Fábián T. Nikolett RDDZXA

(2) $2^{x+3}+4^{1-\frac{x}{2}}=33$

$2^x\cdot 2^3+\frac{2^2}{2^x}=33$

$2^{2x}\cdot 8+4-33\cdot 2^x=0$

$\Delta=33^2-4\cdot 8\cdot 4=1089-128=31^2$

$2^x=\frac{33\pm 31}{16}$ $\left\langle \begin{array}{l} 4 \Rightarrow \underline{x=2} \\ \frac{1}{8} \Rightarrow \underline{x=-3} \end{array}\right.$ $M=\{-3,2\}$

(3c) $3^{x+2}\cdot 2^x-2\cdot 36^x+18=0$

$3^x\cdot 2^x\cdot 3^2-2\cdot 6^{2x}+18$

$9\cdot 6^x-2\cdot 6^{2x}+18=0$

$-2\cdot 6^{2x}+9\cdot 6^x+18=0$

$\Delta=81+4\cdot 2\cdot 18=15^2$

$6^x=\frac{-9\pm 15}{-4}$ $\left\langle \begin{array}{l} 6 \Rightarrow \underline{x=1} \\ \frac{6}{-4}=-\frac{3}{2}<0 \end{array}\right.$ $M=\{1\}$

(3f) $4^{x+1}-9\cdot 2^x+2>0$

$2^2\cdot 2^{2x}-9\cdot 2^x+2>0$

$\Delta=81-4\cdot 4\cdot 2=7^2$

$2^x=\frac{9\pm 7}{4}$ $\left\langle \begin{array}{l} 4 \Rightarrow x=2 \\ \frac{1}{2} \Rightarrow x=-1 \end{array}\right.$ $\underline{M=(-\infty,-1)\cup(2,+\infty)}$

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(8) $3^{2+\log_9 25} + 25^{1-\log_5 2} + 10^{-\lg 4}$

$\left(\begin{array}{l} a^x = b \\ \log_a b = x \end{array} \right.$

$\Rightarrow 9 \cdot 3^{\log_9 25} + 25 \cdot 25^{-\log_5 2} + 10^{-\lg 4} =$

$= 9 \cdot \left(\cancel{9}^{\cancel{\log_9} 25}\right)^{\frac{1}{2}} + 25\left(\cancel{5}^{\cancel{\log_5} 2}\right)^{-2} + \left(\cancel{10}^{\cancel{\lg} 4}\right)^{-1}$

$= 9 \cdot \sqrt{25} + 25 \cdot \frac{1}{4} + \frac{1}{4} = \frac{90+13}{2} = \underline{\underline{\frac{103}{2}}}$

(15b) $\log_{25}\left[\frac{1}{5} \cdot \log_3\left(2 - \log_{\frac{1}{2}} x\right)\right] = -\frac{1}{2}$

$\frac{1}{5} \log_3\left(2 - \log_{\frac{1}{2}} x\right) = \frac{1}{5}$

$\log_3\left(2 - \log_{\frac{1}{2}} x\right) = 1$

$2 - \log_{\frac{1}{2}} x = 3$

$\log_{\frac{1}{2}} x = -1$

$\underline{\underline{x = 2}}$

(15c) $\log_3(x+1) - \log_3(x+10) = 2\log_3 4{,}5 - 4$

$\hookrightarrow \log_3 \frac{x+1}{x+10} = 2\left(\log_3 9 - \log_3 2\right) - 4$

$\log_3 \frac{x+1}{x+10} = \cancel{4} + \log_3 \frac{1}{4} - \cancel{4}$

$\hookrightarrow \frac{x+1}{x+10} = \frac{1}{4}$

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$$4x+4=10+x$$
$$3x=6$$
$$\underline{x=2}$$

15d) $\log_2(x-2)+\log_2(x+3)=1+2\log_4 3$

$$\log_2(x-2)(x+3)=1+2\frac{\log_2 3}{\log_2 4}$$

$$\log_2(x-2)(x+3)=\log_2 2\cdot 3$$

$\hookrightarrow$ $x^2+x-6=6$

$x^2+x-12=0$ $\quad \Delta=1+4\cdot 12=7^2$

$$x_{1,2}=\frac{-1\pm 7}{2} \begin{cases} 3 \\ -4<0 \end{cases}$$

$\underline{M=\{x\}}$

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15e $\log_{32}(2x) - \log_8(4x) + \log_2(x) = 3$

$$\frac{\log_2 2x}{\log_2 32} - \frac{\log_2 4x}{\log_2 8} + \log_2 x = 3$$

$$\frac{\log_2 2x}{5} - \frac{\log_2 4x}{3} + \log_2 x = 3$$

$$\frac{\log_2 2 + \log_2 x}{5} - \frac{\log_2 4 + \log_2 x}{3} + \log_2 x = 3$$ legyen $\log_2 x = a$

$$\frac{1+a}{5} - \frac{2+a}{3} + a = 3$$

$3+3a-10-5a+15a=45$

$13a=52$

$a=4 \Rightarrow \log_2 x = 4$

$\underline{x=16}$

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15g) $x^{(2\lg^2 x - 1{,}5\lg x)} = \sqrt{10}$

$\lg x^{(2\lg^2 x - 1{,}5\lg x)} = \lg\sqrt{10}$

legyen $a = \lg x$

$(2\lg^2 x - 1{,}5\lg x)\cdot\lg x = \lg 10^{\frac{1}{2}}$

$(2a^2 - 1{,}5a)\cdot a = \frac{1}{2}$

$4a^3 - 3a^2 - 1 = 0$

$(a-1)(4a^2 + a + 1) = 0$

$\hookrightarrow 4a^3 - 4a^2 + a^2 - a + -1$

I $a = 1$

$\lg x = 1$

$x = 10$

II $4a^2 + a + 1 = 0$

$\Delta = 1 - 4\cdot 4\cdot 1 = -15 < 0$

$M = \{10\}$

15j) $\log_{\frac{1}{2}} \frac{3-x}{3x-1} \geq 0$

$\log_{\frac{1}{2}} \frac{3-x}{3x-1} \geq \log_{\frac{1}{2}} 1$

$\hookrightarrow \frac{3-x}{3x-1} \geq 1$

$3 - x \geq 3x - 1$

$-4x \geq -4$

$x \leq 1$

$\Rightarrow M = [1, 3)$

1.f $\frac{3-x}{3x-1} > 0$

$3 - x > 0 > 3x - 1$

$3 > 0 > \frac{1}{3}$

$x \in (\frac{1}{3}, 3)$