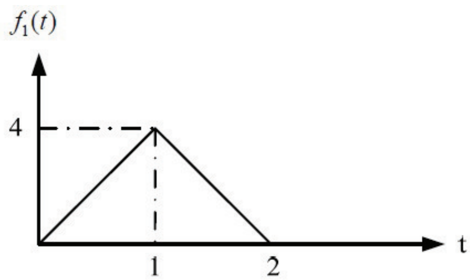
Assignment 2: ME 8930 (LMIs in Optimal and Robust Control)

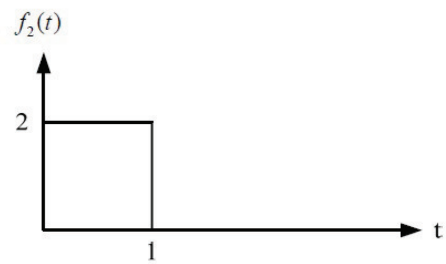
Due on Nov. 3, 2023 by midnight

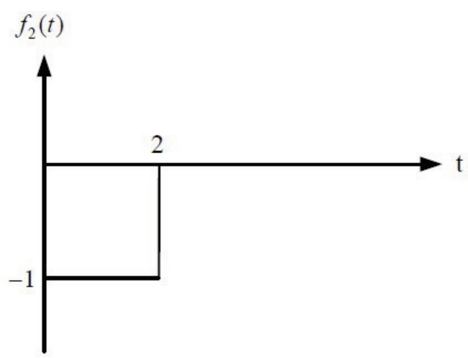
Problem 1: Consider the following signal

$$f(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix}$$

where $f_1(t)$, $f_2(t)$ and $f_3(t)$ are as shown below. Determine the L_1, L_2 and L_∞ norm of the signal $f(t)$.







Problem 2: Consider the following state-space model of the short-period approximation of the F-16 longitudinal dynamics

$$\dot{x} = \begin{bmatrix} -1.01887 & 0.90506 \\ 0.82225 & -1.07741 \end{bmatrix} x + \begin{bmatrix} 0.00203 \\ -0.00164 \end{bmatrix} w_g,$$

with the state vector

$$x = \begin{bmatrix} \alpha \\ q \end{bmatrix},$$

where α is the angle of attack, q is the pitch rate and w_g is the vertical wind gust acting as the disturbance. The vertical acceleration of the aircraft is given by

$$y = 15.87875\alpha + 1.48113q.$$

(a) Compute the energy-to-peak gain (Γ_{ep}) of the system.

(b) Consider a pulse disturbance $w_g(t) = 2$ for $0 \leq t \leq 1$ and $w_g(t) = 0$ for $t > 1$. Calculate the energy of this disturbance signal $\|w_g\|_{L_2}$. Simulate the response of the system using MATLAB. Is the system response consistent with the system gain Γ_{ep}? Why?

(c) Compute the energy-to-energy gain (Γ_{ee}) (H_∞ norm) of the system (by solving the LMI problem in Bounded Real Lemma). Estimate the energy of the response of the system, i.e., $\|y\|_{L_2}$, to the pulse disturbance $w_g(t)$. Is the system response consistent with the system gain Γ_{ee}? Why?

(d) Let $G(s)$ be the transfer function of the above system. Plot $|G(j\omega)|$ as a function of ω. Verify that the peak value of the plot provides the energy-to-energy gain of the system.

Problem 3: Consider the following linear uncertain system

$$\ddot{q}(t) + (5 + 2\delta)\dot{q}(t) + (4 + \delta)q(t) = 0$$

where $\delta(t)$ is a time-varying uncertainty.

1. Write the system in the state-space form

$$\begin{aligned} \dot{x} &= Ax + K\phi \\ \psi &= Mx + H\phi \end{aligned}$$

   with an uncertainty interconnection

$$\phi = \delta(t)\psi.$$

2. Discuss in detail how to find a bound γ such that the uncertain system is stable for all uncertainties $\delta = \delta(t)$ with $|\delta(t)| < \gamma$.

3. Discuss a solution approach for the same problem when the uncertainty δ is time invariant. Then, find an interval for δ that guarantees that the uncertain system is stable.

NOTE: For Problems 2 and 3, please attach your MATLAB (or Python) files and outputs.