② $x = (1,-2,-3,5)$
$y = (-1,2,-1,0)$
$z = (2,-1,1,3)$

a) $\langle x,y\rangle = \langle(1,-2,-3,5),(-1,2,-1,0)\rangle = 1(-1)+(-2)2+(-3)(-1)+5\cdot 0 = -1-4+3 = -2$

b) $\|x\| = \sqrt{\langle x,x\rangle} = \sqrt{\langle(1,-2,-3,5),(1,-2,-3,5)\rangle} = \sqrt{1^2+(-2)^2+(-3)^2+5^2} = \sqrt{1+4+9+25} = \sqrt{39}$

c) $\|x-z\| = \sqrt{\langle x-z,x-z\rangle} = \sqrt{(-1)^2+(-1)^2+(-4)^2+2^2} = \sqrt{1+1+16+4} = \sqrt{22}$

d) $\dfrac{\langle x,z\rangle\cdot y - \langle y,z\rangle\cdot x}{\|y\|^2}$

$\langle x,z\rangle = \langle(1,-2,-3,5),(2,-1,1,3)\rangle = 2+2-3+15 = 16$
$\langle y,z\rangle = \langle(-1,2,-1,0),(2,-1,1,3)\rangle = -2-2-1 = -5$
$\|y\|^2 = \langle y,y\rangle = 1+4+1 = 6$

$\hookrightarrow \dfrac{16(-1,2,-1,0)-(-5)(1,2,-3,5)}{6} = \dfrac{(-16,32,-16,0)-(-5,10,15,-25)}{6} = \dfrac{1}{6}\cdot(-11,22,-31,25)$

e) z irányú egységvektor, z-vel ellentétes irányú egységvektor

$(-1)\dfrac{z}{\|z\|} = -\dfrac{(2,-1,1,3)}{\sqrt{15}} = \dfrac{(-2,1,-1,-3)}{\sqrt{15}}$

$\|z\| = 4+1+1+9 = \sqrt{15}$

③

$u_1 = (1,1,1,1)$
$u_2 = (1,-1,-1,1)$ $\in \mathbb{R}^4$ euklideszi térben
$u_3 = (-1,0,0,1)$

a) $\langle u_1,u_2\rangle = 1-1-1+1 = 0$
$\langle u_2,u_3\rangle = -1+0+0+1 = 0$
$\langle u_3,u_1\rangle = -1+0+0+1 = 0$

↳ ortogonális rendszer

b) $\left\|\sum_{i=1}^{n} x_i\right\|^2 = \|(1,1,1,1)+(1,-1,-1,1)+(1,0,0,1)\|^2 = \|(1,0,0,3)\|^2 = 1+9 = 10$

$\sum_{i=1}^{n}\|x_i\|^2 = \|u_1\|^2+\|u_2\|^2+\|u_3\|^2 = 4+4+2 = 10$