

$$x_p = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\dot{x}_p = \begin{bmatrix} 0 & 10 & 2 \\ -1 & 1 & 0 \\ 0 & 2 & -5 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} w$$

$$y = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}^T x_p + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u$$

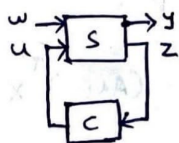
$$z = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T x_p + 2w$$

Now, we know the general form

$$\dot{x}_p = A_p x_p + B_p u + D_p w$$

$$y = C_p x_p + B_y u + D_y w$$

$$z = M_p x_p + D_z w$$



Comparing, we get

$$A_p = \begin{bmatrix} 0 & 10 & 2 \\ -1 & 1 & 0 \\ 0 & 2 & -5 \end{bmatrix} \Rightarrow \bar{A}_p = A_p + 0.2I = \begin{bmatrix} 0 & 10 & 2 \\ -1 & 1 & 0 \\ 0 & 2 & -5 \end{bmatrix} + \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 & 10 & 2 \\ -1 & 1.2 & 0 \\ 0 & 2 & -4.8 \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$C_p = [10 \ 0 \ 0]$$

$$D_p = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$B_y = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$D_y = 0$$

$$M_p = [0 \ 1 \ 0]$$

$$D_z = 2$$

There exists an H_∞ controller of order $n_c \leq n_p$: $\begin{cases} \dot{x}_c = A_c x_c + B_c z \\ u = C_c x_c + D_c z \end{cases}$ such that CL system is stable and

$$\Gamma_{cc} = \max_{w \neq 0} \frac{\|y\|_{L_2}}{\|w\|_{L_2}} < \gamma \quad \text{iff}$$

$$\begin{bmatrix} B_p \\ B_y \end{bmatrix}^\perp \begin{bmatrix} A_p x + x A_p^T + D_p D_p^T & x C_p^T + D_p D_y^T \\ * & D_y D_y^T - \gamma^2 I \end{bmatrix} \begin{bmatrix} B_p \\ B_y \end{bmatrix}^{\perp T} < 0 \quad (c_1)$$

$$\begin{bmatrix} M_p^T \\ D_z^T \end{bmatrix}^\perp \begin{bmatrix} \gamma A_p + A_p^T \gamma + C_p^T C_p & \gamma D_p + C_p^T D_y \\ * & D_y^T D_y - \gamma^2 I \end{bmatrix} \begin{bmatrix} M_p^T \\ D_z^T \end{bmatrix}^{\perp T} < 0 \quad (c_2)$$

$$\begin{bmatrix} x & \gamma I \\ \gamma I & y \end{bmatrix} \geq 0 \quad (c_3)$$

$$\text{rank} \begin{bmatrix} x & \gamma I \\ \gamma I & y \end{bmatrix} \leq n_p + n_c \quad (c_4)$$

- For static state feedback (i.e. $n_c = 0$, $M_p = I$), $C_2 - C_4$ are redundant and need not be checked.
- For full order controller (i.e. $n_p = n_c$) condition (C_4) is redundant and need not be checked.

MATLAB Notation (LMI Control Toolbox)

$$\begin{aligned} \dot{x} &= A x + B_1 w + B_2 u \\ z &= C_1 x + D_{11} w + D_{12} u \\ y &= C_2 x + D_{21} w + D_{22} u \end{aligned}$$

Comparing with the earlier notation:

$$A = A_p, \quad B_1 = D_p, \quad B_2 = B_p$$

$$C_1 = C_p, \quad D_{11} = D_y, \quad D_{12} = B_y$$

$$C_2 = M_p, \quad D_{21} = D_z, \quad D_{22} = 0$$

$$\text{let } A = \bar{A}_p$$

$$B = \begin{bmatrix} D_p & B_p \end{bmatrix}$$

$$C = \begin{bmatrix} C_p \\ M_p \end{bmatrix}$$

$$D = \begin{bmatrix} D_y & B_y \\ D_z & 0 \end{bmatrix}$$

By lumping matrices together:

$$\dot{x}_p = A_p x_p + \begin{bmatrix} D_p & B_p \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}$$

$$\begin{bmatrix} y \\ z \end{bmatrix} = \begin{bmatrix} C_p \\ M_p \end{bmatrix} x_p + \begin{bmatrix} D_y & B_y \\ D_z & 0 \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}$$

How Dynamic Controller Design in MATLAB

$$s = \text{ltisys}(A, B, C, D)$$

$$r = [n_z, n_u]$$

$$\begin{bmatrix} g_{opt} & G \end{bmatrix} = \text{hinflmi}(s, r)$$

$$\gamma_{cc} < \gamma^*$$

controller

$$[A_c, B_c, C_c, D_c] = \text{ltiss}(G)$$

Closed-loop (CL) system:

$$A_{cl} = \begin{bmatrix} A_p + B_p D_c M_p & B_p C_c \\ B_c M_p & A_c \end{bmatrix}$$

$$B_{cl} = \begin{bmatrix} D_p + B_p D_c D_z \\ B_c D_z \end{bmatrix}$$

$$C_{cl} = \begin{bmatrix} C_{pt} B_y D_c M_p & B_y C_c \end{bmatrix}$$

$$D_{cl} = D_y + B_y D_c D_z$$

Verification:

$$\begin{aligned} H_\infty \text{ norm : } S_{cl} &= ss(A_{cl}, B_{cl}, C_{cl}, D_{cl}) \\ \text{hinf_norm} &= \text{hinfnorm}(S_{cl}) \\ \text{hinf_norm} &< \gamma^* (j_{opt}) \end{aligned} \quad \left. \vphantom{\begin{aligned} H_\infty \text{ norm : } S_{cl} &= ss(A_{cl}, B_{cl}, C_{cl}, D_{cl}) \\ \text{hinf_norm} &= \text{hinfnorm}(S_{cl}) \\ \text{hinf_norm} &< \gamma^* (j_{opt}) \end{aligned}} \right\} \text{MATLAB}$$

$\hookrightarrow \rho_{ee} < \gamma^*$

Eigenvalues : $\text{Re}(\lambda_i(A_{cl})) < 0 \leftarrow \text{stability guaranteed}$
(Poles)

$\text{Re}(\lambda_i(A_{cl})) < -0.2 \leftarrow \text{stability with desired performance constraint guaranteed}$

Problem 1

CODE:

```
% PROBLEM 1

close all
clear
clc

% Define the system matrices
Ap = [0, 10, 2; -1, 1, 0; 0, 2, -5];
Ap_bar = Ap + 0.2*eye(3);
Bp = [0; 1; 0];
Cp = [1, 0, 0; 0, 0, 0];
Dp = [1; 0; 1];
By = [0; 1];
Dy = [0; 0];
Mp = [0, 1, 0];
Dz = 2;

% Convert to MATLAB notation
A = Ap_bar;
B1 = Dp;
B2 = Bp;
C1 = Cp;
D11 = Dy;
D12 = By;
C2 = Mp;
D21 = Dz;
D22 = 0;

% LTI system
S = ltisys(A, [B1 B2], [C1; C2], [D11 D12; D21 D22]);

% H-infinity LMI
[gopt, G] = hinflmi(S,[1 1])

% Controller matrices
disp('H $\infty$  controller:')
[Ac, Bc, Cc, Dc] = ltiss(G)
```

```

% Closed-loop system matrices
Acl = [Ap_bar+Bp*Dc*Mp, Bp*Cc; Bc*Mp, Ac];
Bcl = [Dp+Bp*Dc*Dz; Bc*Dz];
Ccl = [Cp+By*Dc*Mp, By*Cc];
Dcl = Dy+By*Dc*Dz;

% Closed-loop system
Scl = ss(Acl, Bcl, Ccl, Dcl);
% Scl = slft(S, G);

% Verification
disp('H $\infty$  norm:')
hinf_norm = hinfnorm(Scl)
% hinf_norm = norminf(Scl)
disp('Closed-loop poles:')
eig_Acl = eig(Acl)
if((hinf_norm < gopt) && all(real(eig_Acl) < -0.2))
    disp('Verification of H $\infty$  norm and pole location constraints successful!')
else
    disp('Verification of H $\infty$  norm and pole location constraints failed!')
end

```

OUTPUT:

Minimization of gamma:

Solver for linear objective minimization under LMI constraints

Iterations : Best objective value so far

1	
2	
3	
4	21.011488
5	17.864947
6	15.385893
7	14.718282
8	14.718282
9	13.310861
10	13.310861
11	11.988542
12	11.988542
13	11.352617
14	11.352617
15	11.146445
16	11.125254
17	11.110262
18	11.110262

```

19          11.110262
***          new lower bound:    10.816366
20          11.081688
***          new lower bound:    11.010885

Result:  feasible solution of required accuracy
         best objective value:    11.081688
         guaranteed relative accuracy:  6.39e-03
         f-radius saturation:  0.188% of R =  1.00e+08

```

```

Optimal Hinf performance:  1.108e+01
gopt = 11.0815

```

```

G = 4x4
-4.9470   -6.4084    0.5375    2.0000
-0.5151  -33.3313    9.0153         0
-0.3907  -34.4295    5.4533         0
         0         0         0        -Inf

```

H ∞ controller:

```

Ac = 2x2
-4.9470   -6.4084
-0.5151  -33.3313

```

```

Bc = 2x1
    0.5375
    9.0153

```

```

Cc = 1x2
-0.3907  -34.4295

```

```

Dc = 5.4533

```

H ∞ norm:

```

hinf_norm = 11.0780

```

Closed-loop poles:

```

eig_Acl = 5x1 complex
-23.2063 + 0.0000i
-1.6126 + 3.4277i
-1.6126 - 3.4277i
-4.9193 + 0.0000i
-4.8744 + 0.0000i

```

Verification of H ∞ norm and pole location constraints successful!

SCREENSHOT:

The screenshot displays the MATLAB R2023b environment with the following components:

- Current Folder:** Lists files including ME_8930_HW4.pdf, Q1.docx, Q1.mlx, Q2.docx, Q2.mlx, Q3.docx, and Q3.mlx.
- Live Editor:** Contains a script for 'Convex-Optimization-for-Control-Design' with the following code:

```
34 % Controller matrices
35 disp('Hw controller:')
36 [Ac, Bc, Cc, Dc] = ltiass(G)
37
38 % Closed-loop system matrices
39 Ac1 = [Ap_bar*Bp*Dc*Mp, Bp*Cc; Bc*Mp, Ac];
40 Bc1 = [Dp*Bp*Dc*Dc; Bc*Dc];
41 Cc1 = [Cp*Bp*Dc*Mp, Bp*Cc];
42 Dc1 = Dp*Bp*Dc*Dc;
43
44 % Closed-loop system
45 Sc1 = ss(Ac1, Bc1, Cc1, Dc1);
46 % Sc1 = sifft(S, G);
47
48 % Verification
49 disp('Hw norm:')
50 hinf_norm = hinfnorm(Sc1)
51 % hinf_norm = norminf(Sc1)
52 disp('Closed-loop poles:')
53 eigAc1 = eig(Ac1)
54 if((hinf_norm < gopt) && all(real(eigAc1) < -0.2))
55     disp('Verification of Hw norm and pole location constraints successful!')
56 else
57     disp('Verification of Hw norm and pole location constraints failed!')
58 end
```
- Workspace:** Displays variables A, Ac, Ac1, Ap, Ap_bar, B1, B2, Bc, Bc1, Bp, By, C1, C2, Cc, Cd, Cp, and D11 with their respective values.
- Command Window:** Shows the execution results, including the controller matrices, closed-loop system matrices, and verification results:

```
-0.3907 -34.4295 5.4533 0
0 0 0 -Inf
Hw controller:
Ac = 2x2
-4.9470 -6.4084
-0.5151 -33.3313
Bc = 2x2
0.5375
9.6333
Cc = 2x2
-0.3907 -34.4295
Dc = 5.4533
Hw norm:
hinf_norm = 11.6780
Closed-loop poles:
eigAc1 = 5x1 complex
-23.2863 + 0.0000i
-1.6126 + 3.4277i
-1.6126 - 3.4277i
-4.9193 + 0.0000i
-4.6744 + 0.0000i
Verification of Hw norm and pole location constraints successful!
```