**AuE 8220 - Autonomy: Mobility and Manipulation**

**Homework 6: Jacobians (Related Design and Control Issues)**

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**Problem 1:**

**A: Planar RR Manipulator**

***1-A-i***

**Concept:** We defined a finely spaced workspace grid for the manipulator, taking into account its minimum () and maximum () reach. We then computed the joint angles at each of the workspace grid points using inverse kinematics for elbow-up configuration:

We then verified our approach by recovering the end-effector positions using forward kinematics:

Following are the two plots:

|  |  |
| --- | --- |
|  | Chart  Description automatically generated |

Next, we computed the Jacobian matrix at each point in workspace grid and performed singular value decomposition (SVD) over it.

We then computed the isotropy index at each point in workspace grid:

Chart, surface chart

Description automatically generated

Intuitively, the plot above shows “degree of isotropy” for joint velocities (i.e., degree of equalness in traveling quickly in all possible directions). We measured the minimum and maximum values of isotropy index:

The minimum value of isotropy index is 0.0000.

The maximum value of isotropy index is 0.6667.

The isotropy index is minimum (zero) at points in workspace where the second link is either fully retracted (overlapping on the first link) or when it is fully outstretched. This is because the manipulator cannot move in the direction of link 2 anymore, it can only travel in a direction that is tangential to the direction of link 2. It is maximum (0.6667) somewhere between these two limits. We analyzed the joint-space as well as task-space coordinates at which the isotropy index was minimum and maximum:

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| Chart, line chart  Description automatically generated | Diagram  Description automatically generated |
| Chart, line chart  Description automatically generated | Chart  Description automatically generated with medium confidence |

**MATLAB Code:** P1\_RR.m lines 13-133

***1-A-ii***

**Concept:** We defined a finely spaced workspace grid for the manipulator, taking into account its minimum () and maximum () reach. We then computed the joint angles at each of the workspace grid points using inverse kinematics for elbow-up configuration:

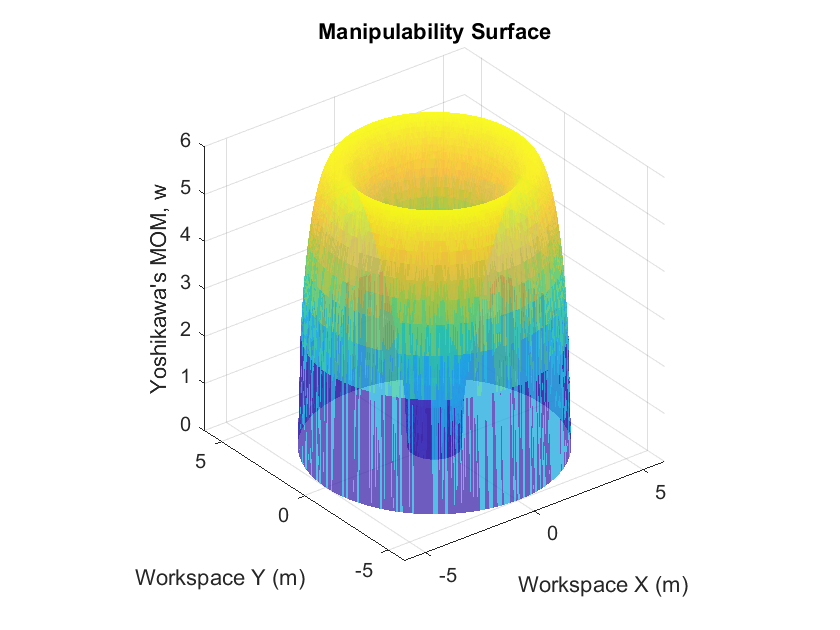
We then verified our approach by recovering the end-effector positions using forward kinematics:

Following are the two plots:

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Next, we computed the Jacobian matrix at each point in workspace grid and performed singular value decomposition (SVD) over it.

We then computed the Yoshikawa’s measure of manipulability (MOM) at each point in workspace grid: ; where .



Intuitively, the plot above shows “combined degree of manipulability” for joint velocities (i.e., cumulative degree of quickness to travel in all possible directions), which is equivalent to the area/volume/hyper-volume of the manipulability ellipse/ellipsoid/hyper-ellipsoid at that point in workspace. We measured the minimum and maximum values of Yoshikawa’s MOM:

The minimum value of MOM is 0.0000.

The maximum value of MOM is 6.0000.

Yoshikawa’s MOM is minimum (zero) at points in workspace where the second link is either fully retracted (overlapping on the first link) or when it is fully outstretched. This is because the manipulator cannot move in the direction of link 2 anymore, it can only travel in a direction that is tangential to the direction of link 2 (as a result, the manipulability ellipse collapses to a line, and hence its area is zero). It is maximum (6) somewhere between these two limits. We analyzed the joint-space as well as task-space coordinates at which Yoshikawa’s MOM was minimum and maximum:

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**MATLAB Code:** P1\_RR.m lines 134-250

***1-A-iii***

**Concept:** We defined a sparsely spaced workspace grid for the manipulator, taking into account its minimum () and maximum () reach. We then computed the joint angles at each of the workspace grid points using inverse kinematics for elbow-up configuration:

We then verified our approach by recovering the end-effector positions using forward kinematics:

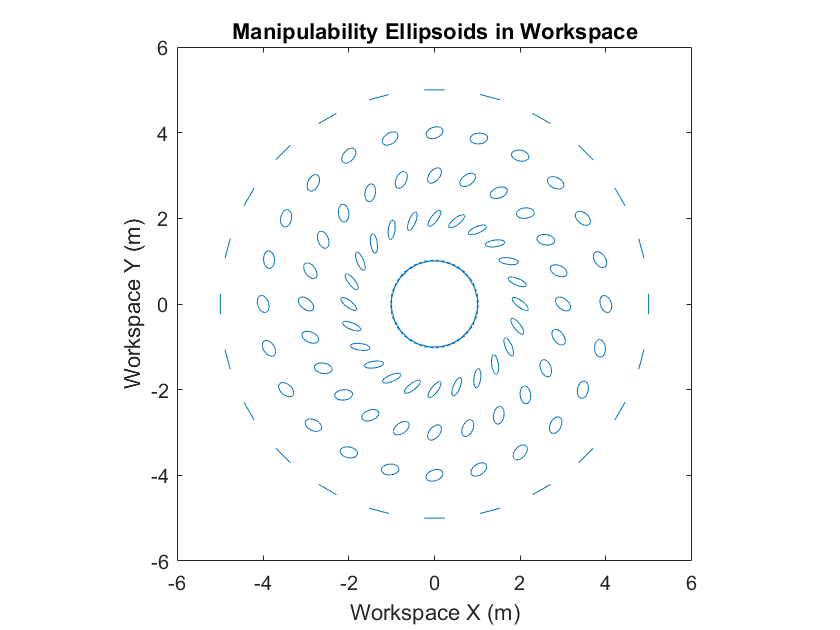
Following are the two plots:

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Next, we computed the Jacobian matrix at each point in workspace grid and performed singular value decomposition (SVD) over it.

We then computed and plotted the scaled manipulability ellipsoids at each point in the workspace grid:

The scale for manipulability ellipsoids is 1:15 m/s.



Intuitively, the plot above shows “degree of manipulability” for joint velocities (i.e., degree of quickness to travel in all possible directions).

**MATLAB Code:** P1\_RR.m lines 251-334

**Problem 1:**

**A: Planar RP Manipulator**

***1-A-i***

**Concept:** We defined a finely spaced workspace grid for the manipulator, taking into account its minimum () and maximum () reach. We then computed the joint angles at each of the workspace grid points using inverse kinematics:

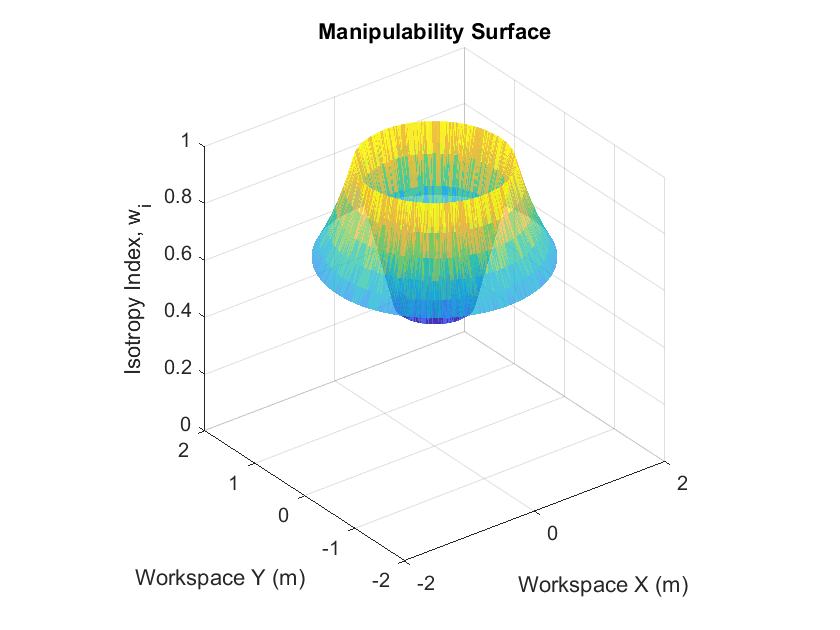
We then verified our approach by recovering the end-effector positions using forward kinematics:

Following are the two plots:

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Next, we computed the Jacobian matrix at each point in workspace grid and performed singular value decomposition (SVD) over it.

We then computed the isotropy index at each point in workspace grid:



Intuitively, the plot above shows “degree of isotropy” for joint velocities (i.e., degree of equalness in traveling quickly in all possible directions). We measured the minimum and maximum values of isotropy index:

The minimum value of isotropy index is 0.5000.

The maximum value of isotropy index is 1.0000.

The isotropy index is minimum (0.5) at points in workspace where the prismatic joint is completely retracted. This is because the manipulator can move fairly quickly in the direction of prismatic joint, but it cannot travel as quickly in a direction that is tangential to the direction of prismatic joint. It is maximum when the prismatic joint is extended half-way. This is because the robot can travel equally quickly in all the directions (isotropy index = 1). We analyzed the joint-space as well as task-space coordinates at which the isotropy index was minimum and maximum:

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**MATLAB Code:** P1\_RR.m lines 13-133

***1-A-ii***

**Concept:** We defined a finely spaced workspace grid for the manipulator, taking into account its minimum () and maximum () reach. We then computed the joint angles at each of the workspace grid points using inverse kinematics:

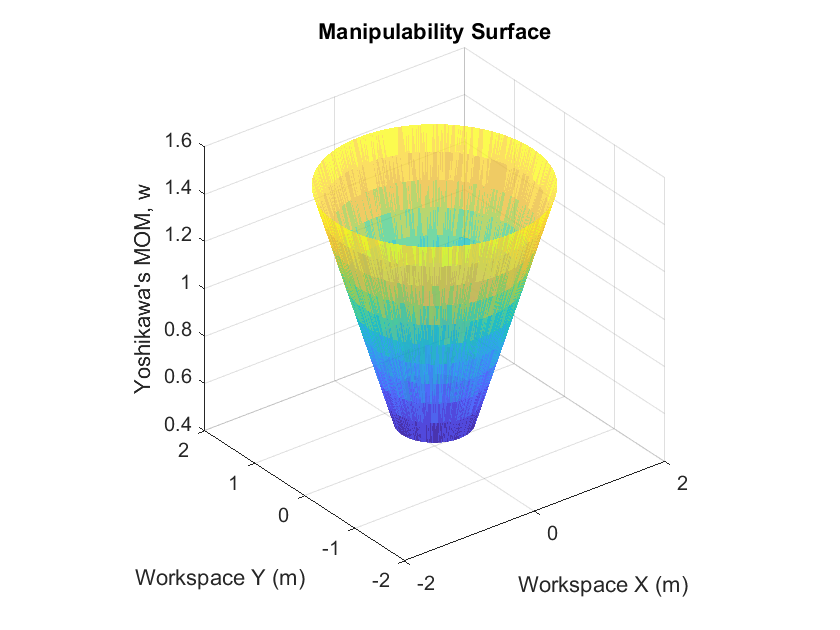
We then verified our approach by recovering the end-effector positions using forward kinematics:

Following are the two plots:

|  |  |
| --- | --- |
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Next, we computed the Jacobian matrix at each point in workspace grid and performed singular value decomposition (SVD) over it.

We then computed the Yoshikawa’s measure of manipulability (MOM) at each point in workspace grid: ; where .



Intuitively, the plot above shows “combined degree of manipulability” for joint velocities (i.e., cumulative degree of quickness to travel in all possible directions), which is equivalent to the area/volume/hyper-volume of the manipulability ellipse/ellipsoid/hyper-ellipsoid at that point in workspace. We measured the minimum and maximum values of Yoshikawa’s MOM:

The minimum value of MOM is 0.5000.

The maximum value of MOM is 1.5000.

Yoshikawa’s MOM is minimum (0.5) at points in workspace where the prismatic joint is fully retracted. This is because the manipulator can move fairly quickly in the direction of prismatic joint, but it cannot travel as quickly in a direction that is tangential to the direction of prismatic joint. It is maximum when the prismatic joint is fully extended. This is because the manipulator is most “manipulable” in a direction that is tangential to the direction of prismatic joint. We analyzed the joint-space as well as task-space coordinates at which Yoshikawa’s MOM was minimum and maximum:

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**MATLAB Code:** P1\_RR.m lines 134-250

***1-A-iii***

**Concept:** We defined a sparsely spaced workspace grid for the manipulator, , taking into account its minimum () and maximum () reach. We then computed the joint angles at each of the workspace grid points using inverse kinematics:

We then verified our approach by recovering the end-effector positions using forward kinematics:

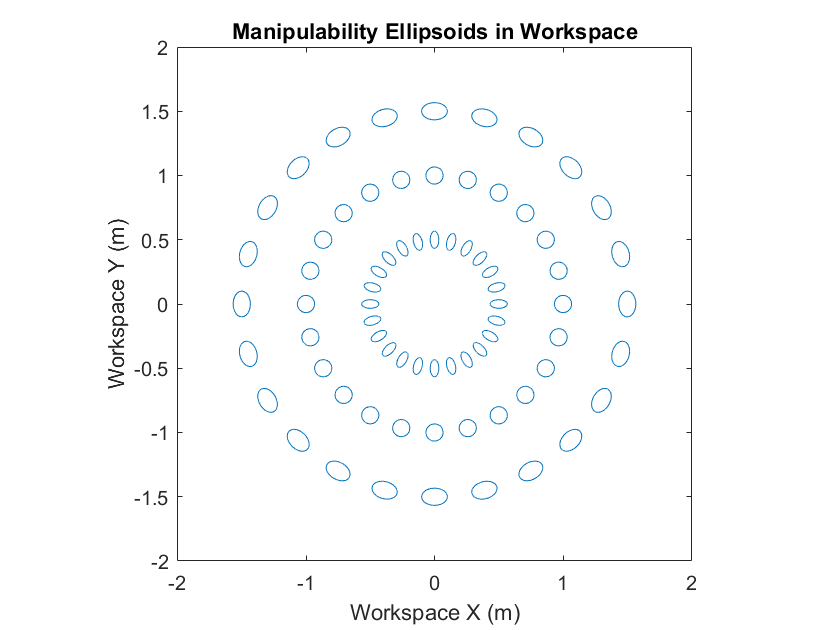
Following are the two plots:

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| --- | --- |
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Next, we computed the Jacobian matrix at each point in workspace grid and performed singular value decomposition (SVD) over it.

We then computed and plotted the scaled manipulability ellipsoids at each point in the workspace grid:

The scale for manipulability ellipsoids is 1:15 m/s.



Intuitively, the plot above shows “degree of manipulability” for joint velocities (i.e., degree of quickness to travel in all possible directions).

**MATLAB Code:** P1\_RR.m lines 251-334