② $A \in \mathbb{R}^{n \times n}$ mátrix inverze

$$A = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix}$$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 \\ 1 & 1 & 1 & \cdots & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 & 0 & 0 & 0 & \cdots & 1 \end{array}\right] \begin{array}{l} \\ (2) - 1\cdot(1) \\ (3) - 1\cdot(1) \\ \vdots \\ (n) - 1\cdot(1) \end{array}$$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 & -1 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 1 & \cdots & 0 & -1 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & 1 & \cdots & 1 & -1 & 0 & 0 & \cdots & 1 \end{array}\right] \begin{array}{l} \\ \\ (3) - 1(2) \\ \vdots \\ (n) - 1\cdot(2) \end{array}$$

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 & -1 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 & -1 & -1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 1 & \cdots & 1 & -1 & -1 & 0 & \cdots & 1 \end{array}\right]$$

$\Downarrow$ ismételjük a lépéseket

$$\left[\begin{array}{ccccc|ccccc} 1 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 & -1 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 & -1 & -1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 & -1 & -1 & \cdots & 0 \end{array}\right]$$

↳ egységmátrix van a baloldalon

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & \cdots & 0 \\ -1 & -1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & -1 & \cdots & 1 \end{bmatrix}$$

3

$$A=\begin{bmatrix} 2 & -1 & 0 & 0 & \cdots & 0 \\ -1 & 2 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2 & -1 & \cdots & 0 \\ \vdots & & \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & -1 & 2 & -1 \\ 0 & \cdots & 0 & 0 & -1 & 2 \end{bmatrix} \qquad \begin{bmatrix} 2 & -1 & 0 & 0 & \cdots & 0 \\ -1 & 2 & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2 & -1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & -1 & 2 & -1 \\ 0 & \cdots & 0 & 0 & -1 & 2 \end{bmatrix} (2)+\frac{1}{2}(1)$$

$$\begin{bmatrix} 2 & -1 & 0 & 0 & \cdots & 0 \\ 0 & \frac{3}{2} & -1 & 0 & \cdots & 0 \\ 0 & -1 & 2 & -1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & -1 & 2 & -1 \\ 0 & \cdots & 0 & 0 & -1 & 2 \end{bmatrix} 3+\frac{2}{3}(2) \qquad \begin{bmatrix} 2 & -1 & 0 & 0 & \cdots & 0 \\ 0 & \frac{3}{2} & -1 & 0 & \cdots & 0 \\ 0 & 0 & \frac{4}{3} & -1 & \cdots & 0 \\ \vdots & & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & -1 & 2 & -1 \\ 0 & \cdots & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$L=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 & 0 \\ 0 & -\frac{2}{3} & 1 & 0 & 0 \\ 0 & 0 & -\frac{3}{4} & 1 & 0 \\ \vdots & & & \vdots & \vdots \\ 0 & 0 & 0 & -\frac{n-1}{n} & 1 \end{bmatrix} \Rightarrow U=\begin{bmatrix} 2 & -1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \frac{3}{2} & -1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \frac{4}{3} & -1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & \frac{5}{4} & \cdots & 0 \\ \vdots & \vdots & & \ddots & & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & & \frac{n+1}{n} \end{bmatrix}$$

7

a) $\varphi : [0,1] \to \mathbb{R}$

$\varphi(x) = rx(1-x)$ $r \in [0,4]$ rögzített konstans

a) φ fg $x=0$ esetén veszi fel a legkisebb és $x=\frac{1}{2}$ esete veszi fel a legnagyobb értéket, $\forall r \in (0,4]$ esetén

Ha $r=0$ akkor a függvény értéke szintén 0.

$r \in (0,4]$:

$\varphi(0) = 0$

$\varphi\left(\frac{1}{2}\right) = r\frac{1}{2}\left(1-\frac{1}{2}\right) = \frac{r}{4} \leq 1$

$\Rightarrow$ igaz, hogy φ önmagába képez, mert $\varphi[[0,1]] = [0,1] \subset [0,$

b) $r \in [0,1)$

$\varphi'(x) = r(1-x) - rx$

$|\varphi'(\xi)| = |r(1-\xi) - r\xi| = |r - 2r\xi| = |r(1-2\xi)| \leq |r| = \underbrace{r}_{q}$

$|x_n - x^*| \leq q^k \cdot |x_0 - x^*| = r^k \cdot (1-0) = r^k$

$\Rightarrow x^* = 0$ fixpont stabil