Fábián T. Nikolett RDDZXA

2

$W := \mathrm{Span}((1,1,-1,0);(1,1,1,-1);(2,1,2,1))$

$x := (-1,1,-2,1)$

a) legyen

$u_1 = (1,1,-1,0)$

$u_2 = (1,1,1,-1)$

$u_3 = (2,1,2,1)$

$\langle u_1, u_2 \rangle = 1 + 1 - 1 + 0 = 1$

$\langle u_2, u_3 \rangle = 2 + 1 + 2 - 1 = 4$

$\langle u_3, u_1 \rangle = 2 + 1 - 2 + 0 = 1$

b) NEM ortogonális rendszer

Fábián T. Nikolett RDDZXA

b) legyen

$b_1 = (1, 1, -1, 0)$

$b_2 = (1, 1, 1, -1)$

$b_3 = (2, 1, 2, 1)$

$$u_1 = b_1 = (1, 1, -1, 0)$$

$$u_2 = b_2 - \frac{\langle b_2, u_1 \rangle}{\langle u_1, u_1 \rangle} \cdot u_1$$

$$= (1, 1, 1, -1) - \frac{1+1-1+0}{1+1+1+0} (1, 1, -1, 0)$$

$$= (1, 1, 1, -1) - \frac{1}{3} (1, 1, -1, 0)$$

$$= \left(\frac{2}{3}, \frac{2}{3}, \frac{4}{3}, -1\right) \sim (2, 2, 4, -3)$$

$$u_3 = b_3 - \frac{\langle b_3, u_1 \rangle}{\langle u_1, u_1 \rangle} u_1 - \frac{\langle b_3, u_2 \rangle}{\langle u_2, u_2 \rangle} \cdot u_2$$

$$= (2, 1, 2, 1) - \frac{2+1 \cdot 2+0}{1+1+1+0} (1, 1, -1, 0) - \frac{4+2+8-3}{4+4+16+9} (2, 2, 4, -3)$$

$$= (2, 1, 2, 1) - \frac{1}{3} (1, 1, -1, 0) - \frac{11}{33} (2, 2, 4, -3)$$

$$= (2, 1, 2, 1) - \left(\frac{1}{3}, \frac{1}{3}, -\frac{1}{3}, 0\right) - \left(\frac{2}{3}, \frac{2}{3}, \frac{4}{3}, -1\right)$$

$$= \left(\frac{6}{3} - \frac{1}{3} - \frac{2}{3}, \frac{3}{3} - \frac{1}{3} - \frac{2}{3}, \frac{6}{3} + \frac{1}{3} - \frac{4}{3}, 1 - 0 + 1\right)$$

$$= (1, 0, 1, 2)$$

Fábián T. Nikolett RDDZXA

O.R.

$u_1 = (1,1,-1,0) \quad u_2 = (2,2,4,-3) \quad u_3 = (1,0,1,2)$

$\|u_1\| = \sqrt{3}$

$\|u_2\| = \sqrt{33}$

$\|u_3\| = \sqrt{1+1+4} = \sqrt{6}$

O.N.R

$e_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, 0\right) \quad e_2 = \left(\frac{2}{\sqrt{33}}, \frac{2}{\sqrt{33}}, \frac{4}{\sqrt{33}}, -\frac{3}{\sqrt{33}}\right) \quad e_3 = \left(\frac{1}{\sqrt{6}}, 0, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$

c) $x = (-1,1,-2,1)$

legyen

$W := \mathrm{Span}(u_1, u_2, u_3)$

$$P(x) = \frac{\langle x, u_1\rangle}{\langle u_1, u_1\rangle} u_1 + \frac{\langle x, u_2\rangle}{\langle u_2, u_2\rangle} u_2 + \frac{\langle x, u_3\rangle}{\langle u_3, u_3\rangle} u_3$$

$$= \frac{-1+1+2+0}{1+1+1+0}(1,1,-1,0) + \frac{-1+1-2-1}{1+1+1+1}(1,1,1,-1) + \frac{-2+1-4+1}{4+1+4+1}(2,1,2,1)$$

$$= \frac{3}{3}(1,1,-1,0) + \frac{-3}{4}(1,1,1,-1) + \frac{-4}{10}(2,1,2,1)$$

$$= (1,1,-1,0) + \left(-\frac{15}{20}, -\frac{15}{20}, -\frac{15}{20}, +\frac{15}{20}\right) + \left(-\frac{16}{20}, -\frac{8}{20}, -\frac{16}{20}, -\frac{8}{20}\right)$$

$$= \left(\frac{20}{20} - \frac{15}{20} - \frac{16}{20}, \frac{20}{20} - \frac{15}{20} - \frac{8}{20}, -\frac{20}{20} - \frac{15}{20} - \frac{16}{20}, 0 + \frac{15}{20} - \frac{8}{20}\right)$$

$$= \left(\frac{-11}{20}, -\frac{3}{20}, -\frac{51}{20}, \frac{7}{20}\right) \sim (-11, -3, -51, 7)$$

$$Q(x) = x - P(x)$$

$$= \begin{pmatrix} -1 \\ 1 \\ -2 \\ 1 \end{pmatrix} - \begin{pmatrix} -11 \\ -3 \\ -51 \\ 7 \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \\ 49 \\ -6 \end{pmatrix}$$