Assignment 4: ME 8930 (LMIs in Optimal and Robust Control)

Due on Nov. 27, 2023 by midnight

Problem 1: Consider the following unstable system

$$\begin{aligned}
\dot{x} &= \begin{bmatrix} 0 & 10 & 2 \\ -1 & 1 & 0 \\ 0 & 2 & -5 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} w \\
y &= \begin{bmatrix} x_1 \\ u \end{bmatrix} \\
z &= x_2 + 2w.
\end{aligned}$$

(1) Design an H_∞ dynamic controller to minimize the H_∞ norm of the closed-loop system from w to y and at the same time place all closed-loop poles to the left of a vertical line with real coordinate at -0.2 in the complex plane.

(2) Verify that the closed-loop system meets the designed H_∞ norm and pole location constraints.

Problem 2: Consider a mechanical structure modelled as a three mass-spring interconnection where u is the force (control) input. Assuming unit masses, the state-space representation of the system is $\dot{x} = Ax + Bu$ where

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -k_1 & k_1 & 0 & 0 & 0 & 0 \\ k_1 & -k_1 - k_2 & k_2 & 0 & 0 & 0 \\ 0 & k_2 & -k_2 & 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

The stiffness values k_1 and k_2 are variable with the nominal values of $k_1 = k_2 = 1$, and they vary in the interval $[1 - \alpha, 1 + \alpha]$ where the allowable perturbation is $\alpha = 0.1$.

(1) Confirm that the open-loop uncertain system is not quadratically stable. Notice that the uncertain system can be considered as an affine system since matrix A can be written as $A = A_0 + k_1 A_1 + k_2 A_2$ (Use "quadstab" in MATLAB).

(2) Design an LQR stabilizing controller for the nominal system using the following MATLAB command:

$$>> K = -lqr(A, B, eye(6), 1) \quad \text{(LQR control with unit weights)}$$

Consider the state-feedback control law $u = Kx$ and compute the closed-loop system. Then, determine if the uncertain closed-loop system is quadratically stable or not when the stiffness vary in the interval as above.

(3) Determine the maximum region of quadratic stability of the closed-loop uncertain system, that is, find the maximum $\alpha = \alpha_{max}$ such that the above uncertain system is quadratically stable for all stiffness perturbations in the interval $[1 - \alpha_{max}, 1 + \alpha_{max}]$. Confirm that the closed-loop system is stable for all upper and lower bounds of the stiffness values $k_i = 1 - \alpha_{max}$ and $k_i = 1 + \alpha_{max}$ ($i = 1, 2$), i.e., confirm stability for all four corner closed-loop systems.

Problem 3: Consider the following system that corresponds to the rigid body motion of a satellite:

$$
\begin{aligned}
\dot{x}_p &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} w \\
y &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\
z &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_p + \begin{bmatrix} 0 & 1 \end{bmatrix} w.
\end{aligned}
$$

Notice that in the above formulation, the system output vector y includes the angular velocity and the control input u. The disturbance input $w(t)$ consists of a plant disturbance and a measurement disturbance signal (and so, it is a vector).

(1) Simulate the response of the open-loop system (i.e., with $u = 0$) to a pulse disturbance w of amplitude 0.1 and duration of 1 sec (for both the plant and the measurement components).

(2) Use "hinflmi" in MATLAB to design a dynamic controller that minimizes the energy-to-energy gain of the closed-loop system.

(3) Compute the closed-loop system and examine/validate the closed-loop system stability and performance.

(4) Simulate the closed-loop system output y to a pulse disturbance w of amplitude 0.1 and duration of 1 sec (same input as part (1)).

NOTE: Please include your MATLAB (or Python) files and outputs.