Fábián T. Nikolett RDD2XA

(1b) $\frac{x^4+5x^2+4}{x^4-16} = \frac{(x^2+4)(x^2+1)}{(x^2+4)(x^2-4)} = \frac{x^2+1}{(x-2)(x+2)}$

(1c) $\frac{2x^2-13x-7}{8x^3+1} = \frac{(2x+1)(x-7)}{(2x+1)(4x^2-2x+1)} = \frac{x-7}{4x^2-2x+1}$

(1e) $\frac{2}{x^2-1} - \frac{3}{x^3-1} = \frac{2(x-1)(x^2+x+1)-3(x-1)(x+1)}{(x-1)(x+1)(x-1)(x^2+x+1)} = \frac{(x-1)(2x^2+2x+2-3x-3)}{(x-1)(x^2-1)(x^2+x+1)} = \frac{2x^2-x-1}{(x+1)(x-1)(x^2+x+1)} = \frac{(x-1)(2x+1)}{(x-1)(x+1)(x^2+x+1)} = \frac{2x+1}{(x+1)(x^2+x+1)}$

(2a) $\frac{\sqrt{x^2+1}-\sqrt{2}}{x^3-1} = \frac{(\sqrt{x^2+1}-\sqrt{2})(\sqrt{x^2+1}+\sqrt{2})}{(x-1)(x^2+x+1)(\sqrt{x^2+1}+\sqrt{2})} = \frac{x^2+1-2}{(x-1)(x^2+x+1)(\sqrt{x^2+1}+\sqrt{2})} = \frac{(x-1)(x+1)}{(x-1)(x^2+x+1)(\sqrt{x^2+1}+\sqrt{2})} = \frac{x+1}{(x^2+x+1)(\sqrt{x^2+1}+\sqrt{2})}$

(2b) $\frac{x^2+x-6}{\sqrt{\sqrt{x}-\sqrt{2}+1}-1} = \frac{(x+3)(x-2)(\sqrt{\sqrt{x}-\sqrt{2}+1}+1)}{\sqrt{x}-\sqrt{2}+1-1} = \frac{(\sqrt{x}-\sqrt{2})(\sqrt{x}+\sqrt{2})(x+3)(\sqrt{\sqrt{x}-\sqrt{2}+1}+1)}{\sqrt{x}-\sqrt{2}} = (\sqrt{x}+\sqrt{2})(x+3)(\sqrt{\sqrt{x}-\sqrt{2}+1}+1)$

(2c) $\frac{x^2-64}{\sqrt[3]{x}-2} = \frac{(x-8)(x+8)}{\sqrt[3]{x}-2} = \frac{(\sqrt[3]{x}-2)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)(x+8)}{\sqrt[3]{x}-2} = (x+8)(\sqrt[3]{x^2}+2\sqrt[3]{x}+4)$

(3b) $|2x-7|+|2x+7| = x+15$

$|2x-7| \begin{cases} 2x-7, & x \geq \frac{7}{2} \\ -2x+7, & x < \frac{7}{2} \end{cases}$ $\quad |2x+7| \begin{cases} 2x+7, & x \geq -\frac{7}{2} \\ -2x-7, & x < -\frac{7}{2} \end{cases}$

1. eset: $x \geq \frac{7}{2}$

$2x-7+2x+7 = x+15$

$3x = 15$

$x = 5$

3. eset: $x < -\frac{7}{2}$

$-2x+7-2x-7 = x+15$

$-5x = 15$

$x = -3 \notin (-\infty; -3{,}5)$

2. eset: $-\frac{7}{2} \leq x < \frac{7}{2}$

$-2x+7+2x+7 = x+15$

$x = -1$

$\Rightarrow M = \{-1, 5\}$

(3e) $|x^2-9|+|x^2-4| = 5$

$|x^2-9| \begin{cases} x^2-9, & x \geq 3 \vee x \leq -3 \\ -x^2+9, & -3 < x < 3 \end{cases}$ $\quad |x^2-4| \begin{cases} x^2-4, & x \geq 2 \vee x \leq -2 \\ -x^2+4, & -2 < x < 2 \end{cases}$

1 eset:

$x^2-9+x^2-4 = 5$

$2x^2 = 18$

$x = \pm 3$

2 eset:

$-x^2+9+x^2-4 = 5$

$5 = 5$

3 eset:

$-x^2+9-x^2+4 = 5$

$-2x^2 = -8$ ×

$x = \pm 2$

$\hookrightarrow -2 < x < 2$

$\Rightarrow M = (-\infty; -3] \cup [3; +\infty)$

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(3f) $|x-2|<3$

$|x-2| \begin{cases} x-2, & x>2 \\ -x+2, & x<2 \end{cases}$

1eset: $(2,+\infty)$
$x-2<3$
$x<5$

2eset: $(-\infty,2)$
$-x+2<3$
$-x<1$
$x>-1$

$\Rightarrow M=(-1,5)$

(3g) $|2x-1|<|x-1|$

$|2x-1| \begin{cases} 2x-1, & x \geq \frac{1}{2} \\ -2x+1, & x<\frac{1}{2} \end{cases}$ $\quad |x-1| \begin{cases} x-1, & x \geq 1 \\ -x+1, & x<1 \end{cases}$

1eset: $[1,+\infty)$
$2x-1<x-1$
$x<0$
$\hookrightarrow \notin [1,+\infty)$

2eset: $(\frac{1}{2},1)$
$2x-1<-x+1$
$3x<2$
$x<\frac{2}{3}$

3eset: $(-\infty,\frac{1}{2}]$
$-2x+1<-x+1$
$-x<0$
$x>0$

$\Rightarrow M=(0,\frac{2}{3})$

(6a) $\sqrt{x+1}-\sqrt{9-x}=\sqrt{2x-12}$

1.f $x \geq -1$
$x \leq 9$
$x \geq 6$
$\Rightarrow x \in [6,9]$

$\hookrightarrow x+1=2x-12+9-x+2\sqrt{9-x}\cdot\sqrt{2x-12}$
$2=2\sqrt{(9-x)(2x-12)} \quad /()^2$

$18x-112-2x^2+12x=0$
$-2x^2+30x-112=0$
$-x^2+15x-56=0$
$D=225-4\cdot 56\cdot 1=225-224=1$
$x_{1,2}=\frac{-15\pm 1}{-2} \begin{cases} 7 \\ 8 \end{cases}$

$M=\{7,8\}$

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(6k) $\sqrt{x^2+4x} > 2-x$

I.f. $x(x+4) \geq 0$

$x \geq 0$ $\quad x \leq -4$ $\quad x \in (-\infty, -4] \cup [0; \infty)$

$x^2+4x > 4-4x+x^2$

$8x > 4$

$x > \frac{1}{2}$ $\quad x \in (\frac{1}{2}, +\infty)$

$0 > 2-x$

$x > 2$ $\quad x \in (2, +\infty)$

$\Rightarrow$ $M = (\frac{1}{2}, +\infty)$

(6m) $\sqrt{3-x} - \sqrt{x+1} > \frac{1}{2}$

I.f. $3-x \geq 0$ $\quad x+1 \geq 0$

$x \leq 3$ $\quad x \geq -1$

$x \in [-1, 3]$

$3-x > \frac{1}{4} + 2 \cdot \frac{1}{2}\sqrt{x+1} + x+1$

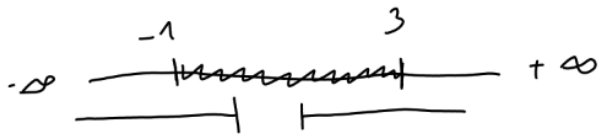
$-2x + \frac{7}{4} > \sqrt{x+1}$

$4x^2 + \frac{49}{16} - 2 \cdot 2x \cdot \frac{7}{4} > x+1$

$4x^2 - 8x + \frac{33}{16} > 0$

$\Delta = 8^2 - 4 \cdot 4 \cdot \frac{33}{16} = 64 - 33 = 31$

$x_{1,2} = \frac{8 \pm \sqrt{31}}{8}$ $\quad 1+\frac{\sqrt{31}}{8}$ $\quad 1-\frac{\sqrt{31}}{8}$



$M = [-1, 1-\frac{\sqrt{31}}{8}) \cup (1+\frac{\sqrt{31}}{8}, 3]$