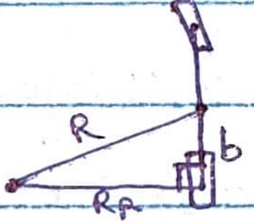
1A. $l = 3.05 \text{ m}$



$a/l = 0.45 \Rightarrow b = l\left(1 - \frac{a}{l}\right)$
$= 1.6775 \text{ m}$

$\Rightarrow R_R = \sqrt{R^2 - b^2}$

$R = 105 \text{ m} \Rightarrow R_R = \sqrt{105^2 - 1.6775^2} = 104.9865991 \text{ m}$

$\delta_{A\,actual} = \tan^{-1}\left(\frac{l}{R_R}\right) = \tan^{-1}\left(\frac{3.05}{104.9865991}\right) = \tan^{-1}\left(0.029051326\right) = \boxed{0.029043158 \text{ rad}}$

Ackermann

$\delta_{A\,approx} = \left(\frac{l}{R}\right) = \frac{3.05}{105} = \boxed{0.029047619 \text{ rad}}$

absolute error → $|\text{error}| = |0.029043158 - 0.029047619| = \boxed{4.460977661 \times 10^{-6} \text{ rad}}$

620×0.3048

1B. $R = 620\ \text{ft} = \cancel{620/3.2808}\ \text{m} = 188.976\ \text{m}$

$V = 70\ \text{mph} = 70 \times 0.44704\ \text{m/s} = 31.2928\ \text{m/s}$

$\therefore$ yaw velocity $= \dot{\psi} = \omega = \dfrac{V}{R} = \dfrac{31.2928}{188.976} = \boxed{0.16559\ \text{rad/s}}$

$\therefore$ lateral acceleration $= a_y = \dfrac{V^2}{R} = R\omega^2 = 188.976(0.16559)^2 = \boxed{5.1818\ \text{m/s}^2}$

16. $l = 2.7 \text{ m}$

$m = 105 \text{ slugs} = 105 \times 14.5939 \text{ kg} = 1532.3598 \text{ kg}$

$a/l = 0.5$

Adapted ISO

$C_{\alpha F} = C_{\alpha R} = 750 \text{ N/deg} = 750 \times \frac{180}{\pi} \text{ N/rad} = 42971.83463 \text{ N/rad}$

$v = 111 \text{ km/h} = 111/3.6 \text{ m/s} = 30.8333 \text{ m/s}$

$R = 232 \text{ m}$

Force-free rolling:

$\tan(\beta_0) = \frac{b}{R_R} \Rightarrow \beta_0 \approx \frac{b}{R}$ (lecture notes)

$\therefore b = l\left(1 - \frac{a}{l}\right) = 2.7(1-0.5) = 2.7 \times 0.5 = 1.35 \text{ m}$

$\therefore \beta_0 = \frac{1.35}{232} = \boxed{0.005818965517 \text{ rad}}$

Rolling with lateral forces:

$|\beta| + |\alpha_R| = \left|\frac{\dot{\psi} b}{v}\right|$ ... (lecture notes)

$\therefore |\beta| + |\alpha_R| = \left|\frac{b}{R}\right|$ ... $\left[\left(\frac{\dot{\psi}}{v}\right) = \frac{1}{R}\right]$

$\therefore b = l\left(1 - \frac{a}{l}\right) = 2.7(1-0.5) = 2.7 \times 0.5 = 1.35 \text{ m}$

$\therefore |F_{y_R}| = \frac{a}{l} \cdot F_y = \left(\frac{a}{l}\right) \cdot m \cdot a_y = \left(\frac{a}{l}\right)\frac{mv^2}{R} \overset{!}{=} C_{\alpha R}\, \alpha_R$

$\therefore |\alpha_R| = \left(\frac{a}{l}\right)\frac{mv^2}{R C_{\alpha R}} = \frac{1532.3598 \times (30.833)^2 \times 0.5}{232 \times 42971.83463} = 0.073063 \text{ rad}$

$\therefore \beta = \frac{b}{R} - \alpha_R = 0.005818965517 - 0.073063 = \boxed{-0.067244 \text{ rad}}$

2.1 By transport theorem $\begin{pmatrix} B = \text{body} \\ I = \text{inertial} \end{pmatrix}$ (Translation + rotation)

velocity $\rightarrow$

$$\underbrace{\frac{d}{dt}\underline{r}^I}_{\substack{\text{vel. in}\\ \text{inertial}\\ \text{coordinates}}} = \underbrace{\frac{d}{dt}\underline{r}^B}_{\substack{\text{vel. in}\\ \text{body}\\ \text{coordinates}}} + \underbrace{\underline{\omega}_{B|I}}_{\substack{\text{ang. vel}\\ \text{of body frame}\\ \text{w.r.t inertial}\\ \text{frame}}} \times \underbrace{\underline{r}^B}_{\substack{\text{position}\\ \text{in body}\\ \text{coordinates}}} + \underbrace{\frac{d}{dt}\underline{r}_B^I}_{\substack{\text{vel. of body frame}\\ \text{w.r.t. inertial}\\ \text{frame}}} \quad - (1)$$

acceleration $\rightarrow$

$$\underbrace{\frac{d^2}{dt^2}\underline{r}^I}_{\substack{\text{acc. in inertial}\\ \text{coordinates}}} = \overbrace{\frac{d^2}{dt^2}\underline{r}^B}^{\text{acc. in body coordinates}} + \overbrace{\underline{\omega}_{B|I} \times (\underline{\omega}_{B|I} \times \underline{r}^B)}^{\text{centripetal acc.}} +$$

$$\underbrace{\dot{\underline{\omega}}_{B|I}}_{\substack{\text{ang. acc. of}\\ \text{body frame w.r.t}\\ \text{inertial frame}}} \times \underbrace{\underline{r}^B}_{\substack{\text{position}\\ \text{in body}\\ \text{coordinates}}} + \underbrace{2\,\underline{\omega}_{B|I} \times \dot{\underline{r}}^B}_{\text{Coriolis acc.}} + \underbrace{\frac{d^2}{dt^2}\underline{r}_B^I}_{\substack{\text{acc. of body frame}\\ \text{w.r.t. inertial frame}}} \quad - (2)$$

Unit conversion:

$90 \text{ km/h} = 90/3.6 \text{ m/s} = 25 \text{ m/s}$

Cross product

Solving for velocity:

vel. of car A in inertial coordinates $\rightarrow$ $\frac{d}{dt}\underline{r}^I = -25\cdot\cos(30^\circ)\,\hat{i} + 25\cdot\sin(30^\circ)\,\hat{j} = -21.6506\,\hat{i} + 12.5\,\hat{j}$ m/s

vel. of car B in inertial coordinates $\rightarrow$ $\frac{d}{dt}\underline{r}_B^I = 25\,\hat{i}$ m/s

ang. vel. of car B in inertial coordinates $\rightarrow$ $\underline{\omega}_{B|I} = \left(\frac{d}{dt}\underline{r}_B^I\right)/100 = 25/100 = -0.25\,\hat{k}$ rad/s $\quad\ldots(\omega = v/r)$

position of car A w.r.t. car B $\rightarrow$ $\underline{r}^B = 50\,\hat{j}$ m

① $\Rightarrow$ $-21.6506\,\hat{i} + 12.5\,\hat{j} = \frac{d}{dt}\underline{r}^B + (-0.25)\hat{k} \times 50\hat{j} + 25\,\hat{i}$

$\therefore \frac{d}{dt}\underline{r}^B = -21.6506\,\hat{i} + 12.5\,\hat{j} - 12.5\,\hat{i} - 25\,\hat{i}$

$\therefore \frac{d}{dt}\underline{r}^B = \boxed{-59.1506\,\hat{i} + 12.5\,\hat{j} \quad \text{m/s}}$

vel. of car A w.r.t car B

Solving for acceleration (in addition to $\underline{\omega}_{B|I} = 0.25\hat{k}$ (rad/s), $\underline{r}^B = 50\hat{j}$ (m) & $\dot{\underline{r}}^B = \frac{d}{dt}\underline{r}^B = -34.1506\hat{i} + 12.5\hat{j}$ (m/s))

acc. of car A in inertial coordinates $\rightarrow$ $\frac{d^2}{dt^2}\underline{r}^I = 0$

acc. of car B in inertial coordinates $\rightarrow$ $\frac{d^2}{dt^2}\underline{r}_B^I = 0 - \left(\frac{d}{dt}\underline{r}_B^I\right)^2/100 = -\left(25^2/100\right) = -6.25\,\hat{j}$ m/s² $\quad\ldots(a = v^2/r)$

no translational acc.   acc. due to rotation

ang. acc. of car B in inertial coordinates $\rightarrow$ $\dot{\underline{\omega}}_{B|I} = 0$

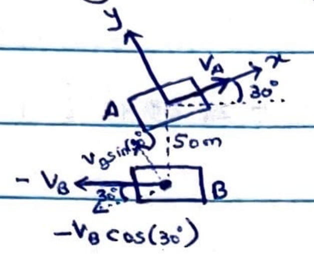
② $\Rightarrow$ $0 = \frac{d^2}{dt^2}\underline{r}^B + \left[-0.25\hat{k} \times (-0.25\hat{k} \times 50\hat{j})\right] + (0 \times 50\hat{j}) + 2\left[-0.25\hat{k} \times (-59.1506\hat{i} + 12.5\hat{j})\right] - 6.25\hat{j}$

$\therefore \frac{d^2}{dt^2}\underline{r}^B = 3.125\,\hat{j} - 29.5753\,\hat{j} - 6.25\,\hat{i} + 6.25\,\hat{j} = \boxed{-6.25\,\hat{i} - 20.2003\,\hat{j} \quad \text{m/s}^2}$

acc. of car A w.r.t. car B

Expressing everything w.r.t car A:

2.2



$V_A = V_B = 25$ m/s

By transport theorem (non-rotating frame of reference) (B = body, I = inertial)

$$\underbrace{\frac{d}{dt}\underline{r}^I}_{\text{vel. in inertial coordinates}} = \underbrace{\frac{d}{dt}\underline{r}^B}_{\text{vel. in body coordinates}} + \underbrace{\frac{d}{dt}\underline{r}_B^I}_{\text{vel. of body frame w.r.t. inertial frame}}$$

$$\therefore \frac{d}{dt}\underline{r}^B = \frac{d}{dt}\underline{r}^I - \frac{d}{dt}\underline{r}_B^I$$

$$= -25\cdot\cos(30^\circ)\hat{i} + 25\cdot\sin(30^\circ)\hat{j} - 25\hat{i}$$

$$= -21.6506\,\hat{i} + 12.5\,\hat{j} - 25\,\hat{i}$$

$$\therefore \frac{d}{dt}\underline{r}^B = \boxed{-46.6506\,\hat{i} + 12.5\,\hat{j} \quad \text{m/s}}$$

⇒ Thus, we can see that velocity of car A as seen by observer moving & rotating with car B (sol^n of 2.1) is not negative of the velocity which car B appears to have to a non-rotating observer in car A. This is because we have a difference of "$\omega \times r$" term when measuring relative velocities from rotating & non-rotating frames of reference.

Side Note ⇒ 2 bodies travelling in opposite direction (towards/away) have relative velocities w.r.t each other in same direction (sign). However, 2 bodies travelling in same direction have relative velocities w.r.t. each other in opposite directions (sign) and hence one is negative of the other.

3 a. Considering translation as well as rotation :

By transport theorem, $\begin{pmatrix} B = \text{body} \\ I = \text{inertial} \end{pmatrix}$

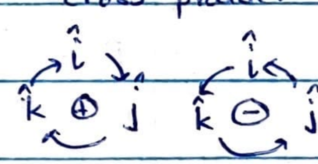
velocity → $\underbrace{\frac{d}{dt}\underline{r}^I}_{\text{vel. in inertial coordinates}} = \underbrace{\frac{d}{dt}\underline{r}^B}_{\text{vel. in body coordinates}} + \underbrace{\underline{\omega}_{B|I}}_{\text{rotational velocity}} \times \underbrace{\underline{r}^B}_{\text{radial dist}} + \underbrace{\frac{d}{dt}\underline{r}^I_B}_{\text{vel. of body frame origin w.r.t inertial frame.}}$ — ①

acceleration → $\underbrace{\frac{d^2}{dt^2}\underline{r}^I}_{\text{acc. in inertial coordinates}} = \underbrace{\frac{d^2}{dt^2}\underline{r}^B}_{\text{acc. in body coordinates}} + \underbrace{\underline{\omega}_{B|I} \times (\underline{\omega}_{B|I} \times \underline{r}^B)}_{\text{centripetal acc.}}$

$+ \underbrace{\underline{\dot{\omega}}_{B|I}}_{\text{rotational acc.}} \times \underbrace{\underline{r}^B}_{\text{radial dist}} + \underbrace{2\,\underline{\omega}_{B|I} \times \dot{\underline{r}}^B}_{\text{Coriolis acc.}}$

$+ \underbrace{\frac{d^2}{dt^2}\underline{r}^I_B}_{\text{acc. of body frame origin w.r.t. inertial frame}}$ — ②

cross product



Solving for velocity :

vel. of aircraft A w.r.t inertial coordinates → $\frac{d}{dt}\underline{r}^I = 100\,\hat{i}$ m/s

vel. of aircraft B w.r.t inertial coordinates → $\frac{d}{dt}\underline{r}^I_B = 150\,\hat{i}$ m/s

ang. vel. of aircraft B w.r.t inertial coordinates → $\underline{\omega}_{B|I} = \left(\frac{d}{dt}\underline{r}^I_B\right)/\rho = 150/400 = 0.375\,\hat{k}$ rad/s ... $\left(\omega = \frac{v}{r}\right)$

position of aircraft A w.r.t aircraft B → $\underline{r}^B = -100\,\hat{j}$ m

① ⇒ $100\,\hat{i} = 150\,\hat{i} + (0.375\,\hat{k} \times (-100)\hat{j}) + \frac{d}{dt}\underline{r}^B$

∴ $\frac{d}{dt}\underline{r}^B = 100\,\hat{i} - 150\,\hat{i} - 37.5\,\hat{i} = \boxed{-87.5\,\hat{i} \text{ m/s}}$

↑ vel. of aircraft A w.r.t. aircraft B

Solving for acceleration (in addition to $\underline{\omega}_{B|I} = \underset{(\text{rad/s})}{0.375\hat{k}}$, $\underline{r}_{\bullet}^{B} = \underset{(\text{m})}{-100\hat{j}}$ & $\dot{\underline{r}}_{\bullet}^{B} = \frac{d}{dt}\underline{r}_{\bullet}^{B} = \underset{(\text{m/s})}{-87.5\hat{i}}$)

acc. of aircraft A w.r.t. inertial coordinates → $\frac{d^2}{dt^2}\underline{r}^{I} = 0$

acc. of aircraft B w.r.t inertial coordinates → $\frac{d^2}{dt^2}\underline{r}_{B}^{I} = \left(\frac{d}{dt}\underline{r}_{B}^{I}\right)^2 / \rho = 150^2/400 = 56.25\hat{j}\ \text{m/s}^2$ ... $(a = v^2/r)$

ing acc. of aircraft B w.r.t inertial coordinates → $\dot{\omega}_{B|I} = 0$

(2) ⇒ $0 = 56.25\hat{j} + \left[0.375\hat{k} \times (0.375\hat{k} \times (-100)\hat{j})\right] + 0 \times (-100)\hat{j} + 2(0.375\hat{k} \times (-87.5)\hat{i}) + \frac{d^2}{dt^2}\underline{r}_{\bullet}^{B}$

$$\therefore \frac{d^2}{dt^2}\underline{r}_{\bullet}^{B} = -56.25\hat{j} - (0.375\hat{k} \times 37.5\hat{i}) + 65.625\hat{j}$$

$$= -56.25\hat{j} - 14.0625\hat{j} + 65.625\hat{j}$$

$$\therefore \frac{d^2}{dt^2}\underline{r}_{\bullet}^{B} = \boxed{-4.6875\hat{j}\ \text{m/s}^2}$$

acc. of aircraft A w.r.t. aircraft B

3b. Considering only translation (non-rotating)

Solving for velocity:

$$\frac{d}{dt}\underline{r}^{I} = \frac{d}{dt}\underline{r}^{B} + \frac{d}{dt}\underline{r}_{B}^{I}$$

$$\therefore \frac{d}{dt}\underline{r}_{\bullet}^{B} = \frac{d}{dt}\underline{r}^{I} - \frac{d}{dt}\underline{r}_{B}^{I} = 100\hat{i} - 150\hat{i} = \boxed{-50\hat{i}\ \text{m/s}}$$

Solving for acceleration:

$$\frac{d^2}{dt^2}\underline{r}^{I} = \frac{d^2}{dt^2}\underline{r}^{B} + \frac{d^2}{dt^2}\underline{r}_{B}^{I}$$

$$\therefore \frac{d^2}{dt^2}\underline{r}_{\bullet}^{B} = \frac{d^2}{dt^2}\underline{r}^{I} - \frac{d^2}{dt^2}\underline{r}_{B}^{I} = 0 - 56.25\hat{j} = \boxed{-56.25\hat{j}\ \text{m/s}^2}$$

⇒ Comparing sol^{n}s of 3a & 3b we can see that the velocity and acceleration which aircraft ~~plane~~ A appears to have to pilot of aircraft B (i.e. $\frac{d}{dt}\underline{r}^B_{\bullet}$ and $\frac{d^2}{dt^2}\underline{r}^B_{\bullet}$ respectively) is not the same with/without considering rotation of pilot of aircraft B. (and that the latter one, i.e. 3b, is clearly an incorrect or erroneous representation of the motion of aircraft B w.r.t. aircraft A).

4. $m = 2100$ kg

$l = 3.3$ m

Adapted ISO $\begin{cases} C_{\alpha F} = 78.9 \times 10^3 \text{ N/rad} \\ C_{\alpha R} = 77.5 \times 10^3 \text{ N/rad} \end{cases}$

$R = 144$ m

$v = 0 \rightarrow 78$ mph $= 0 \rightarrow 78 \times 0.44704$ m/s $= 0 \rightarrow 34.8691$ m/s

$a_1 = 2.046$ m, $b_1 = 1.254$m (38% front weight distribution)

$a_2 = 1.650$ m, $b_2 = 1.650$ m (50% —— 〃 ——)

$a_3 = 1.254$ m, $b_3 = 2.046$ m (62% —— 〃 ——)

$$a_y = \frac{v^2}{R} \quad -(1)$$

$$F_{y_F} = \left(\frac{b}{l}\right) F_y = \left(\frac{b}{l}\right) m \cdot a_y = \left(\frac{b}{l}\right) \frac{mv^2}{R} \overset{!}{=} \alpha_F \cdot C_{\alpha F}$$

$$\therefore \alpha_F = \left(\frac{b}{l}\right) \frac{mv^2}{R C_{\alpha F}} = \left(\frac{b}{l}\right) \frac{ma_y}{C_{\alpha F}} \quad -(2)$$

$$F_{y_R} = \left(\frac{a}{l}\right) F_y = \left(\frac{a}{l}\right) m \cdot a_y = \left(\frac{a}{l}\right) \frac{mv^2}{R} \overset{!}{=} \alpha_R \cdot C_{\alpha R}$$

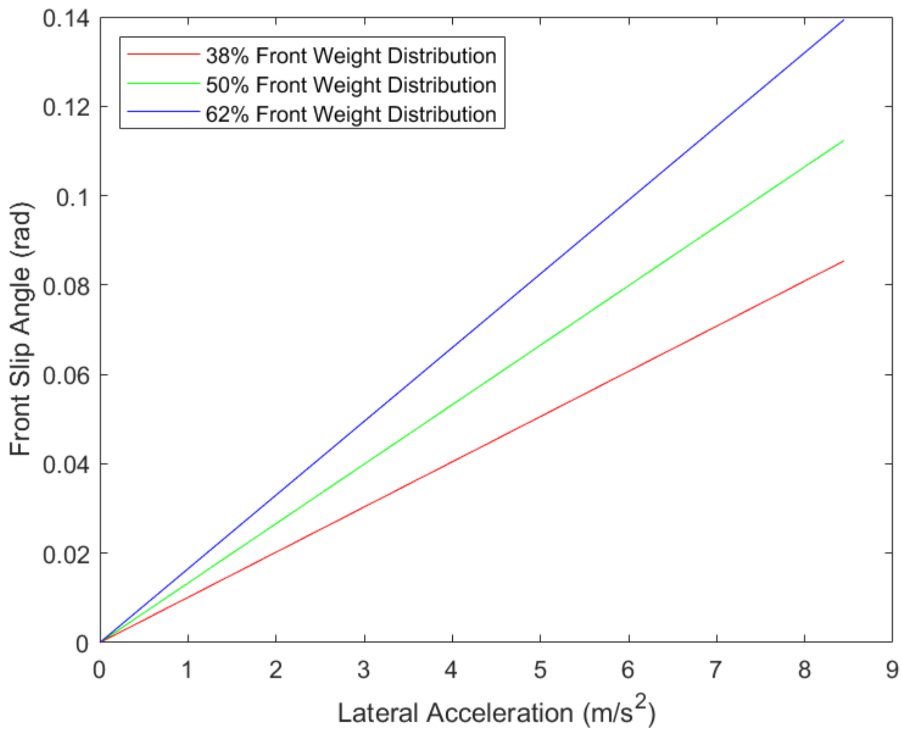
$$\therefore \alpha_R = \left(\frac{a}{l}\right) \frac{mv^2}{R C_{\alpha R}} = \left(\frac{a}{l}\right) \frac{ma_y}{C_{\alpha R}} \quad -(3)$$

$$|\beta| + |\alpha_R| = \left|\frac{b}{R}\right| = \left|\frac{\dot{\psi} b}{v}\right| \quad -(4)$$ ... (lecture notes)

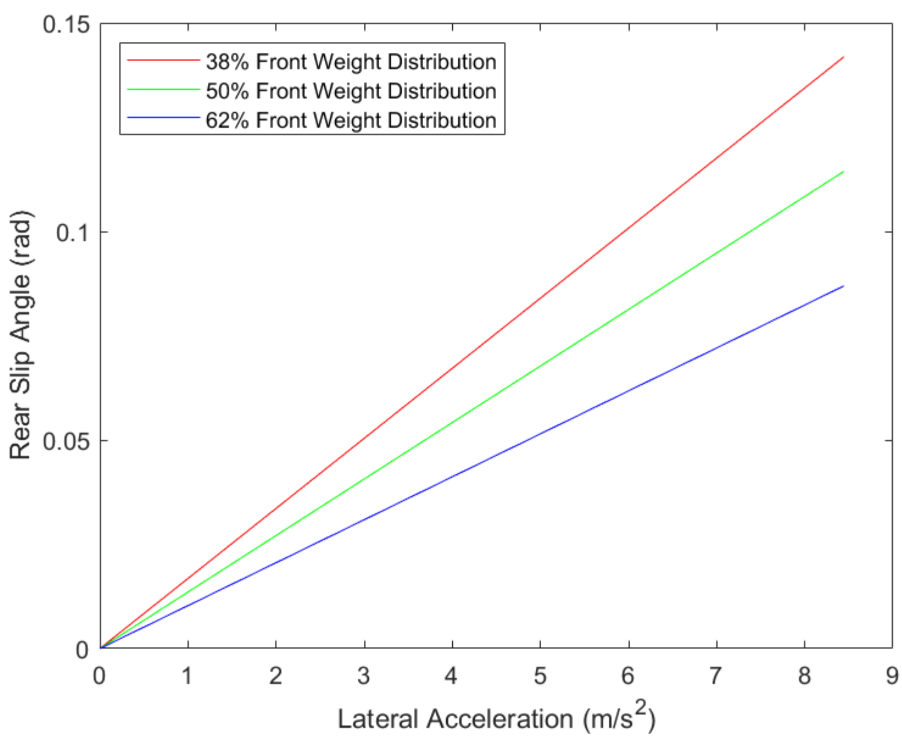
$$|\delta| = |\delta_A| - |\alpha_F| + |\alpha_R| = \left|\frac{l}{R}\right| - |\alpha_F| + |\alpha_R| \quad -(5)$$ ... (lecture notes)

Solved using MATLAB [PTO] →

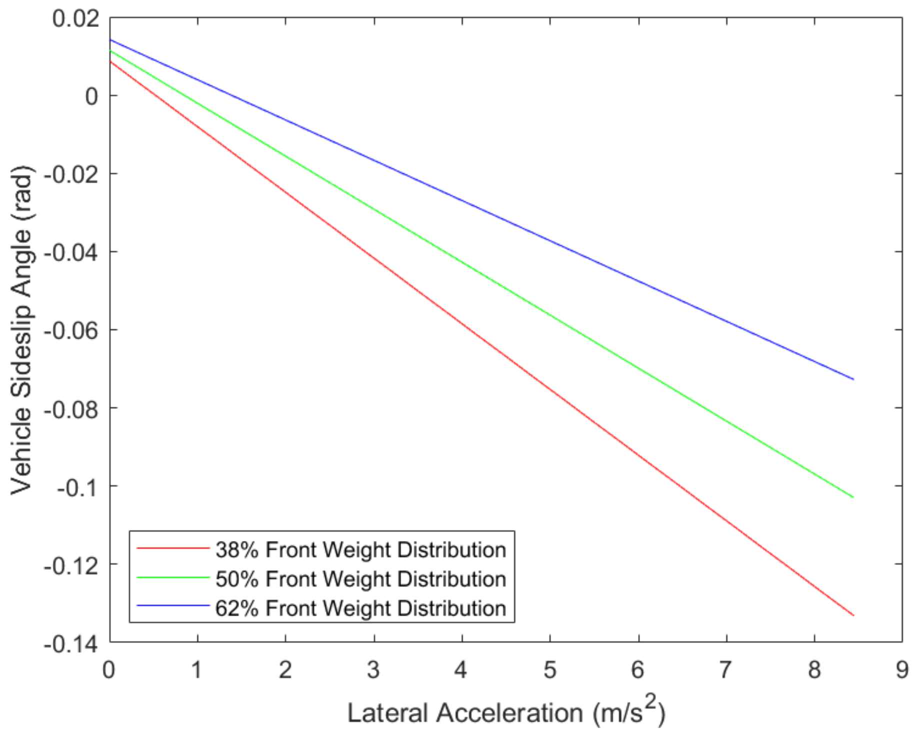
4A. Plot Front Slip Angle vs. Lateral Acceleration



4B. Plot Rear Slip Angle vs. Lateral Acceleration



4C. Plot Vehicle Sideslip Angle (β) vs. Lateral Acceleration



4D. Plot Steer Angle vs. Lateral Acceleration