EMTH1019 Linear Algebra & Statistics for Engineers

Tutorial 5 Vectors

During this workshop, students will work towards the following learning outcomes:

- perform basic vector arithmetic and calculate vector properties.

- calculate the dot product of two vectors, and use it to find angles, projections, and work done.

Vector arithmetic and properties

1. Given the vectors $a = [2, -1, 3]$, $b = [4, 0, -3]$ and $c = [1, -2, 2]$, find:

   (i) $a + b$

   (ii) $3a - 4c$

   (iii) The magnitude of b

   (iv) $\hat{b}$

   (v) A vector in the same direction as b but has the same length of c

2. Given the points $A(2, -3)$ and $B(4, 1)$, find the vectors $a = \overrightarrow{OA}$ and $b = \overrightarrow{AB}$.

Dot product and applications

3. In each of the following cases, find (i) $a.b$, (ii) the angle between a and b, (iii) the scalar projection of a on b, and (iv) the vector projection of a on b.

   (a) $a = [2, -4, \sqrt{5}]$, $b = [-2, 4, -\sqrt{5}]$

   (b) $a = 2i + 10j - 11k$, $b = 2i + 2j + k$

   (c) $a = i + k$, $b = i + j + k$

4. A truss takes the shape of a cube with a member on the main diagonal. Find the angle between this diagonal member and one of the edge members. (Hint: Consider the coordinates of the corners of a simple cube of side length one placed in three space.)

5. Show that the vectors $u = [2, -2, -1]$ and $v = [3, 5, -4]$ are orthogonal to one another.

6. Find the work done by a force $F = 5j$ (magnitude 5 Newtons) in moving an object along the line from the origin to the point $(1, 1, 1)$ (distance in meters).

7. How much work does it take to slide a crate 15m along a loading dock by pulling on it with a 150 Newton force at an angle of $45°$ from the horizontal?

8. Verify that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$, where $\cos \alpha$, $\cos \beta$ and $\cos \gamma$ are the direction cosines of a vector a in 3 space.