

$$\dot{x}_p = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} w$$

$$y = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$z = \begin{bmatrix} 1 & 0 \end{bmatrix} x_p + \begin{bmatrix} 0 & 1 \end{bmatrix} w$$

System matrices

$$A_p = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B_p = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad D_p = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$C_p = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B_y = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad D_y = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M_p = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D_z = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$r = [n_z, n_u]$$

Lumped system matrices

$$A = A_p$$

$$B = \begin{bmatrix} D_p & B_p \end{bmatrix}$$

$$C = \begin{bmatrix} C_p & M_p \end{bmatrix}$$

$$D = \begin{bmatrix} D_y & B_y \\ D_z & 0 \end{bmatrix}$$

Open-loop system

$$S_{ol} = ss(A, B, C, D)$$

Inputs to system:

$$w = \begin{cases} 0.1 & ; 0 \leq t \leq 1 \\ 0 & ; \text{otherwise} \end{cases}$$

$$u = 0 \quad \forall t$$

w is a 2×1 vector since the disturbance input $w(t)$ consists of plant disturbance and measurement disturbance

Simulate open-loop system using "lsim" for $t=0$ to $t=10$ seconds

No Dynamic Controller Design in MATLAB:

$$s = \text{tfss}(A, B, C, D)$$

$$r = [n_z, n_u]$$

$$[G_{opt}, G] = \text{hinf_lmi}(s, r)$$

$$[A_c, B_c, C_c, D_c] = \text{hiss}(G)$$

$\|r\| < 1$

Controller

Closed-loop system:

$$A_{cl} = \begin{bmatrix} A_p + B_p D_c M_p & B_p C_c \\ B_c M_p & A_c \end{bmatrix}$$

$$B_{cl} = \begin{bmatrix} D_p + B_p D_c D_z \\ B_c D_z \end{bmatrix}$$

$$C_{cl} = [C_p + B_y D_c M_p \quad B_y C_c]$$

$$D_{cl} = D_y + B_y D_c D_z$$

Examination / validation of system:

- Stability: Eigenvalues: $\text{Re}(\lambda_i(A_{cl})) < 0 \leftarrow$ stability guaranteed (Poles)
- Performance: H_2 norm: $S_{cl} = \text{ss}(A_{cl}, B_{cl}, C_{cl}, D_{cl})$
(energy-to-energy gain $\|e\|$)
 $\left. \begin{array}{l} \text{hinf_norm} = \text{hinfnorm}(S_{cl}) \\ \text{hinf_norm} \approx g_{opt} \end{array} \right\} \text{MATLAB}$
 $\uparrow \gamma^* \quad (\|e\| < \gamma^*)$

Inputs to closed-loop system:

$$w = \begin{cases} 0.1; & 0 \leq t \leq 1 \\ 0; & \text{otherwise} \end{cases}$$

$$u = 0 \quad \forall t$$

w is a 2×1 vector since the disturbance input $w(t)$ consists of plant disturbance and measurement disturbance

Simulate closed-loop system using "lsim" for $t=0$ to $t=10$ seconds

Problem 3

CODE:

```
% PROBLEM 3

% Clear workspace
close all
clear
clc

% Define the system matrices
Ap = [0 1; 0 0];
Bp = [0; 1];
Dp = [0 0; 1 0];
Cp = [0 1; 0 0];
By = [0; 1];
Dy = [0 0; 0 0];
Mp = [1 0];
Dz = [0 1];

% Lumped system matrices
A = Ap;
B = [Dp Bp];
C = [Cp; Mp];
D = [Dy By; Dz 0]

% State-space system
Sol = ss(A, B, C, D)
% sys_openloop = ss(Ap, Dp, Cp, 0)

% Define the lumped (disturbance + control) input
t = 0:0.01:10;
w_amplitude = 0.1;
w_duration = 1;
w_pulse = w_amplitude * (t >= 0 & t <= w_duration);
u = zeros(size(w_pulse));
w = [w_pulse; w_pulse; u];
% w = [w_pulse; w_pulse];

% Simulate the open-loop system response
```

```

[y_ol, t_out, x_ol] = lsim(Sol, w', t);
% [y_ol, t_out, x_ol] = lsim(Sol, w, t);

% Plot the results
figure;
sgtitle('Open-Loop System Response');
subplot(4, 1, 1);
plot(t, w(2, :), t, w(3, :));
legend('W', 'U');
subplot(4, 1, 2);
plot(t, x_ol(:, 1), t, x_ol(:, 2));
legend('X1', 'X2');
subplot(4, 1, 3);
plot(t, y_ol(:, 1), t, y_ol(:, 2));
legend('Y1', 'Y2');
subplot(4, 1, 4);
plot(t, y_ol(:, 3));
legend('Z');

```

```

% H-infinity LMI
S = ltisys(A, B, C, D);
[gopt, G] = hinflmi(S,[1 1])

% Controller matrices
disp('H $\infty$  controller:')
[Ac, Bc, Cc, Dc] = ltiss(G)

```

```

% Closed-loop system matrices
disp('Closed-loop system:')
Acl = [Ap+Bp*Dc*Mp, Bp*Cc; Bc*Mp, Ac]
Bcl = [Dp+Bp*Dc*Dz; Bc*Dz]
Ccl = [Cp+By*Dc*Mp, By*Cc]
Dcl = Dy+By*Dc*Dz

```

```

% Closed-loop system
Scl = ss(Acl, Bcl, Ccl, Dcl);
% Scl = slft(S, G);

```

```

% Verification
disp('H $\infty$  norm:')
hinf_norm = hinfnorm(Scl)

```

```

% hinf_norm = norminf(Scl)
disp('Closed-loop poles:')
eig_Acl = eig(Acl)
if((hinf_norm < gopt) && all(real(eig_Acl) < 0.0))
    disp('Verification of H $\infty$  norm and pole location constraints successful!')
else
    disp('Verification of H $\infty$  norm and pole location constraints failed!')
end

```

```

% Define the lumped (disturbance + control) input
t = 0:0.01:10;
w_amplitude = 0.1;
w_duration = 1;
w_pulse = w_amplitude * (t >= 0 & t <= w_duration);
u = zeros(size(w_pulse));
w = [w_pulse; w_pulse];

```

```

% Simulate the closed-loop system response
[y_cl, t_out, x_cl] = lsim(Scl, w', t);

```

```

% Plot the results
figure;
sgtitle('Closed-Loop System Response');
subplot(3, 1, 1);
plot(t, w(2, :));
legend('W');
subplot(3, 1, 2);
plot(t, x_cl(:, 1), t, x_cl(:, 2), t, x_cl(:, 3));
legend('X1', 'X2', 'X3');
subplot(3, 1, 3);
plot(t, y_cl(:, 1), t, y_cl(:, 2));
legend('Y1', 'Y2');

```

OUTPUT:

```

D = 3x3
    0    0    0
    0    0    1
    0    1    0

Sol =

    A =
        x1  x2
    x1    0    1
    x2    0    0

```

```

B =
      u1  u2  u3
x1      0   0   0
x2      1   0   1

```

```

C =
      x1  x2
y1      0   1
y2      0   0
y3      1   0

```

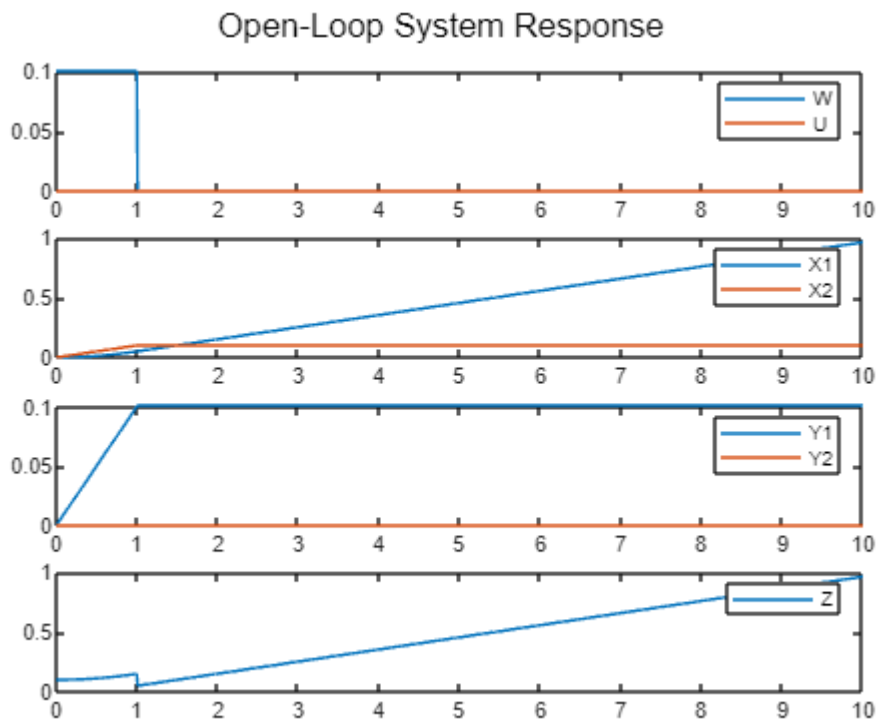
```

D =
      u1  u2  u3
y1      0   0   0
y2      0   0   1
y3      0   1   0

```

Continuous-time state-space model.

[Model Properties](#)



Minimization of gamma:

Solver for linear objective minimization under LMI constraints

Iterations : Best objective value so far

```

1
2          4.331487
3          2.480378
4          2.094585
5          1.984537

```

```

        6          1.984537
        7          1.891906
        8          1.891906
***          new lower bound:      0.208335
        9          1.693528
       10          1.693528
***          new lower bound:      0.773250
       11          1.636185
***          new lower bound:      1.240010
       12          1.636185
***          new lower bound:      1.567929
       13          1.622326
       14          1.620133
***          new lower bound:      1.604742

Result:  feasible solution of required accuracy
         best objective value:      1.620133
         guaranteed relative accuracy: 9.50e-03
         f-radius saturation: 0.399% of R = 1.00e+08

```

```

Optimal Hinf performance: 1.620e+00
gopt = 1.6197
G = 3x3

```

```

    -2.0572    1.5998    1.0000
     2.0785   -1.6165     0
         0         0   -Inf

```

H ∞ controller:

Ac = -2.0572

Bc = 1.5998

Cc = 2.0785

Dc = -1.6165

Closed-loop system:

Acl = 3x3

```

    0    1.0000     0
   -1.6165     0    2.0785
    1.5998     0   -2.0572

```

Bcl = 3x2

```

    0     0
   1.0000  -1.6165
    0     1.5998

```

Ccl = 2x3

```

    0    1.0000     0
   -1.6165     0    2.0785

```

Dcl = 2x2

```

    0     0
    0   -1.6165

```

H ∞ norm:

hinf_norm = 1.6185

Closed-loop poles:

eig_Acl = 3x1 complex

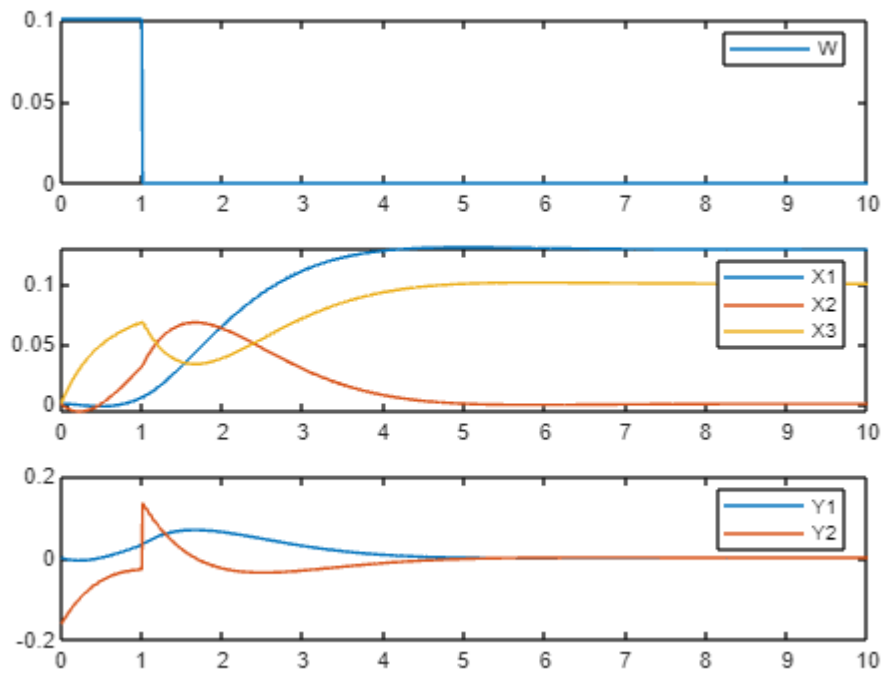
```

   -1.0285 + 0.7471i
   -1.0285 - 0.7471i
    -0.0002 + 0.0000i

```

Verification of H ∞ norm and pole location constraints successful!

Closed-Loop System Response



SCREENSHOT:

