

**CURTIN COLLEGE - EMTH1019 Linear Algebra and Statistics**  
**Final Assignment (Weight 25%, Total Marks = 50)**

**STUDENT DECLARATION**

I declare that this assignment is all my own work.

<b>Student Name</b>	
<b>Student ID</b>	

**Submission Requirements**

- All answers are to be **HANDWRITTEN**.
- Submit a single PDF or docx file. Multiple or zip files are **NOT** acceptable.
- It is your responsibility to ensure all images are upright and legible.
- Marks per question are as indicated in the assignment.
- Sufficient logical working is to be provided. **No working = no marks.**
- Answers to be summarized.
- As this assessment is in lieu of a final exam the results will not be released.
- Late penalties apply.

1. (3 + 3 + 1 + 1 = 8 marks)

Given the planes  $2x - 4y + z = 7$  and  $3x + y - z = 2$  determine:

- The angle between the planes.
- The equation of the line of intersection.
- The coordinates of point that lies on both planes.
- Check that the point from c) satisfies both plane equations.

2. (4 marks)

Determine the shortest distance between the plane  $2x - 4y + z = 7$  and the point  $A(3,1,1)$ . Ensure that you provide a quality sketch that identifies and labels all key information.

3. (4 marks)

Find the point at which the line  $\{x = 1 + t, y = 2t + 5, z = 3t - 1\}$  intersects the plane  $x + 2y + z = 26$ . Check that your answer is valid for both the line and the plane.

4. (1 + 2 + 2 + 3 + 2 = 10 marks)

The following system of equations has infinite solutions.

$$-x_1 + x_2 - 2x_3 = 1$$

$$2x_1 + x_2 - 2x_3 = -2$$

$$-x_1 + 2x_2 - 4x_3 = 1$$

- Write down the augmented matrix of the system.
- Use Gaussian Elimination to reduce  $[A|\mathbf{b}]$  to row echelon form
- Use rank to describe the type of solution.
- Solve the system of equations using an appropriate parameter
- Check the solution by substituting it back into all of the initial equations.

5. (4 marks)

Determine the exact value(s) of  $x$  for which the following set of vectors is

Linearly Independent.  $\left\{ \begin{bmatrix} 2 \\ 1 \\ x \end{bmatrix}, \begin{bmatrix} -2 \\ x \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -3 \end{bmatrix} \right\}$ .

6. (4 marks)

Determine if the following set of vectors are linearly independent or linearly dependent.

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

7. (6 marks)

Determine if the following lines are skew lines.

$$\mathcal{L}_1: \begin{cases} x_1 = 3 + 4t \\ x_2 = 1 + 3t \\ x_3 = 3 + t \\ x_4 = 1 + 2t \end{cases} \quad \mathcal{L}_2: \begin{cases} x_1 = 1 + 3r \\ x_2 = 8 - r \\ x_3 = 3 + 2r \\ x_4 = 1 + r \end{cases}$$

8. (2 + 2 + 4 + 2 = 10 marks)

For the following data points  $\{(0,1), (2,1), (3,2), (4,3), (5,3)\}$  determine the least squares approximating line.

- Plot the data on a linear graph.
- Set up a system of linear equations.
- Solve the normal system of equations  $A^T A x = A^T \mathbf{b}$  using Gaussian Elimination.
- Check your solution against the graph in part a) and check that your solution satisfies the normal system of equations.

End of assessment