1.

$$
\begin{array}{cccccc}
+ & + & + & - & - & - \\
i & j & k & i & j & \\
2 & -1 & 0 & 2 & -1 & \\
1 & 3 & -2 & 1 & 3 &
\end{array}
$$

$$a \times b = i\left((-1)(-2) - (0)(3)\right) + j\left((0)(1) - (2)(-2)\right) + k\left((2)(3) - (-1)(1)\right)$$
$$= i(2 - 0) + j(0 + 4) + k(6 + 1)$$
$$= [2, 4, 7]$$

$(a \times b) \cdot b = [2, 4, 7] \cdot [1, 3, -2] = (2)(1) + (4)(3) + (7)(-2) = 2 + 12 - 14 = 0$

Since $(a \times b) \cdot b = 0$, $a \times b$ and b are perpendicular.

2. Area of triangle $= \frac{1}{2}$ Area of parallelogram

$$= \frac{1}{2}\left\| \overrightarrow{PQ} \times \overrightarrow{PR} \right\|$$

$\overrightarrow{PQ} = [2 - 1, 0 - (-1), -1 - 2] = [1, 1, -3]$

$\overrightarrow{PR} = [0 - 1, 2 - (-1), 1 - 2] = [-1, 3, -1]$

$\overrightarrow{PQ} \times \overrightarrow{PR} = i(-1 - (-9)) + j(3 - (-1)) + k(3 - (-1)) = [8, 4, 4]$

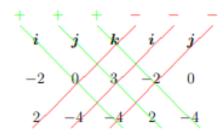$$\text{Area} = \frac{1}{2}\sqrt{(8)^2 + (4)^2 + (4)^2} = \frac{1}{2}\sqrt{64 + 16 + 16} = \frac{\sqrt{96}}{2}$$

3. Area of parallelogram $= \|u \times v\| = \|u\| \, \|v\| \sin\theta$

Given $\|u\| = 16$, $\|v\| = 4$, $\cos\theta = \frac{1}{2} \Rightarrow \theta = \cos^{-1}(\frac{1}{2}) = 60°$

$\therefore$ Area $= (16)(4)\sin(60°) = 64\left(\frac{\sqrt{3}}{2}\right) = 32\sqrt{3}$

4. Coplanar if $a \cdot (b \times c) = 0$

$$
\begin{array}{cccccc}
+ & + & + & - & - & - \\
i & j & k & i & j & \\
-2 & 0 & 3 & -2 & 0 & \\
2 & -4 & -4 & 2 & -4 &
\end{array}
$$

$b \times c = i(0 - (-12)) + j(6 - 8) + k(8 - 0) = [12, -2, 8]$

$a \cdot (b \times c) = [1, 2, -1] \cdot [12, -2, 8] = 12 - 4 - 8 = 0$

So the vectors $a = [1, 2, -1]$, $b = [-2, 0, 3]$ and $c = [2, -4, -4]$ are coplanar.

5. (i) $A+B = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 2+7 & 0+(-5) & -1+1 \\ 4+1 & -5+(-4) & 2+(-3) \end{bmatrix}$

$$= \begin{bmatrix} 9 & -5 & 0 \\ 5 & -9 & -1 \end{bmatrix}$$

(ii) $-4B = -4\begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} = \begin{bmatrix} -4(7) & -4(-5) & -4(1) \\ -4(1) & -4(-4) & -4(-3) \end{bmatrix} = \begin{bmatrix} -28 & 20 & -4 \\ -4 & 16 & 12 \end{bmatrix}$

(iii) $AC = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ This is not possible. The number of columns in A is not the same as the number of rows in C.

(iv) $CB = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 7+2 & -5-8 & 1-6 \\ -14+1 & 10-4 & -2-3 \end{bmatrix} = \begin{bmatrix} 9 & -13 & -5 \\ -13 & 6 & -5 \end{bmatrix}$

(v) $AB^T = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}\begin{bmatrix} 7 & 1 \\ -5 & -4 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 14+0-1 & 2+0+3 \\ 28+25+2 & 4+20-6 \end{bmatrix} = \begin{bmatrix} 13 & 5 \\ 55 & 18 \end{bmatrix}$

(vi) $C - 3I_2 = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1-3 & 2 \\ -2 & 1-3 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -2 & -2 \end{bmatrix}$$

(vii) $C^2 = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1-4 & 2+2 \\ -2-2 & -4+1 \end{bmatrix} = \begin{bmatrix} -3 & 4 \\ -4 & -3 \end{bmatrix}$

6. $A_{6\times 4}\, B_{n\times m} = (AB)_{6\times 8}$
   The number of columns in A, 4, equals the number of rows in B, n, $\therefore\ n = 4$
   The number of columns in AB, 8, equals the number of columns in B, m, $\therefore\ m = 8$
   $\therefore$ order of $B = n \times m = 4 \times 8$

7. $B_{n\times m}\, C_{m\times p} = (BC)_{n\times p}$ Given $n \times p = 4 \times 3$. Hence $n = 4$, i.e., B has 4 rows.

8. $AB = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix} = \begin{bmatrix} 23 & -10+5k \\ -9 & 15+k \end{bmatrix}$

   $BA = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}\begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 23 & 15 \\ 6-3k & 15+k \end{bmatrix}$

   Need $-10+5k = 15 \quad\Rightarrow\quad 5k = 25 \quad\Rightarrow\quad k = 5$
   and $\quad 6-3k = -9 \quad\Rightarrow\quad -3k = -15 \quad\Rightarrow\quad k = 5$

9. $AB = \begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix}\begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} -14+15 & -10+10 \\ 21-21 & 15-14 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   $BA = \begin{bmatrix} -7 & -5 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & 5 \\ -3 & -7 \end{bmatrix} = \begin{bmatrix} -14+15 & -35+35 \\ 6-6 & 15-14 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   Since $AB = BA = I$, the matrices A and B are the inverse of one another.

10. $AB = 0$ If A is invertible, A^{-1} exists. Multiply both sides by A^{-1}, from the left:
    $A^{-1}AB = A^{-1}0 \quad\Rightarrow\quad IB = 0 \quad\Rightarrow\quad B = 0$