

① $P(x)=0$

$$P(x) = x^2 - 6x + 3 \Rightarrow x_{1,2} = \frac{6 \pm \sqrt{36-12}}{2} = \frac{6 \pm \sqrt{24}}{2} = \frac{6 \pm 2\sqrt{6}}{2} = 3 \pm \sqrt{6}$$

$$P(x) = 2x^2 + 7x - 1 \Rightarrow x_{1,2} = \frac{-7 \pm \sqrt{49+8}}{4} = \frac{-7 \pm \sqrt{57}}{4}$$

② $x^2 - 6x + 3$

a) $x_1 + x_2 = \frac{b}{a} = 6$

b) $x_1 x_2 = \frac{c}{a} = 3$

c) $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1 x_2 = 36 - 6 = 30$

d) $|x_1 - x_2| = \sqrt{(x_1 + x_2)^2 - 4x_1 x_2} = \sqrt{36 - 12} = \sqrt{24} = 2\sqrt{6}$

e) $\frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1 x_2} = \frac{6}{3} = 2$

$$2x^2 + 7x - 1$$

a) $x_1 + x_2 = -\frac{7}{2}$

b) $x_1 x_2 = -\frac{1}{2}$

c) $x_1^2 + x_2^2 = \left(-\frac{7}{2}\right)^2 - 2\left(-\frac{1}{2}\right) = \frac{49}{4} + 1 = \frac{53}{4}$

d) $|x_1 - x_2| = \sqrt{\left(\frac{49}{4} + 1\right) - 4\left(-\frac{1}{2}\right)} = \sqrt{\frac{53}{4} + 2} = \sqrt{\frac{59}{4}} = \frac{\sqrt{59}}{2}$

e) $\frac{1}{x_1} + \frac{1}{x_2} = \frac{-\frac{7}{2}}{-\frac{1}{2}} = 7$

③

$$\frac{3x^2 + 7x - 4}{x^2 + 2x - 3} < 2$$

$$\frac{3x^2 + 7x - 4 - 2(x^2 + 2x - 3)}{x^2 + 2x - 3} < 0$$

$$\frac{3x^2 + 7x - 4 - 2x^2 - 4x + 6}{x^2 + 2x - 3} < 0$$

$$\frac{x^2 + 3x + 2}{x^2 + 2x - 3} < 0$$

$$x^2 + 3x + 2 = 0$$

$$\Delta = 9 - 4 = 5$$

$$x_{1,2} = \frac{-3 \pm \sqrt{5}}{2}$$

$$x^2 + 2x - 3 = 0$$

$$\Delta = 4 + 12 = 16$$

$$x_{1,2} = \frac{-2 \pm 4}{2} = -1, 1$$

$$M_1 = (-2, -1) \cap (-\infty, -3) \cup (1, +\infty) = \emptyset$$

$$M_2 = (-\infty, -2) \cup (-1, \infty) \cap (-3, 1) = (-3, -2) \cup (-1, 1)$$

③c

$$\frac{x-1}{x+1} > \frac{3x+4}{1-2x}$$

$$\frac{x-1}{x+1} - \frac{3x+4}{1-2x} > 0$$

$$\frac{(x-1)(1-2x) - (3x+4)(x+1)}{(x+1)(1-2x)} > 0$$

$$\frac{x - 2x^2 - 1 + 2x - 3x^2 - 3x - 4x - 4}{(x+1)(1-2x)} > 0$$

$$\frac{-5x^2 - 4x - 5}{(x+1)(1-2x)} > 0$$

$$-5x^2 - 4x - 5 = 0$$

$$\Delta = 16 - 4 \cdot 50 = -184 < 0$$

$$\hookrightarrow (x+1)(1-2x) < 0$$

$$x+1 < 0 \Rightarrow x < -1$$

$$1-2x < 0 \Rightarrow x > \frac{1}{2}$$

x	-1	0	$\frac{1}{2}$
1	-	0	+
2	+	+	0
3	-	0	-

$$M = (-\infty, -1) \cup \left(\frac{1}{2}, +\infty\right)$$

$$(4) (p^2-1)x^2 + 2(p-1)x + 1 > 0$$

$$\text{für } p=1 \quad (1) \quad \text{für } p=-1$$

$$0+0+1 > 0 \quad \checkmark$$

$$0-4x+1 > 0 \quad \times$$

$$p^2-1 > 0 \quad \text{es } \Delta < 0$$

$$\Delta = (2(p-1))^2 - 4(p^2-1) \cdot 1$$

$$\hookrightarrow 4(p^2-2p+1) - 4p^2 - 4 = 4p^2 - 2p + 1 - 4p^2 + 4 = -8p + 8 < 0 \quad (2) \Rightarrow p > 1$$

$$\Rightarrow (1) \text{ \& } (2) \Rightarrow p \geq 1$$

$$(3) \frac{7-\sqrt{52}}{3} \leq \frac{x+3}{x^2-x+1} \leq \frac{7+\sqrt{52}}{3}$$

$$\frac{7-\sqrt{52}}{3} \leq \frac{x+3}{x^2-x+1} \quad \Delta = 1-4 = -3 \quad \hookrightarrow x^2-x+1 > 0$$

$$(7-\sqrt{52})(x^2-x+1) \leq 3(x+3)$$

$$(7-\sqrt{52})x^2 - (7-\sqrt{52})x + (7-\sqrt{52}) - 3x - 9 \leq 0$$

$$(7-\sqrt{52})x^2 - (10-\sqrt{52})x - 2-\sqrt{52} \leq 0$$

$$7-\sqrt{52} < 0 \quad \text{es } \Delta \leq 0$$

$$\Delta = (10-\sqrt{52})^2 - 4(7-\sqrt{52})(-2-\sqrt{52}) = 100 - 20\sqrt{52} + 52 + 4(14 + 7\sqrt{52} - 2\sqrt{52} - 52)$$

$$\hookrightarrow 162 - 20\sqrt{52} + 56 + 20\sqrt{52} - 208 = 0 \leq 0$$

$$\frac{x+3}{x^2-x+1} < \frac{7+\sqrt{52}}{3}$$

$$(7+\sqrt{52})(x^2-x+1) \leq 3(x+3)$$

$$(7+\sqrt{52})x^2 - (7+\sqrt{52})x + 7+\sqrt{52} - 3x - 9 \leq 0$$

$$(7+\sqrt{52})x^2 - (10+\sqrt{52})x - 2+\sqrt{52} \leq 0$$

$$7+\sqrt{52} > 0 \quad \text{es } \Delta \geq 0$$

$$\Delta = (10+\sqrt{52})^2 - 4(7+\sqrt{52})(-2+\sqrt{52}) = 100 + 20\sqrt{52} + 52 + 4(-14 + 7\sqrt{52} - 2\sqrt{52} + 52)$$

$$\hookrightarrow 152 + 20\sqrt{52} - 162 - 20\sqrt{52} = 0 \geq 0$$