

WORKSHOP 7

This workshop will build on material from Lecture 7: Systems of Linear Equations & Gaussian Elimination.

During this workshop, students will work towards the following learning outcomes:

- solve a system of equations by performing Gaussian elimination on an augmented matrix.
- determine the number of solutions to a system of equations, and describe the case of infinitely many solutions in terms of parameters.

Systems of equations using matrices

1. Find the augmented matrix for each of the following system:

$$\begin{array}{ll}
 \begin{array}{rcl}
 2x_1 - x_2 & = & 3 \\
 \text{(i)} \quad 4x_1 + 3x_2 - x_3 & = & 1 \\
 3x_2 + x_3 & = & 0
 \end{array}
 &
 \begin{array}{rcl}
 x_1 + 2x_2 + x_4 + 2 & = & 0 \\
 \text{(ii)} \quad x_2 - 3x_3 + x_5 & = & -1 \\
 4x_3 - 2x_2 + x_1 + 3x_5 & = & 0
 \end{array}
 \end{array}$$

2. Which of the following matrices are in row echelon form? If it is not in row-echelon form give the reason why.

$$\begin{array}{ll}
 \text{(i)} \quad \begin{bmatrix} 0 & 1 & 7 & 0 & -2 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
 &
 \text{(ii)} \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 2 & 4 & 0 & 1 \\ 0 & -1 & 2 & 5 \end{bmatrix}
 \end{array}$$

3. Determine the rank of the following matrices:

$$\begin{array}{lll}
 \text{(i)} \quad A = \begin{bmatrix} 2 & 1 \\ -4 & 0 \end{bmatrix}
 &
 \text{(ii)} \quad B = \begin{bmatrix} 2 & 3 & -1 \\ -8 & -7 & 6 \\ 6 & -1 & -7 \end{bmatrix}
 &
 \text{(iii)} \quad C = \begin{bmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & 2 & 4 \\ 0 & -9 & 6 & 5 & -6 \end{bmatrix}
 \end{array}$$

4. For the following systems of linear equations write down the augmented matrix of the system then use the Gaussian Elimination method and the concept of rank of a matrix to solve the system of equations.

$$\begin{array}{ll}
 \begin{array}{rcl}
 x_1 + 2x_2 + 3x_3 & = & 6 \\
 \text{(i)} \quad 2x_1 - 3x_2 + 2x_3 & = & 14 \\
 3x_1 + x_2 - x_3 & = & -2
 \end{array}
 &
 \begin{array}{rcl}
 2x_1 - 3x_2 + 2x_3 & = & 1 \\
 \text{(ii)} \quad x_2 - 4x_3 & = & 8 \\
 5x_1 - 8x_2 + 7x_3 & = & 1
 \end{array}
 \end{array}$$

$$\begin{array}{rcl}
 3x_1 + 5x_2 - 4x_3 & = & 7 \\
 \text{(iii)} \quad -3x_1 - 2x_2 + 4x_3 & = & -1 \\
 6x_1 + x_2 - 8x_3 & = & -4
 \end{array}$$

5. Kitchens "R" Us produces refrigerators, dishwashers and stoves at three different factories. Factory A can produce 10 refrigerators, 50 dishwashers and 30 stoves per day. Factory B can produce 20 refrigerators, 40 dishwashers and 10 stoves per day. Factory C can produce 20 refrigerators, 10 dishwashers and 40 stoves per day. Kitchens "R" Us receives an order for 100 refrigerators, 290 dishwashers and 180 stoves. Use matrices to find how many days should each factory be scheduled to fill this order.
6. Determine if the following set of vectors is linearly dependent or independent.

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix} \right\}$$

