$$P < A^T P A + Q - A^T P B (R + B^T P B)^{-1} B^T P A \quad -①$$
$$P > 0 \quad -②$$

Schur Complement

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0 \Leftrightarrow \begin{bmatrix} X - YZ^{-1}Y^T & 0 \\ 0 & Z \end{bmatrix} > 0$$

The given inequality ①② can be expressed as

$$A^T P A + Q - P - A^T P B (R + B^T P B)^{-1} B^T P A > 0 \quad -③$$
$$P > 0 \quad -④$$

Let $X = A^T P A + Q - P$

$Y = A^T P B \Leftrightarrow Y^T = B^T P A$

$Z = R + B^T P B$

Then inequality ③ can be expressed as

$$\begin{bmatrix} A^T P A + Q - P & A^T P B \\ B^T P A & R + B^T P B \end{bmatrix} > 0 \quad -⑤$$

Inequality ④ can be included into ⑤ as follows

$$\begin{bmatrix} A^T P A + Q - P & A^T P B & 0 \\ B^T P A & R + B^T P B & 0 \\ 0 & 0 & P \end{bmatrix} > 0$$