Wednesday, October 11, 2023 9:45 AM

4.3.

a)

$$L \cdot U$$

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 4 & 7 \\ 2 & 5 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \cdot \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 4 & 7 \\ 2 & 5 & 9 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ - & - & - \\ & & \end{bmatrix}$$

$$\begin{array}{cc} & \begin{array}{ccc} a_{11} & & a_{13} \end{array} \\ a_{21} & \begin{bmatrix} 2 & 1 & 3 \\ 4 & 4 & 7 \\ 2 & 5 & 9 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$(a_{21})\ 4 = 2 l_{21} + \cancel{1 \cdot 0} + \cancel{0 \cdot 0}$$

$$l_{21} = 2$$

$$(a_{22})\ 4 = 1 \cdot l_{21} + 1 \cdot u_{22} + \cancel{0 \cdot 0}$$

$$4 = 2 + u_{22}$$

$$u_{22} = 2$$

$$(a_{23})\ 7 = 3 l_{21} + 1 u_{23} + \cancel{0 \cdot u_{33}}$$

$$7 = 6 + u_{23}$$

$$u_{23} = 1$$

(a_{31}) $\quad l_{31} = 1$

(a_{32}) $\quad l_{32} = 2$

(a_{33}) $\quad u_{33} = 4$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{bmatrix}$$

b)

$$b = \begin{bmatrix} -4 \\ 1 \\ 2 \end{bmatrix} \qquad Ax = b$$

$$x = ?$$

$$A = L \cdot U$$

$$Ax = b$$

$$L \cdot \underbrace{Ux}_{y} = b \qquad Ux = y$$

(1) $Ly = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \\ 2 \end{bmatrix}$$

$$1 \cdot y_1 + \cancel{0 \cdot y_2} + \cancel{0 \cdot y_3} = -4 \Rightarrow y_1 = -4$$

$2y_1 + 1\cdot y_2 + \cancel{0\cdot y_3} = 1$

$-8 + y_2 = 1 \Rightarrow y_2 = 9$

$y_1 + 2y_2 + y_3 = 2$

$-4 + 2\cdot 9 + y_3 = 2 \Rightarrow y_3 = -12$

(II) $Ux = y$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & 1 \\ 0 & 0 & 4 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -4 \\ 9 \\ -12 \end{bmatrix}$$

$\cancel{0\cdot x_1} + \cancel{0\cdot x_2} + 4\cdot x_3 = -12$

$4x_3 = -12 \Rightarrow x_3 = -3$

$\cancel{0\cdot x_1} + 2\cdot x_2 + x_3 = 9$

$2x_2 - 3 = 9 \Rightarrow x_2 = 6$

$2x_1 + x_2 + 3x_3 = -4$

$2x_1 + 6 + 3\cdot(-3) = -4$

$2x_1 - 3 = -4 \Rightarrow x_1 = -\frac{1}{2}$

$$x = \begin{bmatrix} -\frac{1}{2} \\ 6 \\ -3 \end{bmatrix}$$ — $Ax = b$ megold. is

4.4.*

$[1 \quad 2 \quad -1]$ $\quad A \in \mathbb{R}^{3\times 3}$

a) $A = \begin{bmatrix} 1 & 2 & \cdot \\ 1 & 3 & 0 \\ 2 & 0 & 0 \end{bmatrix}$ $n = 3$

$D_1 \quad D_2 \quad D_3 = \det(A)$

$D_1 : [1] \quad D_1 = \det([1]) = 1$

$D_2 \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \quad D_2 = \det\left(\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}\right)$

$= 1 \cdot 3 - 1 \cdot 2 = 1$

$D_3 : A \quad D_3 = \det(A)$

$= -1(2 \cdot 3 \cdot (-1)) = 6$

$D_1 = 1 \neq 0 \quad D_2 = 1 \neq 0 \Rightarrow \exists$ LU felb.

$\det(A) = 6 \neq 0 \Rightarrow$ egyért LU. felb

(b) $D_1 = 2 \neq 0 \quad D_2 = 0$

$12 = 4 l_2 + u_1 l_3 + \cancel{1 \cdot 0}$

$12 = 16 + \cancel{0 \cdot l_3}$

$12 = 16$ ↯ $\Rightarrow \nexists$ LU felb

(c) $D_1 = 2 \neq 0 \quad D_2 = 0$

$$16 = 4 l_2 + u_1 \boxed{l_3} + 1 \cdot 0$$

$16 = 16 \to l_3$ szabadon megv.

$$14 = 6 \cdot l_2 + u_2 \cdot l_3 + u_3 \cdot 1$$

$$14 = 6 \cdot 4 - 10 \cdot l_3 + u_3$$

$$-10 = -10 l_3 + u_3$$

$$u_3 = -10 + 10 l_3$$

$$u_3 = 10(l_3 - 1)$$

$$\begin{bmatrix} 2 & 4 & 6 \\ 4 & 8 & 2 \\ 8 & 16 & 14 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & l_3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 6 \\ 0 & 0 & -10 \\ 0 & 0 & 10(l_3-1) \end{bmatrix}$$

$$l_3 \in \mathbb{R}$$

6. 1. a) $P(x) = 3x^4 + 2x^3 + \underbrace{}_{0 \cdot x^2} + x + 2$

$P(\frac{1}{2}) = ?$ $P(\xi$

a_i	3	2	0	1	2
ξ_i	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{7}{4}$	$\frac{7}{8}$	$\frac{15}{16}$
$a_i^{(1)}$	3	$\frac{7}{2}$	$\frac{7}{4}$	$\frac{15}{8}$	$\frac{47}{16}$

1 2 4 8 16

$$P\left(\frac{1}{2}\right) = \frac{47}{16}$$

$$P(x) = 3x^4 + 2x^3 + x + 2$$

$$\left(x - \frac{47}{16}\right) \cdot Q(x)$$

$$\left(x - \frac{47}{16}\right)\left(3x^3 + \frac{7}{2}x^2 + \frac{7}{4}x\right.$$

b) $\deg(\ldots)$

$$P(2) = 68$$

6.2. $P(2) = 68$

$$P'(2) = ?$$

a: | 3 2 0 1 2

ξ_1	2	6	16	32	66
$a_i^{(1)}$	3	8	16	33	68
ξ_1	2	6	28	88	
$a_i^{(2)}$	3	14	44	121	

$\leftarrow P'(2$

$$P(2) = 68 \qquad P'(2) = 121$$

6.3. $\xi = 3$ Taylor pol,

$$P(x) = \underline{a}(x-3)^3 + \underline{b}(x-3)^2 + \underline{c}(x-3) + \underline{d}$$

$$a = \frac{P'''(3)}{3!} \longrightarrow 4\times \text{Horner}$$

$$b = \frac{P''(3)}{2!} \longrightarrow 3\times \text{Horner}$$

$$c = \frac{P'(3)}{1} \longrightarrow 2\times \text{Horn}$$

$$d = P(3) \longrightarrow 1\times \text{Horner}$$

$$P(x) = x^3 - x^2 + x - 1 \qquad \xi =$$

a_i	1	-1	1	-1
ξ_i 3		3	6	21
$a_i^{(1)}$	1	2	7	20 $P($
ξ_i 3		3	15	
$a_i^{(2)}$	1	5	22 $\rightarrow \frac{P'(3}{1!}$	
ξ_i 3		3		
$a_i^{(3)}$	1	8 $\rightarrow \frac{P''(3)}{2!} \rightarrow$		
ξ_i 2				
$a_i^{(4)}$	1 $\rightarrow \frac{P'''(3)}{3!} \rightarrow P''$			

Taylor; $P(x) = 1 \cdot (x-3)^3 + 8$

$+ 22(x-3) +$

$\xi = 3$