$$\dot{x} = \begin{bmatrix} -4+\delta & 2 \\ 1+\delta & -7 \end{bmatrix} x$$

$$\dot{x}(t) = \left(\underbrace{\begin{bmatrix} -4 & 2 \\ 1 & -7 \end{bmatrix}}_{A} + \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \underbrace{\delta(t)}_{\Delta} \right) x(t)$$

$$= A\, x(t) + K\, \phi(t)$$

$\phi(t) = \Delta \psi(t)$

$\psi(t) = M x(t) + H \phi(t)$

$$A = \begin{bmatrix} -4 & 2 \\ 1 & -7 \end{bmatrix} \qquad K = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 \end{bmatrix} \qquad H = 0$$

Problem 1-B

CODE:

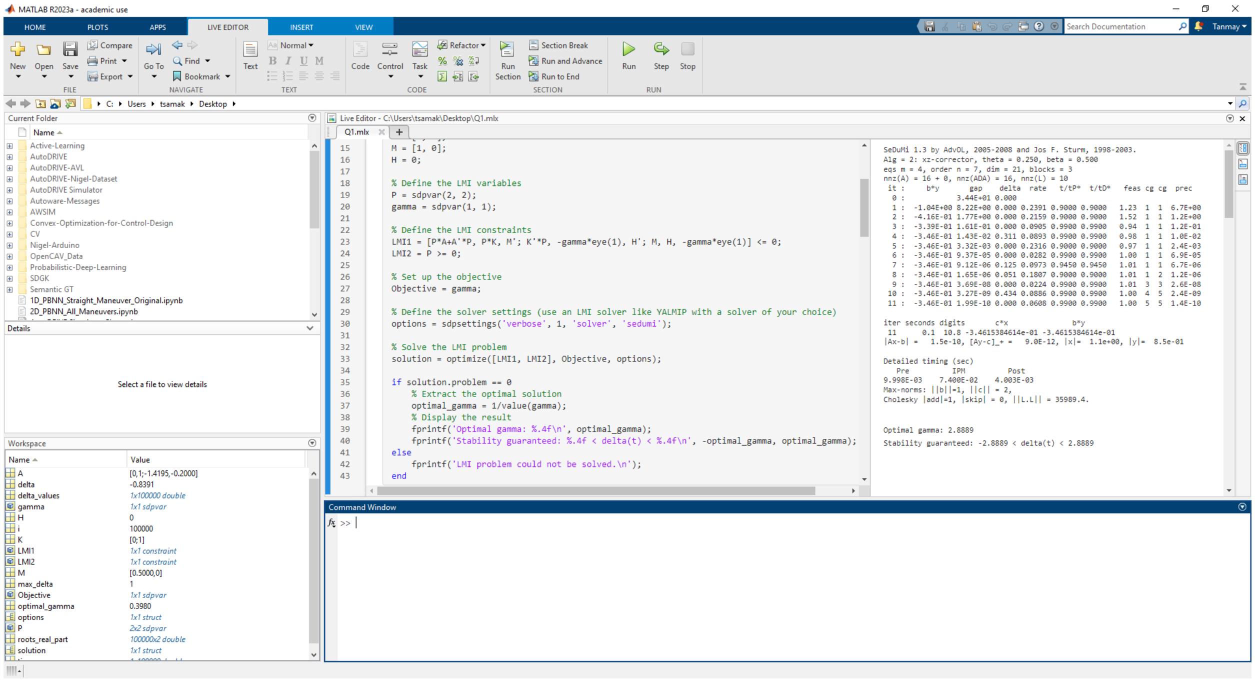
```
% PROBLEM 1-B


% Clear workspace
close all
clear
clc


% Add parser and solver to path
addpath(genpath('C:\Users\tsamak\Downloads\MathWorks\Toolboxes\archives\required\
YALMIP'))
addpath(genpath('C:\Users\tsamak\Downloads\MathWorks\Toolboxes\archives\required\
SeDuMi'))


% Define the system matrices
A = [-4, 2; 1, -7];
K = [1; 1];
M = [1, 0];
H = 0;


% Define the LMI variables
P = sdpvar(2, 2);
gamma = sdpvar(1, 1);


% Define the LMI constraints
LMI1 = [P*A+A'*P, P*K, M'; K'*P, -gamma*eye(1), H'; M, H, -gamma*eye(1)] <= 0;
LMI2 = P >= 0;


% Set up the objective
Objective = gamma;


% Define the solver settings (use an LMI solver like YALMIP with a solver of your
choice)
options = sdpsettings('verbose', 1, 'solver', 'sedumi');


% Solve the LMI problem
solution = optimize([LMI1, LMI2], Objective, options);
```

```
if solution.problem == 0
    % Extract the optimal solution
    optimal_gamma = 1/value(gamma);
    % Display the result
    fprintf('Optimal gamma: %.4f\n', optimal_gamma);
    fprintf('Stability guaranteed: %.4f < delta(t) < %.4f\n', -optimal_gamma,
optimal_gamma);
else
    fprintf('LMI problem could not be solved.\n');
end
```

OUTPUT:

```
Optimal gamma: 2.8889
Stability guaranteed: -2.8889 < delta(t) < 2.8889
```

SCREENSHOT:

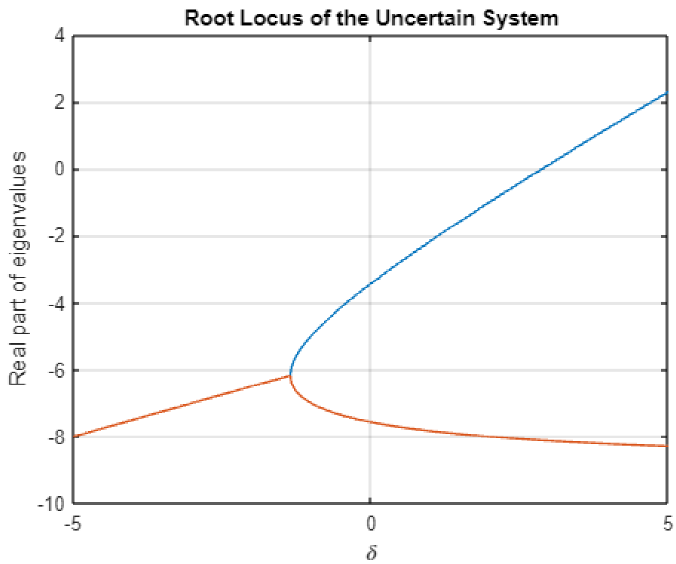


Problem 1-C

CODE:

```
% PROBLEM 1-C


% Define delta limits and granularity
delta_values = linspace(-5, 5, 100000);
roots_real_part = zeros(length(delta_values), 2);
max_delta = -100; % Initialize max_delta for optimization


% Plot root locus for all delta values
for i = 1:length(delta_values)
    delta = delta_values(i);
    A = [-4 + delta, 2; 1 + delta, -7];
    roots_real_part(i, :) = real(eig(A));
    if (all(real(eig(A)) < 0)) && (delta > max_delta)
        max_delta = delta; % Optimize max_delta
    end
end


% Create root locus plot
figure;
plot(delta_values, roots_real_part);
xlabel('\delta');
ylabel('Real part of eigenvalues');
title('Root Locus of the Uncertain System');
grid on;


% Print result
fprintf('Optimal delta: %.4f\n', max_delta);
```

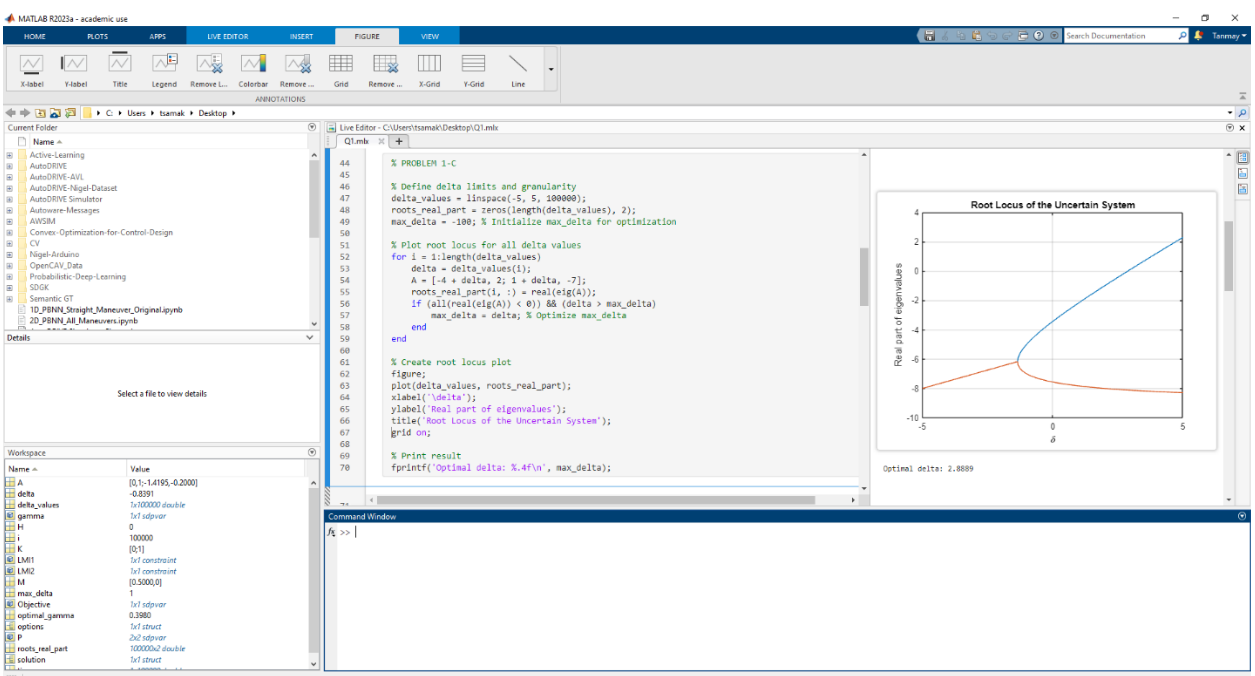
OUTPUT:




```
Optimal delta: 2.8889
```

The maximum value of |δ| such that the system has eigenvalues with negative real part is consistent with the maximum bound for δ that guarantees stability of the uncertain system (based on LMI representation of SGT).

SCREENSHOT:



$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -1+\delta/2 & -0.2 \end{bmatrix} x$$

$$\dot{x}(t) = \left(\underbrace{\begin{bmatrix} 0 & 1 \\ -1 & -0.2 \end{bmatrix}}_{A} + \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix} \underbrace{\delta(t)}_{\Delta} \right) x(t)$$

$$= A\, x(t) + K\, \phi(t)$$

$$\phi(t) = \Delta \psi(t)$$

$$\psi(t) = Mx(t) + H\phi(t)$$

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -0.2 \end{bmatrix} \qquad K = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \qquad H = 0$$

Problem 1-D

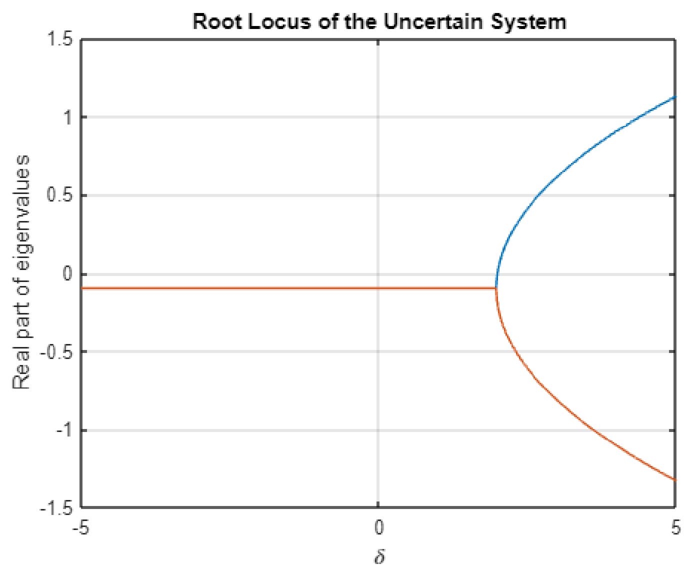
CODE:

```
% PROBLEM 1-D


% Define the system matrices
A = [0, 1; -1, -0.2];
K = [0; 1];
M = [0.5, 0];
H = 0;


% Define the LMI variables
P = sdpvar(2, 2);
gamma = sdpvar(1, 1);


% Define the LMI constraints
LMI1 = [P*A+A'*P, P*K, M'; K'*P, -gamma*eye(1), H'; M, H, -gamma*eye(1)] <= 0;
LMI2 = P >= 0;


% Set up the objective
Objective = gamma;


% Define the solver settings (use an LMI solver like YALMIP with a solver of your
choice)
options = sdpsettings('verbose', 1, 'solver', 'sedumi');


% Solve the LMI problem
solution = optimize([LMI1, LMI2], Objective, options);


if solution.problem == 0
    % Extract the optimal solution
    optimal_gamma = 1/value(gamma);
    % Display the result
    fprintf('Optimal gamma: %.4f\n', optimal_gamma);
    fprintf('Stability guaranteed: %.4f < delta(t) < %.4f\n', -optimal_gamma,
optimal_gamma);
else
    fprintf('LMI problem could not be solved.\n');
end
```

```
% Define delta limits and granularity
delta_values = linspace(-5, 5, 100000);
roots_real_part = zeros(length(delta_values), 2);
max_delta = -100; % Initialize max_delta for optimization


% Plot root locus for all delta values
for i = 1:length(delta_values)
    delta = delta_values(i);
    A = [0, 1; -1 + 0.5*delta, -0.2];
    roots_real_part(i, :) = real(eig(A));
    if (all(real(eig(A)) < 0)) && (delta > max_delta)
        max_delta = delta; % Optimize max_delta
    end
end


% Create root locus plot
figure;
plot(delta_values, roots_real_part);
xlabel('\delta');
ylabel('Real part of eigenvalues');
title('Root Locus of the Uncertain System');
grid on;


% Print result
fprintf('Optimal delta: %.4f\n', max_delta);
```

OUTPUT:

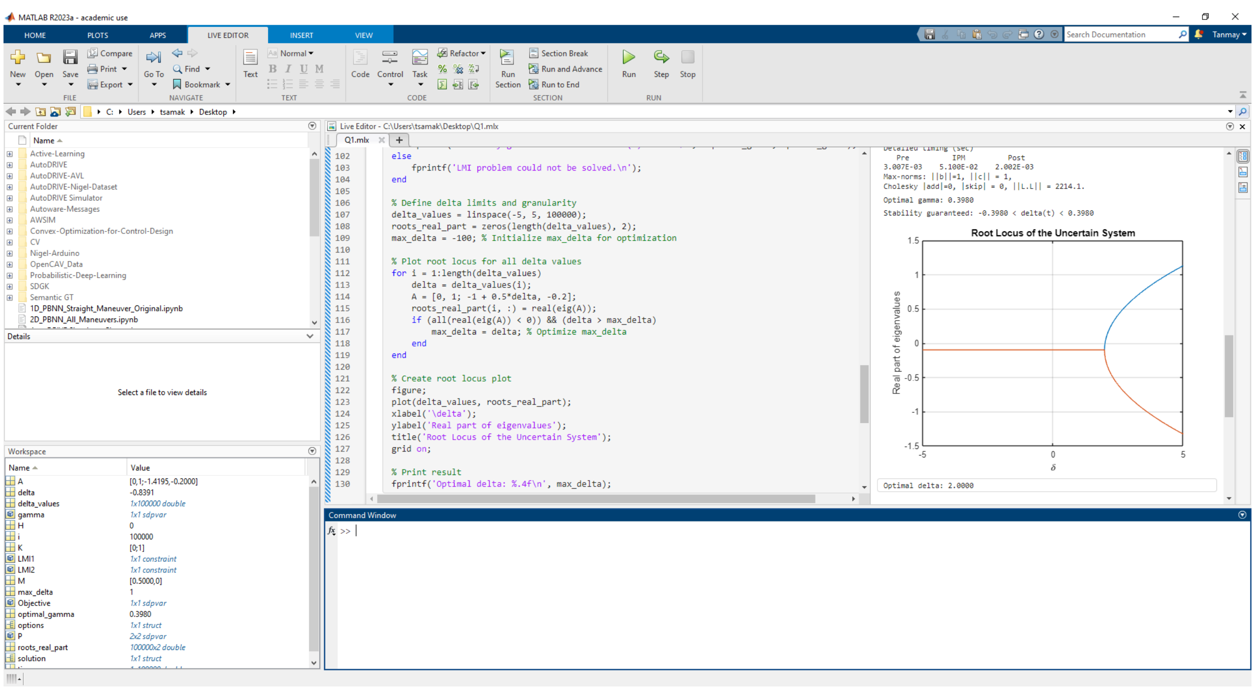
```
Optimal gamma: 0.3980
Stability guaranteed: -0.3980 < delta(t) < 0.3980
```




```
Optimal delta: 2.0000
```

The maximum value of |δ| such that the system has eigenvalues with negative real part is NOT consistent with the maximum bound for δ that guarantees stability of the uncertain system (based on LMI representation of SGT).

SCREENSHOT:



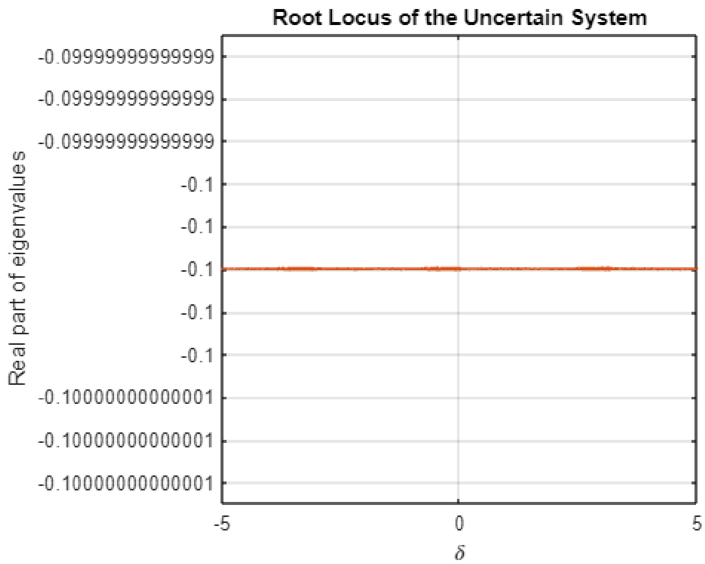
Problem 1-E

CODE:

```
% PROBLEM 1-E


% Define time array for simulation
time = linspace(0, 10, 100000);
roots_real_part = zeros(length(delta_values), 2);
max_delta = -100; % Initialize max_delta for optimization


% Plot root locus for all delta values
for i = 1:length(time)
    delta = cos(time(i));
    A = [0, 1; -1 + 0.5*delta, -0.2];
    roots_real_part(i, :) = real(eig(A));
    if (all(real(eig(A)) < 0)) && (delta > max_delta)
        max_delta = delta; % Optimize max_delta
    end
end


% Create root locus plot
figure;
plot(delta_values, roots_real_part);
xlabel('\delta');
ylabel('Real part of eigenvalues');
title('Root Locus of the Uncertain System');
grid on;


% Print result
fprintf('Optimal delta: %.4f\n', max_delta);
```

OUTPUT:

```
Optimal gamma: 0.3980
Stability guaranteed: -0.3980 < delta(t) < 0.3980
```

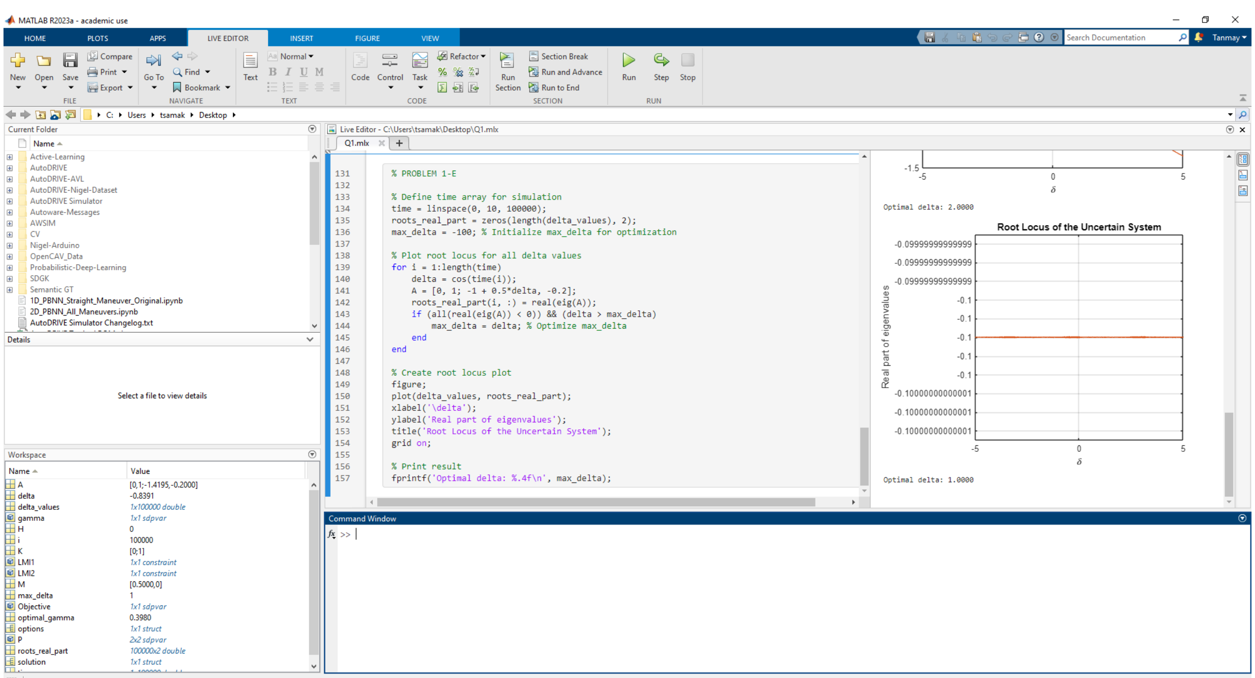



```
Optimal delta: 1.0000
```

The system is not stable throughout the given δ(t) = cos(2t). The system may be stable for -0.3980 < δ(t) < 0.3980, provided it is not already destabilized before that time instant.

The maximum value of |δ| such that the system has eigenvalues with negative real part is NOT consistent with the maximum bound for δ that guarantees stability of the uncertain system (based on LMI representation of SGT). This is likely because Eigenvalue test is not applicable to such time-varying uncertainties δ(t), it can only be applied to time-invariant or very slowly time-varying uncertainties.

SCREENSHOT:



Problem 1-F

The system is guaranteed to be stable between $-0.3980 < \delta(t) < 0.3980$ for time-invariant perturbations. The system is not guaranteed to be stable throughout any time-varying perturbation, but it may be stable for $-0.3980 < \delta(t) < 0.3980$, provided it is not already destabilized before that time instant.

Eigenvalue conditions cannot guarantee stability to time-varying perturbations. They can only be applied to time-invariant or very slowly (slower than system dynamics) time-varying uncertainties.

SGT condition can guarantee stability to time-invariant as well as time-varying perturbations.