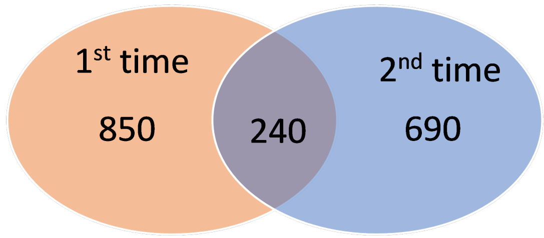
Workshop 1 - Solutions

1. (a) sample space $= \{3, 4, 5, 13, 14, 15, 23, 24, 25, 123, 124, 125, 213, 214, 215\}$
   (b) $\{3, 4, 5\}$
   (c) $\{5, 15, 25, 125, 215\}$
   (d) $\{3, 4, 5, 23, 24, 25\}$

2. Define $A = \{\text{saw it 1st time}\}$ and $B = \{\text{saw it 2nd time}\}$. Then we want $A \cap B$.
   Since 4700 didn't see the movie at all, then $6000 - 4700 = 1300$ did see it, which is $A \cup B$.
   But $A \cup B = A + B - A \cap B$ (which is easier to see in a Venn diagram).
   So then $A \cap B = A + B - A \cup B = 850 + 690 - 1300 = 240$

   Here is the corresponding Venn diagram:



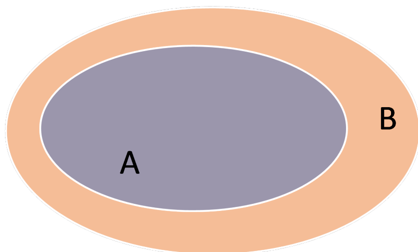
3. (a) By factorisation, $A = \{-4, 2\}$ and $B = \{-3, 2\}$. Hence
   $A \cup B = \{-4, -3, 2\}$
   $A \cap B = \{2\}$.
   (b) Using inequalities, we have then $A = \{x : -4 \leq x \leq 2\}$ and $B = \{x : -3 \leq x \leq 2\}$. Then
   $A \cap B = \{x : -3 \leq x \leq 2\}$
   $A \cup B = \{x : -4 \leq x \leq 2\}$.

4. $A = \{A\clubsuit, A\diamondsuit, A\heartsuit, A\spadesuit\}$
   $B = \{2\spadesuit, 3\spadesuit, \ldots, K\spadesuit, A\spadesuit\}$

   Therefore,

   $A \cap B = \{A\spadesuit\}$
   $A \cup B = \{2\spadesuit, 3\spadesuit, \ldots, K\spadesuit, A\spadesuit, A\clubsuit, A\diamondsuit, A\heartsuit\}$

5. Both circumstances imply that A is contained within B, i.e., $A \subseteq B$. On a Venn diagramm it looks like this:



6. No solution provided.

7. I have done this using De Morgan's laws and the identities given in 6.

(a) Since $(A \cap B)^c = A^c \cup B^c$ we can write

$$(A \cap B^c)^c = A^c \cup B$$

(b) Here, we use De Morgan's laws and then the identity in 6(a).

$$\begin{aligned} B \cup (A \cup B)^c &= B \cup (A^c \cap B^c) \\ &= (B \cup A^c) \cap (B \cup B^c) \\ &= (B \cup A^c) \cap \mathcal{S} \\ &= B \cup A^c \end{aligned}$$

which is the same as the result for part (a).

(c) We use the same strategy here as in part (b):

$$\begin{aligned} A \cap (A \cap B)^c &= A \cap (A^c \cup B^c) \\ &= (A \cap A^c) \cup (A \cap B^c) \\ &= \emptyset \cup (A \cap B^c) \\ &= A \cap B^c \end{aligned}$$