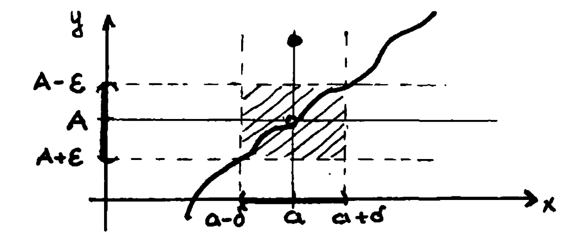
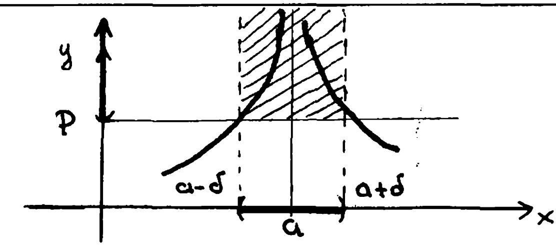
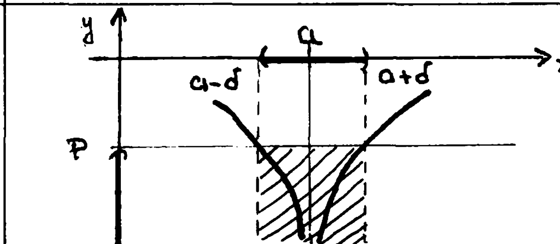
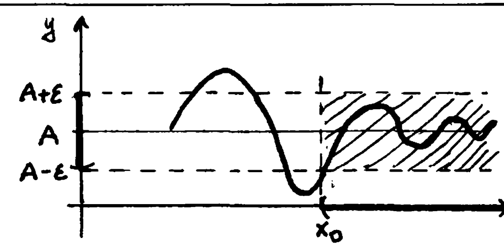
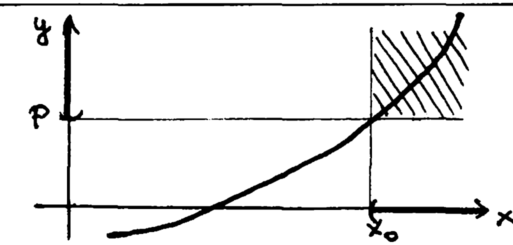
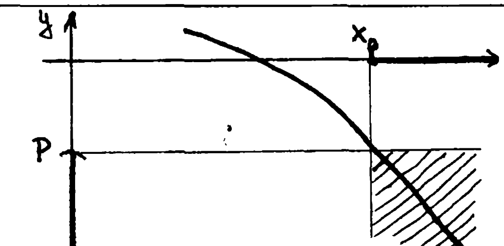
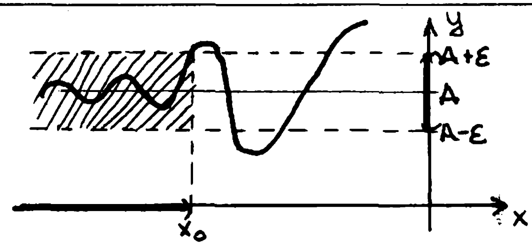
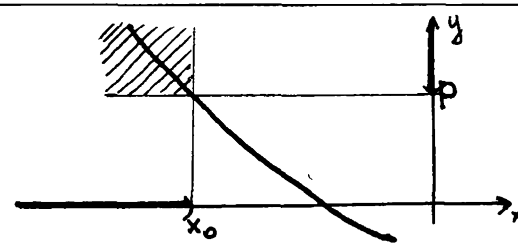
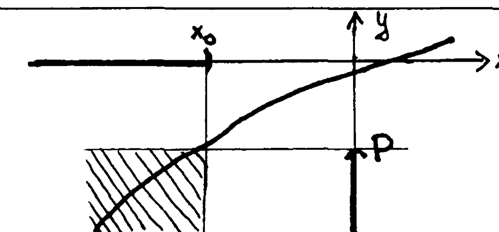
# A $\lim\limits_{x\to a} f(x) = A$ definíciója

Legyen $f$ valós-valós függvény és tegyük fel, hogy $a \in \mathcal{D}_f'$ (azaz $a$ torlódási pontja $\mathcal{D}_f$-nek). Azt mondjuk, hogy az $f$ függvénynek az $a$ **pontban** $A \in \overline{\mathbb{R}}$ **a határértéke**, ha

$$\forall\, \varepsilon > 0\text{-hoz} \quad \exists\, \delta > 0, \quad \text{hogy} \quad \forall\, x \in \big(k_\delta(a) \setminus \{a\}\big) \cap \mathcal{D}_f \quad \text{esetén} \quad f(x) \in k_\varepsilon(A).$$

Attól függően, hogy $a$, illetve $A$ valós szám vagy $\pm\infty$, ezt a definíciót egyenlőtlenségekkel a következőképpen fogalmazhatjuk meg:

| | $A \in \mathbb{R}$ | $A = +\infty$ | $A = -\infty$ |
|---|---|---|---|
| $a \in \mathbb{R}$ |  $\forall \varepsilon > 0\ \exists \delta > 0\ \forall x \in \mathcal{D}_f,\ 0 < \lvert x-a\rvert < \delta:\ \lvert f(x) - A\rvert < \varepsilon$ |  $\forall P > 0\ \exists \delta > 0\ \forall x \in \mathcal{D}_f,\ 0 < \lvert x-a\rvert < \delta:\ f(x) > P$ |  $\forall P < 0\ \exists \delta > 0\ \forall x \in \mathcal{D}_f,\ 0 < \lvert x-a\rvert < \delta:\ f(x) < P$ |
| $a = +\infty$ |  $\forall \varepsilon > 0\ \exists x_0 > 0\ \forall x \in \mathcal{D}_f,\ x > x_0:\ \lvert f(x) - A\rvert < \varepsilon$ |  $\forall P > 0\ \exists x_0 > 0\ \forall x \in \mathcal{D}_f,\ x > x_0:\ f(x) > P$ |  $\forall P < 0\ \exists x_0 > 0\ \forall x \in \mathcal{D}_f,\ x > x_0:\ f(x) < P$ |
| $a = -\infty$ |  $\forall \varepsilon > 0\ \exists x_0 < 0\ \forall x \in \mathcal{D}_f,\ x < x_0:\ \lvert f(x) - A\rvert < \varepsilon$ |  $\forall P > 0\ \exists x_0 < 0\ \forall x \in \mathcal{D}_f,\ x < x_0:\ f(x) > P$ |  $\forall P < 0\ \exists x_0 < 0\ \forall x \in \mathcal{D}_f,\ x < x_0:\ f(x) < P$ |