

4

FABIAN TÍMEA VIKOLETT

RDDZXA

$$\textcircled{4} \quad a) \lim_{n \rightarrow +\infty} \frac{\sqrt{n^3+1} - \sqrt{n^3-n^2}}{\sqrt{4n+1}} = \frac{\sqrt{\frac{n^3}{n^3} + \frac{1}{n^3}} - \sqrt{\frac{n^3}{n^3} - \frac{n^2}{n^3}}}{\sqrt{4\frac{n}{n} + \frac{1}{n}}}$$

$$\begin{aligned} |q| < 1 &\Rightarrow q^n \rightarrow 0 \\ &\text{ÉS} \\ &4n \rightarrow 0 \\ &= \frac{\sqrt{1+0} - \sqrt{1-0}}{\sqrt{4+0}} \\ &= \frac{1-1}{4} = 0 \end{aligned}$$

$$\begin{aligned} b) \lim_{n \rightarrow +\infty} \sqrt[5]{5^{n+1} + n^2 3^n} &= \sqrt[5]{5^n (5 + n^2 (\frac{3}{5}))} \\ &= 5 \cdot \sqrt[5]{5 + n^2 (\frac{3}{5})} = 5 \end{aligned}$$

$$L) \lim_{n \rightarrow +\infty} (5 + n^2 (\frac{3}{5}))^{\frac{1}{5}} = 5 + 0 = 5 \in (0, +\infty)$$

$$\begin{aligned} c) \lim_{n \rightarrow \infty} \left(\frac{n+5}{2n} \right)^{3n+1} &= \left(\left(\frac{\frac{n}{n} + \frac{5}{n}}{\frac{2n}{2n}} \right)^n \right)^3 \cdot \left(\frac{1 + \frac{5}{n}}{1} \right) \\ &= \left(\frac{(1 + \frac{5}{n})^n}{1^n} \right)^3 \cdot \frac{1 + \frac{5}{n}}{1} \\ &= (e^5)^3 \cdot \frac{1+0}{1} \\ &= e^{15} \end{aligned}$$