EMTH1019 Linear Algebra and Statistics for Engineers

Workshop 9 Solutions

1. (a) American heads of household
   (b) 1000
   (c) Hardest household place to clean
   (d) $1000 \times 0.12 = 120$
   (e) Actual percentage could be 5% lower or 5% hihger than quoted.
   (f) Between 30% and 40% of all adults think that Venetian blinds are the hardest to clean.

2. (a) Yes, if the rate increases from 4% to 6%, that i sa 50% increase in the rate : $(6 - 4)/4 = 2/4 = 0.50 = 50\%$. As a percent alone, the 50% is meaningless; it does not give the actual size of the numbers involved.
   (b) The phrase ”50% jump” works much more effectively at getting people’s attention than does ”2% increase”

3. (a) All assembled parts from the assembly line
   (b) infinite
   (c) The parts checked
   (d) Categorical, categorical, numerical.

4. $\bar{x} = (\sum x)/n = (1 + 2 + 1 + 3 + 2 + 1 + 5 + 3)/8 = 18/8 = 2.25$

5. Ranked data: 4.15, 4.25, 4.25, 4.50, 4.60, 4,60, 4.75, 4.90; position of median is$(n + 1)/2 = (8 + 1)/2 = 4.5$, i.e. mean of 4th and 5th values in the ranked data. So, median = $(4.50 + 4.60)/2 = 4.55$.

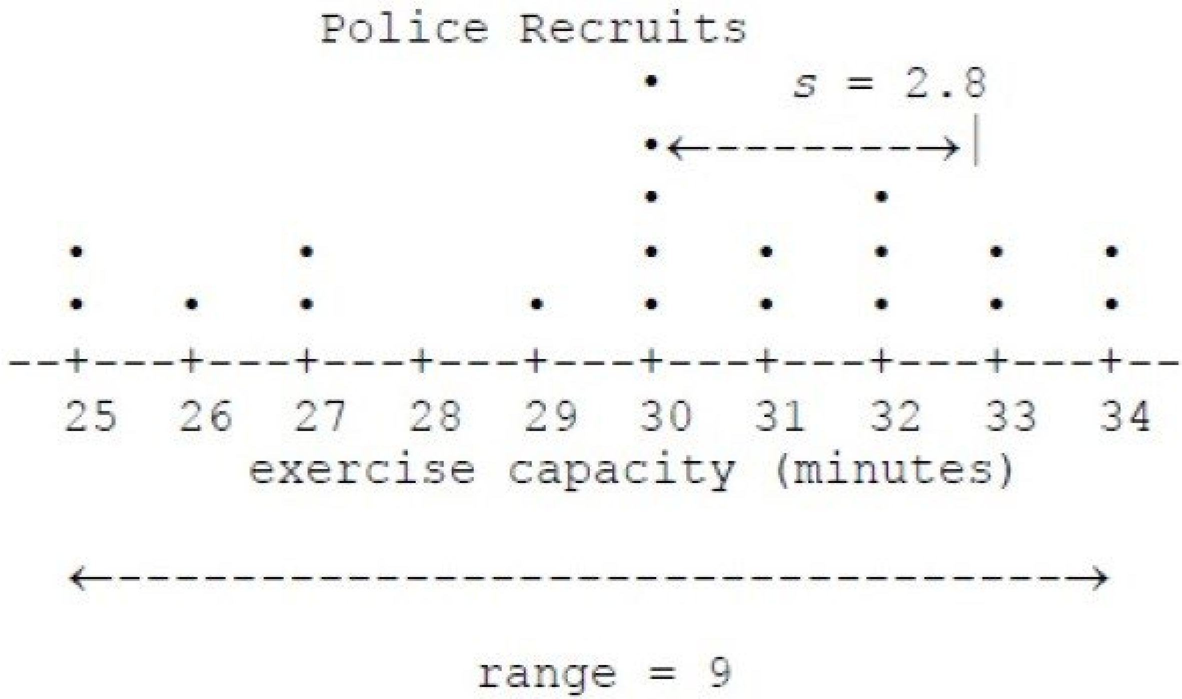
6. (a) mean = $\sum x /_{n} = 402/10 = 40.2$
   (b) ranked data: 28, 29, 33, 40, 41, 42, 44, 48, 48, 49. Position of median is$(n + 1)/2 = (10 + 1)/2 = 5.5$, i.e. mean of the 5th and 6th position, so median = 41.5.
   (c) Mode = 48.

7. The mean is the balance point or the centre of gravity to all the data values. Since the weights of the data values on each side of $\bar{x}$ are equal,$\Sigma(x - \bar{x})$ will give a positive amount and and an equal negative amount, thereby cancelling each other out. Algebraically: $\Sigma(x - \bar{x}) = \Sigma x - n\bar{x} = \Sigma x - n(\Sigma x/n) = \Sigma x - \Sigma x = 0$

8. (a) 9 − 2 = 7

(b) $s^2 = 8.5$

(c) $s = \sqrt{s^2} = 2.9$



9. (a)

(b) $\bar{x} = 601/20 = 30.05$

(c) 34 − 25 = 9

(d) 7.8.(e) 2.8.

(f) see the graph.

(g) Except for the value $x = 30$, the distribution looks rectangular. Range is a little more than 3 standard deviations.

10. (a) Ranked data: 2.6, 2.7, 3.4, 3.6, 3.7, 3.9, 4.0, 4.4, 4.8, 4.8, 4.8, 5.0, 5.1, 5.6, 5.6, 5.8, 6.8, 7.0, 7.0.
$(n+1)\frac{1}{4} = \frac{21}{4} = 5\frac{1}{4}$, $r = 5$, so $Q_1 = y_5 + \frac{1}{4}(y_6 - y_5) = 3.7 + \frac{1}{4}(3.9 - 3.7) = 3.75$

(b) $Q_2 = (y_{10} + y_{11})/2 = (4.8 + 4.8)/2 = 4.8$

(c) $P_{15} = y_3 + \frac{3}{20}(y_4 - y_3) = 3.4 + \frac{3}{20}(3.6 - 3.4) = 3.43$, $P_{33} = y_6 + \frac{93}{100}(y_7 - y_6) = 3.9 + \frac{93}{100}(4.0 - 3.9) = 3.993$, $P_{90} = y_{18} + \frac{9}{10}(y_{19} - y_{18}) = 18 + \frac{9}{10}(7.0 - 6.8) = 6.98$.

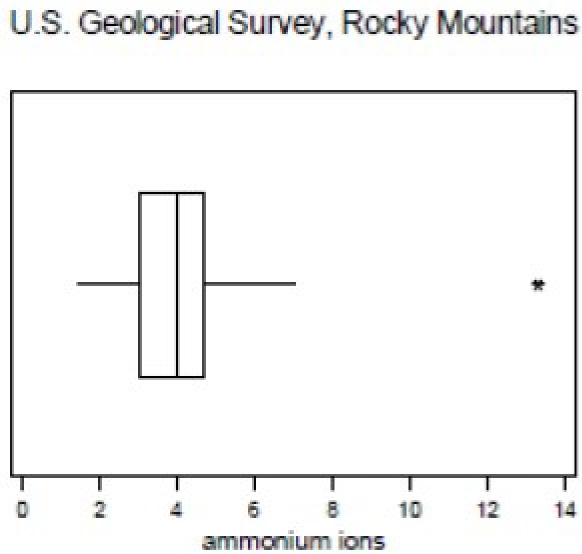
11. (a) Find $Q_1 = 3.0 + \frac{1}{4}(0.1) = 3.025.$

(b) Find $Q_2 = (4.0 + 4.0)/2 = 4.0$.

(c) Find $Q_3 = 4.6 + \frac{3}{4}(0.1) = 4.675.$

(d) Find $P_{30} = 3.1 + 0.9 * 0.1 = 3.19$.

(e) 5-number summary: 1.4,3.025,4.0,4.675,13.3



(f)

12.

(a) $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{6+10+...+4}{9} = 9.2222... \approx 9.22$ (to 2 d.p.). $s^2 = \frac{\sum_{i=1}^{n} x_i^2}{n-1} - \frac{n}{n-1}\bar{x}^2 = 8.69$ (to 2 d.p.). $s = \sqrt{s^2} = 2.95$ (to 2 d.p.)

(b) $Q_1 = 7$, $Q_2 = 10$, $Q_3 = 11.5$. Five Number Summary: 4,7,10,11.5,13, $Range = Max - Min = 13 - 4 = 9$ and $IQR = Q_3 - Q_1 = 11.5 - 7 = 4.5$.

(c)

Stem-and-leaf Plot:

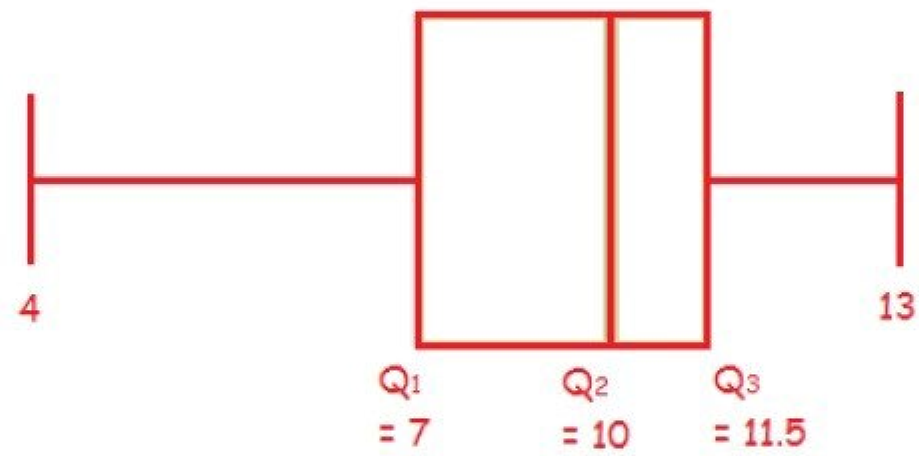
							Frequency	Stem	Leaf				
0 to 5:	4						1	0	4				
6 to 9:	6	8	8			→	3	0	6	8	8		
10 to 15:	10	11	11	12	13		5	1	0	1	1	2	3

Stem width: 10

Each leaf: 1 case(s)

(d) $Q_1 - 1.5IQR = 7 - 1.5(4.5) = 0.25$, $Q_3 + 1.5IQR = 11.5 + 1.5(4.5) = 18.25$

Boxplot:



(e) If we multiply each of the original data by 10 then subtract 3, this is the sameas transforming x into y by using $y = a + bx$ with $a = -3$ and $b = 10$.

New sample mean: $\bar{y} = a + b\bar{x} = -3 + 10 \times 9.2222... \approx 89.22$ (2 d.p.)

New sample variance: $s_y^2 = b^2 s_x^2 = 10^2 \times 8.6944\ldots \approx 869.44$ (2 d.p.)

New sample std: $s_y = |b|s_x = 10 \times 2.9486... \approx 29.49$ (2 d.p.) New

median: $Med(y) = a + bMed(x) = -3 + 10 \times 10 = 97$

New range: $R(y) = |b|R(x) = 10 \times 9 = 90$

New IQR: $IQR(y) = |b|IQR(x) = 10 \times 4.5 = 45$