Chapter 8
Introduction to Graphs

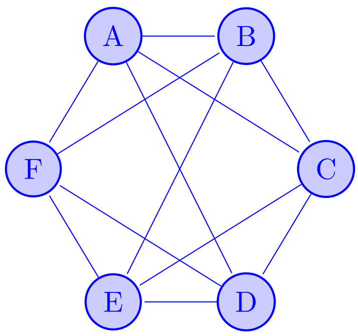
1 Dẫn nhập

Trong phần bài tập này, chúng ta sẽ làm quen với các khái niệm và định nghĩa trong lý thuyết đồ thị . Sinh viên cần ôn lại lý thuyết của chương 8 trước khi làm các bài tập bên dưới.

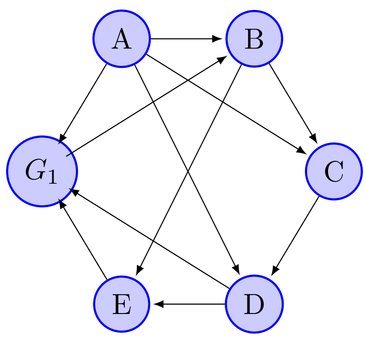
2 Bài tập mẫu

Exercise 1.
Có bao nhiêu cạnh trong một đồ thị vô hướng có 6 đỉnh, mỗi đỉnh có bậc bằng 4?
Lời giải. *Vì tổng các bậc của đồ thị là $6 \times 4 = 24$, nên 2e=24. Do đó, số cạnh trong đồ thị là e=12. Đồ thị có thể được vẽ như hình bên dưới đây:*

□



Exercise 2.
Có bao nhiêu bậc vào và bậc ra của mỗi đỉnh trong đồ thị có hướng G_1 như dưới đây?



Lời giải.

- $deg^-(A) = 0,\ deg^-(B) = 2,\ deg^-(C) = 2,\ deg^-(D) = 2,\ deg^-(E) = 2,\ deg^-(F) = 3,$
- $deg^+(A) = 4,\ deg^+(B) = 2,\ deg^+(C) = 1,\ deg^+(D) = 2,\ deg^+(E) = 1,\ deg^+(F) = 1.$

□

Exercise 3.
Liệu có tồn tại một đơn đồ thị gồm các đỉnh mà có bậc lần lượt là :

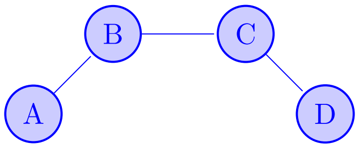
a) 1,1,2,2?

b) 1,1,2,2,3,3,3?

Nếu có hãy vẽ đồ thị đó.

Lời giải.

a) Có tồn tại đơn đồ thị gồm 4 đỉnh mà có bậc lần lượt là 1, 1, 2, và 2.
Đồ thị này được vẽ như sau:



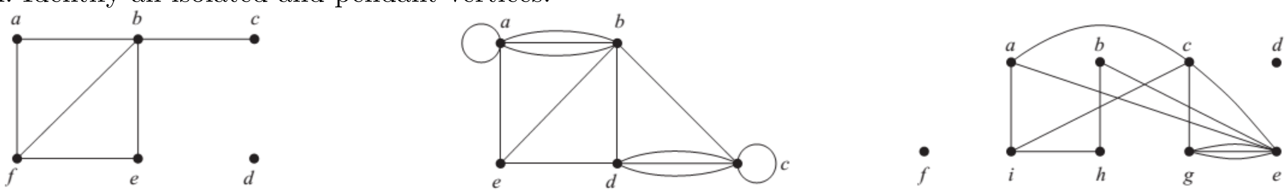
b) Không tồn tại đơn đồ thị gồm 7 đỉnh mà có bậc lần lượt là 1,1,2,2,3,3, và 3 vì tổng số bậc của tất cả các đỉnh là một số lẻ.

□

3 Bài tập cần giải

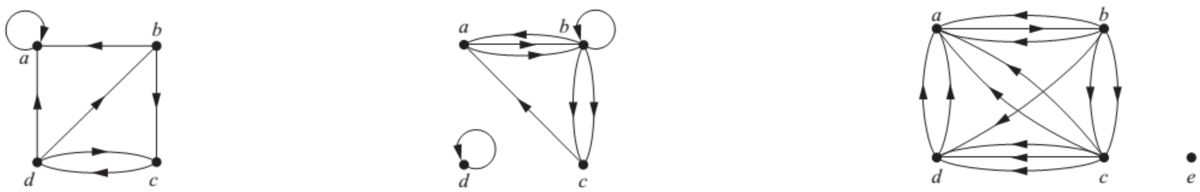
Exercise 4.
For each course at a university, there may be one or more other courses that are its prerequisites. How can a graph be used to model these courses and which courses are prerequisites for which courses? Should edges be directed or undirected? Looking at the graph model, how can we find courses that do not have any prerequisites and how can we find courses that are not the prerequisite for any other courses

Exercise 5.
Find the number of vertices, the number of edges, and the degree of each vertex in the given undirected graph. Identify all isolated and pendant vertices.



Exercise 6.
Can a simple graph exist with 15 vertices each of degree five?

Exercise 7.
Determine the number of vertices and edges and find the in-degree and out-degree of each vertex for the given directed multigraph.



Exercise 8.
Draw these graphs

a) K_7

b) $K_{1,8}$

c) $K_{4,4}$

d) C_7

e) W_7

f) Q_4

Exercise 9.
For which values of n are these graphs bipartite?

a) K_n

b) C_n

c) W_n

d) Q_n

Exercise 10.
How many vertices and how many edges do these graphs have?

a) K_n

b) C_n

c) W_n

d) $K_{m,n}$

e) Q_n

Exercise 11.
Is there any simple graph including vertices that their degree are respectively

a) 3,3,3,3,2

b) 5,4,3,2,1

c) 4,4,3,2,1

d) 4,4,3,3,3

e) 3,2,2,1,0

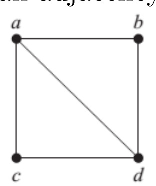
f) 1,1,1,1,1

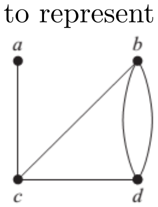
Exercise 12.
How many edges does a graph have if it has vertices of degree 4,3,3,2,2? Draw such a graph

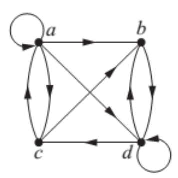
Exercise 13.
The complementary graph $\overline{G}$ of a simple graph G has the same vertices as G. Two vertices are adjacent in $\overline{G}$if and only if they are not adjacent in G. If G has 15 edges and $\overline{G}$ has 13 edges, how many vertices does G have?

Exercise 14.
If the simple graph G has v vertices and e edges, how many edges does $\overline{G}$ have?

Exercise 15.
Use an adjacency list, adjacency matrix to represent the given graph







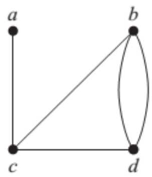
Exercise 16.

a) Draw an directed graph represented by the given adjacency matrix. $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

b) Draw an undirected graph represented by the given adjacency matrix. $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 2 \end{bmatrix}$

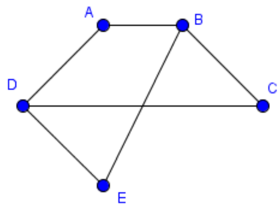
Exercise 17.

Use an incidence matrix to represent the graphs



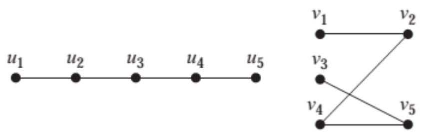
Exercise 18.

Determne whether the following graph is bipartite?

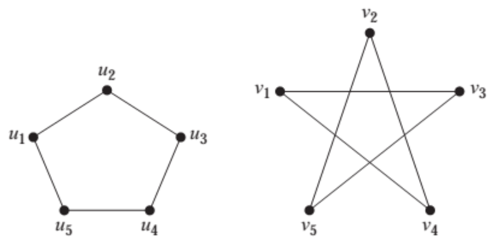


Exercise 19.
Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

a)



b)



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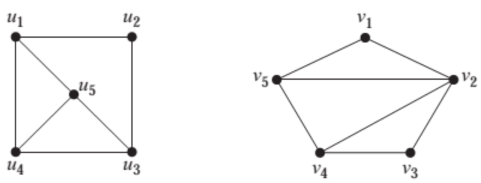

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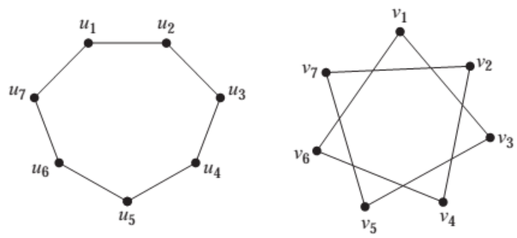
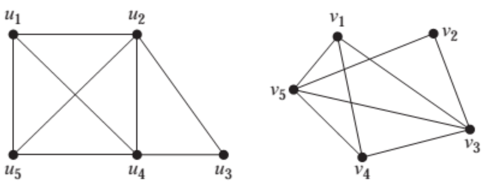
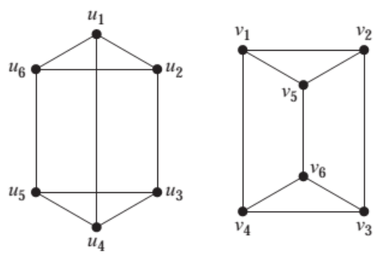


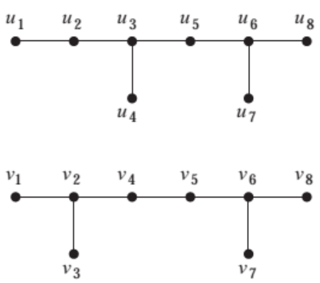
e)



f)



g)



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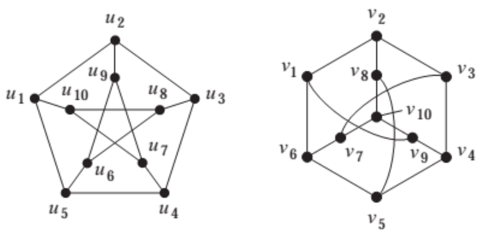
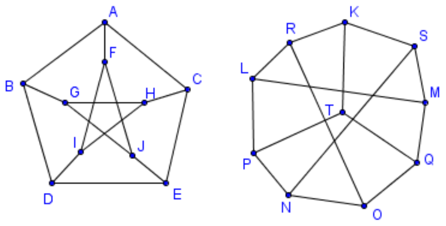
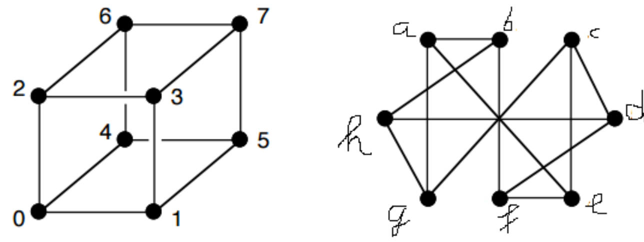
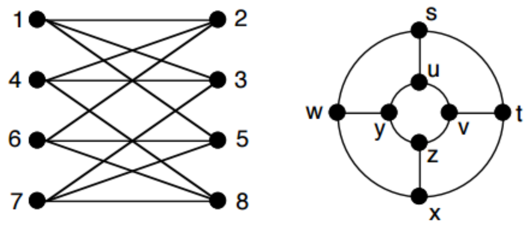
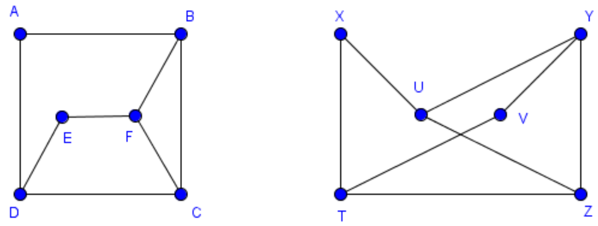


i) 

j) 

k) 

l) 

Exercise 20.

Are the simple graphs with the following adjacency matrices isomorphic?

a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

Exercise 21.

Let G be a graph with v vertices and e edges. Let M be the maximum degree of the vertices of G, and let m be the minimum degree of the vertices of G. Show that $m \leq 2e/v \leq M$.