Wednesday, October 18, 2023 8:36 AM

6.4. $P(x) = x^4 + 2x^3 + x + 2$

1	2	0	1	2

(r)

$$\max_{i=1}^{n} |a_i| = 2$$

$$\max_{i=0}^{n-1} |a_i| = 2$$

$$r = \frac{1}{1 + \frac{2}{2}} = \frac{1}{2}$$

$$R = 1 + \frac{2}{1} = 3$$

$P(x) = 0$

$$x_i \in \left(\frac{1}{2}, 3\right)$$

6.5. $\left.\begin{array}{l} x_i \in \left(\frac{15}{38}, 24\right) \\ x_i \in \mathbb{N} \end{array}\right\} \Rightarrow$

$$\Rightarrow x_i \in \{1, 2, \dots 23\}$$

a_i	1	-9	23	-15
ξ_i	1			
a_i	1	-8	15	0

$P(1) = 0$

$\Downarrow$

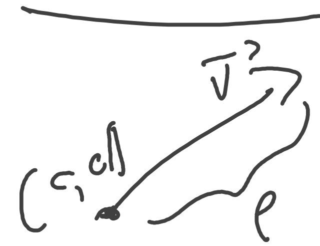
$1 - 9 + 23 - 15$

$$P(x) = (x - 1)\, Q(x)$$

$$Q(x) = x^2 - 8x + 15$$

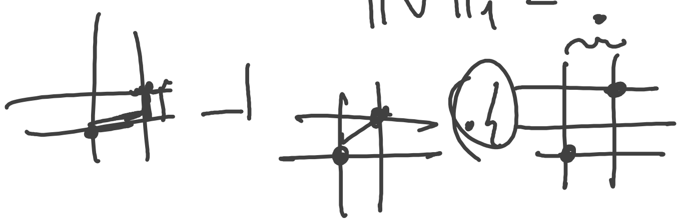
$\dots \quad Q(3) = 0$

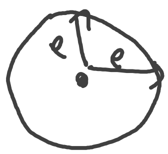
$$Q(x) = (x - 3)(x - 5)$$



$$\|\vec{v}\|_2 = |\vec{v}| = \sqrt{(a-c)^2 + (b-d)^2}$$

$$\|\vec{v}\|_1 =$$





5.3.

a) $A = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

$$\|A\|_1 = \max\{|-1| + |1|, |0| + |2|\}$$

$$= \max\{2, 2\} = 2$$

$$\|A\|_\infty = \max\{1, 3\} = 3$$

$$\|A\|_F = \sqrt{(-1)^2 + 0^2 + 1^2 + 2^2}$$

$$= \sqrt{6}$$

$\|A\|_2 = ?$

$$A^T A = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 \\ 2 & 4 \end{bmatrix}$$

$$\left| \begin{bmatrix} 2-a & 2 \\ 2 & 4-a \end{bmatrix} \right| = (2-a)(4-a)-4$$

$$= a^2 - 6a + 4$$

$$a^2 - 6a + 4 = 0$$

$$a_{1/2} = \frac{6 \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 4}}{2}$$

$$= \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm 2\sqrt{5}}{2}$$

$$= 3 \pm \sqrt{5}$$

$$\lambda_{1/2} = 3 \pm \sqrt{5}$$

$$\|A\|_2 = \left(\max\{3+\sqrt{5}, 3-\sqrt{5}\} \right)^{\frac{1}{2}}$$

$$= \sqrt{3+\sqrt{5}}$$

(6) $B = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$

$$\|B\|_1 = 6$$

$$\|B\|_\infty = 6$$

$$\|B\|_F = \sqrt{16 + 4 + 4 + 16}$$

$$= 140$$

$$\left(\left| \begin{bmatrix} 1-a & 2 & 5 \\ 4 & 3-a & 7 \\ 1 & 1 & 0-a \end{bmatrix} \right| \quad a^3 + \dots = 0 \right)$$

$$Ax = b$$

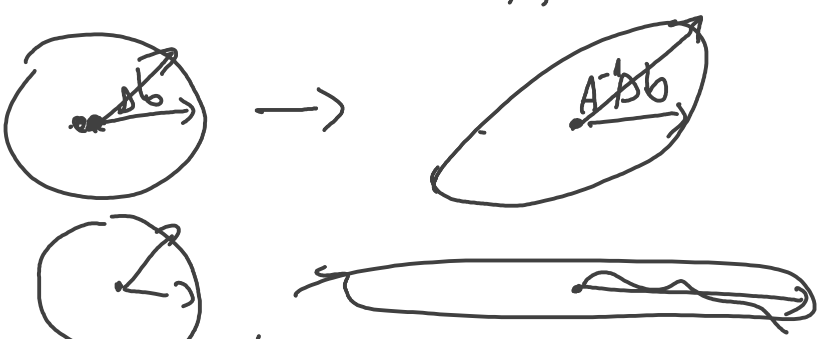
$$Ax = b + \Delta b$$

$$x = A^{-1}(b + \Delta b)$$

$$x = \underbrace{A^{-1}b} + \underbrace{A^{-1}\Delta b}$$

hiba $= A^{-1}\Delta b$

$A^{-1} \cdot \Delta b$



<< cond   >> cond

5.4 $\operatorname{cond} A = \|A\| \cdot \|A^{-1}\|$

$$A = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$\operatorname{cond}_1(A) = \|A\|_1 \cdot \|A^{-1}\|_1$$

$$= \|A^{-1}\|$$

$$= 2 \cdot \|A\|_1$$

$$A^{-1} = \frac{1}{-2} \cdot \begin{bmatrix} 2 & -1 \\ 0 & -1 \end{bmatrix}^T$$

$$\left(A^{-1} = \frac{1}{\det A} \operatorname{adj}(A)^T\right)$$

$$= \begin{bmatrix} -1 & \frac{1}{2} \\ 0 & +\frac{1}{2} \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

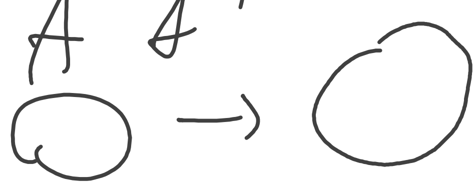
$$\|A^{-1}\|_1 = \max\left\{|-1| + \left|\frac{1}{2}\right|, |0| + \left|\frac{1}{2}\right|\right\}$$

$$= \max\left\{\frac{3}{2}, \frac{1}{2}\right\} = \frac{3}{2}$$

$$\operatorname{cond}_1(A) = 2 \cdot \frac{3}{2} = \underline{3}$$

$$Ax = b$$

$$A \quad A^{-1}$$



$$\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

$$\lambda_{1,2}, B : \left|\begin{bmatrix} 4-a & 2 \\ 2 & 4-a \end{bmatrix}\right| = 0$$

$$\lambda_{1,2} = a_{1,2}$$

$$\lambda_{1,2} \in \{2, 6\}$$

$$\operatorname{cond}_2(B) = \frac{6}{2} = 3$$

4. Tekintsük az

$$A = \begin{bmatrix} \cancel{3} & \cancel{0} & \cancel{3} \\ 0 & 3 & 2 \\ 3 & 2 & 5 \end{bmatrix}$$

mátrixot. Számítsuk ki $\mathrm{cond}_F(A)$-t, és $\mathrm{cond}_2(A)$-t!

$$\mathrm{cond}_F(A) = \|A\|_F \cdot \|A^{-1}\|_F$$

$$\mathrm{cond}_2(A) = \|A\|_2 \cdot \|A^{-1}\|_2$$

$$(\text{szimm.}) \Rightarrow L_2 = \frac{\max \lambda_A}{\min \lambda_A}$$

$$A^{-1} = \frac{1}{\det A} \cdot \mathrm{adj}(A)^T \qquad \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$\mathrm{adj}(A) = \begin{bmatrix} \begin{vmatrix} 3 & 2 \\ 2 & 5 \end{vmatrix} & -\begin{vmatrix} 0 & 2 \\ 3 & 5 \end{vmatrix} & \begin{vmatrix} 0 & 3 \\ 3 & 2 \end{vmatrix} \\ -\begin{vmatrix} 0 & 3 \\ 2 & 5 \end{vmatrix} & \dots & \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 6 & -9 \\ 6 & \dots & \\ & & \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} 11 & 6 & \\ 6 & & \\ -9 & & \dots \end{bmatrix} \qquad \begin{bmatrix} + & - \\ - & + \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \xrightarrow{A^{-1}} \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$$