1. $2x-4y+z-7=0 \qquad 3x+y-z-2=0$

a. $\cos\theta = \frac{a \cdot b}{\|a\|\|b\|} = \frac{2\times3 + -4\times1 + 1\times -1}{\sqrt{2^2+(-4)^2+1^2}\sqrt{3^2+1^2+(-1)^2}} = \frac{1}{\sqrt{21}\sqrt{11}} = \frac{1}{\sqrt{231}}$

$\cos^{-1}\frac{1}{\sqrt{231}} = 86.23^\circ$

b. $n_1 = [2, -4, 1] \qquad a = n_1 \times n_2 = [((-4\times -1)-(1\times1)\ ((1\times3)-(2\times -1))\ ((2\times1)-(3\times -4))]$

$n_2 = [3, 1, -1] \qquad = [3, 5, 14]$

let $x=0$: $\quad L\begin{cases} -4y+z=7 \\ y-z=2 \end{cases} \qquad -4y+z+y-z=7+2$

$-3y=9$

$y=3$

point on line: $[0, -3, -5]$ $\qquad z=y-2$

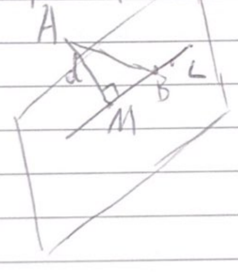
line: $x=3t,\ y=-3+5t,\ z=-5+14t \qquad z=-3-2=-5$

c, $(0, -3, -5)$ from b

d. $2(0)-4(-3)+5=7$

$3(0)+(-3)-(5)=2$

2. from question 1: $2x-4y+z=7$, $\quad L\begin{cases} x=3t \\ y=-3+5t \\ z=-5+14t \end{cases} \qquad A(3,1,1)$




$\cancel{\overrightarrow{AM} = [3t-3, -3+5t, -5+14t]\cdot[3,5,14]=0}$

$\cancel{= 9t+(-9)(15)+25t+(-5\times14)+196t=0}$

$\cancel{\overrightarrow{AM} = [-3+3t, -8+5t, -19+14t]\cdot[3,5,14]=0}$

$\cancel{-9+9t+5(-40)+25t}$

let $x=y=0$: $\quad 2(0)-4(0)+z=7$

$z=7$

$n=[2,-4,1] \qquad |n| = \sqrt{2^2+(-4)^2+1^2} \qquad \overrightarrow{AB} = [3-0,\ 1-0,\ 1-7]$

$= \sqrt{4+16+1} \qquad = [3, 1, -6]$

$= \sqrt{21}$

$d = \left|\frac{[3,1,-6]\cdot[2,-4,1]}{\sqrt{21}}\right| = \left|\frac{6,-4,-6}{\sqrt{21}}\right| = \left|\frac{-4}{\sqrt{21}}\right| = \frac{4}{\sqrt{21}} \approx 0.87$

3. $L \begin{cases} x = 1 + t \\ y = 2t + 5 \\ z = 3t - 1 \end{cases}$ $x + 2y + z = 26$

$$(1+t) + 2(2t+5) + (3t-1) = 26$$
$$1 + t + 4t + 10 + 3t - 1 = 26$$
$$10 + 8t = 26$$
$$8t = 16$$
$$t = 2$$

$L \begin{cases} x = 1 + 2 = 3 \\ y = 2(2) + 5 = 9 \\ z = 3(2) - 1 = 5 \end{cases}$ interact at $(3, 9, 5)$

$(3, 9, 5)$ on plane:

$$3 + 2(9) + 5 = 26$$
$$26 = 26$$

$(3, 9, 5)$ on line:

$3 = 1 + t_1$ = $9 = 2t_2 + 5$ = $5 = 3t_3 - 1$

$2 = t_1$ $\quad 4 = 2t_2$ $\quad 6 = 3t_3$

$t_2 = 2$ $\quad t_3 = 2$

$$t_1 = t_2 = t_3 = 2$$

4\. a. $-x_1+x_2-2x_3=1$

$2x_1+x_2-2x_3=-2$

$-x_1+2x_2-4x_3=1$

b. $\left[\begin{array}{ccc|c} -1 & 1 & -2 & 1 \\ 2 & 1 & -2 & -2 \\ -1 & 2 & -4 & 1 \end{array}\right] \begin{array}{c} \\ R_2+2R_1 \\ R_3-R_1 \end{array} \rightarrow \left[\begin{array}{ccc|c} -1 & 1 & -2 & 1 \\ 0 & 3 & -6 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right] \begin{array}{c} \\ \\ 3R3-R2 \end{array}$

$\downarrow$

$\left[\begin{array}{ccc|c} -1 & 1 & -2 & 1 \\ 0 & 3 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

c. $r(A)=r(A|b)=2<n=3$ $\therefore$ infinitely many solution

d. let $x_3=t$: $3x_2-6x_3=0$ $-1x_1+x_2-2x_3=1$

$3x_2=6x_3$ $-x_1+2t-2t=1$

$x_2=2t$ $x_1=-1$

$[x_1, x_2, x_3]=[-1, 2t, t]$

e. $-(-1)+2t-2(t)=1$ $2(-1)+2t-2(t)=-2$

$1+2t-2t=1$ $-2+2t-2t=-2$

$1=1$ $-2=-2$

$-(-1)+2(2t)-4(t)=1$

$1+4t-4t=1$

$1=1$

5. $\left\{ \begin{bmatrix} 2 \\ 1 \\ x \end{bmatrix} \begin{bmatrix} -2 \\ x \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} \right\}$

$$\det(A) = \begin{vmatrix} 2 & -2 & -1 \\ 1 & x & 1 \\ x & 0 & -3 \end{vmatrix}$$

$$= \cancel{2\begin{vmatrix} x & 1 \\ 0 & -3 \end{vmatrix}} - 2\begin{vmatrix} 1 & 1 \\ x & -3 \end{vmatrix} - x\begin{vmatrix} 2 & -1 \\ x & -3 \end{vmatrix} + 0$$

$$= -2(-3-x) - x(-6+x)$$

$$= 6 + 2x + 6x - x^2$$

$$= 6 + 8x - x^2$$

if $6+8x-x^2=0$, A is linear dependent,

$$6+8x-x^2=0$$

$$x = 4 \pm \sqrt{22}$$

for linear independent : $x \neq 4+\sqrt{22}$ and $x \neq 4-\sqrt{22}$

6. todo: $c_1v_1 + c_2v_2 + c_3v_3 = 0$

$$c_1\begin{bmatrix}1\\0\\3\\0\end{bmatrix} + c_2\begin{bmatrix}1\\1\\4\\5\end{bmatrix} + c_3\begin{bmatrix}0\\1\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}1&1&0&0\\0&1&1&0\\3&4&0&0\\0&5&1&0\end{array}\right] R_3 = R_3 - 3R_1 \quad \left[\begin{array}{ccc|c}1&1&0&0\\0&1&1&0\\0&1&0&0\\0&5&1&0\end{array}\right] \begin{array}{l}R_3 - R_2\\R_4 - 5R_2\end{array} \quad \left[\begin{array}{ccc|c}1&1&0&0\\0&1&1&0\\0&0&-1&0\\0&0&-4&0\end{array}\right] R4 - 4R1$$

$$\left[\begin{array}{ccc|c}1&1&0&0\\0&1&1&0\\0&0&-1&0\\0&0&0&0\end{array}\right]$$

$r(A) = r(A|0) = n = 3 \Rightarrow$ unique solution

$c_1 = 0,\ c2 = 0,\ c_3 = 0 \Rightarrow$ (. I

7. $L_1 \begin{cases} x_1 = 3+4t \\ x_2 = 1+3t \\ x_3 = 3+t \\ x_4 = 1+2t \end{cases}$ $L_2 \begin{cases} x_1 = 1+3r \\ x_2 = 8-r \\ x_3 = 3+2r \\ x_4 = 1+r \end{cases}$

$d_1 = [4,3,1,2]$ $d_2 = [3,-1,2,1]$

$\cancel{3+4t = 1+3r}$ $\cancel{1+3(-\frac{1}{2}+\frac{3}{4}r) = 8-r}$

$\cancel{4t = -2+3r}$

$\cancel{t = -\frac{1}{2}+\frac{3}{4}r}$

Since $d_1 \neq m d_2$, the line are not parallel

$3+t = 3+2r$ $3+4(2r) = 1+3r$

$t = 2r$ $3+8r = 1+3r$

$5r = 1-3$

$r = \cancel{-2}\ -\frac{2}{5}$

$t = 2r$

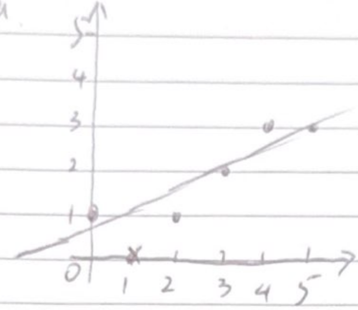
$t = -\frac{4}{5}$

$LHS = 1+2(-\frac{4}{5}) = 1-\frac{8}{5}$ $RHS = 1+1(-\frac{2}{5}) \neq LHS$

so line is not interacted

so 2 line's are skew

№8. a




$$A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \\ 3 \end{bmatrix}$$

b.

$$A^TA = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 14 \\ 14 & 54 \end{bmatrix} \quad (A^TA)^{-1} = \frac{1}{5(54)-(14)(14)} \begin{bmatrix} 54 & -14 \\ -14 & 5 \end{bmatrix}$$

$$= \frac{1}{74} \begin{bmatrix} 54 & -14 \\ -14 & 5 \end{bmatrix}$$

$$\mathrm{pinv}(A) = (A^TA)^{-1}A^T = \frac{1}{74} \begin{bmatrix} 54 & -14 \\ -14 & 5 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 & 5 \end{bmatrix} = \frac{1}{74} \begin{bmatrix} 54 & 26 & 12 & -2 & -16 \\ -14 & -4 & 1 & 6 & 11 \end{bmatrix}$$

$$\hat{x} = \mathrm{pinv}(A)b = \frac{1}{74} \begin{bmatrix} 54 & 26 & 12 & -2 & -16 \\ -14 & -4 & 1 & 6 & 11 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \\ 3 \end{bmatrix} = \frac{1}{74} \begin{bmatrix} 50 \\ 35 \end{bmatrix} = \begin{bmatrix} \frac{50}{74} \\ \frac{35}{74} \end{bmatrix} = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}$$

$$y = \frac{50}{74} + \frac{35}{74}x$$

c. $$A^Tb = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 10 \\ 35 \end{bmatrix}$$

$$A^TA \mid A^Tb = \left[\begin{array}{cc|c} 5 & 14 & 10 \\ 14 & 54 & 35 \end{array}\right] \quad R2 = 14R1 - 5R2$$

$14 \times 14 - 54 \times 5 = 196 - 270 = -74$

$140 - 35 \times 5 = -35$

$$= \left[\begin{array}{cc|c} 5 & 14 & 10 \\ 0 & -74 & -35 \end{array}\right]$$

$-74a_1 = -35$

$a_1 = \frac{35}{74}$

$5(a_0) + 14\left(\frac{35}{74}\right) = 10$

$5a_0 + \frac{490}{74} = 10$

$5a_0 = \frac{250}{74}$

$a_0 = \frac{50}{74}$

$= \frac{25}{37}$

$$y = \frac{25}{37} + \frac{35}{74}x$$

d. plot $y = \frac{25}{37} + \frac{35}{74}$ in to graph, satisfied