①

a) $v_1 = (1,2,2,-1)$

$v_2 = (4,3,9,-4)$

$v_3 = (5,8,9,-5)$

$$\lambda_1 \begin{pmatrix} 1 \\ 2 \\ 2 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 4 \\ 3 \\ 9 \\ -4 \end{pmatrix} + \lambda_3 \begin{pmatrix} 5 \\ 8 \\ 9 \\ -5 \end{pmatrix} = \begin{pmatrix} \lambda_1 + 4\lambda_2 + 5\lambda_3 \\ 2\lambda_1 + 3\lambda_2 + 8\lambda_3 \\ 2\lambda_1 + 9\lambda_2 + 9\lambda_3 \\ -\lambda_1 - 4\lambda_2 - 5\lambda_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\lambda_1 + 4\lambda_2 + 5\lambda_3 = 0$ (1)

$2\lambda_1 + 3\lambda_2 + 8\lambda_3 = 0$ (2)

$2\lambda_1 + 9\lambda_2 + 9\lambda_3 = 0$ (3)

$-\lambda_1 - 4\lambda_2 - 5\lambda_3 = 0$ (4)

$2 \cdot (1) \Rightarrow 2\lambda_1 + 8\lambda_2 + 10\lambda_3 = 0$ $2\lambda + 0 + 0 = 0 \Rightarrow \lambda_1 = 0$

$(2) - 2(1) \Rightarrow -5\lambda_2 - 2\lambda_3 = 0$ $-5\lambda_2 - 2\lambda_2 = 0 \Rightarrow \lambda_2 = \lambda_3 = 0$

$(3) - 2(1) \Rightarrow \lambda_2 - \lambda_3 = 0 \Rightarrow \lambda_2 = \lambda_3$

$(4) + (1) \Rightarrow 0 = 0$

$\Rightarrow \lambda_1 = \lambda_2 = \lambda_3 = 0 \Rightarrow$ független

b) $v_1 = (1,2,3,1)$

$v_2 = (2,2,1,3)$

$v_3 = (-1,2,7,-3)$

$$\lambda_1 \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 2 \\ 1 \\ 3 \end{pmatrix} + \lambda_3 \begin{pmatrix} -1 \\ 2 \\ 7 \\ -3 \end{pmatrix} = \begin{pmatrix} \lambda_1 + 2\lambda_2 - \lambda_3 \\ 2\lambda_1 + 2\lambda_2 + 2\lambda_3 \\ 3\lambda_1 + \lambda_2 + 7\lambda_3 \\ \lambda_1 + 3\lambda_2 - 3\lambda_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$\lambda_1 + 2\lambda_2 - \lambda_3 = 0$ (1)

$2\lambda_1 + 2\lambda_2 + 2\lambda_3 = 0$ (2)

$3\lambda_1 + \lambda_2 + 7\lambda_3 = 0$ (3)

$\lambda_1 + 3\lambda_2 - 3\lambda_3 = 0$ (4) $\lambda_1 + 4\lambda_3 - \lambda_3 = 0$

$2 \cdot (1) = 2\lambda_1 + 4\lambda_2 - 2\lambda_3 = 0$ $\lambda_1 + 3\lambda_3 = 0$

$3 \cdot (1) = 3\lambda_1 + 6\lambda_2 - 3\lambda_3 = 0$ $\lambda_1 = -3\lambda_3 \longrightarrow$ (1)

$(2) - 2(1) = -2\lambda_2 + 4\lambda_3 = 0$ $\Big\}$

$(3) - 3(1) = -5\lambda_2 + 10\lambda_3 = 0$ $\Big\} \Rightarrow \lambda_2 = 2\lambda_3$ $\longrightarrow$

$(4) - (1) = \lambda_2 - 2\lambda_3 = 0$

$-3\lambda_3 + 4\lambda_3 - \lambda_3 = 0$

$-3v_1 + 2v_2 + v_3 = 0$ $\Rightarrow$ összefüggő

SZŰKÍTÉS

②

$v_1 (1, 2, 3, 1)$

$v_2 (2, 2, 1, 3)$

$v_3 (-1, 2, 7, -3)$

L) összefüggő (① b)

$-3v_1 + 2v_2 + v_3 = 0 \implies v_3 = 3v_1 - 2v_2$

$x = \lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3$

$x = \lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 (3v_1 - 2v_2) = (\lambda_1 + 3\lambda_3) v_1 + (\lambda_2 - 2\lambda_3) v_2$

$W = Spam(v_1, v_2, v_3) = Spam(v_1, v_2)$

$\implies v_3$ elhagyható

③ BŐVÍTÉS

$v_1 = (1, -2, 1)$

$v_2 = (2, 1, 0)$

$$\lambda_1 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda_1 + 2\lambda_2 \\ -\lambda_2 + \lambda_2 \\ \lambda_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\lambda_1 + 2\lambda_2 = 0$

$-\lambda_1 + \lambda_2 = 0 \qquad 0 + \lambda_2 = 0$

$\lambda_1 = 0 \qquad\qquad \hookrightarrow \lambda_2 = 0$

$\qquad\qquad \Rightarrow \lambda_1 = \lambda_2 = 0 \quad$ független

a) összefüggő legyen

$$x = v_1 + v_2 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = (3, -1, 1)$$

ha $x \in Span(v_1, v_2)$

$\Rightarrow v_1, v_2, x$ összefüggő

!! nullvektorral is meg lehetett volna csinálni

$\Rightarrow v_1, v_2, 0$ összefüggő

b) független legyen

$x \notin Span(v_1, v_2)$

$$\lambda_1 v_1 + \lambda_2 v_2 = \lambda_1 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \lambda_1 + 2\lambda_2 \\ -2\lambda_1 + \lambda_2 \\ \lambda_1 \end{pmatrix} \in \mathbb{R}^3$$

$\lambda_1, \lambda_2 \in \mathbb{R}$

$x = (a, b, c)$

$\left. \begin{array}{l} \lambda_1 + 2\lambda_2 = a \\ -2\lambda_1 + \lambda_2 = b \\ \lambda_1 = c \end{array} \right\}$ (= minis megoldása

HA $c = \lambda_1 = 0$

$2\lambda_2 = a$

$\lambda_2 = b$ =) ha b=1 minden a ≠ 2 jó megoldás

=) $x = (1, 1, 0)$ ra $x \notin Span(v_1, v_2)$

=) v_1, v_2, x független