AuE-ME 4600-6600: Dynamic Performance of Vehicles
Fall 2022; Midterm Exam – Take Home Portion

1. <u>Power Limit (15 points):</u>

A proving ground test was performed on a long, straight road to determine the terminal velocity of a certain vehicle. During the test, the on-board instrumentation failed after the following data was acquired:

Engine Power at the tires	Vehicle Speed	Acceleration	System Diagnostic Comments
HP	kmph	g	
220	30	0.905	Traction-limited acceleration
220	50	0.895	Traction-limited acceleration
220	---	0.495	Power-limited acceleration
220	100	---	Power-limited acceleration
220	---		Sensor failure; Vehicle Speed
220	---		Sensor failure; Vehicle Speed

The following information is available:

- Vehicle mass: 1450 kg
- Engine power at the tires: always 220 HP
- Resisting forces: aerodynamic drag and no others
- The car's aerodynamicist is away on vacation. When reached by cell phone he said the aerodynamic drag force is perfectly modeled by $F = 0.5 \cdot \rho \cdot v^2 \cdot C_D A$, but he couldn't remember values for C_D or A.
- Assume standard atmospheric density
- Tires: capable of producing 1.1g of lateral acceleration, longitudinal coefficient of friction, μ=0.91

A. Determine this vehicle's terminal velocity.
B. Suppose the driver lifts off the throttle at top speed. What instantaneous deceleration will the vehicle experience solely due to aerodynamic drag?
C. Now suppose the vehicle is cornering at 0.5g, with all four tires operating at 5 deg slip angle. Calculate the total induced drag force. (Note: Assume that the friction limit of the tire is not reached!)
D. Write an equation to predict a new terminal velocity at maximum lateral acceleration, accounting for the induced drag. (Do not solve for the new terminal velocity.)

2. <u>Tire dynamics + Bicycle model (20 points):</u>

The velocity and lateral acceleration instrumentation on an autonomous car at a proving ground has malfunctioned. In this question you are asked to determine the missing information. Here are the vehicle parameters:

- Mass: 1500 kg
- Drag Coefficient: 0.5
- Frontal Area: 2.2 m^2
- Straight Line Top Speed: 240 km/hour
- Tire Print Length: 0.16 m (Note that this is 2a!)
- Lateral Stiffness per Unit Length of the Tire: 6.25 MN/m^2
- Maximum Friction Coefficient: 1.3

The car was being driven on a very large, constant-radius circle as fast as possible when the instrumentation failed – the vehicle was power limited at the time. The only information available after the instrumentation failed was from the steer-by-wire system which needed to overcome a total self-aligning moment of 160 Nm to steer the car.

A. Using the linear (brush) tire model, determine the total lateral force produced by the tires and determine at what slip angle the tires were running (see assumptions)!
B. What was the speed of the vehicle (see hint!)?
C. What was the lateral acceleration and the radius of the circle?
D. What slip angle would result for the lateral force determined in part A if you employed the brush tire model with parabolic pressure distribution?
E. Given a brush tire model with parabolic pressure distribution, which velocity could the car drive on the circle determined in part C before total sliding would occur?
F. Assume the cornering stiffness from part A for the front tires and a CG location resulting with 55% of the weight at the rear axle. What cornering stiffness would be needed for the rear of the vehicle to make it neutral steer?

Assumptions and Hints:

- For part A, assume that all four tires were running at the same slip angle and produced the same lateral force.
- For part B, determine the maximum engine power from the straight-line condition, then account for the induced drag effect (rearward component of the lateral force with respect to the velocity vector) while cornering to determine the speed.
- For part D and E, assume that there is no load transfer, and that the vehicle is perfectly balanced, i.e., the vehicle mass is distributed evenly to all four tires.

3. Tire Dynamics + Semi-Empirical Model (15 points):

Please note that the questions below are all separate parts with separate values given.

A. Non-dimensional Tire Model

The three equations below represent a Nondimensional Tire Model for a certain tire. Expand the model to determine the lateral force at 4 deg slip angle, –1200 lb normal load.

$$\mu = 2 + 0.0004\, F_Z$$

$$c = 0.0003{F_z}^2 + 0.8F_Z$$

$$\bar{F} = \sin(1.4\tan^{-1}(0.7\ \bar{\propto}))$$

Note: B = 0.7, C = 1.4, D = 1, E = 0.

B. Brush Model:

The load of the tire is 4500 N, the contact patch length of the tires is 8 cm, the slip ratio is 0.15, the coefficient of friction is 0.8, and the longitudinal stiffness of the tire is C_x = 1300 N/cm^2. Based on the brush model, calculate the longitudinal force under this slip ratio.

C. Pacejka Model:

The load of the tire is F_Z = 4500 N. The coefficient of road adhesion is μ_p = 1.0 and μ_s = 0.7. When the slip ratio is s = 0.04, the longitudinal force is F_x = 1000 N. When s = 0.2, then F_x reaches the maximum value. Based on the H.B. Pacejka magic formula model, analyze the relationships between the longitudinal force and the slip ratio

4. <u>Tire Data Processing (15)</u>

The table contains tire data recorded at a nonzero inclination angle. Determine the following at –5.55kN normal load.

A. (4pts.) Cornering stiffness

- (4pts.) Left turn lateral friction coefficient (at positive slip angles)
- (3.5pts.) Vertical spring rate (at zero slip angle)
- (3.5pts.) Induced drag at +3 deg slip angle

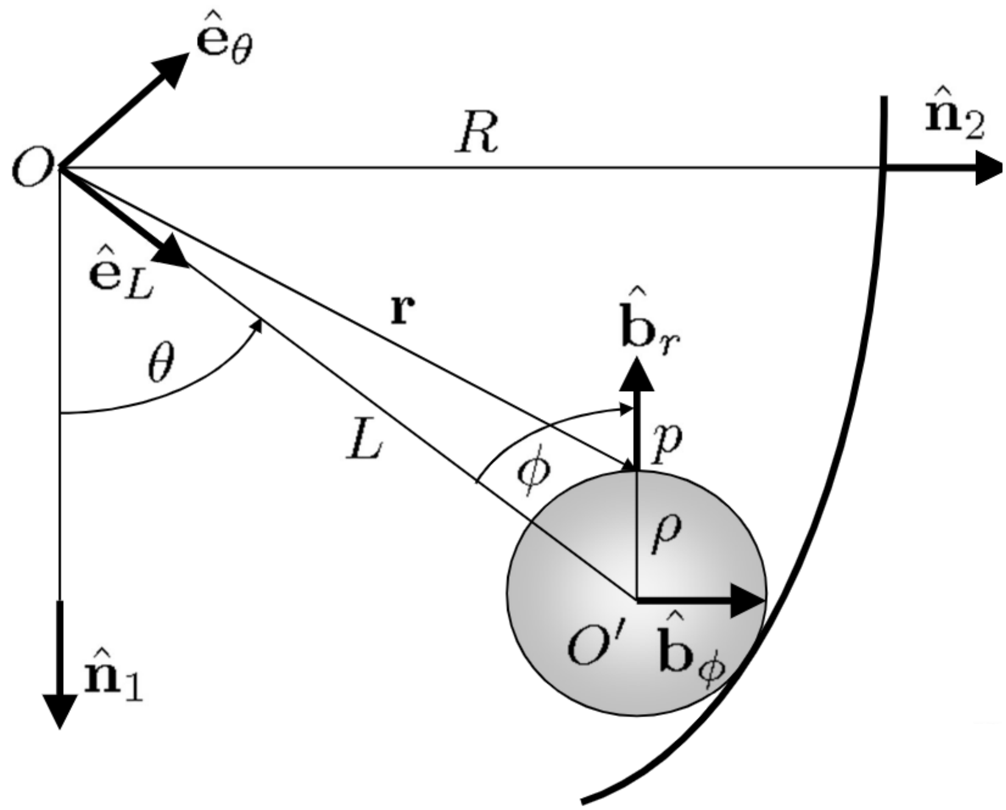
Columns are normal load (FZ), slip angle (SA), inclination angle (IA), inflation pressure (P), lateral force (FY) and loaded radius (RL)

FZ	SA	IA	P	FY	RL
kN	deg	deg	kPa	kN	cm
-3.325	-3	4	221	5.159	27.67
-3.325	-2	4	221	4.400	27.71
-3.325	-1	4	221	2.899	27.76
-3.325	0	4	221	0.160	27.80
-3.325	1	4	221	-2.522	27.77
-3.325	2	4	221	-4.348	27.71
-3.325	3	4	221	-5.439	27.66
-3.325	4	4	221	-5.824	27.63
-3.325	5	4	221	-6.008	27.62
-3.325	6	4	221	-5.935	27.63
-3.325	7	4	221	-5.851	27.63
-3.325	8	4	221	-5.779	27.63
-5.550	-3	4	221	8.042	27.35
-5.550	-2	4	221	6.943	27.43
-5.550	-1	4	221	4.533	27.57
-5.550	0	4	221	0.391	27.66
-5.550	1	4	221	-3.751	27.60
-5.550	2	4	221	-6.941	27.43
-5.550	3	4	221	-8.611	27.30
-5.550	4	4	221	-9.315	27.24
-5.550	5	4	221	-9.374	27.23

-5.550	6	4	221	-9.287	27.24
-5.550	7	4	221	-9.052	27.26
-5.550	8	4	221	-8.902	27.28
-7.775	-3	4	221	10.536	26.99
-7.775	-2	4	221	9.191	27.12
-7.775	-1	4	221	6.271	27.34
-7.775	0	4	221	1.427	27.52
-7.775	1	4	221	-3.967	27.45
-7.775	2	4	221	-8.256	27.19
-7.775	3	4	221	-11.004	26.93
-7.775	4	4	221	-12.163	26.80
-7.775	5	4	221	-12.477	26.76
-7.775	6	4	221	-12.386	26.77
-7.775	7	4	221	-12.187	26.80
-7.775	8	4	221	-11.971	26.82

5. Transport theorem (15 points):

Consider a small disk with radius ρ that rolls without slipping inside a large hoop of radius R as shown. Frame ***N*** is fixed to the hoop, frame $\mathcal{E}$ is fixed to the lever arm that connects the disk to the center of hoop, and the frame ***B*** is fixed to the disk. Point ***O*** is the center of the hoop, point ***O'*** is the center of the disk, and point ***p*** is fixed in the disk. The distance from ***O'*** to ***p*** is ρ. Assuming rolling without slipping and the lever is rotating at a constant rate $\omega = \dot{\theta}$, Determine the inertial acceleration of point ***p*** expressed in $\mathcal{E}$ coordinates. Give answer in terms of ω, L, R, ρ and φ.
(Hint: Refer to Lecture notes)



6. **Racetrack (20 points):**

Assume a racecar has acceleration limits as shown in the g-g diagram below. For simplicity, the maximum driving acceleration is assumed to be constant - a power-limited value estimated for the expected velocity range. Cornering and braking limits are also assumed to be velocity independent.

Determine the minimum time solution for this car to complete a lap of the track below. Solve for a "flying lap" - the car does not start from zero velocity. Each of the two straightaways are 0.45 miles long, and both corners are 180 deg and with radii $R_2 = 180\ m$ and $R_1 = 230\ m$. Assume counterclockwise direction (left turns). Loop is along WXYZ

To simplify the calculations, assume the car accelerates in only one direction at a time - thus holding constant speed while cornering. Assume the transitions between the straightaways and the corners (a change in radius) happen instantaneously. Assume that the corners have a constant radius.

In the end, report the following (note that these quantities are not necessarily computed in this order):

A. Minimum time required for one lap
B. Average speed for one lap
C. The cornering speeds
D. The maximum speed achieved on the straights
E. The location of the braking transition point (measured as a distance from the previous corner)