Assignment 3: ME 8930 (LMIs in Optimal and Robust Control)

Due on Nov. 13, 2023 by midnight

Problem 1: Consider the following linear uncertain system

$$\begin{aligned} \dot{x} &= Ax + K\Phi \qquad (1) \\ \Psi &= Mx + H\Phi \qquad (2) \end{aligned}$$

with the uncertainty interconnection $\Phi = \Delta\Psi$. Use the LMI-based representation of the Small Gain Theorem (SGT) to answer the questions below.

1. Consider the following uncertain system

$$\dot{x} = \begin{bmatrix} -4+\delta & 2 \\ 1+\delta & -7 \end{bmatrix} x$$

   and write the system model in the form of (1)-(2) with $\Delta = \delta$.

2. Use the LMI representation of SGT to compute the maximum bound for δ that guarantees stability of the uncertain system.

3. Plot the root locus of the system as a function of δ (use MATLAB for this). What is the maximum value of $|\delta|$ such that the system has eigenvalues with negative real part? Is this consistent with the result in part 2?

4. Repeat parts 2 and 3 for the following uncertain system

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1+\delta/2 & -0.2 \end{bmatrix} x.$$

5. Simulate this system when $\delta(t) = \cos(2t)$. Is the system stable or unstable for the given $\delta(t)$? Is your answer consistent with the result from the SGT analysis? Why?

6. Comment on the stability of the system to time-invariant and time-varying perturbations. Can eigenvalue conditions guarantee stability to time-varying perturbations? What about the SGT condition?

Problem 2: Consider the following linear plant model

$$\begin{aligned} \dot{x}_p &= \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w \\ y &= \begin{bmatrix} 3 & 1 \end{bmatrix} x_p + u + 2w \\ z &= \begin{bmatrix} 1 & 0 \end{bmatrix} x_p + 2w \end{aligned}$$

and the following controller

$$\begin{aligned}\dot{x}_c &= -4x_c + 2z \\ u &= x_c - 2z.\end{aligned}$$

Using MATLAB commands, determine the closed-loop system equations

$$\begin{aligned}\dot{x}_{cl} &= A_{cl}x_{cl} + B_{cl}w \\ y &= C_{cl}x_{cl} + D_{cl}w.\end{aligned}$$

Then, examine the system stability (whether the closed-loop system is stable or not), and calculate the H_∞ norm of the closed-loop system.

Problem 3: Consider the following linear plant models

$$SYS\,1: \quad \dot{x}_p = \begin{bmatrix} -4 & 1 \\ 0 & 2 \end{bmatrix} x_p + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$SYS\,2: \quad \dot{x}_p = \begin{bmatrix} -3 & 2 \\ 4 & 1 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

For each of the above systems, determine if the system can be stabilized by a static state-feedback control law $u = Kx_p$. For the systems that are stabilizable, determine such a stabilizing control law, i.e., matrix gain K.

NOTE: Please attach your MATLAB (or Python) files and outputs.