

$$\begin{aligned} \textcircled{1} x_1 &= (3, 0, -2, 4) \\ x_2 &= (2, 1, -1, 3) \\ x_3 &= (-1, 4, 2, 0) \\ x_4 &= (-1, 1, 1, -1) \end{aligned}$$

$$a \begin{bmatrix} 3 \\ 0 \\ -2 \\ 4 \end{bmatrix} + b \begin{bmatrix} 2 \\ 1 \\ -1 \\ 3 \end{bmatrix} + c \begin{bmatrix} -1 \\ 4 \\ 2 \\ 0 \end{bmatrix} + d \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3a + 2b - c - d = 0$$

$$b + 4c + d = 0$$

$$-2a - b + 2c + d = 0$$

$$4a + 3b - d = 0 \Rightarrow d = 4a + 3b$$

$$3a + 2b - (-4a - 3b) = 0 \Rightarrow -a - b - c = 0 \Rightarrow c = -a - b$$

$$b + 4c + 4a + 3b = 0 \Rightarrow 4a + 4b + 4c = 0 \quad 4a + 4b - 4a - 4b = 0 \Rightarrow 0a + 0b = 0$$

$$-2a - b + 2c + 4a + 3b = 0 \Rightarrow 2a + 2b + 2c = 0$$

$$2a + 2b - 2a - 2b = 0$$

$\Rightarrow$  a és b szabadon választható !!

$$\begin{aligned} \text{ha } a=1 \\ b=1 \end{aligned} \Rightarrow c = -2$$

$$d = 4a + 3b = 7$$

$$a \begin{bmatrix} 3 \\ 0 \\ -2 \\ 4 \end{bmatrix} + b \begin{bmatrix} 2 \\ 1 \\ -1 \\ 3 \end{bmatrix} + c \begin{bmatrix} -1 \\ 4 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3a + 2b - c = 0 \Rightarrow c = 3a + 2b$$

$$b + 4c = 0$$

$$b + 12a + 8b = 0$$

$$-2a - b + 2c = 0$$

$$-2a - b + 6a + 4b = 0$$

$$4a + 3b = 0$$

$$4a + 3b = 0$$

$$12a + 9b = 0$$

$$4a + 3b = 0$$

$$4a + 3b = 0$$

$$\Rightarrow b = -\frac{4a}{3}$$

$$0a = 0$$

$\Rightarrow$  szabadon választható !!

$$\text{Ha } a=3 \Rightarrow b=-4$$

$$c = 3 \cdot 3 + 2(-4) = 1$$

$\Rightarrow$  bármelyik elhagyható !!

$$W = \text{Span}\{x_1, x_2\}$$

$$a \begin{bmatrix} 3 \\ 0 \\ -2 \\ 4 \end{bmatrix} + b \begin{bmatrix} 2 \\ 1 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3a + 2b = 0$$

$$b = 0 \Rightarrow a = 0$$

$\Rightarrow$  függetlenek

$$-2a - b = 0$$

$$4a + 3b = 0$$

$\hookrightarrow \dim W = 2$

②  $x_1 = (2, 3, 2, 7)$  a)  $x_1, x_2$   
 $x_2 = (0, 1, 0, 1)$   $\dim \mathbb{R}^4 = 4$   $2 < 4 \Rightarrow$  nicht basis  
 $x_3 = (1, 2, 1, 0)$   
 $x_4 = (-1, -5, 2, 0)$  b)  $x_1, x_2, x_3, x_4, x_5$   
 $x_5 = (3, 1, 1, 2)$   $\dim \mathbb{R}^4 = 4$   $5 > 4 \Rightarrow$  nicht basis

(basis ist allerdings  $\mathbb{R}^4$ -em)

c)  $x_1, x_2, x_3, x_4$

$\dim \mathbb{R}^4 = 4$   $4 = 4 \Rightarrow$  lehet basis, de első elemek nem kell a függetlenek-e

$$a \begin{bmatrix} 2 \\ 3 \\ 2 \\ 7 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} + c \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix} + d \begin{bmatrix} -1 \\ -5 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$2a + c - d = 0$

$3a + b + 2c - 5d = 0$

$-2a - c + 2d = 0$

$7a + b = 0 \Rightarrow b = -7a$

$2a + c - d = 0 \Rightarrow d = 2a + c$

$3a - 7a + 2c - 5d = 0 \Rightarrow -4a + 2c - 5d = 0$

$-2a - c + 2d = 0$

$-4a + 2c - 10a - 5c = 0 \Rightarrow -14a - 3c = 0$

$-2a - c + 4a + 2c = 0 \Rightarrow 2a + c = 0 \Rightarrow c = -2a$

$-14a + 6a = 0$

$8a = 0$

$a = 0 \Rightarrow a = b = c = d = 0 \Rightarrow$  független

$\Rightarrow$  BAZIS

③  $W = \text{Span}(x_1, x_2, x_3)$

( $\mathbb{R}^4$ -em)

a)  $x_1 = (1, 2, 2, -1)$

$x_2 = (4, 3, 9, -4)$

$x_3 = (5, 8, 9, -5)$

$$a \begin{bmatrix} 1 \\ 2 \\ 2 \\ -1 \end{bmatrix} + b \begin{bmatrix} 4 \\ 3 \\ 9 \\ -4 \end{bmatrix} + c \begin{bmatrix} 5 \\ 8 \\ 9 \\ -5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$a + 4b + 5c = 0$

$2a + 3b + 8c = 0$

$2a + 9b + 9c = 0$

$-a - 4b - 5c = 0 \Rightarrow a = -4b - 5c$

$-4b - 5c + 4b + 5c = 0 \Rightarrow 0 = 0$

$-8b - 10c + 3b + 8c = 0 \Rightarrow -5b - 2c = 0$

$-8b - 10c + 9b + 9c = 0 \Rightarrow b - c = 0 \Rightarrow b = c$

$-7b = 0$

$b = 0 \Rightarrow a = b = c = 0 \Rightarrow$  független

$\Rightarrow$  BAZIS

b)  $x_1 = (1, 2, 3, 1)$

$x_2 = (2, 2, 1, 3)$

$x_3 = (-1, 2, 7, -3)$

$$a \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} + b \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix} + c \begin{bmatrix} -1 \\ 2 \\ 7 \\ -3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$a + 2b - c = 0 \Rightarrow c = a + 2b$

$2a + 2b + 2c = 0$

$3a + b + 7c = 0$

$a + 3b - 3c = 0$

$2a + 2b + 2a + 4b = 0 \Rightarrow 4a + 6b = 0$

$3a + b + 7a + 14b = 0 \Rightarrow 10a + 15b = 0$

$a + 3b - 3a - 6b = 0 \Rightarrow -2a - 3b = 0$

$\Rightarrow a = -\frac{3b}{2}$

$\hookrightarrow$  összefüggő

$\text{Span}\{x_1, x_2, x_3\} = \text{Span}\{x_1, x_2\}$

$$a \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix} + b \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix} = 0$$

$a + 2b = 0 \Rightarrow a = -2b$

$2a + 2b = 0$

$3a + b = 0$

$a + 3b = 0$

$-4b + 2b = 0$

$-6b + b = 0$

$-2b + 3b = 0$

$\Rightarrow b = 0 \Rightarrow a = b = 0 \Rightarrow$  független

$\Rightarrow$  BAZIS