AuE-ME 4600-6600: Dynamic Performance of Vehicles

Fall 2022 Midterm Exam – Take Home Portion

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Solution 1:

```
clear; close all; clc;
m = 1450; % mass (kg)
Pw = 220*745.6998; % engine power at wheels (W)
rho = 1.225; % air density (kg/m^3)
u = 0.91; % friction coefficient
```

1A.

```
Af = 1.6+(0.00056*(m-765)); % Approximation for Af based on vehicle mass
```

Traction limited acceleration: $Fa \cdot v = (\mu \cdot m \cdot g \cdot v) - Fd \cdot v$

$\Rightarrow m \cdot a = \mu \cdot m \cdot g - 0.5 \cdot \rho \cdot v^2 \cdot Cd \cdot Af$

$\Rightarrow Cd = \dfrac{\mu \cdot m \cdot g - m \cdot a}{0.5 \cdot \rho \cdot v^2 \cdot Af}$

```
v1 = 30/3.6; % velocity reading 1 (m/s)
a1 = 0.905*9.81; % acceleration reading 1 (m/s^2)
v2 = 50/3.6; % velocity reading 2 (m/s)
a2 = 0.895*9.81; % acceleration reading 2 (m/s^2)
Cd1 = ((u*m*9.81)-(m*a1))/(0.5*rho*v1^2*Af); % drag coefficient corresponding to v1, a1
Cd2 = ((u*m*9.81)-(m*a2))/(0.5*rho*v2^2*Af); % drag coefficient corresponding to v2, a2
Cd = (Cd1+Cd2)/2; % mean Cd from Cd1 and Cd2
```

At terminal velocity (power limited acceleration), $a = 0 \Rightarrow Fa = m \cdot a = 0 \Rightarrow 0 = Pw - v \cdot Fd \Rightarrow v = Pw/Fd$

$\Rightarrow v^3 = Pw/(0.5 \cdot \rho \cdot Cd \cdot Af) \Rightarrow v = \sqrt[3]{Pw/(0.5 \cdot \rho \cdot Cd \cdot Af)}$

```
v = nthroot(Pw/(0.5*rho*Cd*Af),3); % terminal velocity (m/s)
fprintf("Terminal velocity, v = %f m/s", v);
```

```
Terminal velocity, v = 53.603682 m/s
```

1B.

```
Fd = 0.5*rho*v^2*Cd*Af; % aerodynamic drag force (N)
d = Fd/m; % decelertation due to aerodynamic drag (m/s^2)
fprintf("Deceleration due to aerodynamic drag, d = %f m/s^2", d);
```

```
Deceleration due to aerodynamic drag, d = 2.110688 m/s^2
```

1C.

```
ay = 0.5*9.81; % lateral acceleration (m/s^2)
Fy = m*ay; % lateral force (N)
alpha = 5*(pi/180); % slip angle (rad)
Fyd = Fy*sin(alpha); % induced drag force (N)
fprintf("Induced drag force, Fyd = %f N", Fyd);
```

```
Induced drag force, Fyd = 619.873431 N
```

1D.

Power equation accounting inducted drag:

$Fa \cdot v = Pw - v \cdot Fd - v \cdot Fyd$

At terminal velocity, $a = 0 \Rightarrow Fa \cdot v = m \cdot a \cdot v = 0 \ \Rightarrow 0 = Pw - v \cdot Fd - v \cdot Fyd$

we know that aerodynamic drag force, $Fd = 0.5 \cdot \rho \cdot v^2 \cdot Cd \cdot Af$

also, we know that induced drag force, $Fyd = Fy \cdot \sin\alpha = m \cdot ay \cdot \sin\alpha = m \cdot \left(\frac{v^2}{R}\right) \cdot \sin\alpha$

$\Rightarrow Pw - v^3(0.5 \cdot \rho \cdot Cd \cdot Af) - v^3\left(\frac{m \cdot \sin\alpha}{R}\right) = 0$

$\Rightarrow Pw - v^3\left(0.5 \cdot \rho \cdot Cd \cdot Af + \frac{m \cdot \sin\alpha}{R}\right) = 0$

$\Rightarrow v^3 = \frac{Pw}{\left(0.5 \cdot \rho \cdot Cd \cdot Af + \frac{m \cdot \sin\alpha}{R}\right)}$

$\Rightarrow v = \sqrt[3]{\frac{Pw}{\left(0.5 \cdot \rho \cdot Cd \cdot Af + \frac{m \cdot \sin\alpha}{R}\right)}}$

Problem 2:

```
clear; close all; clc;
m = 1500; % mass (kg)
Cd = 0.5; % drag coefficient
Af = 2.2; % frontal area (m^2)
v_top = 240/3.6; % top speed (m/s)
a = 0.16/2; % half tire print length (m)
CY = 6.25*1e6; % lateral stiffness per unit length (N/m^2)
u_max = 1.3; % maximum friction coefficient
MZ = 160; % self-aligning moment (Nm)
```

2A.

```
FY = MZ*(3/a); % lateral force (N)
```

```
SA = FY/(2*CY*a^2); % slip angle (rad)
fprintf("Total lateral force produced by the tires, FY = %f N\n" + ...
        "Slip angle at which the tires were running, SA = %f rad", FY, SA);
```

```
Total lateral force produced by the tires, FY = 6000.000000 N
Slip angle at which the tires were running, SA = 0.075000 rad
```

2B.

```
rho = 1.225; % air density (kg/m^3)
Fd = 0.5*rho*v_top^2*Cd*Af; % aerodynamic drag force (N)
Pw = Fd*v_top; % engine power at wheels (W)
Fyd = FY*sin(SA); % induced drag force (N)
v = Pw/(Fd+Fyd); % velocity of the vehicle (m/s) -- P=F*v ==> v=P/F
fprintf("Speed of the vehicle, v = %f m/s", v);
```

```
Speed of the vehicle, v = 57.964087 m/s
```

2C.

```
ay = FY/m; % lateral acceleration (m/s^2)
R = v^2/ay; % radius of circle (m)
fprintf("Lateral acceleration, ay = %f m/s^2\n" + ...
        "Radius of circle, R = %f m", ay, R);
```

```
Lateral acceleration, ay = 4.000000 m/s^2
Radius of circle, R = 839.958836 m
```

2D.

$$\mathrm{Fy}_{\mathrm{parabolic}} = \mu \cdot \mathrm{Fz} \cdot \left[3 \cdot \theta \cdot \sigma_y - 3 \cdot \theta^2 \cdot \sigma_y^2 + \theta^3 \cdot \sigma_y^3\right]$$

w.k.t. $\mathrm{Fy}_{\mathrm{parabolic}} = \mathrm{Fy}_{\mathrm{linear}}$ (from part 2A)

$$\Rightarrow \frac{\mathrm{Fy}_{\mathrm{linear}}}{\mu \cdot \mathrm{Fz}} = \left[3 \cdot \theta \cdot \sigma_y - 3 \cdot \theta^2 \cdot \sigma_y^2 + \theta^3 \cdot \sigma_y^3\right]$$

$$\Rightarrow \theta^3 \cdot \sigma_y^3 - 3 \cdot \theta^2 \cdot \sigma_y^2 + 3 \cdot \theta \cdot \sigma_y - \frac{\mathrm{Fy}_{\mathrm{linear}}}{\mu \cdot \mathrm{Fz}} = 0$$

w.k.t. θ and $\frac{\mathrm{Fy}_{\mathrm{linear}}}{\mu \cdot \mathrm{Fz}}$ are constants (known values), so we can solve above equation for σ_y by finding its roots.

```
FZ = (m*9.81)/4; % normal load on each tire (N)
T = (2*a^2*CY)/(3*u_max*FZ); % theta (intermediate variable)
coeff = [T^3 -3*T^2 3*T -(FY/4)/(u_max*FZ)]; % coefficients of polynomial derived above
SY = roots(coeff); % roots of polynomial
% find real root(s) of polynomial
for i=1:length(SY)
    if isreal(SY(i))
        SY_real = SY(i);
    end
end
SA_parabolic = atan(SY_real); % slip angle (rad)
```

```
fprintf("Slip angle (considering lateral force determined in part A, employing brush tire model
```

```
Slip angle (considering lateral force determined in part A, employing brush tire model with parabolic pressure distr
```

2E.

```
FZ = (m*9.81)/4; % normal load on each tire (N)
FY_parabolic = u_max*FZ; % maximum lateral load each tire can take (N)
ay = FY_parabolic/m; % maximum lateral acceleration each tire can take (m/s^2)
v = sqrt(R*ay); % maximum velocity before total sliding would occur (m/s)
fprintf("Maximum velocity before total sliding occurs (considering radius determined in part C,
```

```
Maximum velocity before total sliding occurs (considering radius determined in part C, employing brush tire model wi
```

2F.

```
a = 0.55; % distance of COG from front (%)
b = 0.45; % distance of COG from rear (%)
CYf = (FY/4)/SA; % cornering stiffness of front tire (N/rad)
CYr = (a/b)*CYf; % cornering stiffness of rear tire (N/rad)
fprintf("Rear cornering stiffness (to make the vehicle neutral-steer under given conditions), C
```

```
Rear cornering stiffness (to make the vehicle neutral-steer under given conditions), CYr = 24444.444444 N/rad
```

Problem 3:

```
clear; close all; clc;
```

3A.

```
FZ = -1200*4.4482; % normal load (N)
SA = 4*pi/180; % slip angle (rad)
u = 2+(0.0004*FZ); % friction coefficient
C = (0.0003*FZ^2)+(0.8*FZ); % cornering stiffness (N/rad)
a_bar = (C*tan(SA))/(u*FZ); % non-dimensional slip angle
F_bar = sin(1.4*atan(0.7*a_bar)); % non-dimensional lateral force (B=0.7, C=1.4, D=1, E=0)
FY = F_bar*u*FZ; % lateral force (N)
fprintf("Lateral force corresponding to 4 deg slip angle and -1200 lb normal load, FY = %f N",
```

```
Lateral force corresponding to 4 deg slip angle and -1200 lb normal load, FY = 277.909578 N
```

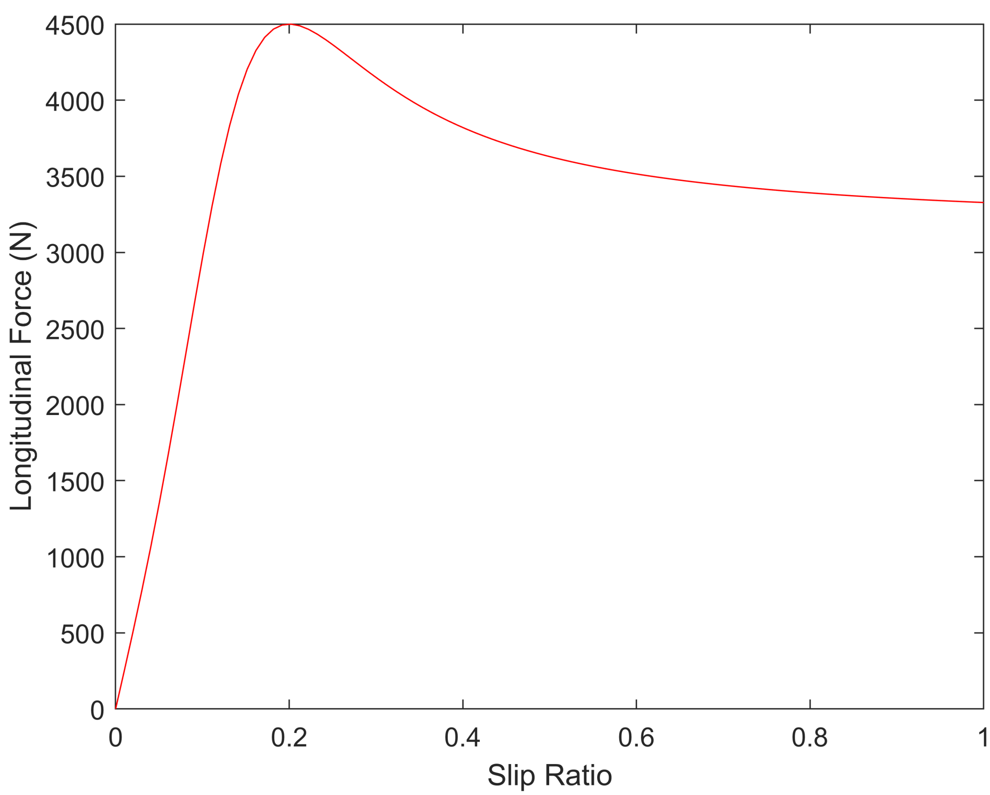
3B.

```
FZ = 4500; % normal load (N)
a = 0.04; % half patch length (m)
K = 0.15; % slip ratio
u = 0.8; % friction coefficient
CX = 1300*10000; % longitudinal stiffness of the tire (N/m^2)
T = (2*a^2*CX)/(3*u*FZ); % theta (intermediate variable)
SX = K/(1+K); % theoretical slip from slip ratio
FX = u*FZ*((3*T*SX)-(3*T^2*SX^2)+(T^3*SX^3)); % longitudinal force (N) based on brush model ass
fprintf("Longitudinal force corresponding to 0.15 slip ratio, FX = %f N", FX);
```

```
Longitudinal force corresponding to 0.15 slip ratio, FX = 3156.490284 N
```

3C.

```
FZ = 4500; % normal load (N)
up = 1.0; % road adhesion (friction coefficient at peak value)
us = 0.7; % friction coefficient at asymptotic value
s1 = 0.04; % slip ratio reading 1
FX1 = 1000; % longitudinal force reading 1
s2 = 0.2; % slip ratio reading 2
FX2 = up*FZ; % peak longitudinal force (reading 2)
s0 = 0; FX0 = 0; % assuming the longitudinal force is 0 at 0 slip ratio
D = FX2; % peak FX
xm = s2; % slip ratio corresponding to peak FX
BCD = (FX1-FX0)/(s1-s0); % slope in linear range (0<=s<=0.04), m = (y2-y1)/(x2-x1)
ys = us*FZ; % asymptotic longitudinal force
C = 1+(1-((2/pi)*asin(ys/D)));
B = BCD/(C*D);
E = ((B*xm)-tan(pi/(2*C)))/((B*xm)-atan(B*xm));
% analysis of relationship between longitudinal force and slip ratio
X = linspace(0,1); % input (slip ratio)
Y = D*sin(C*atan((B*X)-E*((B*X)-atan((B*X))))); % output (longitudinal force corresponding to s
figure()
plot(X,Y,'-','color','red')
xlabel("Slip Ratio")
ylabel("Longitudinal Force (N)")
```

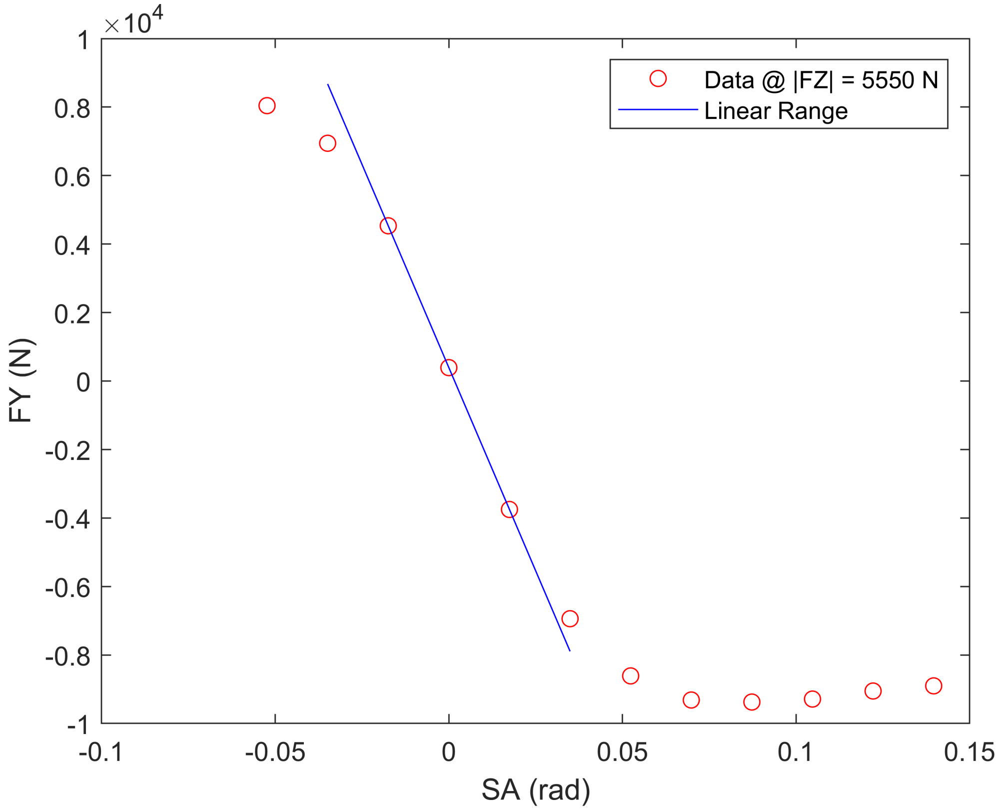


Solution 4:

```
clear; close all; clc;
FZ = [-3.325 -3.325 -3.325 -3.325 -3.325 -3.325 -3.325 -3.325 -3.325 -3.325 -3.325 -3.325 -5.55
SA = [-3 -2 -1 0 1 2 3 4 5 6 7 8 -3 -2 -1 0 1 2 3 4 5 6 7 8 -3 -2 -1 0 1 2 3 4 5 6 7 8].*(pi/18
IA = [4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4].*(pi/180); % inc
P  = [221 221 221 221 221 221 221 221 221 221 221 221 221 221 221 221 221 221 221 221 221 221 2
FY = [5.159 4.4 2.899 0.16 -2.522 -4.348 -5.439 -5.824 -6.008 -5.935 -5.851 -5.779 8.042 6.943
RL = [27.67 27.71 27.76 27.8 27.77 27.71 27.66 27.63 27.62 27.63 27.63 27.63 27.35 27.43 27.57
```

4A.

```
idx = find(FZ==-5550); % filter data (get array indices corresponding to FZ = -5550 N)
SA5550 = SA(idx); % slip angle (rad) corresponding to FZ = -5550 N
FY5550 = FY(idx); % lateral force (N) corresponding to FZ = -5550 N
FZRLpoly = polyfit(SA5550(3:5),FY5550(3:5),1); % fit polynomial (line) to FY vs SA data in line
figure()
plot(SA5550,FY5550, 'o','color','red')
hold on
plot(SA5550(2:6),polyval(FZRLpoly,SA5550(2:6)),'-','color','blue')
xlabel("SA (rad)")
ylabel("FY (N)")
legend("Data @ |FZ| = 5550 N","Linear Range",'Location','NE')
%xlim([0, 0.15])
%ylim([-10000, 0])
%set(gca, 'YDir','reverse')
hold off
```




```
dFYdSApoly = polyder(FZRLpoly); % compute derivative (slope) of polynomial fit to FY vs SA data
C = polyval(dFYdSApoly, 0); % cornering stiffness [value of slope/derivative (dFYdSpoly) at SA
fprintf("Cornering stiffness (at -5550 N normal load), C = %f N/rad", C);
```

```
Cornering stiffness (at -5550 N normal load), C = -237319.118743 N/rad
```

4B.

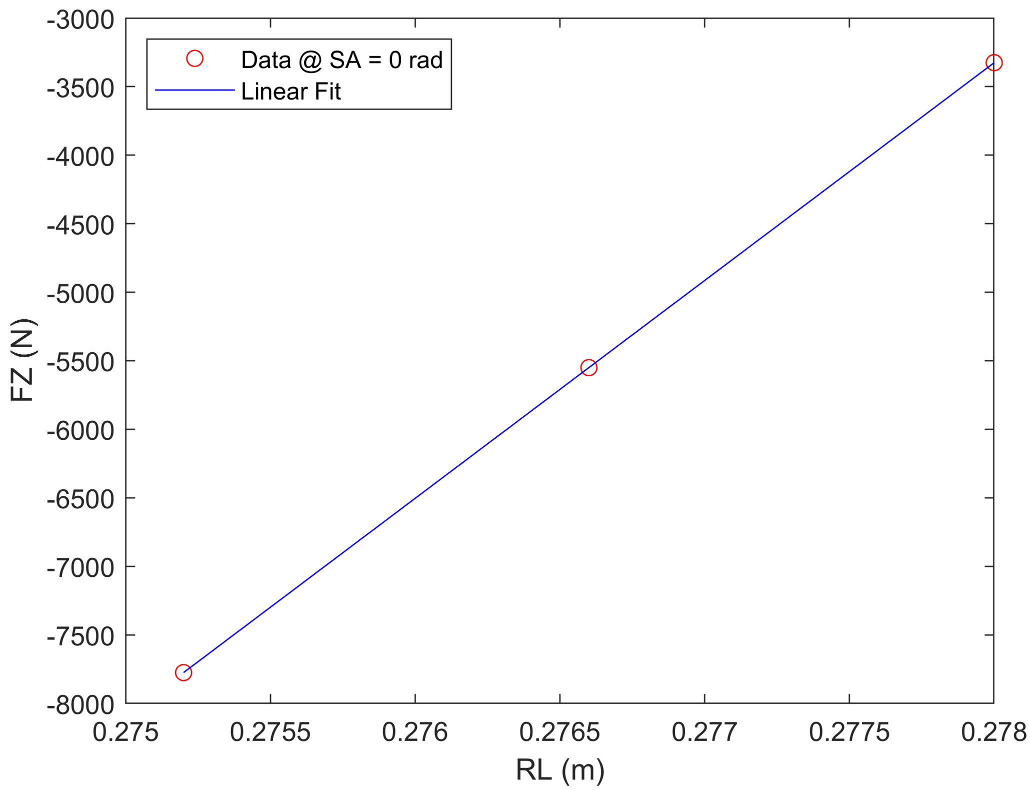
```
% positive SA (left turn) corresponds to negative FY, hence, peak FY will be minimum value
u = min(FY5550)/(-5550); % friction coefficient [absolute value of peak FY divided by normal lo
fprintf("Friction coefficient (at -5550 N normal load), u = %f", u);
```

```
Friction coefficient (at -5550 N normal load), u = 1.689009
```

4C.

```
% disregarding the condition to solve only for FZ = -5550 N because we need multiple FZ values
idx = find(SA==0); % filter data (get array indices corresponding to SA = 0 rad)
FZ0 = FZ(idx); % normal load (N) corresponding to SA = 0 rad
RL0 = RL(idx); % loaded radius (m) corresponding to SA = 0 rad
FZRLpoly = polyfit(RL0,FZ0,1); % fit polynomial (line) to FZ vs RL data at SA = 0 rad
figure()
plot(RL0,FZ0, 'o','color','red')
hold on
plot(RL0,polyval(FZRLpoly,RL0),'-','color','blue')
xlabel("RL (m)")
ylabel("FZ (N)")
legend("Data @ SA = 0 rad","Linear Fit",'Location','NW')
```

```
hold off
```




```
k = polyder(FZRLpoly);
fprintf("Vertical spring rate (at 0 rad slip angle), k = %f N/m", k)
```

```
Vertical spring rate (at 0 rad slip angle), k = 1589285.714286 N/m
```

4D.

```
idx = find(FZ==-5550 & SA==deg2rad(3)); % filter data (get array indices corresponding to FZ =
FY3 = FY(idx); % lateral force (N) corresponding to SA = 3 deg
FYd = FY3*sind(3); % induced drag (N) at 3 deg slip angle
fprintf("Induced drag (at -5550 N normal load and 3 deg slip angle), FYd = %f N", FYd)
```

```
Induced drag (at -5550 N normal load and 3 deg slip angle), FYd = -450.664919 N
```

Problem 5:

The position of point 'p' is given by

$$r = L\hat{e}_L + \rho\hat{b}_r$$

Using transport theorem to find inertial velocity of disk expressed in $\mathcal{E}$ coordinates:

$$\dot{r} \equiv \frac{^{\mathcal{E}}d}{dt} r = \frac{^{B}d}{dt} r + \underline{\omega_{B|\mathcal{E}}} \times r$$

$$= \frac{^{B}d}{dt}(L\hat{e}_L + \rho\hat{b}_r) + (+\dot{\phi}\hat{b}_3) \times (L\hat{e}_L + \rho\hat{b}_r)$$

$$\boxed{\begin{aligned}\omega_{B|\mathcal{E}} &= \dot{\phi}\hat{b}_3 \\ &= -\dot{\phi}\hat{e}_3\end{aligned}}$$

$$\boxed{\begin{aligned}L\hat{e}_L &= -(-L\cos\phi\,\hat{b}_r) \\ &= L\cos\phi\,\hat{b}_r\end{aligned}}$$

$$= \frac{^{B}d}{dt}(\rho + L\cos\phi)\hat{b}_r + \dot{\phi}\hat{b}_3 \times (\rho + L\cos\phi)\hat{b}_r$$

$$= \cancel{\frac{^{B}d}{dt}\rho} + \frac{^{B}d}{dt} L\cos\phi\,\hat{b}_r + \dot{\phi}\hat{b}_3 \times (\rho + L\cos\phi)\hat{b}_r$$

$$= \left(\cos\phi\,\cancel{\frac{^{B}d}{dt}L} + L\frac{^{B}d}{dt}\cos\phi\right)\hat{b}_r + \dot{\phi}\hat{b}_3 \times (\rho + L\cos\phi)$$

$$= -L\sin\phi\,\dot{\phi}\hat{b}_r + \dot{\phi}(\rho + L\cos\phi)(+\hat{b}_\phi)$$

$$= -L\sin\phi \cdot \dot{\phi}\hat{b}_r + (\rho + L\cos\phi)\dot{\phi}\,\hat{b}_\phi$$

Using transport theorem to find inertial acceleration w.r.t. $\mathcal{E}$

$$\ddot{r} \equiv \frac{^{\mathcal{E}}d}{dt}\dot{r} = \frac{^{B}d}{dt}\dot{r} + \cancel{\alpha_{B|\mathcal{E}} \times r} + \omega_{B|\mathcal{E}} \times \dot{r}$$

$$= (-L\cos\phi\,\dot{\phi}^2 - L\sin\phi\,\ddot{\phi})\hat{b}_r + (\rho\ddot{\phi} + L\sin\phi\,\dot{\phi}^2 + L\cos\phi\,\ddot{\phi})$$

$$+ \dot{\phi}\hat{b}_3 \times (-L\sin\phi\,\dot{\phi}\hat{b}_r + (\rho + L\cos\phi)\dot{\phi}\hat{b}_\phi)$$

$$= (-L\cos\phi\,\dot{\phi}^2 - L\sin\phi\,\ddot{\phi})\hat{b}_r + ((\rho + L\cos\phi)\ddot{\phi} + L\sin\phi\,\dot{\phi}^2)$$

$= \rho\dot{\phi}$

$\dot{\phi} = L\dot{\theta} = \frac{L\omega}{\rho}$

$\rho\dot{\phi}$

$\dot{\phi} = 0$

$\frac{L\omega}{\rho}$

$\frac{L\omega}{\rho}$

Solution 6:

```
clear; close all; clc;
R1 = 230; % large corner radius (m)
R2 = 180; % small corner radius (m)
L = 0.45*1609.344; % length of straight (m)
```

6C.

```
ay1 = 1.1*9.81; % lateral acceleration in large corner (m/s^2)
ay2 = 1.2*9.81; % lateral acceleration in small corner (m/s^2)
vc1 = abs(sqrt(R1*ay1)); % velocity in large corner (m/s)
vc2 = abs(sqrt(R2*ay2)); % velocity in small corner (m/s)
fprintf("Velocity in corner 1 (ZW), vc1 = %f m/s\n" + ...
        "Velocity in corner 2 (XY), vc2 = %f m/s", vc1, vc2);
```

```
Velocity in corner 1 (ZW), vc1 = 49.818972 m/s
Velocity in corner 2 (XY), vc2 = 46.032163 m/s
```

6E.

```
% location of transition point (St) lies at the intersection of forward and braking velocity cu
af = 0.5*9.81; % forward acceleration (m/s^2)
ab = 1.3*9.81; % braking acceleration (m/s^2)
St1 = (vc2^2-vc1^2+(2*ab*L))/(2*(af+ab)); % transition point along straight 1 (WX) (m) | vf=vc1
St2 = (vc1^2-vc2^2+(2*ab*L))/(2*(af+ab)); % transition point along straight 2 (YZ) (m) | vf=vc2
fprintf("Location of transition point along straight 1 (WX), St1 = %f m\n" + ...
        "Location of transition point along straight 2 (YZ), St2 = %f m", St1, St2);
```

```
Location of transition point along straight 1 (WX), St1 = 512.759022 m
Location of transition point along straight 2 (YZ), St2 = 533.314578 m
```

6D.

```
% maximum speed on straight is at location of transition point (St)
vm1 = abs(sqrt(vc1^2+(2*af*St1))); % maximum velocity along straight 1 (WX) (m/s)
vm2 = abs(sqrt(vc2^2+(2*af*St2))); % maximum velocity along straight 2 (YZ) (m/s)
fprintf("Maximum velocity along straight 1 (WX), vm1 = %f m/s\n" + ...
        "Maximum velocity along straight 2 (YZ), vm2 = %f m/s", vm1, vm2);
```

```
Maximum velocity along straight 1 (WX), vm1 = 86.672349 m/s
Maximum velocity along straight 2 (YZ), vm2 = 85.736667 m/s
```

6B.

```
% calculate average speed along the 6 zones of the track (W-St1-X-Y-St2-Z-W) to compute net ave
s1 = St1; % distance along W-->St1
v1 = (vc1+vm1)/2; % average speed along W-->St1
s2 = L-St1; % distance along St1-->X
v2 = (vm1+vc2)/2; % average speed along St1-->X
s3 = pi*R2; % distance along X-->Y
v3 = vc2; % average speed along X-->Y
s4 = St2; % distance along Y-->St2
```

```
v4 = (vc2+vm2)/2; % average speed along Y-->St2
s5 = L-St2; % distance along St2-->Z
v5 = (vm2+vc1)/2; % average speed along St2-->Z
s6 = pi*R1; % distance along Z-->W
v6 = vc1; % average speed along Z-->W
va = (s1+s2+s3+s4+s5+s6)/((s1/v1)+(s2/v2)+(s3/v3)+(s4/v4)+(s5/v5)+(s6/v6)); % average speed for
fprintf("Average speed for 1 lap, va = %f m/s", va);
```

```
Average speed for 1 lap, va = 56.538839 m/s
```

6A.

```
% minimum time required for 1 lap can be computed based on the total distance in 1 lap and the
% note: we already solved the racing problem for maximum possible average speed
tm = (s1+s2+s3+s4+s5+s6)/va; % minimum time required for 1 lap
fprintf("Minimum time required for 1 lap, tm = %f s", tm);
```

```
Minimum time required for 1 lap, tm = 48.399695 s
```