Vehicle Dynamics Homework 2 Solutions

Note: entire MATLAB code is presented at end of solutions.

Problem 1:

As mentioned on the first page of Homework 2, there are a total of six target loads spaced apart by 50 lbf. Our first step is to find these target loads at the conditions mentioned before question 1: data at 0 deg inclination angle, 12 psi inflation pressure, and 25 mph roadway speed. After converting the forces into lbf, pressures into psi, and velocities into mph, the indices for the target values at each of the six loads can be found as follows:

```
targetFZ = 50*round(FZ/50); % rounded all loads to the nearest ...
    50 lbf
targetFZlist = unique(targetFZ); % removes all repeats; should ...
    be a 6x1 vector, for the 6 different normal loads

% Target values for each normal load
iiFZ350 = find(targetFZ == targetFZlist(1) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);
iiFZ300 = find(targetFZ == targetFZlist(2) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12); % ends up being empty set; ...
    remove from targetFZlist
iiFZ250 = find(targetFZ == targetFZlist(3) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);
iiFZ150 = find(targetFZ == targetFZlist(4) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);
iiFZ100 = find(targetFZ == targetFZlist(5) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);
iiFZ50 = find(targetFZ == targetFZlist(6) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);

targetFZlist(2) = []; % now have five different normal loads
```

The plot then simply comes from indexing slip angle and lateral force by those indices and putting them on a single plot with a legend to identify which corresponds to each target load.

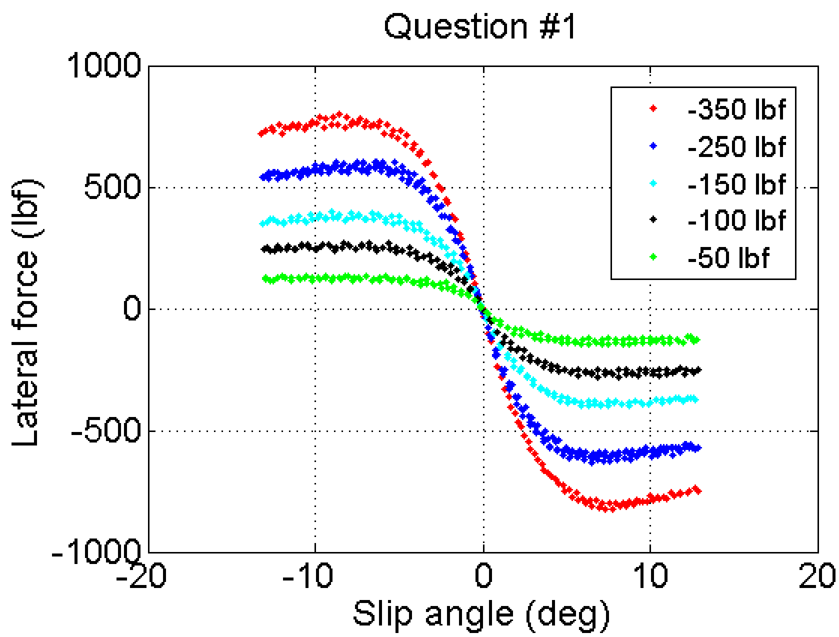

Figure 1: Lateral force vs. slip angle for each normal load.

Problem 2:

Recall from class that cornering stiffness is the slope of the lateral force versus slip angle curve in its linear range. Keeping in mind that the linear range is around 0 *deg*, we can create new indices as follows:

```
%% Question 2
% Limit slip angle to between -1 and 1 deg for linear part
slope1cs = find(targetFZ == targetFZlist(1) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
cs1 = polyfit(SA(slope1cs),FY(slope1cs),1);
slope2cs = find(targetFZ == targetFZlist(2) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
cs2 = polyfit(SA(slope2cs),FY(slope2cs),1);
slope3cs = find(targetFZ == targetFZlist(3) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
cs3 = polyfit(SA(slope3cs),FY(slope3cs),1);
slope4cs = find(targetFZ == targetFZlist(4) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
cs4 = polyfit(SA(slope4cs),FY(slope4cs),1);
slope5cs = find(targetFZ == targetFZlist(5) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
cs5 = polyfit(SA(slope5cs),FY(slope5cs),1);
```

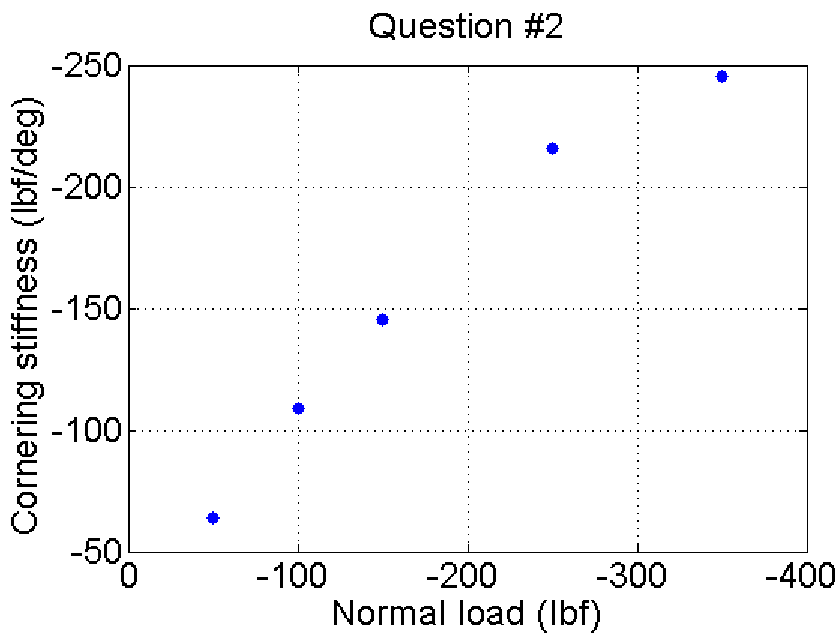

Figure 2: Cornering stiffness vs. Normal load.

Normal load (lbf)	Cornering stiffness (lbf/deg)
-350	-245.382689169168
-250	-216.030935427145
-150	-145.436054969249
-100	-109.202316102064
-50	-63.8904332276433

Table 1: Cornering stiffness at each normal load.

Problem 3:

The tire's friction coefficient, μ, comes from dividing the maximum lateral force by the normal load it bears. In a left hand turn, the lateral forces will be negative. Knowing this, we can find the friction coefficient through:

```
fyPeak = [min(FY(iiFZ350)); min(FY(iiFZ250)); min(FY(iiFZ150)); ...
    min(FY(iiFZ100)); min(FY(iiFZ50))];
mu = abs(fyPeak./targetFZlist);
```

The plot is then simple.

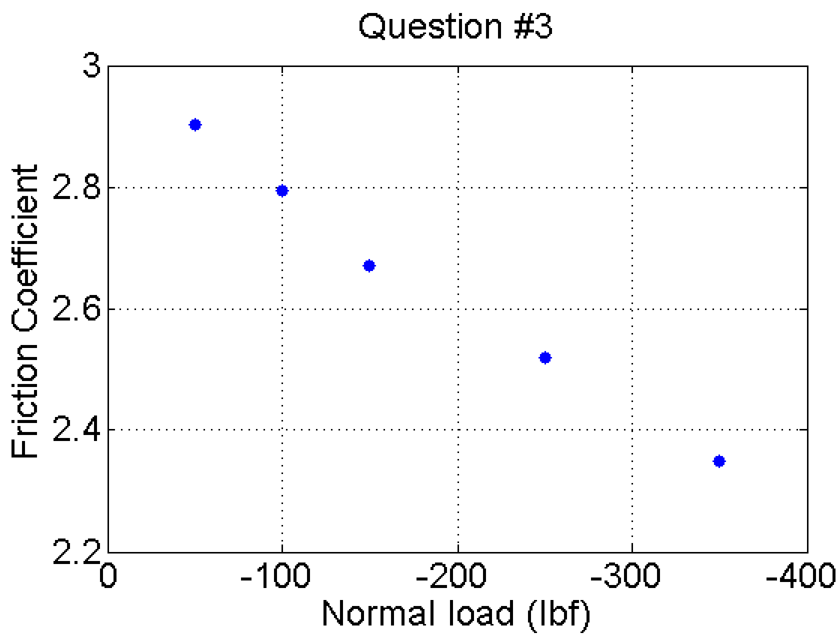

Figure 3: Friction coefficient vs. Normal load.

Normal load (lbf)	Friction coefficient (unitless)
-350	2.34937549349049
-250	2.51948777834732
-150	2.66962118538787
-100	2.7950271074247
-50	2.9036772695766

Table 2: Friction coefficient at each normal load.

Problem 4:

Using the indices developed in the first problem, the following plot can be made:

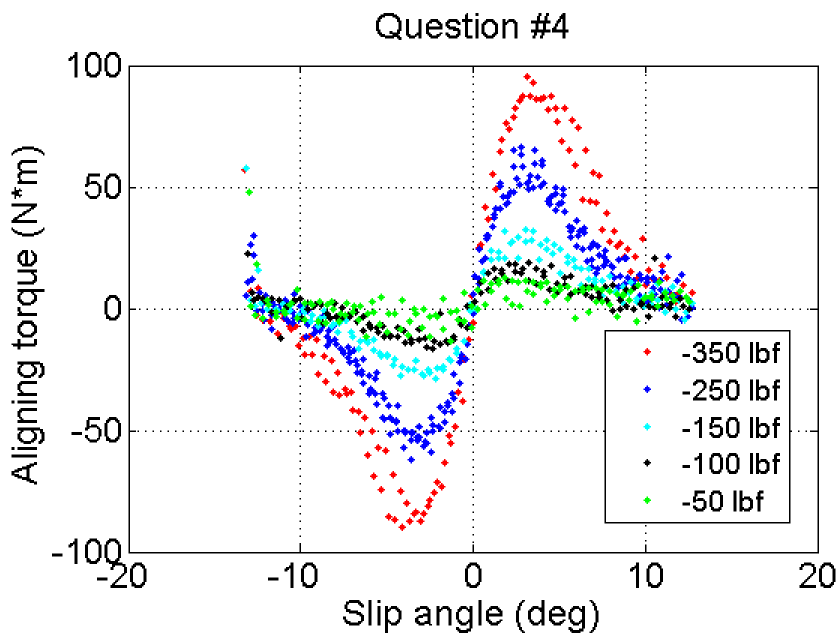

Figure 4: Aligning torque vs. slip angle.

Problem 5:

Restricting slip angle magnitudes to be less than 1 generally limits the lateral force. Indexing this way and fitting the data with a linear fit can be done via:

```
vsrInd = find(round(V) == 25 & round(IA) == 0 & round(P) == 12 & ...
    abs(SA) < 1);
cm2in = 0.393701; % converting cm to inches
RL = cm2in.*RL;

slope_sr = polyfit(RL(vsrInd),FZ(vsrInd),1);

disp(['Vertical Spring Rate at 200 lb. from Linear fit: ...
    ',num2str(slope_sr(1)),' lbf/in'])
```

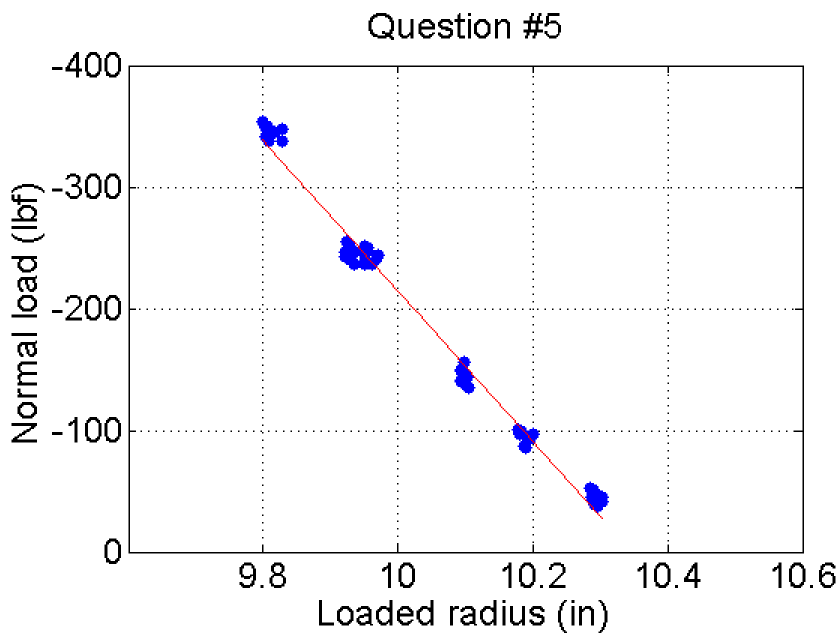

Figure 5: Normal load vs. loaded radius.

Using the linear fit, the vertical spring rate of the tire at a 200 lbf normal load is simply the slope of the line: 617.7769 lbf/in

MATLAB code for solution:

```
% RVD_Homework 3
clear all
clc
close all

format long g

load('tireF15')

% ET is time, sec
% V roadway velocity, kph
% SA slip angle, deg
% IA inclination angle, deg
% P inflation pressure, kPa
% FZ normal load, N
% FY lateral force, N
% MX overturning moment, N*m
% MZ aligning torque, N*m
% RL loaded radius, cm

figure
plot(ET, V, '.')
grid on
title('Roadway speed vs. time', 'FontSize', 18)
xlabel('time (sec)', 'FontSize', 18)
ylabel('(kph)', 'FontSize', 18)
set(gca,'FontSize',18);

figure
plot(ET, IA, '.')
grid on
title('Inclination angle vs. time', 'FontSize', 18)
xlabel('time (sec)', 'FontSize', 18)
ylabel('(deg)', 'FontSize', 18)
set(gca,'FontSize',18);

figure
plot(ET, SA, '.')
grid on
title('Slip angle vs. time', 'FontSize', 18)
xlabel('time (sec)', 'FontSize', 18)
ylabel('(deg)', 'FontSize', 18)
set(gca,'FontSize',18);

figure
plot(ET, FZ, '.')
grid on
title('Normal load vs. time', 'FontSize', 18)
xlabel('time (sec)', 'FontSize', 18)
ylabel('(N)', 'FontSize', 18)
set(gca,'FontSize',18);

figure
plot(ET, P, '.')
```

```
grid on
title('Inflation pressure vs. time', 'FontSize', 18)
xlabel('time (sec)', 'FontSize', 18)
ylabel('(kPa)', 'FontSize', 18)
set(gca,'FontSize',18);

%% Unit conversions
% For questions, use only the data at 0 deg. inclination angle, ...
    12 psi inflation pressure and 25 mph roadway speed
kph2mph = 0.621371;
kPa2psi = 0.145037738;
newton2lbf = 0.224808943;
zDegIA = find(round(IA) == 0);

V = V.*kph2mph;
P = P.*kPa2psi;
FZ = FZ.*newton2lbf;
FY = FY.*newton2lbf;

%% Finding target values
% Finding the target normal loads
targetFZ = 50*round(FZ/50); % rounded all loads to the nearest ...
    50 lbf
targetFZlist = unique(targetFZ); % removes all repeats; should ...
    be a 6x1 vector, for the 6 different normal loads

% Target values for each normal load
iiFZ350 = find(targetFZ == targetFZlist(1) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);
iiFZ300 = find(targetFZ == targetFZlist(2) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12); % ends up being empty set; ...
    remove from targetFZlist
iiFZ250 = find(targetFZ == targetFZlist(3) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);
iiFZ150 = find(targetFZ == targetFZlist(4) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);
iiFZ100 = find(targetFZ == targetFZlist(5) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);
iiFZ50 = find(targetFZ == targetFZlist(6) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12);

targetFZlist(2) = []; % now have five different normal loads

%% Question 1

forLegend = {'-350 lbf'; '-250 lbf'; '-150 lbf'; '-100 lbf'; ...
    '-50 lbf'};

figure
plot(SA(iiFZ350), FY(iiFZ350), 'r .')
hold
plot(SA(iiFZ250), FY(iiFZ250), 'b .')
plot(SA(iiFZ150), FY(iiFZ150), 'c .')
plot(SA(iiFZ100), FY(iiFZ100), 'k .')
plot(SA(iiFZ50), FY(iiFZ50), 'g .')
grid on
h_legend  = legend(forLegend,'Location','Best');
```

```
set(h_legend,'FontSize',14)
title('Question #1', 'FontSize', 18)
xlabel('Slip angle (deg)', 'FontSize', 18)
ylabel('Lateral force (lbf)', 'FontSize', 18)
set(gca,'FontSize',18);

%% Question 2
% Limit slip angle to between -1 and 1 deg for linear part
slope1cs = find(targetFZ == targetFZlist(1) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
cs1 = polyfit(SA(slope1cs),FY(slope1cs),1);
slope2cs = find(targetFZ == targetFZlist(2) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
cs2 = polyfit(SA(slope2cs),FY(slope2cs),1);
slope3cs = find(targetFZ == targetFZlist(3) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
cs3 = polyfit(SA(slope3cs),FY(slope3cs),1);
slope4cs = find(targetFZ == targetFZlist(4) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
cs4 = polyfit(SA(slope4cs),FY(slope4cs),1);
slope5cs = find(targetFZ == targetFZlist(5) & round(V) == 25 & ...
    round(IA) == 0 & round(P) == 12 & abs(SA) < 1);
cs5 = polyfit(SA(slope5cs),FY(slope5cs),1);

cs_all = [cs1(1);cs2(1);cs3(1);cs4(1);cs5(1)];

% Make a table
disp('                 Normal load  Cornering stiffness')
disp('                      lbf                lbf/deg')
disp([targetFZlist cs_all])

figure
plot(targetFZlist, cs_all, '.', 'MarkerSize', 20)
grid on
set(gca,'XDir','reverse')
set(gca,'YDir','reverse')
set(gca, 'FontSize', 18)
xlabel('Normal load (lbf)', 'FontSize', 18)
ylabel('Cornering stiffness (lbf/deg)', 'FontSize', 18)
title('Question #2', 'FontSize', 18)

%% Question 3

fyPeak = [min(FY(iiFZ350)); min(FY(iiFZ250)); min(FY(iiFZ150)); ...
    min(FY(iiFZ100)); min(FY(iiFZ50))];
mu = abs(fyPeak./targetFZlist);

% Make a table
disp('                 Normal load  Friction coefficient')
disp('                      lbf                (unitless)')
disp([targetFZlist mu])

figure
plot(targetFZlist,mu,'.','MarkerSize',20);
grid on
set(gca,'XDir','reverse')
set(gca, 'FontSize', 18)
```

```matlab
xlabel('Normal load (lbf)', 'FontSize', 18)
ylabel('Friction Coefficient', 'FontSize', 18)
title('Question #3', 'FontSize', 18)

%% Question 4

figure
plot(SA(iiFZ350), MZ(iiFZ350), 'r .')
hold
plot(SA(iiFZ250), MZ(iiFZ250), 'b .')
plot(SA(iiFZ150), MZ(iiFZ150), 'c .')
plot(SA(iiFZ100), MZ(iiFZ100), 'k .')
plot(SA(iiFZ50), MZ(iiFZ50), 'g .')
grid on
h_legend  = legend(forLegend,'Location','Best');
set(h_legend,'FontSize',14)
title('Question #4', 'FontSize', 18)
xlabel('Slip angle (deg)', 'FontSize', 18)
ylabel('Aligning torque (N*m)', 'FontSize', 18)
set(gca,'FontSize',18);

%% Question 5

vsrInd = find(round(V) == 25 & round(IA) == 0 & round(P) == 12 & ...
    abs(SA) < 1);
cm2in = 0.393701; % converting cm to inches
RL = cm2in.*RL;

figure
plot(RL(vsrInd), FZ(vsrInd),'.','MarkerSize',20)
grid on
hold on
set(gca,'YDir','reverse','FontSize',18)
ylabel('Normal load (lbf)','FontSize',18)
xlabel('Loaded radius (in)','FontSize',18)
title('Question #5','FontSize',18)

slope_sr = polyfit(RL(vsrInd),FZ(vsrInd),1);

x = linspace(min(RL(vsrInd)),max(RL(vsrInd)),501);  %vector of x ...
    values for plotting the fit

% Plot the linear fit
y1 = polyval(slope_sr,x);
plot(x,y1,'r')

disp(['Vertical Spring Rate at 200 lb. from Linear fit: ...
    ',num2str(slope_sr(1)),' lbf/in'])
```