1c

$f(x) := x^2 - 4x + 3 \quad (x \in \mathbb{R})$

$\hookrightarrow (x-2)^2 - 1$

$f(x) \geq -1 = f(2)$

$k = 2$

$K = \nexists$ ← felülről nem korlátos

1b

$f(x) := x^2 - 4x + 3 \quad \left(\frac{1}{2} \leq x \leq 3\right)$

$\hookrightarrow (x-2)^2 - 1$

$k = 2$

$K = \frac{1}{2}$

3

$2a+b=24$

$$T=ab=a(24-2a)$$
$$=24a-2a^2$$
$$=-2(a-12a)$$
$$=-2((a-6)^2-36)$$
$$=-2(a-6)^2+72$$

$a=6 \quad b=12$

$\Rightarrow T=72$

12a

a) $\lim\limits_{x\to\infty} \frac{x^4-2x^3+x^2+7}{x^3+x+1}=+\infty$

$$\frac{x^4-2x^3+x^2+7}{x^3+x+1} \geq \frac{x^4-2x^3}{3x^3} \geq \frac{\frac{1}{2}x^4+x^3(\frac{1}{2}x-2)}{3x^3} \geq \frac{\frac{1}{2}x^4}{3x^3}=\frac{x}{6} \quad (x>4)$$

$\frac{x}{6}>6 \Leftrightarrow x>6P$

$x>K:=\max\{4,6P\}$

$f(x)\geq\frac{x}{6}>P$

12b

$$\lim_{x\to\infty} \frac{2x^3-x^2+3}{x^3+2x-5} = 2$$

$$|f(x)-2| = \left|\frac{-x^2-4x+13}{x^3+2x-5}\right| = \frac{x^2+4x-13}{x^3+2x-5} \quad (x>3)$$

$$\frac{x^2+4x-13}{x^3+2x-5} \leq \frac{5x^2}{\frac{1}{2}x^3+\frac{1}{2}x^3-5} \leq \frac{5x^2}{\frac{1}{2}x^3} = \frac{10}{x}$$

$$\frac{10}{x} < \varepsilon \iff x < \frac{10}{\varepsilon}$$

$$k := \max\left\{3, \frac{10}{\varepsilon}\right\}$$

12c

$$\lim_{x\to\infty} \frac{x^3+x^2-2x-3}{9-4x^2} = -\infty$$

$$f(x) < p \iff -f(x) > -p$$

$$-p > 0$$

$$\hookrightarrow \ -f(x) = \frac{x^3+x^2-2x-3}{4x^2-9} \geq \frac{x^3-5x}{4x^2} = \frac{\frac{1}{2}x^3+x\left(\frac{1}{2}x^2-5\right)}{4x^2} \geq \frac{\frac{1}{2}x^3}{4x^2} = \frac{x}{8} \quad (x>4)$$

$$\frac{x}{8} > -p \iff x > -8p > 0$$

$$k := \max\{4, -8p\}$$