Problem 2-A

CODE:

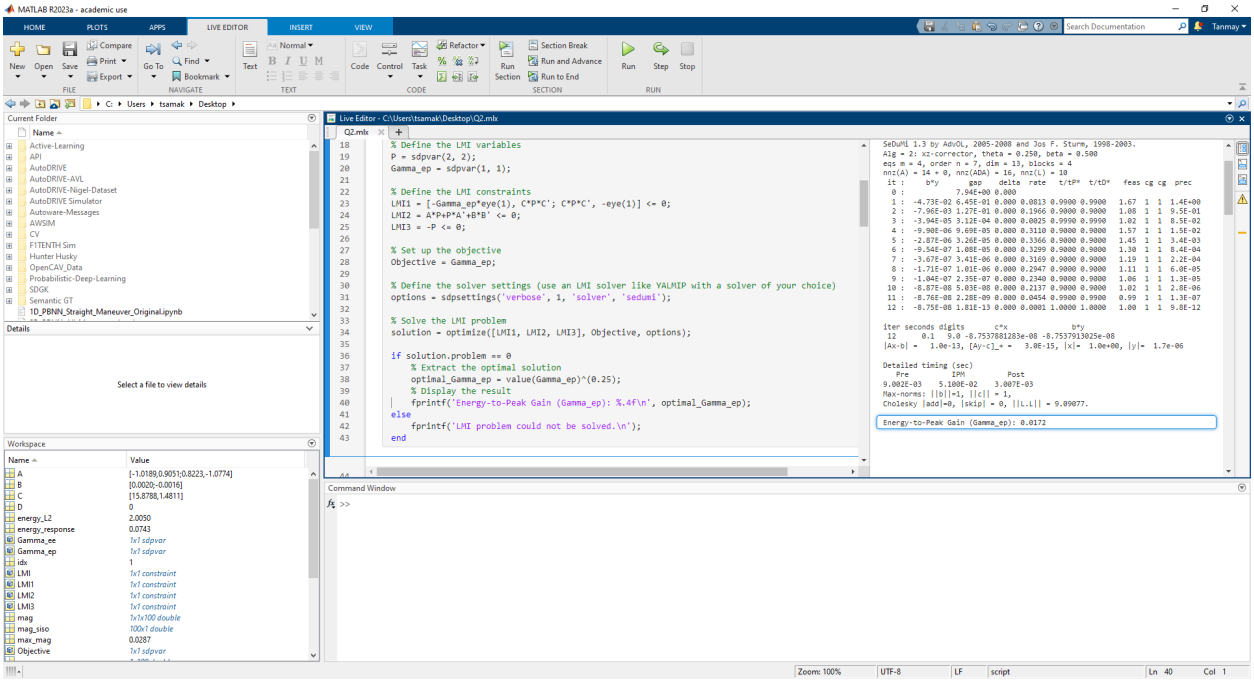
```
% PROBLEM 2-A


% Clear workspace
close all
clear
clc


% Add parser and solver to path
addpath(genpath('C:\Users\tsamak\Downloads\MathWorks\Toolboxes\archives\required\
YALMIP'))
addpath(genpath('C:\Users\tsamak\Downloads\MathWorks\Toolboxes\archives\required\
SeDuMi'))


% Define the system matrices A, B, and C
A = [-1.01887 0.90506; 0.82225 -1.07741];
B = [0.00203; -0.00164];
C = [15.87875, 1.48113];
D = 0;


% Define the LMI variables
P = sdpvar(2, 2);
Gamma_ep = sdpvar(1, 1);


% Define the LMI constraints
LMI1 = [-Gamma_ep*eye(1), C*P*C'; C*P*C', -eye(1)] <= 0;
LMI2 = A*P+P*A'+B*B' <= 0;
LMI3 = -P <= 0;


% Set up the objective
Objective = Gamma_ep;


% Define the solver settings (use an LMI solver like YALMIP with a solver of your
choice)
options = sdpsettings('verbose', 1, 'solver', 'sedumi');


% Solve the LMI problem
```

```
solution = optimize([LMI1, LMI2, LMI3], Objective, options);


if solution.problem == 0
    % Extract the optimal solution
    optimal_Gamma_ep = value(Gamma_ep)^(0.25);
    % Display the result
    fprintf('Energy-to-Peak Gain (Gamma_ep): %.4f\n', optimal_Gamma_ep);
else
    fprintf('LMI problem could not be solved.\n');
end
```

OUTPUT:

```
Energy-to-Peak Gain (Gamma_ep): 0.0172
```

SCREENSHOT:



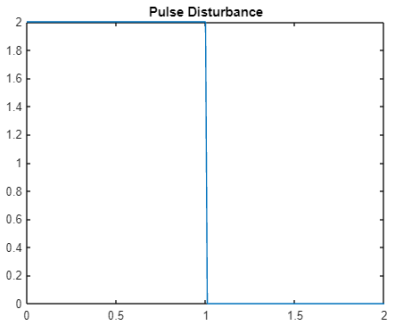
Problem 2-B

CODE:

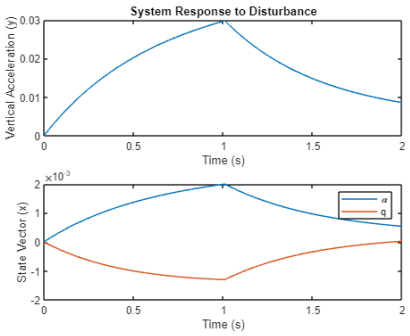
```
% PROBLEM 2-B


% Define the state-space system
A = [-1.01887 0.90506; 0.82225 -1.07741];
B = [0.00203; -0.00164];
C = [15.87875 1.48113];
D = 0;


% Define the disturbance signal w_g(t)
t = 0:0.01:2;  % Time vector from 0 to 2 with a step size of 0.01
w_g = 2 * (t >= 0 & t <= 1);  % Pulse disturbance, 2 for 0 ≤ t ≤ 1, 0 otherwise
plot(t,w_g)
title('Pulse Disturbance')


% Calculate the L2-norm (energy) of the disturbance signal
energy_L2 = sqrt(trapz(t, w_g.^2));
disp(['Energy of the disturbance signal: ', num2str(energy_L2)]);


% Simulate the system response
sys = ss(A, B, C, D);
[y, t, x] = lsim(sys, w_g, t);


% Estimate the energy of the response to a pulse disturbance
energy_response = sqrt(trapz(t, y.^2)); % L2 norm


fprintf('Estimated energy of system response: %f\n', energy_response);


if optimal_Gamma_ep >= energy_response
    fprintf('System response is consistent with Gamma_ep.\n');
else
    fprintf('System response is not consistent with Gamma_ep.\n');
end


% Plot the response
subplot(2, 1, 1);
plot(t, y);
```

```
xlabel('Time (s)');
ylabel('Vertical Acceleration (y)');
title('System Response to Disturbance');
subplot(2, 1, 2);
plot(t, x);
xlabel('Time (s)');
ylabel('State Vector (x)');
legend('\alpha', 'q');
```

OUTPUT:



Disturbance Signal

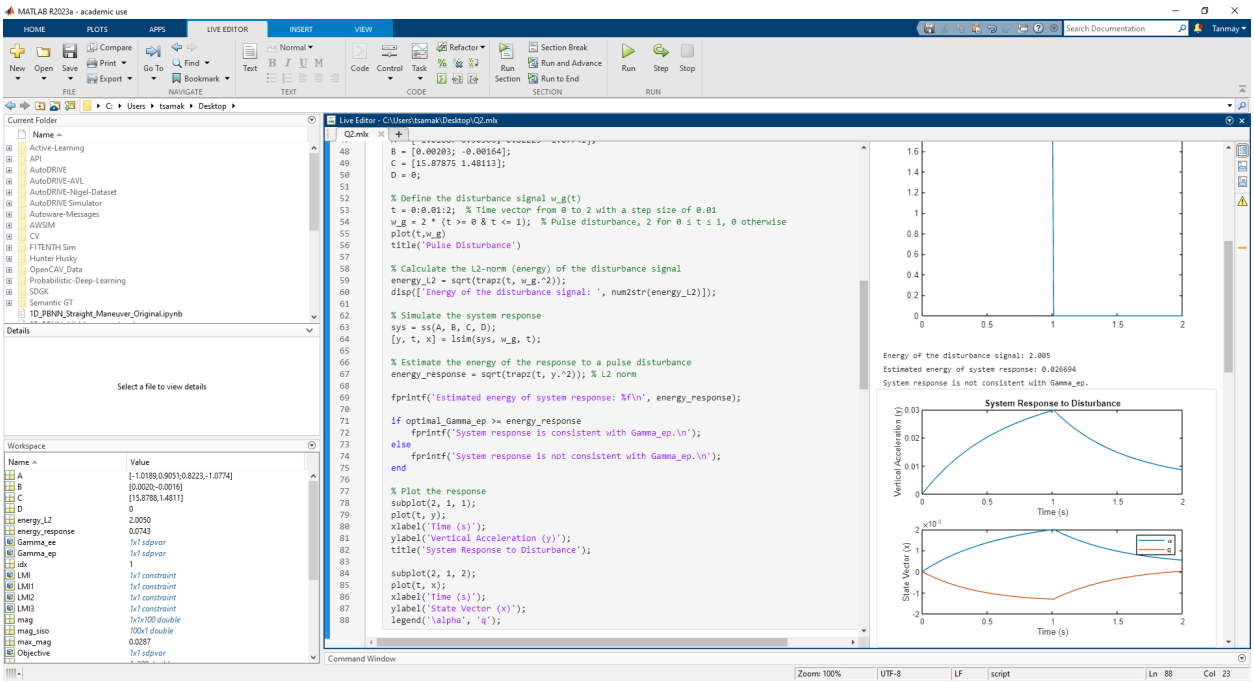


System Response

```
Energy of the disturbance signal: 2.005
Estimated energy of system response: 0.026694
System response is not consistent with Gamma_ep.
```

The inconsistency indicates that the system is not very robust to given disturbance.

SCREENSHOT:



Problem 2-C

CODE:

```
% PROBLEM 2-C


% Define the system matrices A, B, and C
A = [-1.01887 0.90506; 0.82225 -1.07741];
B = [0.00203; -0.00164];
C = [15.87875, 1.48113];
D = 0;


% Define the LMI variables
P = sdpvar(2, 2);
Gamma_ee = sdpvar(1, 1);


% Define the LMI constraints
LMI = [A'*P + P*A, P*B, C'; B'*P, -Gamma_ee*eye(1), D'; C, D, -Gamma_ee*eye(1)]
<= 0;


% Set up the objective
Objective = Gamma_ee;


% Define the solver settings (use an LMI solver like YALMIP with a solver of your
choice)
options = sdpsettings('verbose', 1, 'solver', 'sedumi');


% Solve the LMI problem
solution = optimize(LMI, Objective, options);


if solution.problem == 0
    % Extract the optimal solution
    optimal_Gamma_ee = value(Gamma_ee);

    % Estimate the energy of the response to a pulse disturbance (adjust the time
horizon as needed)
    T = 10; % Adjust the time horizon as needed
    sys = ss(A, B, C, 0);
    t = 0:0.01:T;
    [y, t] = lsim(sys, ones(size(t)), t); % Pulse disturbance input is ones
```

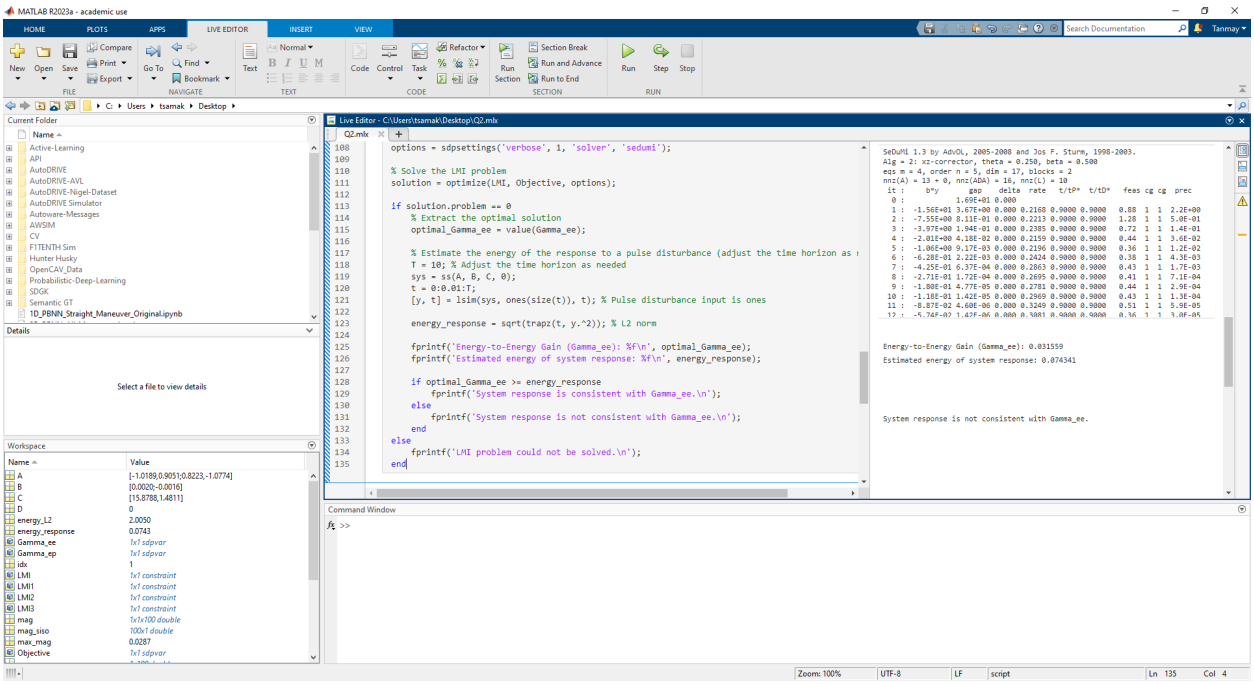
```
    energy_response = sqrt(trapz(t, y.^2)); % L2 norm

    fprintf('Energy-to-Energy Gain (Gamma_ee): %f\n', optimal_Gamma_ee);
    fprintf('Estimated energy of system response: %f\n', energy_response);

    if optimal_Gamma_ee >= energy_response
        fprintf('System response is consistent with Gamma_ee.\n');
    else
        fprintf('System response is not consistent with Gamma_ee.\n');
    end
else
    fprintf('LMI problem could not be solved.\n');
end
```

OUTPUT:

```
Energy-to-Energy Gain (Gamma_ee): 0.031559
Estimated energy of system response: 0.074341
System response is not consistent with Gamma_ee.
```

The inconsistency indicates that the system is not very robust to given disturbance.

SCREENSHOT:



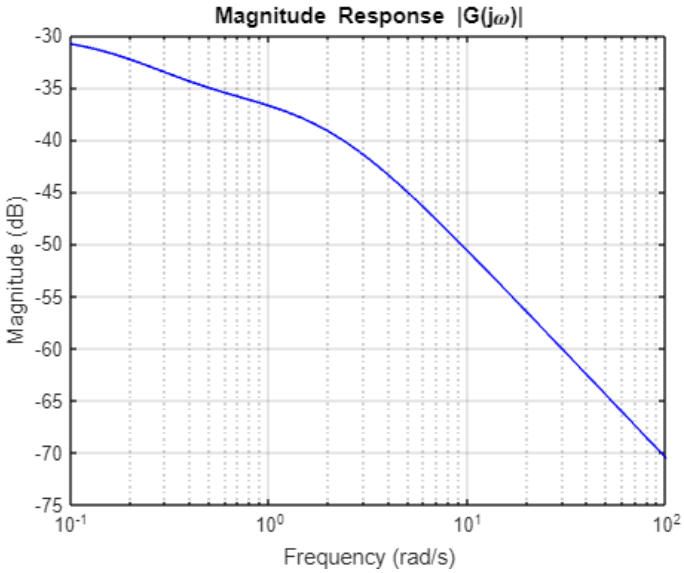
Problem 2-D

CODE:

```
% PROBLEM 2-D


% Define the state-space system
A = [-1.01887 0.90506; 0.82225 -1.07741];
B = [0.00203; -0.00164];
C = [15.87875 1.48113];
D = 0;
sys = ss(A, B, C, D);


% Define a range of frequencies
omega = logspace(-1, 2, 100);  % Adjust the frequency range as needed


% Calculate the frequency response of the system
[mag, ~, ~] = bode(sys, omega);


% Extract the magnitude for the SISO system
mag_siso = squeeze(mag);


% Find the peak magnitude and its corresponding frequency
[max_mag, idx] = max(mag_siso);
peak_freq = omega(idx);


% Plot the magnitude response
figure;
semilogx(omega, 20*log10(mag_siso), 'b');  % Plot in decibels
title('Magnitude Response |G(j\omega)|');
xlabel('Frequency (rad/s)');
ylabel('Magnitude (dB)');
grid on;


% Display the peak magnitude and frequency
fprintf('Peak Magnitude (dB): %.4f dB at Frequency %.4f rad/s\n',
20*log10(max_mag), peak_freq);
fprintf('Energy-to-Energy Gain (Gamma_ee): %.4f\n', max_mag);


if abs(optimal_Gamma_ee-max_mag) <= 0.01
```

```
    fprintf('Peak is equivalent to Gamma_ee.\n');
else
    fprintf('Peak is not equivalent to Gamma_ee.\n');
end
```

OUTPUT:




```
Peak Magnitude (dB): -30.8437 dB at Frequency 0.1000 rad/s
Energy-to-Energy Gain (Gamma_ee): 0.0287
Peak is equivalent to Gamma_ee.
```

SCREENSHOT: