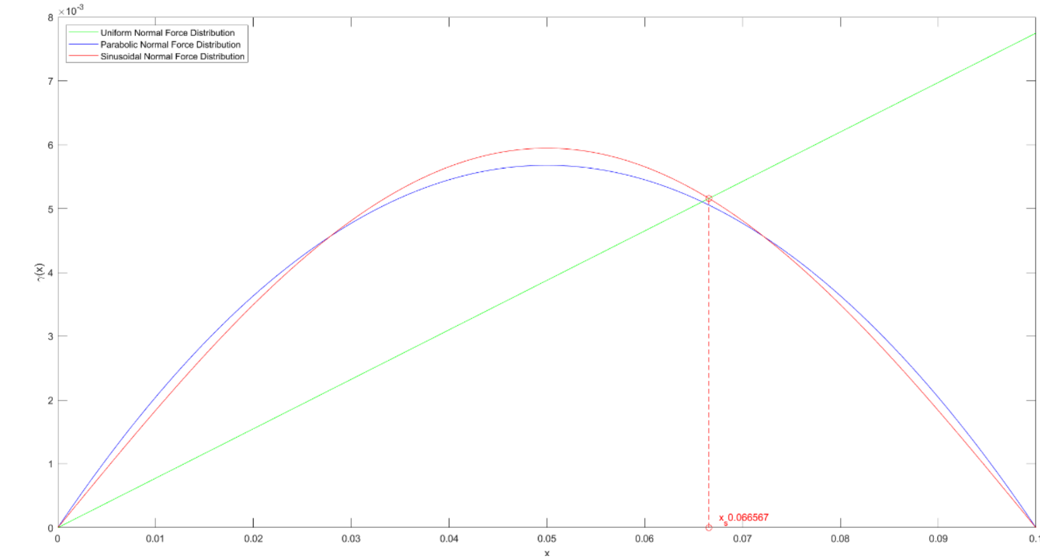
Problem 1

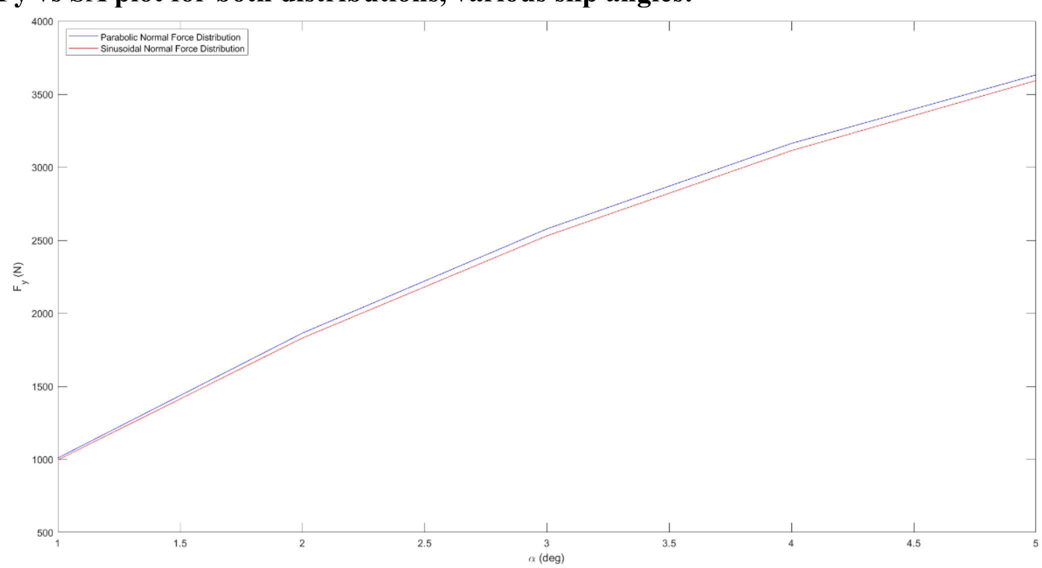
A. Calculate the force, Fy, under this lateral slip assuming a sinusoidal pressure distribution as shown below. Comment on the potential position of xs and is there a discontinuity?



- **Derivation of Fy for sinusoidal distribution is attached at end of this document.**
- **For given data, $Fy = 3332.9$ N**
- **For given lateral slip, $xs = 0.0666$ m. Assuming a sinusoidal pressure distribution, the position of transition point from front end of the tire patch i.e. xs can lie between x=0 and x=10 cm, depending upon the slip angle of tire. There is no discontinuity and there is no gap between the sticking and sliding regions.**

B. Compare this resulting lateral force with the one resulting from the parabolic model. Plot the lateral force Fy, over a range of slip angles, for both pressure distributions.

- **For parabolic model, $Fy = 3379.0$ N (for given data)**
- **Fy vs SA plot for both distributions, various slip angles:**

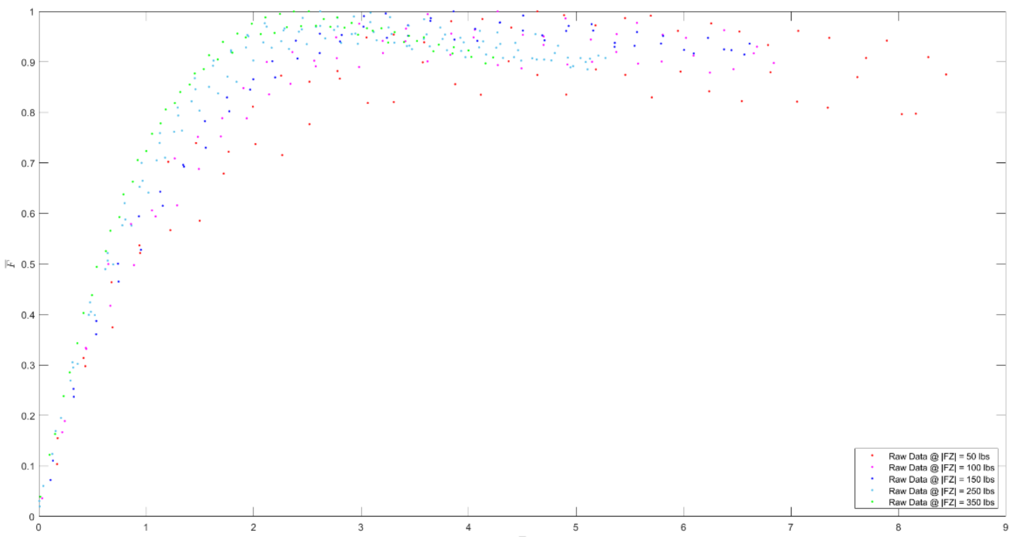


C. Comment about the transition point for the following bilinear distribution (Fig 2) and how it affects sliding region.

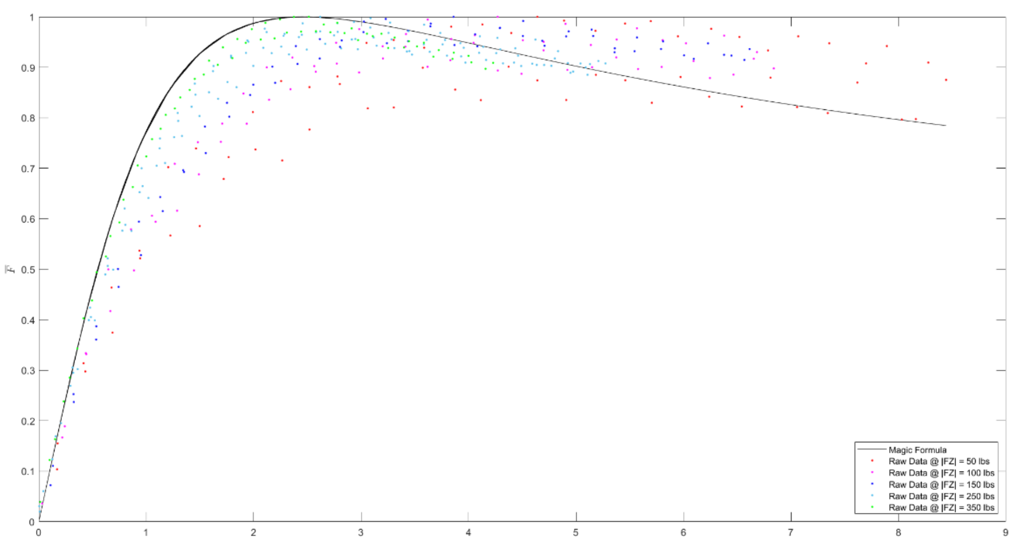
- **The position of transition point from front end of the tire patch i.e. x_s can lie between x>a and x<=2a since for a line with (slip) angle greater than or equal to the angle of the first line of the bilinear distribution, the entire tire patch will be sliding.**

Problem 2

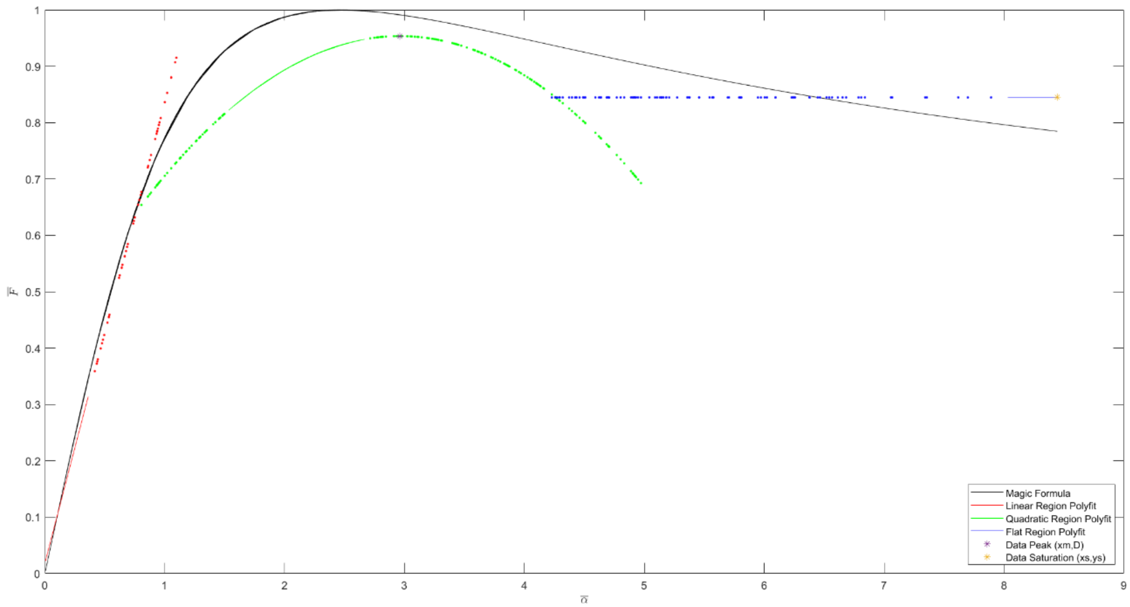
1. Plot nondimensional lateral force vs. nondimensional slip angle.



2. Add a plot of the Magic Formula over the data in question #1

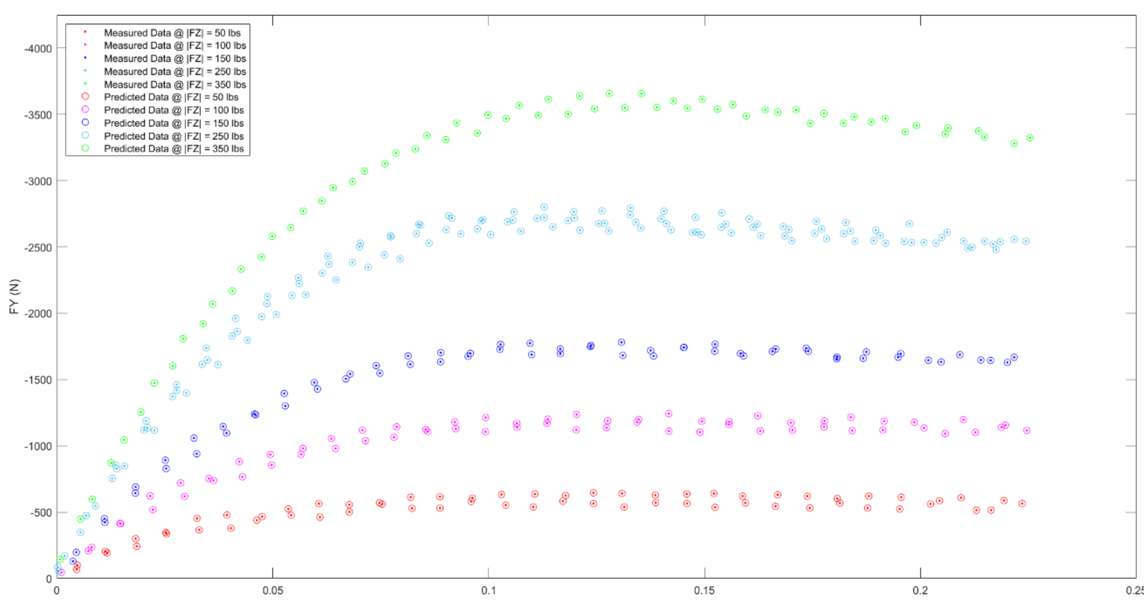


3. How do these coefficients compare with the one derived from the plot in question #1 and from the relationships given in class?

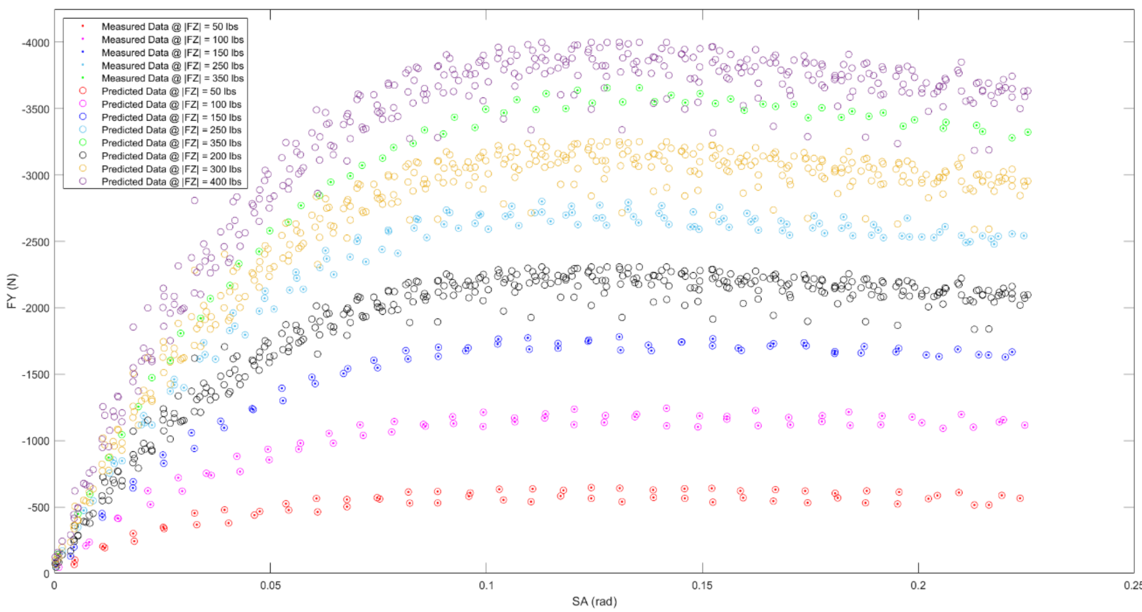


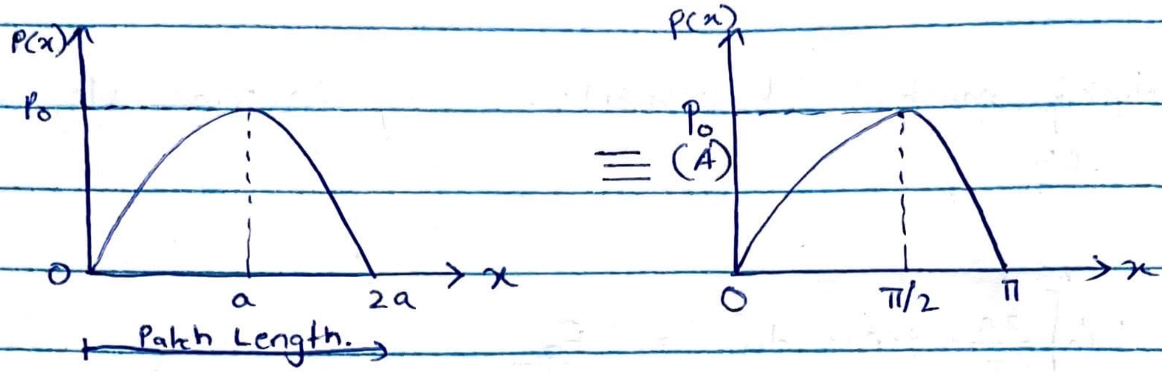
Pacejka Coefficient	B	C	D	E
Given Value	0.6000	1.6667	1.0000	0.2000
Derived Value	0.6560	1.3071	0.9535	-0.7593

4. Expand the nondimensional model at the five measured normal loads. Create a plot of lateral force vs. slip angle, showing both the measured data and the model predictions.



5. Also expand the Nondimensional Model at –200, –300 and –400 lb normal load. Plot the result on the same figure as question #4





- Sinusoidal function : $P(x) = A \sin B(x-h) + k$

Amplitude $= P_0$

Scaling Factor

Horizontal shift $= 0$

Vertical shift $= 0$

$\therefore \quad P(x) = P_0 \sin Bx$

For given case, $x = 2a \equiv x = \pi$

$\therefore \quad P(x) = 0 \quad$ at $x = 2a$

$\therefore \quad P_0 \sin B(2a) = 0$

$\therefore \quad \sin B(2a) = 0 \qquad \because P_0$ cannot be 0

$\therefore \quad 2aB = \pi$

$\therefore \quad B = \dfrac{\pi}{2a}$

i.e. $P(x) = P_0 \sin\left(\dfrac{\pi x}{2a}\right) \qquad — ①$

- Let patch width $= 2b$ (similar to patch length $= 2a$)

$$\therefore \int_0^{2a} 2b\, P(x)\, dx = F_z$$

$$\therefore \int_0^{2a} 2b \cdot P_0 \sin\left(\frac{\pi x}{2a}\right) dx = F_z \qquad \ldots \text{(from ①)}$$

$$\therefore 2bP_0 \int_0^{2a} \sin\left(\frac{\pi x}{2a}\right) dx = F_z$$

Applying method of substitution by intermediate variable

$u = \frac{\pi x}{2a}$,

$$\therefore P_0 \int_0^{2a} \frac{2a}{\pi} \sin u \, du = \frac{F_z}{2b}$$

$$\therefore P_0 \cdot \frac{2a}{\pi} \int_0^{2a} \sin u \, du = \frac{F_z}{2b}$$

$$\therefore P_0 \int_0^{2a} \sin u \, du = \frac{\pi F_z}{4ab}$$

$$\therefore P_0 \left[-\cos u \right]_{x=0}^{2a} = \frac{\pi F_z}{4ab}$$

$$\therefore P_0 \left[-\cos\left(\frac{\pi x}{2a}\right) \right]_0^{2a} = \frac{\pi F_z}{4ab}$$

$$\therefore P_0 \left[(-\cos \pi) - (-\cos 0) \right] = \frac{\pi F_z}{4ab}$$

$$\therefore 2P_0 = \frac{\pi F_z}{4ab}$$

$$\therefore P_0 = \frac{\pi F_z}{8ab} \qquad -②$$

Substituting ② in ①,

$$\therefore P(x) = \frac{\pi F_z}{8ab} \cdot \sin\left(\frac{\pi x}{2a}\right) \qquad -③$$

- Force per unit length,

$$q_z(x) = p(x) \cdot 2b$$

$$\therefore q_z(x) = \frac{\pi F_z}{8ab} \cdot \sin\left(\frac{\pi x}{2a}\right) \cdot 2b \quad \ldots \text{(from 3)}$$

$$\therefore q_z(x) = \frac{\pi F_z}{4a} \cdot \sin\left(\frac{\pi x}{2a}\right) \quad — (4)$$

- Let c = cornering stiffness per unit length, then deflection in sliding region $\gamma_{slide}(x)$, (μ = coeff of friction)

$$c \cdot \gamma_{slide}(x) = \mu \cdot q_z(x) \quad \ldots (dF = \mu N \text{ on each tread})$$

$$\therefore c \cdot \gamma_{slide}(x) = \frac{\pi \mu F_z}{4ac} \cdot \sin\left(\frac{\pi x}{2a}\right) \quad \ldots \text{(from 4)}$$

- Total lateral force on tire,

$$F_y = \int_0^{2a} c \cdot \gamma(x)\, dx$$

equal to area under the curve (solved using MATLAB)