simulation_week6

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```
library(tidyverse)
library(purrr)
library(ggplot2)
library(here)
devtools::load_all()
set.seed(1222)
```

Commonly used kernel functions

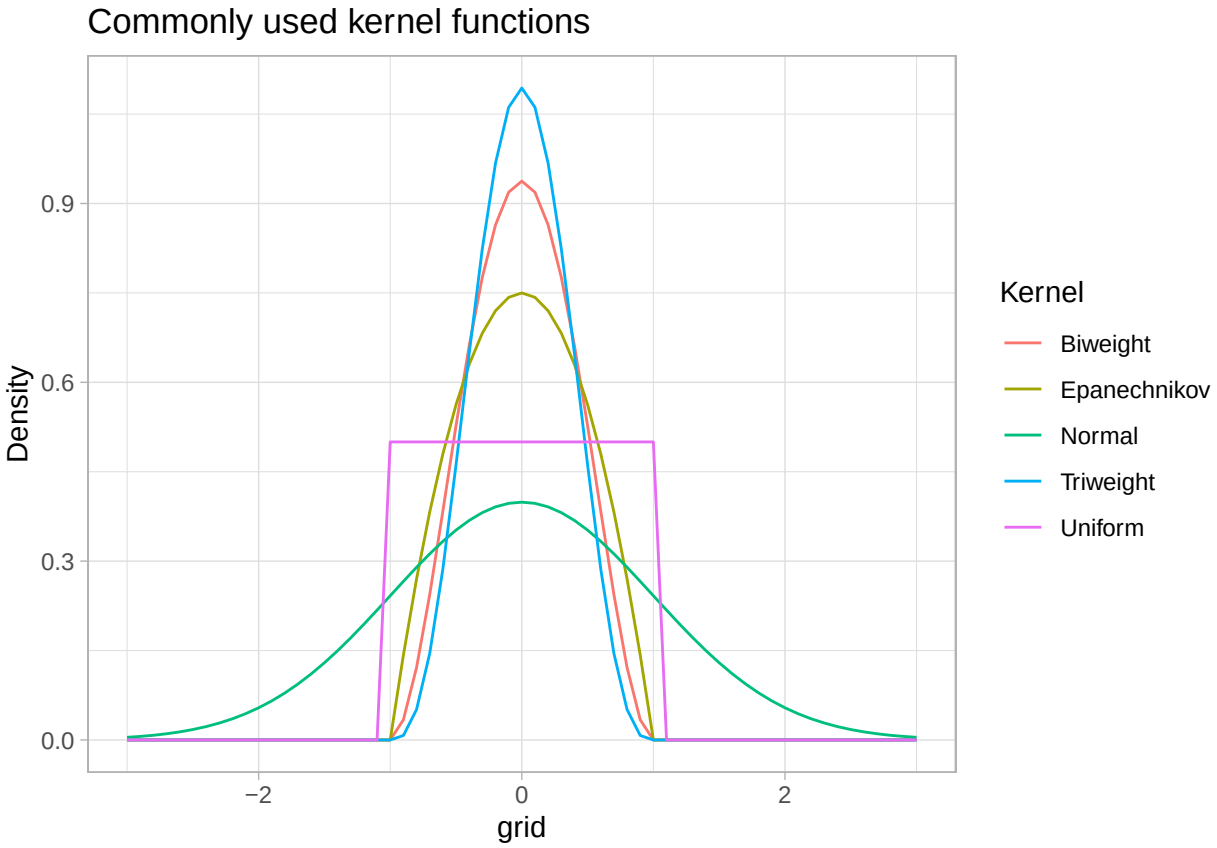
In the beta family kernel density, lambda=0,1,2,3 correspond to the uniform, the Epanech- nikov, the biweight, and the triweight kernel functions, respectively.

```
kernel_beta <- function(x,lambda){(1-x^2)^lambda*(abs(x)<=1)/beta(lambda+1,0.5)}

kernel_comp <- tibble(
  grid = seq(-3,3,by=0.1),
  Normal = dnorm(grid),
  Uniform = kernel_beta(grid,0),
  Epanechnikov = kernel_beta(grid,1),
  Biweight = kernel_beta(grid,2),
  Triweight = kernel_beta(grid,3)
  )

kernel_comp_long <- kernel_comp %>%
  pivot_longer(c(`Normal`, `Uniform`,`Epanechnikov`,
                 `Biweight`,`Triweight`), names_to = "Kernel", values_to = "Density")


ggplot(data=kernel_comp_long) +
  geom_line(aes(x = grid, y = Density, color = Kernel)) +
  labs(title = "Commonly used kernel functions") +
  theme_light()
```



Compute the MADE (Mean Absolute Deviation Errors)

The Mean Absolute Deviation Errors for $\hat{f}(\cdot)$is defined as

$$MADE = \frac{1}{n}\sum_{k=1}^{n}|(\hat{f}(u_k) - f(u_k))|$$

, where $\hat{f}(u_k)$ is the kernel estimte of $f(u_k))$, and $\{u_k\}$ are the frid points taken to be arbitrary within the range of data.

When the sample is generated from the known distribution like exp,beta, gamma, etc., we can use corresponding built-in functions (dexp(), dgamma, etc.) to calculated the true denstities of the grid points. If the sample is not from the familar distributions, we can use the built-in function "denstiy()" in R to get the value of $f(u_k))$s.

Simulation

Explore the simulation results with different sample sizes & kernels

Build simulation functions that can save the MADE.

Note: The follwoing functions are similar to `kde_est_big`, `sim_big_made`, and `save_made` that are used in week 7, which are listed in the `kde_est_functions.R`. However, they are the first versions and are much simpler. Thus, we will not modify this part and keep those fucntions here as the records.

```r
# Define some functions for gievn h=bandwidth,
# simulate over different sample size and kernels

kde_n_ker_est <- function(x, n, ker, grid){
  # x large population
  # n sample sizes
  # ker name of kernels
  # grid grid points
  map2(.x = n, .y = ker,
       ~KDE_est(sample(x, .x, replace = FALSE),
                grid, h= 1.06*100^(-0.2), ker = .y)$f_est)
}

sim_made <- function(ns, kes, x, grid, true_f){
  # generates every combination of parameters (ns:kers)
  simulation_params_big <- list(
  n = ns,
  kernel_type = kers)
  n_ker_est_big <- cross_df(simulation_params_big)


  n_ker_est_big <- n_ker_est_big %>%
  mutate(
    # for each combination of the paras, using the kde_n_ker_est()
    f_ests = kde_n_ker_est(x, n, kernel_type, grid),
    f_true = map(1:length(n), ~true_f) # save the corresponding true density
  ) %>%
  mutate(
    MADE = map2_dbl(.x = f_ests, .y = f_true, ~made(.x,.y)) # calculate the MADE
  )
  return(n_ker_est_big$MADE)
}
```

Simulation

Generate samples from t(3) distribution. Then, using five types of kernel function and different size of samples that are drawn from the large population.

```r
# population
x <- rt(50000, df=3)

# simulaiton parameters
ns <- c(100, 500, 1000, 1500) # sample sizes
kers  <- c("normal", "uniform", "epanech", "biweight", "triweight") # kernel functions
grid <- seq(-5,15, 0.1)

# true density
true_f <- dt(grid, df=3)

# simulation
n.sim <- 100 # number of replicates
sim_rlt <- map(1:n.sim, ~sim_made(ns, kers, x, grid,true_f))
```

Save result

```
made_mat <- matrix(unlist(sim_rlt), nrow = length(ns)*length(kers) )

made_comapre <- tibble(
  n =rep(ns, time=length(kers)),
  kernel_type = rep(kers, each=length(ns)),
  made_mean = apply(made_mat, 1, mean),
  made_sd = apply(made_mat, 1, sd)
)
write_rds(made_mat , here("results", "week6-sim.rds"))
write_rds(made_comapre, here("results", "week6-sim-plot.rds"))
```