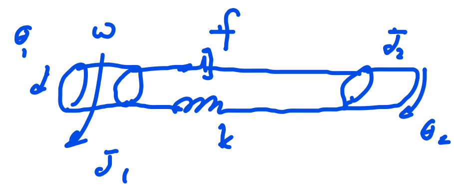
Design Example

This example is adapted from [1] and covered by the demo `sateldem`. The system is a satellite consisting of two rigid bodies (main body and instrumentation module) joined by a flexible link (the "boom"). The boom is modeled as a spring with torque constant k and viscous damping f and finite-element analysis gives the following uncertainty ranges for k and f :

$0.09 \le k \le 0.4$

$$0.09 \le k \le 0.4$$
$$0.0038 \le f \le 0.04:$$



The dynamical equations are

$$\begin{cases} J_1\ddot{\theta}_1 + f(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = T + w \\ J_2\ddot{\theta}_2 + f(\dot{\theta}_2 - \dot{\theta}_1) + k(\theta_2 - \theta_1) = 0 \end{cases}$$

where θ_1 and θ_2 are the yaw angles for the main body and the sensor module, T is the control torque, and w is a torque disturbance on the main body.

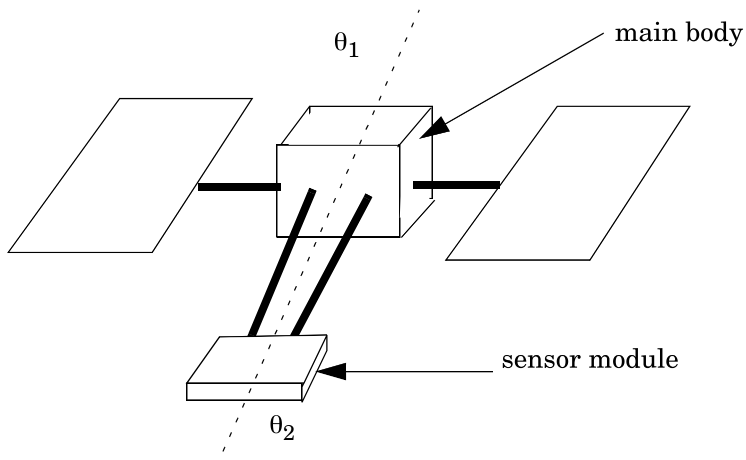


Figure 4-1: Satellite

The control purpose is to minimize the influence of the disturbance w on the angular position θ_2. This goal is expressed through the following objectives:

Γ_{ee}

- Obtain a good trade-off between the RMS gain from w to θ_2 and the H_2 norm of the transfer function from w to

Γ_{ep}

$$\begin{pmatrix} \theta_1 \\ \theta_2 \\ T \end{pmatrix}$$

(LQG cost of control)

- Place the closed-loop poles in the region shown in Figure 4-3 to guarantee some minimum decay rate and closed-loop damping

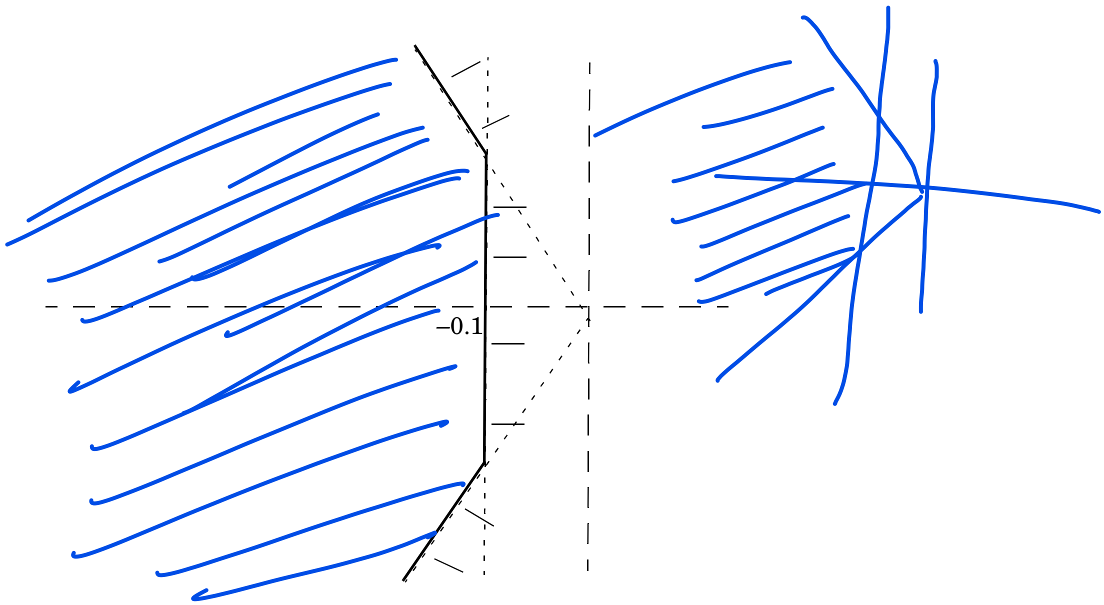


Figure 4-2: Pole placement region

- Achieve these objectives for all possible values of the varying parameters k and f. Since these parameters enter the plant state matrix in an affine manner, we can model the parameter uncertainty by a polytopic system with four vertices corresponding to the four combinations of extremal parameter values (see "From Affine to Polytopic Models" on page 2-20).

To solve this design problem with the LMI Control Toolbox, first specify the plant as a parameter-dependent system with affine dependence on k and f. A state-space description is readily derived from the dynamical equations as:

$x_p = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot\theta_1 \\ \dot\theta_2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & J_1 & 0 \\ 0 & 0 & 0 & J_2 \end{bmatrix} \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \\ \ddot\theta_1 \\ \ddot\theta_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k & k & -f & f \\ k & -k & f & -f \end{bmatrix}}_{A_p} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot\theta_1 \\ \dot\theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} (w + T)$$

$A_p = A_0 + k.A_k + f.A_f$

$$z_1 = \theta_2, \qquad z_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \dot\theta_1 \\ \dot\theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} T$$

y_1, y_2

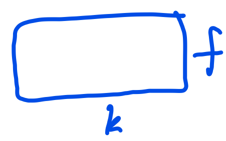
This parameter-dependent model is entered by the commands

```
a0 = [zeros(2) eye(2); zeros(2,4)]
ak = [zeros(2,4) ; [-1 1;1 -1] zeros(2)]
af = [zeros(2,4) ; zeros(2) [-1 1;1 -1]]
e0 = diag([1 1 J1 J2])

b = [0 0;0 0;1 1;0 0] % b = [b1 b2]
c = [0 1 0 0;1 0 0 0;0 1 0 0;0 0 0 0] % c = [c1;c2]
d = [0 0;0 0;0 0;0 1]

% range of parameter values
pv = pvec('box',[0.09 0.4 ; 0.0038 0.04])
                  k             f

% parameter-dependent plant
S  P = psys(pv,[ ltisys(a0,b,c,d,e0) , ...
    ltisys(ak,0*b,0*c,0*d,0) , ...
    ltisys(af,0*b,0*c,0*d,0) ])
```



Next, specify the LMI region for pole placement as the intersection of the half-plane $x < -0.1$ and of the sector centered at the origin and with inner angle $3\pi/4$. This is done interactively with the function `lmireg`:

```
region = lmireg
```

To assess the trade-off between the H_∞ and H_2 performances, first compute the optimal quadratic H_∞ performance subject to the pole placement constraint by

```
gopt = msfsyn(P,[1 1],[0 0 1 0],region)
```

This yields `gopt` ≈ 0. For a prescribed H_∞ performance `g` > 0, the best H_2 performance `h2opt` is computed by

```
[gopt,h2opt,K,Pcl] = msfsyn(P,[1 1],[g 0 0 1],region)
```

Here `obj = [g 0 0 1]` asks to optimize the H_2 performance subject to $\|T_\infty\|_\infty <$ `g` and the pole placement constraint. Repeating this operation for the values `g` $\in \{0.01, 0.1, 0.2, 0.5\}$ yields the Pareto-like trade-off curve shown in Figure 4-4.

By inspection of this curve, the state-feedback gain `K` obtained for `g = 0.1` yields the best compromise between the H_∞ and H_2 objectives. For this choice of `K`, Figure 4-5 superimposes the impulse responses from w to θ_2 for the four combinations of extremal values of k and f.

Finally, the closed-loop poles for these four extremal combinations are displayed in Figure 4-6. Note that they are robustly placed in the prescribed LMI region.

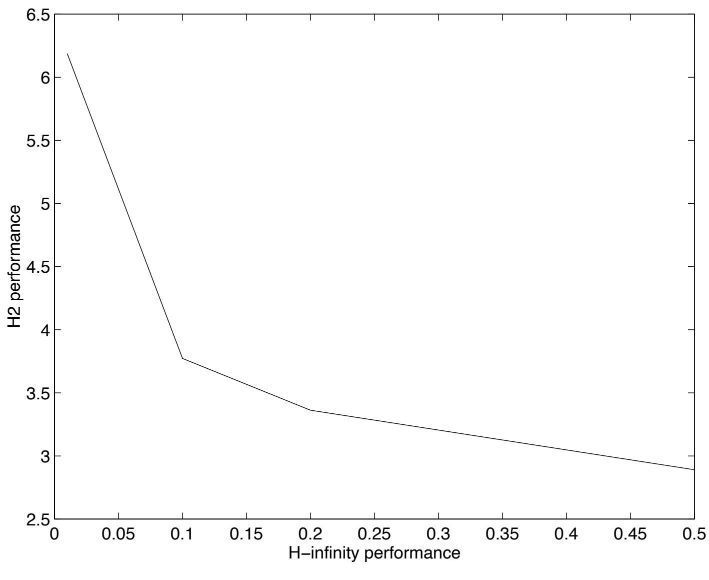


Figure 4-3: Trade-off between the H_∞ and H_2 performances

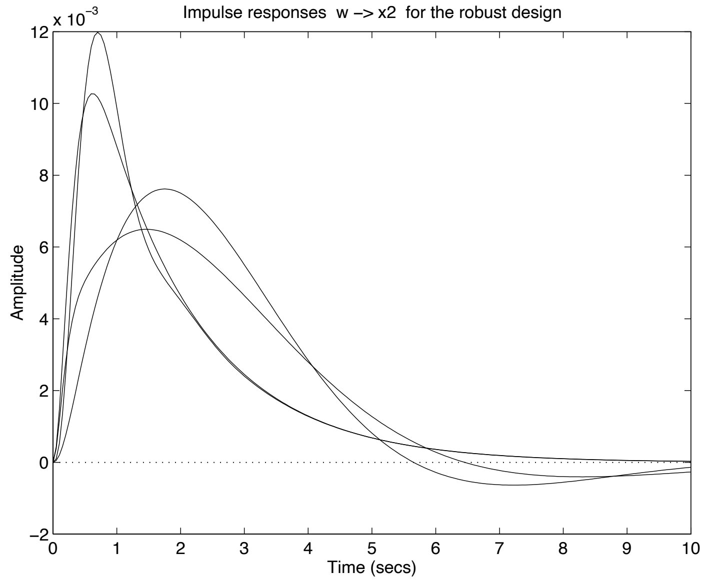


Figure 4-4: Impulse responses for the extremal values of k and f

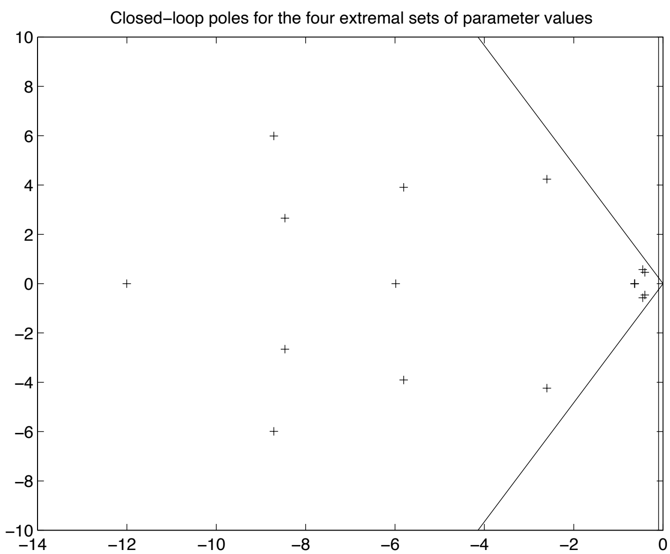


Figure 4-5: Corresponding closed-loop poles

References

[1] Biernacki, R.M., H. Hwang, and S.P. Battacharyya, "Robust Stability with Structured Real Parameter Perturbations," *IEEE Trans. Aut. Contr.*, AC–32 (1987), pp. 495–506.

[2] Boyd, S., L. El Ghaoui, E. Feron, V. Balakrishnan, *Linear Matrix Inequalities in Systems and Control Theory*, SIAM books, 1994.

[3] Chilali, M., and P. Gahinet, "H_∞ Design with Pole Placement Constraints: an LMI Approach," to appear in *IEEE Trans. Aut. Contr*. Also in *Proc. Conf. Dec. Contr.*, 1994, pp. 553–558.

[4] Khargonekar, P.P., and M.A. Rotea,"Mixed H_2/H_∞ Control: A Convex Optimization Approach," *IEEE Trans. Aut. Contr.*, 39 (1991), pp. 824-837.

[5] Scherer, C., "H_∞ Optimization without Assumptions on Finite or Infinite Zeros," *SIAM J. Contr. Opt.*, 30 (1992), pp. 143–166.