

3.2.

$$\left[\begin{array}{cccc|cc} 1 & -1 & 2 & 1 & 3 & 3 \\ 0 & 5 & -3 & -3 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{array} \right] \times$$

$\cancel{x_1} + \cancel{x_2} \cdot 0 + \cancel{x_3} \cdot 0 + \cancel{x_4} \cdot 0 = 0$
 $0 = 0$

$$x_1 - x_2 + 2x_3 + x_4 = 3$$

$$\begin{aligned} 5x_2 - 3x_3 - 3x_4 &= -1 \\ 0 &= 0 \\ 0 &= 0 \end{aligned}$$

$$\begin{cases} x_2 = \frac{3x_3 + 3x_4}{5} \\ x_1 = -2x_3 - x_4 + \frac{3x_3 + 3x_4}{5} \end{cases} \Rightarrow \infty \text{ sol} \\ \text{negold.}$$

$x_0 = 2$ - ellonfan \Rightarrow mins
negold.

3.3

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 4 & 2 & 0 & 1 & 0 \\ -1 & 5 & -2 & 0 & 0 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 0 & -2 & 1 & 0 \\ 0 & 0 & -1 & 7 & -3 & 1 \end{array} \right] \cdot 1$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 8 & -3 & 1 \\ 0 & 2 & 0 & -2 & 1 & 0 \\ 0 & 0 & -1 & 7 & -3 & 1 \end{array} \right] \xrightarrow{(-\frac{1}{2})}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 9 & -\frac{7}{2} & 1 \\ 0 & 2 & 0 & -2 & 1 & 0 \\ 0 & 0 & -1 & 7 & -3 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 9 & -\frac{7}{2} & 1 \\ 0 & 1 & 0 & -1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -7 & 3 & -1 \end{array} \right]$$

$$A \cdot A^{-1} = I_3$$

$$A^{-1} = \begin{bmatrix} 0 & -\frac{7}{2} & 1 \\ -1 & \frac{1}{2} & 0 \\ -7 & 3 & -1 \end{bmatrix}$$

3.4.

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 4 & 7 \\ 2 & 5 & 9 \end{bmatrix}$$

$$b = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 1 \\ 4 & 4 & 7 & 1 \\ 2 & 5 & 9 & 3 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 2 & 5 & 9 & 3 \\ 4 & 4 & 7 & 1 \\ 2 & 1 & 3 & 1 \end{array} \right]$$

YL 2 5 0 1 0 2 1 0 1

I re'szleges

fő.e.k. (RF)

$$\textcircled{4} \begin{bmatrix} 4 & 7 & 1 \\ 2 & 1 & 1 \\ 2 & 5 & 3 \end{bmatrix} \begin{matrix} (-\frac{1}{2}) \\ (-\frac{1}{2}) \\ (-\frac{1}{2}) \end{matrix}$$

II teljes

fő.e.k. (TF)

$$\begin{bmatrix} 9 & 5 & 2 & 3 \\ 7 & 4 & 4 & 1 \\ 3 & 1 & 2 & 1 \end{bmatrix} \begin{matrix} (-\frac{7}{9}) \\ (-\frac{1}{9}) \\ (-\frac{1}{9}) \end{matrix}$$

$$\begin{bmatrix} 4 & 4 & 7 & 1 \\ 0 & -1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 3 & \frac{11}{2} & \frac{5}{2} \end{bmatrix}$$

$$\begin{bmatrix} 9 & 5 & 2 & 3 \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} 4 & 4 & 7 & 1 \\ 0 & 3 & \frac{11}{2} & \frac{5}{2} \\ 0 & -1 & -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{matrix} (-\frac{1}{3}) \\ (-\frac{1}{3}) \\ (-\frac{1}{3}) \end{matrix}$$

$$\begin{bmatrix} 4 & 4 & 7 & 1 \\ 0 & 3 & \frac{11}{2} & \frac{5}{2} \\ 0 & 0 & \frac{4}{3} & \frac{4}{3} \end{bmatrix}$$

$$\frac{4}{3}x_3 = \frac{4}{3} \Rightarrow x_3 = 1$$

$$3x_2 + \frac{11}{2}x_3 = \frac{5}{2} \Rightarrow x_2 = -1$$

$$4x_1 + 4x_2 + 7x_3 = 1 \Rightarrow x_1 = -\frac{1}{2}$$

$$b) \det(A) = 4 \cdot 3 \cdot \frac{1}{3} \cdot (-1)^{-} = 16$$

$$L_2 \cdot L_1 \cdot A = U$$

$$L A = L_2^{-1} U$$

4.1. $\begin{bmatrix} 1 & 2 & 3 \\ -2 & 4 & 7 \\ 1 & 3 & 7 \end{bmatrix} \xrightarrow{R_2 + (-1)R_1} \xrightarrow{R_3 + (-1)R_1} L_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 8 & 13 \\ 0 & 1 & 4 \end{bmatrix} \xrightarrow{R_2 \cdot (-\frac{1}{8})} \xrightarrow{R_2 \leftrightarrow R_3} L_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{1}{8} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 8 & 13 \\ 0 & 0 & \frac{19}{8} \end{bmatrix} = U \quad L_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$L_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{8} & 1 \end{bmatrix}$$

$$L = L_1^{-1} \cdot L_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & \frac{1}{8} & 1 \end{bmatrix}$$

4.2. $L = ? \quad U = ?$

$$\begin{bmatrix} 2 & -5 & 3 \\ 1 & 3 & 7 \\ -4 & 2 & -1 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \xrightarrow{R_3 + (-2)R_2} \xrightarrow{R_1 \cdot \frac{1}{2}} \xrightarrow{R_2 \cdot (-\frac{1}{2})}$$

$$\begin{bmatrix} 2 & -5 & 3 \\ \frac{1}{2} & \frac{11}{2} & \frac{11}{2} \\ 1 & 2 & -1 \end{bmatrix} \cdot \frac{16}{11} \xrightarrow{R_1 \leftrightarrow R_3}$$

$$\begin{bmatrix} -2 & -8 & 5 \end{bmatrix} \quad \leftarrow$$

$$\begin{bmatrix} 2 & -5 & 3 \\ \frac{1}{2} & \frac{11}{2} & \frac{11}{2} \\ -2 & -\frac{16}{11} & 5 \end{bmatrix}$$

$$\rightarrow U = \begin{bmatrix} 2 & -5 & 3 \\ 0 & \frac{11}{2} & \frac{11}{2} \\ 0 & 0 & 5 \end{bmatrix}$$

$$\hookrightarrow L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ -2 & -\frac{16}{11} & 1 \end{bmatrix}$$