

$$\begin{aligned} 1) \quad & x_2 - 3x_3 = -5 \\ & 4x_1 + 5x_2 - 2x_3 = 10 \\ & 2x_1 + 3x_2 - x_3 = 7 \end{aligned}$$

$$A = \begin{bmatrix} 0 & 1 & -3 \\ 4 & 5 & -2 \\ 2 & 3 & -1 \end{bmatrix} \quad b = \begin{pmatrix} -5 \\ 10 \\ 7 \end{pmatrix}$$

$$\begin{array}{ccc|c} 0 & \boxed{1} & -3 & -5 \\ 4 & 5 & -2 & 10 \\ 2 & 3 & -1 & 7 \end{array}$$

$$\begin{array}{ccc|c} 0 & \underline{1} & -3 & -5 \\ 4 & 0 & 13 & 35 \\ \boxed{2} & 0 & 8 & 22 \end{array}$$

$$\begin{array}{ccc|c} 0 & \underline{1} & 0 & -5 \\ 0 & 0 & \boxed{-3} & -9 \\ \underline{1} & 0 & 0 & 11 \end{array}$$

$$\begin{array}{ccc|c} 0 & \underline{1} & 0 & 4 \\ 0 & 0 & \underline{1} & 3 \\ \underline{1} & 0 & 0 & -1 \end{array}$$

↳ regularitás (3x3 as mátrix)
ES 3 megjelölt

$$2a) \quad A = \begin{bmatrix} 5 & 2 & -3 \\ 3 & 1 & -2 \\ 2 & -3 & -4 \end{bmatrix}$$

$$\begin{array}{ccc|ccc} 5 & 2 & -3 & 1 & 0 & 0 \\ 3 & \boxed{1} & -2 & 0 & 1 & 0 \\ 2 & -3 & -4 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{ccc|ccc} -1 & 0 & \boxed{1} & 1 & -2 & 0 \\ 3 & \underline{1} & -2 & 0 & 1 & 0 \\ 11 & 0 & -10 & 0 & 3 & 1 \end{array}$$

$$\begin{array}{ccc|ccc} -1 & 0 & \underline{1} & 1 & -2 & 0 \\ 1 & \underline{1} & 0 & 2 & -3 & 0 \\ \boxed{11} & 0 & 0 & 10 & -17 & 1 \end{array}$$

$$\begin{array}{ccc|ccc} 0 & 0 & \underline{1} & 11 & -19 & 1 \\ 0 & \underline{1} & 0 & -8 & 14 & -1 \\ \underline{1} & 0 & 0 & 10 & -17 & 1 \end{array}$$

$$A^{-1} = \begin{bmatrix} 11 & -19 & 1 \\ -8 & 14 & -1 \\ 10 & -17 & 1 \end{bmatrix}$$

$$2b) \quad A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ -3 & 1 & -7 \end{bmatrix}$$

$$\begin{array}{ccc|ccc} \boxed{1} & 1 & -1 & 1 & 0 & 0 \\ 2 & 0 & 3 & 0 & 1 & 0 \\ -3 & 1 & -7 & 0 & 0 & 1 \end{array}$$

$$\begin{array}{ccc|ccc} \underline{1} & 1 & -1 & 1 & 0 & 0 \\ 0 & -2 & 5 & -2 & 1 & 0 \\ 0 & \boxed{1} & -4 & 3 & 0 & 1 \end{array}$$

$$\begin{array}{ccc|ccc} \underline{1} & 0 & 3 & \frac{1}{4} & 0 & -\frac{1}{4} \\ 0 & 0 & 3 & -\frac{3}{2} & 1 & \frac{1}{2} \\ 0 & \underline{1} & -1 & \frac{3}{4} & 0 & \frac{1}{4} \end{array}$$

↳ mátrix szinguláris \Rightarrow nincs inverze

$$3) \quad A = \begin{bmatrix} 5 & 1 & -7 & -2 \\ 0 & 2 & 1 & 1 \\ 1 & 5 & 1 & 2 \\ -3 & -1 & 4 & 1 \end{bmatrix} \in \mathbb{R}^{4 \times 4}$$

$$\begin{array}{cccc} 5 & 1 & -7 & -2 \\ 0 & 2 & 1 & \boxed{1} \\ 1 & 5 & 1 & 2 \\ -3 & -1 & 4 & 1 \end{array}$$

$$\begin{array}{cccc} 5 & 5 & -5 & 0 \\ 0 & 2 & 1 & \underline{1} \\ \boxed{1} & 1 & -1 & 0 \\ -3 & -3 & 3 & 0 \end{array}$$

$$\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & \underline{1} \\ \underline{1} & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}$$

↳ a mátrix rangja 2
 $2 \neq 4 \Rightarrow$ szinguláris

$$4) \quad A = \begin{bmatrix} 5 & 1 & -7 & -2 \\ 0 & 2 & 1 & 1 \\ 1 & 5 & 1 & 2 \\ -3 & -1 & 4 & 1 \end{bmatrix}$$

↳ szinguláris (első feladat)

$$\text{rang}(A) = 2 < 4$$

$$a) \quad b_1 = (-1, 3, 7, 0)$$

$$\begin{array}{cccc|c} 5 & 1 & -7 & -2 & -1 \\ 0 & 2 & 1 & \boxed{1} & 3 \\ 1 & 5 & 1 & 2 & 7 \\ -3 & -1 & 4 & 1 & 0 \end{array}$$

$$\begin{array}{cccc|c} 5 & 5 & -5 & 0 & 5 \\ 0 & 2 & 1 & \underline{1} & 3 \\ \boxed{1} & 1 & -1 & 0 & 1 \\ -3 & -3 & 3 & 0 & -3 \end{array}$$

$$\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & \underline{1} & 3 \\ 1 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

↳ 2 megjelölt sor végén 0 szám
 \Rightarrow végtelen sok megoldás

$$b) \quad b_2 = (0, 5, 7, -1)$$

$$\begin{array}{cccc|c} 5 & 1 & -7 & -2 & 0 \\ 0 & 2 & 1 & \boxed{1} & 5 \\ 1 & 5 & 1 & 2 & 7 \\ -3 & -1 & 4 & 1 & -1 \end{array}$$

$$\begin{array}{cccc|c} 5 & 5 & -5 & 0 & 10 \\ 0 & 2 & 1 & \underline{1} & 5 \\ \boxed{1} & 1 & -1 & 0 & -3 \\ -3 & -3 & 3 & 0 & -6 \end{array}$$

$$\begin{array}{cccc|c} 0 & 0 & 0 & 0 & 25 \\ 0 & 2 & 1 & \underline{1} & 5 \\ \underline{1} & 1 & -1 & 0 & -3 \\ 0 & 0 & 0 & 0 & -15 \end{array}$$

↳ megjelölt sorok végén 0 szám
 \Rightarrow nincs megoldás