Bài tp chng 5
Relation

1 Dn nhp

Trong bài tp di đây, chúng ta s làm quen vi các kin thc liên quan đn quan h. Sinh viên cn ôn li lý thuyt ca chng 5 trc khi làm bài tp bên di.

2 Bài tp cn gii

Exercise 1.
For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

a) $\{(2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)\}$

b) $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$

c) $\{(2, 4), (4, 2)\}$

d) $\{(1, 2), (2, 3), (3, 4)\}$

e) $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$

f) $\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

Exercise 2.
Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

a) $x + y = 0$

b) $x = \pm y$

c) $x - y$ is a rational number.

d) $x = 2y$

e) $xy \geq 0$

f) $xy = 0$

g) $x = 1$

h) $x = 1$ or $y = 1$

i) $x \equiv y \pmod 7$

Exercise 3.
Let $R_1 = \{(1,2),(2,3),(3,4)\}$ and $R_2 = \{(1,1),(1,2),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(3,4)\}$ be relations from $\{1,2,3\}$ to $\{1,2,3,4\}$. Find

a) $R_1 \cup R_2$

b) $R_1 \cap R_2$

c) $R_1 - R_2$

d) $R_2 - R_1$

Exercise 4.
Let R_1 and R_2 be the "congruent modulo 3" and the "congruent modulo 4" relations, respectively, on the set of integers. That is, $R_1 = \{(a,b)|a \equiv b(\mod 3)\}$ and $R_2 = \{(a,b)|a \equiv b(\mod 4)\}$. Find

a) $R_1 \cup R_2$

b) $R_1 \cap R_2$

c) $R_1 - R_2$

d) $R_2 - R_1$

Exercise 5.
Let R be the relation on the set $\{1,2,3,4,5\}$ containing the ordered pairs (1,1), (1,2), (1,3), (2,3), (2,4), (3,1), (3,4), (3,5), (4,2), (4,5), (5,1), (5,2) and (5,4). Find R^2, R^3.

Exercise 6.
For each of these relations on the set {1,2,3,4}, let

$R_1 = \{(1,1),(2,2),(3,3)\}$
$R_2 = \{(4,2),(2,4),(2,2),(2,3),(3,2),(3,3),(4,4)\}$
$R_3 = \{(1,1),(1,2),(2,1),(2,2),(3,3)\}$
$R_4 = \{(1,1),(1,3),(2,2),(2,3),(3,1),(3,2),(3,3),(4,4)\}$
$R_5 = \{(4,4),(4,1),(4,2),(1,4),(1,1),(1,2),(2,4),(2,2),(3,3)\}$
$R_6 = \{(1,2)\}$.

Find

a) $R_1 \circ R_2, R_1 \circ R_3, R_1 \circ R_4, R_1 \circ R_5, R_1 \circ R_6$

b) $R_2 \circ R_3 \circ R_4 \circ R_6$

c) $(R_3)^2$

d) $(R_3)^4$

e) reflexive closure ca R_2

f) symmetric closure ca $R_1 \circ R_2$

g) transitive closure ca R_6

Exercise 7.
Give an example of a relation on a set {1,2,3,4} that is

a) reflexive and symmetric, but not transitive.

b) reflexive and transitive, but not symmetric.

c) transitive and symmetric, but not reflexive.

Exercise 8.

a) How many relations are there on the set $\{a, b, c, d\}$?

b) How many relations are there on the set $\{a, b, c, d\}$ that contain the pair (a, a)?

Exercise 9.
List the ordered pairs in the relations on $\{1, 2, 3, 4\}$ corresponding to these matrices (where the rows and columns correspond to the integers listed in increasing order).

a) $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

Exercise 10.
Draw the directed graph that represents the relation $\{(a,a),(a,b),(b,c),(c,b),(c,d),(d,a),(d,b)\}$

Exercise 11.
Let R be the relation that contains the pair (a,b) if a and b are cities such that there is a direct non-stop airline flight from a to b. When is (a,b) in

a) R^2 ?

b) R^3 ?

Exercise 12.
Let R be the relation on the set $\{0,1,2,3\}$ containing the ordered pairs (0,1), (1,1), (1,2), (2,0), (2,2) and (3,0). Find the

a) reflexive closure of R.

b) symmetric closure of R.

c) transitive closure of R.

Exercise 13.
Let R be the relation $\{(a,b)|a \text{ divides } b\}$ on the set of integers. What is the symmetric closure of R?

Exercise 14.
Find the smallest relation containing the relation $\{(1,2),(1,4),(3,3),(4,1)\}$ that is

a) reflexive and transitive

b) symmetric and transitive

c) reflexive, symmetric and transitive

Exercise 15.
Which of these relations on $\{0,1,2,3\}$ are equivalence relations? Determine the properties of an equivalence relation that the others lack.

a) $\{(0,0),(1,1),(2,2),(3,3)\}$

b) $\{(0,0),(0,2),(2,0),(2,2),(2,3),(3,2),(3,3)\}$

c) $\{(0,0),(1,1),(1,2),(2,1),(2,2),(3,3)\}$

d) $\{(0,0),(1,1),(1,3),(2,2),(2,3),(3,1),(3,2),(3,3)\}$

Exercise 16.
Which of these collections of subsets are partitions of {1, 2, 3, 4, 5, 6}?

a) {1,2}, {2,3,4}, {4,5,6}

b) {1}, {2, 3, 6}, {4}, {5}

c) {2,4,6}, {1,3,5}

d) {1,4,5}, {2,6}

Exercise 17.
List the ordered pairs in the equivalence relations produced by these partitions of {0, 1, 2, 3, 4, 5}.

a) {0}, {1,2}, {3,4,5}

b) {0,1}, {2,3}, {4,5}

Exercise 18.
Let R be the relation on the set of real numbers such that aRb if and only if $a - b$ is an integer.

a) Is R an equivalence relation?

b) What is the equivalence class of 1 for this equivalence relation?

c) What is the equivalence class of 1/2 for this equivalence relation?

Exercise 19.
Let R be the relation on the set $A = \{1, 2, 3, 4, 5\}$ such that

$$(a, b)R(c, d) \Leftrightarrow a + b = c + d$$

a) Is R an equivalence relation?

b) What is the equivalence class of [(1,3)], [(2,4)], [(1,1)]?

c) Find the partition of set A formed by the equivalence classes of part b.

Exercise 20.
Which of these relations on {0, 1, 2, 3} are partial orderings? Determine the properties of a partial ordering that the others lack.

1. {(0,0), (1,1), (2,2), (3,3) }
2. {(0,0), (1,1), (2,0), (2,2), (2,3), (3,2), (3,3)}
3. {(0,0), (1,1), (1,2), (2,2), (3,3)}
4. {(0,0), (1,1), (1,2), (1,3), (2,2), (2,3), (3,3)}

5. $\{(0,0), (0,1), (0,2), (1,0), (1,1), (1,2), (2,0), (2,2), (3,3)\}$

Exercise 21.
Which of these are posets?

a) $(\mathbb{Z}, =)$

b) $(\mathbb{Z}, \neq)$

c) $(\mathbb{Z}, \geq)$

d) $(\mathbb{Z}, \nmid)$