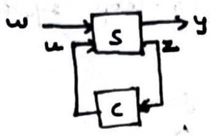
$$\dot{x}_p = A_p x_p + B_p u + D_p w$$

$$y = C_p x_p + B_y u + D_y w$$

$$z = M_p x_p + D_z w$$



Comparing with given $\overbrace{\text{system}}^{\text{F8 Aircraft}}$ equations:

$$A_p = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1.5 & -1.5 & 0 & 0.0057 & 1.5 & 0 & 0 & 0 \\ -12 & 12 & -0.6 & -0.0344 & -12 & 0 & 0 & 0 \\ -0.852 & 0.29 & 0 & -0.014 & -0.29 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.73 & 2.8294625 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1000 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1000 \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0 & 0 \\ 0.16 & 0.8 \\ -19 & -3 \\ -0.0115 & -0.0087 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad D_p = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.1149 & 0 & 0 \\ 4 & 0 & 0 \\ 0 & 1024 & 0 \\ 0 & 0 & 1024 \end{bmatrix}$$

$$C_p = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B_y = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} \quad D_y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_p = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -139.020647321 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & -139.020647321 \end{bmatrix} \quad D_z = \begin{bmatrix} 0 & 142.857142857 & 0 \\ 0 & 0 & 142.857142857 \end{bmatrix}$$

Lumped system matrices:

$A = A_p$

$B = \begin{bmatrix} D_p & B_p \end{bmatrix}$

$C = \begin{bmatrix} C_p & M_p \end{bmatrix}$

$D = \begin{bmatrix} D_y & B_y \\ D_z & 0 \end{bmatrix}$

Open-loop system:

$S_{OL} = ss(A, B, C, D)$

Input to open-loop system:

- $w(t) = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \delta(t)$ unit impulse disturbance

where $\delta(t) = \begin{cases} 1 \; ; & 0 \leq t \leq 0.01 \\ 0 \; ; & t > 0.01 \end{cases}$ Δt

- $u(t) = 0 \quad \forall t$

Open-loop system simulation:

- Simulate open-loop system S_{OL} using "lsim" for $t = 0$ to $t = 10$ sec. with $\Delta t = 0.01$ sec.
- Plot $w(t)$ vs. t ← wind gust disturbance (input)
  Plot $y_1(t)$ vs. t ← aircraft pitch angle (output)

Full order optimal H_∞ dynamic controller design

$S = \text{ltisys}(A, B, C, D)$

$n_z = \text{size}(M_p, 1)$

$n_u = \text{size}(B_p, 2)$

$r = [n_z \;\; n_u]$

$[\text{gopt}, G] = \text{hinflmi}(S, r)$

$\Gamma_{ee} < \gamma^*$ controller

$[A_c, B_c, C_c, D_c] = \text{ltiss}(G)$

Closed-loop system:

$$A_{CL} = \begin{bmatrix} A_p + B_p D_c M_p & B_p C_c \\ B_c M_p & A_c \end{bmatrix} \qquad B_{CL} = \begin{bmatrix} D_p + B_p D_c D_z \\ B_c D_z \end{bmatrix}$$

$$C_{CL} = \begin{bmatrix} C_p + B_y D_c M_p & B_y C_c \end{bmatrix} \qquad D_{CL} = D_y + B_y D_c D_z$$

$$S_{CL} = ss(A_{CL}, B_{CL}, C_{CL}, D_{CL})$$

Design Validation:

- Stability : Eigenvalues (Poles) : $Re(\lambda_i(A_{cl})) < 0$ ← closed loop system stability guaranteed

- Performance: H_∞ norm (Γ_{ee}) : Γ_{ee} = hinfnorm(S_{cl})

  $\underset{\text{hinf\_norm}}{\Gamma_{ee}} < \underset{\text{gopt}}{\gamma^*}$ ← optimal (guaranteed) H_∞ performance

Input to closed-loop system:

- $w(t) = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix} \delta(t)$ ← unit impulse signal

  where $\delta(t) = \begin{cases} 1; & 0 \le t \le \underbrace{0.01}_{\Delta t} \\ 0; & t > 0.01 \end{cases}$

Closed-loop system simulation:

- Simulate closed-loop system S_{cl} using "lsim" for $t = 0$ to $t = 10$ sec. with $\Delta t = 0.01$ sec.

- Plot $w(t)$ vs. t ← wind gust disturbance (input)

  Plot $y_1(t)$ vs. t ← aircraft pitch angle (output)

Problem 3

CODE:

```
% PROBLEM 3


% Clear workspace
close all
clear
clc


% Define the system matrices
Ap = [0 0 1 0 0 0 0 0;
      1.5 -1.5 0 0.0057 1.5 0 0 0;
      -12 12 -0.6 -0.0344 -12 0 0 0;
      -0.825 0.29 0 -0.014 -0.29 0 0 0;
      0 0 0 0 -0.73 2.82940625 0 0;
      0 0 0 0 0 -1.25 0 0;
      0 0 0 0 0 0 -1000 0;
      0 0 0 0 0 0 0 -1000];
Bp = [0 0;
      0.16 0.8;
      -19 -3;
      -0.0115 -0.0087;
      0 0;
      0 0;
      0 0;
      0 0];
Dp = [0 0 0;
      0 0 0;
      0 0 0;
      0 0 0;
      0.1149 0 0;
      4 0 0;
      0 1024 0
      0 0 1024];
Cp = [1 0 0 0 0 0 0 0;
      0 1 0 0 0 0 0 0;
      0 0 0 0 0 0 0 0;
      0 0 0 0 0 0 0 0];
By = [0 0;
      0 0;
      0.01 0;
      0 0.01];
Dy = [0 0 0;
```

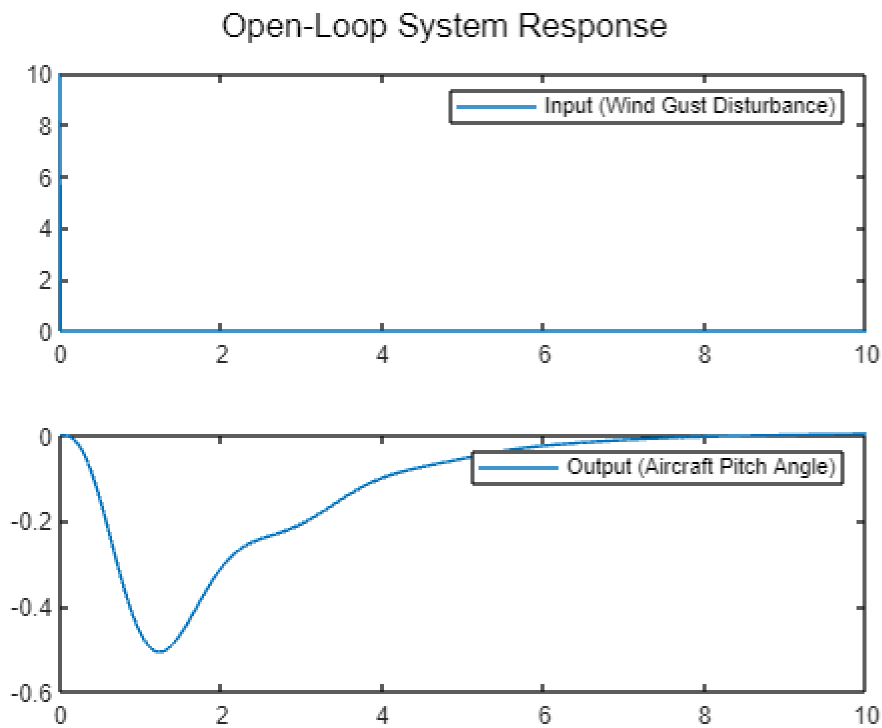
```
      0 0 0;
      0 0 0;
      0 0 0];
Mp = [1 0 0 0 0 0 -139.020647321 0
      0 1 0 0 0 0 0 -139.020647321];
Dz = [0 142.857142857 0
      0 0 142.857142857];


% Lumped system matrices
A = Ap;
B = [Dp Bp];
C = [Cp; Mp];
D = [Dy By; Dz zeros(size(Dz,1), size(By,2))];


% State-space system
Sol = ss(A, B, C, D)


% Define the lumped (disturbance + control) input
t = 0:0.01:10;
w_amplitude = [10; 0; 0];
w_duration = 0.01;
w_impulse = w_amplitude * (t >= 0 & t < w_duration);
u = zeros(size(Bp,2), size(w_impulse,2));
w = [w_impulse; u];


% Simulate the open-loop system response
[y_ol, t_out, x_ol] = lsim(Sol, w', t);


% Plot the results
figure;
sgtitle('Open-Loop System Response');
subplot(2, 1, 1);
plot(t, w(1, :));
legend('Input (Wind Gust Disturbance)');
subplot(2, 1, 2);
plot(t, y_ol(:, 1));
legend('Output (Aircraft Pitch Angle)');


% H∞ controller design
S = ltisys(A, B, C, D);
nz = size(Mp, 1);
nu = size(Bp, 2);
```

```
r = [nz nu];
[gopt, G] = hinflmi(S, r)


% H∞ controller matrices
disp('H∞ controller:')
[Ac, Bc, Cc, Dc] = ltiss(G)


% Closed-loop system matrices
disp('Closed-loop system:')
Acl = [Ap+Bp*Dc*Mp, Bp*Cc; Bc*Mp, Ac]
Bcl = [Dp+Bp*Dc*Dz; Bc*Dz]
Ccl = [Cp+By*Dc*Mp, By*Cc]
Dcl = Dy+By*Dc*Dz


% Closed-loop system
Scl = ss(Acl, Bcl, Ccl, Dcl);


% Verification of designed H∞ controller
disp('H∞ norm:')
hinf_norm = hinfnorm(Scl)
disp('Closed-loop poles:')
eig_Acl = eig(Acl)
if((hinf_norm < gopt) && all(real(eig_Acl) < 0.0))
    disp('Verification of H∞ norm and stability constraints successful!')
else
    disp('Verification of H∞ norm and stability constraints failed!')
end


% Define the lumped (disturbance + control) input
t = 0:0.01:10;
w_amplitude = [10; 0; 0];
w_duration = 0.01;
w_impulse = w_amplitude * (t >= 0 & t < w_duration);
w = w_impulse;


% Simulate the open-loop system response
[y_cl, t_out, x_cl] = lsim(Scl, w', t);


% Plot the results
figure;
sgtitle('Closed-Loop System Response');
subplot(2, 1, 1);
```

```
plot(t, w(1, :));
legend('Input (Wind Gust Disturbance)');
subplot(2, 1, 2);
plot(t, y_cl(:, 1));
legend('Output (Aircraft Pitch Angle)');
```

OUTPUT:

```
Sol =

  A = 
            x1       x2       x3       x4       x5       x6       x7       x8
   x1        0        0        1        0        0        0        0        0
   x2      1.5     -1.5        0   0.0057      1.5        0        0        0
   x3      -12       12     -0.6  -0.0344      -12        0        0        0
   x4   -0.825     0.29        0   -0.014    -0.29        0        0        0
   x5        0        0        0        0    -0.73    2.829        0        0
   x6        0        0        0        0        0    -1.25        0        0
   x7        0        0        0        0        0        0    -1000        0
   x8        0        0        0        0        0        0        0    -1000

  B = 
            u1       u2       u3       u4       u5
   x1        0        0        0        0        0
   x2        0        0        0     0.16      0.8
   x3        0        0        0      -19       -3
   x4        0        0        0  -0.0115  -0.0087
   x5   0.1149        0        0        0        0
   x6        4        0        0        0        0
   x7        0     1024        0        0        0
   x8        0        0     1024        0        0

  C = 
         x1    x2    x3    x4    x5    x6    x7    x8
   y1     1     0     0     0     0     0     0     0
   y2     0     1     0     0     0     0     0     0
   y3     0     0     0     0     0     0     0     0
   y4     0     0     0     0     0     0     0     0
   y5     1     0     0     0     0     0  -139     0
   y6     0     1     0     0     0     0     0  -139

  D = 
          u1     u2     u3     u4     u5
   y1      0      0      0      0      0
   y2      0      0      0      0      0
   y3      0      0      0   0.01      0
   y4      0      0      0      0   0.01
   y5      0  142.9      0      0      0
   y6      0      0  142.9      0      0

Continuous-time state-space model.
```

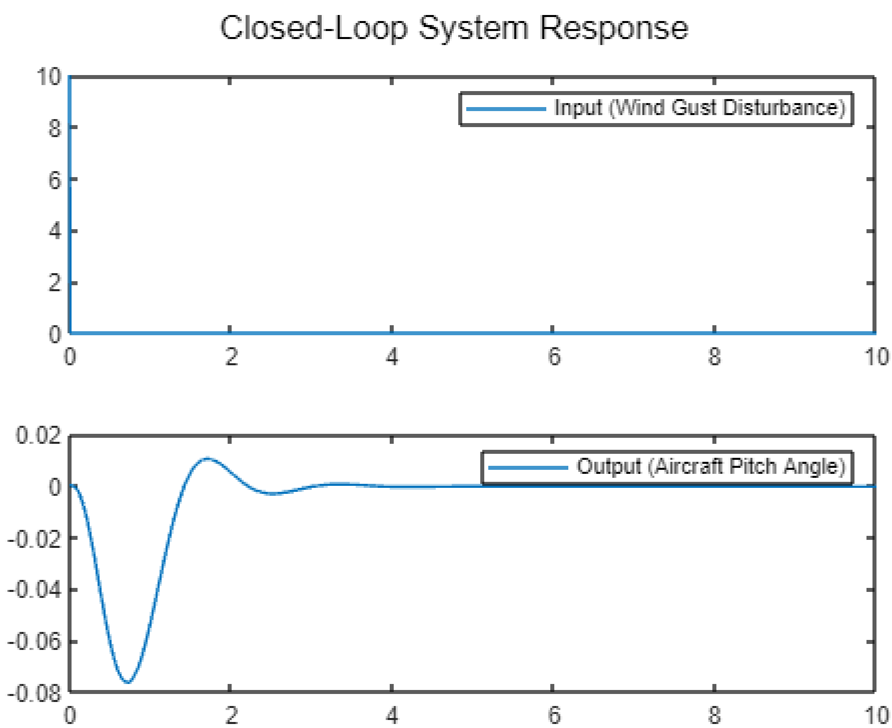



```
Minimization of gamma:

Solver for linear objective minimization under LMI constraints

Iterations    :    Best objective value so far

     1
     2
     3                 3650.305369
     4                  679.874263
     5                  415.718365
     6                  196.917091
     7                  124.993308
     8                  124.993308
     9                   39.263120
    10                   32.197975
    11                   21.790789
    12                   21.790789
    13                    4.993262
    14                    4.993262
    15                    2.508027
    16                    1.610124
    17                    1.610124
    18                    1.224387
    19                    1.224387
    20                    1.224387
    21                    1.003425
    22                    1.003425
    23                    1.003425
    24                    0.941879
```

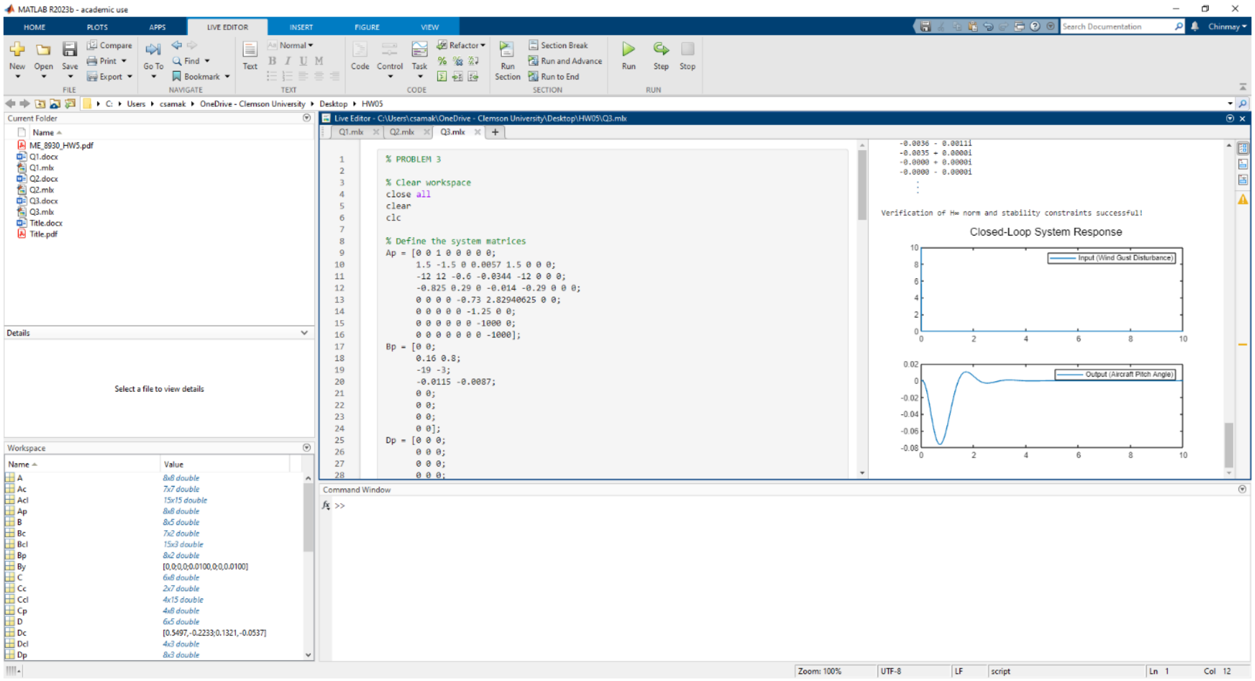
```
    25                   0.941879
    26                   0.941879
    27                   0.941879
    28                   0.912216
    29                   0.912216
    30                   0.912216
    31                   0.896765
    32                   0.896765
    33                   0.896765
***                 new lower bound:     0.820170
    34                   0.896765
    35                   0.887844
    36                   0.887844
    37                   0.887844
***                 new lower bound:     0.867568
    38                   0.884997
    39                   0.883333
* switching to QR
    40                   0.883333
***                 new lower bound:     0.877811

 Result:  feasible solution of required accuracy
          best objective value:     0.883333
          guaranteed absolute accuracy:  5.52e-03
          f-radius saturation:  0.458% of R =  1.00e+08

 Optimal Hinf performance:  8.823e-01
gopt = 0.8823
G = 10×10
   77.0684 -206.4493  -45.4703  -89.5611  -35.5194  -12.4998  106.3172   -
2.3242    3.8637    7.0000
 -220.1100  183.7132    2.3812  116.5260   57.5111    1.5305 -226.7402
4.7605   -1.7312         0
  397.2066 -510.2170  -63.3345 -269.2750 -121.1621  -18.7382  432.1665   -
5.0789   -3.3582         0
    5.7829   -3.5433    1.5761   -4.9621    1.2446    0.7772    7.5619
1.3169    3.6942         0
  293.4949 -341.9114  -33.2695 -188.0955  -88.9407  -10.9885  313.4063   -
1.8555    1.3533         0
   -7.7820    8.1529    0.3504    4.6494    2.2216    0.1100   -8.8267   -
0.2847   -0.0942         0
 -263.0885  307.1698   29.7766  169.0752   79.4155    9.7083 -282.4749
0.6617   -0.6630         0
  -60.3310  101.1714   16.6276   48.7081   21.1076    4.7857  -70.3890
0.5497   -0.2233         0
 -102.4696  -63.6786  -48.4705    1.8408   11.4945  -12.2981  -80.1751
0.1321   -0.0537         0
         0         0         0         0         0         0         0
0         0      -Inf
H∞ controller:
Ac = 7×7
   77.0684 -206.4493  -45.4703  -89.5611  -35.5194  -12.4998  106.3172
 -220.1100  183.7132    2.3812  116.5260   57.5111    1.5305 -226.7402
  397.2066 -510.2170  -63.3345 -269.2750 -121.1621  -18.7382  432.1665
    5.7829   -3.5433    1.5761   -4.9621    1.2446    0.7772    7.5619
  293.4949 -341.9114  -33.2695 -188.0955  -88.9407  -10.9885  313.4063
   -7.7820    8.1529    0.3504    4.6494    2.2216    0.1100   -8.8267
```

```
 -263.0885  307.1698   29.7766  169.0752   79.4155    9.7083 -282.4749
Bc = 7×2
   -2.3242    3.8637
    4.7605   -1.7312
   -5.0789   -3.3582
    1.3169    3.6942
   -1.8555    1.3533
   -0.2847   -0.0942
    0.6617   -0.6630
Cc = 2×7
  -60.3310  101.1714   16.6276   48.7081   21.1076    4.7857  -70.3890
 -102.4696  -63.6786  -48.4705    1.8408   11.4945  -12.2981  -80.1751
Dc = 2×2
    0.5497   -0.2233
    0.1321   -0.0537
Closed-loop system:
Acl = 15×15
10^3 ×
         0         0    0.0010         0         0         0         0
0         0         0         0         0         0         0         0
    0.0017   -0.0016         0    0.0000    0.0015         0   -0.0269
0.0109   -0.0916   -0.0348   -0.0361    0.0093    0.0126   -0.0091   -0.0754
   -0.0228    0.0164   -0.0006   -0.0000   -0.0120         0    1.5070   -
0.6121    1.4537   -1.7312   -0.1705   -0.9310   -0.4355   -0.0540    1.5779
   -0.0008    0.0003         0   -0.0000   -0.0003         0    0.0010   -
0.0004    0.0016   -0.0006    0.0002   -0.0006   -0.0003    0.0001    0.0015
         0         0         0         0   -0.0007    0.0028         0
0         0         0         0         0         0         0         0
         0         0         0         0         0   -0.0013         0
0         0         0         0         0         0         0         0
         0         0         0         0         0         0   -1.0000
0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0   -
1.0000         0         0         0         0         0         0         0
   -0.0023    0.0039         0         0         0         0    0.3231   -
0.5371    0.0771   -0.2064   -0.0455   -0.0896   -0.0355   -0.0125    0.1063
    0.0048   -0.0017         0         0         0         0   -0.6618
0.2407   -0.2201    0.1837    0.0024    0.1165    0.0575    0.0015   -0.2267
Bcl = 15×3
10^3 ×
         0         0         0
         0    0.0277   -0.0112
         0   -1.5486    0.6290
         0   -0.0011    0.0004
    0.0001         0         0
    0.0040         0         0
         0    1.0240         0
         0         0    1.0240
         0   -0.3320    0.5520
         0    0.6801   -0.2473
Ccl = 4×15
    1.0000         0         0         0         0         0         0
0         0         0         0         0         0         0         0
         0    1.0000         0         0         0         0         0
0         0         0         0         0         0         0         0
    0.0055   -0.0022         0         0         0         0   -0.7642
0.3104   -0.6033    1.0117    0.1663    0.4871    0.2111    0.0479   -0.7039
```

```
    0.0013   -0.0005         0         0         0         0   -0.1836
0.0746   -1.0247   -0.6368   -0.4847    0.0184    0.1149   -0.1230   -0.8018
Dcl = 4×3
         0         0         0
         0         0         0
         0    0.7852   -0.3190
         0    0.1887   -0.0766
H∞ norm:
hinf_norm = 0.8813
Closed-loop poles:
eig_Acl = 15×1 complex
10^3 ×
  -0.1048 + 0.0000i
  -0.0311 + 0.0301i
  -0.0311 - 0.0301i
  -0.0016 + 0.0038i
  -0.0016 - 0.0038i
  -0.0036 + 0.0011i
  -0.0036 - 0.0011i
  -0.0035 + 0.0000i
  -0.0000 + 0.0000i
  -0.0000 - 0.0000i
Verification of H∞ norm and stability constraints successful!
```



SCREENSHOT:


MATLAB R2023b - academic use
Live Editor - C:\Users\csamak\OneDrive - Clemson University\Desktop\HW05\Q3.mlx
% PROBLEM 3
% Clear workspace
close all
clear
clc
% Define the system matrices
Ap = [0 0 1 0 0 0 0 0;
1.5 -1.5 0 0.0057 1.5 0 0 0;
-12 12 -0.6 -0.0344 -12 0 0 0;
-0.825 0.29 0 -0.014 -0.29 0 0 0;
0 0 0 0 -0.73 2.82940625 0 0;
0 0 0 0 0 -1.25 0 0;
0 0 0 0 0 0 -1000 0;
0 0 0 0 0 0 0 -1000];
Bp = [0 0;
0.16 0.8;
-19 -3;
-0.0115 -0.0087;
0 0;
0 0;
0 0;
0 0];
Dp = [0 0 0;
0 0 0;
0 0 0;
0 0 0;
-0.0036 - 0.0011i
-0.0035 + 0.0000i
-0.0000 + 0.0000i
-0.0000 - 0.0000i
Verification of H∞ norm and stability constraints successful!
Closed-Loop System Response
Input (Wind Gust Disturbance)
Output (Aircraft Pitch Angle)
Command Window