CURTIN UNIVERSITY
Discipline of Mathematics and Statistics

MATH1015 Linear Algebra 1

MID-SEMESTER TEST

Semester 1, 2021

INSTRUCTIONS: Answer all questions in the spaces provided.

To obtain full marks for a question you must **clearly** show appropriate working.

TIME ALLOWED: 1 hour.

SOLUTIONS

TOTAL MARKS: 50

AIDS ALLOWED:
1. Scientific Calculator.
2. A4 Sheet of handwritten or typed notes (both sides).

Last Name: ______________________

Given Name: ______________________

Student Number: ______________________

Tutor's Name: ______________________

Workshop Day: ______________ Workshop Time: ______________

Declaration: *I hereby undertake not to discuss or divulge the content or format of the test paper with any other person until all tests have been written, and declare that I have no prior knowledge of the contents of the test paper.*

I unconditionally accept any action that may be taken should Curtin University consider that an infringement of the statute No. 10 of the Student Disciplinary Statute has occurred.

Signature: **Date:** / / 2021

Question 1

Given the points $A(2,1,-3)$ and $B(4,0,-1)$, and the vectors $\boldsymbol{c} = [2,1,-1]$, $\boldsymbol{d} = [1,3,-2]$, $\boldsymbol{e} = [-1,2,0]$ and $\boldsymbol{f} = [-2,-6,x]$.

(i) Determine the position vector of the point A. (1 mark)

(ii) Find the vector from point A to point B. (2 marks)

(iii) Express the vector $\boldsymbol{c}$ in terms of the standard unit basis vectors. (1 mark)

(iv) Determine the vector twice the length of $\boldsymbol{d}$ which is also in the opposite direction to $\boldsymbol{d}$. (2 marks)

(v) Find a non-zero vector perpendicular to $\boldsymbol{e}$. (2 marks)

(i). $\vec{OA} = [2,1,-3]$ 1mark

(ii). $\vec{AB} = \vec{OB} - \vec{OA}$

$= [4,0,-1]-[2,1,-3]$ 1 $= [2,-1,2]$ 1

(iii). $\underset{\sim}{c} = 2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k}$ or $2\underset{\sim}{i} + 1\underset{\sim}{j} - 1\underset{\sim}{k}$ 1

(iv). $-2\underset{\sim}{d} = -2[1,3,-2]$

$= [-2,-6,4]$

1mark for -ive
1mark for scalar multi. of 2

(v) Find any vector $\underset{\sim}{x}$ such that

$\underset{\sim}{x} \cdot \underset{\sim}{e} = 0$ 1

$[x_1, x_2, x_3] \cdot [-1,2,0] = 0$

$-1x_1 + 2x_2 = 0$

One possible solution: $x_1 = 2,\ x_2 = 1,\ x_3 = 0$

ie $\underset{\sim}{x} = [2,1,0]$ 1

Any vector that makes dot product $= 0$ is acceptable
Note: must be non-zero so $\underset{\sim}{x} = [0,0,0]$ is not accepted.

Question 1 is continued on the next page...

Question 1 continued

Given the points $A(2,1,-3)$ and $B(4,0,-1)$, and the vectors $\boldsymbol{c} = [2,1,-1]$, $\boldsymbol{d} = [1,3,-2]$, $\boldsymbol{e} = [-1,2,0]$ and $\boldsymbol{f} = [-2,-6,x]$.

(vi) Determine the value of x that makes $\boldsymbol{f}$ parallel to $\boldsymbol{d}$. (2 marks)

(vii) Determine the vector projection of $\boldsymbol{d}$ onto $\boldsymbol{e}$. (3 marks)

(viii) Find a non-zero vector orthogonal to both $\boldsymbol{c}$ and $\boldsymbol{d}$. (3 marks)

(vi). Need $\underset{\sim}{f} = m\underset{\sim}{d}$

$[-2,-6,x] = m[1,3,-2]$

If $m=-2$ (1)

$[-2,-6,x] = [-2,-6,4]$ $\therefore x = 4$ (1)

(vii). Scalar proj.,

$$p = \underset{\sim}{d}\cdot\hat{\underset{\sim}{e}} = \frac{\underset{\sim}{d}\cdot\underset{\sim}{e}}{\|\underset{\sim}{e}\|} = \frac{[1,3,-2]\cdot[-1,2,0]}{\sqrt{(-1)^2+(2)^2+(0)^2}} \text{ (1/2) (1/2)}$$

$$= \frac{-1+6-0}{\sqrt{1+4+0}} = \frac{5}{\sqrt{5}} \text{ (1/2)}$$

Vector proj.,

$$\underset{\sim}{p} = p\hat{\underset{\sim}{e}} = p\frac{\underset{\sim}{e}}{\|\underset{\sim}{e}\|} = \frac{5}{\sqrt{5}}\frac{[-1,2,0]}{\sqrt{5}} \text{ (1/2)} = [-1,2,0] \text{ (1)}$$

(viii). $\underset{\sim}{c}\times\underset{\sim}{d}$ (1) $=$

$$\begin{array}{ccccc} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} & \underset{\sim}{i} & \underset{\sim}{j} \\ 2 & 1 & -1 & 2 & 1 \\ 1 & 3 & -2 & 1 & 3 \end{array}$$

$$= \underset{\sim}{i}((1)(-2)-(-1)(3)) + \underset{\sim}{j}((-1)(1)-(2)(-2)) + \underset{\sim}{k}((2)(3)-(1)(1)) \text{ (1/2) (1/2) (1/2)}$$

$$= \underset{\sim}{i}(-2+3) + \underset{\sim}{j}(-1+4) + \underset{\sim}{k}(6-1)$$

$$= [1,3,5] \text{ (1/2)}$$

or any multiple of this vector is acceptable.

Question 2 is on the next page...

Question 2

Given the matrices,

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 4 \\ -2 & 5 \end{bmatrix}$$

Find, if possible the following. If it's not possible then explain why.

(i) $A - 2B$ (2 marks)
(ii) B^{-1} (3 marks)
(iii) $A^T B$ (3 marks)
(iv) $A \div B$ (2 marks)

(i). $A-2B = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} - 2\begin{bmatrix} -1 & 4 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} - \begin{bmatrix} -2 & 8 \\ -4 & 10 \end{bmatrix}$ 1

$= \begin{bmatrix} 4 & -5 \\ 3 & -10 \end{bmatrix}$ 1

(ii). $B^{-1} = \underbrace{\dfrac{1}{(-1)(5)-(4)(-2)}}_{1} \begin{bmatrix} 5 & -4 \\ -(-2) & -1 \end{bmatrix}$ 1 $= \dfrac{1}{3}\begin{bmatrix} 5 & -4 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} 5/3 & -4/3 \\ 2/3 & -1/3 \end{bmatrix}$ 1

(iii). $A^T B = \underbrace{\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}}_{1} \begin{bmatrix} -1 & 4 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} (2)(-1)+(-1)(-2) & (2)(4)+(-1)(5) \\ (3)(-1)+(0)(-2) & (3)(4)+(0)(5) \end{bmatrix}$

$= \begin{bmatrix} 0 & 3 \\ -3 & 12 \end{bmatrix}$ 1/2 for each correct entry

(iv). $A \div B =$ d.n.e 1

Can't divide matrices. 1

Question 3

Given the following system of linear equations,

$$\begin{aligned} 2x_1 - x_2 - 3x_3 &= -9 \\ -x_1 + 2x_2 + x_3 &= 6 \\ 4x_1 + x_2 + kx_3 &= 2 \end{aligned}$$

(i) Find the augmented matrix $[A|\boldsymbol{b}]$ of the system. (2 marks)

(ii) By using elementary row operations reduce the augmented matrix into row echelon form. (3 marks)

(iii) Using your result from (ii) determine the value of k for which the system has no solution. (1 mark)

(i). $\left[\begin{array}{ccc|c} 2 & -1 & -3 & -9 \\ -1 & 2 & 1 & 6 \\ 4 & 1 & k & 2 \end{array}\right]$ 2

(ii). $\left[\begin{array}{ccc|c} 2 & -1 & -3 & -9 \\ -1 & 2 & 1 & 6 \\ 4 & 1 & k & 2 \end{array}\right] \begin{array}{l} \\ R_2 = 2R_2 + R_1 \\ R_3 = R_3 - 2R_1 \end{array} \sim \left[\begin{array}{ccc|c} 2 & -1 & -3 & -9 \\ 0 & 3 & -1 & 3 \\ 0 & 3 & k+6 & 20 \end{array}\right] \begin{array}{l} \\ \\ R_3 = R_3 - R_2 \end{array}$

1 for Row 2

1 for Row 3

$\sim \left[\begin{array}{ccc|c} 2 & -1 & 3 & -9 \\ 0 & 3 & -1 & 3 \\ 0 & 0 & k+7 & 17 \end{array}\right]$ 1

(iii) For no solution $r(A) \neq r([A|\underset{\sim}{b}])$

$\therefore k + 7 = 0$ ½

$\Rightarrow k = -7$ ½

Question 4

Given the following augmented matrix $[A|\boldsymbol{b}]$, which is in row echelon form, the underlying system of linear equations has an infinite amount of solutions. Determine these infinite solutions (make sure you state the rank of A, the rank of $[A|\boldsymbol{b}]$, the number of variables, as well as the number of parameters required to describe the infinite solutions).

$$\left[\begin{array}{ccccc|c} 2 & 0 & -1 & 1 & 0 & 4 \\ 0 & 0 & 0 & 3 & 0 & -6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

(8 marks)

$r(A) = 2$ ½ $r([A|\underset{\sim}{b}]) = 2$ ½ $n = 5$ 1

Need $n - r = 5 - 2 = 3$ parameters 1

Let $x_2 = r$ 1

$x_3 = s$ 1

$x_5 = t$ 1

Row 2: $3x_4 = -6 \Rightarrow x_4 = -2$ 1

Row 1: $2x_1 - x_3 + x_4 = 4$

$2x_1 - s + (-2) = 4$

$2x_1 = 6 + s$

$x_1 = 3 + s/2$ 1

Question 5

Solve the follow system of linear equations by using the Gauss Jordan method to manipulate the augmented matrix into reduced row echelon form,

$$\begin{aligned} x_1 + x_2 + 3x_3 &= 3 \\ x_1 - x_2 - 2x_3 &= 2 \\ -3x_1 + x_2 + 6x_3 &= -2 \end{aligned}$$

(10 marks)

$$[A|\underset{\sim}{b}] = \left[\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 1 & -1 & -2 & 2 \\ -3 & 1 & 6 & -2 \end{array}\right] \begin{array}{l} R_2 = R_2 - R_1 \\ R_3 = R_3 + 3R_1 \end{array}$$

1 for $[A|\underset{\sim}{b}]$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & -2 & -5 & -1 \\ 0 & 4 & 15 & 7 \end{array}\right] R_3 = R_3 + 2R_2 \sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & -2 & -5 & -1 \\ 0 & 0 & 5 & 5 \end{array}\right] R_3 \div (5)$$

1 for Row 2
1 for Row 3
1 for Row 3

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 3 \\ 0 & -2 & -5 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 = R_1 - 3R_3 \\ R_2 = R_2 + 5R_3 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & -2 & 0 & 4 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R_1 = R_1 + \frac{1}{2}R_2 \\ R_2 \div (-2) \end{array}$$

1 for Row 3
1 for Row 1
1 for Row 2

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

1 for Row 1
1 for Row 2

$$\left.\begin{array}{ll} \text{Row 3:} & x_3 = 1 \\ \text{Row 2:} & x_2 = -2 \\ \text{Row 1:} & x_1 = 2 \end{array}\right\} 1$$

~END OF TEST~