②

$u=(1,2,-1)$
$v=(6,4,2)$
$x=(9,2,7)$
$y=(4,-1,8)$

a) $-2u+3v=-(2,4,-2)+(18,12,6)=(16,8,8)$

b) $W: \text{Span}(u,v)=\{\lambda\cdot u+\eta\cdot v \mid \lambda,\eta\in\mathbb{R}\}$

$\hookrightarrow \lambda\cdot u+\eta\cdot v=\lambda(1,2,-1)+\eta\cdot(6,4,2)$
$=(\lambda,2\lambda,-\lambda)+(6\eta,4\eta,2\eta)$
$=(\lambda+6\eta,2\lambda+4\eta,-\lambda+2\eta)$

$\Rightarrow W=\{(\lambda+6\eta,2\lambda+4\eta,-\lambda+2\eta) \mid \lambda,\eta\in\mathbb{R}\}\subseteq\mathbb{R}^3$

c) $(9,2,7)=(\lambda+6\eta,2\lambda+4\eta,-\lambda+2\eta)$

$9=\lambda+6\eta \quad (1)$
$2=2\lambda+4\eta \quad (2)$
$7=-\lambda+2\eta \quad (3)$

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$2\cdot(1) \Rightarrow 18=2\lambda+12\eta$
$(2)-2(1) \Rightarrow -16=-8\eta$
$(3)+(1) \Rightarrow 16=8\eta$
$\Big\}\Rightarrow \eta=2$

$(1)\Rightarrow 9=\lambda+6\cdot2 \Rightarrow \lambda=-3$
$(2)\Rightarrow 2=2\lambda+4\cdot2 \Rightarrow \lambda=-3$
$(3)\Rightarrow 7=-\lambda+2\cdot2 \Rightarrow \lambda=-3$

$\Downarrow$

$x\in W$

d) $(4,-1,8)=(\lambda+6\eta,2\lambda+4\eta,-\lambda+2\eta)$

$4=\lambda+6\eta \quad (1)$
$-1=2\lambda+4\eta \quad (2)$
$8=-\lambda+2\eta \quad (3)$

$2\cdot(1)\Rightarrow 8=2\lambda+12\eta$
$(2)-2\cdot(1)\Rightarrow -9=-8\eta \Rightarrow \eta=\frac{8}{9}$
$(3)+(1)\Rightarrow 12=8\eta \Rightarrow \eta=\frac{4}{3}$
$\Big\}\Rightarrow \frac{8}{9}\neq\frac{4}{3} \Rightarrow \eta\neq\eta$

$\Downarrow$

$y\notin W$

③

a) $S_5=\{(x-y,3x,2x+y)\in\mathbb{R}^3 \mid x,y\in\mathbb{R}\}\subseteq\mathbb{R}^3$

b) $S_3=\{(x,y,z)\in\mathbb{R}^3 \mid 2x-3y+z=0\}\subseteq\mathbb{R}^3$

c) $W_1=\{(x-y+5z,3x-z,2x+y-7z,-x)\in\mathbb{R}^4 \mid x,y,z\in\mathbb{R}\}\subseteq\mathbb{R}^4$

d) $W_2=\{(x,y,z)\in\mathbb{R}^3 \mid 2x-3y+z=0\}\subseteq\mathbb{R}^3$

a) $(x-y,3x,2x+y)=(x,3x,2x)+(-y,0,y)=x(1,3,2)+y(-1,0,1)$
$\Rightarrow (1,3,2)\ (-1,0,1)$ generátorrendszer

b) $2x-3y+z=0$
$z=-2x+3y$

$\hookrightarrow (x,y,-2x+3y)=(x,0,-2x)+(0,y,3y)=x(1,0,-2)+y(0,1,3)$
$\Rightarrow (1,0,-2)\ (0,1,3)$ generátorrendszer

c) $(x-y+5z,3x-z,2x+y-7z,-x)=(x,3x,2x,-x)+(-y,0,y,0)+(5z,-z,-7z,0)=x(1,3,2,-1)+y(-1,0,1,0)+z(5,-1,-7,0)$
$\Rightarrow (1,3,2,-1)\ (-1,0,1,0)\ (5,-1,-7,0)$ generátorrendszer

d) $x+3y=0$
$x=0-3y$

$\Rightarrow (-3y,y,z)=(-3y,y,0)+(0,0,z)=y(-3,1,0)+z(0,0,1)$
$\Rightarrow (-3,1,0)\ (0,0,1)$ generátorrendszer

④ a) $W_1 = \{(x,y,z)\in\mathbb{R}^3 \mid [2\ \ -3\ \ 5]\cdot\begin{pmatrix}x\\y\\z\end{pmatrix} = (0)\}$

$[2\ \ -3\ \ 5]\cdot\begin{pmatrix}x\\y\\z\end{pmatrix} = 0$

$2x-3y+5z=0$

$x = \frac{3y-5z}{2}$

$\Rightarrow \left(\frac{3y-5z}{2}, y, z\right) = \left(\frac{3y}{2}, y, 0\right) + \left(-\frac{5}{2}z, 0, z\right) = y\left(\frac{3}{2},1,0\right) + z\left(-\frac{5}{2},0,1\right)$

$\Rightarrow \left(\frac{3}{2},1,0\right)\ \left(-\frac{5}{2},0,1\right)$ generátorrendszer

b) $W_2 = \{(x,y,z)\in\mathbb{R}^3 \mid \begin{bmatrix}1 & -2 & 3\\ 2 & 0 & -1\end{bmatrix}\cdot\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}\}$

$\begin{bmatrix}1 & -2 & 3\\ 2 & 0 & -1\end{bmatrix}\cdot\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

$x-2y+3z=0$ $\qquad x-2y+6x=0$

$2x-z=0 \Rightarrow z=2x$ $\qquad y=\frac{7x}{2}$

$\Rightarrow \left(x, \frac{7x}{2}, 2x\right) = x\left(1,\frac{7}{2},2\right)$

$\Rightarrow \left(1,\frac{7}{2},2\right)$ generátorrendszer

c) $W_3 = \{(x,y,z)\in\mathbb{R}^3 \mid \begin{bmatrix}2 & -1 & -2\\ -4 & 2 & 4\end{bmatrix}\cdot\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}\}$

$\begin{bmatrix}2 & -1 & -2\\ -4 & 2 & 4\end{bmatrix}\cdot\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}$

$2x-y-2z=0\ (1)$ $\qquad 2\cdot(1) = 4x-2y-4z=0$

$-4x+2y+4z=0\ (2)$ $\qquad (2)+2(1) = -4x+4x+2y-2y+4z-4z=0$

$y=2x-2z$

$(x, 2x-2z, z) = (x,2x,0)\ (0,-2z,z) = x(1,2,0)+y(0,-2,1)$

$\Rightarrow (1,2,0)\ (0,-2,1)$ generátorrendszer

⑤ $W = \{(2x-y+z,\ y+3z,\ x+y-2z,\ x-y)\in\mathbb{R}^4 \mid x,y,z\in\mathbb{R},\ x+2y+z=0\} \subseteq \mathbb{R}^4$

$x+2y+z=0$

$x=-2y-z$

$(2(-2y-z)-y+z,\ y+3z,\ -2y-z+y-2z,\ -2y-z-y) = (-5y-z,\ y+3z,\ -y-3z,\ -3y-z) = y(-5,1,-1,-3)+z(-1,3,-3,-1)$

$\Rightarrow (-5,1,-1,-3)\ (-1,3,-3,-1)$ generátorrendszer