

EMTH1019 Week 12 Tutorial questions

A sad day. Last tutorial.



Inconsistent systems

1. Use the pseudoinverse $\text{pinv}(A) = (A^T A)^{-1} A^T$ to find the least squares solution for the following inconsistent systems of linear equations.

$$\begin{array}{ll} & \begin{array}{l} 2x_1 - 2x_2 = -1 \\ x_1 - 2x_2 = 0 \\ x_1 + x_2 = 1 \\ -2x_1 + 2x_2 = 0 \end{array} \\ \text{(i)} \quad \begin{array}{l} x_1 + 2x_2 = 3 \\ x_1 + x_2 = 1 \\ 2x_1 + 3x_2 = 3 \end{array} & \text{(ii)} \end{array}$$

You can use $A^T A \mathbf{x} = A^T \mathbf{b}$ if you prefer

2. For the following inconsistent linear systems, solve the normal system of equations $A^T A \mathbf{x} = A^T \mathbf{b}$ using Gaussian Elimination to determine the least squares solution.

$$\begin{array}{ll} \text{(i)} & \begin{array}{l} 2x_1 + x_2 = 2 \\ x_1 + 2x_2 = 0 \\ x_1 + x_2 = -3 \end{array} \end{array}$$

$$\begin{array}{ll} \text{(ii)} & \begin{array}{l} 2x_2 + x_3 = 1 \\ x_1 + x_2 - x_3 = 0 \\ 2x_1 + x_2 = 1 \\ x_1 + x_2 + x_3 = -1 \\ 2x_2 - x_3 = 0 \end{array} \end{array}$$

3. The following system of linear equations is consistent and has a unique solution.

$$x_1 + x_2 + x_3 = 6$$

$$x_1 - x_2 + x_3 = 2$$

$$x_1 + 2x_2 - x_3 = 2$$

- (i) Solve the system $A\mathbf{x} = \mathbf{b}$ using Gaussian Elimination to find this unique solution \mathbf{x} .
- (ii) Determine the least square solution $\hat{\mathbf{x}}$ to the normal system of equations by using Gaussian Elimination.
- (iii) What do you notice when you compare the unique solution \mathbf{x} from (i) to the least squares solution $\hat{\mathbf{x}}$ from (ii).

Least squares lines

4. For each of the following given sets of data points, find the least squares line $y = a_0 + a_1x$ by (a) using the pseudoinverse, and (b) solving the normal system using Gaussian Elimination.
- (i) $(1, 1), (2, 5), (3, 9)$
 - (ii) $(-3, 8), (-1, 5), (1, 3), (3, 0)$
 - (iii) $(-2, 3), (-1, 1), (0, 0), (1, -2), (2, -4)$

Quadratic approximations

5. Find a quadratic least squares approximating polynomial $y = a_0 + a_1x + a_2x^2$ for the data points $(-3, 1)$, $(-2, 0)$, $(0, 1)$, $(2, 3)$, $(3, 5)$.