Bài tập chương 7
Discrete Probability

1 Dẫn nhập

Trong chương bài tập này, chúng ta sẽ luyện tập với bài tập cơ bản về Xác suất rời rạc. Sinh viên cần ôn lại lý thuyết của chương 7 trước khi làm bài tập bên dưới.

2 Bài tập mẫu

Exercise 1.
Xác suất để một chọn ngẫu nhiên một ngày mà ngày hôm đó là ngày trong tháng 4 là bao nhiêu? Biết năm được chọn là năm nhuận (366 ngày).
Lời giải.
Khi chọn một ngày trong năm, chúng ta có thể chọn trong $|S| = 366$ *ngày.*
Số ngày có thể chọn ra thỏa mãn đó là ngày trong tháng 4 là $|E| = 30$ *ngày (vì tháng 4 có 30 ngày).*
Vậy xác suất để chọn được một ngày trong tháng 4 là $30/366 \approx 0.08 = 8\%$. □

Exercise 2.
Xác suất để chúng ta có được 4 mặt số khi thảy đồng xu cân bằng 5 lần là bao nhiêu, nếu lần thảy đầu tiên cho ra mặt số?
Lời giải.
Đây là một dạng bài toán tìm xác suất có điều kiện.
Gọi F *là sự kiện lần thảy đầu tiên xuất hiện mặt số,* E *là sự kiện có 4 mặt số sau tổng cộng 5 lần thảy.*
Sau khi đã thảy lần đầu tiên được mặt số, 4 lần thảy tiếp theo để thành công phải là một trong các trường hợp {SSSH, SSHS, SHSS, HSSS}, trong số 2^4 *khả năng xảy ra.*
Vậy xác suất cần tìm là $\frac{4}{2^4} = \frac{1}{4}$. □

3 Bài tập cần giải

Exercise 3.

a) What is the probability that a randomly selected integer chosen from the first 100 positive integers is odd?

b) What is the probability that the sum of the numbers on two dice is even when they are rolled?

c) What is the probability that a fair die never comes up an even number when it is rolled six times?

d) What is the probability that a positive integer not exceeding 100 selected at random is divisible by 3?

Exercise 4.
What is the probability that Abby, Barry, and Sylvia win the first, second, and third prizes, respectively, in a drawing if 200 people enter a contest and

a) no one can win more than one prize.

b) winning more than one prize is allowed.

Exercise 5.
Suppose that E and F are events such that $p(E) = 0.7$ and $p(F) = 0.5$. Show that $p(E \cup F) \geq 0.7$ and $p(E \cap F) \geq 0.2$.

Exercise 6.
There is a 1% probability for a hard drive to crash. Therefore, it has two backups, each having a 2% probability to crash, and all three components are independent of each other. The stored information is lost only in an unfortunate situation when all three devices crash. What is the probability that the information is saved?

Exercise 7.
A problem on a multiple-choice quiz is answered correctly with probability 0.9 if a student is prepared. An unprepared student guesses between 4 possible answers, so the probability of choosing the right answer is 1/4. Seventy-five percent of students prepare for the quiz. If Mr. X gives a correct answer to this problem, what is the chance that he did not prepare for the quiz?

Exercise 8.
Assume that the probability a child is a boy is 0.51 and that the sexes of children born into a family are independent. What is the probability that a family of five children has

a) exactly three boys?

b) at least one boy?

c) at least one girl?

d) all children of the same sex?

Exercise 9.
What is the conditional probability that a randomly generated bit string of length four contains at least two consecutive 0s, given that the first bit is a 1? (Assume the probabilities of a 0 and a 1 are the same.)

Exercise 10.
What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up tails?

Exercise 11.
Find each of the following probabilities when n independent Bernoulli trials are carried out with probability of success p.

a) the probability of no successes

b) the probability of at least one success

c) the probability of at most one success

d) the probability of at least two successes

Exercise 12.
Suppose that Frida selects a ball by first picking one of two boxes at random and then selecting a ball from this box at random. The first box contains two white balls and three blue balls, and the second box contains four white balls and one blue ball. What is the probability that Frida picked a ball from the first box if she has selected a blue ball?

Exercise 13.
Suppose that a Bayesian spam filter is trained on a set of 500 spam messages and 200 messages that are not spam. The word ”exiting” appears in 40 spam messages and in 25 messages that are not spam. Would an incoming message be rejected as spam if it contains the word ”exiting” and the threshold for rejecting spam is 0.9?

Exercise 14.
Suppose that 8% of the patients tested in a clinic are infected with HIV. Furthermore, suppose that when a blood test for HIV is given, 98% of the patients infected with HIV test positive and that 3% of the patients not infected with HIV test positive. What is the probability that

a) a patient testing positive for HIV with this test is infected with it?

b) a patient testing negative for HIV with this test is infected with it?

Exercise 15.

a) What is the expected number of heads that come up when a fair coin is flipped 5 times?

b) What is the variance of the number of heads that come up when a fair coin is flipped 5 times?

Exercise 16.
The final exam of a discrete mathematics course consists of 50 true/false questions, each worth two points, and 25 multiple-choice questions, each worth four points. The probability that Linda answers a true/false question correctly is 0.9, and the probability that she answers a multiple-choice question correctly is 0.8. What is her expected score on the final?

Exercise 17.
A computer virus is trying to corrupt two files. The first file will be corrupted with probability 0.4. Independently of it, the second file will be corrupted with probability 0.3.

a) Compute the probability mass function (pmf) of X, the number of corrupted files.

b) Draw a graph of its cumulative distribution function (cdf).

Exercise 18.
Every day, the number of traffic accidents has the probability mass function independently of other

x	0	1	2	more than 2
$P(x)$	0.6	0.2	0.2	0

days. What is the probability that there are more accidents on Friday than on Thursday?

Exercise 19.
We throw a coin until a head turns up for the second time, where p is the probability that a throw results in a head and we assume that the outcome of each throw is independent of the previous outcomes. Let X be the number of times we have thrown the coin.

a) Determine $P(X = 2), P(X = 3)$, and $P(X = 4)$.

b) Show that $P(X = n) = (n - 1)p^2(1 - p)^{n-2}$ for $n \geq 2$.

Exercise 20.
Let X_n be the random variable that equals the number of tails minus the number of heads when n fair coins are flipped.

a) What is the expected value of X_n?

b) What is the variance of X_n?