(4)

FÁBIÁN TÍMEA NIKOLETT

RDDZXA

(4) a) $\lim\limits_{n\to+\infty} \dfrac{\sqrt{n^3+1}-\sqrt{n^3-n^2}}{\sqrt{4n+1}} = \dfrac{\sqrt{\frac{n^3}{n^3}+\frac{1}{n^3}}-\sqrt{\frac{n^3}{n^3}-\frac{n^2}{n^3}}}{\sqrt{4\frac{n}{n}+\frac{1}{n}}}$

$|q|<1 \Rightarrow q^n \to 0$

ÉS

$-q^n \to 0$

$= \dfrac{\sqrt{1+0}-\sqrt{1-0}}{\sqrt{4+0}}$

$= \dfrac{1-1}{4} = 0$

b) $\lim\limits_{n\to+\infty} \sqrt[n]{5^{n+1}+n^2 3^n} = \sqrt[n]{5^n\left(5+n^2\left(\frac{3}{5}\right)^n\right)}$

$= 5\cdot\sqrt[n]{5+n^2\left(\frac{3}{5}\right)^n} = 5$

L) $\lim\left(5+n^2\left(\frac{3}{5}\right)^n\right) = 5+0 = 5 \in (0,+\infty)$

c) $\lim\limits_{n\to\infty}\left(\dfrac{n+5}{2n}\right)^{3n+1} = \left(\left(\dfrac{\frac{n}{n}+\frac{5}{n}}{\frac{2n}{2n}}\right)^n\right)^3\cdot\left(\dfrac{1+\frac{5}{n}}{1}\right)$

$= \left(\dfrac{\left(1+\frac{5}{n}\right)^n}{1^n}\right)^3\cdot\dfrac{1+\frac{5}{n}}{1}$

$= \left(e^5\right)^3\cdot\dfrac{1+0}{1}$

$= e^{15}$