AuE-6600 | Dynamic Performance of Vehicles

Homework 4

Problem 1

Given a vehicle with wheelbase l = 3.05 m and a CG position with a/l = 0.45.

1. Determine the error in the Ackermann steering angle if you use the small angle approximation while cornering with a radius of 105m.
2. Determine the yaw velocity and the lateral acceleration of a vehicle which travels on circular track with radius 620 ft and a velocity of 70 mph.
3. Suppose that now the vehicle has a wheelbase of 2.7 m, a mass of 105 slugs and the location of the center of gravity is given by a/l=0.5. Determine the vehicle's slip angle. The cornering stiffness at the front and rear wheels amounts to 750 N/deg when the vehicle is driven with 111 km/h on a circular path of radius 232 m. Compare this vehicle slip angle with the vehicle slip angle β0 for force-free rolling.

Solution 1

1. Determine the error in the Ackermann steering angle if you use the small angle approximation while cornering with a radius of 105m.

```
l = 3.05; % m
a = 0.45*l; % m
b = l - a; % m
R = 105; % m
Rr = sqrt(R^2 - b^2);
deltaA_actual = atan2(l,Rr); % Actual steering angle
deltaA_approx = l/R; % Small angle approximation
deltaA_error = abs(deltaA_actual - deltaA_approx);
fprintf("Actual Ackermann steering angle is %d rad\n" + ...
        "Approximated steering angle is %d rad\n" + ...
        "Error due to small angle approximation is %d rad", ...
        deltaA_actual, deltaA_approx, deltaA_error);
```

```
Actual Ackermann steering angle is 2.904316e-02 rad
Approximated steering angle is 2.904762e-02 rad
Error due to small angle approximation is 4.461025e-06 rad
```

2. Determine the yaw velocity and the lateral acceleration of a vehicle which travels on circular track with radius 620 ft and a velocity of 70 mph.

```
R = 620; % ft
R = R*0.3048; % m
v = 70; % mph
v = v*0.44704; % m/s
W = v/R;
a_y = v^2/R;
fprintf("Yaw velocity of the vehicle is %.4f rad/s\n" + ...
        "Lateral acceleration of the vehicle is %.4f m/s^2\n", ...
```

```
        W, a_y);
```

```
Yaw velocity of the vehicle is 0.1656 rad/s
Lateral acceleration of the vehicle is 5.1818 m/s^2
```

3. Suppose that now the vehicle has a wheelbase of 2.7 m, a mass of 105 slugs and the location of the center of gravity is given by a/l=0.5. Determine the vehicle's slip angle. The cornering stiffness at the front and rear wheels amounts to 750 N/deg when the vehicle is driven with 111 km/h on a circular path of radius 232 m. Compare this vehicle slip angle with the vehicle slip angle β0 for force-free rolling.

```
% Given data
l = 2.7; % m
a = 0.5*l; % m
b = l-a; % m
m = 105; % slugs
m = m*14.594; % kg
c_alpha = 750; % N/deg (adapted ISO convention)
c_alpha = c_alpha*57.29578; % N/rad
v = 111; % km/h
v = v/3.6; % m/s
R = 232; % m

% Force-free rolling
B0 = b/R;
fprintf("Slip angle of vehicle undergoing force-free rolling is %d rad",B0);
```

```
Slip angle of vehicle undergoing force-free rolling is 5.818966e-03 rad
```

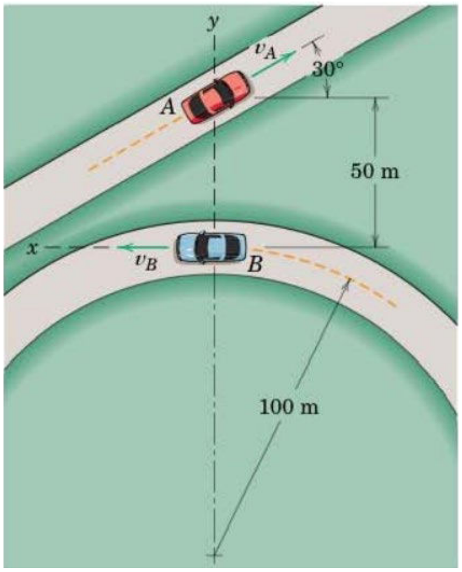
```
% Rolling with lateral forces
a_y = (v^2/R); % Adapted ISO convention
F_y = m*a_y;
F_y_R = (a/l)*F_y;
alpha_R = F_y_R/c_alpha;
B = abs(b/R)-abs(alpha_R); % Adapted ISO convention
fprintf("Slip angle of vehicle rolling with lateral forces is %d rad",B);
```

```
Slip angle of vehicle rolling with lateral forces is -6.724491e-02 rad
```

Problem 2

Each of the two cars A and B is travelling with a constant speed of 90 km/h.

1. Determine the velocity and acceleration of car A as seen by an observer moving and rotating with car B when the cars are in the position shown. The x-y axes are attached to car B.
2. Is this apparent velocity the negative of the velocity which B appears to have to a nonrotating observer in car A. If not, Explain the reason?



Solution 2

1. Determine the velocity and acceleration of car A as seen by an observer moving and rotating with car B when the cars are in the position shown. The x-y axes are attached to car B.

```
R = 100; % m
r = 50; % m
v_a = 90/3.6; % m/s
v_b = 90/3.6; % m/s
r_I_dot = [-v_a*cosd(30);v_a*sind(30);0]; % -v_a*cos(30) i^ + v_a*sin(30) j^ m/s
r_B_I_dot = [v_b;0;0]; % v_b i^ m/s
W_B_I = [0;0;-r_B_I_dot(1)/R]; % v_B/R k^ rad/s
r_B = [0;r;0]; % r j^ m
r_B_dot = r_I_dot - r_B_I_dot - cross(W_B_I, r_B);
fprintf("Instantaneous velocity (m/s) of car A w.r.t. car B is:\n%.4f i\n%.4f\t j\n%.4f\t k", 
        r_B_dot(1),r_B_dot(2),r_B_dot(3));
```

```
Instantaneous velocity (m/s) of car A w.r.t. car B is:
-59.1506 i
12.5000     j
0.0000     k
```

```
r_I_dot_dot = [0;0;0]; % 0 m/s^2
r_B_I_dot_dot = [0;-r_B_I_dot(1)^2/R;0]; % -v_B^2/R j^ m/s^2
W_B_I_dot = [0;0;0]; % 0 m/s^2
r_B_dot_dot = r_I_dot_dot - r_B_I_dot_dot - cross(W_B_I, cross(W_B_I, r_B)) - cross(W_B_I_dot,
fprintf("Instantaneous acceleration (m/s^2) of car A w.r.t. car B is:\n%.4f\t i\n%.4f j\n%.4f\t
        r_B_dot_dot(1),r_B_dot_dot(2),r_B_dot_dot(3));
```

```
Instantaneous acceleration (m/s^2) of car A w.r.t. car B is:
-6.2500     i
-20.2003 j
0.0000     k
```

2. Is this apparent velocity the negative of the velocity which B appears to have to a nonrotating observer in car A. If not, Explain the reason?

```
% Calculate velocity of car B w.r.t. car A (expressed w.r.t. car B)
v_a = 90/3.6; % m/s
v_b = 90/3.6; % m/s
v_a = [-v_a*cosd(30);v_a*sind(30);0]; % -v_a*cos(30) i^ + v_a*sin(30) j^ m/s
v_b = [v_b;0;0]; % v_b i^ m/s
v_b_a = v_b - v_a;
fprintf("Instantaneous velocity (m/s) of car B w.r.t. car A is:\n%.4f\t i\n%.4f j\n%.4f\t k", .
        v_b_a(1),v_b_a(2),v_b_a(3));
```

```
Instantaneous velocity (m/s) of car B w.r.t. car A is:
46.6506     i
-12.5000 j
0.0000     k
```

```
% Calculate negative of the velocity which car A appears to have to a non-rotating observer in
v_a = 90/3.6; % m/s
v_b = 90/3.6; % m/s
r_I_dot_nonrot = [-v_a*cosd(30);v_a*sind(30);0]; % -v_a*cos(30) i^ + v_a*sin(30) j^ m/s
r_B_I_dot_nonrot = [v_b;0;0]; % v_b i^ m/s
r_B_dot_nonrot = r_I_dot_nonrot - r_B_I_dot_nonrot;
fprintf("Negative of the velocity of car A w.r.t. non-rotating observer in car B is:\n%.4f\t i\
        -r_B_dot_nonrot(1),-r_B_dot_nonrot(2),-r_B_dot_nonrot(3));
```

```
Negative of the velocity of car A w.r.t. non-rotating observer in car B is:
46.6506     i
-12.5000 j
-0.0000     k
```

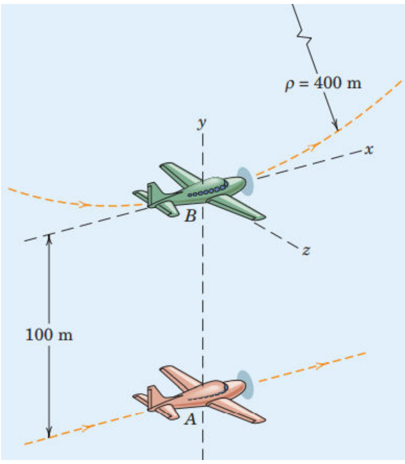
Note: The instantaneous velocity of car B w.r.t. car A (expressed w.r.t. car B) is equal to the negative of velocity which car A appears to have to a ***non-rotating observer*** in car B (expressed w.r.t. car B).

Answer: The instantaneous velocity of car A w.r.t. car B, will not be equal to the negative of the instantaneous velocity of car B w.r.t. car A. This is because car B is undergoing circular motion (i.e. rotating frame) and thus, we need to have additional corrective terms in order to express the velocity/acceleration of car A measured w.r.t. car B, which is/are not required when measuring velocity/acceleration of car B w.r.t. car A. This is why the results differ.

Problem 3

Aircraft B has a constant speed of 150 m/s as it passes the bottom of a circular loop of 400-m radius. Aircraft A flying horizontally in the plane of the loop passes 100 m directly below B at a constant speed of 100 m/s.

1. Determine the instantaneous velocity and acceleration which A appears to have to the pilot of B, who is fixed to his rotating aircraft.
2. Compare your results for part (a) with the case of erroneously treating the pilot of aircraft B as nonrotating.



Solution 3

1. Determine the instantaneous velocity and acceleration which A appears to have to the pilot of B, who is fixed to his rotating aircraft.

```
R = 400; % m
r = 100; % m
r_I_dot = [100;0;0]; % 100 i^ m/s
r_B_I_dot = [150;0;0]; % 150 i^ m/s
W_B_I = [0;0;r_B_I_dot(1)/R]; % v_B/R k^ rad/s
r_B = [0;-r;0]; % -r j^ m
r_B_dot = r_I_dot - r_B_I_dot - cross(W_B_I, r_B);
fprintf("Instantaneous velocity (m/s) of plane A w.r.t. plane B is:\n%.4f i\n%.4f\t j\n%.4f\t k
        r_B_dot(1),r_B_dot(2),r_B_dot(3));
```

```
Instantaneous velocity (m/s) of plane A w.r.t. plane B is:
-87.5000 i
0.0000     j
0.0000     k
```

```
r_I_dot_dot = [0;0;0]; % 0 m/s^2
r_B_I_dot_dot = [0;r_B_I_dot(1)^2/R;0]; % v_B^2/R j^ m/s^2
W_B_I_dot = [0;0;0]; % 0 m/s^2
r_B_dot_dot = r_I_dot_dot - r_B_I_dot_dot - cross(W_B_I, cross(W_B_I, r_B)) - cross(W_B_I_dot,
fprintf("Instantaneous acceleration (m/s^2) of plane A w.r.t. plane B is:\n%.4f\ti\n%.4f\tj\n%.
        r_B_dot_dot(1),r_B_dot_dot(2),r_B_dot_dot(3));
```

```
Instantaneous acceleration (m/s^2) of plane A w.r.t. plane B is:
0.0000    i
-4.6875    j
0.0000    k
```

2. Compare your results for part (a) with the case of erroneously treating the pilot of aircraft B as nonrotating.

```
r_I_dot_nonrot = [100;0;0]; % 100 i^ m/s
r_B_I_dot_nonrot = [150;0;0]; % 150 i^ m/s
r_B_dot_nonrot = r_I_dot_nonrot - r_B_I_dot_nonrot;
fprintf("Instantaneous velocity (m/s) of plane A w.r.t. non-rotating observer in plane B is:\n%
        r_B_dot_nonrot(1),r_B_dot_nonrot(2),r_B_dot_nonrot(3));
```

```
Instantaneous velocity (m/s) of plane A w.r.t. non-rotating observer in plane B is:
-50.0000 i
0.0000     j
0.0000     k
```

```
r_I_dot_dot_nonrot = [0;0;0]; % 0 m/s^2
r_B_I_dot_dot_nonrot = [0;r_B_I_dot_nonrot(1)^2/R;0]; % v_B_nonrot^2/R j^ m/s^2
r_B_dot_dot_nonrot = r_I_dot_dot_nonrot - r_B_I_dot_dot_nonrot;
fprintf("Instantaneous acceleration (m/s^2) of plane A w.r.t. non-rotating observer in plane B
        r_B_dot_dot_nonrot(1),r_B_dot_dot_nonrot(2),r_B_dot_dot_nonrot(3));
```

```
Instantaneous acceleration (m/s^2) of plane A w.r.t. non-rotating observer in plane B is:
0.0000     i
-56.2500 j
0.0000     k
```

```
r_B_dot_error = r_B_dot-r_B_dot_nonrot;
fprintf("Error in computing instantaneous velocity (m/s) of plane A w.r.t. rotating/non-rotatin
        r_B_dot_error(1),r_B_dot_error(2),r_B_dot_error(3));
```

```
Error in computing instantaneous velocity (m/s) of plane A w.r.t. rotating/non-rotating observer in plane B is:
-37.5000 i
0.0000     j
0.0000     k
```

```
r_B_dot_dot_error = r_B_dot_dot-r_B_dot_dot_nonrot;
fprintf("Error in computing instantaneous acceleration (m/s^2) of plane A w.r.t. rotating/non-r
        r_B_dot_dot_error(1),r_B_dot_dot_error(2),r_B_dot_dot_error(3));
```

```
Error in computing instantaneous acceleration (m/s^2) of plane A w.r.t. rotating/non-rotating observer in plane B is
0.0000    i
51.5625    j
0.0000    k
```

The resulting instantaneous velocity and acceleration of plane A w.r.t. plane B, when considering plane B to be rotating and non-rotating respectively, are not the same since plane B cannot be considered an inertial frame when it is undergoing circular motion. Thus, we need to have additional corrective terms in order to express the velocity/acceleration of plane A measured w.r.t. plane B, which is why the results differ.

Problem 4

Consider the following Bicycle Model vehicle:

- Mass: 2100 kg
- Wheelbase: 3.3 m
- Weight Distribution: Defined below
- Front Cornering Stiffness: −78.9 kN/rad
- Rear Cornering Stiffness: −77.5 kN/rad

This vehicle is being driven on a 144 m radius (left turn) skid pad at ever increasing speed, from zero to 78 mph.

For the weight distributions 38% front (a=2.046 m, b=1.254 m), 50% front (a=1.65 m, b=1.65 m), and 62% front (a=1.254 m, b=2.046 m), answer the following:

1. Plot Front Slip Angle vs. Lateral Acceleration
2. Plot Rear Slip Angle vs. Lateral Acceleration
3. Plot Vehicle Sideslip Angle (β) vs. Lateral Acceleration
4. Plot Steer Angle vs. Lateral Acceleration

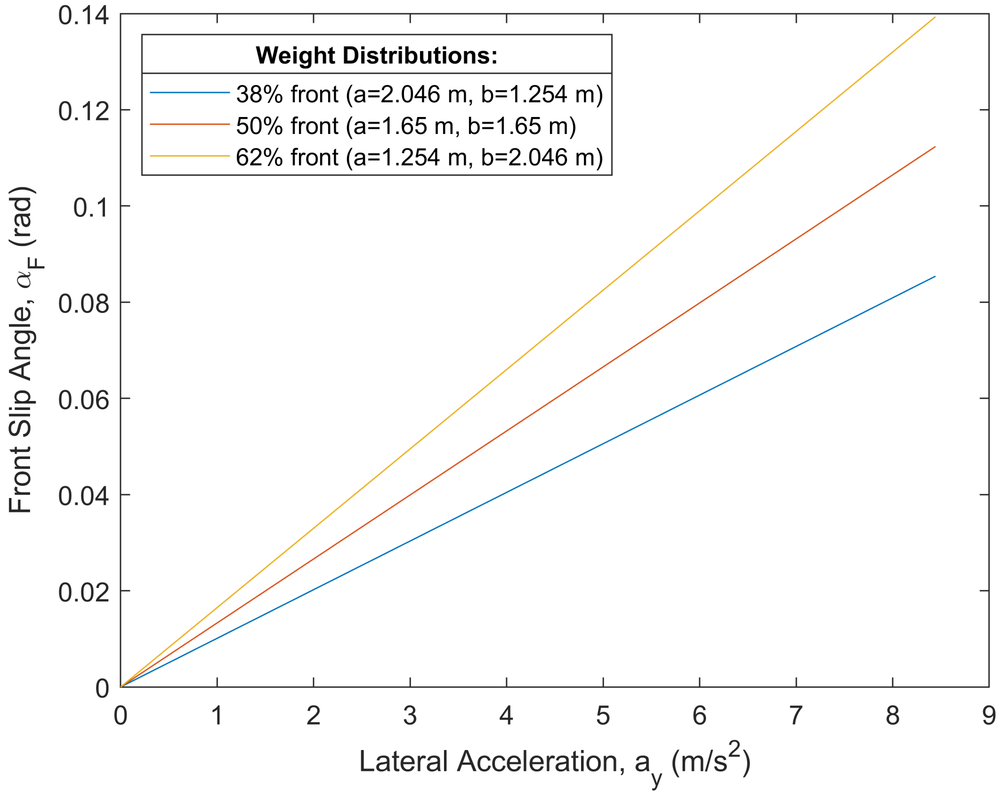
You should plot the results for all three models in one figure for each of the questions.

Solution 4

```
% Given data
m = 2100; % kg
l = 3.3; % m
c_alpha_F = 78.9e3; % N/rad (adapted ISO convention)
c_alpha_R = 77.5e3; % N/rad (adapted ISO convention)
R = 144; % m
v = 0:1e-4:(78*0.44704); % m/s
a_1 = 2.046; b_1 = 1.254; % m (for 38% front weight distribution)
a_2 = 1.650; b_2 = 1.650; % m (for 50% front weight distribution)
a_3 = 1.254; b_3 = 2.046; % m (for 62% front weight distribution)
% Calculate lateral acceleration and lateral force
a_y = (v.^2/R); % Adapted ISO convention
F_y = m*a_y;
```

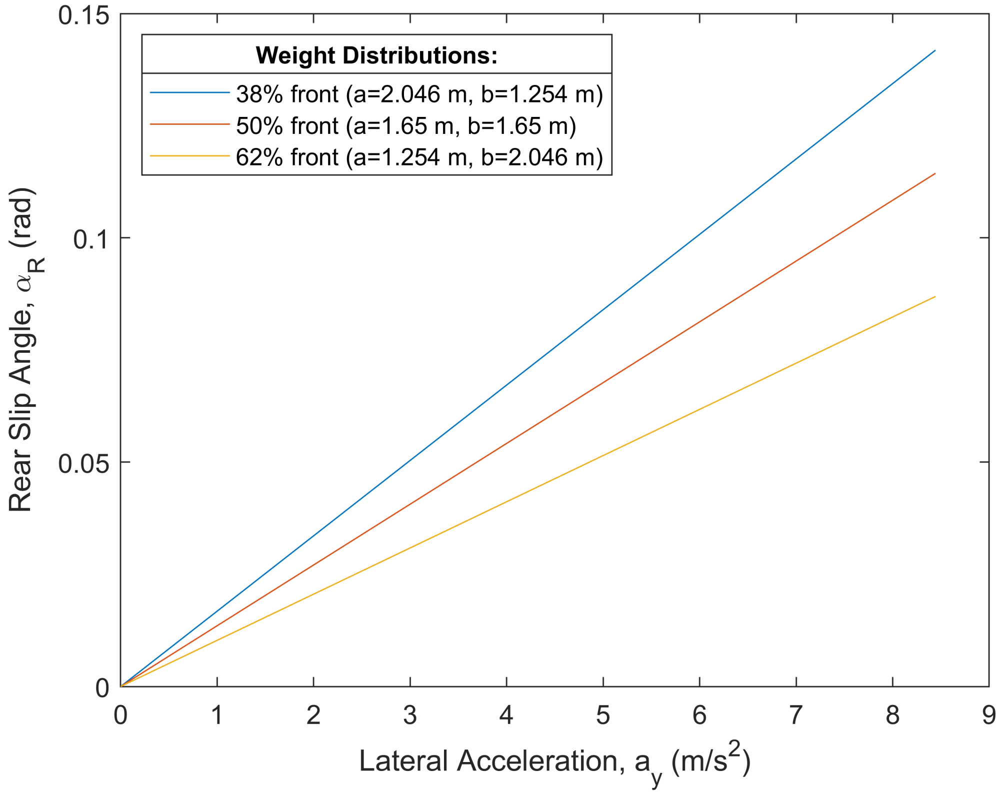
1. Plot Front Slip Angle vs. Lateral Acceleration

```
F_y_F_1 = (b_1/l)*F_y;
alpha_F_1 = F_y_F_1/c_alpha_F;
F_y_F_2 = (b_2/l)*F_y;
alpha_F_2 = F_y_F_2/c_alpha_F;
F_y_F_3 = (b_3/l)*F_y;
alpha_F_3 = F_y_F_3/c_alpha_F;
figure(1)
plot(a_y,alpha_F_1)
hold on
plot(a_y,alpha_F_2)
plot(a_y,alpha_F_3)
hold off
xlabel("Lateral Acceleration, a_y (m/s^2)")
ylabel("Front Slip Angle, \alpha_F (rad)")
lgd = legend("38% front (a=2.046 m, b=1.254 m)", "50% front (a=1.65 m, b=1.65 m)", "62% front (
title(lgd,'Weight Distributions:')
```



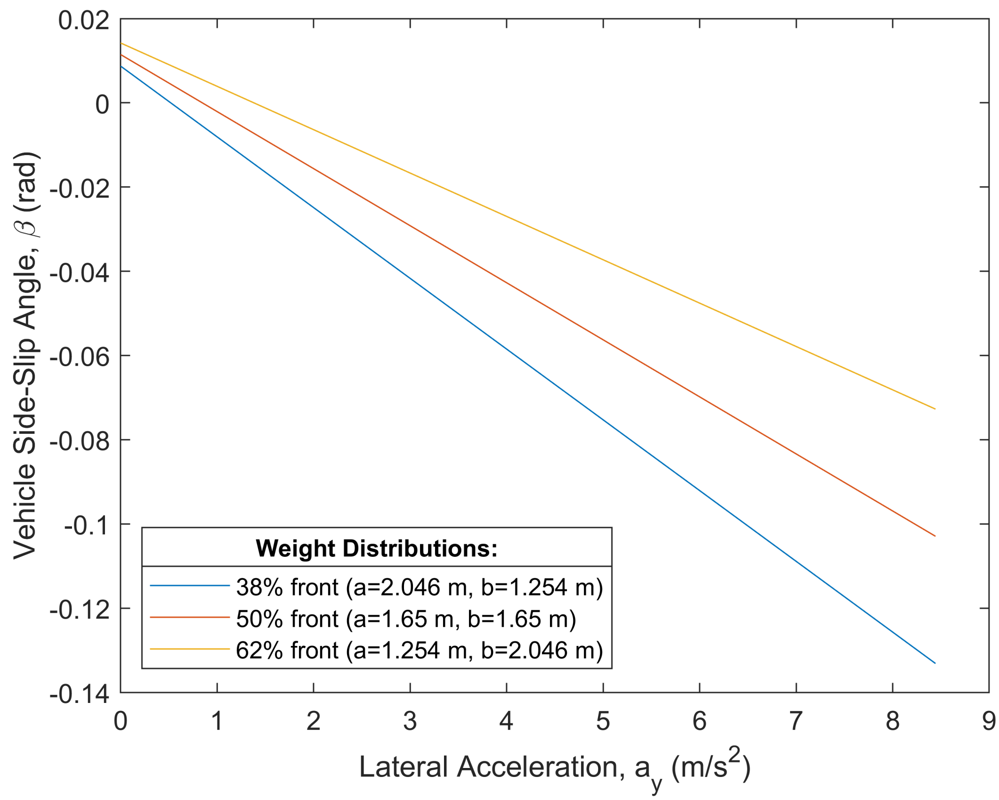
2. Plot Rear Slip Angle vs. Lateral Acceleration

```
F_y_R_1 = (a_1/l)*F_y;
alpha_R_1 = F_y_R_1/c_alpha_R;
F_y_R_2 = (a_2/l)*F_y;
alpha_R_2 = F_y_R_2/c_alpha_R;
F_y_R_3 = (a_3/l)*F_y;
alpha_R_3 = F_y_R_3/c_alpha_R;
figure(2)
plot(a_y,alpha_R_1)
hold on
plot(a_y,alpha_R_2)
plot(a_y,alpha_R_3)
hold off
xlabel("Lateral Acceleration, a_y (m/s^2)")
ylabel("Rear Slip Angle, \alpha_R (rad)")
lgd = legend("38% front (a=2.046 m, b=1.254 m)", "50% front (a=1.65 m, b=1.65 m)", "62% front (
title(lgd,'Weight Distributions:')
```



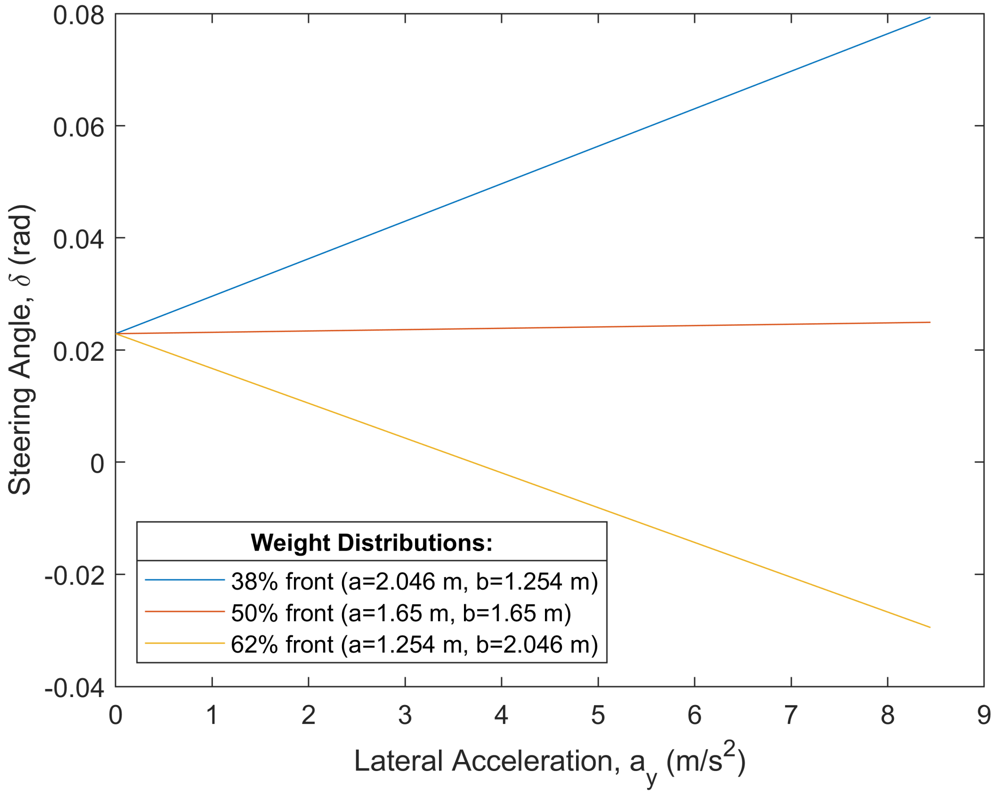
3. Plot Vehicle Sideslip Angle (β) vs. Lateral Acceleration

```
beta_1 = abs(b_1/R)-abs(alpha_R_1); % Adapted ISO convention
beta_2 = abs(b_2/R)-abs(alpha_R_2); % Adapted ISO convention
beta_3 = abs(b_3/R)-abs(alpha_R_3); % Adapted ISO convention
figure(3)
plot(a_y,beta_1)
hold on
plot(a_y,beta_2)
plot(a_y,beta_3)
hold off
xlabel("Lateral Acceleration, a_y (m/s^2)")
ylabel("Vehicle Side-Slip Angle, \beta (rad)")
lgd = legend("38% front (a=2.046 m, b=1.254 m)", "50% front (a=1.65 m, b=1.65 m)", "62% front (
title(lgd,'Weight Distributions:')
```



4. Plot Steer Angle vs. Lateral Acceleration

```
delta_1 = abs(l/R) - abs(alpha_F_1) + abs(alpha_R_1); % Adapted ISO convention
delta_2 = abs(l/R) - abs(alpha_F_2) + abs(alpha_R_2); % Adapted ISO convention
delta_3 = abs(l/R) - abs(alpha_F_3) + abs(alpha_R_3); % Adapted ISO convention
figure(4)
plot(a_y,delta_1)
hold on
plot(a_y,delta_2)
plot(a_y,delta_3)
hold off
xlabel("Lateral Acceleration, a_y (m/s^2)")
ylabel("Steering Angle, \delta (rad)")
lgd = legend("38% front (a=2.046 m, b=1.254 m)", "50% front (a=1.65 m, b=1.65 m)", "62% front (
title(lgd,'Weight Distributions:')
```


0.08
0.06
0.04
0.02
0
-0.02
-0.04
Steering Angle, δ (rad)
0
1
2
3
4
5
6
7
8
9
Lateral Acceleration, a_y (m/s^2)
Weight Distributions:
38% front (a=2.046 m, b=1.254 m)
50% front (a=1.65 m, b=1.65 m)
62% front (a=1.254 m, b=2.046 m)