COMP1002
DATA STRUCTURES AND ALGORITHMS

LECTURE 7: HASH TABLES

Curtin University

Discipline of Computing

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This Week

- Hashtables and hashing
- Properties of a good hashtable
- Collisions and collision handling methods
  - Open addressing (probing) approaches
  - Separate chaining
- Hashtable Big-O analysis
- Some good hash functions and poor hash functions

Hash Tables – Hashing

- A hash table stores each data element using an associated key
  - The key is later used to find the element efficiently
  - Hash tables convert the key into an index via an arithmetic function and then place the data at this index
  - This conversion is referred to as "hashing": applying an arithmetic function to a key to map it to a location (index) in an array for storing the data associated with that key
  - The arithmetic function is called the hashing function
  - The location it maps a key to is called the hash index

A Simple Example

- Simple hash function: add up the letters in a string
  - How is this possible? Because chars are actually numbers
    - The application just reinterprets them as symbols for the user
  - Char values are encoded according to Unicode standard
    - ASCII is a subset of Unicode for the European alphabets
      - Unicode is actually an extension of ASCII in order to handle non-Latin languages such as Mandarin, Japanese, Arabic, etc
    - So let's have a look at the ASCII character set…

ASCII Character Set

Ctrl	Dec	Hex	Char	Code	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char	Dec	Hex	Char
^@	0	00		NUL	32	20		64	40	@	96	60	`	128	80	Ç	160	A0	á	192	C0	└	224	E0	α
^A	1	01		SOH	33	21	!	65	41	A	97	61	a	129	81	ü	161	A1	í	193	C1	┴	225	E1	ß
^B	2	02		STX	34	22	"	66	42	B	98	62	b	130	82	é	162	A2	ó	194	C2	┬	226	E2	Γ
^C	3	03		ETX	35	23	#	67	43	C	99	63	c	131	83	â	163	A3	ú	195	C3	├	227	E3	π
^D	4	04		EOT	36	24	$	68	44	D	100	64	d	132	84	ä	164	A4	ñ	196	C4	─	228	E4	Σ
^E	5	05		ENQ	37	25	%	69	45	E	101	65	e	133	85	à	165	A5	Ñ	197	C5	┼	229	E5	σ
^F	6	06		ACK	38	26	&	70	46	F	102	66	f	134	86	å	166	A6	ª	198	C6	╞	230	E6	μ
^G	7	07		BEL	39	27	'	71	47	G	103	67	g	135	87	ç	167	A7	º	199	C7	╟	231	E7	τ
^H	8	08		BS	40	28	(	72	48	H	104	68	h	136	88	ê	168	A8	¿	200	C8	╚	232	E8	Φ
^I	9	09		HT	41	29	)	73	49	I	105	69	i	137	89	ë	169	A9	⌐	201	C9	╔	233	E9	Θ
^J	10	0A		LF	42	2A	*	74	4A	J	106	6A	j	138	8A	è	170	AA	¬	202	CA	╩	234	EA	Ω
^K	11	0B		VT	43	2B	+	75	4B	K	107	6B	k	139	8B	ï	171	AB	½	203	CB	╦	235	EB	δ
^L	12	0C		FF	44	2C	,	76	4C	L	108	6C	l	140	8C	î	172	AC	¼	204	CC	╠	236	EC	∞
^M	13	0D		CR	45	2D	-	77	4D	M	109	6D	m	141	8D	ì	173	AD	¡	205	CD	═	237	ED	φ
^N	14	0E		SO	46	2E	.	78	4E	N	110	6E	n	142	8E	Ä	174	AE	«	206	CE	╬	238	EE	∈
^O	15	0F		SI	47	2F	/	79	4F	O	111	6F	o	143	8F	Å	175	AF	»	207	CF	╧	239	EF	∩
^P	16	10		DLE	48	30	0	80	50	P	112	70	p	144	90	É	176	B0	░	208	D0	╨	240	F0	≡
^Q	17	11		DC1	49	31	1	81	51	Q	113	71	q	145	91	æ	177	B1	▒	209	D1	╤	241	F1	±
^R	18	12		DC2	50	32	2	82	52	R	114	72	r	146	92	Æ	178	B2	▓	210	D2	╥	242	F2	≥
^S	19	13		DC3	51	33	3	83	53	S	115	73	s	147	93	ô	179	B3	│	211	D3	╙	243	F3	≤
^T	20	14		DC4	52	34	4	84	54	T	116	74	t	148	94	ö	180	B4	┤	212	D4	╘	244	F4	⌠
^U	21	15		NAK	53	35	5	85	55	U	117	75	u	149	95	ò	181	B5	╡	213	D5	╒	245	F5	⌡
^V	22	16		SYN	54	36	6	86	56	V	118	76	v	150	96	û	182	B6	╢	214	D6	╓	246	F6	÷
^W	23	17		ETB	55	37	7	87	57	W	119	77	w	151	97	ù	183	B7	╖	215	D7	╫	247	F7	≈
^X	24	18		CAN	56	38	8	88	58	X	120	78	x	152	98	ÿ	184	B8	╕	216	D8	╪	248	F8	°
^Y	25	19		EM	57	39	9	89	59	Y	121	79	y	153	99	Ö	185	B9	╣	217	D9	┘	249	F9	∙
^Z	26	1A		SUB	58	3A	:	90	5A	Z	122	7A	z	154	9A	Ü	186	BA	║	218	DA	┌	250	FA	·
^[	27	1B		ESC	59	3B	;	91	5B	[	123	7B	{	155	9B	¢	187	BB	╗	219	DB	█	251	FB	√
^\	28	1C		FS	60	3C	<	92	5C	\	124	7C			156	9C	£	188	BC	╝	220	DC	▄	252	FC	ⁿ
^]	29	1D		GS	61	3D	=	93	5D	]	125	7D	}	157	9D	¥	189	BD	╜	221	DD	▌	253	FD	²
^^	30	1E	▲	RS	62	3E	>	94	5E	^	126	7E	~	158	9E	₧	190	BE	╛	222	DE	▐	254	FE	■
^-	31	1F	▼	US	63	3F	?	95	5F	_	127	7F	⌂	159	9F	ƒ	191	BF	┐	223	DF	▀	255	FF	

ASCII Character Set

- So each letter can be converted into a number…
  - Range of ASCII values is 0-255
  - Each character can then be represented using a byte = 2^8

- Examples:
  - **"0"** (zero) is 48 in decimal   ← numbers are all less than letters
  - **"9"** is 57
  - **"A"** is 65 **"Z"** is 90
  - **"a"** is 97   ← this is how "apple" is less than "bear" and "zombie"
  - **"z"** is 122
  - **"!"** is 33

- Hex values are more interesting:
  - 0011 0001 is **"1"** – clear bits 4-5 to convert from character to num (0000 0001)
  - 0100 0001 is **"A"** and 0110 0001 is **"a"** – set or clear bit 5 to convert

ASCII Character Set

- Notice the blank chars early on…
  - These are special 'control' chars, some of which have been defined as special 'escape characters' \x in Java/Python/C/C++
  - Examples:
    - 10 is LF (line feed), which is '\n' in
    - 13 is CR (carriage return), which is '\r'
      - Under Linux, end-of-line is \n. Under Windows, eol is \r\n
    - 0 is NULL which is '\0'
      - Particularly important in C for null-terminated strings
    - 9 is HT (horizontal tab) == '\t'

- Not all these control chars have escape chars
  - These are typically unprintable characters (don't show up) or produce odd symbols depending on the display program
  - *e.g.* Notepad will show a □

A Simple Example

- Back to the example: hash function for string keys

```
FUNCTION hashFunction IMPORT key (string) EXPORT hashIndex (integer)
    hashIndex ← 0
    FOR ii ← 0 TO len(key)-1 DO
        hashindex ← hashindex + key.charAt(ii)

    hashIndex ← hashindex % hashArray.length     // We'll discuss this line later
```

- Java
  - key.charAt(ii) returns the char at index ii (zero-based)
  - Since chars are numbers, we can add them to an int arithmetically.
  - So "abcde" would be:
    - hashIdx = 'a'+'b'+'c'+'d'+'e'; // =495
- Python
  - Use ord() function to get ASCII values
    - hashindex = hashindex + ord(key[ii])

Hashing

- So a hash table has two major components:
  - Array: (table) to store the data
  - Hash function: to map keys to integer indexes in the array

- When a new element is to be added, both a key and data must be provided to the hash table
  - The hash table hashes the key and stores both the key and data at the calculated hash index
    - Key must be unique
  - Thus time complexity is the time it takes to perform the hash calculation $O(?)$ plus the $O(1)$ array access time
    - Accessing data in the hash table is the same

Hash Table – Example

Table

	0
	1
	2
Fred Flintstone	3
	4
	5
	6
Barney Rubble	7
	8
	9
	10

hash("Flintstone")
returns 3

"Flintstone" ➤ hash function ➤

hash("Rubble")
returns 7

"Rubble" ➤ hash function ➤

Keyed on last name

Hash function converts each key to a (hopefully different!) hash index

⋮

	n-1
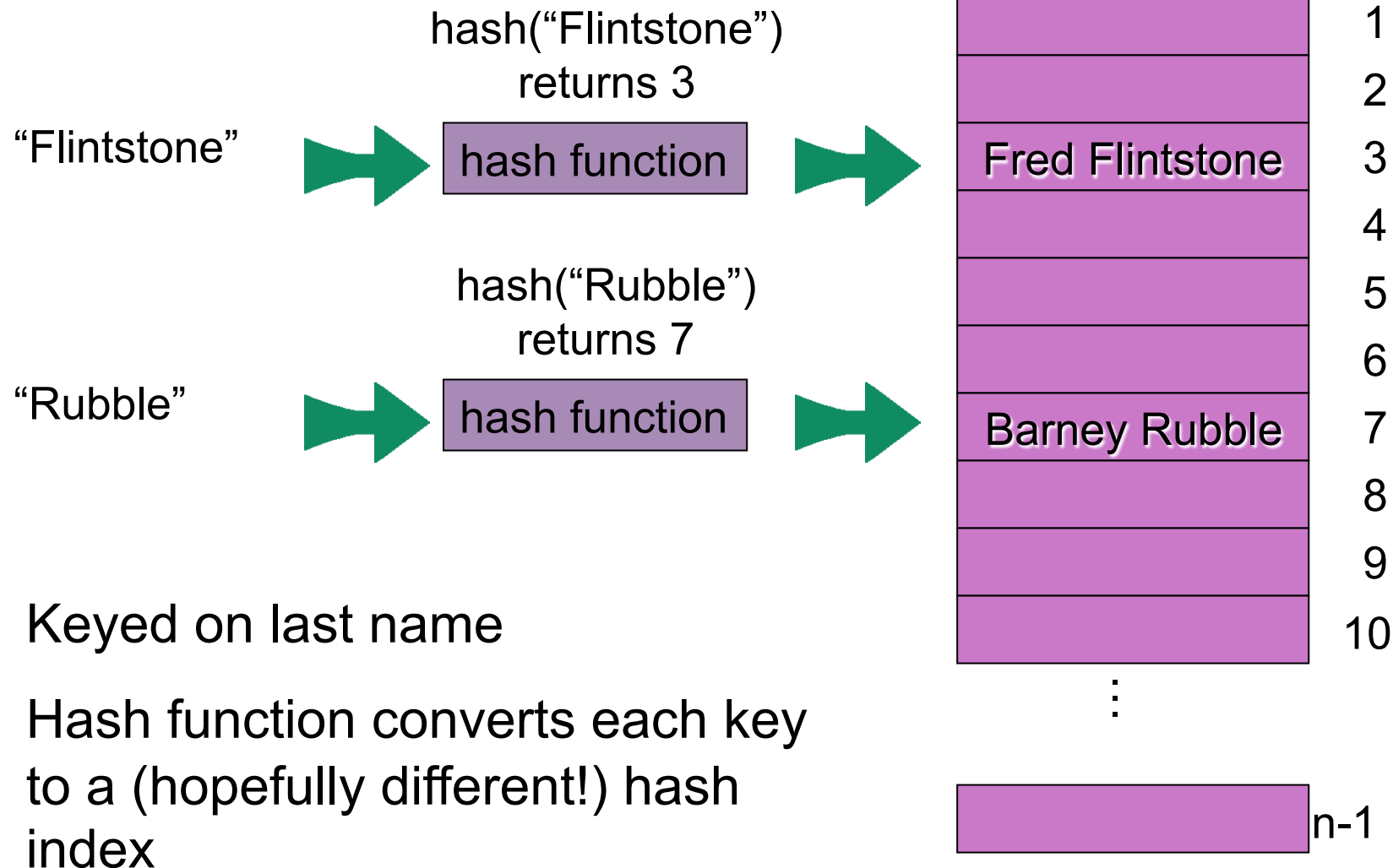
HASH FUNCTIONS

Properties of a *Good* Hash Function

- A good hash function should:
  1. Return indexes that fit within the size of the array
     - *i.e.,* [0 .. arrayLength-1]
  2. Be fast to compute
     - The hash function is a critical factor in access time
  3. Be repeatable (*i.e.,* always return same index) for a given key
  4. Distribute keys evenly over the full range of the array
     - This is to minimise collisions, a major issue in hash tables
- Properties 1–3 are easy to ensure, Property 4 is not
  - You don't know what keys you'll be getting in advance, so it's not possible to ensure they will be evenly distributed

Property 1: Hash Indexes Must Fit Table

- Let's consider a simple hash function that operates on integer keys (*e.g.,* student IDs)

  - A bad implementation would be:

```
CLASS HashTable
Class field : hashArray

FUNCTION hashFunction IMPORT key (integer) EXPORT hashIndex (integer)
hashIndex ← key
```

- What's the problem with it?

  - Key can be larger than table.length or possibly negative, this will make hashIndex out of the table's index range.

Property 1: Hash Indexes Must Fit Table

- So why not make the table big enough to handle any integer key that we expect from the application?
  - That's a waste of space: there are far fewer actual data items than the number of possible key values
    - *e.g.,* Student IDs are large numbers that track *all* students ever (~15 million), but there are only ~50,000 current students
    - We thus only want to reserve space for 50,000, not all 15m
- What the hash function must do is 'compress' the range of the key down into the range of the table
  - Easy to do! Just take the modulus of the hashIndex
    - Although there are better and worse modulus values…

Property 1: Hash Indexes Must Fit Table

- The obvious modulus is the table's length, since any integer's remainder will then fit inside the table

```
FUNCTION hashFunction IMPORT key (integer) EXPORT hashIndex (integer)
    hashIndex ← key % table.length
```

- Prime numbers often make good modulo values
  - Primes will usually distribute the keys most evenly
    - … although it does depend somewhat on the hash function used
  - So when allocating the table, size it with a prime number
    - ideal prime sizes are as far as possible from powers of 2
      - 769, 1543, 3079, 6151, 12289, 24593, 49157, 98317 *etc.*
  - If the user is allowed to specify the max size of the hash table in the constructor, round the max up to the nearest prime number
    - Finding the next prime can be done by incrementing a starting candidate value and testing for primality – an $O(N^{1/2})$ problem

Finding the Next Prime

```
FUNCTION findNextPrime IMPORT startVal (integer) EXPORT prime (integer)

IF (startVal % 2 == 0) THEN
    primeVal ← startVal + 1            ← Even numbers are never prime, so make it odd
ELSE
  primeVal ← startVal
ENDIF
primeVal ← primeVal - 2               ← Pre-decrement since we will be looking for the NEXT prime,
                                         but want to include startVal as a candidate

isPrime ← FALSE
WHILE (NOT isPrime)                    ← There is always a prime number to be found
    primeVal ← primeVal + 2           ← Next candidate

    // Test if primeVal candidate is actually a prime number
    ii ← 3
    isPrime = TRUE
    WHILE (ii*ii <= primeVal) AND (isPrime)    ← No need to check beyond sqrt(primeVal)
        IF (primeVal % ii == 0) THEN              (if the values up to ii are not whole divisors, the final
            isPrime ← FALSE                        (ii-1)/primeVal percentile won't be divisors either)
        ELSE
            ii ← ii + 2                        ← Skip testing with even numbers
        ENDIF
    ENDWHILE
ENDWHILE
```

Finding the Next Prime (improved)

```
FUNCTION findNextPrime IMPORT startVal (integer) EXPORT prime (integer)

IF (startVal % 2 == 0) THEN
    primeVal ← startVal -1                    ← Even numbers are never prime, so make it odd 1 less as primeval+2
ELSE
  primeVal ← startVal
ENDIF

isPrime ← FALSE
DO                                // Test if primeVal candidate is actually a prime number
    primeVal ← primeVal + 2                   ← Next candidate
    ii ← 3
    isPrime = TRUE
    rootVal = SQRT primeVal                   ← No need to check beyond sqrt(primeVal)
    DO
        IF (primeVal % ii == 0) THEN              (if the values up to ii are not whole divisors, the final
            isPrime ← FALSE                        (ii-1)/primeVal percentile won't be divisors either)
        ELSE
            ii ← ii + 2                        ← Skip testing with even numbers
        ENDIF
    WHILE (ii <= rootVal) AND (isPrime)
WHILE (NOT isPrime)                           ← There is always a prime number to be found
```

Property 2: Fast to Compute

- In general, access time in a hash table is:
  - (speed of hash function) + (speed of array access)
    - Array access is O(1), so the hash function is the limiting factor

- Fortunately, hash functions can be pretty fast
  - The hash function only needs to operate on the key
  - Thus it will take constant time regardless of how many elements exist in the hash table (exact time depends on hash func)
    - Constant time complexity: $O(k) \rightarrow \approx O(1)$
  - *e.g.,* consider string keys (such as LastName)
    - Time complexity = $O(L) \approx O(1)$, where L = average name length
    - Independent of table size N, hence stays O(1) regardless of N

Property 3: Repeatable

- A hash function is useless if it returns different hash indexes every time for the same key
  - It won't be able to remember where it stored keys/values!

- Solution: Avoid using time-varying values in the calculation of the hash function
  - *e.g.,* no random numbers, date/time, or hash table statistics
- Issue: Table size must be used as the modulo!
  - Resizing the table means hash function modulo changes
  - Thus you must re-hash *all existing entries* after a resize
    - *i.e.,* rebuild the hash table from scratch, re-inserting each existing element again into the new hash table

Property 4: Distributes Keys Evenly

- The toughest property to achieve
  - It is very difficult to even *evaluate* that a hash function distributes evenly
    - Need to perform statistical tests with a large set of sample keys
  - The problem is that you don't quite know what keys you will be receiving from an application
    - You can often see (in the code) if a hash function will be poor, but guaranteeing a hash function will be good is more difficult
  - Fortunately, much effort has already gone into developing hash functions that work well for a range of typical keys
    - *e.g.,* dictionary words, names, *etc.*

Collisions

- Let's take a little detour and discuss some issues affecting hash tables
  - This will help with understanding how Property 4 (Evenly Distributed) can be best achieved
    - … or at least in realising when it *isn't* achieved!
- A collision is when two (or more) keys map to the same hash index
  - Not a good thing, but must be expected to occur at least sometimes since the hash function 'compresses' the range of the key into the range of the hash table size
    - Unless you are lucky, some keys will hash to the same index

Collisions

- Since collisions are inevitable, we must handle them

- **First**: Never fill up the hashtable
  - The fuller the table, the more likely a collision
  - Try to avoid going over about 50% full (rough rule of thumb)
    - *i.e.,* allocate about twice as much space as you expect to use
    - But don't go too sparsely-populated (< 10%) – waste of space!

- **Second**: Use a collision-handling method
  - There are four typical approaches, split into two broad categories: open addressing and separate chaining

Collision Handling Methods

- Open Addressing: Upon a collision, jump forward ('probe') a set amount to a new index and try again
  - If the new index is also used, repeat the probe until an empty index is found

  - 1. Linear Probing – probe by step size of one every time
  - 2. Quadratic Probing – probe forward by $(probeNum)^2$
    - *i.e.,* probe first by 1, then 4, then 9, 16, 25, 36, 49, …
  - 3. Double Hashing – use a *secondary* (different) hash function on the key to generate the probe step length

- 4. Separate Chaining: Key-value pairs are added to a linked list anchored at the colliding hash index

Property 4: Distributes Keys Evenly

- Why do we need even (*i.e.,* uniform) distribution?
  - Consider the opposite case: all keys hash to index 0
  - This means all keys end up at the same position
    - *i.e.,* every key collides – worst case!
    - Inserting the Nth item takes O(N) probes (for linear/quad probing)
      - Sep-chaining is O(1), double-hashing will vary but won't be good
    - Accessing an item takes (on average) N/2 probes = O(N)
      - Even for separate chaining and (probably) double-hashing
    - Terrible: we should be getting O(1) from our hash table on both!
  - Uniform = each key maps to a different index

Example Hash Functions

- Let's take a look at a couple of hash functions proposed for use in hashtables
  - Only given in Java and here for your interest – not examinable!

  - These operate on either Strings or byte arrays so that they can apply to any type of data
    - A string can be considered a byte array that is limited to text values (alphanumerics, punctuation and symbols)
  - They are by no means 'optimal', since optimality will depend on your actual keys being used
    - And nobody can predict that in advance!
  - Still, they are pretty good at distributing 'average' keys evenly amongst the table
    - Code assumes that hashArray is the hash table array

Example Hash Functions

- Java's hashCode() implementation for String class

```java
private int hash(String key)
{
    int hashIdx = 0;

    for (int ii = 0; ii < key.length(); ii++) {
        hashIdx = (31 * hashIdx) + key.charAt(ii);
    }
    return hashIdx % hashArray.length;
}
```

- Strange (often prime) numbers like 31 are popular
- A variant is the 'Bernstein' hash function
  - Uses 33 instead of 31 – apparently this is even more effective in practice - hash functions are black magic!

Example Hash Functions

- Another fairly simple but good hash function

```
private int hash(byte[] key)
{
    int a = 63689;
    int b = 378551;
    int hashIdx = 0;

    for (int ii = 0; ii < key.length; ii++)
    {
        hashIdx = (hashIdx * a) + key[ii];
        a *= b;
    }
    return hashIdx % hashArray.length;
}
```

- Non-commutative multiplications by weird numbers a and b are apparently important here
  - Again, only borne out in practice

Example Hash Functions

- FNV Hash

```
private int hash(byte[] key)
{
    long hashIdx = 2166136261;

    for (long ii = 0; ii < key.length; ii++) {
        hashIdx = (hashIdx * 16777619) ^ key[ii];        ← ^ is the XOR operator
    }
    return (int)(hashIdx % hashArray.length);
}
```

- More complex and does a lot of integer overflow
  - It's not the complexity that makes it better, but in practice this is a pretty good hash
  - See http://www.isthe.com/chongo/tech/comp/fnv/

Example Hash Functions

- Shift-Add-XOR Hash

```java
private int hash(byte[] key)
{
    int hashIdx = 0;
    for (int ii = 0; ii < len; ii++) {
        hashIdx = hashIdx ^ ( (hashIdx << 5) + (hashIdx << 2) + key[ii] );
    }
    return hashIdx % hashArray.length;
}
```

- Bit shifting is also pretty common in good hash functions
    - It's a fast way to take a number to a power-of-2
    - But it's not the bit-shifting itself that makes it a good hash!

Example Hash Functions

- ELF Hash

```
private int hash(byte[] key)
{
    int hashIdx = 0;
    int g;
    for (int ii = 0; ii < len; ii++) {
        hashIdx = (hashIdx << 4) + key[ii];
        g = hashIdx & 0xF0000000L;
        if (g != 0) {
            hashIdx = hashIdx ^ (g >> 24);        ← ^ is the XOR operator
        }
        hashIdx = hashIdx & ~g;
    }
    return hashIdx % hashArray.length;
}
```

- Another bit-shifter: quite old, and reasonably good too

Poor Hash Functions

- Take-the-first-value:

```
private int hash(byte[] key)
{
    hashIdx = key[0];
    return hashIdx % hashArray.length;
}
```

- Very poor:
  - Collisions with keys that start with the same value
  - Hash indexes will also only be between 0…255
    - Means slots past 255 can never be used

Poor Hash Functions

- Add-them-all-up:

```java
private int hash(byte[] key)
{
    int hashIdx = 0;
    for (int ii = 0; ii < len; ii++) {
        hashIdx += key[ii];
    }
    return hashIdx % hashArray.length;
}
```

- Poor:
  - Collisions when two keys are an anagram of one another
    - Eg: "fred", "derf", "rfde" all hash to the same index
  - Changing the add to a multiply doesn't help any either – anagrams still produce the same hash index

Poor Hash Functions

- XOR-them-all-together:

```
private int hash(byte[] key)
{
    hashIdx = key[0];
    for (int ii = 0; ii < len; ii++) {
        hashIdx = hashIdx ^ key[ii];          ← ^ is the XOR operator
    }
    return hashIdx % hashArray.length;
}
```

- Poor:
  - Hash indexes will also only be between 0…255
    - Means slots past 255 can never be used

COLLISION HANDLING

Linear Probing

- Upon a collision, add 1 to the index and try again
  - Repeatedly probe by 1 until a free slot (bucket) is found
  - Remember to wrap-around the array!
    - *i.e.,* if hash index == tableLen-1, then stepping by 1 will exceed the table size. In this case, we must 'wrap' and probe to index 0

- Linear probing is simple to implement, but has some unfortunate consequences if collisions build up
  - See the next slides for an example

Linear Probing – Example
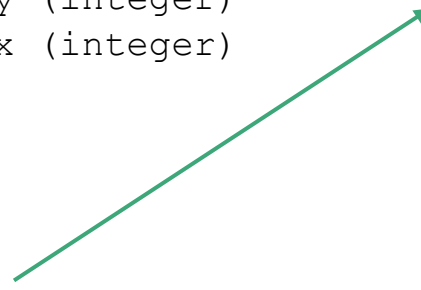
- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key (integer)
                EXPORT hashIndex (integer)
  hashIndex ← key % 11
```

  - Keys:

15 →

Table

Keys	Data	
		0
		1
		2
		3
15	<data>	4
		5
		6
		7
		8
		9
		10

Linear Probing – Example

Table (ignoring data)

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key (integer)
                EXPORT hashIndex (integer)
  hashIndex ← key % 11
```

- Keys:

15

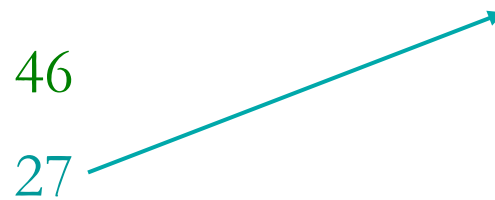|	
	0
	1
	2
	3
15	4
	5
	6
	7
	8
	9
	10

Linear Probing – Example

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key (integer)
              EXPORT hashIndex (integer)
  hashIndex ← key % 11
```

- Keys:

15

46

Table

	0
	1
46	2
	3
15	4
	5
	6
	7
	8
	9
	10

Linear Probing – Example

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key (integer)
                EXPORT hashIndex (integer)
  hashIndex ← key % 11
```
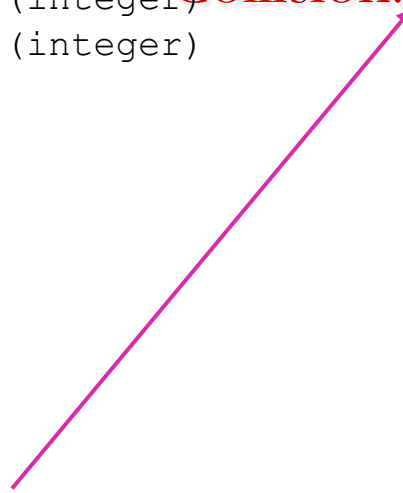
- Keys:

15

46

27

Table

	0
	1
46	2
	3
15	4
27	5
	6
	7
	8
	9
	10

Linear Probing – Example

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key (integer)
              EXPORT hashIndex (integer)
  hashIndex ← key % 11
```

- Keys:

15

46

27

13

Table

Collision!

	0
	1
46	2
	3
15	4
27	5
	6
	7
	8
	9
	10

Linear Probing – Example

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key (integer)
                EXPORT hashIndex (integer)
  hashIndex ← key % 11
```
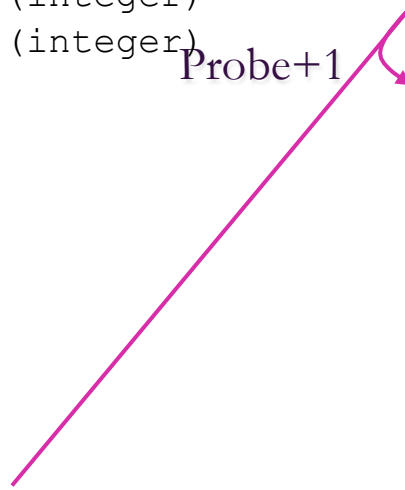
- Keys:

15

46

27

13

Table

	0
	1
46	2
13	3
15	4
27	5
	6
	7
	8
	9
	10

Probe+1

Linear Probing – Example

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key (integer)
               EXPORT hashIndex (integer)
  hashIndex ← key % 11
```

- Keys:

15

46

27

13

37

Table

	0
	1
46	2
13	3
15	4
27	5
37	6
	7
	8
	9
	10

Probe+1

Probe+2

Linear Probing – Analysis

- When collisions occur, linear probing tends to create clusters of filled slots
  - Since it probes one step at a time, it always fills in gaps
  - … thus creating blocks of filled indexes
    - This is called primary clustering
  - Primary clusters will slow down access time
    - Need multiple probing steps to find end of the primary cluster
      - And then insertion at the end makes the cluster even larger/worse!
      - *e.g.,* try inserting 24 into the previous table
    - Can make fast O(1) insertion degrade considerably, especially if table gets pretty full

Primary Clusters

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key (integer)
                 EXPORT hashIndex (integer)
  hashIndex ← key % 11
```

- Keys:

15

46

27

13

37

Table

	0
	1
46	2
13	3
15	4
27	5
37	6
	7
	8
	9
	10

Primary cluster from probing

Primary Clusters

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key (integer)
               EXPORT hashIndex (integer)
  hashIndex ← key % 11
```

- Keys:

15

46

27

13

37

24

O(5) insertion
when N=5!

Table

	0
	1
46	2
13	3
15	4
27	5
37	6
24	7
	8
	9
	10

Quadratic Probing

- Exactly like linear probing, except that the step-size increases at each probe iteration
  - In particular, stepSize = probeNum^2
    - *i.e.,* probe1 = +1, probe2 = +4, probe3 = +9, probe4 = +16, *etc.*
  - Hence called *quadratic* probing (due to squaring of probe)

- The increasing step-size means that more gaps will be left in the table when probing
  - Avoids (or at least greatly reduces) primary clusters
  - Step size increases by > 1 as well: means that a probe can 'leap' past a cluster

Quadratic Probing

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key (integer)
                EXPORT hashIndex (integer)
  hashIndex ← key % 11
```

- Keys:

15

46

27

13

37

Table

	0
	1
46	2
13	3
15	4
27	5
	6
	7
37	8
	9
	10

Probe+1

Probe+4

Quadratic Probing – Analysis

- Doesn't get primary clusters since step-size > 1
  - Ensures that gaps are left between step jumps
- <u>Not</u> inefficient: deal with (multi-)wrap-around via modulo
  - newHashIdx = (origHashIdx + probeNum^2) % tblSize

- But does get secondary clusters
  - Multiple keys mapping to the same hash index: the *initial* collision then occurs on the same place as a previous value's *initial* collision, so the new value will follow the exact same path as the previous collider
- Also, can't guarantee it will visit all slots
  - Step size always increasing: may miss empty slots

Secondary Clusters

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key
                EXPORT hashIndex
   hashIndex ← key % 11
```

- Keys:

15

46

27

18

37

26

Table

	0
	1
46	2
13	3
15	4
27	5
	6
	7
37	8
26	9
	10

Probe+16
(almost
wraps twice!)

Probe+1

Probe+4

Probe+9
(wraps around,
finally past the
trail left by 37!)

Double Hashing

- Quadratic probing's increasing-step-size is an issue
  - Cannot guarantee that all slots will be visited
    - So if only one free slot, the quadratic stepping might miss it
      - Although hash tables aren't usually *that* full
- The secondary clustering is also an issue: inefficient

- Double hashing seeks to solve these problems
  - Calculate a step size based on the *key*
    - Each key will have its own step-size increment
      - Greatly reduces secondary clustering
    - Step-size for a given key won't change, thus with a prime-sized table it is *guaranteed* to be able to visit all slots
      - Because no common divisor between step-size and table-size

Double Hashing

- Step-size calc is done by a *second* hash function
  - Simple hash functions are good enough: we aren't overly concerned with even distribution since it's just step-size
  - Define a maximum step-size and make that the modulo
  - Must not produce 0 step sizes though
  - A good secondary hash function is:

```
FUNCTION stepHash IMPORT key (integer)
             EXPORT hashStep (integer)
hashStep ← MAX_STEP – ( key % MAX_STEP ) // Step size will be between 1 and maxStep
```

  - MAX_STEP should be small-ish; certainly << table size!
  - Use a prime number for MAX_STEP

Double Hashing

Table

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key
              EXPORT hashIndex
  hashIndex ← key % 11
```

Secondary hash:

15

```
FUNCTION stepHash IMPORT key
              EXPORT hashStep
  hashStep ← 5 - (key % 5)
```

46

27

- Keys:

13

	0
	1
46	2
	3
15	4
27	5
13	6
	7
	8
	9
	10

Probe+2

Probe+4

Double Hashing

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key
              EXPORT hashIndex
  hashIndex ← key % 11
```

Secondary hash:

```
FUNCTION stepHash IMPORT key
              EXPORT hashStep
  hashStep ← 5 - (key % 5)
```
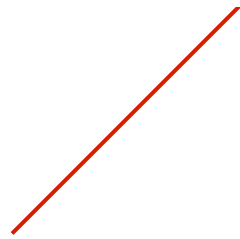
- Keys:

15

46

27

13

37

Probe+3

Table

	0
	1
46	2
	3
15	4
27	5
13	6
37	7
	8
	9
	10

Double Hashing

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key
               EXPORT hashIndex
  hashIndex ← key % 11
```

Secondary hash:

```
FUNCTION stepHash IMPORT key
              EXPORT hashStep
  hashStep ← 5 - (key % 5)
```

- Keys:

15

46

27

13

37

8

Table

	0
	1
46	2
	3
15	4
27	5
13	6
37	7
8	8
	9
	10

Double Hashing

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key
             EXPORT hashIndex
  hashIndex ← key % 11
```

Secondary hash:

```
FUNCTION stepHash IMPORT key
             EXPORT hashStep
  hashStep ← 5 - (key % 5)
```

- Keys:

15

46

27

13

37

8

26

Table

	0
26	1
46	2
	3
15	4
27	5
13	6
37	7
8	8
	9
	10

Probe+4

Probe+8
(wraps around)

Double Hashing – Analysis

- No primary OR secondary clustering
  - Since step sizes will often be > 1, gaps will be left when probing, thus primary clusters are unlikely
  - Each key has its own step-size, so no secondary clusters

- Having a prime table length is important here:
  - Guarantees that an empty slot will be found, since no common divisor between table length and *any* step size
    - Otherwise 'resonances' can occur when wrapping-around to the beginning of the table during probing – you'd keep visiting the same slots on each pass, and hence miss other slots

Separate Chaining

- The previous approaches used probing on collisions
- Separate chaining uses an entirely different method
  - Make each hashtable entry a pointer to a linked list
  - When a collision occurs, the new key-value pair is simply added to the linked list
    - That way, all items are stored at their key's hash index position
    - Collisions just mean that a chain (linked list) of items are stored at the colliding hash index position

Separate Chaining – Analysis

- This makes for quite different behaviour
  - Must 'probe' by traversing across the linked lists
  - No limits on the amount of items you can put in
    - Although performance still degrades as the table fills up
  - O(1) insertion … *no matter how full the table is*
    - Just insert the new item at the front of the linked list
    - But still can degrade to O(N) for access/removal
      - Just like open addressing, but now traversing the linked list
  - But makes things more complicated
    - Every entry is now a linked list : extra coding
    - Space overhead of 'next' pointers is also unavoidable: every entry is now data + next pointer

Separate Chaining

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key
                EXPORT hashIndex
  hashIndex ← key % 11
```
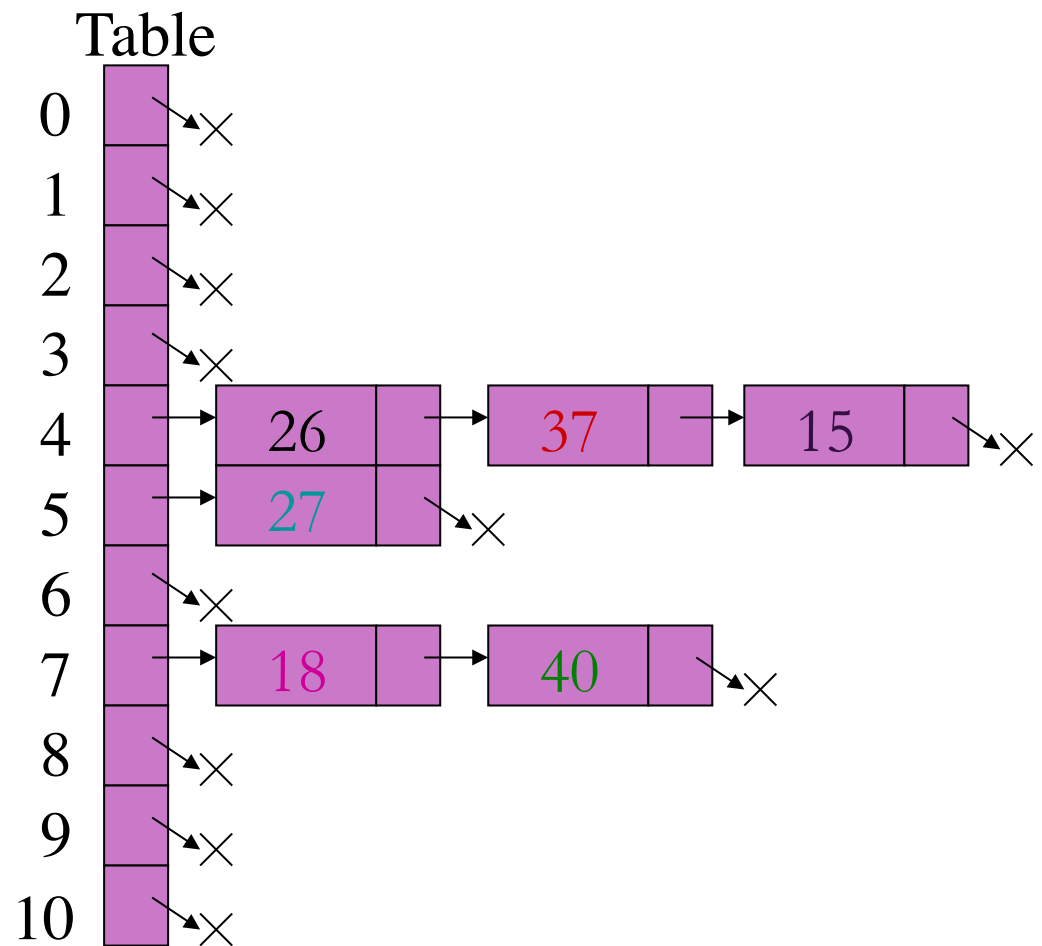
- Keys:

15

40

27

18

37

26

Table

0	→ ✗
1	→ ✗
2	→ ✗
3	→ ✗
4	→ [26] → [37] → [15] → ✗
5	→ [27] → ✗
6	→ ✗
7	→ [18] → [40] → ✗
8	→ ✗
9	→ ✗
10	→ ✗

ACCESSING AND REMOVING

Acessing Values in a Hashtable

- So far, we've focused on inserting key-value pairs
- But accessing a value already in the hashtable is almost the same:
    - The main difference is that instead of hashing and finding a free slot, we instead hash and find the key
    - Thus we *must* store the key as well as the value in the table
        - Otherwise we won't know whether we are at the right value

Accessing Values in a Hashtable

- **Separate chaining:** hash the key to retrieve the hash index, then search the linked list to find that key

- **Open addressing:** hash the key and probe (according to the probing algorithm used in insertion) until we find that key
  - We can abort early if we hit an empty slot, since this indicates that the key isn't even in the table
- Depending on how many collisions have occurred, access time can vary from O(1) to O(N)
  - But it will generally be at the lower end: O(1)
  - …unless the load factor of hashtable is high

Removal of Items From a Hash Table

- Removal in separate chaining is easy enough:
  - Just find the item to remove (same as accessing) and delete it from the linked list it resides in

- Open addressing *seems* easy:
  - Just probe to the item (same as accessing) and make the slot empty so that it is free for subsequent inserts
  - There's a small problem with this:
    - Accessing relies on finding empty slots to conclude that the key is not in the table and so abort probing
    - But now we just removed an item that *may* have been in the middle of a chain of historical probes

Open Addr Removing Issue

Table

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key
              EXPORT hashIndex
  hashIndex ← key % 11
```
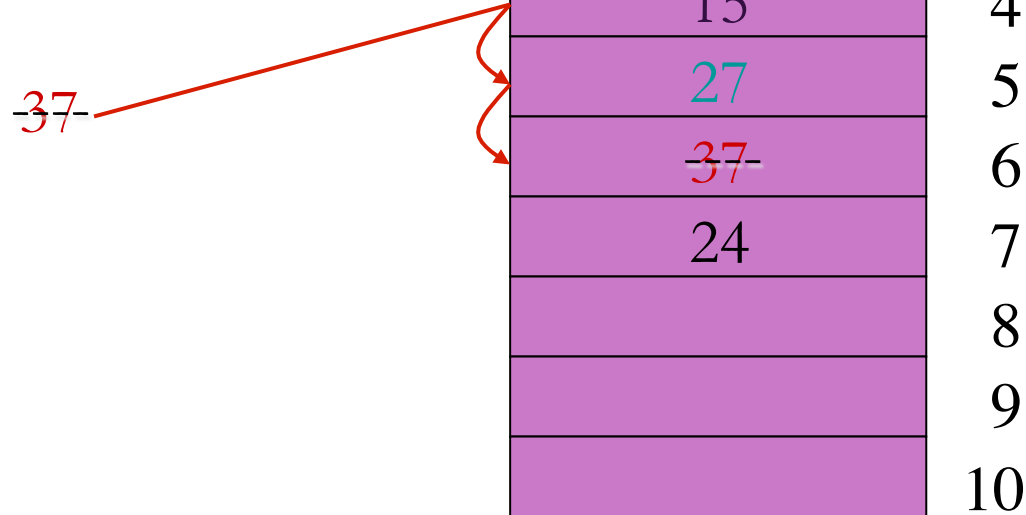
- Assume linear probing

	0
	1
46	2
13	3
15	4
27	5
37	6
24	7
	8
	9
	10

Open Addr Removing Issue

Table

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key
                EXPORT hashIndex
  hashIndex ← key % 11
```

- Delete 37

	0
	1
46	2
13	3
15	4
27	5
37	6
24	7
	8
	9
	10

-37-

Open Addr Removing Issue

Table

- Basic (poor!) hash function:

```
FUNCTION hashFunction IMPORT key
               EXPORT hashIndex
  hashIndex ← key % 11
```

- Delete 37

- Now access 24
     ?!?!?!

-37-

24

??What?
There's no 24??

	0
	1
46	2
13	3
15	4
27	5
	6
24	7
	8
	9
	10

Removal Solution

- The solution for removal is to mark a removed slot as "free-but-formerly-used"
  - Tells access probes that the key it is looking for might be found further down due to the removal
    - Thus access probes can only stop once it encounters a "free-and-never-been-used" slot
    - This does not affect insert: insert still adds to first free slot

  - This can be a problem for hashtables that experience a lot of adds and removes (*e.g.,* due to long lifetime)
    - Eventually, *all* slots will be marked as "free-but-formerly-used"
      - re-build the hashtable!
    - Which means access time for keys that aren't in the table degenerates to O(N):
      - only if the key exists can it stop early

Linear Probing – PseudoCode

```
CLASS DSAHashEntry
FIELD key (String)                                    ← Can be any appropriate data type
FIELD value (Object)                                  ← So that we can store any kind of data values!
FIELD state (int)                                     ← 0 = never used, 1 = used, -1 = formerly-used

CONSTRUCTOR IMPORT nothing
key ← ""
value ← null
state ← 0
                                                      There are other ways to do this!

CONSTRUCTOR IMPORT inKey, inValue
key ← inKey
value ← inValue
state ← 1


--------------------------------------------------------------------


CLASS DSAHashTable
FIELD hashArray (DSAHashEntry array)                  ← The actual hashtable
FIELD count (int)                                     ← Number of items inserted in the hashtable

CONSTRUCTOR IMPORT tableSize

actualSize ← NextPrime(tableSize)
allocate hashArray[actualSize]
FOR ii ← 0 TO actualSize-1 DO                         ← Initialise entries
   hashArray [ii] ← allocate HashEntry
ENDFOR
```

Linear Probing – PseudoCode

```
METHOD get IMPORT inKey EXPORT retValue

hashIdx ← hash(inKey)
origIdx ← hashIdx                                    ← Just in case the hashtable is 100% full!
found ← FALSE
giveUp ← FALSE

WHILE (NOT found) AND (NOT giveUp) DO
    IF (hashArray[hashIdx].state = 0) THEN           ← Stop if we hit a never-used entry
        giveUp = TRUE
    ELSEIF (hashArray[hashIdx].key = inKey)          ← Check if this is the key we want
        found = TRUE
    ELSE
        hashIdx ← (hashIdx + 1) % hashArray.length   ← Probe, handling wrap-around
        IF (hashIdx = origIdx) THEN
            giveUp = TRUE                            ← Stop if we have checked all nodes
        ENDIF
    ENDIF
ENDWHILE

IF (NOT found) THEN
    ABORT                                            ← ie: throw an exception
ENDIF
retValue = hashArray [hashIdx].value
```
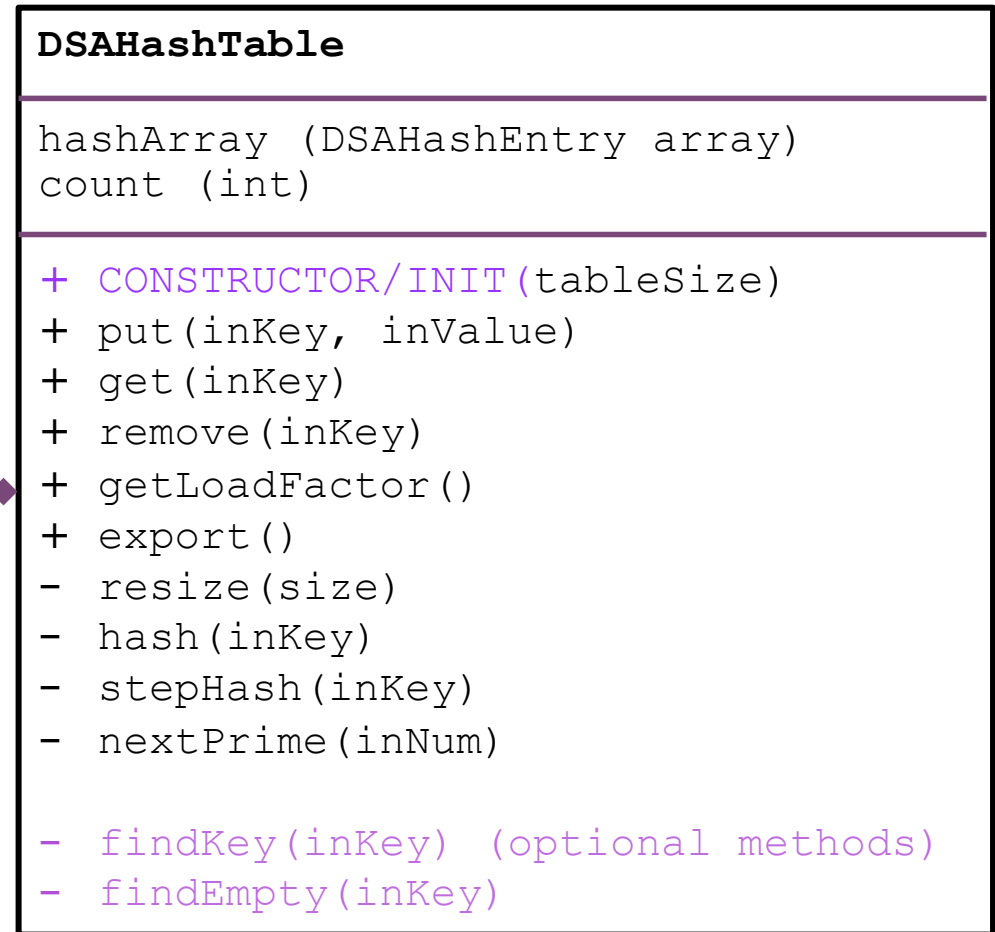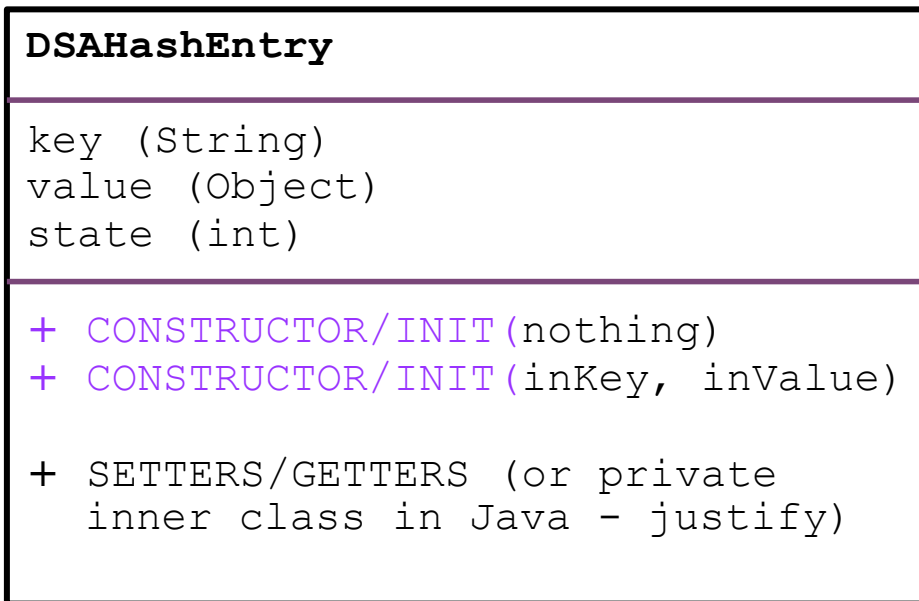
Linear Probing – PseudoCode

- You work out the other methods in the practical
- UML Class diagrams hint:

```
DSAHashTable
─────────────────────────────────────
hashArray (DSAHashEntry array)
count (int)
─────────────────────────────────────
+ CONSTRUCTOR/INIT(tableSize)
+ put(inKey, inValue)
+ get(inKey)
+ remove(inKey)
+ getLoadFactor()
+ export()
- resize(size)
- hash(inKey)
- stepHash(inKey)
- nextPrime(inNum)

- findKey(inKey) (optional methods)
- findEmpty(inKey)
```

```
DSAHashEntry
─────────────────────────────────────
key (String)
value (Object)
state (int)
─────────────────────────────────────
+ CONSTRUCTOR/INIT(nothing)
+ CONSTRUCTOR/INIT(inKey, inValue)

+ SETTERS/GETTERS (or private
  inner class in Java - justify)
```

- + indicates "public" methods and "–" is used for private methods.
- In Python, everything is public, but we can use an underscore to show something is "private" e.g. _hash(inKey)

Load Factor

- Notice collisions are more likely if the table has more items in it
  - More items means less unused slots

- Load factor is a measure of how 'full' a hash table is

$$LF = \frac{numItems}{tableCapacity}$$

  - Has important consequences for the frequency of collisions in open-addressing (probing) approaches
    - Similar for separate chaining but not as catastrophic, as we will see

Load Factor and Collisions – Open Addr

- In <u>open addressing</u>, load factor directly indicates the proportion of used slots
  - LF = 0.0: collision is impossible (nothing to collide with!)
  - LF = 1.0: collision is guaranteed (all slots are filled!)
    - And worse: no free slots means you can't insert the new item!
  - LF cannot exceed 1.0 <u>in open addressing approaches</u>
    - Can't put in more items than there are table slots!

  - Collisions will increase as we move from LF = 0.0 to 1.0
    - Exactly what the curve looks like depends on the hash function; better (uniform) hash functions will only get bad at higher LFs

Load Factor and Collisions – Sep. Chain

- Separate chaining is not so badly affected
    - Insertion in separate chaining is *always* O(1)
    - Only access/deletion time is affected
    - Consider: LF = 1.0 indicates that each slot has, on average, 1 entry in the linked list for that slot
        - Some entries will have more, some entries will have none
    - Thus access time will be, on average, O(2)
        - In comparison: with probing methods, access time depends on the number of probes, which is often >> 1 when LF → 1.0

    - Access time increases as LF increases: approx O(LF)
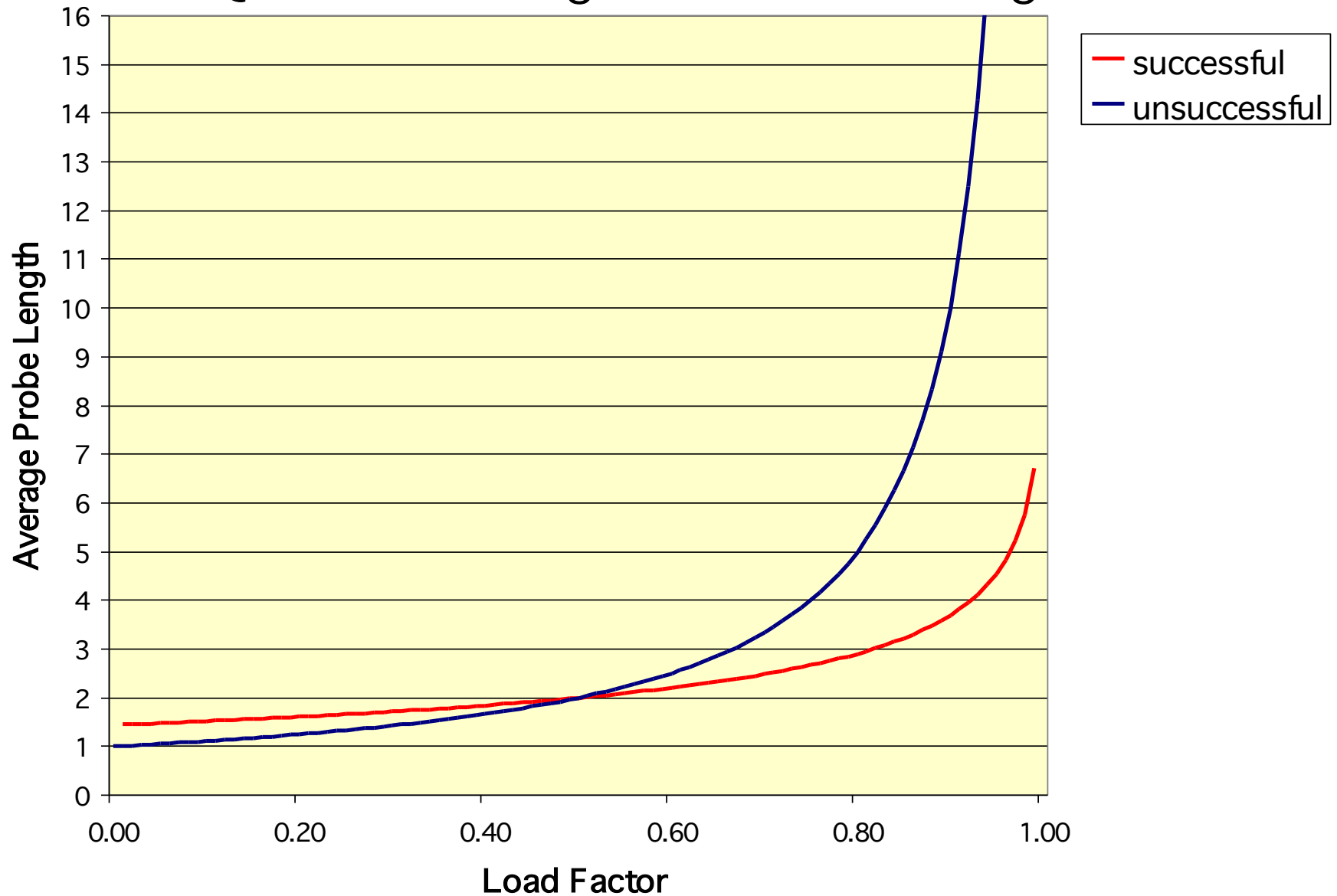   - But only on average: some individual slots may be very full

Load Factor Impact:
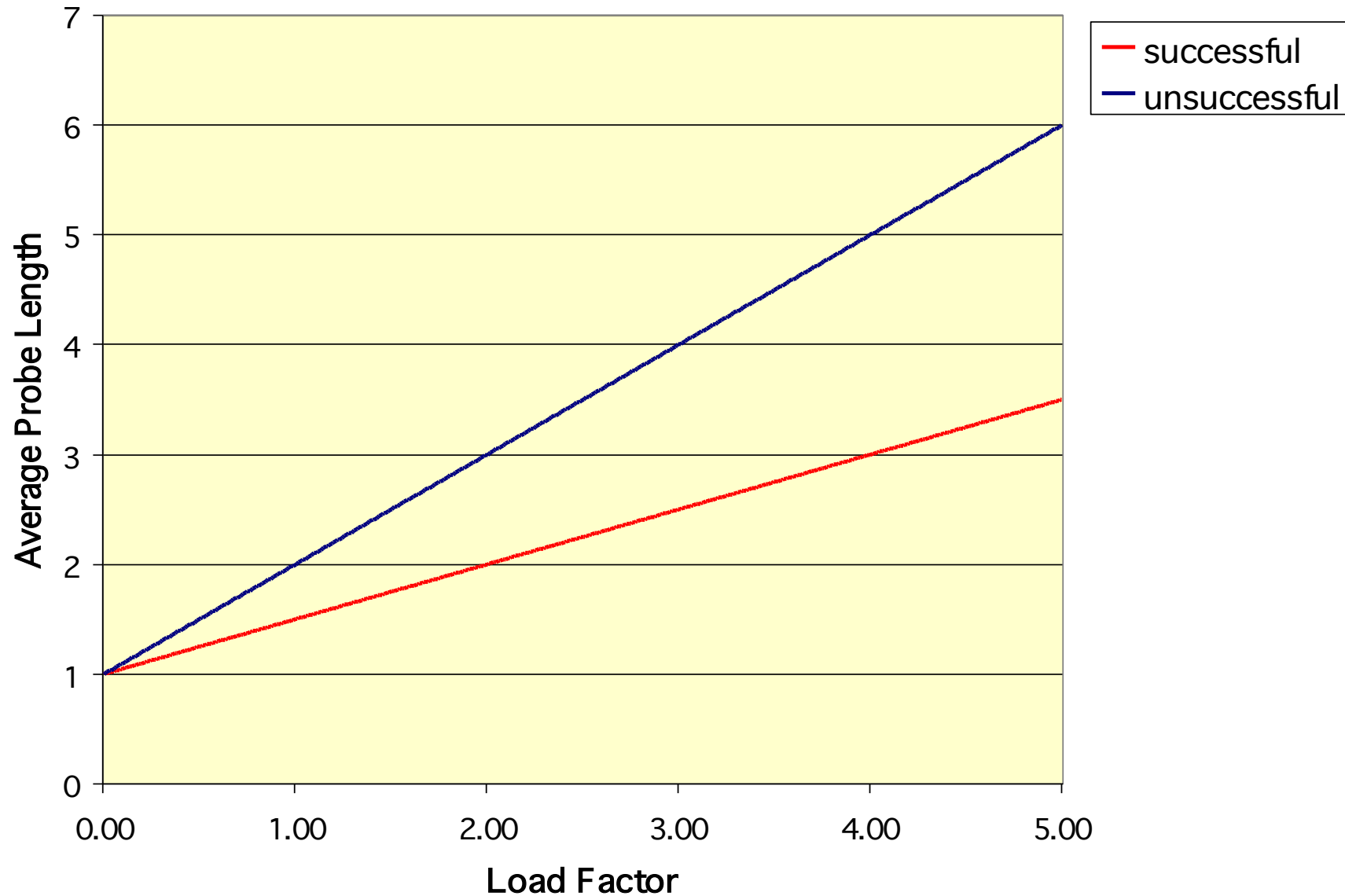

Linear Probe Performance
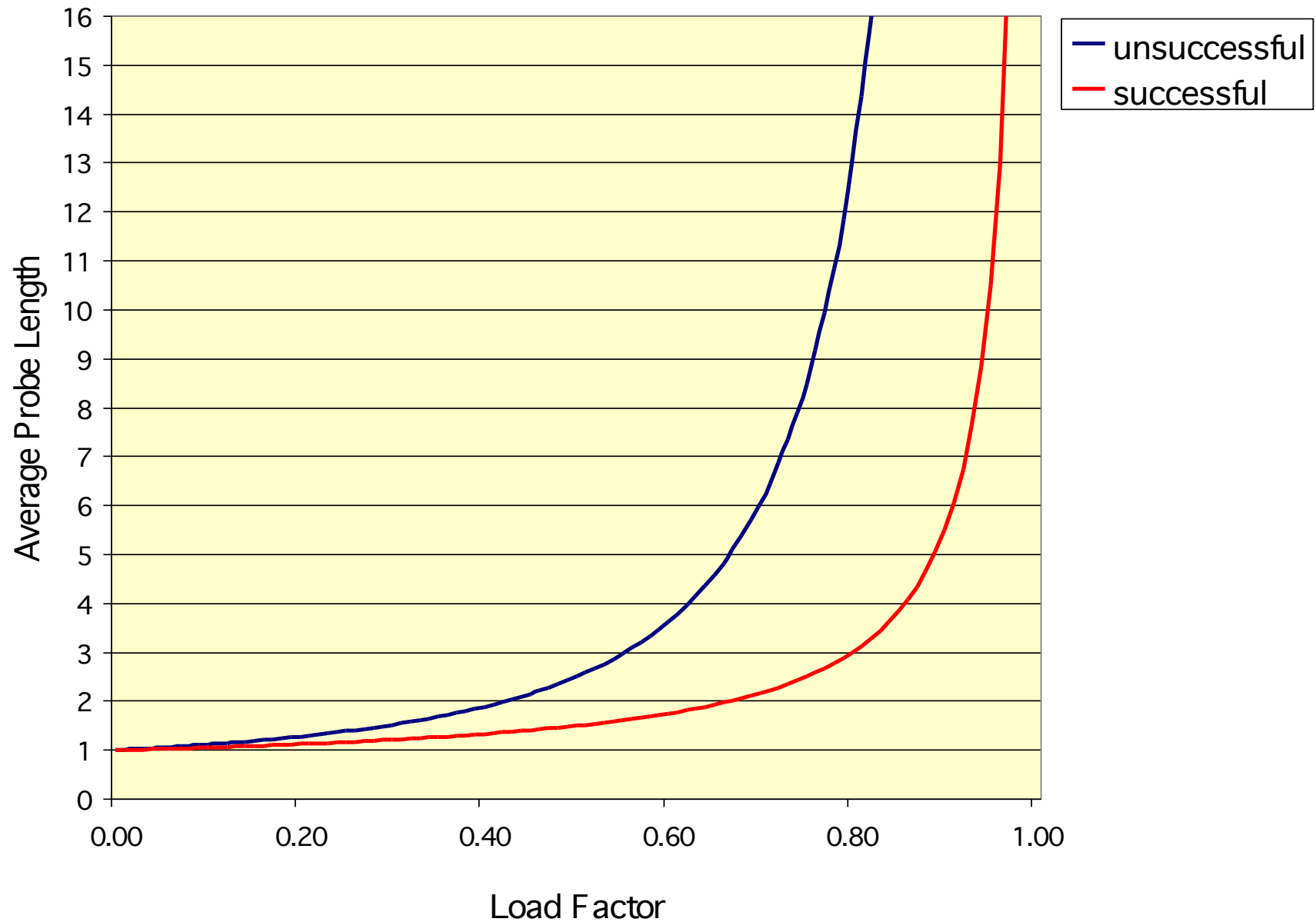
Load Factor Impact:

Quadratic Probing and Double Hashing

Load Factor Impact:

Separate-Chaining Performance

Performance Conclusions

- For open addressing, make sure the hash table isn't too full
  - *i.e.,* Keep the load factor below 0.6 or 0.7

- But don't make the table too big
  - LF < 0.4 means 60% of the table is unused (wasted) space

- Not so much of a problem for separate chaining
  - Overall, separate chaining tends to be less 'brittle', but also more complex to implement since you need the linked lists

Collision-Handling Comparison

- Separate chaining pros (vs open addressing methods)
  - ☑ O(1) insertion regardless of how full the table is
  - ☑ No limit on number of items that can be stored
    - *i.e.,* load factor can exceed 1.0
  - ☑ Performance is better with higher load factors
  - ☑ No need to specially mark slots with removed items
    - The linked list will automatically 'close up' the gap

- Separate chaining cons
  - ☒ More complicated data structure needed (array of linked lists)
  - ☒ Node pointer memory overhead for each element
  - ☒ Access time tends to be slower when load factor is low

Collision-Handling Comparison

- Linear probing (vs other open addressing approaches):
  - ☑ Simplest open addressing approach
  - ☑ Will always find a free slot
  - ☒ Suffers from primary clustering
    - This can make insertion/accessing quite slow if it gets bad
- Quadratic probing
  - ☑ Largely eliminates primary clustering
  - ☒ Suffers from secondary clustering
  - ☒ May fail to find free slots (eg: if there are only a few left)
    - … and so also can't tell when to give up looking for a key!

Collision-Handling Comparison

- Double Hashing:
  - ☑ No problems with primary or secondary clustering
  - ☑ Will always find a free slot
    - … as long as the table size is prime
  - ☒ Needs a secondary hash function (extra complexity)

Next Week

- Heaps