Chapter 9
Introduction to Graphs

Discrete Structures for Computing on January 4, 2023

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Contents

Course outcomes

	Course learning outcomes
L.O.1	Understanding of logic and discrete structures
	L.O.1.1 – Describe definition of propositional and predicate logic L.O.1.2 – Define basic discrete structures: set, mapping, graphs
L.O.2	Represent and model practical problems with discrete structures
	L.O.2.1 – Logically describe some problems arising in Computing L.O.2.2 – Use proving methods: direct, contrapositive, induction L.O.2.3 – Explain problem modeling using discrete structures
L.O.3	Understanding of basic probability and random variables
	L.O.3.1 – Define basic probability theory L.O.3.2 – Explain discrete random variables
L.O.4	Compute quantities of discrete structures and probabilities
	L.O.4.1 – Operate (compute/ optimize) on discrete structures L.O.4.2 – Compute probabilities of various events, conditional ones, Bayes theorem

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Contents

Motivations

The need of the graph

- Representation/Storing
- Searching/sorting
- Optimization

Its applications

- Electric circuit/board
- Chemical structure
- Networking
- Map, geometry, . . .

- Graph theory is useful for analysing "things that are connected to other things".
- Some difficult problems become easy when represented using a graph.

Contents

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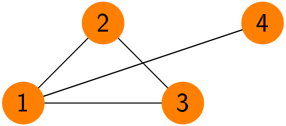
Graph

Definition

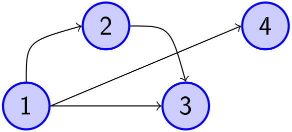
A graph (*đồ thị*) G is a pair of (V, E), which are:

- V – nonempty set of vertices (nodes) (*đỉnh*)
- E – set of edges (*cạnh*)

A graph captures abstract relationships between vertices.



Undirected graph



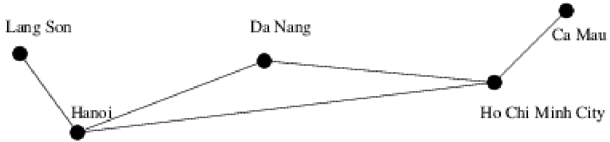
Directed graph

Undirected Graph (Đồ thị vô hướng)

Definition (Simple graph (đơn đồ thị))

- Each edge connects two different vertices, and
- No two edges connect the same pair of vertices

An edge between two vertices u and v is denoted as $\{u, v\}$

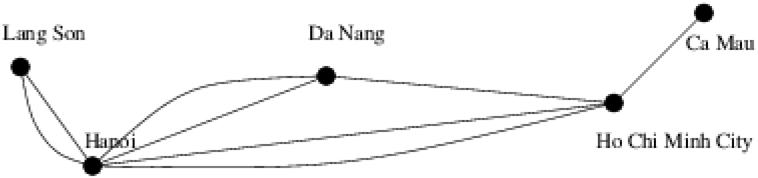


Undirected Graph

Definition (Multigraph (đa đồ thị))

Graphs that may have multiple edges connecting the same vertices.

An unordered pair of vertices $\{u, v\}$ are called multiplicity m (*bội* m) if it has m different edges between.

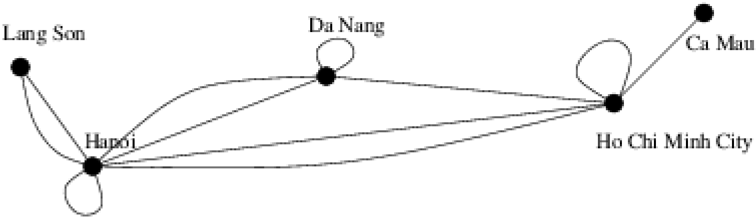


Undirected Graph

Definition (Pseudograph (giả đồ thị))

Are multigraphs that have

- loops (*khuyên*)– edges that connect a vertex to itself



Terminologies For Undirected Graph

Neighborhood

In an undirected graph $G = (V, E)$,

- two vertices u and $v \in V$ are called **adjacent** (*liền kề*) if they are **end-points** (*điểm đầu mút*) of edge $e \in E$, and
- e is **incident with** (*cạnh liên thuộc*) u and v
- e is said to **connect** (*cạnh nối*) u and v;

The degree of a vertex

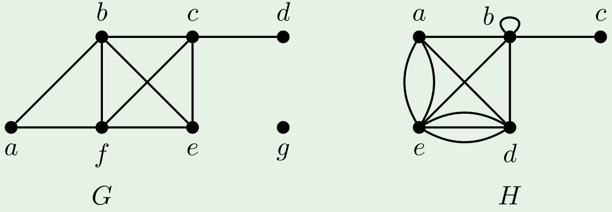
The **degree of a vertex** (*bậc của một đỉnh*), denoted by $\deg(v)$ is the number of edges incident with it, except that a loop contributes twice to the degree of that vertex.

- isolated vertex (*đỉnh cô lập*): vertex of degree 0
- pendant vertex (*đỉnh treo*): vertex of degree 1

Example

Example

What are the degrees and neighborhoods of the vertices in these graphs?



Solution

In G, *deg*(a) = *2*, *deg*(b) = *deg*(c) = *deg*(f) = *4*, *deg*(d) = *1*, ...
Neiborhoods of these vertices are
$N(a) = \{b, f\}, N(b) = \{a, c, e, f\}, \ldots$
In H, *deg*(a) = *4*, *deg*(b) = *deg*(e) = *6*, *deg*(c) = *1*, ...
Neiborhoods of these vertices are
$N(a) = \{b, d, e\}, N(b) = \{a, b, c, d, e\}, \ldots$

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Basic Theorems

Theorem (The Handshaking Theorem)

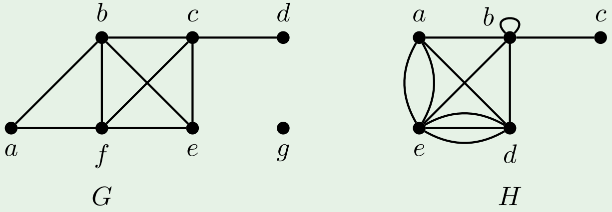
Let $G = (V, E)$ be an undirected graph with m edges. Then

$$2m = \sum_{v \in V} deg(v)$$

(Note that this applies even if multiple edges and loops are present.)

Example

What are the degrees and neighborhoods of the vertices in these graphs?



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Prove that ...

Theorem

An undirected graph has an even number of odd-degree vertices.

...

If the number of vertices in an undirected graph is an odd number, then there exists an even-degree vertex.

...

If the number of vertices in an undirected graph is an odd number, then the number of vertices with even degree is odd.

...

If the number of vertices in an undirected graph is an even number, then the number of vertices with even degree is even.

Exercise

Exercise (1)

Is there any undirected simple graph including four vertices that their degrees are respectively 1, 1, 2, 2 ?

Exercise (2)

Is there any undirected simple graph including six vertices that their degree are respectively 2, 3, 3, 3, 3, 3 ?

Exercise (3)

An undirected simple graph G has 15 edges, 3 vertices of degree 4 and other vertices having degree 3. What is the number of vertices of the graph G?

Exercise (4)

Is it possible that each person has exactly 5 friends in the same group of 9 people ?

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Exercise

Exercise (6)

Give an undirected simple graph $G = (V, E)$ with $|V| = n$, show that

a) $\forall v \in V$, $\deg(v) < n$,

b) there does not exist simultaneously both a vertex of degree 0 and a vertex of degree $(n - 1)$,

c) deduce that there are at least two vertices of the same degree.

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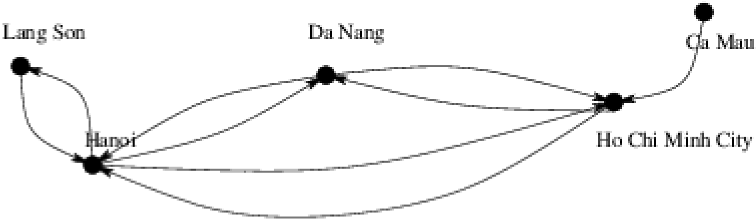
Directed Graph

Definition (Directed Graph (đồ thị có hướng))

A directed graph G is a pair of (V, E), in which:

- V – nonempty set of vertices
- E – set of directed edges (*cạnh có hướng*, arcs)

A directed edge start at u and end at v is denoted as (u, v).



Terminologies for Directed Graph

Neighborhood

In an directed graph $G = (V, E)$,

- u is said to be **adjacent to** (*nối tới*) v and v is said to be adjacent from (*được nối từ*) u if (u, v) is an arc of G, and
- u is called **initial vertex** (*đỉnh đầu*) of (u, v)
- v is called **terminal** (*đỉnh cuối*) or **end vertex** of (u, v)
- the initial vertex and terminal vertex of a loop are the same.

The degree of a vertex

In a graph G with directed edges:

- **in-degree** (*bậc vào*) of a vertex v, denoted by $\deg^-(v)$, is the number of edges with v as their terminal vertex.
- **out-degree** (*bậc ra*) of a vertex v, denoted by $\deg^+(v)$, is the number of edges with v as their initial vertex.

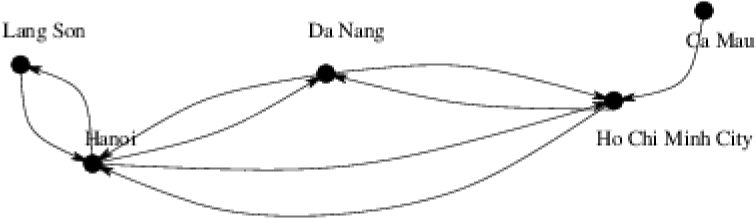
Note: a loop at a vertex contributes 1 to both the in-degree and the out-degree of this vertex.

Basic Theorem

Theorem

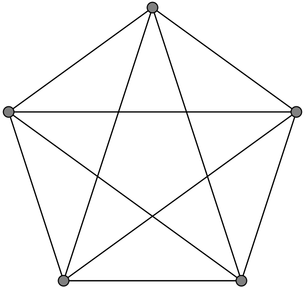
Let $G = (V, E)$ be a graph with directed edges. Then

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|.$$

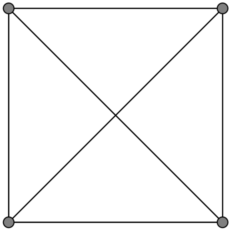


Complete Graphs

A complete graph (*đồ thị đầy đủ*) on n vertices, K_n, is a simple graph that contains exactly one edge between each pair of distinct vertices.



K_5



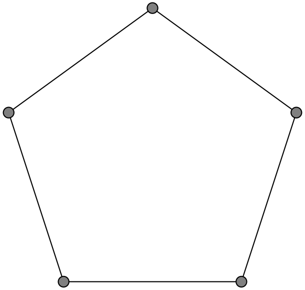
K_4

Exercise

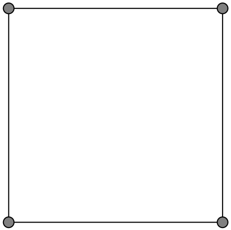
What is the largest number of edges a undirected simple graph with 10 vertices can have? K_n can have?

Cycles

A cycle (*đồ thị vòng*) C_n, $n \geq 3$, consists of n vertices $v_1, v_2, \ldots, v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_{n-1}, v_n\}$, and $\{v_n, v_1\}$.



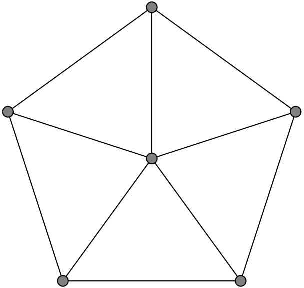
C_5



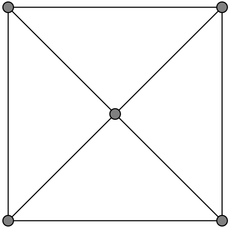
C_4

Wheels

We obtain a wheel (*đồ thị hình bánh xe*) W_n when we add an additional vertex to a cycle C_n, for $n \geq 3$, and connect this new vertex to each of the n vertices in C_n.



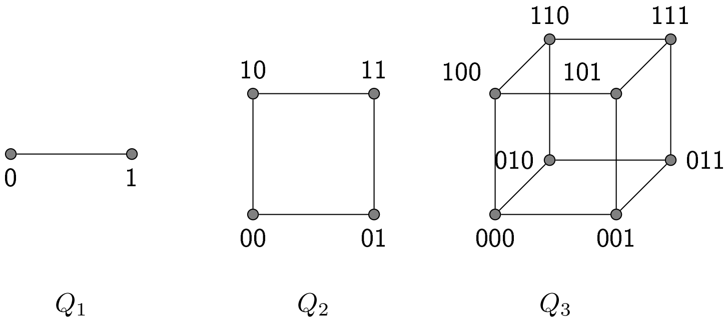
W_5



W_4

n-cube

An n-dimensional hypercube (*khối* n *chiều*), Q_n, is a graph that has vertices representing the 2^n bit strings of length n. Two vertices are adjacent iff the bit strings that they represent differ in exactly one bit position.



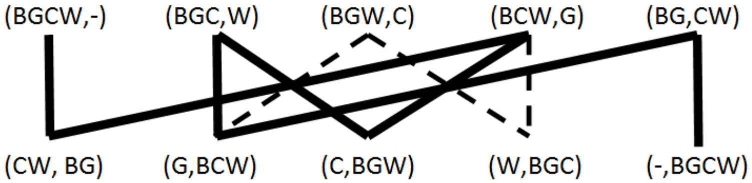
What's about Q_4?

Applications of Special Graphs

- Local networks topologies
  - Star, ring, hybrid
- Parallel processing
  - Linear array
  - Mesh network

Graph

- One goat, a cabbage and a wolf are on a side of river; a boatman wishes to transport them to the other side but, his boat being too small, he could transport only one of them at once.
- How does he proceed not to leave them together without surveillance: the wolf and the goat, as well as the goat and the cabbage?



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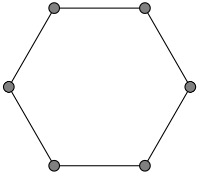
Bipartite Graphs

Definition

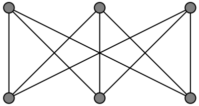
A simple graph G is called bipartite (*đồ thị phân đôi*) if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2 (so that no edge in G connects either two vertices in V_1 or two vertices in V_2)

Example

C_6 is bipartite



C_6



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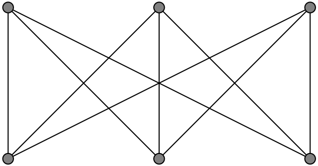
Complete Bipartite Graphs

Definition

A complete bipartite $K_{m,n}$ is a graph that

- has its vertex set partitioned into two subsets of m and n vertices, respectively,
- with an edge between two vertices iff one vertex is in the first subset and the other is in the second one

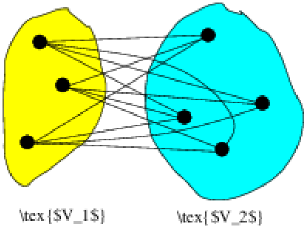


$K_{3,3}$

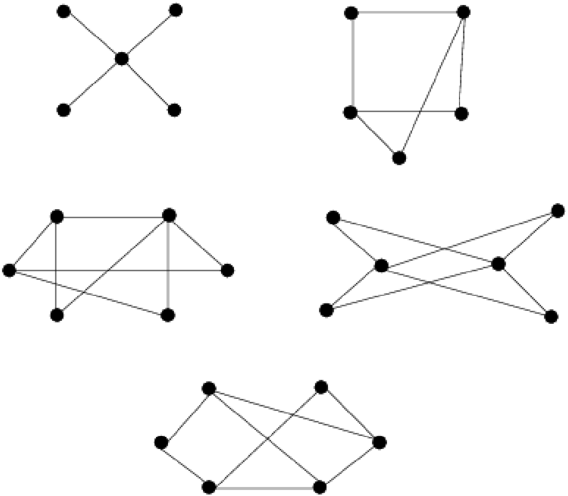
Bipartite graphs

Example (Bipartite graphs?)

- C_6
- C_n
- K_3
- K_n
- the following graph



Bipartie graph



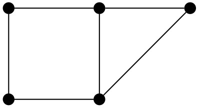
New Graph From Old

Definition

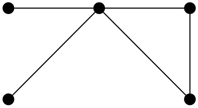
A **subgraph** (*đồ thị con*) of a graph $G = (V, E)$ is a graph $H = (W, F)$ where $W \subseteq V$ and $F \subseteq E$.

Definition

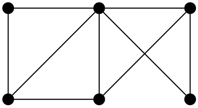
The **union** (*hợp*) of two simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a simple graph with vertex set $V_1 \cup V_2$ and edge set $E_1 \cup E_2$. The union of G_1 and G_2 is denoted by $G_1 \cup G_2$.



G_1

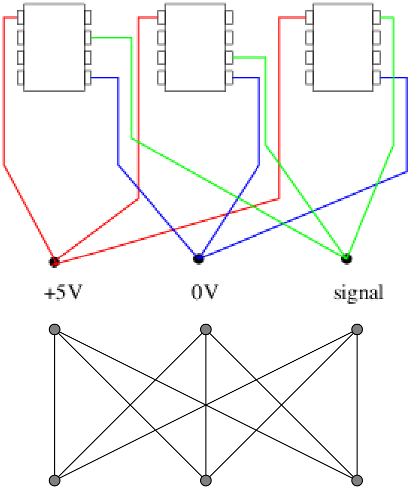


G_2



$G_1 \cup G_2$

Planar Graphs



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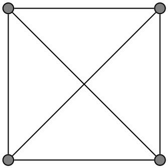
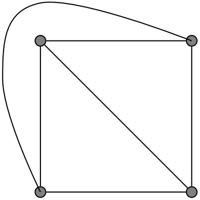
Planar Graphs

Definition

- A graph is called planar (*phẳng*) if it can be drawn in the plane without any edges crossing.
- Such a drawing is called planar representation (*biểu diễn phẳng*) of the graph.



K_4

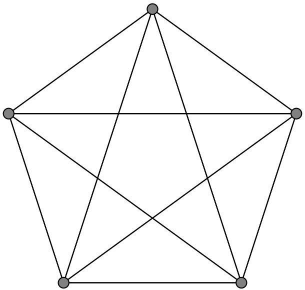


K_4 with no crossing

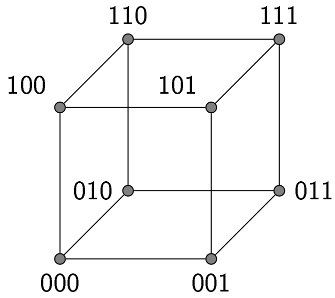
Example

Example

- Is K_5 planar?
- Is Q_3 planar?



K_5



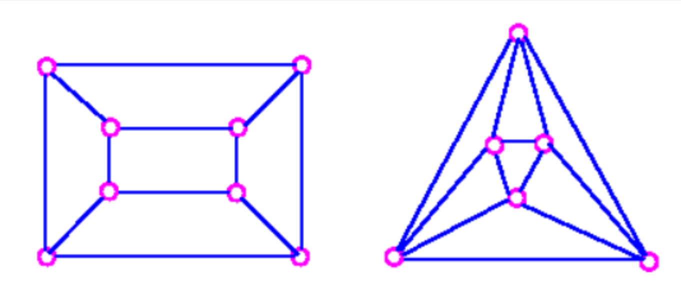
Q_3

Important Corollaries

Corollary

- *If* G *is a* **connected planar simple graph** *with* e *edges and* v *vertices where* $v \geq 3$, *then* $e \leq 3v - 6$.
- *If* G *is a* **connected planar simple graph** *with* e *edges and* v *vertices where* $v \geq 3$, *and no circuits of length* 3, *then* $e \leq 2v - 4$.

Example



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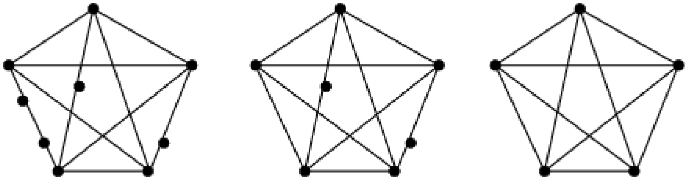
Elementary Subdivision

Definition

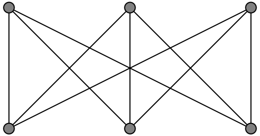
- Given a planar graph G, an elementary subdivision (*phân chia sơ cấp*) is removing an edge $\{u, v\}$ and adding a new vertex w together with edges $\{u, w\}$ and $\{w, v\}$.
- Graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are called homeomorphic (*đồng phôi*) if they can obtained from the same graph by a sequence of elementary subdivisions.



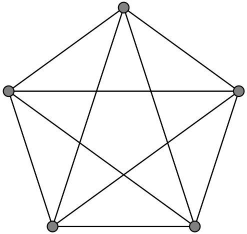
Kuratowski's Theorem

Theorem

A graph is nonplanar iff it contains a subgraph homeomorphic to $K_{3,3}$ *or* K_5.



$K_{3,3}$
Non-planar



K_5
Non-planar

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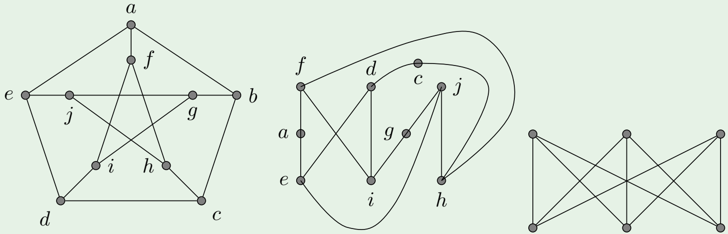
Example

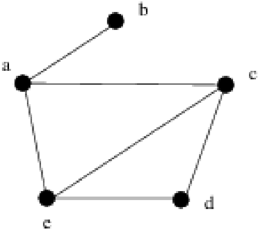
A graph is nonplanar iff it contains a subgraph homeomorphic to $K_{3,3}$.

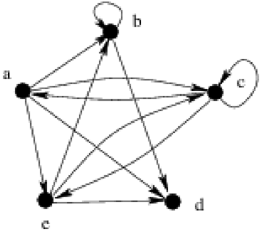


Adjacency Lists (Danh sách kề)

Vertex	Adjacent vertices
a	b, c, e
b	a
c	a, d, e
d	c, e
e	a, c, d

Initial vertex	Terminal vertices
a	b, c, d, e
b	b, d
c	a, c, e
d	c, e
e	b, c, d



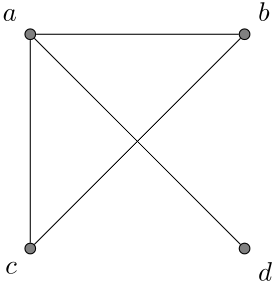


Adjacency Matrices

Definition

Adjacency matrix (*Ma trận kề*) A_G of $G = (V, E)$

- Dimension $|V| \times |V|$
- Matrix elements
  $$a_{ij} = \begin{cases} 1 & \text{if } (v_i, v_j) \in E \\ 0 & \text{otherwise} \end{cases}$$



$$\begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \left[\begin{array}{cccc} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Examples

Example

Give the graph defined by the following adjacency matrix

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 0 & 1 & 1 & 0 \\ B & 0 & 0 & 0 & 1 & 0 \\ C & 1 & 0 & 0 & 1 & 0 \\ D & 1 & 1 & 1 & 0 & 1 \\ E & 0 & 0 & 0 & 1 & 0 \end{array}$$

Adjacency Matrices

Example

Give the directed graph defined by the following adjacency matrix

$$\begin{array}{c c} & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

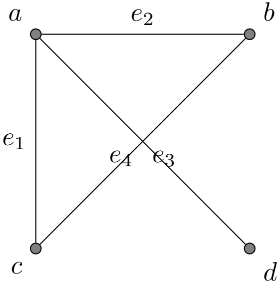
Incidence Matrices

Definition

Incidence matrix (*ma trận liên thuộc*) M_G of $G = (V, E)$

- Dimension $|V| \times |E|$
- Matrix elements

$$m_{ij} = \begin{cases} 1 & \text{if } e_j \text{ is incident with } v_i \\ 0 & \text{otherwise} \end{cases}$$

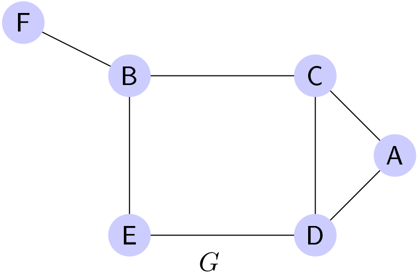


$$\begin{array}{c c} & \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Examples

Example

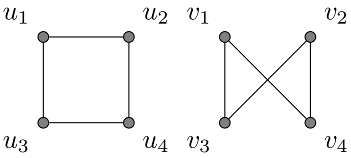
Give incidence matrix according to the following graph



Graph Isomorphism

Definition

$G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are **isomorphic** (*đẳng cấu*) if there is a **one-to-one function** f from V_1 to V_2 with the property that a and b are adjacent in G_1 iif $f(a)$ and $f(b)$ are adjacent in G_2, for all a and b in V_1. Such a function f is called an **isomorphism** (*một đẳng cấu*).
(i.e. there is a one-to-one correspondence between vertices of the two graphs that preserves the adjacency relationship.)

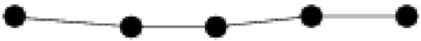


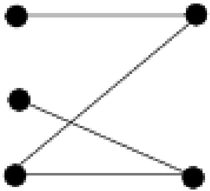
Isomorphism function $f : U \longrightarrow V$ with

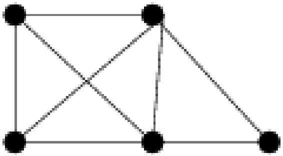
$$f(u_1) = v_1 \qquad f(u_2) = v_4$$
$$f(u_3) = v_3 \qquad f(u_4) = v_2$$

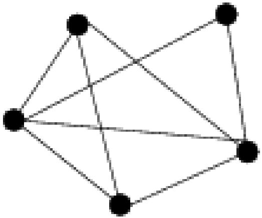
Contents

Isomorphism ?

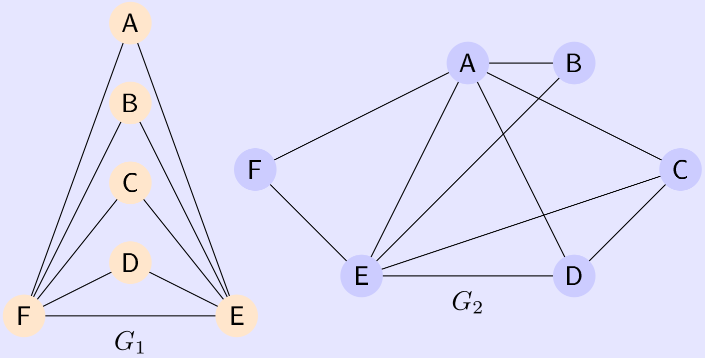




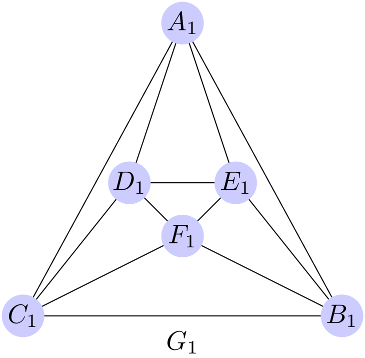


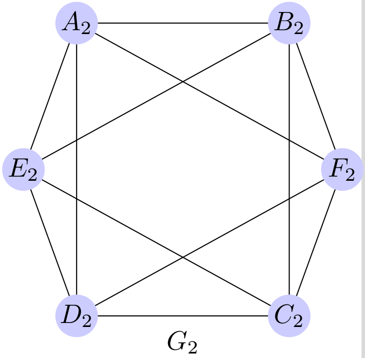


Isomorphism?



Isomorphism?





Isomorphism

Are the simple graphs with the following adjacency matrices isomorphic ?

1. $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
2. $\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$
3. $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$

Isomorphism

Determine whether the graphs (without loops) with the incidence matrices are isomorphic.

- $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$
- $\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix}$
- Extend the definition of isomorphism of simple graphs to undirected graphs containing loops and multiple edges.
- Define isomorphism of directed graphs

Revision

Đồ thị G_1 và G_2 tương ứng với các ma trận liền kề và liên thuộc dưới đây.

$$(G_1)\quad \begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & 0 & 1 & 1 & 1 & 1 \\ B & 1 & 0 & 0 & 1 & 0 \\ C & 1 & 0 & 0 & 1 & 0 \\ D & 1 & 1 & 1 & 0 & 1 \\ E & 1 & 0 & 0 & 1 & 0 \end{array}$$

$$(G_2)\quad \begin{array}{c|ccccccc} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\ \hline A & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ B & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ C & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ D & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ E & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}$$

Hãy cho biết quan hệ của hai đồ thị G_1 và G_2.

A) Đẳng cấu

B) Không đẳng cấu

C) Thứ tự

D) Tương đương

Revision

Xét đồ thị đầy đủ K_5 và đồ thị phân đôi đầy đủ $K_{3,2}$. Khi đó ta có:

- A) $K_{3,2}$ và K_5 là không đẳng cấu.
- B) $K_{3,2}$ và K_5 có cùng số đỉnh.
- C) $K_{3,2}$ và K_5 có cùng số cạnh.
- D) $K_{3,2}$ và K_5 là đẳng cấu.
- E) Các đáp án khác đều sai.

Revision

Chọn phát biểu đúng với đồ thị đơn vô hướng (*undirected simple graph*) có n đỉnh.

- A) Bậc của một đỉnh bất kỳ trong đồ thị nhỏ hơn $n-2$.
- B) Tồn tại một đỉnh trong đồ thị có bậc là 1.
- C) Không thể chứa đỉnh cô lập.
- D) Tồn tại hai đỉnh trong đồ thị có cùng số bậc.
- E) Các đáp án khác đều sai.

Prove that ...

...

There are 101 invited people in a party.
Suppose that A knows $B \Rightarrow B$ knows A.
Prove that

1. at least one people knows an even number of other people.
2. at least two people who know the same number of people (but not considering himself).

A chess tournament of n persons plays according to the circle competition. Prove that at any moment of the tournament there are always two players having identical number of games played. And if $n \geq 4$, at any intermediate moment of the tournament, there are always two players having identical number of games that they are the winner.

Revision

In a tournament with n teams participated $(n \geq 4)$, $n+1$ competition games were happening. Prove that there exists a team that has played at least three matches.

With any four of the n people $(n \geq 4)$, there exists a person who knows the three others. Prove that there exists a person who knows all $n-1$ others.

In a party of 6 people, prove that there are 3 people who know each other or 3 people who do not know each other.

During a summer vacation, 7 friends are vacationing away. They promised each other that during the holidays each person must write to exactly three of them. Prove that there is someone who does not write back to the his sender.

Revision

Trong kỳ Hoa Sơn luận võ, năm vị cao thủ đã gặp nhau để xác định danh hiệu đệ nhất: Đông Tà, Tây Độc, Nam Đế, Bắc Cái và Trung Thần Thông.
Để phân biệt thắng thua thì họ đấu từng cặp đôi và không giới hạn thời gian. Nhà vô địch là người có nhiều trận thắng nhất.
Đông Tà không thể đánh bại Nam Đế, nhưng ông ta đã đánh bại Tây Độc.
Do dùng nhiều sức trong mỗi trận đấu nên Nam Đế chỉ thắng hai trận đầu tiên.
Bắc Cái chỉ thắng được Nam Đế.
Tây Độc không thể chiến thắng Trung Thần Thông, nhưng lại chiến thắng Nam Đế và Bắc Cái.
Riêng Trung Thần Thông chỉ bị thất bại một trận đấu.
Hãy cho biết Trung Thần Thông đã bị đánh bại bởi vị nào?

A) Nam Đế
B) Nam Đế hoặc Đông Tà
C) Đông Tà
D) Tây Độc

Revision

Một dự án gồm các công việc A, B, C, D, E, F và G cần thực hiện. Thời lượng (theo ngày) cần thiết để xử lý các công việc lần lượt là

p_A	p_B	p_C	p_D	p_E	p_F	p_G
5	2	6	7	9	3	2

Ta ký hiệu

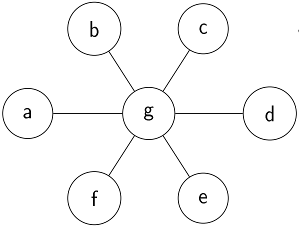
$$X_1 + X_2 + \ldots + X_n \preceq Y_1 + Y_2 + \ldots + Y_m$$

biểu diễn các công việc X_i $(i = 1, \ldots, n)$ đều cần hoàn thành trước khi khởi động các công việc Y_k $(k = 1, \ldots, m)$.
Xét thời gian bắt đầu khởi động dự án là 0. Dự án được gọi là "kết thúc" khi tất cả các công việc trong dự án đều hoàn thành.
Biết rằng: $A \preceq B + C$; $B + C \preceq D$; $C \preceq F + G$; $E \preceq F$.
Hỏi dự án này sẽ kết thúc sớm nhất vào ngày nào?

Nguyen An Khuong, Tran Tuan Anh, Nguyen Tien Thinh, Mai Xuan Toan, Tran Hong Tai

Revision

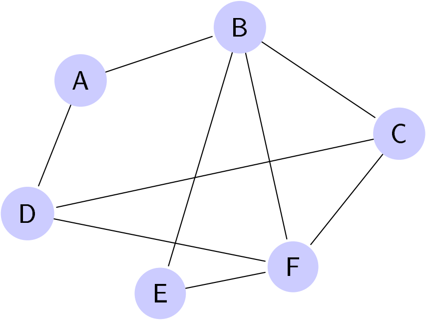
Một ban chỉ huy quân sự muốn thiết lập một mặt trận gồm các cứ điểm a, b, ..., g. Các cứ điểm này và đường nối trực tiếp giữa chúng tạo nên một đồ thị đơn vô hướng.



Do số lượng thiết giáp có giới hạn nên ban chỉ huy quyết định chỉ đồn trú thiết giáp tại một số cứ điểm mà thôi. Tuy nhiên, để đảm bảo tính tác chiến nhanh chóng nên ta yêu cầu: nếu cứ điểm nào không có đồn trú thiết giáp thì ít nhất một cứ điểm bên cạnh (đỉnh kề) phải có đơn vị thiết giáp đồn trú.

Hỏi có bao nhiêu cách triển khai thiết giáp đến các cứ điểm cho mặt trận như đồ thị bên ?

Revision



a) Hãy xác định danh sách kề, ma trận kề và ma trận liên thuộc của đồ thị trên.

b) Hãy cho biết đồ thị này có phải là đồ thị phân đôi không. Nếu có, hãy vẽ lại dưới dạng một đồ thị phân đôi.

Revision

Do khói, bụi và hơi nước bốc lên từ một miệng núi lửa bên dưới mặt sông băng Eyjafjallajokull ở Iceland vào ngày thứ tư (14/04/2010), hơn 90.000 chuyến bay ở châu Âu đã bị hủy. Đây cũng là một minh chứng về sự bất ổn của thiên nhiên có thể gây tổn hại tới công việc kinh doanh toàn cầu.

Để giảm thiểu thiệt hại về kinh tế, cơ quan quản lý tối ưu hóa và lập lịch đường bay EuroControl cố gắng tiếp tục duy trì một số đường bay đi và đến Việt Nam, liên quan đến các thành phố lớn như: Hồ Chí Minh (A), Paris (B), Berlin (C), và London (D). Tuy nhiên, do ảnh hưởng của môi trường thiên nhiên nói trên, chỉ có một vài chuyến bay có thể hoạt động: từ A hướng đến B và D, từ B hướng đến C, từ C hướng đến A và D, từ D hướng đến B.

a) Hãy vẽ đồ thị có hướng tương ứng.

b) Viết ma trận kề M cho đồ thị có hướng này

c) Hãy tính $M + M^2 + M^3$ và cho biết ý nghĩa của ma trận này.

Nguyen An Khuong, Tran Tuan Anh, Nguyen Tien Thinh, Mai Xuan Toan, Tran Hong Tai

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Revision

Hai lớp được định nghĩa như các hình bên dưới trái. Hãy vẽ đồ thị biểu diễn hàm thành viên có thể được gọi từ một hàm.

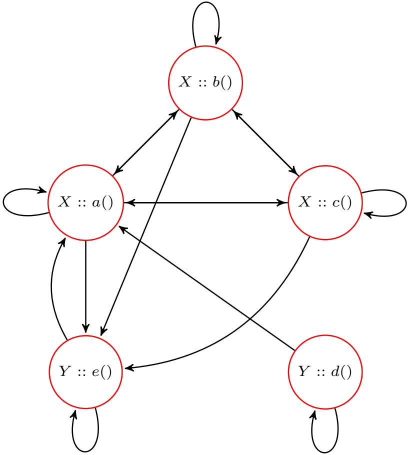
class X

public:
X();
~ X();
a();
protected:
b();
private:
c();

class Y

public:
Y();
~ Y();
e();
private:
d();

(*Chú ý* : Một cung từ hàm u đến hàm v biểu diễn rằng v có thể được gọi bởi u.) Ví dụ.



Có bao nhiêu đường đi đơn khác nhau từ đỉnh X::a() đến đỉnh Y::d() ?