

Vehicle Dynamics Homework 2 Solutions

Note: entire MATLAB code is presented at end of solutions.

Problem 1:

As mentioned on the first page of Homework 2, there are a total of six target loads spaced apart by 50 *lbf*. Our first step is to find these target loads at the conditions mentioned before question 1: data at 0 *deg* inclination angle, 12 *psi* inflation pressure, and 25 *mph* roadway speed. After converting the forces into *lbf*, pressures into *psi*, and velocities into *mph*, the indices for the target values at each of the six loads can be found as follows:

```
1 targetFZ = 50*round(FZ/50); % rounded all loads to the nearest ...  
   50 lbf  
2 targetFZlist = unique(targetFZ); % removes all repeats; should ...  
   be a 6x1 vector, for the 6 different normal loads  
3  
4 % Target values for each normal load  
5 iiFZ350 = find(targetFZ == targetFZlist(1) & round(V) == 25 & ...  
   round(IA) == 0 & round(P) == 12);  
6 iiFZ300 = find(targetFZ == targetFZlist(2) & round(V) == 25 & ...  
   round(IA) == 0 & round(P) == 12); % ends up being empty set; ...  
   remove from targetFZlist  
7 iiFZ250 = find(targetFZ == targetFZlist(3) & round(V) == 25 & ...  
   round(IA) == 0 & round(P) == 12);  
8 iiFZ150 = find(targetFZ == targetFZlist(4) & round(V) == 25 & ...  
   round(IA) == 0 & round(P) == 12);  
9 iiFZ100 = find(targetFZ == targetFZlist(5) & round(V) == 25 & ...  
   round(IA) == 0 & round(P) == 12);  
10 iiFZ50 = find(targetFZ == targetFZlist(6) & round(V) == 25 & ...  
   round(IA) == 0 & round(P) == 12);  
11  
12 targetFZlist(2) = []; % now have five different normal loads
```

The plot then simply comes from indexing slip angle and lateral force by those indices and putting them on a single plot with a legend to identify which corresponds to each target load.

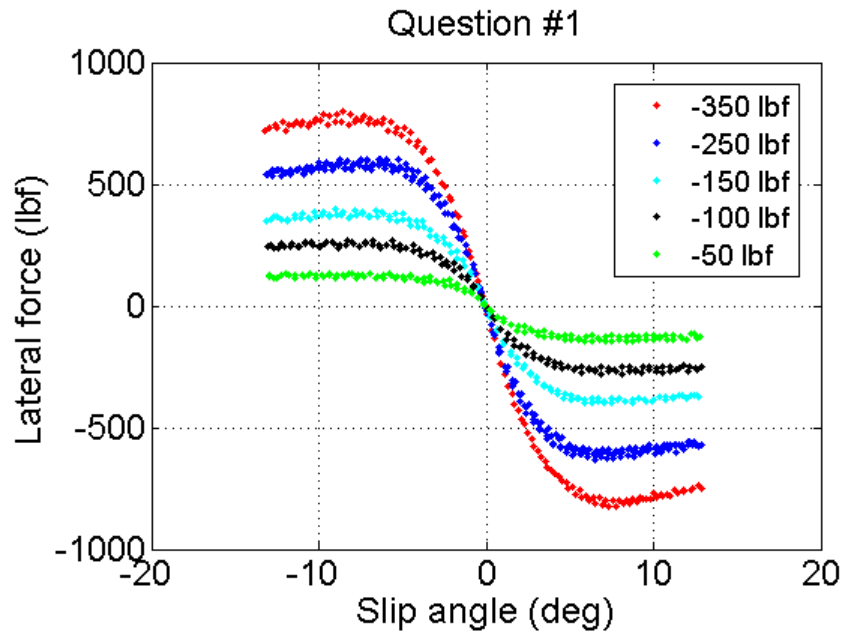


Figure 1: Lateral force vs. slip angle for each normal load.

Problem 2:

Recall from class that cornering stiffness is the slope of the lateral force versus slip angle curve in its linear range. Keeping in mind that the linear range is around 0 deg, we can create new indices as follows:

```

1 %% Question 2
2 % Limit slip angle to between -1 and 1 deg for linear part
3 slope1cs = find(targetFZ == targetFZlist(1) & round(V) == 25 & ...
4     round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
5 cs1 = polyfit(SA(slope1cs),FY(slope1cs),1);
6 slope2cs = find(targetFZ == targetFZlist(2) & round(V) == 25 & ...
7     round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
8 cs2 = polyfit(SA(slope2cs),FY(slope2cs),1);
9 slope3cs = find(targetFZ == targetFZlist(3) & round(V) == 25 & ...
10    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
11 cs3 = polyfit(SA(slope3cs),FY(slope3cs),1);
12 slope4cs = find(targetFZ == targetFZlist(4) & round(V) == 25 & ...
13    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
14 cs4 = polyfit(SA(slope4cs),FY(slope4cs),1);
15 slope5cs = find(targetFZ == targetFZlist(5) & round(V) == 25 & ...
16    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
17 cs5 = polyfit(SA(slope5cs),FY(slope5cs),1);

```

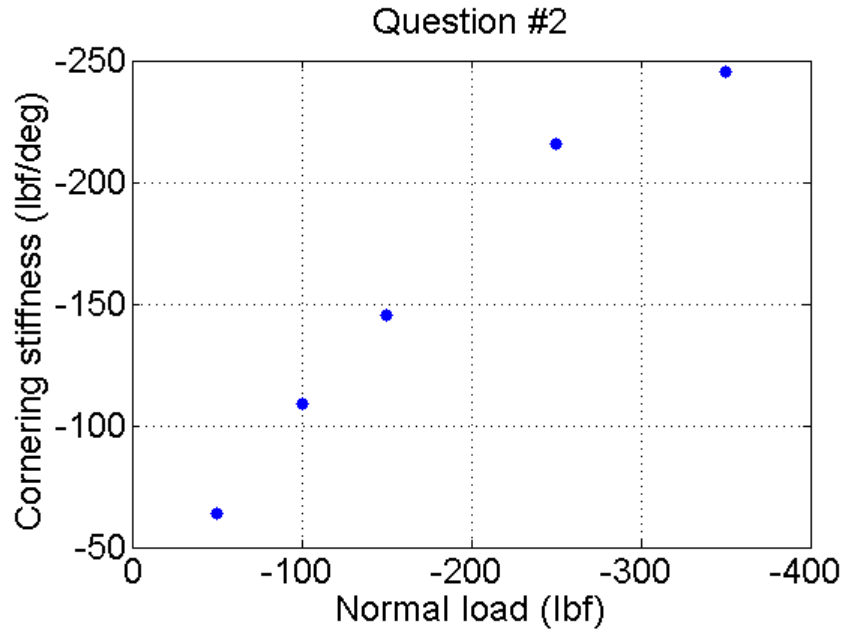


Figure 2: Cornering stiffness vs. Normal load.

Normal load (<i>lbf</i>)	Cornering stiffness (<i>lbf/deg</i>)
-350	-245.382689169168
-250	-216.030935427145
-150	-145.436054969249
-100	-109.202316102064
-50	-63.8904332276433

Table 1: Cornering stiffness at each normal load.

Problem 3:

The tire's friction coefficient, μ , comes from dividing the maximum lateral force by the normal load it bears. In a left hand turn, the lateral forces will be negative. Knowing this, we can find the friction coefficient through:

```

1  fyPeak = [min(FY(iiFZ350)); min(FY(iiFZ250)); min(FY(iiFZ150)); ...
             min(FY(iiFZ100)); min(FY(iiFZ50))];
2  mu = abs(fyPeak./targetFZlist);

```

The plot is then simple.

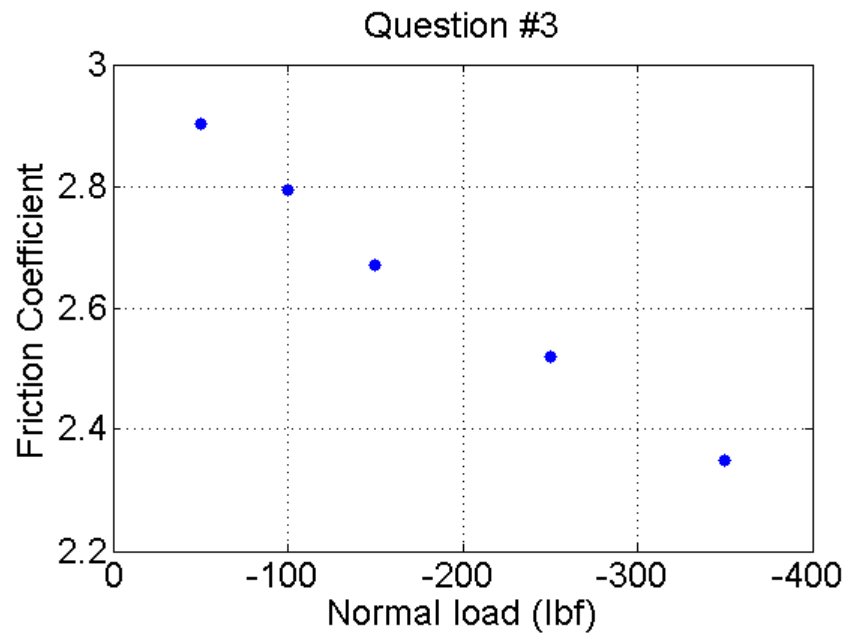


Figure 3: Friction coefficient vs. Normal load.

Normal load (<i>lbf</i>)	Friction coefficient (unitless)
-350	2.34937549349049
-250	2.51948777834732
-150	2.66962118538787
-100	2.7950271074247
-50	2.9036772695766

Table 2: Friction coefficient at each normal load.

Problem 4:

Using the indices developed in the first problem, the following plot can be made:

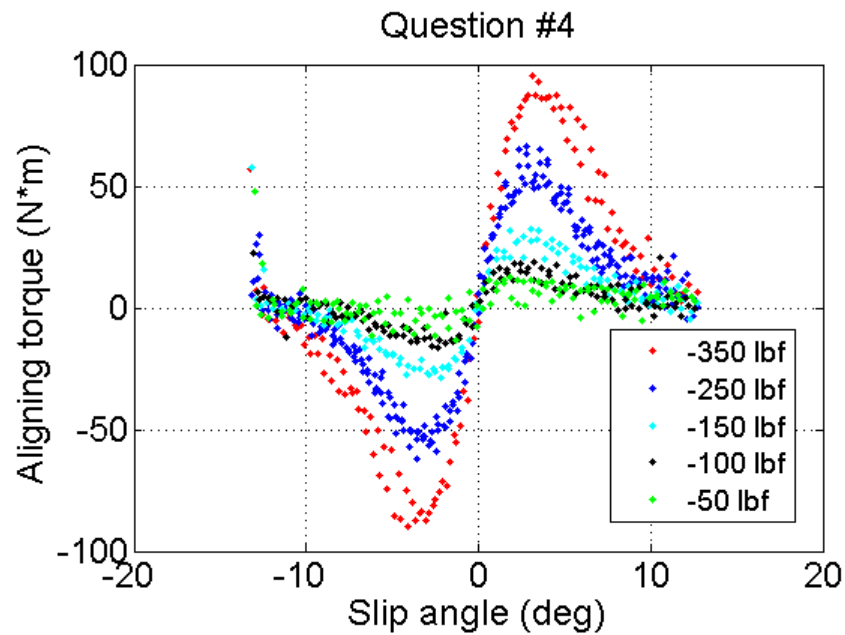


Figure 4: Aligning torque vs. slip angle.

Problem 5:

Restricting slip angle magnitudes to be less than 1 generally limits the lateral force. Indexing this way and fitting the data with a linear fit can be done via:

```

1 vsrInd = find(round(V) == 25 & round(IA) == 0 & round(P) == 12 & ...
    abs(SA) < 1);
2 cm2in = 0.393701; % converting cm to inches
3 RL = cm2in.*RL;
4
5 slope_sr = polyfit(RL(vsrInd),FZ(vsrInd),1);
6
7 disp(['Vertical Spring Rate at 200 lb. from Linear fit: ...
    ',num2str(slope_sr(1)),' lbf/in'])

```

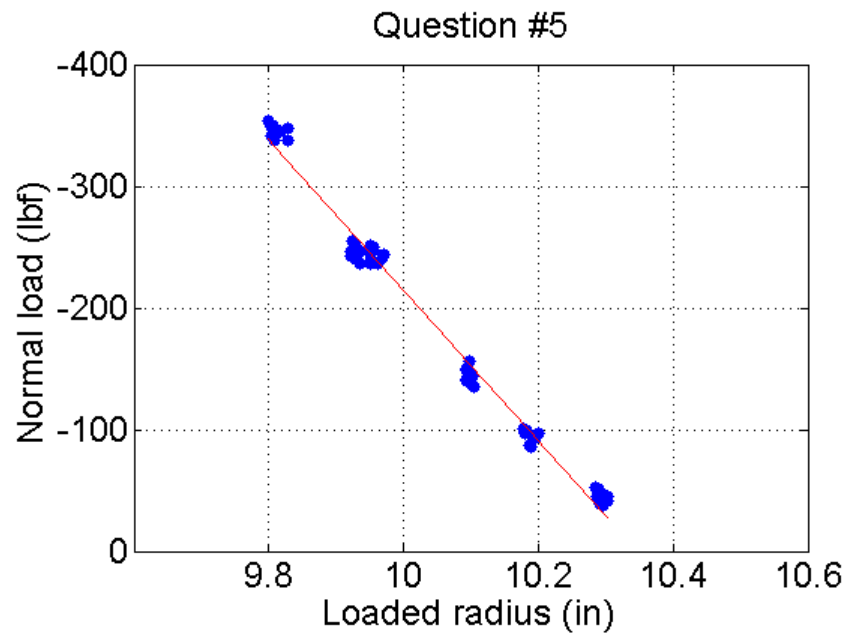


Figure 5: Normal load vs. loaded radius.

Using the linear fit, the vertical spring rate of the tire at a 200 *lbf* normal load is simply the slope of the line: 617.7769 *lbf/in*

MATLAB code for solution:

```
1 % RVD_Homework 3
2 clear all
3 clc
4 close all
5
6 format long g
7
8 load('tireF15')
9
10 % ET is time, sec
11 % V roadway velocity, kph
12 % SA slip angle, deg
13 % IA inclination angle, deg
14 % P inflation pressure, kPa
15 % FZ normal load, N
16 % FY lateral force, N
17 % MX overturning moment, N*m
18 % MZ aligning torque, N*m
19 % RL loaded radius, cm
20
21 figure
22 plot(ET, V, '.')
23 grid on
24 title('Roadway speed vs. time', 'FontSize', 18)
25 xlabel('time (sec)', 'FontSize', 18)
26 ylabel('(kph)', 'FontSize', 18)
27 set(gca, 'FontSize', 18);
28
29 figure
30 plot(ET, IA, '.')
31 grid on
32 title('Inclination angle vs. time', 'FontSize', 18)
33 xlabel('time (sec)', 'FontSize', 18)
34 ylabel('(deg)', 'FontSize', 18)
35 set(gca, 'FontSize', 18);
36
37 figure
38 plot(ET, SA, '.')
39 grid on
40 title('Slip angle vs. time', 'FontSize', 18)
41 xlabel('time (sec)', 'FontSize', 18)
42 ylabel('(deg)', 'FontSize', 18)
43 set(gca, 'FontSize', 18);
44
45 figure
46 plot(ET, FZ, '.')
47 grid on
48 title('Normal load vs. time', 'FontSize', 18)
49 xlabel('time (sec)', 'FontSize', 18)
50 ylabel('(N)', 'FontSize', 18)
51 set(gca, 'FontSize', 18);
52
53 figure
54 plot(ET, P, '.')
55
```

```

55 grid on
56 title('Inflation pressure vs. time', 'FontSize', 18)
57 xlabel('time (sec)', 'FontSize', 18)
58 ylabel('(kPa)', 'FontSize', 18)
59 set(gca,'FontSize',18);
60
61 %% Unit conversions
62 % For questions, use only the data at 0 deg. inclination angle, ...
    12 psi inflation pressure and 25 mph roadway speed
63 kph2mph = 0.621371;
64 kPa2psi = 0.145037738;
65 newton2lbf = 0.224808943;
66 zDegIA = find(round(IA) == 0);
67
68 V = V.*kph2mph;
69 P = P.*kPa2psi;
70 FZ = FZ.*newton2lbf;
71 FY = FY.*newton2lbf;
72
73 %% Finding target values
74 % Finding the target normal loads
75 targetFZ = 50*round(FZ/50); % rounded all loads to the nearest ...
    50 lbf
76 targetFZlist = unique(targetFZ); % removes all repeats; should ...
    be a 6x1 vector, for the 6 different normal loads
77
78 % Target values for each normal load
79 iiFZ350 = find(targetFZ == targetFZlist(1) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);
80 iiFZ300 = find(targetFZ == targetFZlist(2) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12); % ends up being empty set; ...
    remove from targetFZlist
81 iiFZ250 = find(targetFZ == targetFZlist(3) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);
82 iiFZ150 = find(targetFZ == targetFZlist(4) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);
83 iiFZ100 = find(targetFZ == targetFZlist(5) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);
84 iiFZ50 = find(targetFZ == targetFZlist(6) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);
85
86 targetFZlist(2) = []; % now have five different normal loads
87
88 %% Question 1
89
90 forLegend = {'-350 lbf'; '-250 lbf'; '-150 lbf'; '-100 lbf'; ...
    '-50 lbf'};
91
92 figure
93 plot(SA(iiFZ350), FY(iiFZ350), 'r .')
94 hold
95 plot(SA(iiFZ250), FY(iiFZ250), 'b .')
96 plot(SA(iiFZ150), FY(iiFZ150), 'c .')
97 plot(SA(iiFZ100), FY(iiFZ100), 'k .')
98 plot(SA(iiFZ50), FY(iiFZ50), 'g .')
99 grid on
100 hLegend = legend(forLegend,'Location','Best');

```



```

101 set(hlegend,'FontSize',14)
102 title('Question #1', 'FontSize', 18)
103 xlabel('Slip angle (deg)', 'FontSize', 18)
104 ylabel('Lateral force (lbf)', 'FontSize', 18)
105 set(gca,'FontSize',18);
106
107 %% Question 2
108 % Limit slip angle to between -1 and 1 deg for linear part
109 slope1cs = find(targetFZ == targetFZlist(1) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
110 cs1 = polyfit(SA(slope1cs),FY(slope1cs),1);
111 slope2cs = find(targetFZ == targetFZlist(2) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
112 cs2 = polyfit(SA(slope2cs),FY(slope2cs),1);
113 slope3cs = find(targetFZ == targetFZlist(3) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
114 cs3 = polyfit(SA(slope3cs),FY(slope3cs),1);
115 slope4cs = find(targetFZ == targetFZlist(4) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
116 cs4 = polyfit(SA(slope4cs),FY(slope4cs),1);
117 slope5cs = find(targetFZ == targetFZlist(5) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
118 cs5 = polyfit(SA(slope5cs),FY(slope5cs),1);
119
120 cs_all = [cs1(1);cs2(1);cs3(1);cs4(1);cs5(1)];
121
122 % Make a table
123 disp('          Normal load   Cornering stiffness')
124 disp('          lbf          lbf/deg')
125 disp([targetFZlist cs_all])
126
127 figure
128 plot(targetFZlist, cs_all, '.', 'MarkerSize', 20)
129 grid on
130 set(gca,'XDir','reverse')
131 set(gca,'YDir','reverse')
132 set(gca, 'FontSize', 18)
133 xlabel('Normal load (lbf)', 'FontSize', 18)
134 ylabel('Cornering stiffness (lbf/deg)', 'FontSize', 18)
135 title('Question #2', 'FontSize', 18)
136
137 %% Question 3
138
139 fyPeak = [min(FY(iiFZ350)); min(FY(iiFZ250)); min(FY(iiFZ150)); ...
    min(FY(iiFZ100)); min(FY(iiFZ50))];
140 mu = abs(fyPeak./targetFZlist);
141
142 % Make a table
143 disp('          Normal load   Friction coefficient')
144 disp('          lbf          (unitless)')
145 disp([targetFZlist mu])
146
147 figure
148 plot(targetFZlist,mu, '.', 'MarkerSize',20);
149 grid on
150 set(gca,'XDir','reverse')
151 set(gca, 'FontSize', 18)

```

```

152 xlabel('Normal load (lbf)', 'FontSize', 18)
153 ylabel('Friction Coefficient', 'FontSize', 18)
154 title('Question #3', 'FontSize', 18)
155
156 %% Question 4
157
158 figure
159 plot(SA(iiFZ350), MZ(iiFZ350), 'r .')
160 hold
161 plot(SA(iiFZ250), MZ(iiFZ250), 'b .')
162 plot(SA(iiFZ150), MZ(iiFZ150), 'c .')
163 plot(SA(iiFZ100), MZ(iiFZ100), 'k .')
164 plot(SA(iiFZ50), MZ(iiFZ50), 'g .')
165 grid on
166 h_legend = legend(forLegend, 'Location', 'Best');
167 set(h_legend, 'FontSize', 14)
168 title('Question #4', 'FontSize', 18)
169 xlabel('Slip angle (deg)', 'FontSize', 18)
170 ylabel('Aligning torque (N*m)', 'FontSize', 18)
171 set(gca, 'FontSize', 18);
172
173 %% Question 5
174
175 vsrInd = find(round(V) == 25 & round(IA) == 0 & round(P) == 12 & ...
176             abs(SA) < 1);
177 cm2in = 0.393701; % converting cm to inches
178 RL = cm2in.*RL;
179
180 figure
181 plot(RL(vsrInd), FZ(vsrInd), '.', 'MarkerSize', 20)
182 grid on
183 hold on
184 set(gca, 'YDir', 'reverse', 'FontSize', 18)
185 ylabel('Normal load (lbf)', 'FontSize', 18)
186 xlabel('Loaded radius (in)', 'FontSize', 18)
187 title('Question #5', 'FontSize', 18)
188
189 slope_sr = polyfit(RL(vsrInd), FZ(vsrInd), 1);
190
191 x = linspace(min(RL(vsrInd)), max(RL(vsrInd)), 501); %vector of x ...
192                                     values for plotting the fit
193
194 % Plot the linear fit
195 y1 = polyval(slope_sr, x);
196 plot(x, y1, 'r')
197
198 disp(['Vertical Spring Rate at 200 lb. from Linear fit: ...
199       ', num2str(slope_sr(1)), ' lbf/in'])

```