1a

$$\begin{aligned} x_2 - 3x_3 &= -5 \\ 4x_1 + 5x_2 - 2x_3 &= 10 \\ 2x_1 + 3x_2 - x_3 &= 7 \end{aligned}$$

$\Rightarrow$ 3x3-as mátrix

$n = 3,\ m = 3$

a) $A = \begin{bmatrix} 0 & 1 & -3 \\ 4 & 5 & -2 \\ 2 & 3 & -1 \end{bmatrix} \quad b = \begin{pmatrix} -5 \\ 10 \\ 7 \end{pmatrix}$

b)

$$\left.\begin{array}{ccc|c} 0 & \boxed{1} & -3 & -5 \\ 4 & 5 & -2 & 10 \\ 2 & 3 & -1 & 7 \end{array}\right.$$

← végig kell osztani a sort a kiválasztott elemmel → 1 lesz

$$\left(\begin{array}{ccc|c} 0 & \underline{1} & -3 & -5 \\ 4 & 0 & 13 & 35 \\ 2 & 0 & 8 & 22 \end{array}\right)$$

← osztás után az oszlopot nullázzuk
$\Rightarrow$ kivonjuk a sorokból a generáló sor valahányszorosát

$$\left.\begin{array}{ccc|c} 0 & \underline{1} & -3 & -5 \\ 4 & 0 & 13 & 35 \\ \boxed{2} & 0 & 8 & 22 \end{array}\right.$$

$$\left(\begin{array}{ccc|c} 0 & \underline{1} & -3 & -5 \\ 0 & 0 & -3 & -9 \\ \underline{1} & 0 & 4 & 11 \end{array}\right)$$

← osztás majd nullázás

$$\left.\begin{array}{ccc|c} 0 & \underline{1} & -3 & -5 \\ 0 & 0 & \boxed{-3} & -9 \\ \underline{1} & 0 & 4 & 11 \end{array}\right.$$

$$\left(\begin{array}{ccc|c} 0 & \underline{1} & 0 & 4 \\ 0 & 0 & \underline{1} & 3 \\ \underline{1} & 0 & 0 & -1 \end{array}\right)$$

$r = 3$
↳ megjelölt elemek

$x_2 = 4$
$x_3 = 3$
$x_1 = -1$

$$\left.\begin{array}{ccc|c} \underline{1} & 0 & 0 & -1 \\ 0 & \underline{1} & 0 & 4 \\ 0 & 0 & \underline{1} & 3 \end{array}\right.$$

c) $r = 3 \Rightarrow rang(A) = 3$

d) $x_1 = -1 \quad x_2 = 4 \quad x_3 = 3$

↳ $x = (x_1, x_2, x_3) = (-1, 4, 3)$

e) $M = \{(-1, 4, 3)\} \subset \mathbb{R}^3$

f) $x_1 = 0 \quad x_2 = 0 \quad x_3 = 0$

↳ $(0, 0, 0) \in \mathbb{R}^3$

$M_h = \{(0,0,0)\} = ker(A) \subseteq \mathbb{R}^3$

nincs bázisa

$\dim M_h = 0$ ← szabad ismeretlenek száma

15) $-3x_1 + x_2 + x_3 - x_4 - 2x_5 = 2$

$2x_1 - x_2 + x_5 = 0$

$-x_1 + x_2 + 2x_3 + x_4 - x_5 = 8$

$x_2 + x_3 + 2x_4 = 6$

1

a) $A = \begin{bmatrix} -3 & 1 & 1 & -1 & -2 \\ 2 & -1 & 0 & 0 & 1 \\ -1 & 1 & 2 & 1 & -1 \\ 0 & 1 & 1 & 2 & 0 \end{bmatrix} \quad b = \begin{pmatrix} 2 \\ 0 \\ 8 \\ 6 \end{pmatrix}$

b) $\left(\begin{array}{ccccc|c} -3 & 1 & \boxed{1} & -1 & -2 & 2 \\ 2 & -1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 2 & 1 & -1 & 8 \\ 0 & 1 & 1 & 2 & 0 & 6 \end{array}\right)$

$\left(\begin{array}{ccccc|c} -3 & 1 & \underline{1} & -1 & -2 & 2 \\ 2 & -1 & 0 & 0 & \boxed{1} & 0 \\ 1 & -1 & 0 & 3 & 3 & 4 \\ 3 & 0 & 0 & 3 & 2 & 4 \end{array}\right)$

$\left(\begin{array}{ccccc|c} 1 & -1 & \underline{1} & -1 & 0 & 2 \\ 2 & -1 & 0 & 0 & \underline{1} & 0 \\ \boxed{-1} & 2 & 0 & 3 & 0 & 4 \\ -1 & 2 & 0 & 3 & 0 & 4 \end{array}\right)$

$\left(\begin{array}{ccccc|c} 0 & 1 & \underline{1} & 2 & 0 & 6 \\ 0 & 3 & 0 & +6 & \underline{1} & 8 \\ \underline{1} & -2 & 0 & -3 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$

$\left(\begin{array}{ccccc|c} 0 & 1 & \underline{1} & 2 & 0 & 6 \\ 0 & 3 & 0 & 6 & 1 & 8 \\ \underline{1} & -2 & 0 & -3 & 0 & -4 \end{array}\right) \begin{array}{l} \Rightarrow x_3 = 6 - x_2 - 2x_4 \\ \Rightarrow x_5 = 8 - 3x_2 - 6x_4 \\ \Rightarrow x_1 = -4 + 2x_2 + 3x_4 \end{array}$

$x_2 \in \mathbb{R} \quad x_4 \in \mathbb{R}$

c) $r = 3 \Rightarrow \text{rang}(A) = 3$

d)

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -4 + 2x_2 + 3x_4 \\ x_2 \\ 6 - x_2 - 2x_4 \\ x_4 \\ 8 - 3x_2 - 6x_4 \end{pmatrix} = \begin{pmatrix} -4 \\ 0 \\ 6 \\ 0 \\ 8 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ -1 \\ 0 \\ -3 \end{pmatrix} x_2 + \begin{pmatrix} 3 \\ 0 \\ -2 \\ 1 \\ -6 \end{pmatrix} x_4$$

$\hookrightarrow x^0 = (-4, 0, 6, 0, 8) \quad v_2 = (2, 1, -1, 0, -3) \quad v_4 = (3, 0, -2, 1, -6)$

e) $M = \{ x^0 + x_2 v_2 + x_4 v_4 \mid x_2, x_4 \in \mathbb{R} \}$

f) $x_2 \in \mathbb{R} \quad x_4 \in \mathbb{R} \quad x_1 = 2x_2 + 3x_4 \quad x_3 = -x_2 - 2x_4 \quad x_5 = -3x_2 - 6x_4$

$M_h = \ker(A) = \{ x_2 v_2 + x_4 v_4 \mid x_2, x_4 \in \mathbb{R} \} = \text{Span}(v_2, v_4)$

M_h bázisa: v_2, v_4

$\dim M_h = \dim \ker(A) = 2$

1c
$$2x_1+3x_2-x_3+2x_4=-1$$
$$x_1+4x_2-4x_3+3x_4=2$$
$$4x_1+x_2+5x_3=1$$

a) $A=\begin{bmatrix}2&3&-1&2\\1&4&-4&3\\4&1&5&0\end{bmatrix}$ $b=\begin{pmatrix}-1\\2\\1\end{pmatrix}$

b)
$$\left(\begin{array}{cccc|c}2&3&-1&2&-1\\1&4&-4&3&2\\4&\boxed{1}&5&0&1\end{array}\right)$$

$$\left(\begin{array}{cccc|c}-10&0&-16&\boxed{2}&-4\\-15&0&-24&3&-2\\4&\underline{1}&5&0&1\end{array}\right)$$

$$\left(\begin{array}{cccc|c}-5&0&-8&\underline{1}&-2\\0&0&0&0&4\\4&\underline{1}&5&0&1\end{array}\right)$$

c) rang(A) = 2

d) $\emptyset$ megoldás

e) $M=\emptyset$

f)
$$\left(\begin{array}{cccc|c}2&3&-1&2&0\\1&4&-4&3&0\\4&\boxed{1}&5&0&0\end{array}\right)$$

$$\left(\begin{array}{cccc|c}-10&0&-16&\boxed{2}&0\\-15&0&-24&3&0\\4&\underline{1}&5&0&0\end{array}\right)$$

$$\left(\begin{array}{cccc|c}-5&0&-8&\underline{1}&0\\0&0&0&0&0\\4&\underline{1}&5&0&0\end{array}\right)$$

$$\left(\begin{array}{cccc|c}-5&0&-8&\underline{1}&0\\4&\underline{1}&5&0&0\end{array}\right)$$

$x_1\in\mathbb{R}$ $x_3\in\mathbb{R}$ $x_2=-4x_1-5x_3$ $x_4=5x_1+8x_3$

$$x=\begin{pmatrix}x_1\\x_2\\x_3\\x_4\end{pmatrix}=\begin{pmatrix}x_1\\-4x_1-5x_3\\x_3\\5x_1+8x_3\end{pmatrix}=\begin{pmatrix}1\\-4\\0\\5\end{pmatrix}x_1+\begin{pmatrix}0\\-5\\1\\8\end{pmatrix}x_3$$

$M_h=\ker(A)=\{x_1v_1+x_3v_3 \mid x_1,x_3\in\mathbb{R}\}=\mathrm{Span}(v_1,v_3)$

M_h bázisa: $v_1=(1,-4,0,5)$
$v_2=(0,-5,1,8)$

$\dim M_h=2$

②

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 \\ 2 & 3 & 1 & 1 \\ 1 & 0 & 8 & 2 \end{bmatrix}$$

$$\ker(A) = \{ x \in \mathbb{R}^4 \mid Ax = 0 \}$$

$$\left[\begin{array}{cccc|c} \boxed{1} & 1 & 3 & 1 & 0 \\ 2 & 3 & 1 & 1 & 0 \\ 1 & 0 & 8 & 2 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} \underline{1} & 1 & 3 & 1 & 0 \\ 0 & \boxed{1} & -5 & -1 & 0 \\ 0 & -1 & 5 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} \underline{1} & 0 & 8 & 2 & 0 \\ 0 & \underline{1} & -5 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x_3 \in \mathbb{R}$ $\quad x_4 \in \mathbb{R}$ $\quad x_1 = -8x_3 - 2x_4$ $\quad x_2 = 5x_3 + x_4$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -8x_3 - 2x_4 \\ 5x_3 + x_4 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -8 \\ 5 \\ 1 \\ 0 \end{pmatrix} x_3 + \begin{pmatrix} -2 \\ 1 \\ 0 \\ 1 \end{pmatrix} x_4$$

$\ker(A)$ bázisa: $v_3 = (-8, 5, 1, 0)$

$v_4 = (-2, 1, 0, 1)$

$\dim \ker(A) = 2$