⑤

a) $A := \begin{bmatrix} 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & 1 \\ 4 & -4 & 4 & 0 \\ -2 & 2 & -2 & 1 \end{bmatrix}$  $b := \begin{pmatrix} 2 \\ 0 \\ 4 \\ -3 \end{pmatrix}$

$$\left[\begin{array}{cccc|c} \boxed{1} & -1 & 1 & -1 & 2 \\ 1 & -1 & 1 & 1 & 0 \\ 4 & -4 & 4 & 0 & 4 \\ -2 & 2 & -2 & 1 & -3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & -1 & 2 \\ 0 & 0 & 0 & \boxed{2} & -2 \\ 0 & 0 & 0 & 4 & -4 \\ 0 & 0 & 0 & -1 & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -1 & 1 & 0 & 1 \\ 0 & 0 & 0 & \boxed{1} & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

↳ $x_2 \in \mathbb{R}$, $x_3 \in \mathbb{R}$, $x_1 = 1 + x_2 - x_3$, $x_4 = -1$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 + x_2 - x_3 \\ x_2 \\ x_3 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} + x_2 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

b)

$$a \begin{pmatrix} 1 \\ 1 \\ 1 \\ -2 \end{pmatrix} + b \begin{pmatrix} -1 \\ -1 \\ -4 \\ 2 \end{pmatrix} + c \begin{pmatrix} 1 \\ 1 \\ 4 \\ -2 \end{pmatrix} + d \begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ c \\ 0 \end{pmatrix}$$

$a - b + c - d = 0 \quad \Rightarrow d = a - b + c$

$a - b + c + d = 0$

$4a - 4b + 4c = 0$

$-2a + 2b - 2(+d) = 0$

$a - b + c + a - b + c = 0 \Rightarrow 2a - 2b + 2c = 0$

$4a - 4b + 4c = 0$

$-2a + 2b - 2c + a - b + c = 0 \Rightarrow -a + b - c = 0$

$\Rightarrow b = a + c$

$\hookrightarrow$ összefüggő

$M_h = \{ x_2 v_2 + x_3 v_3 \mid x_2, x_3 \in \mathbb{R} \} = Span(v_2, v_3)$

$M_h$ bázisa : $v_2, v_3$

c) $r = 2 \Rightarrow rang(A) = 2$

$\dim ker(A) = 2$