$$\dot{x}_p = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} w \qquad \Rightarrow n_p = 2$$

$$y = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$z = \begin{bmatrix} 1 & 0 \end{bmatrix} x_p + \begin{bmatrix} 0 & 1 \end{bmatrix} w$$

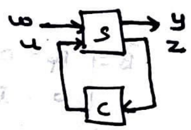
Optimal H_∞ controller design $(n_c = 1)$ $\hookrightarrow$ reduced order $(n_c < n_p)$

$\Gamma_{ee} < \gamma$

$\hookrightarrow \gamma = 3, 4, 5$

General form

$$\dot{x}_p = A_p x_p + B_p u + D_p w$$
$$y = C_p x_p + B_y u + D_y w$$
$$z = M_p x_p + D_z w$$



Comparing given system equations with general form

$$A_p = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B_p = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad D_p = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$C_p = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B_y = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad D_y = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$M_p = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad D_z = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

* Use above system matrices to obtain X and Y using Standard LMI Method and/or Alternating Projection Method (discussed later).

Confirm that $\text{rank}\begin{bmatrix} R \end{bmatrix} = \text{rank}\begin{bmatrix} X & \gamma I \\ \gamma I & Y \end{bmatrix} = \underset{n_p + n_c}{3}$

$\hookrightarrow$ This guarantees existance of first order controller

Perform SVD of $\underbrace{Y - \gamma^2 X^{-1}}_{n_p \times n_p}$

$$\text{SVD}(Y - \gamma^2 X^{-1}) = U \Sigma V^T = n_p\Big\{ \begin{bmatrix} \overset{n_c}{U_1} & \overset{n_p - n_c}{U_2} \end{bmatrix} \begin{bmatrix} \overset{n_c \times n_c}{\Sigma_1} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}$$

$$\text{Let} \left\{ \begin{aligned} Y_{11} &\in \mathbb{R}^{2\times 2} = Y \\ Y_{22} &\in \mathbb{R}^{1\times 1} = I \\ Y_{12} &\in \mathbb{R}^{1\times 2} = U_1 \Sigma_1^{1/2} \end{aligned} \right\} \Rightarrow Y - \gamma^2 X^{-1} = Y_{12} Y_{22}^{-1} Y_{12}^T$$

Construct the augmented matrix

$$P = \begin{bmatrix} Y & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix}$$

Find unknown controller parameters by solving the General Matrix Inequality

$$\Gamma G \Lambda + (\Gamma G \Lambda)^T + Q < 0$$

where $\Gamma = \begin{bmatrix} PB \\ 0 \\ H \end{bmatrix}$, $\Lambda = \begin{bmatrix} M & E & 0 \end{bmatrix}$, $Q = \begin{bmatrix} PA + A^T P & PD & C^T \\ D^T P & -\gamma^2 I & F^T \\ C & F & -I \end{bmatrix}$

where:

$A = \begin{bmatrix} A_p & 0 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} B_p & 0 \\ 0 & I \end{bmatrix}$

$M = \begin{bmatrix} M_p & 0 \\ 0 & I \end{bmatrix}$ $E = \begin{bmatrix} D_z \\ 0 \end{bmatrix}$

$H = \begin{bmatrix} B_y & 0 \end{bmatrix}$ $D = \begin{bmatrix} D_p \\ 0 \end{bmatrix}$

$C = \begin{bmatrix} C_p & 0 \end{bmatrix}$ $F = D_y$

$$G = \begin{array}{c} \\ \left[\begin{array}{c:c} D_c & C_c \\ \hdashline B_c & A_c \end{array}\right] \begin{array}{l} \}\, n_u \\ \}\, n_c \end{array} \\ \begin{array}{cc} \underbrace{}_{n_z} & \underbrace{}_{n_c} \end{array} \end{array}$$

unknown

i.e. Use $\underbrace{P \text{ and } A, B, C, D, E, F, H, M}_{\Gamma,\ \Lambda,\ Q}$ to solve "basiclmi"

$$G = \text{basiclmi}\,(Q, \Gamma^T, \Lambda)$$

Use partitioned form of controller matrix G to obtain A_c, B_c, C_c and D_c.

Standard LMI Method:

$$\min \ \operatorname{trace}(X+Y)$$

X, Y, γ ← usually unknown/variable but known in this problem $[\gamma = 3,4,5]$

$$\text{s.t.} \quad \begin{bmatrix} B_p \\ B_y \end{bmatrix}^{\perp} \begin{bmatrix} A_pX + XA_p^T + D_pD_p^T & XC_p^T + D_pD_y^T \\ C_pX + D_yD_p^T & D_yD_y^T - \gamma^2 I \end{bmatrix} \begin{bmatrix} B_p \\ B_y \end{bmatrix}^{\perp T} < 0 \qquad (C_1)$$

$$\begin{bmatrix} M_p^T \\ D_2^T \end{bmatrix}^{\perp} \begin{bmatrix} YA_p + A_p^TY + C_p^TC_p & YD_p + C_p^TD_y \\ D_p^TY + D_y^TC_p & D_y^TD_y - \gamma^2 I \end{bmatrix} \begin{bmatrix} M_p^T \\ D_2^T \end{bmatrix}^{\perp T} < 0 \qquad (C_2)$$

$$\begin{bmatrix} X & \gamma I \\ \gamma I & Y \end{bmatrix} \geq 0 \qquad (C_3)$$

Alternating Projection Method:

non-convex constraint →

$$\operatorname{rank} \begin{bmatrix} X & \gamma I \\ \gamma I & Y \end{bmatrix} \leq n_p + n_c \qquad (C_4)$$

Idea: Iterate by projecting on $\overbrace{\text{LMIs } (C_1 - C_3)}^{\text{first space}}$ and the $\underbrace{\text{rank constraint } (C_4)}_{\text{second space}}$ back & forth until convergence is met.

Approach:

Working in $(C_1)-(C_3)$ space

Solve $(C_1)-(C_3)$ as an LMI problem to obtain X and Y

Let $R = \begin{bmatrix} X & \gamma I \\ \gamma I & Y \end{bmatrix}$

for iteration = 1 : max_iterations (say 100)

- Enforce the rank constraint (if not satisfied)

  $U, \Sigma, V^T = \operatorname{svd}(R)$; $\Sigma = \begin{bmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_k \end{bmatrix}$ with $\sigma_1 > \sigma_2 > \dots > \sigma_{k-1} > \sigma_k$

  If we replace smallest σ_i of R (i.e. σ_k) by zero we get

  $R' = U \begin{bmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \sigma_{k-1} & \vdots \\ 0 & \cdots & 0 \end{bmatrix} V^T$ whose rank is one less than original R.

  Instead of replacing one σ_i with 0, we can replace multiple (m) so that rank constraint is satisfied.

- Else (if rank constraint is satisfied)

  $R' = R$

- $\min\limits_{R'} \| R - R' \|$ ← If $\| R - R' \| <$ tolerance $\Rightarrow$ break

  s.t. $\begin{cases} R' \overset{X,Y}{\leftarrow} \text{is solution to } (C_1)-(C_3) \\ \operatorname{rank}(R') = k-m \end{cases}$ ··· (using SVD)

Projection onto C_4 space & working in C_4 space

- $\min\limits_{R_1} \|R_1 - R'\|$ ← If $\|R_1 - R'\| <$ tolerance ⇒ break

s.t. $R_1 \in$ LMIs (C_1)-(C_3)

R_1 is the closest approximation to R' used for projection

Now;

$\because \|R_1 - R'\| < \bar{\gamma} \Rightarrow \begin{bmatrix} -\bar{\gamma}^2 I & R_1 - R' \\ R_1 - R' & -I \end{bmatrix} < 0$

$\therefore \min\limits_{R_1, \hat{\gamma}} \hat{\gamma} \quad \dots \quad (\hat{\gamma} = \bar{\gamma}^2)$

s.t. $R_1 \in$ LMIs (C_1)-(C_3)

$$\begin{bmatrix} -\hat{\gamma} I & R - R' \\ R - R' & -I \end{bmatrix} < 0$$

↓

Get R_1

Projection (back) onto (C_1)-(C_3) space

↓

Set $R = R_1$

↓

Continue to go back and forth by iterating while projecting on LMIs (C_1)-(C_3) space and rank constraint (C_4) space until convergence is met (i.e. termination criteria based on tolerance or max-iterations is reached).

Problem 2

CODE:

```
% PROBLEM 2


% Clear workspace
close all
clear
clc


% Add parser and solver to path
addpath(genpath('C:\Users\csamak\Downloads\MathWorks\Toolboxes\archives\required\
YALMIP'))
addpath(genpath('C:\Users\csamak\Downloads\MathWorks\Toolboxes\archives\required\
SeDuMi'))


for g = 3:1:5 % Gamma (γ)
    disp('-----------------------------------')
    if g == 3
        disp('Standard LMI Method | CASE 1: γ = 3')
    elseif g == 4
        disp('Standard LMI Method | CASE 2: γ = 4')
    elseif g == 5
        disp('Standard LMI Method | CASE 3: γ = 5')
    end
    disp('-----------------------------------')


    % Define the system matrices
    Ap = [0 1; 0 0];
    Bp = [0; 1];
    Dp = [0 0; 1 0];
    Cp = [0 1; 0 0];
    By = [0; 1];
    Dy = [0 0; 0 0];
    Mp = [1 0];
    Dz = [0 1];

    % Define the LMI variables
    X = sdpvar(2, 2);
    Y = sdpvar(2, 2);

    % Define the LMI constraints
```

```
    C1 = null([Bp; By]')' * [(Ap*X + X*Ap' + Dp*Dp') (X*Cp' + Dp*Dy'); (Cp*X + 
Dy*Dp') (Dy*Dy' - g^2*eye(size(Dy)))] * null([Bp; By]') <= 0;
    C2 = null([Mp'; Dz']')' * [(Y*Ap + Ap'*Y + Cp'*Cp) (Y*Dp + Cp'*Dy); (Dp'*Y + 
Dy'*Cp) (Dy'*Dy - g^2*eye(size(Dy)))] * null([Mp'; Dz']') <= 0;
    C3 = [X g*eye(size(Dy)); g*eye(size(Dy)) Y] >= 0;

    % Set up the objective
    Objective = trace(X + Y);

    % Define the solver settings (use an LMI solver like YALMIP with a solver of 
your choice)
    options = sdpsettings('verbose', 1, 'solver', 'sedumi');

    % Solve the LMI problem
    solution = optimize([C1, C2, C3], Objective, options);

    % 1. Obtain X and Y by solving above optimization problem
    if solution.problem == 0
        % Extract the optimal solutions
        X = value(X);
        Y = value(Y);
        % Display the results
        disp('X*:');
        disp(X);
        disp('Y*:');
        disp(Y);
    else
        fprintf('LMI problem could not be solved.\n');
    end

    % 2. Confirm that rank([X γ*I; γ*I Y]) = 3 and compute terms Y12 and Y22
    if rank([X g*eye(size(Dy)); g*eye(size(Dy)) Y]) == 3
        disp('rank([X γ*I; γ*I Y]) = 3')
        disp('Solving LMI problem resulted in reduced order controller with nc = 
1')
    elseif rank([X g*eye(size(Dy)); g*eye(size(Dy)) Y]) == 4
        disp('rank([X γ*I; γ*I Y]) = 4')
        disp('Solving LMI problem resulted in full order controller with nc = 2')
    else
        disp('rank([X γ*I; γ*I Y]) < 3')
        disp('Solving LMI problem resulted in zero order controller with nc = 0')
    end
    [U, S, V] = svd(Y - g^2*inv(X));
    u1 = U(:,1); % Extract u1 so that u1 = np x nc
    S1 = S(1,1); % Extract S1 so that S1 = nc x nc
    disp('Extracted u1 & Σ1 so as to obtain reduced order controller with nc = 
1')
    Y12 = u1*(S1^0.5)
```

```
    Y22 = eye(size(Y12', 1), size(Y12, 2))

    % 3. Define augmented matrix P & find controller by solving general matrix
inequality
    P = [Y Y12; Y12' Y22]
    A = [Ap [0; 0]; [0 0] 0];
    B = [Bp [0; 0]; 0 1];
    C = [Cp [0; 0]];
    D = [Dp; [0 0]];
    E = [Dz; [0 0]];
    F = Dy;
    H = [By [0; 0]];
    M = [Mp 0; [0 0] 1];
    Gamma = [P*B; [0 0]; [0 0]; H] % Γ
    Lambda = [M E [0; 0] [0; 0]] % Λ
    Q = [P*A+A'*P P*D C'; D'*P -g^2*eye(2) F'; C F -eye(2)] % Q
    G = basiclmi(Q, Gamma', Lambda); % Solve general matrix inequality
    if size(G) ~= 0
        disp('Controller matrix (G):')
        disp(G)
        fprintf('Ac = %f', value(G(2,2)))
        fprintf('Bc = %f', value(G(2,1)))
        fprintf('Cc = %f', value(G(1,2)))
        fprintf('Dc = %f', value(G(1,1)))
    end
end
```

```
% Clear workspace
close all
clear
clc


% Add parser and solver to path
addpath(genpath('C:\Users\csamak\Downloads\MathWorks\Toolboxes\archives\required\
YALMIP'))
addpath(genpath('C:\Users\csamak\Downloads\MathWorks\Toolboxes\archives\required\
SeDuMi'))


for g = 3:1:5 % Gamma (γ)
    disp('----------------------------------------------')
    if g == 3
        disp('Alternating Projection Method | CASE 1: γ = 3')
    elseif g == 4
        disp('Alternating Projection Method | CASE 2: γ = 4')
```

```
    elseif g == 5
        disp('Alternating Projection Method | CASE 3: γ = 5')
    end
    disp('---------------------------------------------')


    % Define the system matrices
    Ap = [0 1; 0 0];
    Bp = [0; 1];
    Dp = [0 0; 1 0];
    Cp = [0 1; 0 0];
    By = [0; 1];
    Dy = [0 0; 0 0];
    Mp = [1 0];
    Dz = [0 1];

    % Define the LMI variables
    X = sdpvar(2, 2);
    Y = sdpvar(2, 2);

    % Define the LMI constraints
    C1 = null([Bp; By]')' * [(Ap*X + X*Ap' + Dp*Dp') (X*Cp' + Dp*Dy'); (Cp*X + Dy*Dp') (Dy*Dy' - g^2*eye(size(Dy)))] * null([Bp; By]') <= 0;
    C2 = null([Mp'; Dz']')' * [(Y*Ap + Ap'*Y + Cp'*Cp) (Y*Dp + Cp'*Dy); (Dp'*Y + Dy'*Cp) (Dy'*Dy - g^2*eye(size(Dy)))] * null([Mp'; Dz']') <= 0;
    C3 = [X g*eye(size(Dy)); g*eye(size(Dy)) Y] >= 0;

    % Set up the objective
    Objective = trace(X + Y);

    % Define the solver settings (use an LMI solver like YALMIP with a solver of your choice)
    options = sdpsettings('verbose', 1, 'solver', 'sedumi');

    % Alternating projection method
    max_iterations = 100;
    tolerance = 1e-22;

    % Working in C1-C3 space
    solution = optimize([C1, C2, C3], Objective, options);
    if solution.problem == 0
        X = value(X);
        Y = value(Y);
    end
    R = [X, g*eye(size(Dy)); g*eye(size(Dy)), Y];

    for iteration = 1:max_iterations
        [U, S, V] = svd(R);
```

```
        if not(rank(R) == 3)
            S(size(S,1), size(S,2)) = 0;
            R_prime = U*S*V';
        else
            R_prime = R;
        end
        % Check for convergence between R and R_prime
        if norm(R - R_prime) < tolerance
            disp(['Converged at iteration ', num2str(iteration)]);
            break;
        end
        % Projection onto C4 space & working in C4 space
        R1 = sdpvar(size(R,1), size(R,2)); % Define the LMI variable
        g_sqr = sdpvar(1, 1); % Define the LMI variable
        LMI_R1 = [-g_sqr*eye(size(R)) R1-R_prime; R1-R_prime -eye(size(R))] <= 0;
% Define the LMI constraint
        Objective_R1 = g_sqr; % Set up the objective
        solution_R1 = optimize(LMI_R1, Objective_R1, options); % Solve the LMI
problem
        if solution_R1.problem == 0
            R1 = value(R1); % Extract the optimal solution
        end
        % Check for convergence between R1 and R_prime
        if norm(R1 - R_prime) < tolerance
            disp(['Converged at iteration ', num2str(iteration)]);
            break;
        end
        % Projection onto C1-C3 space
        R = R1;
    end

    % Display the results
    disp('X*:');
    disp(X);
    disp('Y*:');
    disp(Y);

    % Display rank information
    if rank(R_prime) == 3
        disp('Solving LMI problem resulted in a reduced-order controller with nc
= 1');
    else
        disp('Rank constraint not satisfied.');
        return; % Exit if the rank constraint is not satisfied
    end

    % Extract u1 & Σ1 to obtain reduced order controller with nc = 1
    [U, S, V] = svd(Y - g^2*inv(X));
```

```
    u1 = U(:, 1);
    S1 = S(1, 1);
    Y12 = u1 * (S1^0.5);
    Y22 = eye(size(Y12', 1), size(Y12, 2));

    % Define augmented matrix P for the controller
    P = [Y, Y12; Y12', Y22];

    % Obtain the controller matrix G
    A = [Ap, [0; 0]; [0 0], 0];
    B = [Bp, [0; 0]; 0, 1];
    C = [Cp, [0; 0]];
    D = [Dp; [0 0]];
    E = [Dz; [0 0]];
    F = Dy;
    H = [By, [0; 0]];
    M = [Mp, 0; [0 0], 1];
    Gamma = [P * B; [0 0]; [0 0]; H]; % Γ
    Lambda = [M, E, [0; 0], [0; 0]]; % Λ
    Q = [P * A + A' * P, P * D, C'; D' * P, -g^2 * eye(2), F'; C, F, -eye(2)]; %
Q
    G = basiclmi(Q, Gamma', Lambda); % Solve general matrix inequality
    if size(G) ~= 0
        disp('Controller matrix (G):')
        disp(G)
        fprintf('Ac = %f', value(G(2,2)))
        fprintf('Bc = %f', value(G(2,1)))
        fprintf('Cc = %f', value(G(1,2)))
        fprintf('Dc = %f', value(G(1,1)))
    end
end
```

OUTPUT:

```
------------------------------------
Standard LMI Method | CASE 1: γ = 3
------------------------------------
SeDuMi 1.3 by AdvOL, 2005-2008 and Jos F. Sturm, 1998-2003.
Alg = 2: xz-corrector, theta = 0.250, beta = 0.500
eqs m = 6, order n = 11, dim = 35, blocks = 4
nnz(A) = 20 + 0, nnz(ADA) = 36, nnz(L) = 21
 it :     b*y       gap    delta  rate   t/tP*  t/tD*   feas cg cg  prec
  0 :            2.71E+01 0.000
  1 :  -5.65E+00 9.11E+00 0.000 0.3364 0.9000 0.9000   1.41  1  1  3.2E+00
  2 :  -1.25E+01 2.87E+00 0.000 0.3153 0.9000 0.9000   0.91  1  1  9.6E-01
  3 :  -1.70E+01 6.35E-01 0.000 0.2209 0.9000 0.9000   0.66  1  1  2.5E-01
  4 :  -1.98E+01 5.13E-02 0.000 0.0808 0.9900 0.9900   0.78  1  1  2.1E-02
  5 :  -2.01E+01 1.90E-03 0.000 0.0369 0.9900 0.9900   0.99  1  1  8.3E-04
  6 :  -2.01E+01 1.03E-04 0.407 0.0545 0.9900 0.9900   1.00  1  1  4.5E-05
  7 :  -2.01E+01 7.99E-06 0.254 0.0773 0.9900 0.9900   1.00  1  1  3.5E-06
```

```
  8 :  -2.01E+01 7.20E-07 0.438 0.0902 0.9900 0.9900   1.00  1  1  3.2E-07
  9 :  -2.01E+01 1.51E-07 0.000 0.2100 0.9000 0.9000   1.00  2  2  6.7E-08
 10 :  -2.01E+01 1.86E-08 0.017 0.1233 0.9450 0.9450   1.00  2  2  8.3E-09
 11 :  -2.01E+01 1.61E-09 0.060 0.0865 0.9900 0.9900   1.00  2  2  7.2E-10

iter seconds digits       c*x               b*y
 11      0.2   9.8 -2.0062574492e+01 -2.0062574496e+01
|Ax-b| =   4.8e-10, [Ay-c]_+ =   9.5E-10, |x|=  1.1e+01, |y|=  1.1e+01

Detailed timing (sec)
   Pre          IPM          Post
3.090E-01    5.140E-01    3.300E-02
Max-norms: ||b||=1, ||c|| = 9,
Cholesky |add|=0, |skip| = 0, ||L.L|| = 6.52152.
X*:
    6.9382   -1.0201
   -1.0201    3.9239
Y*:
    4.2969   -2.3744
   -2.3744    4.9036
rank([X γ*I; γ*I Y]) = 4
Solving LMI problem resulted in full order controller with nc = 2
Extracted u1 & Σ1 so as to obtain reduced order controller with nc = 1
Y12 = 2×1
   -1.7170
    1.5871
Y22 = 1
P = 3×3
    4.2969   -2.3744   -1.7170
   -2.3744    4.9036    1.5871
   -1.7170    1.5871    1.0000
Gamma = 7×2
   -2.3744   -1.7170
    4.9036    1.5871
    1.5871    1.0000
         0         0
         0         0
         0         0
    1.0000         0
Lambda = 2×7
     1     0     0     0     1     0     0
     0     0     1     0     0     0     0
Q = 7×7
         0    4.2969         0   -2.3744         0         0         0
    4.2969   -4.7488   -1.7170    4.9036         0    1.0000         0
         0   -1.7170         0    1.5871         0         0         0
   -2.3744    4.9036    1.5871   -9.0000         0         0         0
         0         0         0         0   -9.0000         0         0
         0    1.0000         0         0         0   -1.0000         0
         0         0         0         0         0         0   -1.0000
   Warning in BASICLMI: the solvability conditions are not satisfied
-----------------------------------
Standard LMI Method | CASE 2: γ = 4
-----------------------------------
SeDuMi 1.3 by AdvOL, 2005-2008 and Jos F. Sturm, 1998-2003.
Alg = 2: xz-corrector, theta = 0.250, beta = 0.500
eqs m = 6, order n = 11, dim = 35, blocks = 4
```

```
nnz(A) = 20 + 0, nnz(ADA) = 36, nnz(L) = 21
 it :     b*y       gap    delta  rate   t/tP*  t/tD*   feas cg cg  prec
  0 :            4.74E+01 0.000
  1 :  -7.71E+00 1.53E+01 0.000 0.3239 0.9000 0.9000   1.46  1  1  2.8E+00
  2 :  -1.67E+01 4.33E+00 0.000 0.2820 0.9000 0.9000   1.04  1  1  8.1E-01
  3 :  -2.02E+01 8.97E-01 0.000 0.2072 0.9000 0.9000   0.83  1  1  1.8E-01
  4 :  -2.23E+01 4.34E-02 0.000 0.0484 0.9900 0.9900   0.85  1  1  8.9E-03
  5 :  -2.25E+01 2.43E-03 0.160 0.0560 0.9675 0.9675   1.00  1  1  5.0E-04
  6 :  -2.25E+01 2.68E-04 0.198 0.1101 0.9450 0.9450   1.00  1  1  5.5E-05
  7 :  -2.25E+01 2.09E-05 0.319 0.0779 0.9900 0.9900   1.00  1  1  4.3E-06
  8 :  -2.25E+01 9.96E-06 0.205 0.4772 0.9000 0.9000   1.00  1  1  2.1E-06
  9 :  -2.25E+01 2.48E-06 0.000 0.2490 0.9000 0.9000   1.00  1  1  5.2E-07
 10 :  -2.25E+01 2.29E-07 0.369 0.0922 0.9900 0.9900   1.00  2  2  4.9E-08
 11 :  -2.25E+01 8.19E-08 0.171 0.3580 0.9000 0.9000   1.00  2  2  1.8E-08
 12 :  -2.25E+01 6.92E-09 0.149 0.0845 0.9900 0.9900   1.00  2  2  1.5E-09
 13 :  -2.25E+01 6.74E-10 0.139 0.0974 0.9900 0.9900   1.00  2  2  1.5E-10

iter seconds digits       c*x               b*y
 13      0.0  10.2 -2.2456285268e+01 -2.2456285269e+01
|Ax-b| =   8.7e-11, [Ay-c]_+ =   2.7E-10, |x|=  7.9e+00, |y|=  1.2e+01

Detailed timing (sec)
   Pre          IPM          Post
1.200E-02    3.201E-02    2.997E-03
Max-norms: ||b||=1, ||c|| = 16,
Cholesky |add|=0, |skip| = 0, ||L.L|| = 4.35694.
X*:
    6.9145   -0.8214
   -0.8214    4.9001
Y*:
    4.9551   -2.0759
   -2.0759    5.6866
rank([X γ*I; γ*I Y]) = 4
Solving LMI problem resulted in full order controller with nc = 2
Extracted u1 & Σ1 so as to obtain reduced order controller with nc = 1
Y12 = 2×1
   -1.6106
    1.5346
Y22 = 1
P = 3×3
    4.9551   -2.0759   -1.6106
   -2.0759    5.6866    1.5346
   -1.6106    1.5346    1.0000
Gamma = 7×2
   -2.0759   -1.6106
    5.6866    1.5346
    1.5346    1.0000
         0         0
         0         0
         0         0
    1.0000         0
Lambda = 2×7
     1     0     0     0     1     0     0
     0     0     1     0     0     0     0
Q = 7×7
         0    4.9551         0   -2.0759         0         0         0
    4.9551   -4.1518   -1.6106    5.6866         0    1.0000         0
```

```
         0   -1.6106         0    1.5346         0         0         0
   -2.0759    5.6866    1.5346  -16.0000         0         0         0
         0         0         0         0  -16.0000         0         0
         0    1.0000         0         0         0   -1.0000         0
         0         0         0         0         0         0   -1.0000
Controller matrix (G):
   -2.6758    1.3083
    8.5579   -5.0717
Ac = -5.071654
Bc = 8.557912
Cc = 1.308284
Dc = -2.675828
-----------------------------------
Standard LMI Method | CASE 3: γ = 5
-----------------------------------
SeDuMi 1.3 by AdvOL, 2005-2008 and Jos F. Sturm, 1998-2003.
Alg = 2: xz-corrector, theta = 0.250, beta = 0.500
eqs m = 6, order n = 11, dim = 35, blocks = 4
nnz(A) = 20 + 0, nnz(ADA) = 36, nnz(L) = 21
 it :     b*y       gap    delta  rate   t/tP*  t/tD*   feas cg cg  prec
  0 :            7.35E+01 0.000
  1 :  -1.03E+01 2.28E+01 0.000 0.3097 0.9000 0.9000   1.48  1  1  2.6E+00
  2 :  -2.10E+01 5.93E+00 0.000 0.2602 0.9000 0.9000   1.11  1  1  7.3E-01
  3 :  -2.36E+01 1.27E+00 0.000 0.2149 0.9000 0.9000   0.92  1  1  1.6E-01
  4 :  -2.57E+01 5.44E-02 0.000 0.0427 0.9900 0.9900   0.87  1  1  7.1E-03
  5 :  -2.58E+01 2.00E-03 0.024 0.0368 0.9900 0.9900   1.00  1  1  2.6E-04
  6 :  -2.58E+01 2.24E-04 0.206 0.1119 0.9450 0.9450   1.00  1  1  2.9E-05
  7 :  -2.58E+01 1.85E-05 0.480 0.0825 0.9900 0.9900   1.00  1  1  2.4E-06
  8 :  -2.58E+01 8.75E-06 0.164 0.4733 0.9000 0.9000   1.00  1  1  1.2E-06
  9 :  -2.58E+01 1.63E-06 0.000 0.1859 0.9000 0.9000   1.00  1  1  2.2E-07
 10 :  -2.58E+01 1.36E-07 0.471 0.0837 0.9900 0.9900   1.00  1  2  1.8E-08
 11 :  -2.58E+01 2.69E-08 0.000 0.1976 0.9000 0.9000   1.00  2  2  3.6E-09
 12 :  -2.58E+01 1.17E-09 0.359 0.0434 0.9900 0.9900   1.00  2  2  1.6E-10

iter seconds digits       c*x               b*y
 12      0.0  10.1 -2.5756604760e+01 -2.5756604762e+01
|Ax-b| =   8.3e-11, [Ay-c]_+ =   4.2E-10, |x|=  6.6e+00, |y|=  1.3e+01

Detailed timing (sec)
   Pre          IPM          Post
3.300E-02    2.400E-02    2.002E-03
Max-norms: ||b||=1, ||c|| = 25,
Cholesky |add|=0, |skip| = 0, ||L.L|| = 4.93398.
X*:
    7.4856   -0.7233
   -0.7233    5.8495
Y*:
    5.8149   -1.9387
   -1.9387    6.6066
rank([X γ*I; γ*I Y]) = 4
Solving LMI problem resulted in full order controller with nc = 2
Extracted u1 & Σ1 so as to obtain reduced order controller with nc = 1
Y12 = 2×1
   -1.5604
    1.5103
Y22 = 1
P = 3×3
```

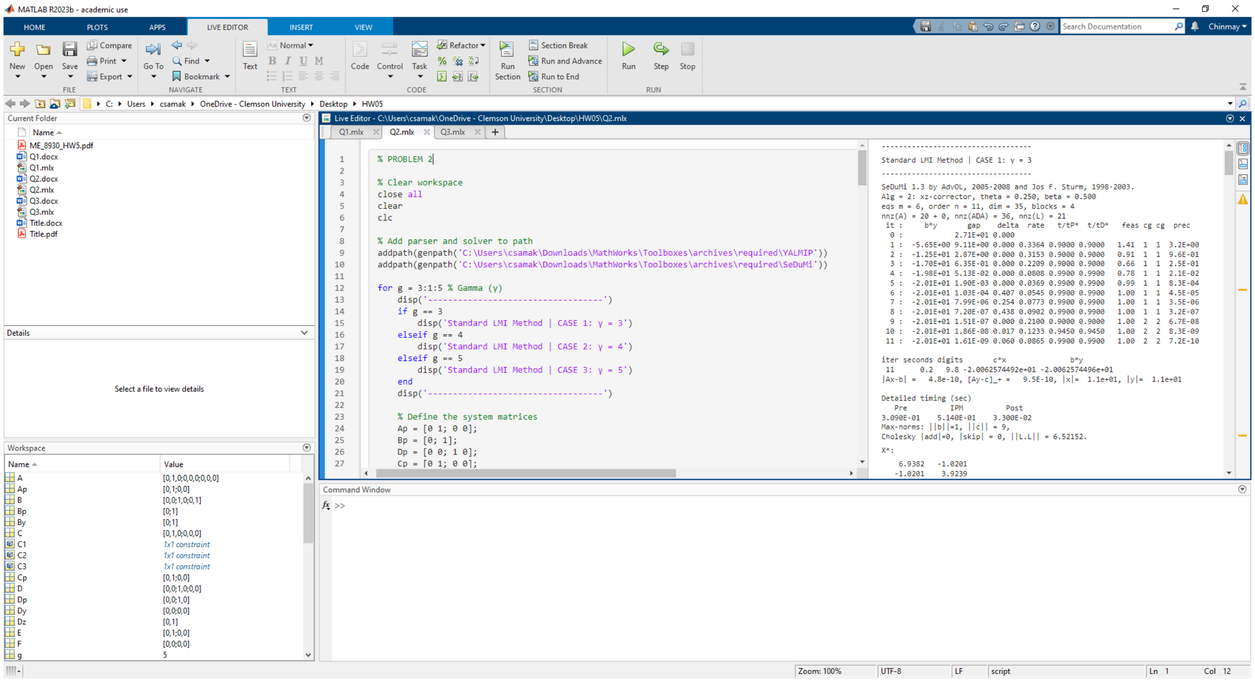
```
    5.8149   -1.9387   -1.5604
   -1.9387    6.6066    1.5103
   -1.5604    1.5103    1.0000
Gamma = 7×2
   -1.9387   -1.5604
    6.6066    1.5103
    1.5103    1.0000
         0         0
         0         0
         0         0
    1.0000         0
Lambda = 2×7
     1     0     0     0     1     0     0
     0     0     1     0     0     0     0
Q = 7×7
         0    5.8149         0   -1.9387         0         0         0
    5.8149   -3.8774   -1.5604    6.6066         0    1.0000         0
         0   -1.5604         0    1.5103         0         0         0
   -1.9387    6.6066    1.5103  -25.0000         0         0         0
         0         0         0         0  -25.0000         0         0
         0    1.0000         0         0         0   -1.0000         0
         0         0         0         0         0         0   -1.0000
Controller matrix (G):
   -3.1340    1.6176
   11.9541   -7.5012
Ac = -7.501234
Bc = 11.954090
Cc = 1.617626
Dc = -3.134028
---------------------------------------------
Alternating Projection Method | CASE 1: γ = 3
---------------------------------------------
SeDuMi 1.3 by AdvOL, 2005-2008 and Jos F. Sturm, 1998-2003.
Alg = 2: xz-corrector, theta = 0.250, beta = 0.500
eqs m = 6, order n = 11, dim = 35, blocks = 4
nnz(A) = 20 + 0, nnz(ADA) = 36, nnz(L) = 21
 it :     b*y       gap    delta  rate   t/tP*  t/tD*   feas cg cg  prec
  0 :            2.71E+01 0.000
  1 :  -5.65E+00 9.11E+00 0.000 0.3364 0.9000 0.9000   1.41  1  1  3.2E+00
  2 :  -1.25E+01 2.87E+00 0.000 0.3153 0.9000 0.9000   0.91  1  1  9.6E-01
  3 :  -1.70E+01 6.35E-01 0.000 0.2209 0.9000 0.9000   0.66  1  1  2.5E-01
  4 :  -1.98E+01 5.13E-02 0.000 0.0808 0.9900 0.9900   0.78  1  1  2.1E-02
  5 :  -2.01E+01 1.90E-03 0.000 0.0369 0.9900 0.9900   0.99  1  1  8.3E-04
  6 :  -2.01E+01 1.03E-04 0.407 0.0545 0.9900 0.9900   1.00  1  1  4.5E-05
  7 :  -2.01E+01 7.99E-06 0.254 0.0773 0.9900 0.9900   1.00  1  1  3.5E-06
  8 :  -2.01E+01 7.20E-07 0.438 0.0902 0.9900 0.9900   1.00  1  1  3.2E-07
  9 :  -2.01E+01 1.51E-07 0.000 0.2100 0.9000 0.9000   1.00  2  2  6.7E-08
 10 :  -2.01E+01 1.86E-08 0.017 0.1233 0.9450 0.9450   1.00  2  2  8.3E-09
 11 :  -2.01E+01 1.61E-09 0.060 0.0865 0.9900 0.9900   1.00  2  2  7.2E-10

iter seconds digits       c*x               b*y
 11      0.0   9.8 -2.0062574492e+01 -2.0062574496e+01
|Ax-b| =   4.8e-10, [Ay-c]_+ =   9.5E-10, |x|=  1.1e+01, |y|=  1.1e+01

Detailed timing (sec)
   Pre          IPM          Post
1.100E-02    3.900E-02    2.997E-03
```

```
Max-norms: ||b||=1, ||c|| = 9,
Cholesky |add|=0, |skip| = 0, ||L.L|| = 6.52152.
SeDuMi 1.3 by AdvOL, 2005-2008 and Jos F. Sturm, 1998-2003.
Alg = 2: xz-corrector, theta = 0.250, beta = 0.500
eqs m = 11, order n = 9, dim = 65, blocks = 2
nnz(A) = 20 + 0, nnz(ADA) = 121, nnz(L) = 66
 it :     b*y       gap    delta  rate   t/tP*  t/tD*   feas cg cg  prec
  0 :            8.16E+00 0.000
  1 :  -2.50E+00 3.13E+00 0.000 0.3836 0.9000 0.9000   2.27  1  1  2.7E+00
  2 :   1.31E-01 8.96E-01 0.000 0.2861 0.9000 0.9000   3.42  1  1  3.8E-01
  3 :   5.07E-03 1.99E-02 0.000 0.0222 0.9900 0.9900   1.25  1  1  1.2E-01
  4 :   1.19E-07 3.91E-07 0.000 0.0000 1.0000 1.0000   1.01  1  1  2.3E-05
  5 :   5.71E-14 1.85E-13 0.442 0.0000 1.0000 1.0000   1.00  1  1  1.1E-11

iter seconds digits       c*x               b*y
  5      0.0   8.9  1.1776667857e-13  5.7097292825e-14
|Ax-b| =   3.0e-14, [Ay-c]_+ =   5.7E-14, |x|=  5.0e-01, |y|=  1.1e+01

Detailed timing (sec)
   Pre          IPM          Post
5.501E-02    2.299E-02    9.002E-03
Max-norms: ||b||=1, ||c|| = 1.387637e+01,
Cholesky |add|=0, |skip| = 0, ||L.L|| = 1.
Converged at iteration 2
X*:
    6.9382   -1.0201
   -1.0201    3.9239
Y*:
    4.2969   -2.3744
   -2.3744    4.9036
Solving LMI problem resulted in a reduced-order controller with nc = 1
   Warning in BASICLMI: the solvability conditions are not satisfied
---------------------------------------------
Alternating Projection Method | CASE 2: γ = 4
---------------------------------------------
SeDuMi 1.3 by AdvOL, 2005-2008 and Jos F. Sturm, 1998-2003.
Alg = 2: xz-corrector, theta = 0.250, beta = 0.500
eqs m = 6, order n = 11, dim = 35, blocks = 4
nnz(A) = 20 + 0, nnz(ADA) = 36, nnz(L) = 21
 it :     b*y       gap    delta  rate   t/tP*  t/tD*   feas cg cg  prec
  0 :            4.74E+01 0.000
  1 :  -7.71E+00 1.53E+01 0.000 0.3239 0.9000 0.9000   1.46  1  1  2.8E+00
  2 :  -1.67E+01 4.33E+00 0.000 0.2820 0.9000 0.9000   1.04  1  1  8.1E-01
  3 :  -2.02E+01 8.97E-01 0.000 0.2072 0.9000 0.9000   0.83  1  1  1.8E-01
  4 :  -2.23E+01 4.34E-02 0.000 0.0484 0.9900 0.9900   0.85  1  1  8.9E-03
  5 :  -2.25E+01 2.43E-03 0.160 0.0560 0.9675 0.9675   1.00  1  1  5.0E-04
  6 :  -2.25E+01 2.68E-04 0.198 0.1101 0.9450 0.9450   1.00  1  1  5.5E-05
  7 :  -2.25E+01 2.09E-05 0.319 0.0779 0.9900 0.9900   1.00  1  1  4.3E-06
  8 :  -2.25E+01 9.96E-06 0.205 0.4772 0.9000 0.9000   1.00  1  1  2.1E-06
  9 :  -2.25E+01 2.48E-06 0.000 0.2490 0.9000 0.9000   1.00  1  1  5.2E-07
 10 :  -2.25E+01 2.29E-07 0.369 0.0922 0.9900 0.9900   1.00  2  2  4.9E-08
 11 :  -2.25E+01 8.19E-08 0.171 0.3580 0.9000 0.9000   1.00  2  2  1.8E-08
 12 :  -2.25E+01 6.92E-09 0.149 0.0845 0.9900 0.9900   1.00  2  2  1.5E-09
 13 :  -2.25E+01 6.74E-10 0.139 0.0974 0.9900 0.9900   1.00  2  2  1.5E-10

iter seconds digits       c*x               b*y
 13      0.0  10.2 -2.2456285268e+01 -2.2456285269e+01
```

```
|Ax-b| =   8.7e-11, [Ay-c]_+ =   2.7E-10, |x|=  7.9e+00, |y|=  1.2e+01

Detailed timing (sec)
   Pre          IPM          Post
2.100E-02    2.800E-02    2.002E-03
Max-norms: ||b||=1, ||c|| = 16,
Cholesky |add|=0, |skip| = 0, ||L.L|| = 4.35694.
SeDuMi 1.3 by AdvOL, 2005-2008 and Jos F. Sturm, 1998-2003.
Alg = 2: xz-corrector, theta = 0.250, beta = 0.500
eqs m = 11, order n = 9, dim = 65, blocks = 2
nnz(A) = 20 + 0, nnz(ADA) = 121, nnz(L) = 66
 it :     b*y       gap    delta  rate   t/tP*  t/tD*   feas cg cg  prec
  0 :            8.16E+00 0.000
  1 :  -2.49E+00 3.13E+00 0.000 0.3838 0.9000 0.9000   2.28  1  1  2.7E+00
  2 :   1.30E-01 8.94E-01 0.000 0.2857 0.9000 0.9000   3.42  1  1  3.8E-01
  3 :   5.01E-03 1.97E-02 0.000 0.0221 0.9900 0.9900   1.25  1  1  1.2E-01
  4 :   1.17E-07 3.85E-07 0.000 0.0000 1.0000 1.0000   1.01  1  1  2.3E-05
  5 :   3.01E-14 9.08E-14 0.202 0.0000 1.0000 1.0000   1.00  1  1  5.5E-12

iter seconds digits       c*x               b*y
  5      0.0   9.4  5.0899527759e-14  3.0144842876e-14
|Ax-b| =   1.7e-14, [Ay-c]_+ =   3.0E-14, |x|=  5.0e-01, |y|=  1.3e+01

Detailed timing (sec)
   Pre          IPM          Post
2.997E-03    1.001E-02    9.958E-04
Max-norms: ||b||=1, ||c|| = 1.382892e+01,
Cholesky |add|=0, |skip| = 0, ||L.L|| = 1.
Converged at iteration 2
X*:
    6.9145   -0.8214
   -0.8214    4.9001
Y*:
    4.9551   -2.0759
   -2.0759    5.6866
Solving LMI problem resulted in a reduced-order controller with nc = 1
Controller matrix (G):
   -2.6758    1.3083
    8.5579   -5.0717
Ac = -5.071654
Bc = 8.557912
Cc = 1.308284
Dc = -2.675828
---------------------------------------------
Alternating Projection Method | CASE 3: γ = 5
---------------------------------------------
SeDuMi 1.3 by AdvOL, 2005-2008 and Jos F. Sturm, 1998-2003.
Alg = 2: xz-corrector, theta = 0.250, beta = 0.500
eqs m = 6, order n = 11, dim = 35, blocks = 4
nnz(A) = 20 + 0, nnz(ADA) = 36, nnz(L) = 21
 it :     b*y       gap    delta  rate   t/tP*  t/tD*   feas cg cg  prec
  0 :            7.35E+01 0.000
  1 :  -1.03E+01 2.28E+01 0.000 0.3097 0.9000 0.9000   1.48  1  1  2.6E+00
  2 :  -2.10E+01 5.93E+00 0.000 0.2602 0.9000 0.9000   1.11  1  1  7.3E-01
  3 :  -2.36E+01 1.27E+00 0.000 0.2149 0.9000 0.9000   0.92  1  1  1.6E-01
  4 :  -2.57E+01 5.44E-02 0.000 0.0427 0.9900 0.9900   0.87  1  1  7.1E-03
  5 :  -2.58E+01 2.00E-03 0.024 0.0368 0.9900 0.9900   1.00  1  1  2.6E-04
```

```
  6 :  -2.58E+01 2.24E-04 0.206 0.1119 0.9450 0.9450   1.00  1  1  2.9E-05
  7 :  -2.58E+01 1.85E-05 0.480 0.0825 0.9900 0.9900   1.00  1  1  2.4E-06
  8 :  -2.58E+01 8.75E-06 0.164 0.4733 0.9000 0.9000   1.00  1  1  1.2E-06
  9 :  -2.58E+01 1.63E-06 0.000 0.1859 0.9000 0.9000   1.00  1  1  2.2E-07
 10 :  -2.58E+01 1.36E-07 0.471 0.0837 0.9900 0.9900   1.00  1  2  1.8E-08
 11 :  -2.58E+01 2.69E-08 0.000 0.1976 0.9000 0.9000   1.00  2  2  3.6E-09
 12 :  -2.58E+01 1.17E-09 0.359 0.0434 0.9900 0.9900   1.00  2  2  1.6E-10

iter seconds digits       c*x               b*y
 12      0.0  10.1 -2.5756604760e+01 -2.5756604762e+01
|Ax-b| =   8.3e-11, [Ay-c]_+ =   4.2E-10, |x|=  6.6e+00, |y|=  1.3e+01

Detailed timing (sec)
   Pre          IPM          Post
2.002E-03    2.700E-02    2.002E-03
Max-norms: ||b||=1, ||c|| = 25,
Cholesky |add|=0, |skip| = 0, ||L.L|| = 4.93398.
SeDuMi 1.3 by AdvOL, 2005-2008 and Jos F. Sturm, 1998-2003.
Alg = 2: xz-corrector, theta = 0.250, beta = 0.500
eqs m = 11, order n = 9, dim = 65, blocks = 2
nnz(A) = 20 + 0, nnz(ADA) = 121, nnz(L) = 66
 it :     b*y       gap    delta  rate   t/tP*  t/tD*   feas cg cg  prec
  0 :            8.33E+00 0.000
  1 :  -2.75E+00 3.18E+00 0.000 0.3812 0.9000 0.9000   2.25  1  1  2.7E+00
  2 :   1.48E-01 9.32E-01 0.000 0.2935 0.9000 0.9000   3.43  1  1  3.9E-01
  3 :   6.51E-03 2.33E-02 0.000 0.0250 0.9900 0.9900   1.28  1  1  1.2E-01
  4 :   1.86E-07 5.57E-07 0.000 0.0000 1.0000 1.0000   1.01  1  1  3.1E-05
  5 :   4.75E-14 1.27E-13 0.301 0.0000 1.0000 1.0000   1.00  1  1  6.9E-12

iter seconds digits       c*x               b*y
  5      0.0   9.2  7.8883843350e-14  4.7484765648e-14
|Ax-b| =   2.1e-14, [Ay-c]_+ =   4.8E-14, |x|=  5.0e-01, |y|=  1.5e+01

Detailed timing (sec)
   Pre          IPM          Post
2.299E-02    9.998E-03    1.006E-03
Max-norms: ||b||=1, ||c|| = 1.497125e+01,
Cholesky |add|=0, |skip| = 0, ||L.L|| = 1.
Converged at iteration 2
X*:
    7.4856   -0.7233
   -0.7233    5.8495
Y*:
    5.8149   -1.9387
   -1.9387    6.6066
Solving LMI problem resulted in a reduced-order controller with nc = 1
Controller matrix (G):
   -3.1340    1.6176
   11.9541   -7.5012
Ac = -7.501234
Bc = 11.954090
Cc = 1.617626
Dc = -3.134028
```

SCREENSHOT:


MATLAB R2023b - academic use
Live Editor - C:\Users\csamak\OneDrive - Clemson University\Desktop\HW05\Q2.mlx
% PROBLEM 2
% Clear workspace
close all
clear
clc
% Add parser and solver to path
addpath(genpath('C:\Users\csamak\Downloads\MathWorks\Toolboxes\archives\required\YALMIP'))
addpath(genpath('C:\Users\csamak\Downloads\MathWorks\Toolboxes\archives\required\SeDuMi'))
for g = 3:1:5 % Gamma (γ)
disp('----------------------------------')
if g == 3
disp('Standard LMI Method | CASE 1: γ = 3')
elseif g == 4
disp('Standard LMI Method | CASE 2: γ = 4')
elseif g == 5
disp('Standard LMI Method | CASE 3: γ = 5')
end
disp('----------------------------------')
% Define the system matrices
Ap = [0 1; 0 0];
Bp = [0; 1];
Dp = [0 0; 1 0];
Cp = [0 1; 0 0];
Standard LMI Method | CASE 1: γ = 3
SeDuMi 1.3 by AdvOL, 2005-2008 and Jos F. Sturm, 1998-2003.
Alg = 2: xz-corrector, theta = 0.250, beta = 0.500
eqs m = 6, order n = 11, dim = 35, blocks = 4
nnz(A) = 20 + 0, nnz(ADA) = 36, nnz(L) = 21
it : b*y gap delta rate t/tP* t/tD* feas cg cg prec
0 : 2.71E+01 0.000
1 : -5.65E+00 9.11E+00 0.000 0.3364 0.9000 0.9000 1.41 1 1 3.2E+00
2 : -1.25E+01 2.87E+00 0.000 0.3153 0.9000 0.9000 0.91 1 1 9.6E-01
3 : -1.70E+01 6.35E-01 0.000 0.2209 0.9000 0.9000 0.66 1 1 2.5E-01
4 : -1.98E+01 5.13E-02 0.000 0.0808 0.9900 0.9900 0.78 1 1 2.1E-02
5 : -2.01E+01 1.90E-03 0.000 0.0369 0.9900 0.9900 0.99 1 1 8.3E-04
6 : -2.01E+01 1.03E-04 0.407 0.0545 0.9900 0.9900 1.00 1 1 4.5E-05
7 : -2.01E+01 7.99E-06 0.254 0.0773 0.9900 0.9900 1.00 1 1 3.5E-06
8 : -2.01E+01 7.20E-07 0.438 0.0902 0.9900 0.9900 1.00 1 1 3.2E-07
9 : -2.01E+01 1.51E-07 0.000 0.2100 0.9000 0.9000 1.00 2 2 6.7E-08
10 : -2.01E+01 1.86E-08 0.017 0.1233 0.9450 0.9450 1.00 2 2 8.3E-09
11 : -2.01E+01 1.61E-09 0.060 0.0865 0.9900 0.9900 1.00 2 2 7.2E-10
iter seconds digits c*x b*y
11 0.2 9.8 -2.0062574492e+01 -2.0062574496e+01
|Ax-b| = 4.8e-10, [Ay-c]_+ = 9.5E-10, |x|= 1.1e+01, |y|= 1.1e+01
Detailed timing (sec)
Pre IPM Post
3.090E-01 5.140E-01 3.300E-02
Max-norms: ||b||=1, ||c|| = 9,
Cholesky |add|=0, |skip| = 0, ||L.L|| = 6.52152.
X*:
6.9382 -1.0201
-1.0201 3.9239
Command Window