

Full Name: \_\_\_\_\_

Student Number: \_\_\_\_\_

### Test 1

Total Marks: 67

Time: 100 minutes

#### Instructions:

1. This test has 8 pages. If you need more space, add pages to your submission and make a note on the relevant question page.
2. The test has two sections. Section 1 has 10 multiple-choice questions, where no working is required. Section 2 has 5 questions, where full working should be shown.
3. This is an individual assessment. You may use any calculation technology, and any reference material, but the answers must be your own work.
4. When complete, submit your answers as a single PDF document via the Test 1 submission link on Moodle.

## Section 1

This section contains 10 questions. For each question, indicate the best answer. Working is not needed.

1. **[2 marks]** In a radioactive decay process, alpha particles are emitted independently at an average rate of 8 per minute. What is the chance that fewer than 5 alpha particles will be emitted in the next 30 seconds?  
(A) 0.0996  
(B) 0.1954  
(C) 0.6288  
(D) 0.7851
2. **[2 marks]** The random variable  $X \sim \text{Bin}(12, p)$  has mean 4.8. What is  $P(X > 6)$ ?  
(A) 0.1582  
(B) 0.1766  
(C) 0.6652  
(D) 0.8418
3. **[2 marks]** The continuous random variable  $X$  has a uniform distribution with  $E(X) = 0$  and  $\text{Var}(X) = \frac{16}{3}$ . What is the upper limit of  $X$ ?  
(A) 2.309  
(B) 3  
(C) 4  
(D) 5.657
4. **[2 marks]** The random variable  $X \sim \text{Exponential}(2)$ . What is  $\text{Var}(4X + 3)$ ?  
(A) .5  
(B) 1  
(C) 4  
(D) 8
5. **[2 marks]** Events  $A$  and  $B$  are independent, with  $P(A) = 0.4$  and  $P(A \cap B) = 0.3$ . Find  $P(A \cup B)$ .  
(A) 0.66  
(B) 0.7  
(C) .75  
(D) 0.85

6. **[2 marks]**  $X$  is a random variable with  $E(X^2) = 5$ . Which of the following could be true?
- (A)  $E(X) = 3.5$
  - (B)  $E(X) = 5$
  - (C)  $\text{Var}(X) = 5$
  - (D)  $\text{Var}(X) = 25$
7. **[2 marks]** A discrete random variable  $X$  takes only positive values, and  $P(X = 2) = 0.2$ ,  $P(X = 3) = 0.3$ . Which of the following must be true?
- (A)  $P(X = 1) > 0.1$
  - (B)  $P(X \leq 2) < 0.5$
  - (C)  $P(X > 3) \leq 0.5$
  - (D)  $P(X \leq 4) = 1$
8. **[1 mark]** If the random variable  $X$  has  $E(X) = \text{SD}(X) = 5$ , what is  $E(X^2)$ ?
- (A) 5
  - (B) 25
  - (C) 30
  - (D) 50
9. **[1 mark]** Which of the following can be used to test whether events  $A$  and  $B$  are independent?
- (A)  $P(A \cup B) = P(A) + P(B)$
  - (B)  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  - (C)  $P(A \cap B) = P(A)P(B)$
  - (D)  $P(A|B) = P(B)$
10. **[1 mark]** Which of the following is a valid probability distribution for a discrete random variable  $X$ ?
- (A)  $P(X = 1) = 0.3, P(X = 2) = 0.1, P(X = 3) = 0.5$
  - (B)  $P(X = 1) = 0.3, P(X = 2) = 0.2, P(X = 3) = 0.5$
  - (C)  $P(X = 1) = 0.3, P(X = 2) = 0.8, P(X = 3) = -0.1$
  - (D)  $P(X = 1) = 0.3, P(X = 2) = 0.3, P(X = 3) = 0.5$

## Section 2

This section contains five questions. **Provide full working.**

1. **[9 Marks]** During a disease epidemic, it is estimated that 9% of the population have the disease. A new diagnostic test has been proposed which detects the disease in 92% of cases when the disease is present, and returns a negative result in 94% of cases when the disease is not present.
  - (a) **[4 marks]** Structure the given information using either a tree diagram, a Venn diagram or a contingency table. A tree diagram must assign probabilities to each branch and to the end of each path, a Venn diagram must assign probabilities to each area, and a contingency table must have totals for rows and columns.
  - (b) **[2 marks]** When the test is used in this population, a person receives a positive result (test says disease is present). What is the chance that the person really has the disease?
  - (c) **[3 marks]** The person from part (b) was told to get tested again with the same test. Assuming successive tests on the same person are independent, what would be the chance that the person was diseased if the second result was also positive?

2. [11 Marks] A production line produces 50 complex items of equipment each week. At the end of the week 10 items are removed for testing. Let  $X$  be the number of defective items in the testing sample. If 7 items in each week's output are defective -

(a) [3 marks] What is the distribution of  $X$ ? Give the name and parameter values.

(b) [2 marks] Calculate  $Var(X)$

(c) [3 marks] Calculate  $P(X \geq 3)$

(d) [3 marks] How do the answers to parts (a) and (b) change if the sampling is done with replacement?

3. **[11 Marks]** An archer practicing on a target has probability 0.2 of scoring a bulls-eye on each attempt. Assume that the attempts are independent. The archer continues until she scores 4 bulls-eyes, not necessarily consecutively. Let  $X$  be the number of misses before the first hit and let  $Y$  be the number of misses before the fourth hit.

(a) **[3 marks]** State the distributions of  $X$  and  $Y$ .

(b) **[2 marks]** Find the mean and variance of  $Y$

(c) **[3 marks]** Write down the probability of exactly 7 attempts as a probability statement about  $Y$ . Evaluate this probability.

(d) **[3 marks]** Find the probability that there will be more than 6 misses before the first hit.

4. **[10 Marks]** Let  $X$  be a continuous random variable with CDF

$$F(x) = ax^3 - 3x^2, \quad 0 \leq x \leq 1$$

- (a) **[1 mark]** What is the value of  $a$ ?

- (b) **[2 marks]** Find  $f$ , the probability density function of  $X$

- (c) **[3 marks]** Compute  $P(0.3 < X < 0.6)$

- (d) **[4 marks]** Find the expectation and variance of  $X$ .

5. **[9 Marks]**  $X$  is normally distributed with mean 47.4 and standard deviation 5.71.

(a) **[2 marks]** Calculate  $P(40 < X < 52)$ .

(b) **[2 marks]** Find  $P(X \geq 55)$ .

(c) **[3 marks]** Find the quartiles of the distribution.

(d) **[2 marks]** Calculate  $E(3X^2)$ .

**END OF TEST**