$$m_1\ddot{q}_1 + k_1 q_1 - k_1 q_2 = f(t) + w(t)$$
$$m_2\ddot{q}_2 - k_1 q_1 + (k_1+k_2) q_2 - k_2 q_3 = 0$$
$$m_3\ddot{q}_3 - k_2 q_2 + k_2 q_3 = 0$$

$$x_p = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} \qquad y = q_3$$

$$\dot{x}_p = A_p x_p + B_p u + D_p w$$
$$y = C_p x_p$$

$z = x_p$ (full state feedback)

$u = G x_p$ (Static state feedback control)

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -k_1/m_1 & k_1/m_1 & 0 & 0 & 0 & 0 \\ k_1/m_2 & \frac{(-k_1-k_2)}{m_2} & k_2/m_2 & 0 & 0 & 0 \\ 0 & k_2/m_3 & -k_2/m_3 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/m_1 \\ 0 \\ 0 \end{bmatrix} f(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/m_1 \\ 0 \\ 0 \end{bmatrix} w(t)$$

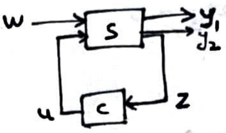
$$q_3 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$

$$A_p = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-k_1}{m_1} & \frac{k_1}{m_1} & 0 & 0 & 0 & 0 \\ \frac{k_1}{m_2} & \frac{(-k_1-k_2)}{m_2} & \frac{k_2}{m_2} & 0 & 0 & 0 \\ 0 & \frac{k_2}{m_3} & \frac{-k_2}{m_3} & 0 & 0 & 0 \end{bmatrix} \qquad B_p = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} \qquad D_p = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix}$$

$$C_p = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

General form:

$$\dot{x}_p = A_p x_p + B_p u + D_p w$$



$$y_1 = C_{p_1} x_p + B_{y_1} u + D_y w$$
$$y_2 = C_{p_2} x_p + B_{y_2} u$$
$$z = M_p x_p \qquad + D_z w$$
$$z = x_p \text{ (full state feedback)}$$

Comparing with earlier notation $\begin{cases} \dot{x}_p = A_p x_p + B_p u + D_p w \\ y = C_p x_p \end{cases}$

general form ↘ ↙ earlier notation

$A_p = A_p$
$B_p = B_p$
$D_p = D_p$
$C_{p_2} = C_p$
$B_{y_2} = 0$
$D_y = 0$

we only consider Γ_{ep} from w to y_2 to design an optimal H_2 controller so as to minimize Γ_{ep} and ensure CL system stability by placing the CL poles in LHP such that $Re(z) < -1$.

desired region for CL-Poles used in "lmireg".

Lumped system representation

$$\left.\begin{aligned} A &= A_p \\ B &= [D_p \;\; B_p] \\ C &= [C_{p_1} \;\; C_{p_2}]^T \\ D &= \begin{bmatrix} D_y & B_{y_1} \\ 0 & B_{y_2} \end{bmatrix} \end{aligned}\right\} S$$

$r = \text{size}(B_{y_2}) = [\text{size}(y_2) \;\; \text{size}(u)]$

$$\begin{array}{cccccc} obj = [& 0 & 0 & 0 & 1 &] \\ & \gamma_1 & \gamma_2 & \alpha & \beta & \end{array}$$

$\Rightarrow \min \Gamma_{ep} \Rightarrow$ optimal H_2 control design

$\min \; (\alpha \Gamma_{ee}^2 + \beta \Gamma_{ep}^2)$

s.t. $\Gamma_{ee} < \gamma_1$

$\Gamma_{ep} < \gamma_2$

$$[\gamma_1^*, \gamma_2^*, G_1, S_{cL}, X] = \text{msfsyn}(S, r, obj, region)$$

γ_1^*: Γ_{ee} of CL system
γ_2^*: Γ_{ep} of CL system
G_1: static state feedback control gain ($u = G_1 x_p$)
S_{cL}: CL system representation
X: Lyapunov parameter matrix

$A_{CL} = A_p + B_p \cdot G_1 \quad \ldots \quad \begin{cases} u = kx \\ \Rightarrow A_{CL} = A + BK \end{cases}$

Design validation:

- performance

→ h2_norm = norm2 (S_{CL})
(Γ_{ep})

$$\hookrightarrow \Gamma_{ep} = \| CPC^T \|^{1/2} = \left(tr(CPC^T)\right)^{1/2}$$

where $AP + PA^T + BB^T = 0$ (Lyapunov equation)
$P > 0$

$\Gamma_{ep} < \gamma_2^*$

- Stability

→ CL poles (z) : $Re\left(\lambda_i\left(A_{CL}\right)\right) = Re(z) < 0$ ← Stability (CL) guaranteed

$Re(z) < -1$ ← pole placement (CL) in desired region.

System simulation:

$$w(t) = \begin{cases} 100 & 0 \leq t \leq 2 \\ 0 & t > 2 \end{cases}$$

$u(t) = 0 \quad \forall t$

simulate open (S_{OL}) and closed (S_{CL}) loop systems using "lsim" for $t = 0$ to $t = 10$ seconds with $\Delta t = 0.01$ sec.

Uncertain system:

10% uncertainty in k_2 (nominal $k_2 = 4000$ N/m)

i.e. $3600 \leq k_2 \leq 4400$ N/m

```
for k2 = 3600:1:4400
    h2_norm = norm2(Scl)  ← using G1
plot(k2, h2_norm)  ← Γep vs. k2 with G1
```

Robust state-feedback controller design:

Affine system representation

$A = A_0 + k_2 A_1$

$$A_0 = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{-k_1}{m_1} & \frac{k_1}{m_1} & 0 & 0 & 0 & 0 \\ \frac{k_1}{m_2} & \frac{-k_1}{m_2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{-1}{m_2} & \frac{1}{m_2} & 0 & 0 & 0 \\ 0 & \frac{1}{m_3} & \frac{-1}{m_3} & 0 & 0 & 0 \end{bmatrix}$$

$B_0 = B$, $C_0 = C$, $D_0 = D$

$P = [k_2]$; $k_2 \in [3600, 4400]$
= pvec ('box', [3600, 4400])

S_0 = ltisys (A_0, B_0, C_0, D_0, 1)

S_1 = ltisys (A_1, 0, 0, 0, 0)

affsys = psys (P, [S_0, S_1])

r = size (B_{y_2}) = [size (y_2) size (u)]

obj = [0 0 0 1] ← optimal H_2 control design (min Γ_{ep})

region = lmireg ← LHP with Re(z) < -1

[γ_1^*, γ_2^*, G_2, S_{cl}, X] = msfsyn (affsys, r, obj, region)

Γ_{ep} of CL system (guaranteed H_2 performance)

Robust static state feedback control gain ($u = G_2 x_p$)

CL system representation obtained

$A_{cl} = A_p + B_p G_2$

($A_{cl} = A + BK$, $u = kx$)

for k_2 = 3600 : 1 : 4400

h2_norm = norm2 (S_{cl}) ← using G_2

plot (k_2, h2_norm) ← Γ_{ep} vs. k_2 with G_2

Problem 1

CODE:

```
% PROBLEM 1


close all
clear
clc


% Define the system parameters
m1 = 20; % kg
m2 = 40; % kg
m3 = 30; % kg
k1 = 2000; % N/m
k2 = 4000; % N/m


% Define the system matrices
Ap = [0 0 0 1 0 0;
      0 0 0 0 1 0;
      0 0 0 0 0 1;
      -k1/m1 k1/m1 0 0 0 0;
      k1/m2 (-k1-k2)/m2 k2/m2 0 0 0;
      0 k2/m3 -k2/m3 0 0 0]
Bp = [0; 0; 0; 1/m1; 0; 0]
Dp = [0; 0; 0; 1/m1; 0; 0]
Cp2 = [0 0 1 0 0 0]
By2 = 0
Dy = 0
```

```
% Lumped system representation
A = Ap;
B = [Dp Bp];
C = [zeros(size(Cp2)); Cp2];
D = [Dy zeros(size(By2)); zeros(size(Dy)) By2];
S = ltisys(A, B, C, D)


% State-feedback controller G1 to minimize the energy-to-peak gain Γep
r = size(By2);
obj = [0 0 0 1]; % Optimal H2 control design objective
region = lmireg; % h (half-plane) --> l (LHP, i.e., Re(z) < x0) --> -1 (x0 = -1)
[g1opt, g2opt, G1, Scl, X] = msfsyn(S, r, obj, region)
```

```
% Closed-loop system
Acl = Ap + Bp*G1
h2_norm = norm2(Scl)
% Scl = ss(Acl, [Dp zeros(size(Bp))], C, D)
% h2_norm = norm(Scl, 2)


% Verification
if((h2_norm < g2opt) && all(real(eig(Acl)) < -1.0))
    disp('Verification of H2 norm and pole location constraints successful!')
else
    disp('Verification of H2 norm and pole location constraints failed!')
end
```

```
% Define the lumped (disturbance + control) input
t = 0:0.01:10;
w_amplitude = 100.0;
w_duration = 2;
w_pulse = w_amplitude * (t >= 0 & t <= w_duration);
u = zeros(size(w_pulse));
w = [w_pulse; u];


% Simulate the open-loop system response
Sol = ss(A, B, C, D)
[y_ol, t_out, x_ol] = lsim(Sol, w', t);


% Plot the results
figure;
sgtitle('Open-Loop System Response');
subplot(3, 1, 1);
plot(t, w(1, :), t, w(2, :));
legend('W', 'U');
subplot(3, 1, 2);
plot(t, x_ol(:, 1), ...
     t, x_ol(:, 2), ...
     t, x_ol(:, 3), ...
     t, x_ol(:, 4), ...
     t, x_ol(:, 5), ...
     t, x_ol(:, 6) ...
     );
legend('X1', 'X2', 'X3', 'X4', 'X5', 'X6');
subplot(3, 1, 3);
plot(t, y_ol(:, 2));
```

```
legend('Y1');


% Simulate the closed-loop system response
Scl = ss(Acl, [Dp zeros(size(Bp))], C, D)
[y_cl, t_out, x_cl] = lsim(Scl, w', t);


% Plot the results
figure;
sgtitle('Closed-Loop System Response');
subplot(3, 1, 1);
plot(t, w(1, :), t, w(2, :));
legend('W', 'U');
subplot(3, 1, 2);
plot(t, x_cl(:, 1), ...
     t, x_cl(:, 2), ...
     t, x_cl(:, 3), ...
     t, x_cl(:, 4), ...
     t, x_cl(:, 5), ...
     t, x_cl(:, 6) ...
     );
legend('X1', 'X2', 'X3', 'X4', 'X5', 'X6');
subplot(3, 1, 3);
plot(t, y_cl(:, 2));
legend('Y1');
```

```
% Define uncertain system parameters
m1 = 20; % kg
m2 = 40; % kg
m3 = 30; % kg
k1 = 2000; % N/m
i = 1;
for k2 = 3600:1:4400 % N/m
    % Define the system matrices
    Ap = [0 0 0 1 0 0;
          0 0 0 0 1 0;
          0 0 0 0 0 1;
          -k1/m1 k1/m1 0 0 0 0;
          k1/m2 (-k1-k2)/m2 k2/m2 0 0 0;
          0 k2/m3 -k2/m3 0 0 0];
    Bp = [0; 0; 0; 1/m1; 0; 0];
    Dp = [0; 0; 0; 1/m1; 0; 0];
    Cp2 = [0 0 1 0 0 0];
    By2 = 0;
    Dy = 0;
```

```
    % Lumped system representation
    A = Ap;
    B = [Dp Bp];
    C = [zeros(size(Cp2)); Cp2];
    D = [Dy zeros(size(By2)); zeros(size(Dy)) By2];
    % Closed-loop system
    Acl = Ap + Bp*G1;
    Scl = ltisys(Acl, [Dp zeros(size(Bp))], C, D);
    h2_norm(i) = norm2(Scl);
    % Scl = ss(Acl, [Dp zeros(size(Bp))], C, D);
    % h2_norm(i) = norm(Scl, 2);
    i = i+1;
end


figure;
sgtitle('Uncertain System with Controller G_1');
plot(3600:1:4400, h2_norm)
xlabel('Spring Constant k_2')
ylabel('Energy to Peak Gain Γ_ep')
```

```
% Define the system parameters
m1 = 20; % kg
m2 = 40; % kg
m3 = 30; % kg
k1 = 2000; % N/m


% System matrices in affine form
A0 = [0 0 0 1 0 0;
      0 0 0 0 1 0;
      0 0 0 0 0 1;
      -k1/m1 k1/m1 0 0 0 0;
      k1/m2 -k1/m2 0 0 0 0;
      0 0 0 0 0 0]
A1 = [0 0 0 0 0 0;
      0 0 0 0 0 0;
      0 0 0 0 0 0;
      0 0 0 0 0 0;
      0 -1/m2 1/m2 0 0 0;
      0 1/m3 -1/m3 0 0 0]
B0 = B
C0 = C
D0 = D


% Uncertain LTI system in affine form
```

```
S0 = ltisys(A0, B0, C0, D0, 1)
S1 = ltisys(A1, zeros(size(B0)), zeros(size(C0)), zeros(size(D0)), 0)


% Parameter vector
P = pvec('box', [3600, 4400]) % k2


% Affine system
affsys = psys(P, [S0, S1])


% State-feedback controller G2 to minimize the energy-to-peak gain Γep
r = size(By2);
obj = [0 0 0 1]; % Optimal H2 control design objective
region = lmireg; % h (half-plane) --> l (LHP, i.e., Re(z) < x0) --> -1 (x0 = -1)
[g1opt, g2opt, G2, Scl, X] = msfsyn(affsys, r, obj, region)


disp('Optimal guaranteed energy-to-peak gain for the uncertain system is:')
disp(g2opt)
```

```
% Define uncertain system parameters
m1 = 20; % kg
m2 = 40; % kg
m3 = 30; % kg
k1 = 2000; % N/m
i = 1;
for k2 = 3600:1:4400 % N/m
    % Define the system matrices
    Ap = [0 0 0 1 0 0;
          0 0 0 0 1 0;
          0 0 0 0 0 1;
          -k1/m1 k1/m1 0 0 0 0;
          k1/m2 (-k1-k2)/m2 k2/m2 0 0 0;
          0 k2/m3 -k2/m3 0 0 0];
    Bp = [0; 0; 0; 1/m1; 0; 0];
    Dp = [0; 0; 0; 1/m1; 0; 0];
    Cp2 = [0 0 1 0 0 0];
    By2 = 0;
    Dy = 0;
    % Lumped system representation
    A = Ap;
    B = [Dp Bp];
    C = [zeros(size(Cp2)); Cp2];
    D = [Dy zeros(size(By2)); zeros(size(Dy)) By2];
    % Closed-loop system
```

```
    Acl = Ap + Bp*G2;
    Scl = ltisys(Acl, [Dp zeros(size(Bp))], C, D);
    h2_norm(i) = norm2(Scl);
    % Scl = ss(Acl, [Dp zeros(size(Bp))], C, D);
    % h2_norm(i) = norm(Scl, 2);
    i = i+1;
end


figure;
sgtitle('Uncertain System with Controller G_2');
plot(3600:1:4400, h2_norm)
xlabel('Spring Constant k_2')
ylabel('Energy to Peak Gain Γ_ep')
```

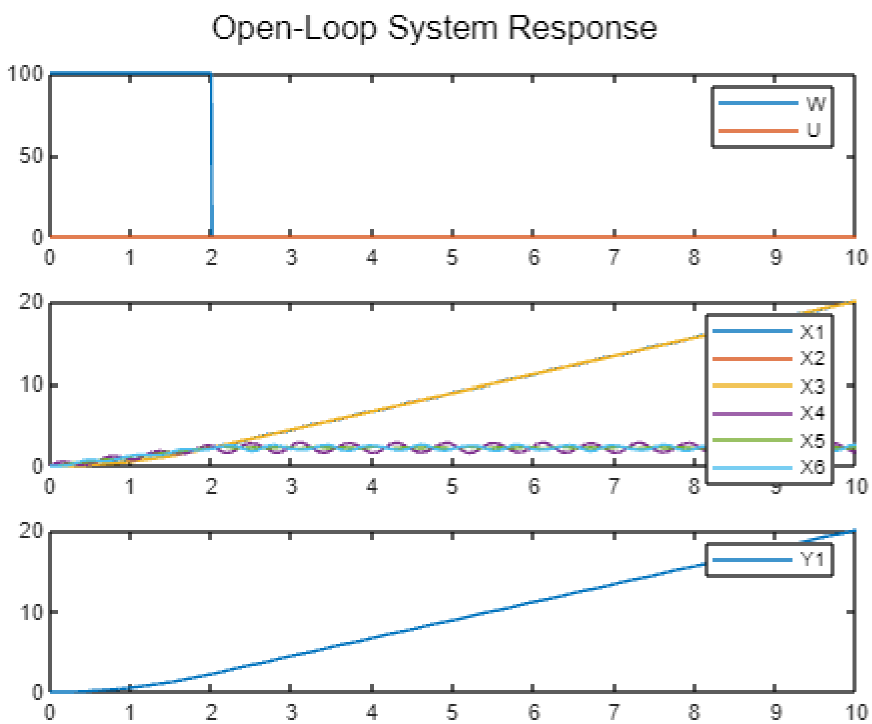
OUTPUT:

```
Ap = 6×6
         0         0         0    1.0000         0         0
         0         0         0         0    1.0000         0
         0         0         0         0         0    1.0000
 -100.0000  100.0000         0         0         0         0
   50.0000 -150.0000  100.0000         0         0         0
         0  133.3333 -133.3333         0         0         0
Bp = 6×1
         0
         0
         0
    0.0500
         0
         0
Dp = 6×1
         0
         0
         0
    0.0500
         0
         0
Cp2 = 1×6
     0     0     1     0     0     0
By2 = 0
Dy = 0
S = 9×9
         0         0         0    1.0000         0         0         0
0    6.0000
         0         0         0         0    1.0000         0         0
0         0
         0         0         0         0         0    1.0000         0
0         0
 -100.0000  100.0000         0         0         0         0    0.0500
0.0500         0
   50.0000 -150.0000  100.0000         0         0         0         0
0         0
```

```
         0  133.3333 -133.3333         0         0         0         0
0         0
         0         0         0         0         0         0         0
0         0
         0         0    1.0000         0         0         0         0
0         0
         0         0         0         0         0         0         0
0      -Inf
Select a region among the following:

h)   Half-plane
d)   Disk
c)   Conic sector
e)   Ellipsoid
p)   Parabola
s)   Horizontal strip
m)   Matrix description of the LMI region
q)   Quit


Select a region among the following:

h)   Half-plane
d)   Disk
c)   Conic sector
e)   Ellipsoid
p)   Parabola
s)   Horizontal strip
m)   Matrix description of the LMI region
q)   Quit
 Optimization of   0.000 * G^2 +  1.000 * H^2 :
 ----------------------------------------------

 Solver for linear objective minimization under LMI constraints

 Iterations   :    Best objective value so far

     1
     2
     3
     4
     5
     6
     7
     8                  0.123781
     9                  0.101980
    10                  0.101980
    11                  0.028857
    12                  0.028857
    13                  0.028857
    14                  0.012000
    15                  0.012000
    16                  0.010372
    17                  0.010372
    18                  0.010372
    19              6.093835e-03
    20              6.093835e-03
```

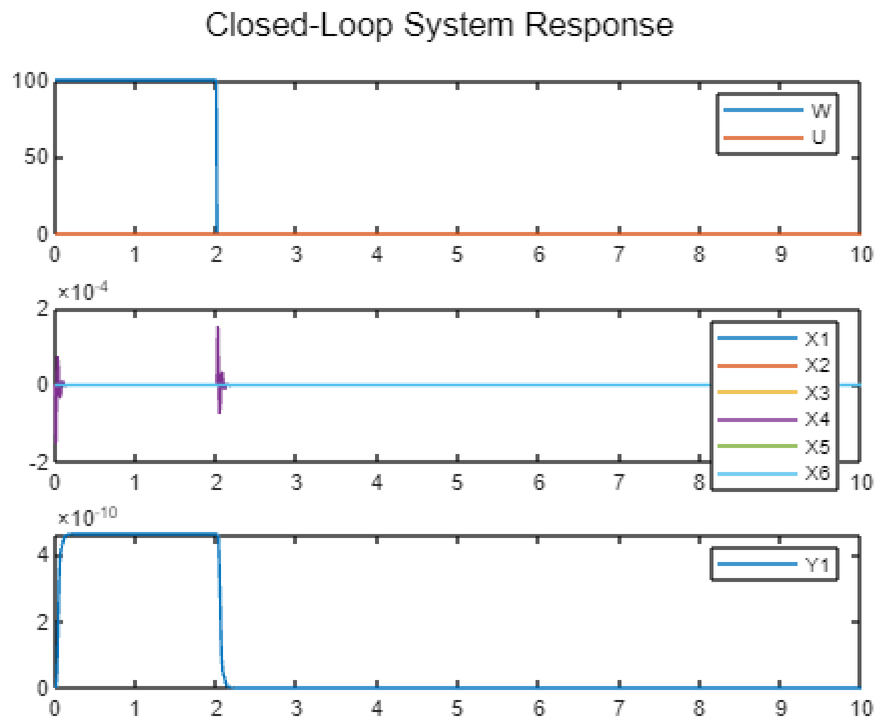
```
    21                6.093835e-03
    22                3.043563e-03
    23                3.043563e-03
    24                3.043563e-03
    25                1.240950e-03
    26                1.240950e-03
    27                1.231455e-03
    28                3.550594e-04
    29                2.305720e-04
    30                2.305720e-04
    31                1.078822e-04
    32                1.078822e-04
    33                1.078822e-04
    34                5.183917e-05
    35                5.183917e-05
    36                5.183917e-05
    37                2.614067e-05
    38                2.614067e-05
    39                2.614067e-05
    40                1.606369e-05
    41                1.606369e-05
    42                1.596526e-05
    43                3.082070e-06
    44                3.082070e-06
    45                2.599298e-06
    46                2.599298e-06
    47                2.599298e-06
    48                9.165308e-07

 Result:  reached the target for the objective value
          best objective value: 9.165308e-07
          f-radius saturation:  0.520% of R =  1.00e+10

 Guaranteed   H2 performance: 9.57e-04

g1opt =

     []
g2opt = 9.5736e-04
G1 = 1×6
10^11 ×
   -0.0008   -0.2312   -1.9312   -0.0000   -0.0024   -0.0996
Scl = 9×8
10^9 ×
         0         0         0    0.0000         0         0         0
0.0000
         0         0         0         0    0.0000         0         0
0
         0         0         0         0         0    0.0000         0
0
   -0.0038   -1.1562   -9.6558   -0.0000   -0.0120   -0.4980    0.0000
0
    0.0000   -0.0000    0.0000         0         0         0         0
0
         0    0.0000   -0.0000         0         0         0         0
0
```

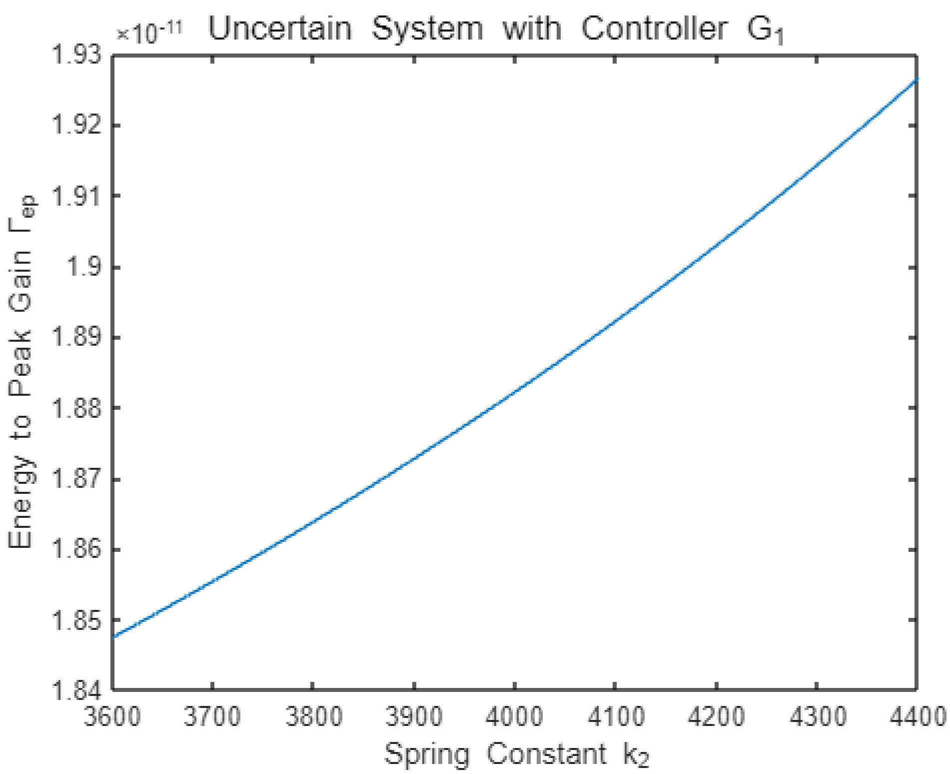
```
         0         0         0         0         0         0         0
0
         0         0    0.0000         0         0         0         0
0
         0         0         0         0         0         0         0      -
Inf
X = 6×6
10^6 ×
    0.0003    0.0000   -0.0000   -0.0080   -0.0002   -0.0000
    0.0000    0.0000    0.0000    0.0000   -0.0000   -0.0000
   -0.0000    0.0000    0.0000   -0.0000    0.0000   -0.0000
   -0.0080    0.0000   -0.0000    1.3121   -0.0003   -0.0000
   -0.0002   -0.0000    0.0000   -0.0003    0.0002    0.0000
   -0.0000   -0.0000   -0.0000   -0.0000    0.0000    0.0000
Acl = 6×6
10^9 ×
         0         0         0    0.0000         0         0
         0         0         0         0    0.0000         0
         0         0         0         0         0    0.0000
   -0.0038   -1.1562   -9.6558   -0.0000   -0.0120   -0.4980
    0.0000   -0.0000    0.0000         0         0         0
         0    0.0000   -0.0000         0         0         0
h2_norm = 1.8822e-11
Verification of H2 norm and pole location constraints successful!
Sol =

  A =
           x1      x2      x3      x4      x5      x6
   x1       0       0       0       1       0       0
   x2       0       0       0       0       1       0
   x3       0       0       0       0       0       1
   x4    -100     100       0       0       0       0
   x5      50    -150     100       0       0       0
   x6       0   133.3  -133.3       0       0       0

  B =
         u1    u2
   x1     0     0
   x2     0     0
   x3     0     0
   x4  0.05  0.05
   x5     0     0
   x6     0     0

  C =
       x1  x2  x3  x4  x5  x6
   y1   0   0   0   0   0   0
   y2   0   0   1   0   0   0

  D =
       u1  u2
   y1   0   0
   y2   0   0

Continuous-time state-space model.
```




```
Scl =

  A = 
                x1          x2          x3          x4          x5          x6
   x1            0           0           0           1           0           0
   x2            0           0           0           0           1           0
   x3            0           0           0           0           0           1
   x4   -3.805e+06  -1.156e+09  -9.656e+09   -1.15e+04  -1.205e+07   -4.98e+08
   x5           50        -150         100           0           0           0
   x6            0       133.3      -133.3           0           0           0

  B = 
          u1    u2
   x1      0     0
   x2      0     0
   x3      0     0
   x4   0.05     0
   x5      0     0
   x6      0     0

  C = 
        x1  x2  x3  x4  x5  x6
   y1    0   0   0   0   0   0
   y2    0   0   1   0   0   0

  D = 
        u1  u2
   y1    0   0
   y2    0   0

Continuous-time state-space model.
```

Model Properties






```
A0 = 6×6
        0         0         0         1         0         0
        0         0         0         0         1         0
        0         0         0         0         0         1
```

```
  -100   100     0     0     0     0
    50   -50     0     0     0     0
     0     0     0     0     0     0
A1 = 6×6
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0   -0.0250    0.0250         0         0         0
         0    0.0333   -0.0333         0         0         0
B0 = 6×2
         0         0
         0         0
         0         0
    0.0500    0.0500
         0         0
         0         0
C0 = 2×6
     0     0     0     0     0     0
     0     0     1     0     0     0
D0 = 2×2
     0     0
     0     0
S0 = 9×9
         0         0         0    1.0000         0         0         0
0    6.0000
         0         0         0         0    1.0000         0         0
0         0
         0         0         0         0         0    1.0000         0
0         0
 -100.0000  100.0000         0         0         0         0    0.0500
0.0500         0
   50.0000  -50.0000         0         0         0         0         0
0         0
         0         0         0         0         0         0         0
0         0
         0         0         0         0         0         0         0
0         0
         0         0    1.0000         0         0         0         0
0         0
         0         0         0         0         0         0         0
0      -Inf
S1 = 9×9 complex
   0.0000 - 1.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
6.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 - 1.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 - 1.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 - 1.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i
```

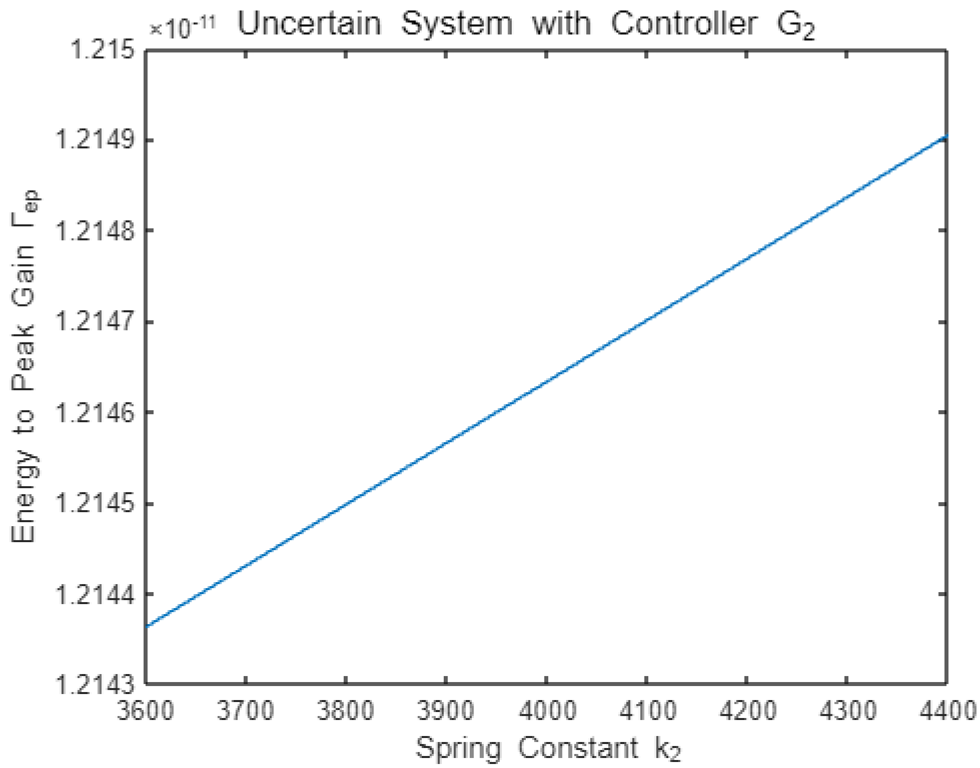
```
   0.0000 + 0.0000i  -0.0250 + 0.0000i   0.0250 + 0.0000i   0.0000 + 0.0000i
0.0000 - 1.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0333 + 0.0000i  -0.0333 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 - 1.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
-Inf + 0.0000i
P = 2×5
          1        3600        4400           0           0
          1           0           0           0           0
affsys = 9×26 complex
10³ ×
     -Inf + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0010 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0060 + 0.0000i   0.0000 + 0.0000i
0.0000 - 0.0010i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0060 + 0.0000i   0.0010 + 0.0000i   3.6000 + 0.0000i   4.4000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0020 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0010 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 - 0.0010i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0010 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0020 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0010 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 - 0.0010i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0060 + 0.0000i   0.0000 + 0.0000i  -0.1000 + 0.0000i   0.1000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0001 + 0.0000i   0.0001 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 - 0.0010i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0020 + 0.0000i   0.0000 + 0.0000i   0.0500 + 0.0000i  -0.0500 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i  -0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 - 0.0010i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0020 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
```

```
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i  -0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 - 0.0010i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0100 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0010 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i     -Inf + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
-Inf + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
0.0000 + 0.0000i   0.0000 + 0.0000i
Select a region among the following:

h)   Half-plane
d)   Disk
c)   Conic sector
e)   Ellipsoid
p)   Parabola
s)   Horizontal strip
m)   Matrix description of the LMI region
q)   Quit


Select a region among the following:

h)   Half-plane
d)   Disk
c)   Conic sector
e)   Ellipsoid
p)   Parabola
s)   Horizontal strip
m)   Matrix description of the LMI region
q)   Quit
 Optimization of   0.000 * G^2 +  1.000 * H^2 :
 ----------------------------------------------

 Solver for linear objective minimization under LMI constraints

 Iterations   :    Best objective value so far

     1
     2
```

3	
4	
5	
6	
7	
8	
9	
10	
11	0.174617
12	0.084142
13	0.084142
14	0.084142
15	0.035606
16	0.035606
17	0.035606
18	0.019613
19	0.019613
20	0.019613
21	0.011597
22	0.011597
23	0.011597
24	7.440620e-03
25	7.440620e-03
26	7.440620e-03
27	4.761071e-03
28	4.761071e-03
29	4.761071e-03
30	3.057379e-03
31	3.057379e-03
32	3.057379e-03
33	2.094639e-03
34	2.094639e-03
35	2.094639e-03
36	1.728979e-03
37	1.728979e-03
38	1.728979e-03
39	1.401557e-03
40	1.401557e-03
41	1.401557e-03
42	7.898234e-04
43	7.898234e-04
44	7.898234e-04
45	3.528895e-04
46	3.528895e-04
47	3.528895e-04
48	1.357855e-04
49	1.357855e-04
50	1.357855e-04
51	5.390543e-05
52	5.390543e-05
53	5.390543e-05
54	2.140225e-05
55	2.140225e-05
56	2.140225e-05
57	1.012459e-05
58	1.012459e-05
59	1.012459e-05

```
    60               5.006723e-06
    61               5.006723e-06
    62               5.006723e-06
    63               2.216772e-06
    64               2.216772e-06
    65               2.216772e-06
    66               9.611413e-07

 Result:  reached the target for the objective value
          best objective value: 9.611413e-07
          f-radius saturation:  0.243% of R =  1.00e+10

 Guaranteed   H2 performance: 9.80e-04

g1opt =

     []
g2opt = 9.8038e-04
G2 = 1×6
10^11 ×
   -0.0009   -0.2480   -1.9988   -0.0000   -0.0028   -0.0997
Scl = 9×19
10^9 ×
      -Inf         0         0         0         0    0.0000         0
0         0    0.0000         0         0         0         0    0.0000
0         0         0    0.0000
    0.0000         0         0         0         0         0    0.0000
0         0         0         0         0         0         0         0
0.0000         0         0         0
    0.0000         0         0         0         0         0         0
0.0000         0         0         0         0         0         0         0
0    0.0000         0         0
    0.0000         0   -0.0046   -1.2400   -9.9938   -0.0000   -0.0138   -
0.4987    0.0000         0         0   -0.0046   -1.2400   -9.9938   -0.0000
-0.0138   -0.4987    0.0000         0
    0.0000         0    0.0000   -0.0000    0.0000         0         0
0         0         0         0    0.0000   -0.0000    0.0000         0
0         0         0         0
    0.0000         0         0    0.0000   -0.0000         0         0
0         0         0         0         0    0.0000   -0.0000         0
0         0         0         0
    0.0000         0         0         0         0         0         0
0         0         0         0         0         0         0         0
0         0         0         0
         0         0         0         0    0.0000         0         0
0         0         0         0         0         0    0.0000         0
0         0         0         0
         0         0         0         0         0         0         0
0         0      -Inf         0         0         0         0         0
0         0         0      -Inf
X = 6×6
10^5 ×
    0.0013    0.0000    0.0000   -0.0411   -0.0008   -0.0000
    0.0000    0.0000    0.0000    0.0000   -0.0000   -0.0000
    0.0000    0.0000    0.0000   -0.0000    0.0000   -0.0000
   -0.0411    0.0000   -0.0000    5.9055    0.0058    0.0001
   -0.0008   -0.0000    0.0000    0.0058    0.0007    0.0000
```

```
   -0.0000   -0.0000   -0.0000    0.0001    0.0000    0.0000
Optimal guaranteed energy-to-peak gain for the uncertain system is:
   9.8038e-04
```



SCREENSHOT: