Wednesday, November 22, 2023 10:19 AM

9.1.

$$\ell_k(x) = \prod_{j=0, j\neq k}^{n} \frac{x - x_j}{x_k - x_j} \quad (k = 0, 1 \ldots, n).$$

a) $f(x) = \sqrt{x}$ $x_{\overline{1,3}} = 1, 4, 9$

$$\ell_{\overline{0,2}}(x) = \frac{\cancel{(x-1)}(x-4)(x-9)}{\cancel{(0-1)}(0-4)(0-9)}$$

$$\ell_0(x) = \frac{(x-4)(x-9)}{(1-4)(1-9)} = \frac{1}{24}(x-4)(x-9)$$

$$\ell_1(x) = \frac{(x-1)(x-9)}{(4-1)(4-9)} = -\frac{1}{15}(x-1)(x-9)$$

$$\ell_2(x) = \frac{(x-1)(x-4)}{(9-1)(9-4)} = \frac{1}{40}(x-1)(x-4)$$

$$L_2(x) = f(x_0)\cdot \ell_0(x) + f(x_1)\cdot \ell_1(x) + f(x_2)\,\ell_2(x)$$

$$= \sqrt{1}\cdot\frac{1}{24}(x-4)(x-9) + \sqrt{4}\cdot\left(-\frac{1}{15}\right)(x-1)(x-9) + \sqrt{9}\cdot\frac{1}{40}(x-1)(x-4)$$

b) $f(x) = \sqrt{x}$ $x_{\overline{0,2}} = \{1, 4, 9\}$

x_i	$f(x_i)$	$f[x_i, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$

1	1
4	2
9	3

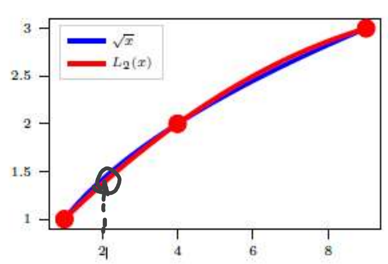
$$\frac{2-1}{4-1} = \frac{1}{3}$$

$$\frac{3-2}{9-4} = \frac{1}{5} \qquad \frac{\frac{1}{5}-\frac{1}{3}}{9-1} = -\frac{1}{60}$$

$$\omega_{-1}(x) = 1$$
$$\omega_0(x) = (x-1)$$
$$\omega_1(x) = (x-1)(x-4)$$
$$\omega_2(x) = (x-1)(x-4)(x-9)$$

$$N_2(x) = 1 \cdot \omega_{-1}(x) + \frac{1}{3}\omega_0(x) - \frac{1}{60}\omega_1(x)$$
$$= 1 + \frac{1}{3}(x-1) - \frac{1}{60}(x-1)(x-4)$$

d)



$$N_2(2) = 1 + \frac{1}{3}(2-1) - \frac{1}{60}(2-1)(2-4)$$
$$= 1 + \frac{1}{3} + \frac{1}{30} = \frac{41}{30}$$

$$N_2(2) \approx 1{,}3666$$

$$f(2) = \sqrt{2} = 1{,}4142\ldots$$

$$|f(2) - N_2(2)| = 0{,}0475$$

$x = 2$

e)

$$|f(x) - p_n(x)| \le \frac{M_{n+1}}{(n+1)!} \cdot |\omega_n(x)|, \text{ ahol}$$
$$M_{n+1} := \|f^{(n+1)}\|_\infty := \|f^{(n+1)}\|_{C[a;b]} := \max_{\xi \in [a;b]} |f^{(n+1)}(\xi)|.$$

$N_2(x)$ – másodfokú $\Rightarrow n = 2$

$$|\omega_2(x)| = |(x-1)(x-4)(x-9)|$$

$$|\omega_2(2)| = |(2-1)(2-4)(2-9)|$$
$$= |1 \cdot (-7)(-2)| = 14$$

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$
$$f'(x) = \frac{1}{2} x^{-\frac{1}{2}}$$
$$f''(x) = -\frac{1}{4} x^{-\frac{3}{2}}$$
$$f'''(x) = \frac{3}{8} x^{-\frac{5}{2}}$$

$$\|f'''(x)\|_\infty \qquad x \in [1, 9]$$

$$f'''(x) = \frac{3}{8} \cdot \frac{1}{\sqrt{x^5}}$$

$\hookrightarrow$ mon csökkenő

$\Downarrow$

$$M_3 = \|f'''(x)\|_\infty = f'''(1) = \frac{3}{8}$$

$$|f(x) - p_n(x)| \le \frac{M_{n+1}}{(n+1)!} \cdot |\omega_n(x)|, \text{ ahol}$$
$$M_{n+1} := \|f^{(n+1)}\|_\infty := \|f^{(n+1)}\|_{C[a;b]} := \max_{\xi \in [a;b]} |f^{(n+1)}(\xi)|.$$

$x = 2$

$$|f(2) - N_2(2)| \leq \frac{M_3}{3!} \cdot |\omega_2(2)|$$

$$|f(2) - N_2(2)| \leq \frac{\cancel{3}}{8} \cdot \frac{1}{\cancel{6}_{2}} \cdot \cancel{14}^{7} = \frac{7}{8}$$

becsült h: 0.875

tényleges h. $0{,}0475$

$x \in [1, 9]$

$$\omega_2(x) = (x-1)(x-4)(x-9)$$

$$\|\omega_2\|_\infty = ?$$

$$\omega_2(x) = x^3 - 14x^2 + 49x - 36$$

$$\omega_2'(x) = 3x^2 - 28x + 49$$

$$3x^2 - 28x + 49 = 0$$

$$x_{1/2} = \frac{28 \pm \sqrt{28^2 - 4 \cdot 3 \cdot 49}}{2 \cdot 3} = \frac{14 \pm 7}{3} \quad x_1 = 7, \quad x_2 = \frac{7}{3}$$

$$\omega_2(x_1) = \omega_2(7) = (7-1)(7-4)(7-9) = -36$$

$$\omega_2(x_2) = \omega_2\left(\frac{7}{3}\right) = \frac{400}{27} \approx 14{,}8$$

$$\|\omega_2(x)\|_\infty = 36 \quad \left| \{ |\omega_2(x_1)| \} \right.$$

$x \in [1, 9]$

$$|f(x) - N_2(x)| \leq \frac{M_3}{3!} \cdot \|\omega_2\|_\infty$$

$$= \frac{3}{8} \cdot \frac{1}{6} \cdot 36$$

$$= \frac{9}{4}$$

9.2.

x_i	$f(x_i)$	$f[x_i, x_{i+1}]$	...		
-2	-15				
-1	-4	$\frac{-4-(-15)}{-1-(-2)} = 11$			
0	-1	$\frac{-1-(-4)}{0-(-1)} = 3$	$\frac{3-11}{0-(-2)} = -4$		
1	0	$\frac{0-(-1)}{1-0} = 1$	$\frac{1-3}{1-(-1)} = -1$	$\frac{-1-(-4)}{1-(-2)} = 1$	
2	5	$\frac{5-0}{2-1} = 5$	$\frac{5-1}{2-0} = \frac{4}{2} = 2$	$\frac{2-(-1)}{2-(-1)} = 1$	$\frac{1-}{2-}$

$\omega_0(x) = (x-(-2))$

$$N_4(x) = -15 +$$
$$+11(x+2) -$$
$$-4(x+2)(x+1) +$$
$$+1(x+2)(x+1)(x-0) +$$
$$+0 \cdot (x+2)(x+1)(x-0)(x-1)$$

$$= x^3 - x^2 + x - 1$$