Section 1 (Multiple-choice Questions)

Question 1

Let A be the set of x for which $x^2 + 3x - 4 = 0$; Let B be the set of x where $B = \{2, 3\}$. Find $P(A \cup B)$.

(A) $\{1, 2, 3, -4\}$

(B) $\{1, 2, 3, 4\}$

(C) $\{1, -2, 3, 4\}$

(D) None of above.

[1.5 Marks]

Question 2

In how many ways can 7 people be lined up to get on a bus?

(A) 720

(B) 823543

(C) 5040

(D) None of above.

[1.5 Marks]

Question 3

There are 7 professors and 6 associate professors in a school. We select 2 people from them randomly. What is the probability that one person is a professor and another person is an associate professor?

(A) 0.245

(B) 0.167

(C) 0.538

(D) None of above.

[1.5 Marks]

Question 4

There are 2 ambulances in a town operating independently. The probability that a specific ambulance is available when needed is 0.97. What is the probability that neither is available when needed?

(A) 0.0009

(B) 0.06

(C) 0.125

(D) None of above.

[1.5 Marks]

Question 5

Discrete random variable X has uniform distribution with sample space $S = \{2, 4, 6, 8, 10\}$. Find $E(2X + 1)$.

(A) 11

(B) 13

(C) 12

(D) None of above.

[1.5 Marks]

Question 6

A professor records the values of several variables for each student in her class. These include the variables listed below. The number of lectures the student missed (Lectures). Attending or not attending an in-class test (Attendance). Final Grade for the topic (Fail, Pass, Credit, Distinction, or High Distinction) (Grade). The total number of marks earned in the class (i.e, the total of the marks on all assessments in the course. The maximum number of marks possible is 500) (Marks). The types of variables Lectures, Attendance, Grade and Marks are respectively:

(A) Discrete, nominal, ordinal and continuous.

(B) Nominal, ordinal, continuous and discrete.

(C) Ordinal, nominal, ordinal and discrete.

(D) Nominal, continuous, ordinal and continuous.

[1.5 Marks]

Question 7

One type of digital signal will be transmitted to a receiver 2 times per minute on average. What is the probability the receiver will receive 3 times of the signal from 11:35 to 11:36?

(A) 0.15

(B) 0.20

(C) 0.18

(D) None of above.

[1.5 Marks]

Question 8

Suppose that X and Y are random variables with Var(X) = 2, Var(Y) = 4 and Cov(X,Y)=2. Find Var($X+Y$).

(A) 5

(B) 10

(C) 27

(D) None of above.

[1.5 Marks]

Question 9

A fruit grower claims that $\frac{1}{3}$ of his peach crop has been contaminated by medfly infestation. Find the probability that among 4 peaches inspected by this grower, all 4 have been contaminated by the medfly?

(A) 0.4

(B) 0.02

(C) 0.012

(D) None of above.

[1.5 Marks]

Question 10

A television station is interested in predicting whether or not voters in its listening area are in favour of federal funding for abortions. It asks its viewers to phone in and indicate whether they are in favour of or opposed to this. Of the 2241 viewers who phoned in, 1574 (70.24%) were opposed to federal funding for abortions. The number 70.24% is a ___.

(A) parameter

(B) population

(C) sample

(D) statistic

[1.5 Marks]

Question 11

Assume the songs on an iPod have a mean play length of 100 seconds with a standard deviation of 10 seconds. The distribution of play times is highly skewed. If I take samples of size 50, the distribution of sample mean play lengths is likely to be:

(A) highly skewed with a mean of 100.

(B) normally distributed with a mean of 100.

(C) highly skewed with a mean of 10.

(D) normally distributed with a mean of 10.

[1.5 Marks]

Question 12

A random sample of 15 eleven-year-old boys was obtained with a mean height of 142 centimetres and a standard deviation of 5 centimetres. Which of the following could be the 99% confidence interval based on that data? The following information may be useful:
t(df=14, confidence level=0.99) = 2.977

(A) 142 ± 2.27

(B) 142 ± 2.77

(C) 142 ± 2.98

(D) 142 ± 3.84

[1.5 Marks]

Mark	
Cumulative	

(A total of 18 marks for section one.)

~~Section 2 is on the next page.~~

Section Two (Show your working)

Question 1

An urn contains 7 red balls and 5 blue balls. We sample 4 balls without replacement.

(a) What is the probability that 3 red balls in the sample?

[4 Marks]

(b) What is the probability that the number of blue balls in the sample is less than 2?

[4 Marks]

Mark	
Cumulative	

(A total of 8 marks for this question.)
Question 2 is on the next page.

Question 2

In a certain plant, there are three machines, $A1$, $A2$ and $A3$, make 20%, 30% and 50%, respectively, of the products. From the experience, it is known that the defective rates are 0.02, 0.03, and 0.04, respectively, for the products made by each machine. Suppose that a finished product is selected randomly.

(a) Given that the product is defective, what is the probability that the product is made by machine $A1$?

[5 Marks]

(b) What is the probability that it is defective?

[3 Marks]

Mark	
Cumulative	

(A total of 8 marks for this question.)
Question 3 is on the next page.

Question 3

The finished inside diameter of piston rings is normally distributed with a mean of 5 centimeters and a standard deviation of 0.02 centimeter.

(a) What is the probability that the inside diameter of a random selected piston ring exceeding 5.005 centimeter?

[4 Marks]

(b)Below which value of the inside diameter will 15% of the piston rings fall?

[4 Marks]

Mark	
Cumulative	

(A total of 8 marks for this question.)
Question 4 is on the next page.

Question 4

Suppose X and Y are discrete random variables. If the joint probability distribution function is given by

$$f(x, y) = \frac{(x + y)}{15}, \quad for \quad x = 1, 2, 3; \quad y = 0, 1$$

(a) Calculate $P(X > Y)$.

[4 Marks]

(b) Calculate $P(Y | X = 2)$.

[4 Marks]

Mark	
Cumulative	

(A total of 8 marks for this question.)
Question 5 is on the next page.

Question 5

In a random sample of $n = 1000$ families owning television sets in the city of Hamilton, Canada, it was found that $x = 560$ subscribed to HBO.

(a) Find a 95% confidence interval for the actual proportion of families in this city who subscribe to HBO.

[3 Marks]

(b) How large a sample is required if we want to be 95% confident that our estimate of p is within 0.02?

[3 Marks]

Mark	
Cumulative	

(A total of 6 marks for this question.)
Question 6 is on the next page.

Question 6

A health spa claims that a new exercise program will reduce a person's waist size over a 5-day period. The waist sizes of 6 men who participated in this exercise program are recorded before and after the 5-day period in the following table. Test the relevant hypothesis to see if the spa is effective. Use a significance level of 5%. Make sure you do all of the following:

- State the null and alternative hypotheses.

- State the p-value of the test.

- Draw your conclusions.

Hint. Copy the following into R console or a chunk in Rmd, to read and analyse the data.
before <-c(90.4, 95.5, 98.7, 112.5, 103.5, 85.6)
after <-c(91.5, 93.6, 97.7, 110.4, 101.2, 84.1)

	Men					
	1	2	3	4	5	6
Waist size before	90.4	95.5	98.7	112.5	103.5	85.6
Waist size after	91.5	93.6	97.7	110.4	101.2	84.1

[8 Marks]

Mark	
Cumulative	

(A total of 8 marks for this question.)
Question 7 is on the next page.

Question 7

In a study conducted by the Department of Mechanical Engineering at Virginia Polytechnic Institute, the steel rods supplied by two different companies were compared. Ten sample springs were made out of the steel rods supplied by each company and the bounciness were studied. The data are as follows. Can we conclude that there is no difference between the steel rods supplied by the two companies? Use a significance level of 5%.
Company A:

$$9.3 \quad 8.9 \quad 6.7 \quad 8.5 \quad 8.6 \quad 6.8 \quad 8.0 \quad 6.9 \quad 9.1 \quad 7.0$$

Company B:

$$11.0 \quad 9.7 \quad 9.6 \quad 10.2 \quad 10.1 \quad 10.9 \quad 9.5 \quad 10.1 \quad 9.2 \quad 9.6$$

Hint. Copy the following into R console or a chunk in Rmd, to read and analyse the data.
A <-c(9.3, 8.9, 6.7, 8.5, 8.6, 6.8, 8.0, 6.9, 9.1, 7.0)
B <-c(11.0, 9.7, 9.6, 10.2, 10.1, 10.9, 9.5, 10.1, 9.2, 9.6)

(a) Write down the null and alternative hypotheses.

[2 Marks]

(c) Calculate the p-value.

[4 Marks]

(d) Based on this p-value, express your conclusion to your analysis.

[2 Marks]

(e) Construct a 99% confidence interval for the difference between the population means.

[2 Marks]

Mark	
Cumulative	

(A total of 10 marks for this question.)
Question 8 is on the next page.

Question 8

The grades of 100 students in a statistic course for a particular semester were recorded as follows. Test the hypothesis, at 0.05 level of significance, that the distribution of grades is uniform.

Grade	A	B	C	D
Frequency	22	27	30	21

[8 Marks]

Mark	
Cumulative	

(A total of 8 marks for this question.)
Question 9 is on the next page.

Question 9

In an experiment to study the dependence of hypertension on smoking habits, the following data were taken on 183 individuals. Test the hypothesis that the presence or absence of hypertension is independent of smoking habits. Use a 0.05 level of significance. You must

- state the hypotheses;

- compute the p-value;

- draw conclusion.

	Smoking habits			
	Nonsmokers	Moderate Smokers	Heavy Smokers	Total
Hypertension	20	36	32	88
No hypertension	54	26	15	95
Total	74	62	47	183

[8 Marks]

Mark	
Cumulative	

(A total of 8 marks for this question.)

Question 10 is on the next page.

Question 10

A study was made on the amount of converted sugar(mg) in a certain process at various temperatures (degree Celsius). The data were recorded as follows:

Temperature	Converted Sugar
1.0	8.1
1.1	7.8
1.2	8.5
1.3	9.8
1.4	9.5
1.5	9.6
1.6	9.9
1.7	10.2
1.8	10.3
1.9	10.5
2.0	10.8

Hint. Copy the following into R console or a chunk in Rmd, to read and analyse the data.
Temperature <-c(1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0)
Sugar<-c(8.1, 7.8, 8.5, 9.8, 9.5, 9.6, 9.9, 10.2, 10.3, 10.5, 10.8)
TSlm=lm(Temperature ∼ Sugar)
summary(TSlm)

(a) Identify the predictor and response variables in this scenario?

[2 Marks]

(b) Estimate the linear regression model.

[3 Marks]

(c) From (b), is there evidence that a higher temperature will lead to more converted sugar?

[3 Marks]

(d) From (b), what is the predicted converted sugar for a temperature of 1.45 Celsius?

[2 Marks]

Mark	
Cumulative	

(A total of 10 marks for this question.)