Static State Feedback Problem

Find feedback (gain) matrix K such that

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$u(t) = Kx(t)$$

is stable

i.e. Find K such that $A+BK$ is Hurwitz

LMI representation:
(BMI)

Find $X>0$, K :

$$X(A+BK) + (A+BK)^T X < 0$$

↳ Above problem is bilinear in K and X

Equivalent problem:

Find $P>0$, Z :

$$AP + PA^T + BZ + Z^T B^T < 0$$

with $u(t) = \underbrace{ZP^{-1}}_{K} x(t)$

system (A,B) is static state feedback stabilizable iff sol^n to this problem exists. (i.e. some $P>0$ and Z exist that satisfy this LMI)

Python Code:

```
import cvxpy as cp
import numpy as np

# Define system matrices for SYS1
A1 = np.array([[-4, 1], [0, 2]])
B1 = np.array([[1], [0]])

# Define system matrices for SYS2
A2 = np.array([[-3, 2], [4, 1]])
B2 = np.array([[0], [1]])

# Define dimensions
n = 2  # Dimension of state vector
m = 1  # Dimension of control input

# Define the decision variables
P1 = cp.Variable((n, n), symmetric=True)
Z1 = cp.Variable((m, n))

P2 = cp.Variable((n, n), symmetric=True)
Z2 = cp.Variable((m, n))

# Define the LMI constraints for SYS1
constraints1 = [A1@P1 + P1@A1.T + B1@Z1 + Z1.T@B1.T << 0, P1 >> 0.0001*np.eye(n)]

# Define the LMI constraints for SYS2
constraints2 = [A2@P2 + P2@A2.T + B2@Z2 + Z2.T@B2.T << 0, P2 >> 0.0001*np.eye(n)]

# Create an optimization problem for SYS1
problem1 = cp.Problem(cp.Minimize(0), constraints1)

# Create an optimization problem for SYS2
problem2 = cp.Problem(cp.Minimize(0), constraints2)

# Solve the LMI for SYS1
problem1.solve()

# Solve the LMI for SYS2
problem2.solve()

# Check the optimization results for SYS1
if problem1.status == cp.OPTIMAL:
```

```python
    print('SYS1 is stabilizable')
    K1 = Z1.value @ np.linalg.inv(P1.value)
    print("K1 = ", K1)
    Acl1 = A1 + B1@K1
    print("Acl1 = A1 + B1@K1 =\n", Acl1)
    print("Eigenvalues of Acl1:", np.linalg.eig(Acl1)[0])
else:
    print('SYS1 is not stabilizable')
    K1 = None

# Check the optimization results for SYS2
if problem2.status == cp.OPTIMAL:
    print('\nSYS2 is stabilizable')
    K2 = Z2.value @ np.linalg.inv(P2.value)
    print("K2 = ", K2)
    Acl2 = A2 + B2@K2
    print("Acl2 = A2 + B2@K2 =\n", Acl2)
    print("Eigenvalues of Acl2:", np.linalg.eig(Acl2)[0])
else:
    print('SYS2 is not stabilizable')
    K2 = None
```

Output:

SYS1 is not stabilizable

SYS2 is stabilizable

K2 = [[-5.862771 -1.10285985]]

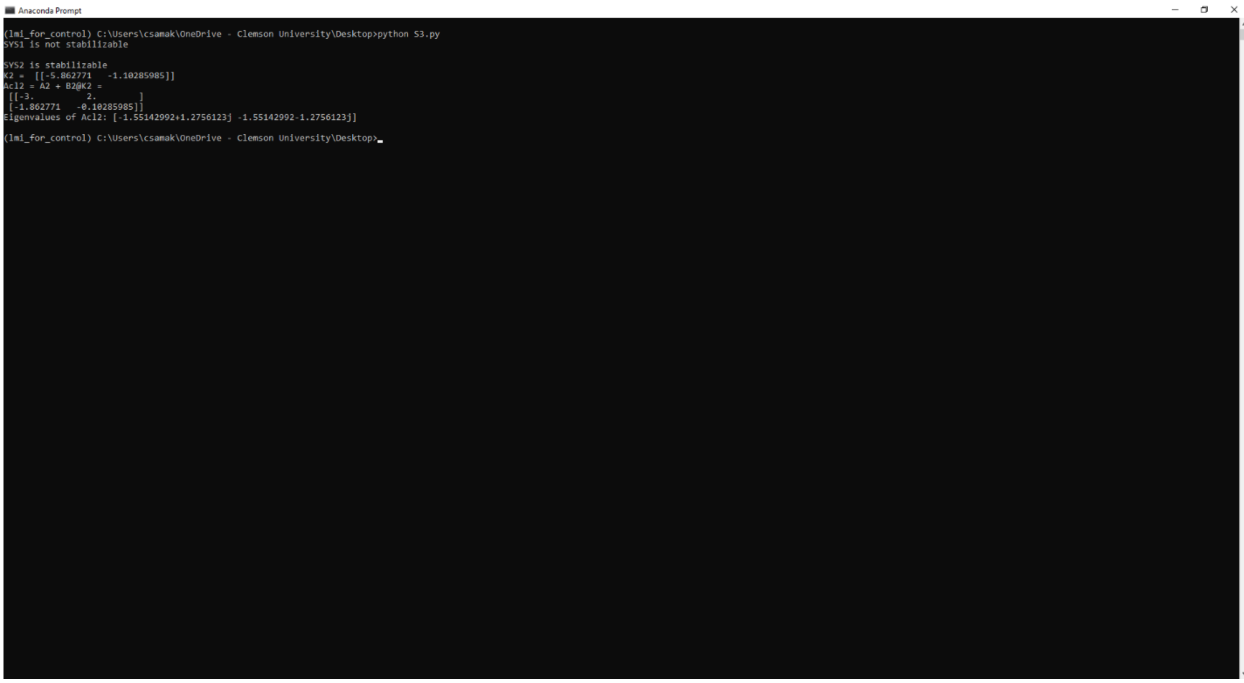
Acl2 = A2 + B2@K2 =

```
 [[-3.         2.        ]
  [-1.862771  -0.10285985]]
```

Eigenvalues of Acl2: [-1.55142992+1.2756123j -1.55142992-1.2756123j]

Since real parts of eigenvalues of Acl2 are negative, it is successfully verified that system SYS2 is in fact stabilized using static state feedback control.

Screenshot:


Anaconda Prompt
(lmi_for_control) C:\Users\csamak\OneDrive - Clemson University\Desktop>python S3.py
SYS1 is not stabilizable
SYS2 is stabilizable
K2 = [[-5.862771   -1.10285985]]
Acl2 = A2 + B2@K2 =
[[-3.   2.   ]
[-1.862771   -0.10285985]]
Eigenvalues of Acl2: [-1.55142992+1.2756123j -1.55142992-1.2756123j]
(lmi_for_control) C:\Users\csamak\OneDrive - Clemson University\Desktop>