

EMTH1019 Linear Algebra & Statistics for Engineers

Tutorial 10 Lines & Planes in 3 Space

During this workshop, students will work towards the following learning outcomes:

- find the vector, parametric and cartesian equations of a line.
- determine if lines are parallel, intersecting or skew.
- find the general equation of a plane.
- find the line of intersection between planes.
- calculate distances between points, lines and planes.

Lines in \mathbb{R}^3

1. Find the Cartesian equations of the line passing through the points $(3, 5, -2)$ and $(2, 1, -1)$.
2. Find the vector and parametric equations for the line passing through the point $(2, 5, -2)$ and is parallel to the line $\frac{x+4}{3} = \frac{y}{-2} = z-3$.
3. Show that the line through the points $(7, 2, 2)$ and $(1, 4, -2)$ is parallel to the line $x = -2 + 3t$, $y = 1 - t$, $z = 4 + 2t$.
4. Find the shortest distance from the point $(0, 0, 12)$ to the line $x = 4t$, $y = -2t$, $z = 2t$.
5. Determine whether the following lines are parallel, intersecting, or skew. If they intersect find the point of intersection, or if they are skew then find the shortest distance between them.

$$(i) \quad L_1 \begin{cases} x = 3 + 2t \\ y = -1 + 4t \\ z = 2 - t \end{cases} \quad L_2 \begin{cases} x = 1 + 4\tau \\ y = 1 + 2\tau \\ z = -3 + 4\tau \end{cases}$$

$$(ii) \quad L_1 \begin{cases} x = 1 + 2t \\ y = -1 - t \\ z = 3t \end{cases} \quad L_2 \begin{cases} x = 2 - \tau \\ y = 3\tau \\ z = 1 + \tau \end{cases}$$

Planes in \mathbb{R}^3

6. Find the equation of the plane containing the points $P(0, 1, 1)$, $Q(1, 0, 1)$ and $R(1, 1, 0)$.
7. Find the equation of the plane passing through the point $P(6, 0, -2)$ and which contains the line $x = 4 - 2t$, $y = 3 + 5t$, $z = 7 + 4t$.
8. Find the parametric equations for the line of intersection of the planes $3x - 2y + z = 1$ and $2x + y - 3z = 3$.
9. For each of the following pairs of planes, decide whether they are parallel, perpendicular or neither.
 - (i) $x + z = 1$, $y + z = 1$.
 - (ii) $-8x - 6y + 2z = 1$, $z = 4x + 3y$.
 - (iii) $x + 4y - 3z = 1$, $-3x + 6y + 7z = 0$.
10. Find the angle between the planes $x + y + z = 0$ and $x + 2y + 3z = 1$.
11. Find the distance from the point $P(3, -2, 7)$ to the plane $4x - 6y + z = 5$.