1a

$$f(x) := \sqrt{\frac{2x^3-1}{x}}$$

$x \Rightarrow x \in \mathbb{R}$

$2x^3-1 \Rightarrow x \in \mathbb{R}$

$\frac{2x^3-1}{x} \Rightarrow x \in \mathbb{R}\setminus\{0\}$

$\sqrt{\frac{2x^3-1}{x}} \Rightarrow x \in \mathbb{R}\setminus\{0\} \wedge \frac{2x^3-1}{x} \geq 0$

$$D = \left\{x \in \mathbb{R} \mid x \neq 0 \wedge \frac{2x^3-1}{x} \geq 0\right\} = \left\{x \in \mathbb{R} \mid \frac{2x^3-1}{x} \geq 0\right\}$$

$\underline{\text{I}}$ eset: $2x^3-1 \geq 0 \wedge x > 0$

$2x^3-1 \geq 0 \Leftrightarrow 2x^3 = 1 \Leftrightarrow x \geq \frac{1}{\sqrt[3]{2}}$

$\hookrightarrow x \in \left[\frac{1}{\sqrt[3]{2}}, +\infty\right)$

$\underline{\text{II}}$ eset: $2x^3-1 \leq 0 \wedge x < 0$

$2x^3-1 \leq 0 \Leftrightarrow x \leq \frac{1}{\sqrt[3]{2}}$

$\hookrightarrow x \in (-\infty, 0)$

$$D = (-\infty, 0) \cup \left(\frac{1}{\sqrt[3]{2}}, +\infty\right)$$

1b)

$f(x) := \sqrt{\lg(x^2-5x+7)}$

$x^2-5x+7 \Rightarrow x \in \mathbb{R}$

$\lg(x^2-5x+7) \Rightarrow x^2-5x+7 > 0$

$\sqrt{\lg(x^2-5x+7)} \Rightarrow x^2-5x+7 > 0 \wedge \lg(x^2-5x+7) \geq 0$

$D = \{x \in \mathbb{R} \mid x^2-5x+7 > 0 \wedge \lg(x^2-5x+7) \geq 0\}$

$x^2-5x+7 = 0$

$\hookrightarrow x_{1,2} = \frac{5 \pm \sqrt{-3}}{2} \notin \mathbb{R}$

$\forall x \in \mathbb{R}: x^2-5x+7 > 0$

$\lg(x^2-5x+7) \geq 0$

$\hookrightarrow x^2-5x+7 \geq 10^0$

$x^2-5x+6 \geq 0$

$x^2-5x+6 \Leftrightarrow (x-2)(x-3) = 0 \Leftrightarrow x \in \{2,3\}$

$\hookrightarrow x \in (-\infty, 2] \cup [3, +\infty)$

$D = (-\infty, 2] \cup [3, +\infty)$

4a

$f(x) := 2(x+3)^2 - 1$

1) x^2 függvényt ábrázoljuk <-> x^2
2) eltoljuk balra 3-al <-> $(x+3)^2$
3) $2\times$ nyújtjuk függőlegesen <-> $2(x+3)^2$
4) letoljuk 1-el <-> $2(x+3)^2-1$

4b

$f(x) := -x^2+5x+3$

$-x^2+5x+3 = -\left(x-\frac{5}{2}\right)^2 + \frac{37}{4}$

1) ábrázoljuk x^2-et <-> x^2
2) eltoljuk jobbra $\frac{5}{2}$-vel <-> $\left(x-\frac{5}{2}\right)^2$
3) tükrözzük x tengelyre <-> $-\left(x-\frac{5}{2}\right)^2$
4) toljuk fel $\frac{37}{4}$-el <-> $-\left(x-\frac{5}{2}\right)^2+\frac{37}{4}$

4g

$f(x) := \frac{4x-1}{2x-1}$

$\frac{4x-1}{2x-1} = 2\cdot\frac{4x-1}{4x-2} = 2\,\frac{4x-2+1}{4x-2} = 2\left(1+\frac{1}{4x-2}\right) = 2+\frac{1}{2x-1} = \frac{1}{2}\cdot\frac{1}{x-\frac{1}{2}}+2$

1) ábrázoljuk $\frac{1}{x}$-et <-> $\frac{1}{x}$
2) jobbra toljuk $\frac{1}{2}$-vel <-> $\frac{1}{x-\frac{1}{2}}$
3) összenyomjuk 1/2 szeresére függőlegesen <-> $\frac{1}{2}\cdot\frac{1}{x-\frac{1}{2}}$
4) eltoljuk felfele 2-vel <-> $\frac{1}{2}\cdot\frac{1}{x-\frac{1}{2}}+2$

5a

$f(x) := x$

$g(x) := \sin x$

$D_{f+g} = D_f \cap D_g = \mathbb{R} \cap \mathbb{R} = \mathbb{R} \quad (f+g)(x) = x + \sin x$

$D_{f-g} = D_f \cap D_g = \text{-//-} = \mathbb{R} \quad (f-g)(x) = x - \sin x$

$D_{f \cdot g} = \text{—— // ——} = \mathbb{R} \quad (f \cdot g)(x) = x \cdot \sin x$

$D_{\frac{f}{g}} = \{ x \in D_f \cap D_g \mid g(x) \neq 0 \}$

$= \{ x \in \mathbb{R} \mid x \neq k\pi, k \in \mathbb{Z} \} \quad \frac{f}{g}(x) = \frac{x}{\sin x}$

$D_{\frac{g}{f}} = \{ x \in D_g \cap D_f \mid f(x) \neq 0 \}$

$= \{ x \in \mathbb{R} \mid x \neq 0 \} = \mathbb{R} \setminus \{0\} \quad \frac{g}{f}(x) = \frac{\sin x}{x}$