptionValue["SearchDepth"];

  samples = generateRandomSamples[0.2, 0.95, {depth, OptionValue["SampleNumber"]}];

  ns = Length[samples[[1]]];
  dim = ImageDimensions[image]-1-1;
  Do[
    If[Or @@ Map[Negative, dim - pos[[k, 1]]], Break[]];
    If[Or @@ Map[Negative, dim - pos[[k, 2]]], Break[]];
    costs = listableCost[
      theta[[k]] + Transpose[samples[[1]]],
      Transpose@Table[theta[[k]], {ns}],
      Transpose@Table[pos[[k]], {ns}],
      image,
      "CurvatureConstraint" → c,
      "ExpansionConstraint" → OptionValue["ExpansionConstraint"],
      "DiameterConstraint" → OptionValue["DiameterConstraint"],
      "Length" → l,
      "VesselDiameter" → OptionValue["VesselDiameter"]
    ];
    inttheta = (# + theta[[k]]) & /@ samples[[1]];
    intpixels = computePixels[pos[[k]], #, l] & /@ inttheta;
    costs2 = listableCost[
      Transpose[inttheta + samples[[2]]],
      Transpose@inttheta,
      Transpose@intpixels,
      image,
      "CurvatureConstraint" → c,
      "ExpansionConstraint" → OptionValue["ExpansionConstraint"],
      "DiameterConstraint" → OptionValue["DiameterConstraint"],
      "Length" → l,
      "VesselDiameter" → OptionValue["VesselDiameter"]
    ];
    costs = costs + 0.7*costs2;
    {th1, th2} = theta[[k]] + First@samples[[1, Ordering[ costs, 1 ]]];
    {pixel1, pixel2} = computePixels[pos[[k]], {th1, th2}, l];

    pos = Append[pos, {pixel1, pixel2}];
    theta = Append[theta, {th1, th2}]
  ,
    {k, OptionValue[MaxIterations]}
  ];
  {pos, theta}
];

computePixels[pos_, theta_, l_] := Module[
  {u, pixel1, pixel2, diff, proj},
  u = #[Mean[theta]] & /@ {Cos, Sin};

```

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    pixel1 = endpixel[pos[[1]], theta[[1]], 1];
    pixel2 = endpixel[pos[[2]], theta[[2]], 1];
    diff = pixel2-pixel1;
    proj = diff.u;
    pixel1 = endpixel[pos[[1]], theta[[1]], 1 + proj/2];
    pixel2 = endpixel[pos[[2]], theta[[2]], 1 - proj/2];
    {pixel1, pixel2}
];

Clear[listableCost]
Clear[ilistableCost]
Options[listableCost] = {
  "Length" → 5,
  "CurvatureConstraint" → 0,
  "ExpansionConstraint" → 0,
  "DiameterConstraint" → 0,
  "VesselDiameter" → 4.5
};

listableCost[{theta1_, theta2_}, {theta01_, theta02_}, {pixel01_, pixel02_},
  imagegradient_, opts:OptionsPattern[]] := Module[
  {c1, c2, l, totalcost, lambda, mu, gamma, pdist},
  l = OptionValue["Length"];
  lambda = OptionValue["CurvatureConstraint"];
  mu = OptionValue["ExpansionConstraint"];
  gamma = OptionValue["DiameterConstraint"];
  c1 = ilistableCost[imagegradient, pixel01, theta1, l];
  c2 = ilistableCost[imagegradient, pixel02, theta2, l];

  totalcost = -(c1+c2);
  totalcost = totalcost + lambda/l^2*((theta1-theta01)^2 + (theta2-theta02)^2);
  totalcost = totalcost + mu/l^2*(theta2-theta1)^2;
  pdist = pixelDistance[{pixel01, pixel02}, {theta1, theta2}, l];
  totalcost = totalcost + gamma*(pdist-OptionValue["VesselDiameter"])^2;
  totalcost = MapThread[If[2.<#1<8., #2, #2+10^4.] &, {pdist, totalcost}];
  totalcost
];

ilistableCost[image_, pixel0_, theta_, l_] := Module[
  {pixel, pos, val, dim, imagedata},
  pixel = endpixel[pixel0, theta, l];
  pos = MapThread[getLinePixelPositions, {pixel0, pixel}];

  dim = ImageDimensions[image];
  pos = Reverse[Transpose[#]] & /@ pos;
  pos[[All, 1]] = dim[[2]] - pos[[All, 1]];
  pos = Transpose /@ pos;
  imagedata = ImageData[image];
  val = Extract[imagedata, #] & /@ pos;
  (Total /@ val)/l
];

pixelDistance[{pixel01_, pixel02_}, {theta01_List, theta02_List}, length_] := Module[
  {pixel1, pixel2},
  pixel1 = endpixel[pixel01, theta01, length];
  pixel2 = endpixel[pixel02, theta02, length];
  N[Norm /@ (pixel2-pixel1)]
];

endpixel[pixel1_, theta_, l_] := Round[pixel1+l*{Cos[theta], Sin[theta]}];

```

```

endpixel[pixel1_, theta_List, l_] := Round[pixel1+l*Transpose[{Cos[theta], Sin[theta]}]];

getLinePixelPositions[pixel0_, pixel_] := Module[
  {pos, posrev},
  pos = igetLinePixelPositions[pixel0, pixel];
  posrev = Reverse /@ igetLinePixelPositions[Reverse[pixel0], Reverse[pixel]];
  Union[pos, posrev]
];

igetLinePixelPositions[pixel0_, pixel_] := Module[
  {diff, slope, xlist, ylist},
  diff = pixel - pixel0;
  If[diff[[1]]==0, Return[{}]];
  slope = diff[[2]]/diff[[1]];
  xlist = Range @@ Sort[{0, diff[[1]]}];
  ylist = Round[xlist*slope];
  Transpose[{xlist, ylist} + pixel0]
];

Clear[viewVesselOnImage3]
Options[viewVesselOnImage3] = {
  "Length" → 5,
  "MaxIterations" → 10,
  "CurvatureConstraint" → 0,
  "DiameterConstraint" → 0,
  "ExpansionConstraint" → 0,
  "VesselDiameter" → 4.5
};

viewVesselOnImage3[image_, {pixel01_, pixel02_}, {theta01_, theta02_}, opts:OptionsPattern[]] :=
Module[
  {pos, pixelPositions, theta},
  {pixelPositions, theta} = findPath[image, Round[{pixel01, pixel02}], {theta01, theta02},
    "CurvatureConstraint" → OptionValue["CurvatureConstraint"],
    "ExpansionConstraint" → OptionValue["ExpansionConstraint"],
    "DiameterConstraint" → OptionValue["DiameterConstraint"],
    "Length" → OptionValue["Length"],
    "MaxIterations" → OptionValue["MaxIterations"],
    "VesselDiameter" → OptionValue["VesselDiameter"]
  ];
  pos = {#1, Reverse[#2]} & @@@ Partition[pixelPositions, 2, 1];
  pos = Partition[Flatten[#, 1], 2, 1, 1] & /@ pos;
  Show[image, Graphics[{EdgeForm[Opacity[0.6, Red]], Opacity[0], Polygon /@ pos}]]
];

locatorIcon[pos_, r_, color_] := Graphics[
  {color, Opacity[0.5], Circle[pos, r],
  Line[pos+r*# & /@ {{-1., 0.}, {1., 0.}}],
  Line[pos+r*# & /@ {{0., -1.}, {0., 1.}}]}
];

findAngle[p1_, p2_, p3_] := Module[
  {angle},
  angle = ConstantArray[N[ArcTan[(p3-Mean[{p1,p2}])[[1]], (p3-Mean[{p1,p2}])[[2]]], 2];
  angle
];

endpixel2[pixel1_, theta_, l_] := Round[pixel1-l*{Cos[theta], Sin[theta]}];

Clear[g, interface]
interface[image_] :=

```

```

Manipulate[
  Style[Show[g,
    locatorIcon[p1, 1.6, Red],
    locatorIcon[p2, 1.6, Red],
    locatorIcon[p3, 1.6, Blue],
    Graphics[{
      Red, Line[{p1, endpixel2[p1, findAngle[p1, p2, p3][[1]], length]}],
      Line[{p2, endpixel2[p2, findAngle[p1, p2, p3][[1]], length]}]
    }, PlotRange -> All]
  ], Magnification -> magnification],
  {{p1, {10, 10}}, Locator, Appearance -> None},
  {{p2, {20, 20}}, Locator, Appearance -> None},
  {{p3, {30, 30}}, Locator, Appearance -> None},
  Button["Start",
    g = viewVesselOnImage3[image, {p1, p2}, findAngle[p1, p2, p3],
      "CurvatureConstraint" -> curvatureConstraint,
      "ExpansionConstraint" -> expansionConstraint,
      "DiameterConstraint" -> diameterConstraint,
      "VesselDiameter" -> vesselDiameter,
      MaxIterations -> iteration,
      "Length" -> length
    ];
  ],
  {{curvatureConstraint, 1.5}, 0, 10, 0.5},
  {{expansionConstraint, 4}, 0, 10, 1},
  {{diameterConstraint, 0.02}, 0, 0.1, 0.01},
  {{vesselDiameter, 4.5}, 2, 7, 0.5},
  {{iteration, 50}, 1, 100, 5},
  {{length, 5}, 1, 10, 1},
  {{magnification, 3}, 1, 10},
  ControlPlacement -> Left
];

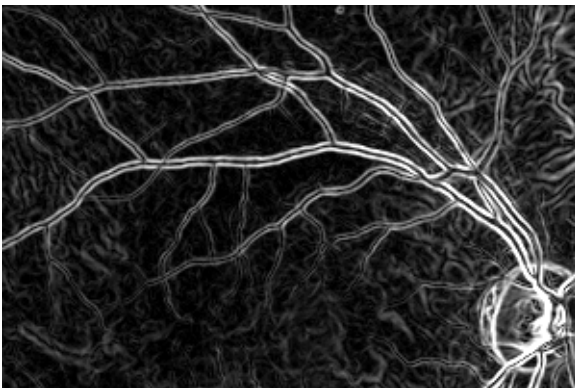
```

### ▼ Result

The demonstration will be shown on a portion of the filtered image.

In[74]:= **gradzoom = ImageTake[grad, {1, 200}, {100, 400}]**

Out[74]=



The way of using the user interface is:

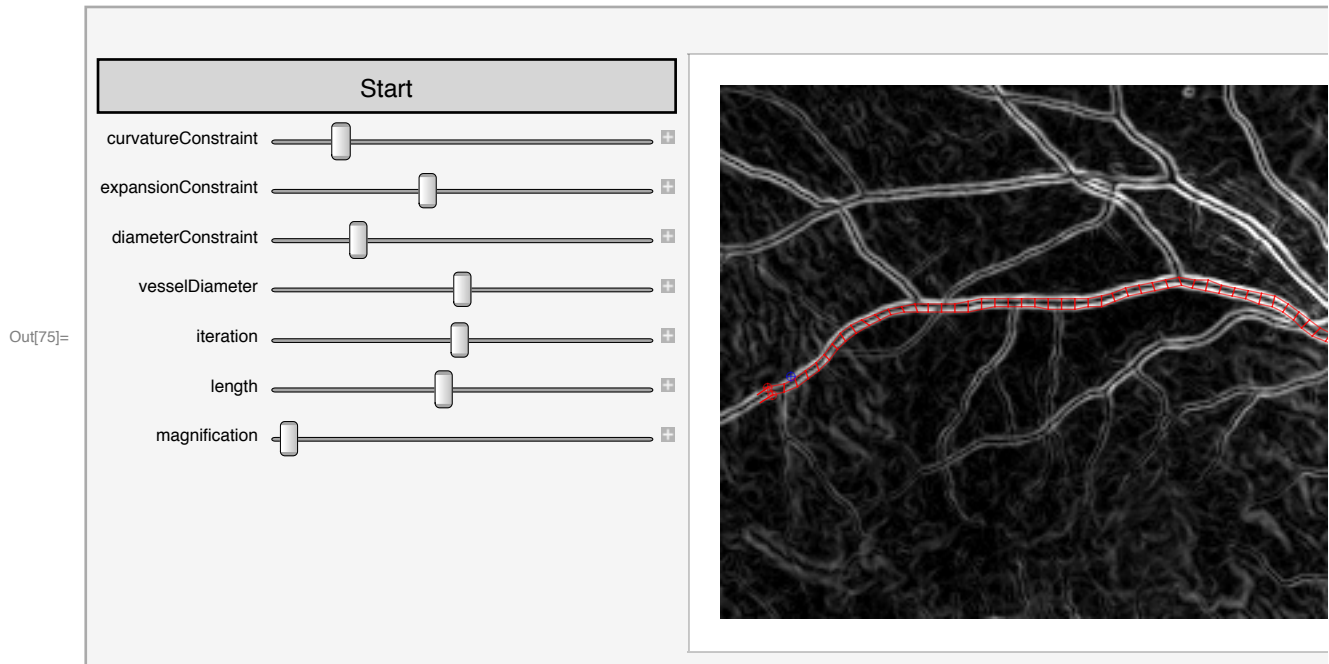
1. Click “Start” button.
2. Move two red dots at each side of desired vessel.
3. Move the blue dot so that the two short lines on each red dots align with the vessel

walls.

4. Press “Start” to see the result with default settings.

5. Adjust the parameters to get the best result.

```
In[75]:= interface[gradzoom]
```



## Conclusions

The user needs to select two points at each side of one vessel (red dots), and a third position to indicate an initial angle (blue dot). By changing the constraint parameters on curvature, expansion, and diameter, our model can trace the vessel without the disturbance of other branching vessels.

## Future Directions

Ideally, the constraint parameters can be optimized automatically. It would be more useful for ophthalmologists if future project can specify branching network, and compute measurements for targeted pathologies with high accuracy. The results of future algorithm will be compared with standard references marked by trained clinicians. We can possibly apply the same technique to 3D OCT image to reconstruct 3D vessel structure.

## References

[1] The Eye Diseases Prevalence Research Group\*. Causes and Prevalence of Visual Impairment Among Adults in the United States. Arch Ophthalmol. 2004;122(4):477-485. doi:10.1001/archoph.122.4.477.



- [2] Odstrcilik, J.; Kolar, R.; Budai, A.; Horneegger, J.; Jan, J.; Gazarek, J.; Kubena, T.; Cernosek, P.; Svoboda, O.; Angelopoulou, E., "Retinal vessel segmentation by improved matched filtering: evaluation on a new high-resolution fundus image database," Image Processing, IET , vol.7, no.4, pp.373,383, June 2013
- [3] Marin, D.; Aquino, A.; Gegundez-Arias, M.E.; Bravo, J.M., "A New Supervised Method for Blood Vessel Segmentation in Retinal Images by Using Gray-Level and Moment Invariants-Based Features," Medical Imaging, IEEE Transactions on , vol.30, no.1, pp.146,158, Jan. 2011
- [4] Staal, J.; Abramoff, M.D.; Niemeijer, M.; Viergever, M.A.; van Ginneken, B., "Ridge-based vessel segmentation in color images of the retina," Medical Imaging, IEEE Transactions on , vol.23, no.4, pp.501,509, April 2004
- [5] Sample color fundus image with manually selected vessel structure. DRIVE database.  
J.J. Staal, M.D. Abramoff, M. Niemeijer, M.A. Viergever, B. van Ginneken, "Ridge based vessel segmentation in color images of the retina", IEEE Transactions on Medical Imaging, 2004, vol. 23, pp. 501-509.

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## Other

Last Modified: Thursday, July 16, 2015

Insert date...