Fábián T. Nikolett RDDZXA

1)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix} \in \mathbb{R}^{3\times 3}$$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 1 & 1 \\ 1 & -\lambda & -1 \\ 0 & 1 & 2-\lambda \end{bmatrix}$$

$$\det(A-\lambda I) = (1-\lambda)\begin{vmatrix} -\lambda & -1 \\ 1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} 1 & -1 \\ 0 & 2-\lambda \end{vmatrix} + 1 \begin{vmatrix} 1 & -\lambda \\ 0 & 1 \end{vmatrix}$$

$$= (1-\lambda)\left(-\lambda(2-\lambda)-(-1)\right) - (2-\lambda) + 1$$

$$= (1-\lambda)(-2\lambda+\lambda^2+1) - 2 + \lambda + 1$$

$$= (1-\lambda)(\lambda^2-2\lambda+1) - (1-\lambda)$$

$$= (1-\lambda)(\lambda^2-2\lambda+1-1)$$

$$= (1-\lambda)(\lambda^2-2\lambda)$$

$$= (1-\lambda)\lambda(\lambda-2)$$

Sajátértékek: $\lambda_1 = 1$ $\lambda_2 = 0$ $\lambda_3 = 2$

$a(1) = 1$
$a(0) = 1$
$a(2) = 1$

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$$\lambda = 1 \Rightarrow \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & -1 \\ 0 & 1 & 1 \end{bmatrix} x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$x_2 + x_3 = 0$$

$$x_1 - x_2 - x_3 = 0 \Rightarrow x_1 = x_2 + x_3$$

$$x_2 + x_3 = 0 \Rightarrow x_2 = -x_3$$

$\hookrightarrow x_1 = 0$

$x_2 = -x_3$

$x_3 \in \mathbb{R}$

$$W_1 = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} : x_3 \in \mathbb{R} \right\}$$

$\hookrightarrow$ sajátvektor $(0, -1, 1)$

$$\dim W_1 = 1 \Rightarrow g(1) = 1$$

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$\lambda = 0$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix} x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x_1 + x_2 + x_3 = 0$

$x_1 - x_3 = 0 \Rightarrow x_1 = x_3$

$x_2 + 2x_3 = 0$

$$\left.\begin{matrix} x_2 + 2x_3 = 0 \\ x_2 + 2x_3 = 0 \end{matrix}\right\} \Rightarrow x_2 = -2x_3$$

$\hookrightarrow x_1 = x_3$

$x_2 = -2x_3$

$x_3 \in \mathbb{R}$

$$W_0 = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 \\ -2x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} : x_3 \in \mathbb{R} \right\}$$

$\hookrightarrow$ sajátvektor $(1, -2, 1)$

$\dim W_0 = 1 \Rightarrow g(0) = 1$

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$\lambda = 2$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 1 & 0 \end{bmatrix} x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$-x_1 + x_2 + x_3 = 0$$
$$x_1 - x_2 - x_3 = 0$$
$$x_2 = 0$$

$$\left.\begin{aligned} -x_1 + x_3 &= 0 \\ x_1 - x_3 &= 0 \end{aligned}\right\} \Rightarrow x_1 = x_3$$

$\hookrightarrow x_1 = x_3$
$x_2 = 0$
$x_3 \in \mathbb{R}$

$$W_2 = \left\{ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3 \\ 0 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} : x_3 \in \mathbb{R} \right\}$$

$\hookrightarrow$ sajátvektor $(1, 0, 1)$

$\dim W_2 = 1 \Rightarrow g(2) = 1$

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$a(1) = g(1) = 1$

$a(0) = g(0) = 1 \quad \Rightarrow$ VAN sajátbázisa

$a(2) = g(2) = 1$

$\Downarrow$

diagonalizálható

$$C = \begin{bmatrix} 0 & 1 & 1 \\ -1 & -2 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$C^{-1} \cdot A C = D$$

diagonális alak: $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$