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(3a)

$x=0$ és $y=0 \Rightarrow x^2+y^2=0$ igaz

$0^2+0^2=0$

$x^2+y^2=0 \Rightarrow x=y=0$ igaz

(3b)

$xy=xz \Rightarrow y=z$ Hamis

$0 \cdot 2 = 0 \cdot 3$

$y=z \Rightarrow xy=xz$ igaz

(3c)

$x>y^2 \Rightarrow x>0$ igaz

$x>0 \Rightarrow x>y^2$ Hamis

$1 \not> 4$

(3d)

$x^2+y^2=12x+16y-75 \Rightarrow 25 \leq x^2+y^2 \leq 225$

$x^2+y^2=12x+16y-75$

$x^2-12x+y^2-16y+75=0$

$(x-6)^2+(y-8)^2-25=0$

$(x-6)^2+(y-8)^2=25$ igaz

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$25 \le x^2+y^2 \le 225 \Rightarrow x^2+y^2 = 12+16y-75$ Hamis

(5a) $\frac{1}{n} < 0{,}01 \quad (n \in \mathbb{N}^+)$

$\forall n \in \mathbb{N}^+ : \frac{1}{n} < 0{,}01$ Hamis

ha $n=2$ $\frac{1}{2} \not< 0{,}01$

↳ $\exists n \in \mathbb{N}^+ : \frac{1}{n} \ge 0{,}01$ Igaz

(5b) $\exists n \in \mathbb{N}^+ : \frac{1}{n} \ge 0{,}01$ Igaz

↳ $\forall n \in \mathbb{N}^+ : \frac{1}{n} < 0{,}01$ Hamis

(5c) $\forall n \ge N : \frac{1}{n} < 0{,}01 \quad (N \in \mathbb{N}^+)$

↳ nyitott kijelentés N nincs megadva $\Rightarrow$ Igaz és Hamis

(5d) $\exists N \in \mathbb{N}^+ : \forall n \ge N : \frac{1}{n} < 0{,}01$ Igaz

↳ $\forall N \in \mathbb{N} : \exists n \ge N : \frac{1}{n} < 0{,}01$ Hamis

(6a) $n \in \mathbb{N} \quad \frac{n^2}{10n-7} > 100$

$\exists n \in \mathbb{N} : \frac{n^2}{10n-7} > 100$ Igaz

↳ $\forall n \in \mathbb{N} : \frac{n^2}{10n-7} > 100$ Hamis

(6b) $\forall n \in \mathbb{N} : \frac{n^2}{10n-7} > 100$ Hamis

↳ $\exists n \in \mathbb{N} : \frac{n^2}{10n-7} > 100$ Igaz

(6c) $\exists N \in \mathbb{N} : \forall n > N : \frac{n^2}{10n-7} > 100$ Igaz

↳ $\forall N \in \mathbb{N} : \exists n > N : \frac{n^2}{10n-7} > 100$ Hamis

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26) $\frac{2n^3+3}{n^5-3n^4-7n^3+2n^2-10n+1} \leq 0{,}05$ NEM

$n^5-3n^4-7n^3-10n = n^5-(3n^4+7n^3+10n) > n^5-(3n^4+7n^4+10n^4) = n^5-20n^4 = \frac{1}{2}n^5+\frac{1}{2}n^5-20n^4 = \frac{1}{2}n^5+n^4(\frac{1}{2}n-20) \geq \frac{1}{2}n^5$
↳ $n \geq 40$

↳ $\frac{5n^3}{\frac{1}{2}n^5} = \frac{10}{n^2}$

ha $n \geq 40$

$\frac{10}{n^2} < 0{,}05$

$n \geq 15$ $\Rightarrow N = 40$

↳ $\forall N \in \mathbb{N}: \exists n \in \mathbb{N}: \frac{2n^3+3}{n^5-3n^4-7n^3+2n^2-10n+1} \geq 0{,}05$

27) $\frac{2n^6-20n^5+3n^4-6n^3+13}{13n^3+100n^2+200} > 230$ NEM

$2n^6-20n^5-6n^3 = 2n^6-(20n^5+6n^3) > 2n^6-(20n^5+6n^5) = 2n^6-26n^5 = n^6+n^6-26n^5 = n^6+n^5(n-26) \geq n^6$
↳ $n \geq 26$

$13n^3+100n^2+200 < 13n^3+100n^3+200n^3 = 313n^3$

↳ $\frac{n^6}{313n^3} = \frac{n^3}{313}$

ha $n \geq 26$

$\frac{n^3}{313} > 230$

$n \leq 42$ $\Rightarrow N = 42$

↳ $\forall N \in \mathbb{N}: \exists n \in \mathbb{N}: \frac{2n^6-20n^5+3n^4-6n^3+13}{13n^3+100n^2+200} \leq 230$