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1) $\operatorname{tg}\frac{\pi}{12} = \frac{\sin\frac{\pi}{12}}{\cos\frac{\pi}{12}} = \frac{\sin\left(\frac{4\pi}{12}-\frac{3\pi}{12}\right)}{\cos\left(\frac{4\pi}{12}-\frac{3\pi}{12}\right)} = \frac{\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right)} = \frac{\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\sin\frac{\pi}{4}}{\cos\frac{\pi}{3}\cos\frac{\pi}{4}+\sin\frac{\pi}{3}\sin\frac{\pi}{4}} = \frac{\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}-\frac{1}{2}\cdot\frac{\sqrt{2}}{2}}{\frac{1}{2}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}} = \frac{\frac{\sqrt{6}-\sqrt{2}}{4}}{\frac{\sqrt{2}+\sqrt{6}}{4}} = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{2}+\sqrt{6}} = \frac{\sqrt{2}(\sqrt{3}-1)}{\sqrt{2}(1+\sqrt{3})} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3-2\sqrt{3}+1}{3-1} = \frac{4-2\sqrt{3}}{2} = \underline{2-\sqrt{3}}$

4a) $\sin 4x = \sin x$

$4x = x + 2k\pi$

$3x = 2k\pi$

$\underline{x = k\frac{2\pi}{3}}$

$4x + x = \pi + 2k\pi$

$5x = \pi + 2k\pi$

$\underline{x = \frac{\pi}{5} + k\frac{2\pi}{2}}$

4d) $\cos 2x - 3\cos x + 2 = 0$

$\cos^2 x - \sin^2 x - 3\cos x + 2 = 0$

$\cos^2 x - (1-\cos^2 x) - 3\cos x + 2 = 0$

$2\cos^2 x - 3\cos x + 1 = 0$

$\Delta = 9 - 4\cdot 2 = 1$

$\cos x = \frac{3\pm 1}{4}$

$1 \Rightarrow \underline{0 + 2k\pi}$

$\frac{1}{2} \Rightarrow \underline{\frac{\pi}{3} + 2k\pi}$

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(4e) $ctg x - tg x = 2\sqrt{3}$

$$\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = 2\sqrt{3}$$

$$\frac{\cos^2 x - \sin^2 x}{\sin x \cdot \cos x} = 2\sqrt{3}$$

$$\cos^2 x - \sin^2 x = 2\sqrt{3} \sin x \cdot \cos x$$

$$\cos 2x = \sqrt{3} \sin 2x$$

$$\frac{\cos 2x}{\sin 2x} = \sqrt{3}$$

$$ctg\, 2x = \sqrt{3}$$

$$\hookrightarrow 2x = \frac{\pi}{6} + 2k\pi$$

$$\underline{x = \frac{\pi}{12} + k\frac{\pi}{2}}$$

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4.
$$\sqrt{2}\sin x\cos\frac{x}{2}=\sqrt{1+\cos x}$$

$$\sin x\cos\frac{x}{2}=\sqrt{\frac{1+\cos x}{2}}$$ $\quad a=\frac{x}{2}$

$$\sin 2a\cos a=\sqrt{\frac{1+\cos 2a}{2}}$$ $\sqrt{\cos^2 a}$

$$\sin 2a\cos a=|\cos a|$$

$\underline{I}$ ha $\cos a=0$

$$\sin 2a\cdot 0=0$$
$$\sin 2a=0 \Rightarrow \underline{a=\frac{\pi}{2}+k\pi}$$

$\underline{II}$ ha $\cos a<0$

$$\sin 2a\cos a=-\cos a$$
$$\sin 2a=-1 \Rightarrow \underline{a=-\frac{\pi}{4}+2k\pi}$$

$\underline{III}$ ha $\cos>0$

$$\sin 2a\cos a=\cos a$$
$$\sin 2a=1 \Rightarrow \underline{a=\frac{\pi}{4}+2k\pi}$$

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(4m) $\cos 2x = \cos x - \sin x$

$\cos^2 x - \sin^2 x = \cos x - \sin x$

$(\cos x - \sin x)(\cos x + \sin x) = \cos - \sin x$

I $\cos x - \sin x = 0$

$\cos x = \sin x$

$\sin\left(\frac{\pi}{2} - x\right) = \sin x$

$\hookrightarrow \frac{\pi}{2} - x = x + 2k\pi$ $\qquad$ $\underline{x = \frac{\pi}{4} + k\pi}$

$\frac{\pi}{2} - x + x = \pi + 2k\pi$ $\qquad$ $0 = \frac{\pi}{2} \cdot k2\pi$ $\emptyset$

II $\cos x + \sin x = 1$

$\frac{1}{\sqrt{2}} \cos x + \frac{1}{\sqrt{2}} \sin x = \frac{1}{\sqrt{2}}$

$\sin\frac{\pi}{4} \cos x + \cos\frac{\pi}{4} \sin x = \frac{1}{\sqrt{2}}$

$\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\hookrightarrow x + \frac{\pi}{4} = \frac{\pi}{4} + 2k\pi$ $\qquad$ $\Rightarrow \underline{x = k2\pi}$

$x + \frac{\pi}{4} = \frac{3\pi}{4} + 2k\pi$ $\qquad$ $\Rightarrow \underline{x = \frac{\pi}{2} + 2k\pi}$

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7a $2\sin^2 x - \sin x - 1 > 0$

$D = 1 + 8 = 3^2$

$\sin x = \frac{1 \pm 3}{4} \begin{cases} 1 \\ -\frac{1}{2} \end{cases} \Rightarrow (-\infty, -\frac{1}{2}) \cup (1, +\infty)$

$\sin x < -\frac{1}{2} \Rightarrow -\frac{5\pi}{6} + 2k\pi < x < -\frac{\pi}{6} + 2k\pi$

$\sin x > 1 \quad \emptyset$

7c $\frac{2\sin x + 1}{2\cos x} \le 0$

I $\begin{cases} 2\sin x + 1 \le 0 \Rightarrow \sin x \le -\frac{1}{2} \Rightarrow -\frac{5\pi}{6} + 2k\pi \le x \le -\frac{\pi}{6} + 2k\pi \\ 2\cos x > 0 \Rightarrow \cos x > 0 \Rightarrow -\frac{\pi}{2} + 2k\pi < x < \frac{\pi}{2} + 2k\pi \end{cases}$

$\Rightarrow -\frac{\pi}{2} + 2k\pi < x \le -\frac{\pi}{6} + 2k\pi$

II $\begin{cases} 2\sin x + 1 \ge 0 \Rightarrow \sin x \ge -\frac{1}{2} \Rightarrow -\frac{\pi}{6} + 2k\pi \le x \le \frac{7\pi}{6} + 2k\pi \\ 2\cos x < 0 \Rightarrow \cos x < 0 \Rightarrow \frac{\pi}{2} + 2k\pi < x < \frac{3\pi}{2} + 2k\pi \end{cases}$

$\Rightarrow \frac{\pi}{2} + 2k\pi < x \le \frac{7\pi}{6} + 2k\pi$