Exercise 9
Connectivity

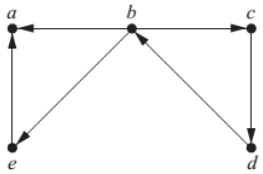
1 Introduction

2 Exercise

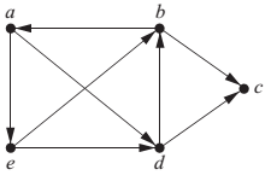
Question 1.

Determine whether each of these graphs is strongly connected and if not, whether it is weakly connected.

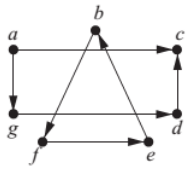
a)



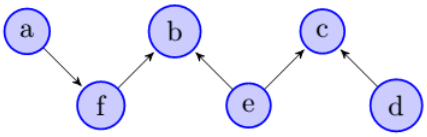
b)



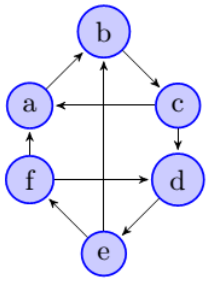
c)



d)



e)



Question 2.

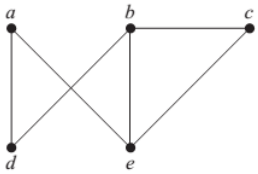
Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are paths?

a) a, e, b, c, b

b) a, e, a, d, b, c, a

c) e, b, a, d, b, e

d) c, b, d, a, e, c



Question 3.

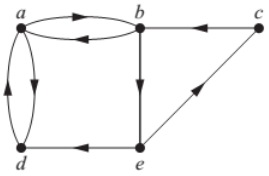
Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are paths?

a) a, b, e, c, b

b) a, d, a, d, a

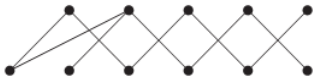
c) a, d, b, e, a

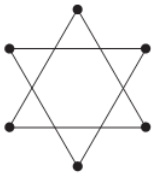
d) a, b, e, c, b, d, a



Question 4.
Whether the given graph is connected.

a) 

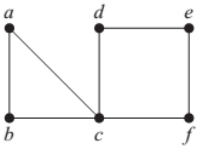
b) 

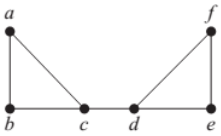
c) 

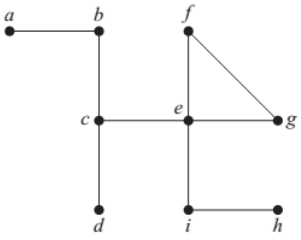
Question 5.
How many connected components does each of the graphs in above Exercise have?

Question 6.
Find the number of paths of length n between two different vertices in K_4 if n is a) 2 b) 3

Question 7.
Find all the cut vertices of the given graph

a) 

b) 

c) 

Question 8.
Find all the cut edges of above graphs

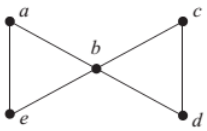
Question 9.
Find all the cut vertices, cut edges of the graphs

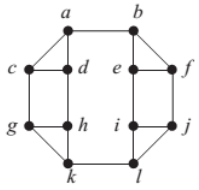
a) C_n, where $n \geq 3$

b) W_n where $n \geq 3$

c) $K_{m,n}$ where $m \geq 2, n \geq 2$

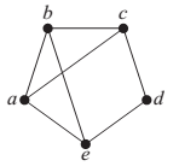
Question 10.
For each of these graphs, find $\kappa(G), \lambda(G)$

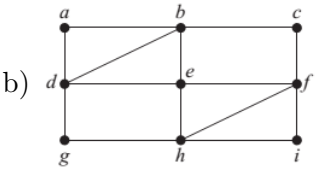
a) 

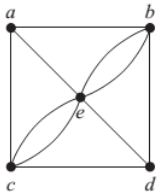
b) 

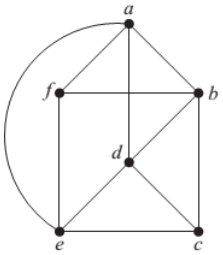
Question 11.
Construct a graph G with $\kappa(G) = 1, \lambda(G) = 2$, and $\min_{v \in V} deg(v) = 3$.

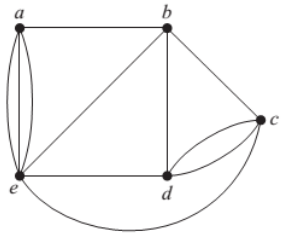
Question 12.
Determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

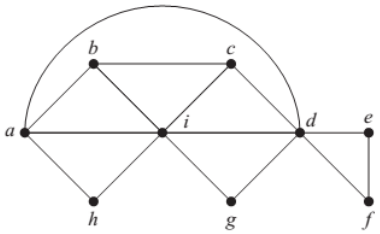
a) 

b) 

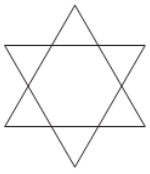
c) 

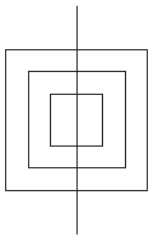
d) 

e) 

f) 

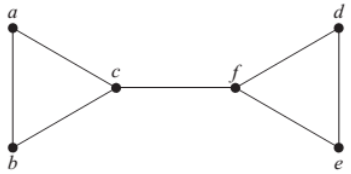
Question 13.
Determine whether the picture shown can be drawn with a pencil in a continuous motion without lifting the pencil or retracing part of the picture.

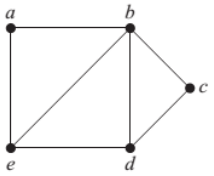
a) 

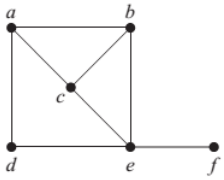
b) 

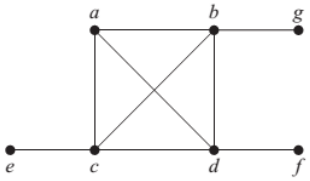
Question 14.
For which values of n do these graphs have an Euler circuit? a) K_n b) C_n c) W_n d) Q_n

Question 15.
Determine whether the given graph has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.

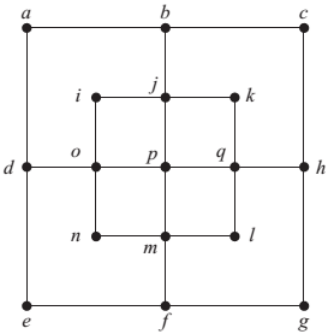
a) 

b) 

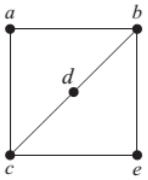
c) 

d) 

e)



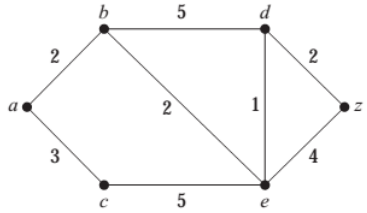
f)



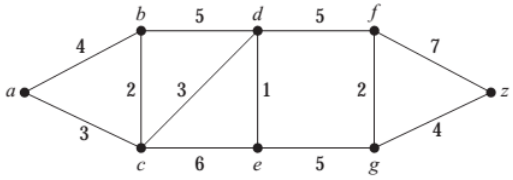
Question 16.
For which values of n do these graphs have an Hamilton circuit? a) K_n b) C_n c) W_n

Question 17.
Find the length of a shortest path between a and z in the given weighted graph.

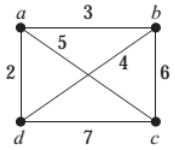
a)



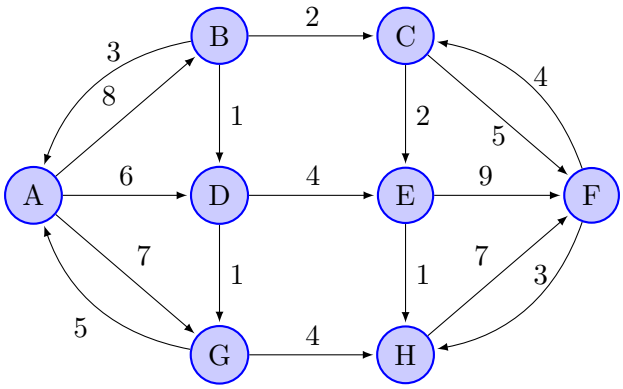
b)



Question 18.
Solve the traveling salesperson problem for this graph by finding the total weight of all Hamilton circuits and determining a circuit with minimum total weight.

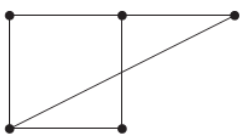


Question 19.
Use Floyd-Warshall to find shortest path between two vertices in a weighted graph.



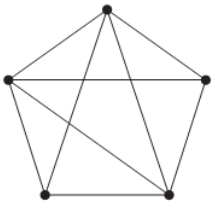
Question 20.
Draw the given planar graph without any crossings.

a)

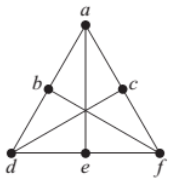


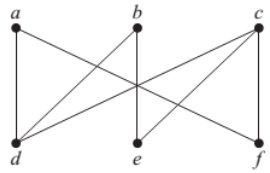
b)

c)

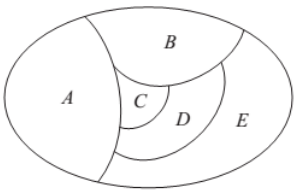


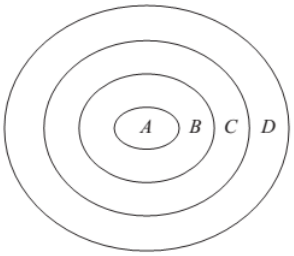
Question 21.
Determine whether the given graph is planar. If so, draw it so that no edges cross

a) 

b) 

Question 22.
Construct the dual graph for the map shown. Then find the number of colors needed to color the map so that no two adjacent regions have the same color.

a) 

b) 

Question 23.
What is the chromatic number of W_n?