# Tutorial 11 Euclidean Vector Spaces

## Euclidean vector spaces

1. Given the vectors $\boldsymbol{a} = [1, 2, 0, 2]$ and $\boldsymbol{b} = [-2, 0, 1, 1]$, find:

   (i) $\boldsymbol{a} + 2\boldsymbol{b}$

   (ii) The unit vector $\hat{\boldsymbol{b}}$

   (iii) A vector in the same direction as $\boldsymbol{b}$ but has the same length of $\boldsymbol{a}$

2. Given the points $A(2, 4, 3, -1, 1)$ and $B(3, 1, 1, 0, -2)$ in $\mathbb{R}^5$, find the distance between the points $A$ and $B$.

3. For the vectors $\boldsymbol{a} = [4, 1, -2, 2]$ and $\boldsymbol{b} = [1, 0, 3, 2]$ determine the vector projection of $\boldsymbol{a}$ on $\boldsymbol{b}$.

4. Find the angle between the hyperplanes $2x_1 - x_2 - 2x_3 + x_4 = -1$ and $x_1 + 3x_2 - x_4 = 2$.

## Vector subspaces

5. For each of the following sets of vectors, determine whether or not it is a subspace of $\mathbb{R}^3$, giving reasons for your answer.

(i) $V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid x - 2y = 0 \right\}$ (ii) $U = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid x^2 = 2y \right\}$

(iii) $W = \left\{ \begin{bmatrix} 2 \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid y, z \in \mathbb{R} \right\}$

Do on Thursday in lecture

**Linear combinations**

6. Let $\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\boldsymbol{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\boldsymbol{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \in \mathbb{R}^3$. Show that $\boldsymbol{w} = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$ is a linear combination of $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$.

7. Let $\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\boldsymbol{v}_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$ and $\boldsymbol{v}_3 = \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix} \in I\!R^3$. Show that $\boldsymbol{w} = \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix}$ can not be written as a linear combination of $\boldsymbol{v}_1$, $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$.

# Linearly Independent (LI) & Linearly Dependent (LD)

**Scalar Multiples**

If you only have 2 vectors and they are scalar multiples, then the set is **LD**.

**Linear Combinations**

For a set of vectors $\{v_1, v_2, v_3, \ldots\}$ if the only solution to

$C_1 v_1 + C_2 v_2 + C_3 v_3 \ldots = 0$ is $C_1 = C_2 = C_3 = 0$ (trivial solution) then the set of vectors is **LI**

- Any other solution for $C_1, C_2, C_3, \ldots$ then **LD.** (e.g. infinite solutions or non-trivial solutions)

**Determinant Method**

If the vectors make a square matrix and the determinant$= 0$, then **LD**.

**More vectors than the space then LD**

If you are in 4 space but have 5 vectors then the set is **LD**

**BUT**…if you have 4 vectors in 5 space then you need to check using the Linear Combination method.

## Linear dependence / independence

8. For each of the following sets of vectors, decide whether they are l.i. or l.d.

(i) $\left\{\begin{bmatrix} -10 \\ 15 \end{bmatrix}, \begin{bmatrix} 4 \\ -6 \end{bmatrix}\right\}$ (ii) $\left\{\begin{bmatrix} 7 \\ 3 \end{bmatrix}, \begin{bmatrix} 21 \\ 12 \end{bmatrix}\right\}$

(iii) $\left\{\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}\right\}$ (iv) $\left\{\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right\}$

(v) $\left\{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}\right\}$ (vi) $\left\{\begin{bmatrix} 1 \\ -1 \\ 2 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \\ -4 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 6 \\ -2 \end{bmatrix}\right\}$