* Problem 1:

Based on the given plots,

$$f_1(t) = \begin{cases} 4t & 0 \leq t \leq 1 \\ -4(t-2) & 1 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

$$f_2(t) = \begin{cases} 2 & 0 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_3(t) = \begin{cases} -1 & 0 \leq t \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

where, $$f(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix}$$

L1 Norm:

$$\|f(t)\|_{L1} = \int_{-\infty}^{\infty} \sum_{i=1}^{n} |f_i(t)|\,dt$$

$$= \int_{-\infty}^{\infty} \sum_{i=1}^{3} |f_i(t)|\,dt \quad \text{since } n=3 \text{ in our case}$$

$$= \cancelto{0}{\int_{-\infty}^{0} \sum_{i=1}^{3} |f_i(t)|\,dt} + \int_{0}^{1} \sum_{i=1}^{3} |f_i(t)|\,dt$$

$$+ \int_{1}^{2} \sum_{i=1}^{3} |f_i(t)|\,dt + \cancelto{0}{\int_{2}^{\infty} \sum_{i=1}^{3} |f_i(t)|\,dt}$$

$$= 0 + \int_{0}^{1} (4t + 2 + 1)\,dt + \int_{1}^{2} -4(t-2) + 1\,dt + 0$$

$$= \int_{0}^{1} (4t+3)\,dt + \int_{1}^{2} -4t + 8 + 1\,dt$$

$$= \int_{0}^{1} (4t+3)\,dt + \int_{1}^{2} (-4t+9)\,dt$$

$$= \left[\cancelto{2}{4}t^2/\cancel{2} + 3t\right]_0^1 - \left[\cancel{4}t^2/\cancel{2} - 9t\right]_1^2$$

$$= \left[2t^2 + 3t\right]_0^1 - \left[2t^2 - 9t\right]_1^2$$

$$\cancel{= (3-0) - [(8-14)-(2-7)]} = (2+3-0) -$$

$$[(8-18)-(2-9)]$$

$$\|f(t)\|_{L1} = 5 - [(-10) - (-7)]$$

$$= 5 - (-3)$$

$$= 8$$

L2 Norm :

$$\|f(t)\|_{L2} = \left(\int_{-\infty}^{\infty} \sum_{i=1}^{n} |f_i(t)|^2 \, dt \right)^{1/2}$$

$$= \left[\left(\int_{-\infty}^{0} \sum_{i=1}^{3} |f_i(t)|^2 \, dt \right) + \left(\int_{0}^{1} \sum_{i=1}^{3} |f_i(t)|^2 \, dt \right) + \left(\int_{1}^{2} \sum_{i=1}^{3} |f_i(t)|^2 \, dt \right) + \left(\int_{2}^{\infty} \sum_{i=1}^{3} |f_i(t)|^2 \, dt \right) \right]^{1/2}$$

$$= \left(0 + \int_{0}^{1} (4t)^2 + 2^2 + (-1)^2 \, dt + \int_{1}^{2} (-4t+8)^2 + (-1)^2 \, dt + 0 \right)^{1/2}$$

$$= \left(\int_{0}^{1} (16t^2 + 4 + 1) \, dt + \int_{1}^{2} (16t^2 + 64 - 64t + 1) \, dt \right)^{1/2}$$

$$= \left(\int_{0}^{1} (16t^2 + 5) \, dt + \int_{1}^{2} (16t^2 - 64t + 65) \, dt \right)^{1/2}$$

$$\|f(t)\|_{L2} = \left(\left[16\frac{t^3}{3} + 5t\right]_0^1 + \left[16\frac{t^3}{3} - 64\frac{t^2}{2} + 65t\right]_1^2 \right)^{1/2}$$

$$= \left[(16/3 + 5) + (128/3 - 256/2 + 130) - (16/3 - 64/2 + 65)\right]^{1/2}$$

$$= \left[(10.3334) + (42.6667 + 2) - (5.3334 + 33)\right]^{1/2}$$

$$= \left[(10.3334) + (44.6667) - (38.3334)\right]^{1/2}$$

$$= (16.6667)^{1/2}$$

$$= 4.0825$$

<u>L_∞ Norm:</u>

$$\|f(t)\|_{L\infty} = \max_{-\infty < t < \infty} (|f_i(t)|)$$

$$= \max\left(\max_{-\infty < t \leq 0} (|f_i(t)|), \max_{0 \leq t \leq 1} (|f_i(t)|), \max_{1 \leq t \leq 2} (|f_i(t)|), \max_{2 \leq t < \infty} (|f_i(t)|) \right)$$

$$= \max\left(0, \max_{0 \leq t \leq 1} (4t, 2, 1), \max_{1 \leq t \leq 2} (8-4t, 0, 1), 0\right)$$

$$= \max(0, 4, 4, 0)$$

$$= 4$$