EMTH1019
Tutorial 5 Vector basics

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1. Given the vectors $\boldsymbol{a} = [2, -1, 3]$, $\boldsymbol{b} = [4, 0, -3]$ and $\boldsymbol{c} = [1, -2, 2]$, find:

   (i) $\boldsymbol{a} + \boldsymbol{b}$

   (ii) $3\boldsymbol{a} - 4\boldsymbol{c}$

   (iii) The magnitude of $\boldsymbol{b}$

   (iv) $\hat{\boldsymbol{b}}$

   (v) A vector in the same direction as $\boldsymbol{b}$ but has the same length of $\boldsymbol{c}$

2. Given the points $A(2, -3)$ and $B(4, 1)$, find the vectors $\boldsymbol{a} = \overrightarrow{OA}$ and $\boldsymbol{b} = \overrightarrow{AB}$.

3. In each of the following cases, find (i) $\boldsymbol{a}.\boldsymbol{b}$, (ii) the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$, (iii) the scalar projection of $\boldsymbol{a}$ on $\boldsymbol{b}$, and (iv) the vector projection of $\boldsymbol{a}$ on $\boldsymbol{b}$.

   (a) $\boldsymbol{a} = [2, -4, \sqrt{5}]$, $\boldsymbol{b} = [-2, 4, -\sqrt{5}]$

3. In each of the following cases, find (i) $\boldsymbol{a}.\boldsymbol{b}$, (ii) the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$, (iii) the scalar projection of $\boldsymbol{a}$ on $\boldsymbol{b}$, and (iv) the vector projection of $\boldsymbol{a}$ on $\boldsymbol{b}$.

(b) $\boldsymbol{a} = 2\boldsymbol{i} + 10\boldsymbol{j} - 11\boldsymbol{k}$, $\boldsymbol{b} = 2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$

5. Show that the vectors $\boldsymbol{u} = [2, -2, -1]$ and $\boldsymbol{v} = [3, 5, -4]$ are orthogonal to one another.

6. Find the work done by a force $\boldsymbol{F} = 5\boldsymbol{j}$ (magnitude 5 Newtons) in moving an object along the line from the origin to the point $(1, 1, 1)$ (distance in meters).

7. How much work does it take to slide a crate 15m along a loading dock by pulling on it with a 150 Newton force at an angle of 45° from the horizontal?

4. A truss takes the shape of a cube with a member on the main diagonal. Find the angle between this diagonal member and one of the edge members. (Hint: Consider the coordinates of the corners of a simple cube of side length one placed in three space.)