Assignment 1: ME 8930 (LMIs in Optimal and Robust Control)

Due on Oct. 4, 2023 by midnight

Problem 1: Represent the inequalities

$$\begin{aligned} &P < A^T P A + Q - A^T P B (R + B^T P B)^{-1} B^T P A \\ &P > 0 \end{aligned}$$

where $R = R^T > 0$, as a <u>single</u> linear matrix inequality (in terms of the variable P).

Problem 2: Consider the unforced system $\dot{x} = Ax$, where the system matrix A can be either (i) or (ii) below

$$(i) \quad A = \begin{bmatrix} -7 & 5 \\ 3 & -4 \end{bmatrix}$$

$$(ii) \quad A = \begin{bmatrix} -6 & 4 & -2 \\ 3 & -8 & 1 \\ -1 & 5 & -7 \end{bmatrix}$$

Write code for an LMI feasibility problem to determine if each of the systems with the given A matrix has eigenvalues to the left of the vertical line $s = -2$ in the complex plane. Then, confirm your results by finding the eigenvalues of A for both cases in MATLAB (or Python).

Problem 3: Consider the following LTI systems

$$SYS1: \quad \dot{x}_p = \begin{bmatrix} -4 & 1 \\ 0 & 2 \end{bmatrix} x_p + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$SYS2: \quad \dot{x}_p = \begin{bmatrix} -3 & 2 \\ 4 & 1 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Determine if the two systems above can be stabilized by a static state-feedback control law $u = Kx_p$. For the systems that are stabilizable, determine such a stabilizing control law, i.e., matrix gain K.

NOTE: For Problems 2 and 3, please attach your MATLAB (or Python) files and outputs.