Exercises

1. A certain type of thread is manufactured with a mean tensile strength of 78.3 and a standard deviation of 5.6 kilograms. How is the variance of the sample mean changed when the sample size is

   (a) increased from 64 to 196?

   (b) decreased from 784 to 49?

2. The heights of 1000 students are approximately normally distributed with a mean of 174.5 cm and a standard deviation of 6.9 cm. Suppose 200 random samples of size 25 are drawn from this population and the means recorded to the nearest tenth of a centimetre. Determine

(a) the mean and standard deviation of the sampling distribution of $\bar{X}$;

(b) The number sample means between 172.5cm and 175.8cm inclusive;

(c) the number of sample means below 172.0 cm.

3. The random variable X, representing the number of cherries in a cherry puff, has the following probability distribution:

x	4	5	6	7
$P(X = x)$	0.2	0.4	0.3	0.1

(a) Find the mean μ and the variance σ^2 of X.

(b) Find the mean $\mu_{\bar{x}}$ and the variance $\sigma^2_{\bar{x}}$ of the mean $\bar{X}$ for random samples of 36 cherry puffs.

(c) Find the probability that the average number of cherries in 36 cherry puffs will be less than 5.5.

$\mu = 40$ $\sigma = 2$

4. If a certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms, what is the probability that a random sample of 36 of these resistors will have a combined resistance of more than 1458 ohms?

$n = 36$ $\sum x = 1458$

Average resistant : $\bar{x} = \frac{\sum x}{n} = \frac{1458}{36} = 40.5$

Assume normal distribution

$P(\bar{X} > 40.5) = P\left(Z > \frac{40.5 - 40}{2/\sqrt{36}}\right)$

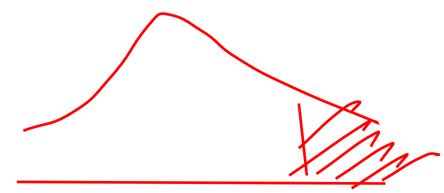
$\boxed{Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$

$= P(Z > 1.5)$

$= 1 - P(Z < 1.5)$

$= 1 - 0.9332$

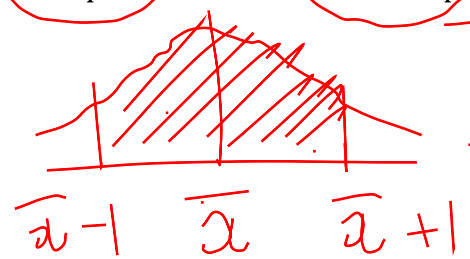
$= 0.0668$



1.5

5. Shear strength measurements for spot welds of a certain type have been found to have a standard deviation of approximately 10 psi. If 100 test welds are to be measured, find the approximate probability that the sample mean will be within 1 psi of the true population mean.

$\sigma = 10$

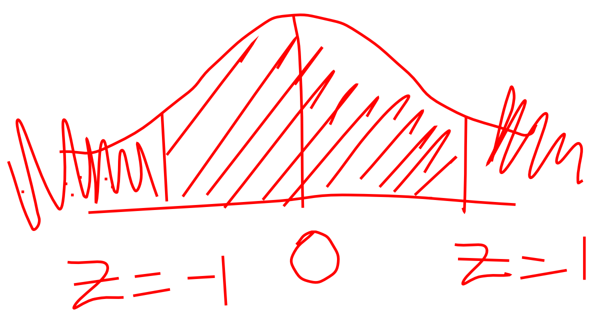


$n = 100 \quad , \quad \sigma = 10$

$C.I = \bar{x} \pm \dfrac{z\sigma}{\sqrt{n}}$

$\text{margin of error} = \dfrac{z\sigma}{\sqrt{n}} = 1$

$\dfrac{z \times 10}{\sqrt{100}} = 1 \Rightarrow z = 1$



$P(-1 < Z < 1)$

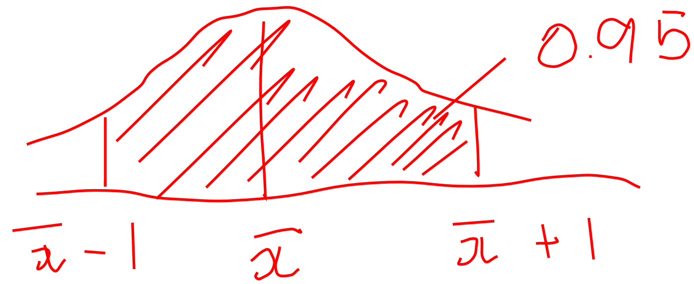
$= 1 - 2 \times P(Z < -1)$

$= 1 - 2 \times 0.1587 = 0.6826$

6. If shear strength measurements have a standard deviation of 10 psi, how many test welds should be used in the sample if the sample mean is to be within 1 psi of the population mean with probability approximately 0.95?

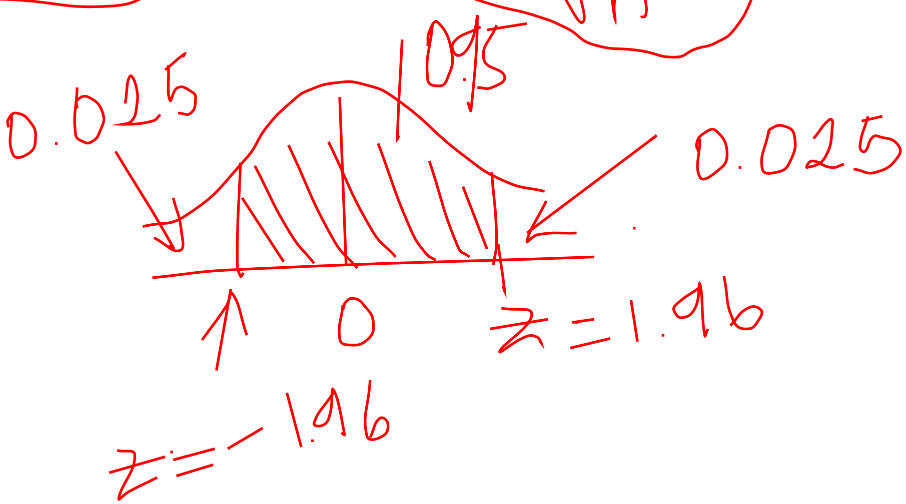
$n = ?$

$\sigma = 10$, $n = ?$



$$\boxed{CI = \bar{X} \pm \frac{z\sigma}{\sqrt{n}}}$$

$$\text{margin of error} = \frac{z\sigma}{\sqrt{n}} = 1$$



$$\frac{z \times 10}{\sqrt{n}} = 1$$

$$\frac{1.96 \times 10}{\sqrt{n}} = 1$$

$$n = 385$$

$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

$\mu = 14$ $\sigma = 2$

7. The fracture strengths of a certain type of glass have a mean of 14 (thousands of pounds per square inch) and a standard deviation of 2. What is the probability that the mean fracture strength for 100 pieces of this glass exceeds 14.5?

$\bar{x}$ $n = 100$

Assume normal distribution

$P(\bar{x} > 14.5) = P\left(Z > \frac{14.5 - 14}{2/\sqrt{100}}\right) = P(Z > 2.5) = 1 - P(Z < 2.5)$

population mean $= \mu$

sample mean $= \bar{x}$

population std. dev $= \sigma$

sample std. dev $= S$

population $Z = \frac{X - \mu}{\sigma}$

sample $Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}}$

8. A manufacturing process produces cylindrical component parts for the automotive industry. The parts must have a mean diameter of 5.0 mm. The engineer involved hypothesises that the population mean is 5.0 mm. $\mu = 5$

$n = 100$

An experiment is conducted in which 100 parts produced are randomly selected and the part diameter is measured. The population standard deviation is σ = 0.1 mm. The experiment indicates a sample mean diameter of $\bar{x} = 5.027\ mm$=.

Does this sample information appear to support or refute the engineer's conjecture?

This exercise reflects the kind of problem often posed and solved with hypothesis testing introduced next week.

The question can be rephrased as follows: How likely is it that we can obtain $\bar{x} \geq 5.027$ with n = 100 if the population mean is μ = 5.0? If this probability suggests that $\bar{x}$ = 5.027 is not unreasonable, the hypothesis that the average is 5mm is accepted. if the probability is quite low, we could argue that the data does not support the hypothesis that μ = 5.0.

$$P(\bar{X} > 5.027) = P\left(Z > \frac{5.027 - 5}{0.1/\sqrt{100}}\right) = P(Z > 2.7)$$
$$= 1 - P(Z < 2.7)$$
$$= 1 - 0.9965 = 0.0035$$

This probability is very small (rare event). But we still able to obtain such a sample with mean=5.027. Therefore we have evidence to doubt the null hypothesis that population mean = 5.0mm population mean $\mu = 200$, $\sigma = 10$ $n = 25$

9. Resistors of a certain type have resistances that average 200 ohms with a standard deviation of 10 ohms. Twenty-five of these resistors are to be used in a circuit.

$\bar{x}$ $n = 25 \rightarrow$ sample

(a) Find the probability that the average resistance of the 25 resistors is between199 and 202 ohms. $P(199 < \bar{x} < 202)$

(b) Find the probability that the *total* resistance of the 25 resistors does not exceed 5100 ohms.

(c) What assumptions are necessary for the answers in (a) and (b) to be good approximations?

(a) Sample $n = 25 \Rightarrow Z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

$$P(199 < \bar{x} < 202) = P\left(\frac{199 - 200}{10/\sqrt{25}} < Z < \frac{202 - 200}{10/\sqrt{25}}\right) = P(-0.5 < Z < 1)$$

$$= P(Z < 1) - P(Z < -0.5)$$

$$= 0.8413 - 0.3085 = 0.5328$$

(b)

$$P(\Sigma x \leq 5100) = P\left(\bar{x} \leq \frac{5100}{25}\right) = P(\bar{x} \leq 204) = P\left(Z \leq \frac{204 - 200}{10/\sqrt{25}}\right)$$

$$= P(Z \leq 2) = 0.9772$$

c) Assume that the distribution is normal

$n = 36$ (sample mean)

10. The average zinc concentration recovered from a sample of measurements taken in 36 different locations in a river is found to be 2.6 grams per millilitre. Suppose that it is known that $\sigma = 0.3$. Find

$\bar{x} = 2.6$

(a) the 95% confidence interval for the mean zinc concentration in the river

(b) the 99% confidence interval for the mean zinc concentration in the river

(c) How large a sample is required if we want to be 95% confident that our estimate of μ is off by less than 0.05?

?

$\bar{x} = 2.6$

(a) 95% confidence interval $= \bar{x} \pm \dfrac{z * \sigma}{\sqrt{n}}$

$\dfrac{1-0.95}{2} = 0.025$

0.95

mean

from z table
$z = -1.96$

$= 2.6 \pm \dfrac{1.96 * 0.3}{\sqrt{36}} = 2.6 \pm 0.098$

(b) 99% confidence interval $= \bar{x} \pm \dfrac{z * \sigma}{\sqrt{n}} = 2.6 \pm \dfrac{2.57 * 0.3}{\sqrt{36}}$

$= 2.6 \pm 0.1285$

$\dfrac{1-0.99}{2} = 0.005$

0.99

$z = -2.57$

(c)

95%

$\mu - 0.05 \quad \mu \quad \mu + 0.05$

margin of error

$95\% \rightarrow z = -1.96$

margin of error $= \dfrac{z * \sigma}{\sqrt{n}} = 0.05$

$\dfrac{1.96 * 0.3}{\sqrt{n}} = 0.05$

$\Rightarrow n = 139$

11. An important property of plastic clays is the percent of shrinkage on drying. Fora certain type of plastic clay 45 test specimens showed an average shrinkage percentage of 18.4. Suppose that it is known that the population standard deviation is 1.2. Estimate the true average percent of shrinkage for specimens of this type in a 98% confidence interval.