Wednesday, December 06, 2023 10:17 AM

$\sum_{k=0}^{n} A_k f(x_k) = \underline{A_0} f(\underset{\sim}{x_0}) + \underline{A_1} \cdot f(\underset{\sim}{x_1}) + \dots + \underline{A_n} f(x_n)$

11.1). $\frac{1}{2}\left[f\left(\frac{1}{3}\right)+f\left(\frac{2}{3}\right)\right] =$

$$= \underbrace{\frac{1}{2}}_{A_0} f(\underbrace{\tfrac{1}{3}}_{x_0}) + \underbrace{\frac{1}{2}}_{A_1} \cdot f(\underbrace{\tfrac{2}{3}}_{x_1})$$

i) pont. t. jobb oldala

$$A_k = \int_a^b \ell_k(x) w(x)\, dx \quad (k = 0, \dots, n)$$

? $A_0 = \int_0^1 \ell_0(x)\, dx$

? $A_1 = \int_0^1 \ell_1(x)\, dx$

$$\ell_i(x) = \frac{(x - x_k) \dots}{(x_i - x_k) \dots}$$

$$\ell_0(x) = \frac{x - x_1}{x_0 - x_1} = \frac{x - \frac{2}{3}}{\frac{1}{3} - \frac{2}{3}} =$$

$$= (-3) \cdot \left(x - \frac{2}{3}\right)$$

$$\ell_1(x) = \frac{x - x_0}{x_1 - x_0} = \frac{x - \frac{1}{3}}{\frac{2}{3} - \frac{1}{3}}$$

$$= 3\left(x - \frac{1}{3}\right)$$

$\int_0^1 \ell_0(x)\, dx = \int_0^1 (-3)\left(x - \frac{2}{3}\right) dx$

$$\int_0 \ell_0(x)\,dx = \int_0 (\ldots)$$

$$= (-3)\int_0^1 x - \frac{2}{3}\,dx$$

$$= (-3)\left[\frac{x^2}{2}\Big|_0^1 - \frac{2}{3}x\Big|_0^1\right]$$

$$= (-3)\left(\frac{1}{2} - \frac{2}{3}\right) = \frac{1}{2} = A_0 \quad \checkmark$$

$$\int_0^1 \ell_1(x)\,dx = \frac{1}{2} = A_1 \quad \checkmark$$

$\Rightarrow$

$$\xrightarrow[\text{p.t. alapján}]{\text{jobb oldala}} \int_0^1 f(x)\,dx \approx \frac{1}{2}\left[f\left(\frac{1}{3}\right) + f\left(\frac{2}{3}\right)\right]$$

interpolációs

ii) $ax+b$

$\{1, x\}$

ax^2+bx+c

$\{1, x, x^2\}$

$$\forall f \quad \int_0^1 f(x)\,dx = \sum A_k f(x_k)$$

2 alappont (x_0, x_1) $\Rightarrow$ elsőfokú

pol kész

$(ax+b)$

$\Updownarrow$

$\ell(x)=1$ és $\ell(x)=x$

$f(x) = 1$ és g

fgvekre ellenőrizzük a pt. bal oldalát

I $f = 1$

$$\int_0^1 1 \, dx = \frac{1}{2}\left[f\left(\frac{1}{3}\right) + f\left(\frac{2}{3}\right)\right]$$

$$x \Big|_0^1 = \frac{1}{2}(1+1)$$

$$1 = \frac{1}{2} \cdot 2 \quad \checkmark$$

II $f = x$

$$\int_0^1 x \, dx = \frac{1}{2}\left[f\left(\frac{1}{3}\right) + f\left(\frac{2}{3}\right)\right]$$

$$\frac{x^2}{2} \Big|_0^1 = \frac{1}{2}\left(\frac{1}{3} + \frac{2}{3}\right)$$

$$\frac{1}{2} = \frac{1}{2} \cdot 1 \quad \checkmark$$

I, II $\Longrightarrow$ pt. bal oldala kv. f. interp.

11.3. (a)

$$\int_1^2 \frac{1}{x} \, dx \approx g. \qquad \frac{1}{x} \hookrightarrow f$$

$$\int_a^b f \approx (b-a) \cdot f\left(\frac{a+b}{2}\right) =: E(f)$$

$$g \approx (2-1) \cdot f\left(\frac{1+2}{2}\right) =$$

$$= 1 \cdot f\left(\frac{3}{2}\right)$$

$$\frac{1}{} = \frac{2}{}$$

$$= 1 \cdot \frac{\quad}{\frac{3}{2}} = \frac{2}{3}$$

(b)

$$\int_a^b f \approx \frac{b-a}{2} \cdot (f(a) + f(b)) =: T(f)$$

$$g \approx \frac{2-1}{2}\left(f(1) + f(2)\right) =$$

$$= \frac{1}{2}\left(\frac{1}{1} + \frac{1}{2}\right) = \frac{3}{4}$$

(c)

$$\int_a^b f \approx \frac{b-a}{6} \cdot \left(f(a) + 4 \cdot f\left(\frac{a+b}{2}\right) + f(b)\right) =: S(f)$$

$$g \approx \ldots$$

hibabecslés:

(a)

$$\int_a^b f - E(f) = \frac{(b-a)^3}{24} \cdot f''(\eta).$$

$$M_2 := \max_{x \in [a,b]} |f''(x)|$$

$$\left| \int_a^b f - E(f) \right| \leq \frac{(b-a)^3}{24} M_2$$

$$f(x) = \frac{1}{x} = x^{-1}$$

$$f'(x) = -x^{-2}$$

$$f''(x) = 2x^{-3} = 2\frac{1}{x^3}$$

mon. csökkő

$$\Downarrow$$

$$M_2 = f''(a)$$

$$= 2 \cdot \frac{1}{1^3} = 2$$

$$\left|\int_a^b f - E(f)\right| \leq \frac{(2-1)^3}{24} \cdot 2 =$$

$$= \frac{1}{12}$$

(b) (c) ....