$P < A^T P A + Q - A^T P B (R + B^T P B)^{-1} B^T P A$

$P > 0$

$R = R^T > 0$

Above inequalities can be rewritten as

$A^T P A + Q - P - A^T P B (R + B^T P B)^{-1} B^T P A > 0$ — ①

$P > 0$ — ②

$R = R^T > 0$

Let (comparing ① with ⑦):

$X = R + B^T P B \iff X^{-1} = (R + B^T P B)^{-1}$ — ③

$Y = B^T P A \iff Y^T = A^T P B$ — ④

$Z = A^T P A + Q - P$ — ⑤

Since $R > 0$, we know that $X > 0$

Applying Schur complement:

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0 \iff \begin{bmatrix} X & 0 \\ 0 & Z - Y^T X^{-1} Y \end{bmatrix} \quad - ⑥$$

i.e. $Z - Y^T X^{-1} Y > 0$; $X > 0$ — ⑦

$$\begin{bmatrix} R + B^T P B & B^T P A \\ A^T P B & A^T P A + Q - P \end{bmatrix} > 0 \quad - ⑧$$

Adding ② to ⑧ while maintaining symmetry of matrix ~~LMI~~ we get the single LMI in terms of P:

$$\begin{bmatrix} R + B^T P B & B^T P A & 0 \\ A^T P B & A^T P A + Q - P & 0 \\ 0 & 0 & P \end{bmatrix} > 0$$