

Solⁿ 1

1.1 Power equation:

$$F_A \cdot v = P_w - v \cdot F_D$$

↑ ↑ ↑ ↑ ↓
 acc. vel. power vel. aero
 force at wheels drag force

$$\textcircled{1} \text{ terminal vel. } \Rightarrow \text{acc} = 0 \Rightarrow F_A = 0 \Rightarrow F_A \cdot v = 0$$

$$\Rightarrow 0 = P_w - v \cdot F_D$$

$$\Rightarrow v F_D = P_w$$

$$\Rightarrow F_D = \frac{P_w}{v}$$

given $\begin{cases} P_w = 230 \text{ hp} = 230 \times 745.6998 = 171510.954 \text{ watts} \\ v = 290 \text{ km/hr} = \frac{290}{3.6} = 80.5556 \text{ m/s} \end{cases}$

$$\Rightarrow F_D = \frac{171510.954}{80.5556} = 2129.1015 \text{ N}$$

Ans: F_D = 2129.1015 N

1.2 Power efficiency:

$$P_w = \eta_d P_e$$

→ $\eta_d = 100 - 12.5 \%$

↑ ↑ ↑
 Power at wheels engine
 at wheels efficiency power

Power equation:

$$F_A \cdot v = P_w - v \cdot F_D$$

$$\textcircled{2} \text{ terminal vel } \Rightarrow \text{acc.} = 0 \Rightarrow F_A = 0 \Rightarrow F_A \cdot v = 0$$

$$\Rightarrow P_w = v \cdot F_D$$

Aero drag:

$$F_D = \frac{1}{2} \rho v^2 C_D A_f$$

↓ ↓ ↑ ↗
 Aero. drag force Air density Apparent vel. Drag coeff.
 Frontal area

$$\Rightarrow P_w = v F_D = v \cdot \left(\frac{1}{2} \rho v^2 C_D A_f \right) = \frac{1}{2} \rho v^3 C_D A_f$$

$$\Rightarrow C_D A_f = \frac{2 P_w}{\rho v^3}$$

(i) given $\begin{cases} P_e = 310 \text{ hp} = 310 \times 745.6998 = 231166.938 \text{ watts} \\ \eta_d = \frac{100 - 12.5}{100} = 0.875 \\ v = 290 \text{ km/hr} = \frac{290}{3.6} = 80.5556 \text{ m/s} \\ \rho = 1.225 \text{ kg/m}^3. \quad (\text{from lecture notes}) \end{cases}$

$$\Rightarrow P_w = 0.875 \times 231166.938 = 202271.0708 \text{ N}$$

$$\Rightarrow C_D A_f = \frac{2 \times 202271.0708}{1.225 \times (80.5556)^3} = 0.6317 \text{ m}^2$$

Ans: $C_D A_f = 0.6317 \text{ m}^2$

(ii) given $\begin{cases} w = 1750 \text{ mm} = 1.75 \text{ m} \\ h = 1210 \text{ mm} = 1.21 \text{ m} \end{cases}$

$$A_f \approx (79-84)\% \text{ of } w \times h \quad (\text{from lecture notes})$$

$$\Rightarrow A_f = \left(\frac{79+84}{2} \right) \% \text{ of } w \times h = 81.5 \% \text{ of } w \times h$$

$$= \frac{81.5}{100} \times 1.75 \times 1.21 = 1.7258 \text{ m}^2$$

$$\Rightarrow C_D = \frac{C_D \cdot A_f}{A_f} = \frac{0.6317}{1.7258} = 0.366$$

Ans: $C_D = 0.366$

1.3 Power eqⁿ @ terminal vel (from solⁿ 1.2 and 1.1)

$$P_w = v F_D = v \cdot \left(\frac{1}{2} \rho v^2 C_D A_F \right) = \frac{1}{2} \rho v^3 C_D A_F$$

$$\therefore \Rightarrow C_D = \frac{2 P_w}{\rho v^3 A_F}$$

given $\left\{ P_w = 1160 \text{ hp} = 1160 \times 745.698 = 865011.768 \text{ watts} \right.$

$$\left. \rho = 1.225 \text{ kg/m}^3 \text{ (lecture notes)} \right.$$

$$v = 278 \text{ mph} = 278 \times 0.447 = 124.266 \text{ m/s}$$

$$A_F = 20.16 \text{ ft}^2 = 20.16 \times 0.0929 = 1.8729 \text{ m}^2$$

$$\Rightarrow C_D = \frac{2 \times 865011.768}{1.225 \times (124.266)^3 \times 1.8729} = 0.393$$

Ans: $C_D = 0.393$

1.4 Power eqⁿ @ terminal vel (from solⁿ 1.1, 1.2, 1.3)

$$P_w - v F_D = 0$$

\Rightarrow For achieving the same speed (go as fast) "v" $\rightarrow V_{\text{terminal}} = v_k$ (let)

$$\underbrace{P_w_M - v F_D M}_{M = \text{Mercedes Uhlenhaut Coupe}} = \underbrace{P_w_K - v F_D K}_{K = \text{Koenigsegg Agera RS}} = 0$$

M = Mercedes
Uhlenhaut
Coupe

K = Koenigsegg
Agera RS

$$\Rightarrow P_w_M = P_w_K - v_k F_{D_K} + v_k F_{D_M}$$

Required power

given $\left\{ \begin{array}{l} P_{W_k} = 1160 \text{ hp} = 1160 \times 745.6998 = \frac{865011.768}{865357.68} \text{ watts} \\ v_k = 278 \text{ mph} = 278 \times 0.447 = 124.266 \text{ m/s} \end{array} \right.$

Aero drag

$$F_D = \frac{1}{2} \rho v^2 C_D A_F$$

$$\Rightarrow F_{D_k} = \frac{1}{2} \times 1.225 \times (124.266)^2 \times 0.393 \times 1.8729$$

from solⁿ 1.3

$$\Rightarrow F_{D_k} = 6961.7411 \cdot N$$

similarly, $F_{D_M} = \frac{1}{2} \times 1.225 \times (124.266)^2 \times 0.366 \times 1.7258$

from solⁿ 1.2 (ii)

$$\Rightarrow F_{D_M} = 5974.2347 \text{ N}$$

$$\Rightarrow P_{W_M} = \frac{865011.768}{865357.68} - (124.66 \times 6961.7411) + (124.66 \times 5974.2347)$$

$$= \frac{865011.768}{865357.68} - \frac{867850.645}{865357.68} + \frac{744748.0977}{865357.68}$$

$$= \frac{742255.1322}{865357.68} \text{ watts} = 741909.2207 \text{ watts}$$

$$\Rightarrow P_{W_M} = \frac{742255.1322}{745.6998} = \frac{741909.2207}{745.6998} = 994.92 \text{ hp}$$

Ans: $P_{W_M} = 994.92 \text{ hp}$

Solⁿ 2

$$2.C \quad a_y = \frac{v_{\text{corner}}^2}{R}$$

$$\Rightarrow v_{\text{corner}} = \sqrt{a_y \cdot R}$$

$$\text{given } \left\{ \begin{array}{l} a_y = 1.2 g = 1.2 \times 9.81 = 11.772 \text{ m/s}^2 \\ 2\pi R = 0.35 \text{ miles} = 0.35 \times 1609.344 = 563.2704 \text{ m} \end{array} \right.$$

$$\Rightarrow R = \frac{563.2704}{3.14} = 179.2945 \text{ m}$$

$$\Rightarrow v_{\text{corner}} = \sqrt{11.772 \times 179.2945}$$

$$= \sqrt{11772 \times 179.2945} \approx 45.94$$

$$= 45.94 \text{ m/s}$$

Ans: $v_{\text{corner}} = 45.94 \text{ m/s}$

2.E From lecture notes,

$$S_T = (v_{\text{brake}})^2 - (v_{\text{accel}})^2 + 2 |a_{\text{brake}}| \cdot L$$

$$L = \frac{2 (a_{\text{accel}} + |a_{\text{brake}}|)}{0.4 \text{ miles}} = 0.4 \times 1609.344 = 643.7376 \text{ m}$$

$$\text{given } \left\{ \begin{array}{l} v_{\text{brake}} = v_{\text{corner}} = 45.94 \text{ m/s} \\ v_{\text{accel}} = v_{\text{corner}} = 45.94 \text{ m/s} \end{array} \right.$$

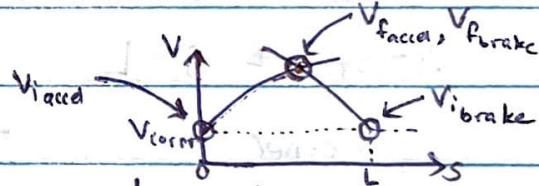
$$|a_{\text{brake}}| = 1.2g = 1.2g = 1.2 \times 9.81 = 11.772 \text{ m/s}^2$$

$$a_{\text{accel}} = 0.5g = 0.5 \times 9.81 = 4.905 \text{ m/s}^2$$

$$\Rightarrow S_T = \frac{(45.94)^2 - (45.94)^2 + (2 \times 11.772 \times 643.7376)}{2 \times (4.905 + 11.772)}$$

$$= 454.403 \text{ m}$$

Ans: $S_T = 454.403 \text{ m}$



2.D Max. speed achieved on straight is at point S_T

$$V_{max}^2 = V_{corner}^2 + 2 \cdot a_{accel} \cdot S_T \quad \dots (v^2 = u^2 + 2as)$$

$$\begin{aligned}\Rightarrow V_{max} &= \sqrt{V_{corner}^2 + 2 \cdot a_{accel} \cdot S_T} \\ &= \sqrt{(45.94)^2 + (2 \times 4.905 \times 454.403)} \\ &= \pm 81.044 \\ &= 81.044 \text{ m/s}\end{aligned}$$

Ans: $V_{max} = 81.044 \text{ m/s}$

2.B For const. acc., $V_{avg} = \frac{V_{final} + V_{initial}}{2}$

3 parts: (2 for straight + 1 for corner):

$$S=0 \rightarrow S=S_T \Rightarrow V_1 = \frac{V_{max} + V_{corner}}{2} = \frac{81.044 + 45.94}{2} = 63.492 \text{ m/s}$$

$$S=S_T \rightarrow S=L \Rightarrow V_2 = \frac{V_{corner} + V_{max}}{2} = \frac{45.94 + 81.044}{2} = 63.492 \text{ m/s}$$

Corner $\Rightarrow V_3 = V_{corner}$ (const vel.) = 45.94 m/s

3 more (same) parts for remainder track (which we can ignore since race circuit is symmetrical).

$$\begin{aligned}\Rightarrow V_{avg} &= \frac{V_1 + V_2 + V_3}{3} \\ &= \frac{63.492 + 63.492 + 45.94}{3} \\ &= 57.641 \text{ m/s}\end{aligned}$$

Ans: $V_{avg} = 57.641 \text{ m/s}$

2.A Since we solved this problem for maximum average velocity (minimum time) in order to "win" the race:

$$t_{min} = \frac{\text{dist}}{V_{avg}}$$

given dist = 1.5 miles = $1.5 \times 1609.344 = 2414.016$ m

$$\Rightarrow t_{\min} = \frac{2414.016}{57.641} = 41.8802 \text{ sec}$$

Ans: $t_{\min} = 41.8802 \text{ sec.}$

Solⁿ 3

$$3.C \quad a_y = \frac{V_{\text{corner}}^2}{R}$$

$$\Rightarrow V_{\text{corner}} = \sqrt{a_y \cdot R}$$

Solving programatically in MATLAB, given $a_y = 1.2 g$ and $\pi R = 0.35$ miles, we get $V_{\text{corner}} = 45.9419$ m/s

Ans: $V_{\text{corner}} = 45.9419 \text{ m/s}$

3E Acceleration on straight is variable

@ begining of straight:

$$t(0) = 0$$

$$v(0) = V_{\text{corner}}$$

$$s(0) = 0$$

choose $\Delta t = 0.001$, let $t_i = i \cdot \Delta t$ with $i = 0, 1, 2, \dots$

Power eqⁿ: $F_A \cdot v = P_w - v \cdot F_D \Rightarrow P_w = F_A \cdot v + F_D \cdot v$

$$\Rightarrow P_w = m \cdot a_{\text{accel}}(t_i) \cdot v(t_i) + \left[\frac{1}{2} \cdot S \cdot v_{(t_i)}^2 \cdot C_0 A_F \right] \cdot v(t_i)$$

$$\Rightarrow a_{\text{accel}}(t_i) = \frac{P_w - \left[\frac{1}{2} \cdot S \cdot v_{(t_i)}^2 \cdot C_0 A_F \right]}{m \cdot v(t_i)}$$

$a_{\text{accel}}(v)$ \leftarrow accel. as function of vel
 (t_i)

Euler forward:

→ solved in MATLAB

$$\begin{bmatrix} v \\ s \end{bmatrix}_{(t_i+1)} = \begin{bmatrix} v \\ s \end{bmatrix}_{(t_i)} + \Delta t \cdot \begin{bmatrix} a_{\text{accel}}(v) \\ v \end{bmatrix}_{(t_i)}$$

calculate over $s = [0, L]$ i.e. length of straight.

Deceleration (braking) over straight is constant

$$\Rightarrow \left(v_{\text{brake}}\right)^2_{(t_i)} = (V_{\text{corner}})^2 + 2 |a_{\text{brake}}| s_{(t_i)} \dots (v^2 = u^2 + 2as)$$

The iteration where velocities from above two equations become equal (very close) is the transition point and the distance corresponding to this is the distance from $s=0$ to $s=s_T$. Thus, iterations can be stopped.

Ans: $s_T = \boxed{613.7667 \text{ m}}$ 527.1507 m

3.D Max. speed achieved on straight is at $s=s_T$. Since we stop iterations of Euler forward algorithm in solⁿ 3.E upon reaching $s=s_T$, the last velocity value in the array is max. velocity (solved in MATLAB)

Ans: $\boxed{V_{\text{max}} = 72.1287 \text{ m/s}}$ 69.6750 m/s

3.B Similar to solⁿ 2.B, we have 3 parts:

$$s=0 \rightarrow s=s_T \text{ (non-uniform accel)} \Rightarrow v_1 = \frac{\sum_{i=0}^n v_{(t_i)}}{n} \quad (t_i = i \cdot \Delta t)$$

$$s=s_T \rightarrow s=L \begin{cases} \text{(uniform accel)} \\ \text{Corner} \end{cases} \Rightarrow v_2 = \frac{V_{\text{corner}} + V_{\text{max}}}{2}$$

$$\Rightarrow v_3 = V_{\text{corner}}$$

v_1 can be found by taking arithmetic mean of all elements

of velocity array as a result of Solⁿ 3.E Euler forward algorithm. v_2 and v_3 can be simply calculated by substituting v_{corner} and v_{max} found in Solⁿ 3.C and Solⁿ 3.D (solved in MATLAB)

Ans: $v_{avg} = \underline{55.4903}$ mls 54.4951 m/s

3.A Since we solved for minimum time to "win" the race

$$t_{min} = \underline{\text{dist}} \leftarrow \text{given dist} = 1.5 \text{ miles}$$

$$v_{avg} \leftarrow \text{Sol}^n 3.B$$

Ans: $t_{min} = \underline{43.5034}$ sec. 44.2979 m/s

Solⁿ 4.

4.1 Solve $v_t = \frac{v_0}{\beta} \tan\left(1 - \frac{t}{T} \tan^{-1}(\beta)\right) + v_{wind}$ for $\beta = [1.1, 1.4, 1.8, 2.2, 2.6, 3.0]$

from $t=0$ to $t=t_{final}$ and compute $\frac{v_t}{v_0}$ and $\frac{t}{T}$ for each value, where $v_0 = 27.8049$ mls and $T = 122.31$ sec.

Ans: Solved in MATLAB

4.2 Since $v_0 = 27.8049$ mls and $T = 122.31$ sec (from given data), for each value of velocity ' v ' & corresponding timestamp ' t ' we can find (and plot) $\frac{v_t}{v_0}$ and $\frac{t}{T}$ (solved in MATLAB). Visually observing (or using least squares) we can determine ~~approx~~: β that corresponds to real world data.

Ans: $\beta = 1.8$

4.3

(a) Drag coeff.

$$C_D = \frac{2 \cdot m \cdot \beta \cdot \tan^{-1}(\beta)}{(V_0 + V_{wind}) \cdot g \cdot A_F}$$

given $m = 78.8$ slugs, $V_0 = 27.8049$ m/s, $V_{wind} = 4.8$ mph,

$g = 1.225$ kg/m³, $A_F = 22.17$ ft², $\beta = 1.8$ (soln 4.2)

we can compute C_D (solved in MATLAB)

Ans: $C_D = 0.4446$

(b) Rolling resistance

$$R_x = \frac{(V_0 + V_{wind})m \cdot \tan^{-1}(\beta)}{\beta \cdot T}$$

given $m = 78.8$ slugs, $V_0 = 27.8049$ m/s, $V_{wind} = 4.8$ mph,

$\beta = 1.8$ (soln 4.2), $T = 122.31$ sec

we can compute R_x (solved in MATLAB)

Ans: $R_x = 178.337$ N