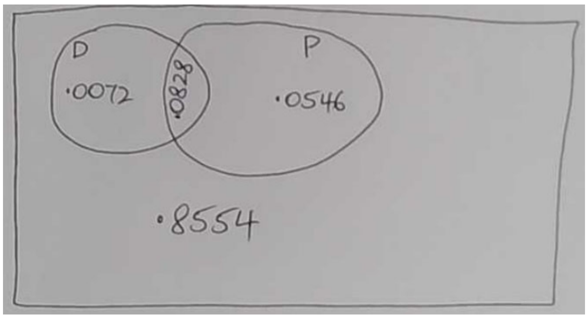
IPDA1005 Test 1 2022 Trimester3

SOLUTIONS

Section 1

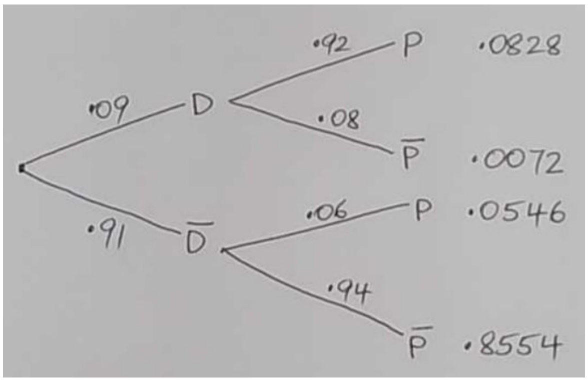
Question	Correct	Notes
1	C	Rate per 30 seconds = 4, so X ~ Poisson(4) Need $P(X \le 4) = .6288$
2	A	Mean = np = 4.8 = 12 × p. Hence p = 4.8 / 12 = 0.4. X ~ Bin(12, .4) $P(X > 6) = 1 - P(X \le 6) = 1 - .8418 = .1582$
3	C	If X ~ U(a,b) then var(X) = $\frac{(b-a)^2}{12}$. If E(X) = 0, then $a = -b$, so var(X) = $\frac{(2b)^2}{12} = \frac{b^2}{3}$. If var(X) = 16/3, then b = 4.
4	C	if X ~ Exponential(2) then var(X) = ¼. Var(4X+3) = 16 var(X) = 16/4 = 4
5	D	Since independent, $P(A \cap B) = P(A)\,P(B)$. i.e., .3 = .4 × P(B). P(B) = .75 $P(A \cup B) = P(A) + P(B) - P(A \cap B) = .4 + .75 - .3 = .85$
6	C	$var(X) = E(X^2) - \mu^2$, so var(X) cannot exceed $E(X^2) = 5$. Also, $var(X) \ge 0$, so $\mu^2 \le E(X^2) = 5$. Thus, $E(X) < \sqrt{5} \approx 2.236$.
7	C	Others *may* be true. Only C *must* be true, since $P(X \le 3) \ge .5$.
8	D	$var(X) = E(X^2) - [E(X)]^2$. We have E(X) = 5, var(X) = 25, so $E(X^2) = 25 + 25 = 50$
9	C	Other options address other concepts (A, B) or are misstated (D).
10	B	Must sum to 1 with all probs non-negative. Only B does this.

Section 2

<table>
<tr><td></td><td></td><td>Solution</td></tr>
<tr><td>1</td><td>(a)</td><td>


Using -
D = diseased
P = positive test result

<table>
<tr><td></td><td>D</td><td>$\bar{D}$</td><td>Total</td></tr>
<tr><td>P</td><td>.0828</td><td>.0546</td><td>.1374</td></tr>
<tr><td>$\bar{P}$</td><td>.0072</td><td>.8554</td><td>.8626</td></tr>
<tr><td>Total</td><td>.09</td><td>.91</td><td>1</td></tr>
</table>



Symbols must be defined.
Labelling must be correct.
Totals completed on table.
End-path probabilities completed on tree diagram.
All four probs provided on Venn diagram.
[4]</td></tr>
<tr><td></td><td>(b)</td><td>P(diseased | positive) = .0828 / .1374 = .6026 (.603 is OK)
[2. 0 if conditional prob not recognised.
1 mark if only one of the component probs is correct]</td></tr>
<tr><td></td><td>(c)</td><td>Redo calculation with P(D) = .6026. This gives –
P(P|D) = .6026 × .92 = .5544 [1]
P(P| $\bar{D}$) = (1-.6026)(1-.94) = .0238 [1]
P(D|P) = .5544 / (.5544 + .0238) = .9588 [1]</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>2</td><td>(a)</td><td>X ~ Hyper(10, 7, 50) [assume n, M, N. Must show numbers. 3]</td></tr>
<tr><td></td><td>(b)</td><td>Var(X) = $n\frac{M}{N}\left(1-\frac{M}{N}\right)\left(\frac{N-n}{N-1}\right)=10\frac{7}{50}\left(1-\frac{7}{50}\right)\left(\frac{40}{49}\right)=.9829$ [2]</td></tr>
<tr><td></td><td>(c)</td><td>P(X ≥ 3) = 1 – P(X ≤ 2) = 1 - .8674 = .1326 [3]</td></tr>
<tr><td></td><td>(d)</td><td>X ~ Bin(10, .14) [2]
Var(X) = 10 × .14 × .86 = 1.204 [1]</td></tr>
<tr><td></td><td></td><td></td></tr>
<tr><td>3</td><td>(a)</td><td>X ~ Geom(.2) OR X ~ Negbin(1, .2) [1]
Y ~ Negbin(4, .2) [2]</td></tr>
<tr><td></td><td>(b)</td><td>mean = E(Y) = $\frac{r(1-p)}{p}=\frac{4\times.8}{.2}=16$ [1]
var(Y) = $\frac{r(1-p)}{p^2}=\frac{4\times.8}{.04}=80$ [1]</td></tr>
<tr><td></td><td>(c)</td><td>P(7 attempts) = P(Y = 3) [1]
$P\left(Y=3\right)=\binom{6}{3}.2^4.8^3=20\times.0016\times.512=.0164$ [2]</td></tr>
<tr><td></td><td>(d)</td><td>P(X>6) = 1 – P(X ≤ 6) = 1 – (1 – $.8^7$) = .2097 [3]</td></tr>
</table>

4	(a)	$F(1) = ax^3 - 3x^2 = 1$. Hence $a = 4$. [1]
	(b)	$f(x) = \frac{d}{dx}F(x) = 12x^2 - 6x$ [2]
	(c)	It is wrong as this part yields a NEGATIVE probability. $F(.6) = -.216$ [1] $F(.3) = -.162$ [1] $P(.3 < X < .6) = F(.6) - F(.3) = -.216 - (-.162) = -.054$ [1]
	(d)	This part yields a NEGATIVE variance, so wrong again. $E(X) = \int_0^0 x(12x^2 - 6x)dx = 6\int_0^0 (2x^3 - x^2)dx = 6\left[\frac{x^4}{2} - \frac{x^3}{3}\right]_0^1 = 6\left(\frac{1}{2} - \frac{1}{3}\right) = 1$ [1.5] $E(X^2) = \int_0^0 x^2(12x^2 - 6x)dx = 6\int_0^0 (2x^4 - x^3)dx = 6\left[\frac{2x^5}{5} - \frac{x^4}{4}\right]_0^1 = 6\left(\frac{2}{5} - \frac{1}{4}\right) = \frac{9}{10}$ [1.5] $Var(X) = E(X^2) - E(X)^2 = .9 - 1 = -.1$ [1]
5	(a)	Using rounded Z-scores and tables: $P(40 < X < 52) = P\left(\frac{40 - 47.4}{5.71} < Z < \frac{52 - 47.4}{5.71}\right)$ $= P(-1.30 < Z < .81)$ $= P(Z < .81) - P(Z < -1.30)$ [1] $= .7910 - .0968$ $= .6942$ [1, total = 2] Using exact z-scores (e.g., in *R*): P(X < 52) – P(X < 40) = .7898 - .0975 =.6923
	(b)	Using rounded Z-scores and tables: $P(X \geq 55) = 1 - P(X < 55)$ $= 1 - P\left(Z < \frac{55 - 47.4}{5.71}\right)$ $= 1 - P(Z < 1.33)$ [1] $= 1 - .9082 = .0918$ [1, total = 2] Using exact z-scores (e.g., in *R*): P(X ≥ 55) = 1 – P(X < 55) = 1 - .9084 =.0916
	(c)	By definition, quartiles divide a distribution into chunks of 25%. Since a normal distribution is symmetric, we need to find only the lower quartile. That is, find x so that P(X ≤ x) = .25 **From tables**, the closest left-tailed probability to 0.25 0.2514, for which the corresponding Z-score is –0.67. [1] This gives x = μ + zσ = 47.4 – .67 × 5.71 = 43.6. [1] Thus, Q1 = 43.6 [1] By symmetry Q3 = 47.4 + .67 × 5.71 = 51.2. **From *R***, the qnorm function gives Q1 = 43.55 and Q3 = 51.25
	(d)	We have Var(X) = 5.71^2 = E(X^2) – E(X)^2, so E(X^2) = 5.71^2 + 47.4^2 = 2279.364 [1] Hence E(3X^2) = 3E(X^2) = 3 × 2279.364 = 6838.092 [1]