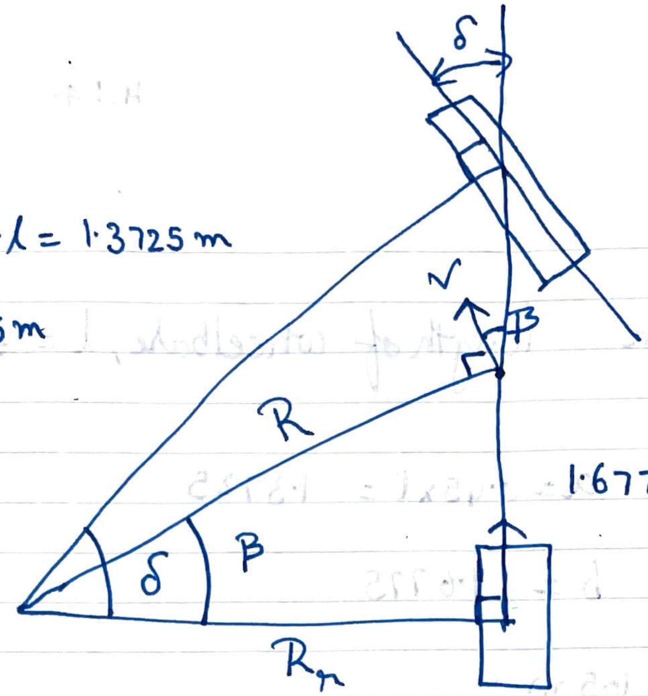
1) $l = 3.05\,m$ $\quad a = 0.45 \times l = 1.3725\,m$



$b = 1.6275\,m$

$R = 105\,m$

$R^2 = R_r^2 + b^2$

$R_r = \sqrt{R^2 - b^2} = 104.9865991\,m$

→ Small angle approximation, $\hat{\delta}_A = \dfrac{l}{R_r}$

$= \dfrac{3.05}{104.9865991}$

$= 0.0290513268$ rad / 1.6645
18415°

→ arc tan formulation, $\delta_A = \tan^{-1}(l/R_r)$

$= \tan^{-1}\left(\dfrac{3.05}{104.9865991}\right)$

$= 0.02904315803$ rad /

1.664050379°

Error in steering angle $= |\delta_A - \hat{\delta}_A|$

$= (0.02904315803 - 0.0290513268)$

$= -8.16877 \times 10^{-6}$ rad $/ -4.68036 \times 10^{-4}\,°$

% change in steering angle $= \dfrac{|\delta_A - \hat{\delta}_A|}{\delta_A} \times 100$

$= 0.0281\,\%$

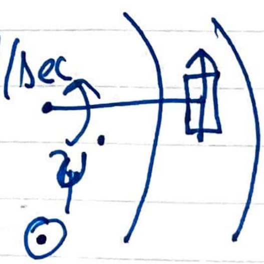
with small angle approximation, steering angle increased by 0.028%

2) $R = 620$ ft $= 188.976$ m $V = 70$ mph $\times$

$V = 70 \times 1.6 \times \dfrac{5}{18} = 31.12$ m/s

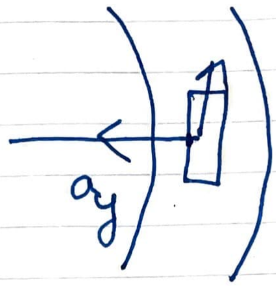
$\dot{\psi}$, Yaw velocity $= \dfrac{V}{R} = \dfrac{31.11}{188.976} = 0.1646$ rad/sec

$= 9.432°$/sec



Lateral acceleration, $a_y = \dfrac{V^2}{R}$

$= \dfrac{31.11 \times 31.11}{188.976}$



$= 5.1214\ m/sec^2$

$= 0.522\ g's$

g = acceleration due to gravity in m/s^2

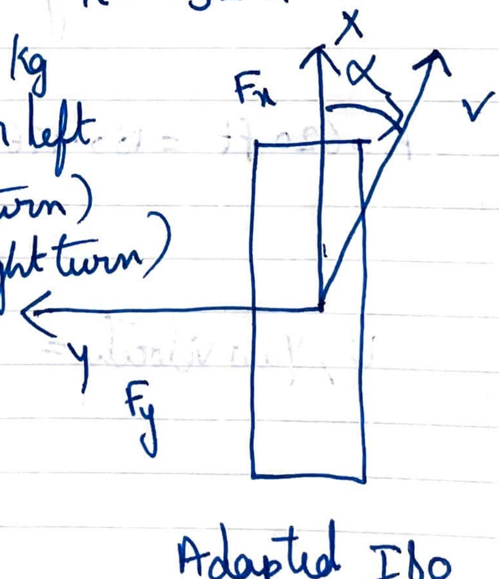
c) Given $l = 2.7\,m$ $a = b = 1.35\,m$ $C_{\alpha f} = C_{\alpha r} = 750\ N/deg$

$V = 111\ km/h = 30.83\ m/s$ $R = 232\,m$

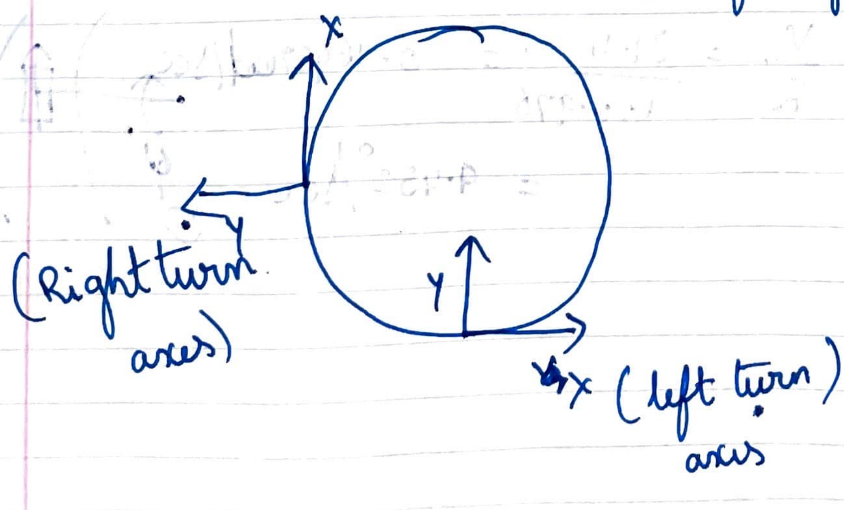
$m = 105$ slugs $= 105 \times 14.5939 = 1532.36\ kg$

$a_y = \dfrac{V_x^2}{R} = 4.097\ m/s^2$ (+ve for left turn)

(−ve for right turn)



Adapted ISO



Total lateral force, $F_y = ma_y$

$= 1532.36 \times 4.097$

$6279.336\,N$ (+ve left turn, -ve right turn

From bicycle model

$F_{yf} + F_{yr} = ma_y$ $\quad I_z\ddot{\psi} = aF_y - bF_r$ $\quad \ddot{\psi} = 0$

$\Rightarrow F_{yf} = \frac{b}{a} F_{yr}$

$$F_{yf} = F_{yr} = \frac{6279.336}{2} = 3139.668\,N$$

$$\alpha_r = \frac{F_{yR}}{C_\alpha} = \frac{3139.668}{750\,N/deg} = 4.186^\circ / 0.07306\,rad$$

+ve left turn

-ve right turn

Vehicle side slip angle is given by

$$|\beta| + |\alpha_r| = \left|\frac{\dot{\psi} b}{V}\right|$$

$$\beta + \alpha_r = \frac{\dot{\psi} b}{V} \Rightarrow \beta = \frac{1.35}{232} - 0.07306$$

$\beta = -0.06724$ rad or -3.85° : left turn

0.06724 rad | 3.85° right turn

In force free rolling, β_o doesnt have slip angles

$$\boxed{\beta_o = \frac{b}{r} = 0.005819 \text{ rad or } 0.33^\circ}$$

2)

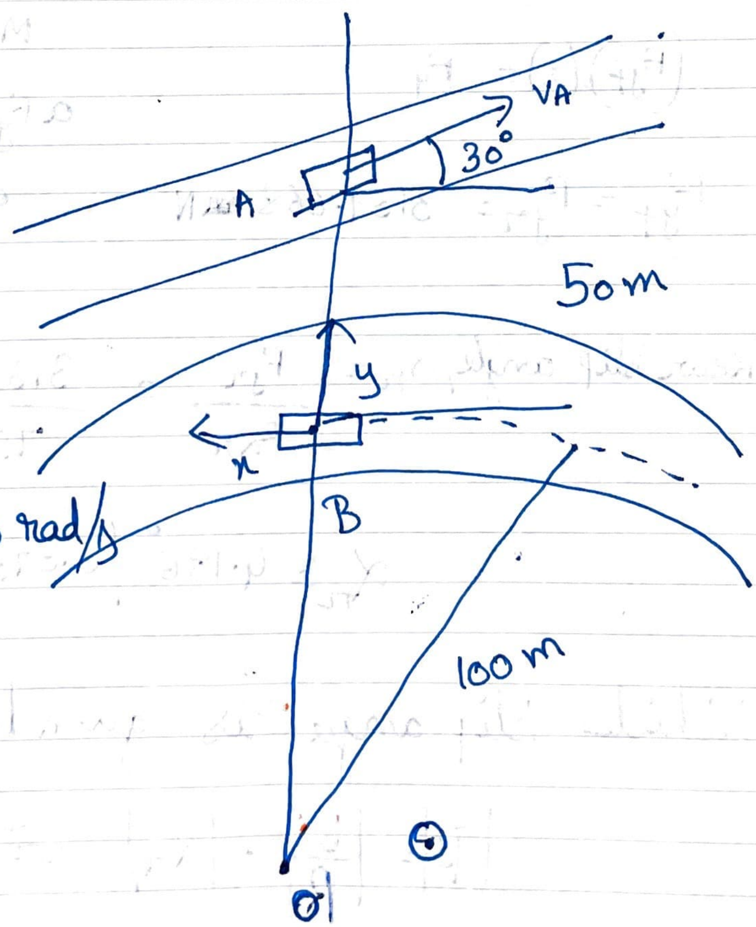
$R = 100\,m$

$V_A = V_B = 90\,km/h$

$= 25\,m/s$

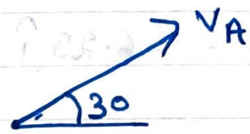
$\omega = \frac{V_B}{R} = \frac{25}{100} = 0.25$ rad/s

$\bar{\omega} = -0.25\,\hat{k}$



$r_{A/B} = 50j$

Vehicle 'A' is Just moving with a velocity whereas Vehicle 'B' has frame attached to it at CG rotating along with vehicle



$\omega_B = -0.25\hat{k}$

Velocity :-

$$V_A = V_B + \omega \times r_{A/B} + \frac{{}^{B}d r_{B/B}}{dt}$$

$$\frac{-25\sqrt{3}}{2}\hat{i} + \frac{25}{2}\hat{j} = 25\hat{i} + \left(-0.25\hat{k} \times 50\hat{j}\right) + V_{rel}$$

$$V_{rel} = \frac{-25\sqrt{3}}{2}\hat{i} - 25\hat{i} + 12.5\hat{j} - 12.5\hat{i}$$

$$V_{rel} = \left(-21.65 - 25 - 12.5\right)\hat{i} + 12.5\hat{j}$$

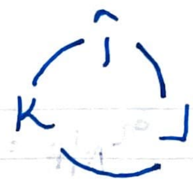
$$= -59.15\hat{i} + 12.5\hat{j}$$

Acceleration :-

$a_A = 0 \quad V_A =$ Constant

$$a_A = a_B + \dot{\omega} \times r + \omega \times (\omega \times r) + a_{rel} + 2\omega \times V_{rel}$$

$$a_B = \frac{v^2}{R} = \frac{25\times 25}{100} = 6.25$$



$$a_B = -6.25\hat{j}$$

ω = constant

$\dot{\omega} = 0$

$$0 = -6.25\hat{j} + 0 + \left(-0.25\hat{k}\times\left(-0.25\hat{k}\times 50\hat{j}\right)\right) + 2\left(-0.25\hat{k}\times V_{rel}\right)$$

$$+a_{rel}$$

$\cancel{a_{rel} =}$

$$2\left(\omega\times V_{rel}\right) = 2\times -0.25\hat{k}\times\left(-59.15\hat{i} + 12.5\hat{j}\right)$$

$$= -0.5\hat{k}\times\left(-59.15\hat{i} + 12.5\hat{j}\right)$$

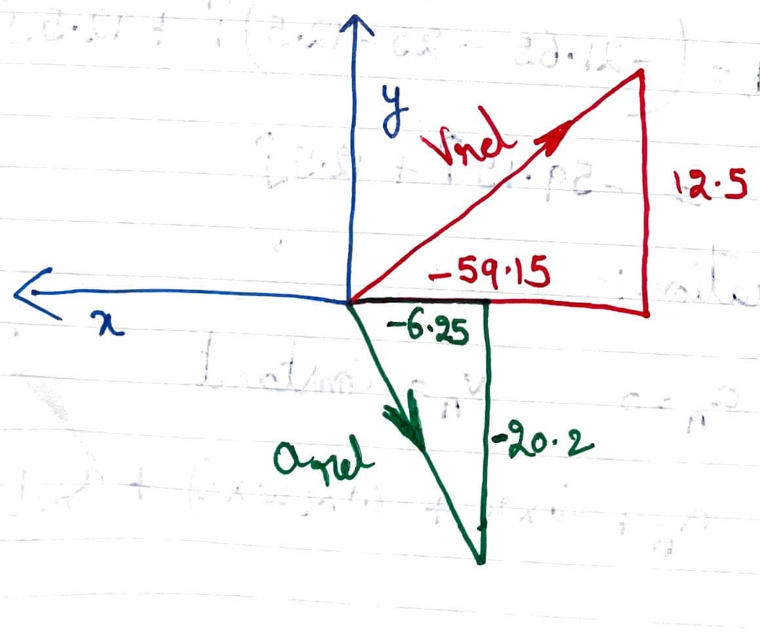
$$= \frac{59.15}{2}\hat{j} + \frac{12.5}{2}\hat{i}$$

$$a_{rel} = 6.25\hat{j} + \frac{50}{16}\hat{j} + -\frac{59.15}{2}\hat{j} - \frac{12.5}{2}\hat{i}$$

$$= +6.25\hat{j} + 3.125\hat{j} - 29.575\hat{j} - 6.25\hat{i}$$

$$\boxed{a_{rel} = -6.25\hat{i} - 20.2\hat{j}}$$

$$V_{rel} = -59.15\hat{i} + 12.5\hat{j}$$



B) From a non-rotating observer in Car A who is Just translating, V_{rel}

$$V_{rel} = V_B - V_A$$ using the earlier axes

$$= 25\hat{i} - \left(\frac{-25\sqrt{3}}{2}\hat{i} + \frac{25}{2}\hat{j}\right)$$

$$V_{rel} = 25 + \frac{25\sqrt{3}}{2}\hat{i} - 12.5\hat{j}$$

$$= 46.65\hat{i} - 12.5\hat{j}$$

As you can notice $(V_{rel})_{translating}$ axes differs from rotating axes rotating velocity by $(\omega \times r)$

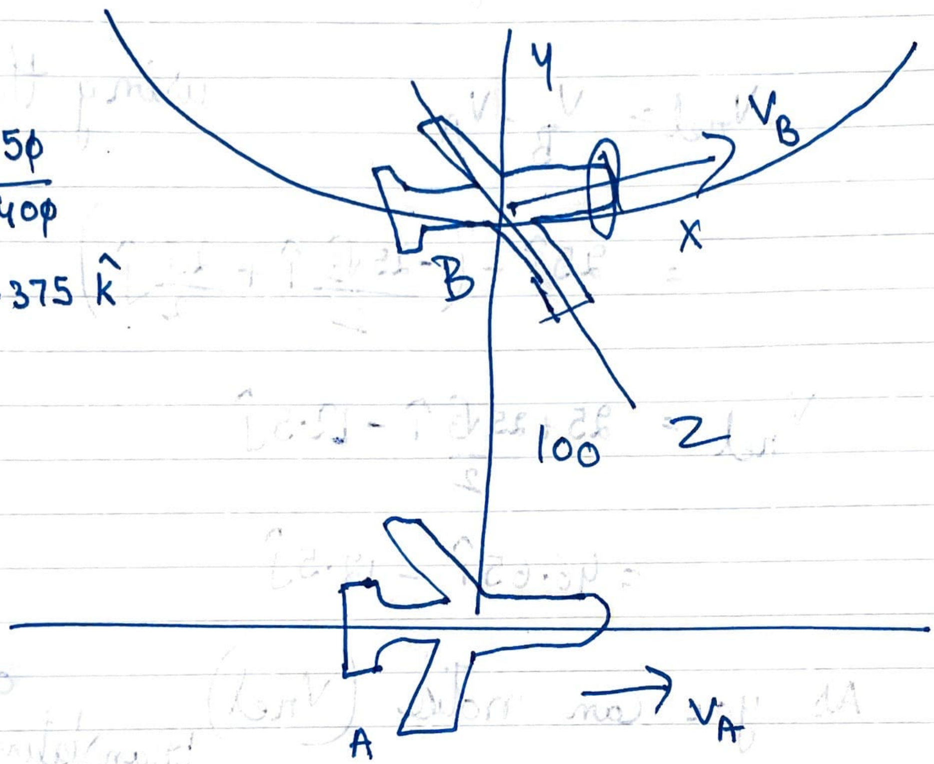
- It is not negative of $(V_{rel})_{rotating}$

3) $R = 400m$ $V_B = 150 m/s$. $V_A = 100 m/sec$

$r_{A/B} = -100\hat{j}$

$\omega = \frac{V_B}{R} = \frac{150}{400}$

$= 0.375\hat{k}$



Velocity

$V_A = V_B + \omega \times r + V_{rel}$

$100\hat{i} = 150\hat{i} + 0.375\hat{k} \times -100\hat{j} + V_{rel}$

$V_{rel} = -50\hat{i} - 37.5\hat{i} = -87.5\hat{i}$

acceleration

$a_A = a_B + \dot{\omega} \times r + \omega \times (\omega \times r) + 2(\omega \times V_{rel}) + a_{rel}$

$\cancel{a_A}$ $\quad \dot{\omega} = 0 \quad$ Constant Speed of v_A and v_B

$a_A = 0$

$$\boxed{a_B = \frac{v^2}{R}\hat{j} = 56.25\hat{j}} \qquad \omega = \frac{v_B}{R} \qquad \boxed{\dot{\omega} = \frac{\dot{v}_B}{R} = 0}$$

$$a_{rel} = -(\omega \times (\omega \times r)) - 2\omega \times v_{rel} - a_B$$

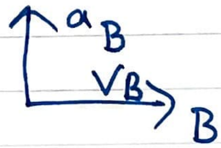
$$= -\left[0.375\hat{k} \times (0.375\hat{k} \times -100\hat{j})\right] - 2\left[0.375\hat{k} \times [-87.5\hat{i}]\right] - 56.25\hat{j}$$

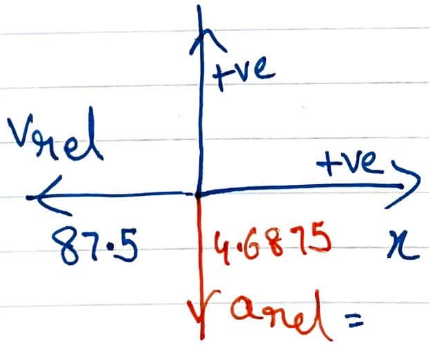
$$= \left[0.375\hat{k} \times (37.5\hat{i})\right] + 2\left[32.8125\hat{j}\right] - 56.25\hat{j}$$

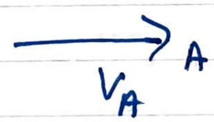
$$= -14.0625\hat{j} + 65.625\hat{j} - 56.25\hat{j}$$

$$a_{rel} = 51.5625\hat{j} - 56.25\hat{j}$$

$$\boxed{a_{rel} = -4.6875\hat{j}}$$







b) It Just becomes translating axes

$$V_{rel} = V_A - V_B = -50\hat{i}$$

$$a_{rel} = a_A - a_B = 0 - 56.25\hat{j} = -56.25\hat{j}$$

As you can ~~n~~notice $(V_{rel})_{nR} \neq (V_{rel})_R$

$$(a_{rel})_{nR} \neq (a_{rel})_R$$

Rotation frame makes a lot of difference in Values

```
clc; clear all; close all;


mass=2100; % mass of vehicle in kg
cf=78900; %N/rad
cr=77500; %N/rad
% cf=77900/57.29; %N/deg
% cr=76500/57.29; %N/deg

R=144; l=3.3; dA=l/R; % Radius, wheelbase and ackermann angle
a60=1.254; b60=2.046; % Distance between axles and C.G for 60% front weight distribution
a50=1.65; b50=1.65; % Distance between axles and C.G for 50% front distribution
a40=2.046; b40=1.254; % Distance between axles and C.G for 40% front distribution

v_init=0; v_final=34.87; % Initial and final velocities in m/s
v=linspace(v_init,v_final,1000); % Velocity vector

ay=[];
saf60=[];sar60=[];
saf50=[];sar50=[];
saf40=[];sar40=[];
B60=[];B50=[];B40=[];
D60=[];D50=[];D40=[];
dAlpha60=[];dAlpha50=[];dAlpha40=[];
c60=0;c50=0;c40=0;

for i=1:length(v)
    accY=v(i)^2/R;
    ay=[ay,accY];

    % 0.6 weight distribution
    alphaF60=(b60*mass*v(i)^2)/(l*R*cf); % Front slip angle
    alphaR60=(a60*mass*v(i)^2)/(l*R*cr); % Rear slip angle
    steer_char60=alphaF60-alphaR60; % Difference in slip angles
    beta60=(b60/R)-alphaR60; % Vehicle slip angle
    delta60=dA+alphaF60-alphaR60; % Steering angle

    if c60==0
        if abs(alphaF60)>6*0.0174533
            disp('At 60% weight distribution, the front saturates first')
            display(v(i))
            display(steer_char60)
            c60=1;
        elseif abs(alphaR60)>6*0.0174533
            disp('At 60% weight distribution, the rear saturates first')
            display(v(i))
            display(steer_char60)
            c60=1;
        end
    end

    saf60=[saf60,alphaF60];
    sar60=[sar60,alphaR60];
    dAlpha60=[dAlpha60,steer_char60];
    B60=[B60,beta60];
    D60=[D60,delta60];
```

```
% 0.5 weight distribution
    alphaF50=(b50*mass*v(i)^2)/(l*R*cf);
    alphaR50=(a50*mass*v(i)^2)/(l*R*cr);
    steer_char50=alphaF50-alphaR50;
    beta50=(b50/R)-alphaR50;
    delta50=dA+alphaF50-alphaR50;

    if c50==0
        if abs(alphaF50)>6*0.0174533
            disp('At 50% weight distribution, the front saturates first')
            display(v(i))
            display(steer_char50)
            c50=1;
        elseif abs(alphaR50)>6*0.0174533
            disp('At 50% weight distribution, the rear saturates first')
            display(v(i))
            display(steer_char50)
            c50=1;
        end
    end

    saf50=[saf50,alphaF50];
    sar50=[sar50,alphaR50];
    dAlpha50=[dAlpha50,steer_char50];
    B50=[B50,beta50];
    D50=[D50,delta50];

    % 0.4 weight distribution
    alphaF40=(b40*mass*v(i)^2)/(l*R*cf);
    alphaR40=(a40*mass*v(i)^2)/(l*R*cr);
    steer_char40=alphaF40-alphaR40;
    beta40=(b40/R)-alphaR40;
    delta40=dA+alphaF40-alphaR40;

    if c40==0
        if abs(alphaF40)>6*0.0174533
            disp('At 40% weight distribution, the front saturates first')
            display(v(i))
            display(steer_char40)
            c40=1;
        elseif abs(alphaR40)>6*0.0174533
            disp('At 40% weight distribution, the rear saturates first')
            display(v(i))
            display(steer_char40)
            c40=1;
        end
    end

    saf40=[saf40,alphaF40];
    sar40=[sar40,alphaR40];
    dAlpha40=[dAlpha40,steer_char40];
    B40=[B40,beta40];
    D40=[D40,delta40];

end
```

```
ug60=D60(end)-D60(1)/(ay(end)-ay(1))
% ug50=D50(end)-D50(1)/(ay(end)-ay(1))
ug50=D50(1)-D50(end)/(ay(1)-ay(end))
ug40=D40(end)-D40(1)/(ay(end)-ay(1))

figure(1)
plot(ay,saf60,'r')
hold on
plot(ay,saf50,'b')
plot(ay,saf40,'k')
xlabel('Lateral acceleration (m/s^2)')
ylabel('Slip angle (rad)')
title('Front slip angle vs lateral acceleration')
grid on; grid minor;
legend('@60% weight distribution','@50% weight distribution','@40% weight
distribution','Location','Best')

figure(2)
plot(ay,sar60,'r')
hold on
plot(ay,sar50,'b')
plot(ay,sar40,'k')
xlabel('Lateral acceleration (m/s^2)')
ylabel('Slip angle (rad)')
title('Rear slip angle vs lateral acceleration')
grid on; grid minor;
legend('@60% weight distribution','@50% weight distribution','@40% weight
distribution','Location','Best')

figure(3)
plot(ay,B60,'r')
hold on
plot(ay,B50,'b')
plot(ay,B40,'k')
xlabel('Lateral acceleration (m/s^2)')
ylabel('Body slip angle (rad)')
title('Body slip angle vs lateral acceleration')
grid on; grid minor;
legend('@60% weight distribution','@50% weight distribution','@40% weight
distribution','Location','Best')

figure(4)
plot(ay,D60,'r')
hold on
plot(ay,D50,'b')
plot(ay,D40,'k')
xlabel('Lateral acceleration (m/s^2)')
ylabel('Steering slip angle (rad)')
title('Steering slip angle vs lateral acceleration')
grid on; grid minor;
legend('@60% weight distribution','@50% weight distribution','@40% weight
distribution','Location','Best')
```

Figures

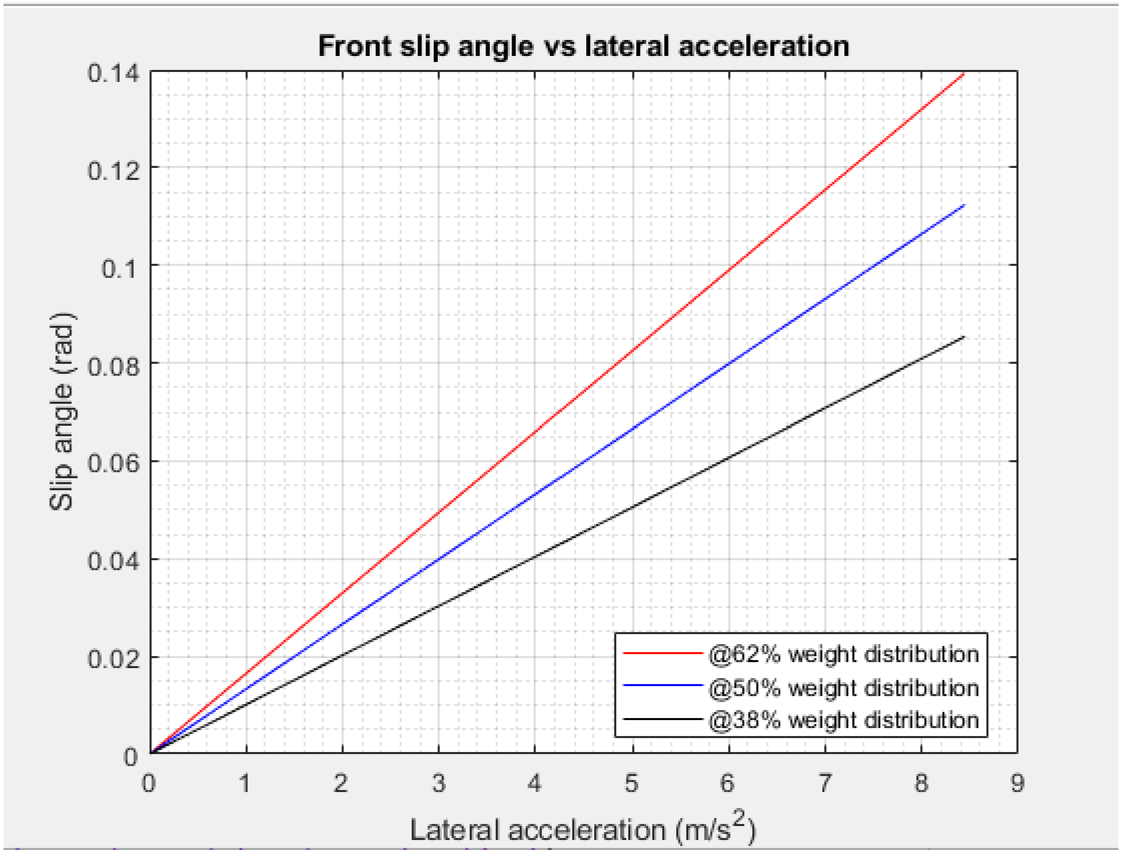


Figure 1 Front slip angle vs lateral acceleration

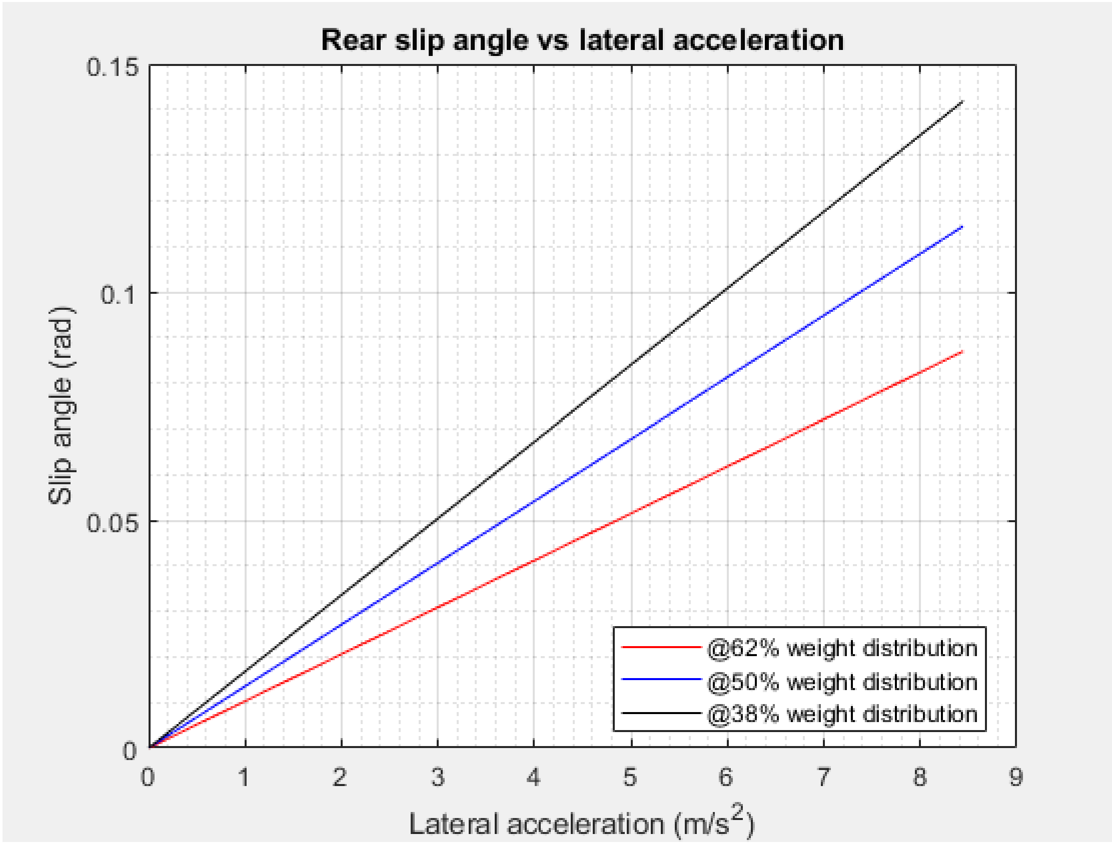


Figure 2 Rear Slip angle vs lateral acceleration

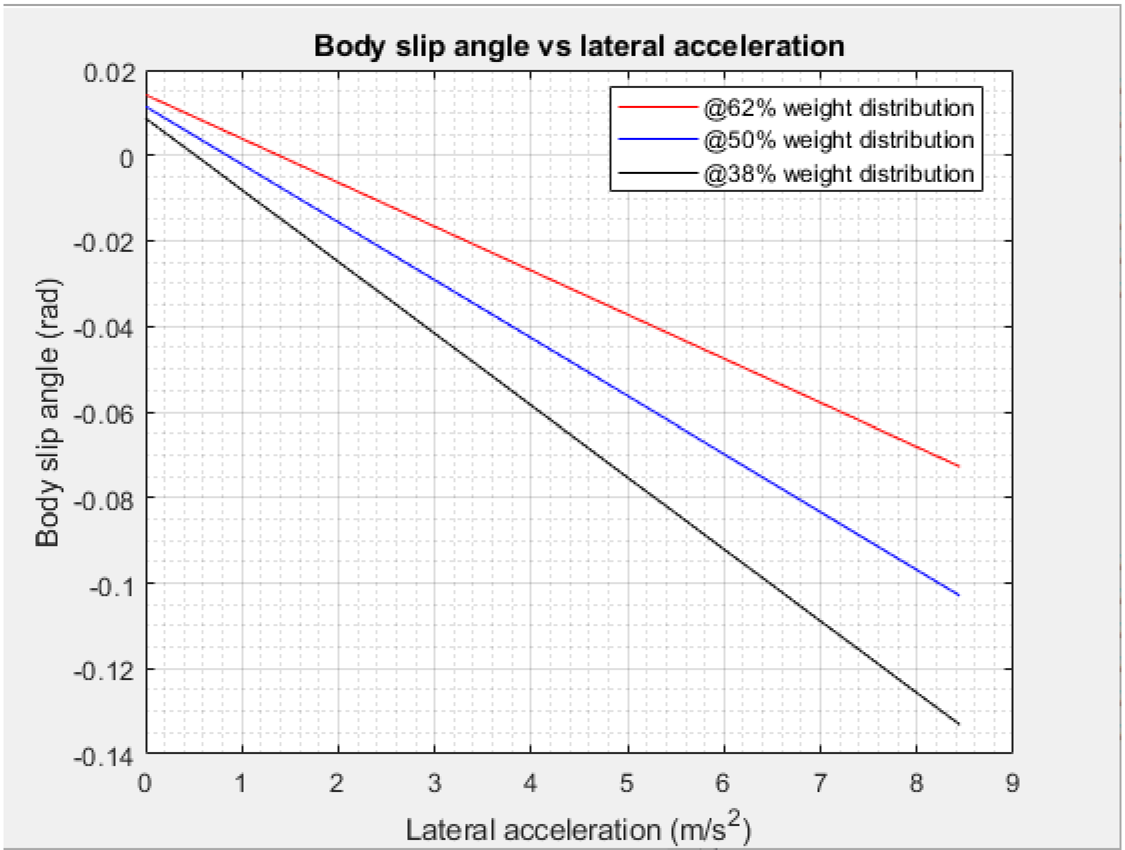


Figure 3 Vehicle Slip angle vs lateral acceleration

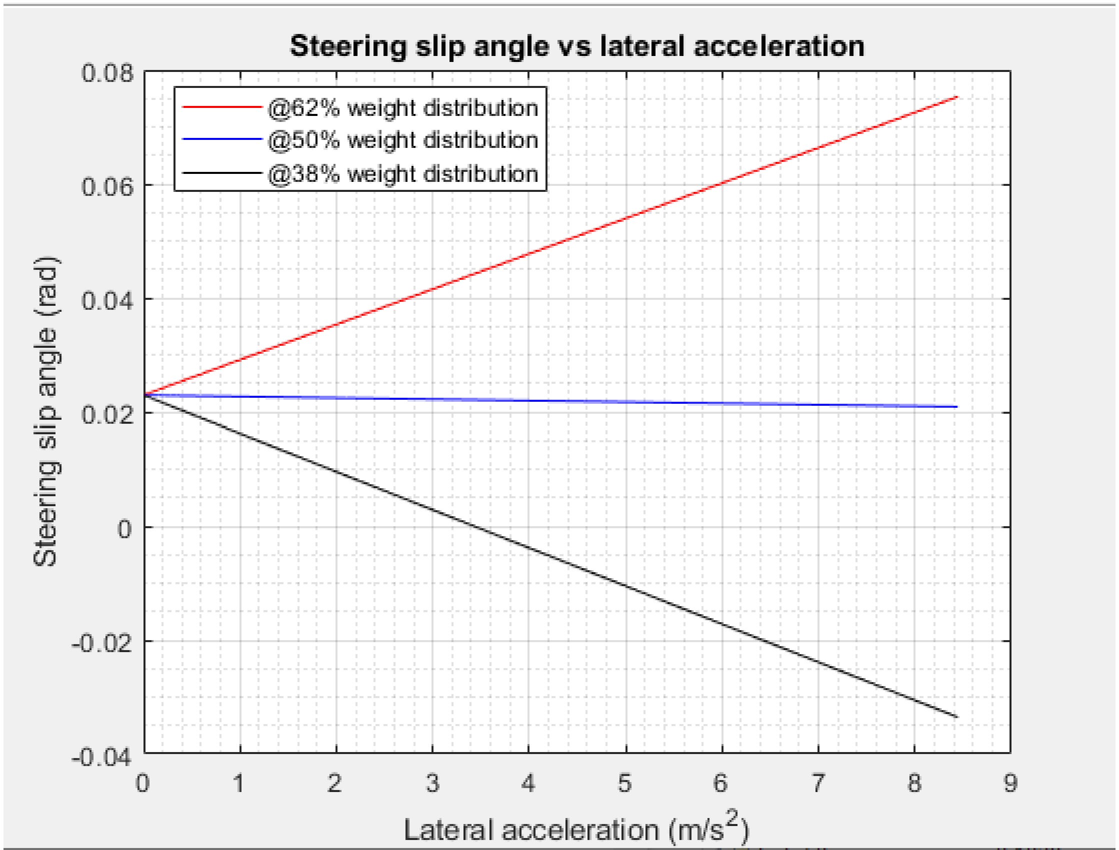


Figure 4 Steering angle vs lateral acceleration