EMTH1019 Tutorial 10

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Lines in $\mathbb{R}^3$

1. Find the Cartesian equations of the line passing through the points $(3, 5, -2)$ and $(2, 1, -1)$.

2. Find the vector and parametric equations for the line passing through the point $(2, 5, -2)$ and is parallel to the line $\dfrac{x+4}{3} = \dfrac{y}{-2} = z - 3$.

3. Show that the line through the points $(7, 2, 2)$ and $(1, 4, -2)$ is parallel to the line $x = -2 + 3t$, $y = 1 - t$, $z = 4 + 2t$.

4. Find the shortest distance from the point $(0, 0, 12)$ to the line $x = 4t$, $y = -2t$, $z = 2t$.

Determine whether the following lines are parallel, intersecting, or skew. If they intersect find the point of intersection, or if they are skew then find the shortest distance between them.

(i) $L_1 \begin{cases} x &= 3+2t \\ y &= -1+4t \\ z &= 2-t \end{cases}$ $L_2 \begin{cases} x &= 1+4\tau \\ y &= 1+2\tau \\ z &= -3+4\tau \end{cases}$

Determine whether the following lines are parallel, intersecting, or skew. If they intersect find the point of intersection, or if they are skew then find the shortest distance between them.

(ii) $L_1 \begin{cases} x &= 1+2t \\ y &= -1-t \\ z &= 3t \end{cases}$ $L_2 \begin{cases} x &= 2-\tau \\ y &= 3\tau \\ z &= 1+\tau \end{cases}$

Planes in $\mathbb{R}^3$

6. Find the equation of the plane containing the points $P(0,1,1)$, $Q(1,0,1)$ and $R(1,1,0)$.

7. Find the equation of the plane passing through the point $P(6, 0, -2)$ and which contains the line $x = 4 - 2t$, $y = 3 + 5t$, $z = 7 + 4t$.

8. Find the parametric equations for the line of intersection of the planes $3x-2y+z=1$ and $2x+y-3z=3$.

9. For each of the following pairs of planes, decide whether they are parallel, perpendicular or neither.

   (i) $x + z = 1$, $y + z = 1$.

   (ii) $-8x - 6y + 2z = 1$, $z = 4x + 3y$.

   (iii) $x + 4y - 3z = 1$, $-3x + 6y + 7z = 0$.

10. Find the angle between the planes $x + y + z = 0$ and $x + 2y + 3z = 1$.

11. Find the distance from the point $P(3, -2, 7)$ to the plane $4x - 6y + z = 5$.