$$f(t) = \begin{bmatrix} f_1(t) \\ f_2(t) \\ f_3(t) \end{bmatrix}$$

$$f_1(t) = \begin{cases} 4t, & 0 \le t \le 1 \\ -4(t-2), & 1 \le t \le 2 \\ 0, & \text{otherwise} \end{cases} \; ; f_2(t) = \begin{cases} 2, & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases} \; ; f_3(t) = \begin{cases} -1, & 0 \le t \le 2 \\ 0, & \text{otherwise} \end{cases}$$

<u>L_1 norm:</u>

$$\|f(t)\|_{L_1} = \int_{-\infty}^{\infty} \sum_{i=1}^{n} |f_i(t)|\, dt$$

$$= \int_{-\infty}^{\infty} \sum_{i=1}^{3} |f_i(t)|\, dt$$

$$= \cancelto{0}{\int_{-\infty}^{0} \sum_{i=1}^{3} |f_i(t)|\, dt} + \int_{0}^{1} \sum_{i=1}^{3} |f_i(t)|\, dt + \int_{1}^{2} \sum_{i=1}^{3} |f_i(t)|\, dt + \cancelto{0}{\int_{2}^{\infty} \sum_{i=1}^{3} |f_i(t)|\, dt}$$

$$= 0 + \int_0^1 (4t+2+1)\, dt + \int_1^2 (-4t+8+1)\, dt + 0$$

$$= \left[4\frac{t^2}{2} + 3t\right]_0^1 + \left[-4\frac{t^2}{2} + 9t\right]_1^2$$

$$= \left[2t^2 + 3t\right]_0^1 + \left[-2t^2 + 9t\right]_1^2$$

$$= \left[(2+3) - 0\right] + \left[(-8+18) - (-2+9)\right]$$

$$= [5] + [10 - 7]$$

$$= 5 + 3$$

$$= 8$$

$\underline{L_2 \text{ norm:}}$

$$\|f(t)\|_{L_2} = \left(\int_{-\infty}^{\infty}\sum_{i=1}^{n}|f_i(t)|^2\,dt\right)^{1/2}$$

$$= \left(\int_{-\infty}^{\infty}\sum_{i=1}^{3}|f_i(t)|^2\,dt\right)^{1/2}$$

$$= \left(\cancelto{0}{\int_{-\infty}^{0}\sum_{i=1}^{3}|f_i(t)|^2\,dt} + \int_{0}^{1}\sum_{i=1}^{3}|f_i(t)|^2\,dt + \int_{1}^{2}\sum_{i=1}^{3}|f_i(t)|^2\,dt + \cancelto{0}{\int_{2}^{\infty}\sum_{i=1}^{3}|f_i(t)|^2\,dt}\right)^{1/2}$$

$$= \left(\int_{0}^{1}\left((4t)^2+2^2+1^2\right)dt + \int_{1}^{2}\left((8-4t)^2+(1)^2\right)dt\right)^{1/2}$$

$$= \left(\int_{0}^{1}\left(16t^2+4+1\right)dt + \int_{1}^{2}\left(64-64t+16t^2+1\right)dt\right)^{1/2}$$

$$= \left(\int_{0}^{1}\left(16t^2+5\right)dt + \int_{1}^{2}\left(16t^2-64t+65\right)dt\right)^{1/2}$$

$$= \left(\left[16\frac{t^3}{3}+5t\right]_{0}^{1} + \left[16\frac{t^3}{3}-64\frac{t^2}{2}+65t\right]_{1}^{2}\right)^{1/2}$$

$$= \left(\left[\left(\frac{16}{3} + 5 \right) - (0) \right] + \left[\left(\frac{128}{3} - 128 + 130 \right) - \left(\frac{16}{3} - 32 + 65 \right) \right] \right)^{1/2}$$

$$= \left(\left[\frac{16}{3} + 5 \right] + \left[\left(\frac{128}{3} + 2 \right) - \left(\frac{16}{3} + 33 \right) \right] \right)^{1/2}$$

$$= \left(\left[\frac{16}{3} + 5 \right] + \left[\frac{112}{3} - 31 \right] \right)^{1/2}$$

$$= \left(\frac{128}{3} - 26 \right)^{1/2}$$

$$\approx (42.667 - 26)^{1/2}$$

$$\approx \sqrt{16.667}$$

$$\approx 4.0825$$

L_∞ norm:

$$\|f(t)\|_{L_\infty} = \max_{-\infty < t < \infty} \max_i \left(|f_i(t)| \right)$$

$$= \max\left(\cancelto{0}{\max_{-\infty < t \le 0} \left(|f_1(t)|, |f_2(t)|, |f_3(t)| \right)}, \max_{0 \le t \le 1} \left(|f_1(t)|, |f_2(t)|, |f_3(t)| \right), \max_{1 \le t \le 2} \left(|f_1(t)|, |f_2(t)|, |f_3(t)| \right), \cancelto{0}{\max_{2 \le t < \infty} \left(|f_1(t)|, |f_2(t)|, |f_3(t)| \right)} \right)$$

$$= \max\left(\max_{0 \le t \le 1} \left(|4t|, |2|, |-1| \right), \max_{1 \le t \le 2} \left(|-4(t-2)|, 0, |-1| \right), 0 \right)$$

$$= \max\left(\max(4, 2, 1), \max(4, 0, 1), 0 \right)$$

$$= \max(4, 4, 0)$$

$$= 4$$