1A. $W = \Sigma F_{zr} = LF + RF + LR + RR$ (N)

$X_{cg} = l \left(\frac{LF+RF}{W}\right)$ (m)

$Y_{cg} = t \left(\frac{RF+RR}{W}\right)$ (m)

1B. $D = (LF + RR) - (RF + LR)$ (N)

2A. Untripped rollover

$a_y = \underbrace{\frac{t}{2h}}_{SSH} g$

t = track width
h = height of CG
a_y = lateral acc. @ which inner wheels lift off

$\Rightarrow \frac{1}{h} = \frac{2 a_y}{g t} \Rightarrow h = \frac{g t}{2 a_y}$

$g = 9.81$ m/s^2

2B. $\Delta W_y = \frac{m a_y h}{t} = \frac{W A_y h}{t}$

$\left\{ \begin{array}{l} W = mg \\ A_y = a_y / g \end{array} \right.$

h/2

2C. $K_{\phi *} = \frac{1}{2} K_* t^2 + K_{\phi *_{Aux}}$

* Front/Rear

roll stiffness; spring stiffness; auxillary roll stiffness via anti-roll bar

- Since lateral load transfer is to be evenly distributed between front and rear, $K_{\phi f} = K_{\phi r}$
- Since suspension springs in rear are 1.25 times stiffer than ones in front, anti-roll bar needs to be added in the front to compensate for less stiff springs.

$\Rightarrow \frac{1}{2} K_f t^2 + K_{\phi f_{Aux}} = \frac{1}{2} K_r t^2$

$\Rightarrow \frac{1}{2} K_f t^2 + K_{\phi f_{Aux}} = \frac{1}{2}(1.25\, K_f)\, t^2$

$\Rightarrow \frac{1}{2} K_f (1.6)^2 + K_{\phi f Aux} = \frac{1}{2}(1.25\, K_f)(1.6)^2$

$\Rightarrow K_{\phi f_{Aux}} = 1.6\, K_f - 1.28\, K_f$

$\Rightarrow K_{\phi f Aux} = 0.32\, K_f$

2D.

$LF + RF + LR + RR - W = 0$

$LF + RF - W(X_{cg}/l) = 0$

$RF + RR - W(Y_{cg}/t) = 0$

$(LF + RR) - (RF + LR) - D = 0$

4 eq^{n}s in 4 unknowns (solved using MATLAB's symbolic toolbox)

3A. $h_{cr} = h - \left(h_f + \frac{(h_r - h_f)a}{l}\right)$ (m)

$a_y = v(\dot{\beta} + \dot{\psi})$ (m/s²)

$\phi = \frac{m_s a_y h_{cr}}{K_r - m_s h_{cr} g}$ ← roll angle (rad)

$\Delta F_{zf\#} = S_f \pm \frac{k_f \phi}{t} + \frac{m_s a_y b h_f}{l t}$ (N)

$\Delta F_{zr\#} = S_r \pm \frac{k_r \phi}{t} + \frac{m_s a_y a h_r}{l t}$ (N)

left/right

$\pm \Rightarrow$ right, left

$\alpha_f = \delta - \frac{\dot{\psi} a}{v} - \beta$

$\alpha_r = \frac{\dot{\psi} b}{v} - \beta$

ISO → Adapted ISO

$F_{y*\#} = -\text{nonlintire}(\alpha_*, F_{z*\#}, v)$

* front/rear

\# left/right

$F_{y*} = \sum F_{y*\#}$ → e.g. $F_{yf} = F_{yfl} + F_{yfr}$

$$\left.\begin{aligned} \dot{\beta} &= \frac{F_{yf} + F_{yr}}{mv} - \dot{\psi} \\ \ddot{\psi} &= \frac{F_{yf} \cdot a - F_{yr} \cdot b}{I_z} \end{aligned}\right\} \dot{x} = \begin{bmatrix} \dot{\beta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix}$$

3B. $h_{cr} = h - \left(h_f + \frac{(h_r - h_f)a}{l}\right)$ (m)

$I_\phi = I_{xx} + m_s h_{cr}^2$ ← parallel axis theorem ($kg \cdot m^2$)

$a_y = v(\dot{\beta} + \dot{\psi})$ (m/s²)

Making small angle approximation:

$\ddot{\phi} = \frac{1}{I_\phi}\left(m_s a_y h_{cr} + m_s g h_{cr} \phi - K_r \phi - B_r \dot{\phi}\right)$ (rad/s²)

$\dot{\phi}_{t+1} = \dot{\phi}_t + \ddot{\phi}_t \, dt$ ← forward Euler update (rad/s)

$\phi_{t+1} = \phi_t + \dot{\phi}_t \, dt$ ← forward Euler update (rad)

$\Delta F_{zf\#} = S_f \pm \frac{k_f \phi}{t} + \frac{m_s a_y b h_f}{l t}$ (N)

$\Delta F_{zr\#} = S_r \pm \frac{k_r \phi}{t} + \frac{m_s a_y a h_r}{l t}$ (N)

left/right

$+ \rightarrow$ right

$- \rightarrow$ left

$\alpha_f = \delta - \frac{\dot{\psi} a}{v} - \beta$

$\alpha_r = \frac{\dot{\psi} b}{v} - \beta$

ISO → Adapted ISO

$F_{y*\#} = -\text{nonlintire}(\alpha_*, F_{z*\#}, v)$

* front/rear

\# left/right

$F_{y*} = \sum F_{y*\#}$ → e.g. $F_{yf} = F_{yfl} + F_{yfr}$

$$\left.\begin{aligned} \dot{\beta} &= \frac{F_{yf} + F_{yr}}{mv} - \dot{\psi} \\ \ddot{\psi} &= \frac{F_{yf} \cdot a - F_{yr} \cdot b}{I_z} \end{aligned}\right\} \dot{x} = \begin{bmatrix} \dot{\beta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix}$$

ANSWER 1A:

The vehicle's CG is at (1.458615 m , 0.685767 m) as measured from the rear axle and left side.

ANSWER 1B:

The vehicle's diagonal weight (a.k.a. cross weight or wedge) is 222.410946 N.

ANSWER 2A:

The height of CG must be 0.888889 m to cause un-tripped rollover of the vehicle at 0.9g lateral acceleration.

ANSWER 2B:

Total load transfer in vehicle when CG height is half of the rollover height will be 5000.00 N.

ANSWER 2C:

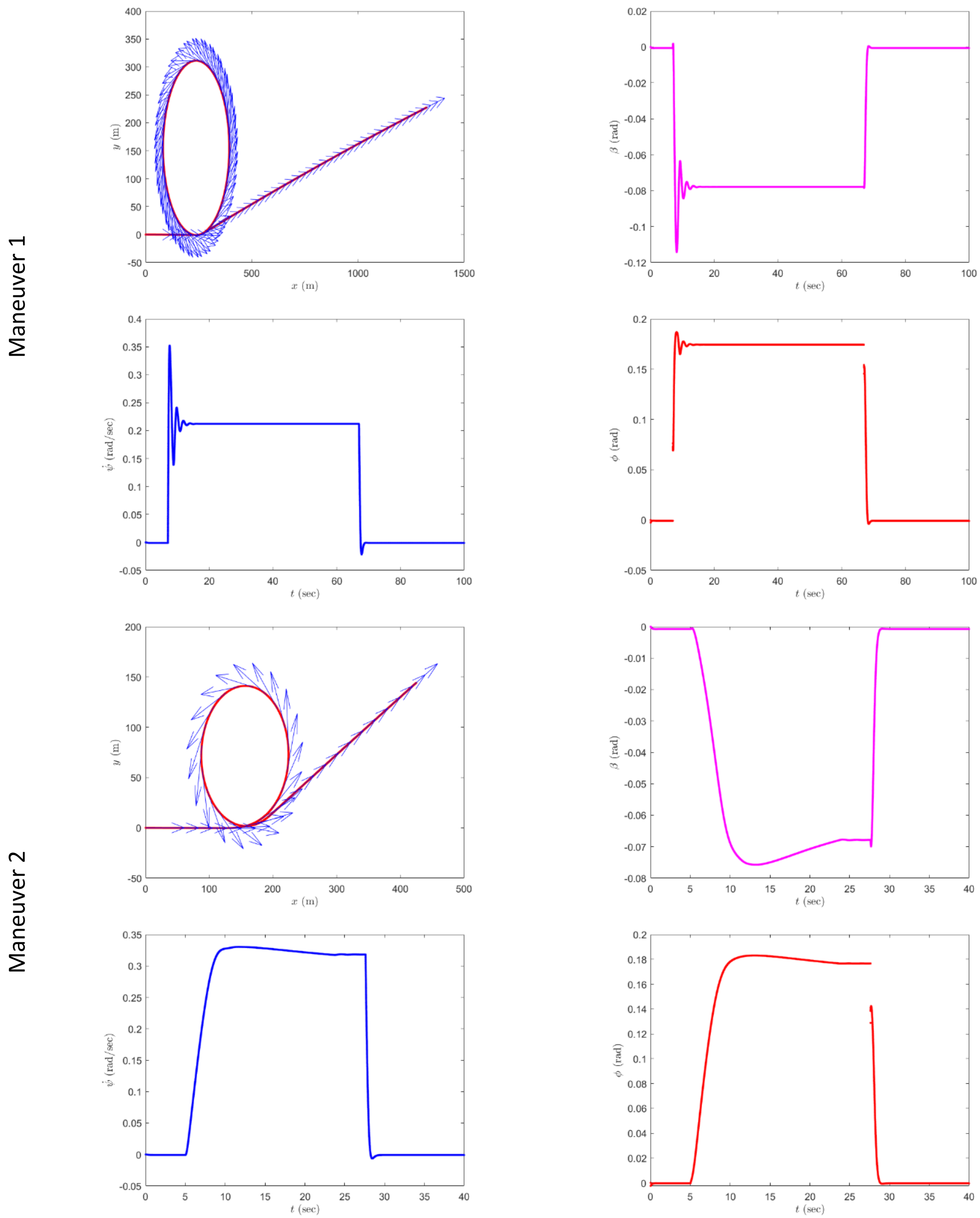
Since the lateral load transfer is to be evenly distributed between front and rear, and since suspension springs in rear are 25% (1.25 times) stiffer than ones in the front, anti-roll bar needs to be added in the front (to compensate for less stiff springs) with torsional stiffness of **0.32 times front spring stiffness (Kf)**.

ANSWER 2D:

The static wheel loads for given vehicle configuration are:

- LF = 6006.25 N
- RF = 5593.75 N
- LR = 4393.75 N
- RR = 4006.25 N

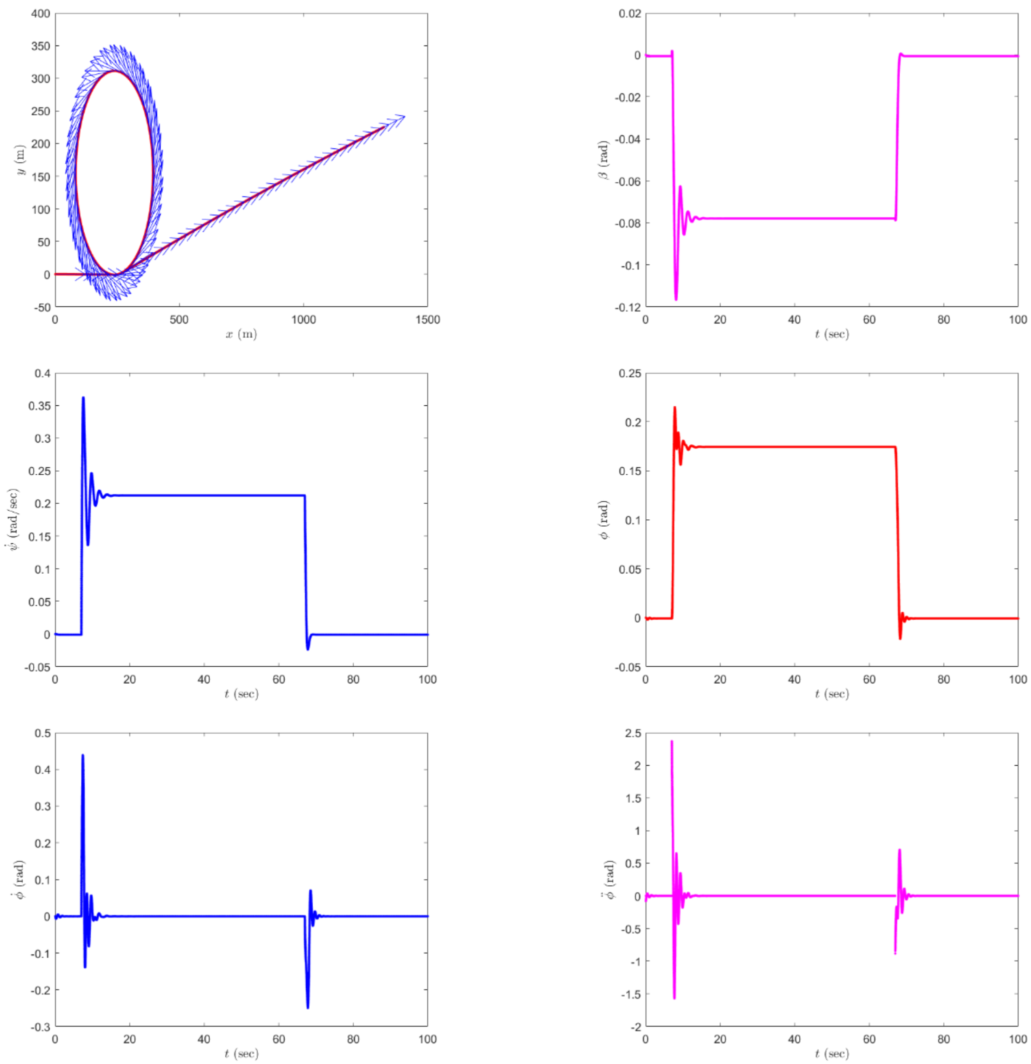
ANSWER 3A:



NOTE: The above plots were generated assuming sprung mass of the vehicle as 100% of the total vehicle mass.

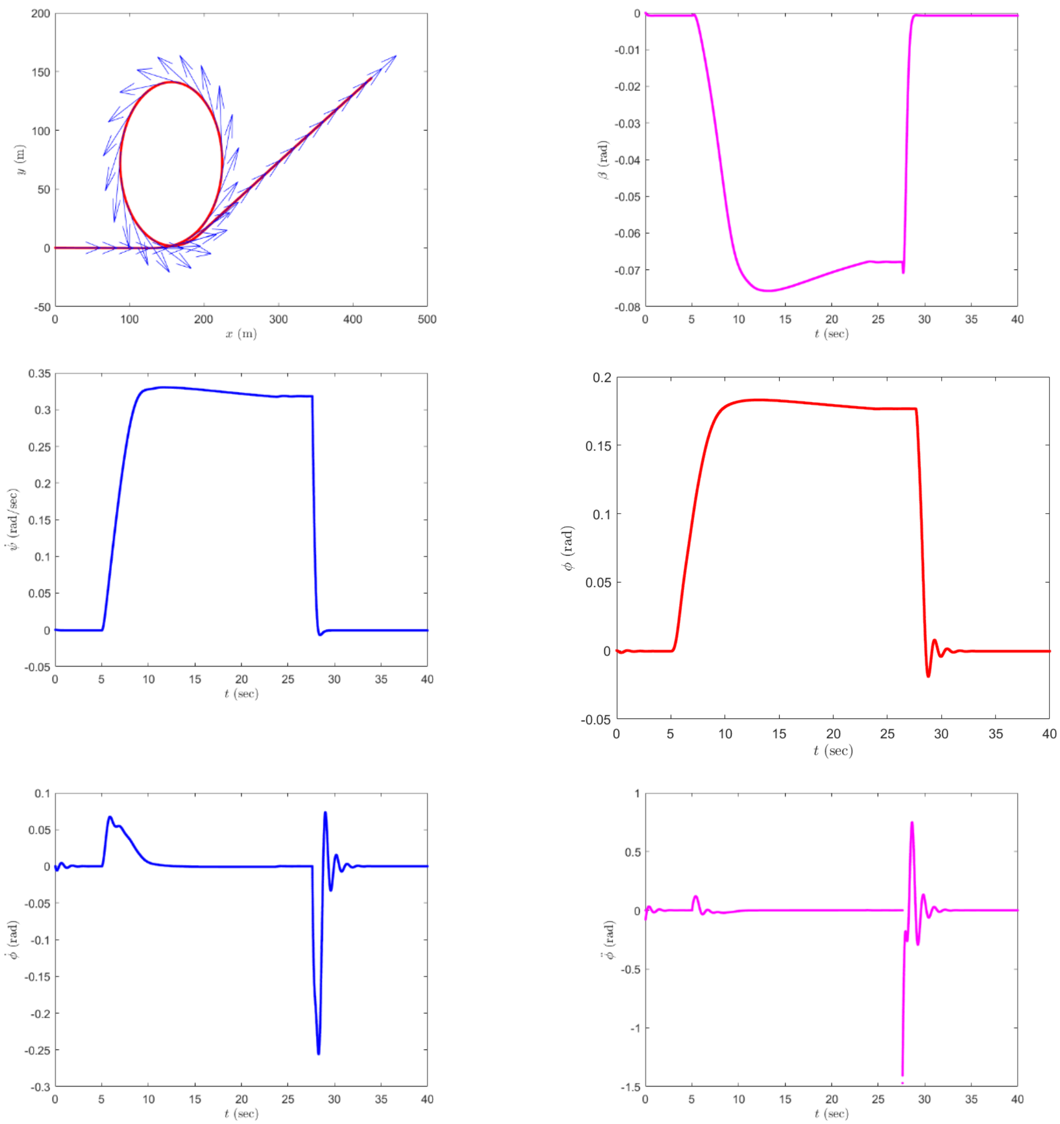
ANSWER 3B:

Maneuver 1



NOTE: The above plots were generated assuming sprung mass of the vehicle as 100% of the total vehicle mass.

Maneuver 2



NOTE: The above plots were generated assuming sprung mass of the vehicle as 100% of the total vehicle mass.

ANSWER 3C:

Major difference between solutions of 3A and 3B is that 3A assumes only steady-state roll (i.e. there are no transients) and its effects on load transfer. On the other hand, 3B uses second order roll dynamics (angular acceleration about the roll axis) at each point in time and uses it to compute the roll angle with numerical integration of the derivative terms, thereby capturing the transients and using them to compute load transfer at each point in time.

Since we are accounting for transient roll dynamics in 3B and not in 3A, we can easily notice a larger peak value of roll angle in 3B plots. Furthermore, we can observe that the roll angle plots display more realistic transient behavior (oscillations) as compared to 3A plots and that there is no sudden increase/decrease in roll angle w.r.t. time in 3B since it gradually changes accounting for the transients (whereas in 3A the roll angle abruptly increases/decreases in a single time step which is clearly observable from 3A plots), which is expected. Finally, this small difference in roll angle update causes minor (the difference could be larger for more aggressive maneuvers and different vehicle parameters) deviation in vehicle trajectory and state propagation.