(2a)

$$\frac{1}{2-3i} = \frac{2+3i}{(2-3i)(2+3i)} = \frac{2+3i}{4-9i^2} = \frac{2+3i}{13} = \frac{2}{13} + \frac{3i}{13}$$

(2b)

$$\frac{1+5i}{3+2i} = \frac{(1+5i)(3-2i)}{(3+2i)(3-2i)} = \frac{3+15i-10i^2-2i}{9-4i^2} = \frac{13+13i}{13} = 1+i$$

(2c)

$$(1-2i)(5+i) = 5-10i-2i^2+i = 7-9i$$

(2e)

$$(2-i)^2+(2+i)^3 = (4-4i+i^2)+(2^3+3\cdot 2^2\cdot i+3\cdot 2\cdot i^2+i^3) = (3-4i)+(8+12i-6-i) = 5+7i$$

(2g)

$$\frac{1+i}{3-i}+\frac{3-i}{1+i} = \frac{(1+i)^2+(3-i)^2}{(3-i)(1+i)} = \frac{1+2i+i^2+9-6i+i^2}{3+3i-i-i^2} = \frac{8-4i}{4+2i} = \frac{4-2i}{2+i} = \frac{(4-2i)(2-i)}{(2+i)(2-i)} = \frac{8-4i-4i+2i^2}{4-i^2} = \frac{6-8i}{5} = \frac{6}{5}-\frac{8i}{5}$$

(2h)

$$\frac{(1+i)^2}{1-i}+\frac{(1-i)^3}{(1+i)^2} = \frac{(1+i)^2(1+i)^2+(1-i)^3(1-i)}{(1-i)(1+i)^2} = \frac{(1+2i+i^2)(1+2i+i^2)+(1-2i+i^2)(1-2i+i^2)}{(1-i^2)(1+i)} = \frac{4i^2+4i^2}{1+i-i^2-i^3} = \frac{-8}{2+2i} = \frac{4(1-i)}{(1+i)(1-i)} = \frac{4-4i}{1-i^2} = \frac{4-4i}{2} = 2-2i$$

(3a)

$x^3-x^2+8x+10=0 \quad z_1=1+3i \quad z_2=1-3i$

z_1

$$(1+3i)^3-(1+3i)^2+8(1+3i)+10=0$$

$$(1+3i)^2(1+3i)-(1+6i+9i^2)+8+24i+10=0$$

$$(6i-8)(1+3i)-(6i-8)+24i+18=0$$

$$6i+18i^2-8-24i-6i+8+24i+16=0$$

$$0=0 \text{ w}$$

z_2

$$(1-3i)^3-(1-3i)^2+8(1-3i)+10=0$$

$$(1-3i)^2(1-3i)-(1^2-6i+9i^2)+8-24i+10=0$$

$$(-6i-8)(1-3i)+6i+8+16-24i=0$$

$$-6i+18i^2-8+24i+6i+8+18-24i=0$$

$$0=0 \text{ w}$$

(12a)

$$x^3-1=(x-1)(x^2+x+1)=0$$

1) $x_1=1$

$$x^2+x+1=0$$

$$\Delta = 1-4 = 3i^2$$

$$x_{2,3} = \frac{-1\pm\sqrt{3}i}{2} \quad < \quad -\frac{1}{2}-\frac{\sqrt{3}i}{2} \quad ; \quad -\frac{1}{2}+\frac{\sqrt{3}i}{2}$$

(12b) $x^4 - 1 = (x^2-1)(x^2+1) = (x-1)(x+1)(x^2+1) = 0$

$x_1 = 1$, $x_2 = -1$ $\quad x^2 = -1$, $x_{3,4} = \pm i$

(12c)

$x^2 - 2\sqrt{2}x + 5 = 0$

$\Delta = 8 - 4 \cdot 5 = 12 i^2$

$x_{1,2} = \dfrac{2\sqrt{2} \pm 2\sqrt{3}i^2}{2}$

(12d)

$x^3 - 9x^2 + 18x + 28 = 0$

Legyen $x = -1$

$-1 - 9 - 18 + 28 = 0$ igaz $\Rightarrow x_1 = -1$

$(x+1)(x^2 + 10x + 28)$

$\Delta = 100 - 4 \cdot 28 = 12 i^2$

$x_{2,3} = \dfrac{-10 \pm \sqrt{12 i^2}}{2}$ $\quad -5 + \sqrt{3}i$, $-5 - \sqrt{3}i$