EMTH1019 Linear Algebra & Statistics for Engineers Tutorial 2

Learning outcomes for this session

At the end of this session, you should be able to

- 1. Distinguish between discrete and continuous random variables.
- 2. Determine the mean and variance of random variables.
- 3. Define probability functions and probability density functions.
- 4. Recognise and apply some basic probability distributions.
- Make sure you can do the star questions 12 (a hard question because of the algebra), 13 (the solution was done with cumulative tables)
- There are more questions at the end of the lecture slides.
- 1. A car pooling study shows that the number of passengers, *X*, in a car (excluding the driver) is likely to assume the values 0,1,2,3 and 4 with probabilities given by the table



- (a) Determine the probability of at least two passengers in a car.
- (b) Find the cumulative distribution function of *X* and sketch it.
- 2. Consider a small post office with a single staff member operating a single postal counter. Suppose that the probability p_k , that there are k customers in the post office, is given by $p_k = p_0 p^k$, k = 0,1,2,... where 0 .
 - (a) Show that $p_0 = 1 p$.
 - (b) Determine the probability that a newly arriving customer has to wait to be served.
- 3. The pdf of random variable X is given by

$$f(x) = \begin{cases} \frac{2}{9}(x+1), & -1 < x < 2\\ 0, & \text{elsewhere} \end{cases}$$

- (a) Sketch f(x).
- (b) Find the cdf of *X* and sketch it.
- (c) Calculate P(X < 0.25) and show this on your two sketches.

The cdf of a random variable Y is given by

$$F(y) = \begin{cases} 0, & \text{if } y < -1\\ \frac{1}{2}(1+y), & \text{if } -1 \le y < 0\\ \frac{1}{2}(1+y^2), & \text{if } 0 \le y < 1\\ 1, & \text{if } y \ge 1 \end{cases}$$

- (a) Find the pdf of Y.
- (b) Sketch f(y) and F(y).
- (c) Calculate $P(Y \le 0.8)$ and show this on your two sketches.



- Refer to Question 1. Calculate (i) E(X) (ii) $E(X^2)$
- (iii) Var(X)
- 6. Using the pdf in Question 3, find: E(X) and $E(X^2)$, and hence Var(X).
- 7. Suppose that an antique jewellery dealer is interested in purchasing a gold necklace for which the probabilities are 0.22, 0.36, 0.28, and 0.14, respectively, that she will be able to sell it for a profit of \$250, sell it for a profit of \$150, break even, or sell it for a loss of \$150. What is the expected profit?
 - 8. The density function of the continuous random variable X, the total number of hours, in units of 100 hours, that a family runs a vacuum cleaner over a period of one year, is given by

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, 1 \le x < 2 \\ 0, & elsewhere \end{cases}$$

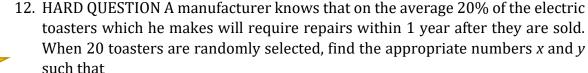
Find the average number of hours per year that families run their vacuum cleaners.

9. Let *X* be a random variable with the following probability distribution:

Find $\mu_{g(X)}$, where $g(X) = (2X + 1)^2$.



- 10. Suppose that the probabilities are 0.4, 0.3, 0.2, and 0.1, respectively, that 0, 1,2,or 3 power failures will strike a certain subdivision in any given year. Find the mean and variance of the random variable X representing the number of power failures striking the subdivision.
- 11. According to a survey by the Administrative Management Society, one-half of U.S. companies give employees four weeks of vacation after they have been with the company for 15 years. Find the probability that among 6 companies surveyed at random, the number that give employees 4 weeks of vacation after 15 years of employment is
 - (a) anywhere from 2 to 5.
 - (b) fewer than 3.





- (a) The probability that at least *x* of them will require repairs is less than 0.5.
- (b) The probability that at least *y* of them will *not* require repairs is greater than 0.8.
- 13. The average number of field mice per acre in a 5-acre wheat field is estimated to be 12. Find the probability that fewer than 7 field mice are found



- (a) on a given acre;
- (b) on two of the next 3 acres inspected.
- 14. Changes in airport procedures require considerable planning. Arrival rates of aircraft is an important factor that must be taken into account. Suppose small aircraft arrive at a certain airport, according to a Poisson process, at the rate of 6 per hour. The Poisson parameter for arrivals for a period of t hours is $\lambda = 6t$.
 - (a) What is the probability that exactly 4 small aircraft arrive during a 1-hourperiod?
 - (b) What is the probability that at least 4 arrive during a 1-hour period?
 - (c) If we define a working day as 12 hours, what is the probability that at least75 small aircraft arrive during a day?



- 15. Given a standard normal distribution, find the value of *k* such that
 - (a) P(Z < k) = 0.0427
 - (b) P(Z > k) = 0.2946
 - (c) P(-0.93 < Z < k) = 0.7235
- 16. Given a standard normal distribution, find the area under the curve which lies
 - (a) to the left of z = 1.43
 - (b) to the right of z = -0.89
 - (c) between z = -2.16 and z = -0.65
 - (d) to the left of z = -1.39
 - (e) to the right of z = 1.96
 - (f) between z = -0.48 and z = 1.74
- 17. A soft drink machine is regulated so that it discharges an average of 200 millilitres per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 millilitres,
 - (a) what fraction of the cups will contain more than 224 millilitres?



- (b) what is the probability that a cup contains between 191 and 209 millilitres?
- (c) how many cups will probably overflow if 230 millilitre cups are used for the next 1000 drinks?
- (d) below what value do we get the smallest 25% of the drinks?
- 18. The fracture strengths of a certain type of glass average 14 (thousands of pounds per square inch) and have standard deviation of 2.



- (a) What is the probability that the average fracture strength for 100 pieces of this glass exceeds 14.5?
- (b) Find the interval that includes the average fracture strength for 100 pieces of this glass with probability 0.95.