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$a^2+ab+b^2=3\left(\frac{a+b}{2}\right)^2+\left(\frac{a-b}{2}\right)^2$

$3\,\frac{a^2+2ab+b^2}{4}+\frac{a^2-2ab+b^2}{4}$

$\Rightarrow \frac{3a^2+6ab+b^2+a^2-2ab+b^2}{4}=\frac{4a^2+4ab+4b^2}{4}=a^2+ab+b^2$

$a^3-b^3=?$ ha $a-b=2$ $a+b=\sqrt{5}$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$\hookrightarrow (a-b)\left[3\left(\frac{a+b}{2}\right)^2+\left(\frac{a-b}{2}\right)^2\right]$

$\Rightarrow 2\left[3\left(\frac{\sqrt{5}}{2}\right)^2+\left(\frac{2}{2}\right)^2\right]=2\cdot\left(\frac{3\cdot 5}{4}+1\right)=2\cdot\frac{19}{4}=\frac{19}{2}$

3b

$\frac{a}{a^3+a^2b+ab^2+b^3}+\frac{b}{a^3-a^2b+ab^2-b^3}+\frac{1}{a^2-b^2}-\frac{1}{a^2+b^2}-\frac{a^2+3b^2}{a^4-b^4}=0$

$a^4-b^4=(a^2-b^2)(a^2+b^2)$

$\to (a+b)\cdot(a^3+ab^2-a^2b-b^3)$

$\to (a-b)(a^3+ab^2+a^2b+b^3)$

$\to \frac{a(a-b)+b(a+b)+a^2+b^2-(a^2-b^2)-(a^2+3b^2)}{a^4-b^4}$

$\Rightarrow \frac{\cancel{a^2}-\cancel{ab}+\cancel{ab}+\cancel{b^2}+\cancel{a^2}+\cancel{b^2}-\cancel{a^2}+\cancel{b^2}-\cancel{a^2}-\cancel{3b^2}}{a^4-b^4}=\frac{0}{a^4-b^4}=0$

4b

$a+b+c=0 \quad \Rightarrow a=-(b+c)$

$a^3+b^3+c^3=3abc$

$-(b+c)^3+b^3+c^3=-(b^3+3b^2c+3bc^2+c^3)+b^3+c^3$

$\Rightarrow -\cancel{b^3}-3b^2c-3bc^2-\cancel{c^3}+\cancel{b^3}+\cancel{c^3}$

$\hookrightarrow -3b^2c-3bc^2=-3bc(b+c)=3(-(b+c))bc=3abc$

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6

$$E(x,y) = \frac{x^3 - x - y^3 + y + xy^2 - x^2y}{x^3 + x - y^3 - y + xy^2 - x^2y}$$

$$\rightarrow x^3 - x - y^3 + y + xy^2 - x^2y = (x^3 - y^3) - (x-y) - (x-y)xy$$

$$\Rightarrow (x-y)(x^2 + \cancel{xy} + y^2 - 1 - \cancel{xy}) = (x-y)(x^2+y^2-1)$$

$$\rightarrow x^3 + x - y^3 - y + xy^2 - x^2y = (x^3 - y^3) + (x-y) - (x-y)xy$$

$$\Rightarrow (x-y)(x^2 + \cancel{xy} + y^2 + 1 - \cancel{xy}) = (x-y)(x^2+y^2-1)$$

$$E(x,y) = \frac{(x-y)(x^2+y^2-1)}{(x-y)(x^2+y^2+1)} = \underline{\frac{x^2+y^2-1}{x^2+y^2+1}}$$

ha $x = \frac{k(1-z^2)}{1+z^2}$ $\quad y = \frac{2kz}{1+z^2}$

$$E = \frac{\left(\frac{k(1-z^2)}{1+z^2}\right)^2 + \left(\frac{2kz}{1+z^2}\right)^2 - 1}{\left(\frac{k(1-z^2)}{1+z^2}\right)^2 + \left(\frac{2kz}{1+z^2}\right)^2 + 1} = \frac{\frac{k^2(1-2z^2+z^4) + 4k^2z^2}{1+2z^2+z^4} - 1}{\frac{k^2(1-2z^2+z^4)+4k^2z^2}{1+2z^2+z^4} + 1}$$

$$\rightarrow \frac{\frac{k^2(1-2z^2+z^4+4z^2)}{1+2z^2+z^4} - 1}{\frac{k^2(1-2z^2+z^4+4z^2)}{1+2z^2+z^4} + 1} = \underline{\frac{k^2-1}{k^2+1}}$$

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(5)

$$f(x) = \frac{1-x}{1+x}$$

$$g(x) = \frac{1+x}{1-x} \qquad f(g(x)) \cdot g(f(x)) + 1 = 0$$

$$f(g(x)) = f\left(\frac{1+x}{1-x}\right) = \frac{1 - \frac{1+x}{1-x}}{1 + \frac{1+x}{1-x}} = \frac{\frac{\cancel{1}-x-\cancel{1}-x}{1-x}}{\frac{1-\cancel{x}+1+\cancel{x}}{1-x}} = \frac{-2x}{2} = -x$$

$$g(f(x)) = g\left(\frac{1-x}{1+x}\right) = \frac{1 + \frac{1-x}{1+x}}{1 - \frac{1-x}{1+x}} = \frac{\frac{1+\cancel{x}+1-\cancel{x}}{1+x}}{\frac{\cancel{1}+x-\cancel{1}+x}{1+x}} = \frac{2}{2x} = \frac{1}{x}$$

$$\hookrightarrow f(g(x)) \cdot g(f(x)) + 1 = -x \cdot \frac{1}{x} + 1 = -1 + 1 = 0$$

(12c)

$$\left(\sqrt{x} - \frac{\sqrt{xy}+y}{\sqrt{x}+\sqrt{y}}\right) \cdot \left(\frac{\sqrt{x}}{\sqrt{x}+\sqrt{y}} + \frac{\sqrt{y}}{\sqrt{x}-\sqrt{y}} + \frac{2\sqrt{xy}}{x-y}\right)$$

$$\hookrightarrow \frac{x+\cancel{\sqrt{xy}}-\cancel{\sqrt{xy}}-\cancel{y}}{\sqrt{x}+\sqrt{y}} \cdot \frac{x-\cancel{\sqrt{xy}}+y+\cancel{\sqrt{xy}}+2\sqrt{xy}}{\cancel{x-y}} = \frac{x+2\sqrt{xy}+y}{\sqrt{x}+\sqrt{y}} = \frac{(\sqrt{x}+\sqrt{y})^2}{\sqrt{x}+\sqrt{y}} = \sqrt{x}+\sqrt{y}$$

(13)

$$E(x,y) = \left(\frac{x^{-\frac{1}{6}} - \frac{5}{\sqrt[6]{y}}}{\frac{1}{x^{\frac{1}{3}}} - y^{-\frac{1}{3}}} - 5\,\frac{x^{-\frac{1}{6}} - y^{-\frac{1}{6}}}{x^{-\frac{1}{3}} - \sqrt[3]{y^{-1}}}\right)^{-1} \cdot \frac{6\sqrt[6]{x}}{\sqrt[3]{x} - \sqrt[3]{y}}$$

legyen $a = \sqrt[6]{x}$

$b = \sqrt[6]{y}$

$$\hookrightarrow \frac{\frac{1}{\sqrt[6]{x}} - \frac{5}{\sqrt[6]{y}}}{\frac{1}{\sqrt[3]{x}} - \frac{1}{\sqrt[3]{y}}} = \frac{\frac{1}{a} - \frac{5}{b}}{\frac{1}{a^2} - \frac{1}{b^2}}$$

$$\frac{\frac{1}{\sqrt[6]{x}} - \frac{1}{\sqrt[6]{y}}}{\frac{1}{\sqrt[3]{x}} - \frac{1}{\sqrt[3]{y}}} = \frac{\frac{1}{a} - \frac{1}{b}}{\frac{1}{a^2} - \frac{1}{b^2}}$$

$$\Rightarrow \frac{\frac{1}{a} - \frac{5}{b} - \frac{5}{a} + \frac{5}{b}}{\frac{1}{a^2} - \frac{1}{b^2}} = \frac{\frac{-4}{a}}{\frac{1}{a^2} - \frac{1}{b^2}} = -\frac{4}{a} \cdot \frac{a^2b^2}{b^2-a^2}$$

$$E = \frac{\cancel{b^2-a^2}}{\cancel{-}4\cancel{a}b^2} \cdot \frac{6\cancel{a}}{\cancel{a^2-b^2}} = \frac{3}{2b^2} = \frac{3}{2\sqrt[3]{y}}$$

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$x_0 = 3$

$P(x) = 2x^3 - 4x^2 - 18$

$P(3) = 2\cdot 3^3 - 4 3^2 - 18 = 2\cdot 27 - 4\cdot 9 - 18 = 56 - 36 - 18 = 0$

$$\begin{aligned} P(x) - P(3) &= 2x^3 - 4x^2 - 18 - (2\cdot 3^3 - 4 3^2 - 18) \\ &= 2(x^3 - 3^3) - 4(x^2 - 3^2) - 18 + 18 \\ &= 2(x-3)(x^2 + 3x + 9) - 4(x-3)(x+3) \\ &= (x-3)(2x^2 + 6x + 18 - 4x - 12) \\ &= \underline{(x-3)(2x^2 + 2x + 6)} \end{aligned}$$

19c

$x_0 = -1$

$P(x) = 2x^4 - 5x^3 - 6x^2 + 3x + 2$

$P(-1) = 2 + 5 - 6 - 3 + 2 = 0$

$$\begin{aligned} P(x) - P(-1) &= 2(x^4 - 1) - 5(x^3 + 1) - 6(x^2 - 1) + 3(x+1) + 2 - 2 \\ &= 2(x-1)(x+1)(x^2+1) - 5(x+1)(x^2 - x + 1) - 6(x-1)(x+1) + 3(x+1) \\ &= (x+1)(2x^3 + 2x - 2x^2 - 2 - 5x^2 + 5x - 5 - 6x + 6 + 3) \\ &= \underline{(x+1)(2x^3 - 7x^2 + x + 2)} \end{aligned}$$

20a

$2x^2 + x + 4$

$(x+3)(2x-5) = 2x^2 + 6x - 5x - 15 = 2x^2 + x - 15$

$\hookrightarrow 2x^2 + 6x - 5x + 4$

$\underline{k = -15}$