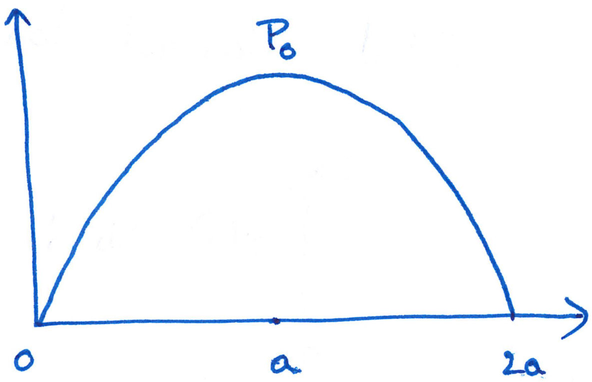
$F_z = 5200\,N$ $\quad 2a = 10\,cm$ $\quad \sigma_y = \tan\alpha = 0.0775$ $\quad c_y = 1250\,N/cm^2$

$\mu = 0.91$

$\rightarrow$ Derivation of Sinusoidal Pressure distribution



Sin graph is 0 at 0 and π

and maximum at $\pi/2$

$P_0 \sin\left(\frac{\pi}{2a} x\right)$ satisfies the

the Condition

$$P(x) = P_0 \sin(kx)$$

At $x = 0$ $\quad P(x) = 0 \Rightarrow P(0) = 0$

$x = \cancel{\pi}\, 2a$ $\quad P(2a) = 0$

$x = a$ $\quad P(a) = P_0$

$$\cancel{P_0} \sin(ka) = \cancel{P_0}$$

$$\sin(ka) = 1$$

$$\sin(ka) = \sin(\pi/2)$$

$$ka = 2n\pi + \pi/2$$

Here $n = 0$

$$ka = \pi/2 \Rightarrow \boxed{k = \pi/2a}$$

$$P(x) = P_0 \sin\left(\frac{\pi}{2a} x\right)$$

$q_z(x) =$ Force per unit length along the contact Patch

Total Normal load is given by pressure distribution

$$\int_0^{2a} P(x) \cdot 2b \cdot dx = F_z$$

$$\int_0^{2a} P_0 \sin\left(\frac{\pi}{2a} x\right) 2b \, dx = F_z$$

$$P_0 \left[\frac{-\cos\left(\frac{\pi}{2a} x\right)}{\pi/2a} \right]_0^{2a} \cdot 2b = F_z$$

$$\frac{P_0 \left[-\cos(\pi) - \left(-\cos(0)\right) \right] 2b}{\pi/2a} = F_z$$

$$P_0 \left[1+1\right] 2b \cdot 2a = \pi F_z$$

$$\boxed{P_0 = \frac{\pi F_z}{8ab}}$$

$$P(x) = \frac{\pi F_z}{8ab} \sin\left(\frac{\pi}{2a} x\right)$$

Force per unit length :-

$$q_z(x) = P(x) \cdot 2b$$

$$q_z(x) = \frac{\pi F_z}{4a} \sin\left(\frac{\pi}{2a} x\right)$$

1) Sliding regions deflection profile :-

$$c\,\partial_{slide}(x) = \mu q_z(x)$$

$$c\,\partial_{slide}(x) = \frac{\mu \pi F_z}{4a} \sin\left(\frac{\pi}{2a} x\right)$$

$$\partial_{slide}(x) = \frac{\mu \pi F_z}{4ac} \sin\left(\frac{\pi}{2a} x\right)$$

2) Sliding initiation Point

$$\partial(x_s) = \cancel{\mu \pi F}\ \mu q_z(x_s)$$

$$\partial(x_s) = \frac{\mu \pi F_z}{4ac} \sin\left(\frac{\pi}{2a} x\right)$$

Deflection profile from sticking region should intersect with sliding region profile

$$\delta(x_s) = \frac{\mu \pi F_z}{4ac} \sin\left(\frac{\pi}{2a} x_s\right)$$

σ_y

$$\sigma_y \cdot x_s = \frac{\mu \pi F_z}{4ac} \sin\left(\frac{\pi}{2a} x_s\right) \qquad \delta(x_s) = \frac{\delta_{max}}{x_s} \cdot x$$

Let $\frac{\mu \pi F_z}{4ac} = \cancel{k}\theta$

$$\sigma_y x_s = \theta \sin\left(\frac{\pi}{2a} x_s\right)$$

using Taylors series for $\sin ax$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots\dots$$

$$\sin(ax) = ax - \frac{a^3 x^3}{3!} + \frac{a^5 x^5}{5!} \dots\dots$$

neglecting higher order terms from 3rd term

$$\sigma_y \cancel{x_s} = \theta \left[\frac{\pi}{2a} \cancel{x_s} - \left(\frac{\pi}{2a}\right)^3 \frac{x_s^{\cancel{3}}}{3!} \right]$$

$$\frac{\sigma_y}{\theta} = \frac{\pi}{2a} \left[1 - \left(\frac{\pi}{2a}\right)^2 \frac{x_s^2}{3!} \right]$$

$$\frac{2a\,\sigma_y}{\pi\,\theta} = 1 - \frac{\pi^2\, x_s^2}{4a^2 \times 6}$$

$$\frac{\pi^2 x_s^2}{24a^2} = 1 - \frac{2a\,\sigma_y}{\pi\,\theta}$$

$$x_s^2 = \frac{24a^2}{\pi^2} \left[1 - \frac{2a}{\pi} \cdot \frac{\sigma_y}{\theta} \right]$$

$$x_s = \pm \sqrt{\frac{24a^2}{\pi^2} \left[1 - \frac{2a}{\pi} \frac{\sigma_y}{\theta} \right]}$$

neglecting -ve value

$$x_s = \sqrt{\frac{24a^2}{\pi^2} \left[1 - \frac{2a}{\pi} \frac{(\sigma_y)}{\theta} \right]} \qquad \sigma_y > 0$$

lateral Force

$$F_y = \int_0^{x_s} c\sigma_y x\,dx + \int_{x_s}^{2a} \frac{\mu \pi F_z}{4a} \sin\left(\frac{\pi}{2a}x\right)$$

$$= c\sigma_y \left[\frac{x^2}{2}\right]_0^{x_s}$$

$$= \frac{c\sigma_y}{2}(x_s)^2 + \frac{\mu\pi F_z}{4a}\left[\frac{-\cos\left(\frac{\pi}{2a}x\right)}{\pi/2a}\right]_{x_s}^{2a}$$

$$= \frac{1}{2}c\sigma_y (x_s)^2 + \frac{\mu\pi F_z}{4a}\left[\frac{1 - \{-\cos\left(\frac{\pi}{2a}x_s\right)}{\pi/2a}\right]$$

$$F_y = \frac{1}{2}c\sigma_y x_s^2 + \frac{\mu\pi F_z}{4a}\left[\frac{1+\cos\left(\frac{\pi}{2a}x_s\right)}{\pi/2a}\right]$$

2) $F_2 = 5200\,N$ $\quad 2a = 10\times10^{-2}\,m$ $\quad \sigma_y = 0.0775$

$C = 1250\times10^4\,N/m^2$ $\quad \mu = 0.91$

For given Values,

$$\theta = \frac{\mu \pi F_2}{4aC}$$

$$x_s = \sqrt{\frac{24a^2}{\pi^2}\left[1 - \frac{2a\sigma_y}{\pi\theta}\right]}$$

$$\theta = \frac{0.91\times3.14\times5200}{4\times5\times10^{-2}\times1250\times10^4}$$

$$= 0.0059$$

$$= 0.0596\,m =$$

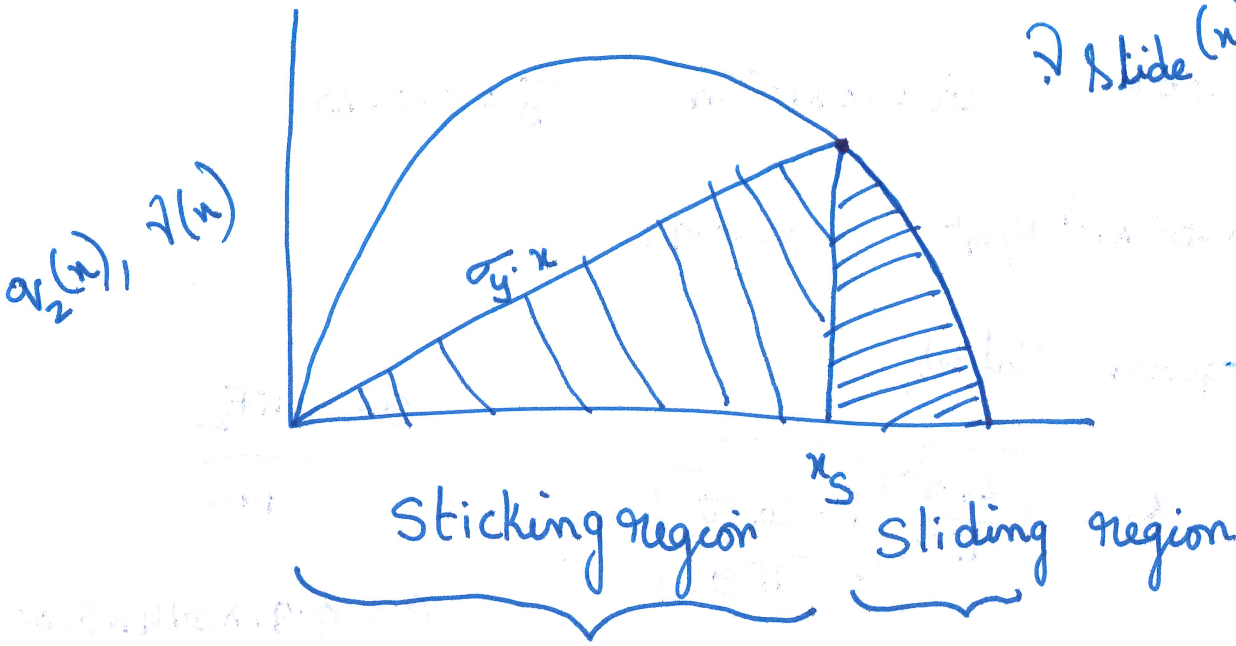
$$\boxed{x_s = 5.96\,cm}$$

$$F_y = \frac{1}{2}c\sigma_y x_s^2 + \frac{\mu\pi F_2}{4a}\left[\frac{1+\cos\left(\frac{\pi}{2a}x_s\right)}{\pi/2a}\right]$$

$$\boxed{F_y = 3383.2\ N}$$

We Can also use graphical analysis to solve the Problem

$$q_{slide}(x) = \frac{\mu \pi F_z}{4ac} \sin\left(\frac{\pi x}{2a}\right)$$



using this approach we see that sliding region's initiation occurs at 6.7 cm

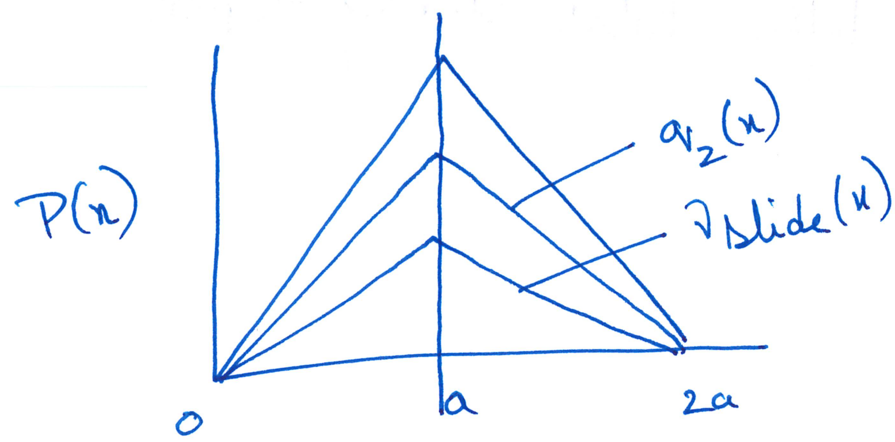
Lateral Force Can be ~~as~~ obtained from area under $q_z(x)$ and x

$\boxed{F_y = \cancel{336}\ 3336.0\ N}$ $\boxed{x_s = 6.7\,cm}$

The Position of x_s is after the Center of contact Patch and the tire is not completely in Sliding region. If the Value of σ_y is greater than Slope of Sliding region's deflection profile

Then the contact patch enters total sliding region.

3) Comment on position of x_s for bilinear distribution



Scalar multiply of pressure distribution

From the pressure distribution, $q_z(x)$ and $\gamma_{slide}(x)$ will be bilinear

The position of x_s can't be between [0, a] If tire is to ~~be~~ have sticking region along the contact patch. The pressure distribution gives you an idea of maximum force per unit length and sliding regions profile. Thus slope of graph determines the position of x_s and also maximum deflection to be experienced by tread elements

If x_s lies b/n $[0, a]$, then tire is in full sliding region as you reached maximum possible slip angle before total sliding region

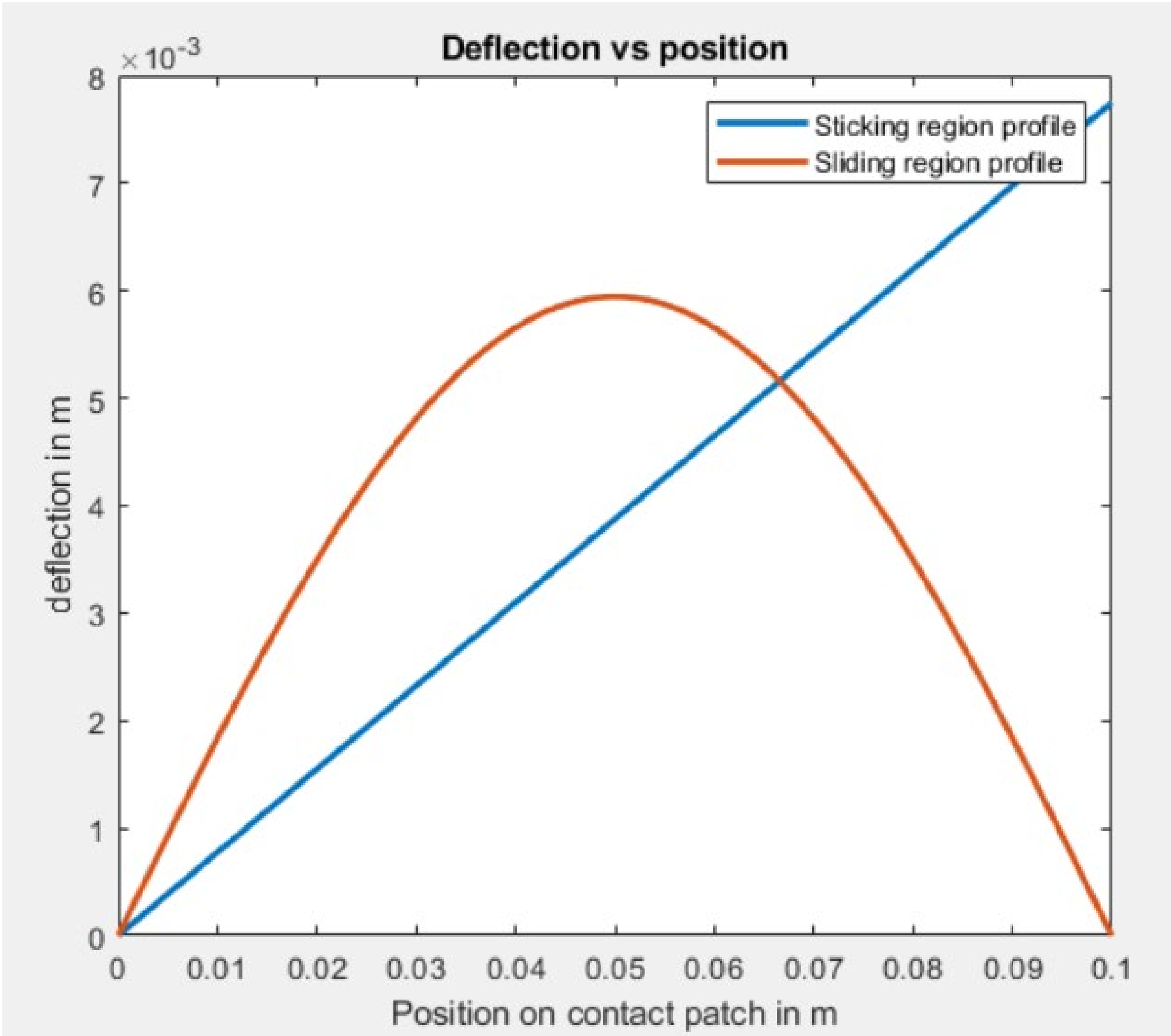


Figure 1 Deflection profiles for position

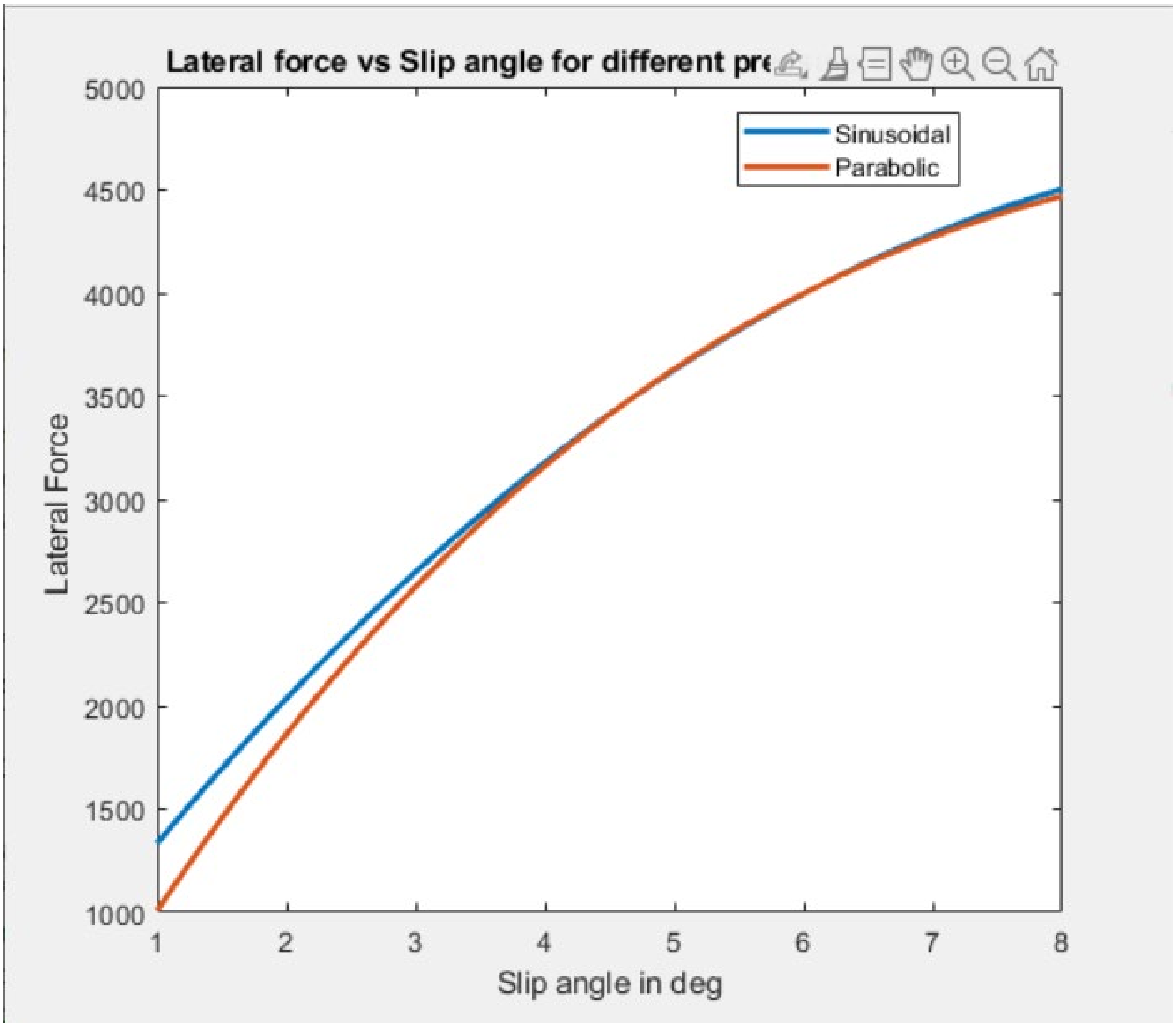


Figure 2 Lateral force vs slip angle from analytical approach

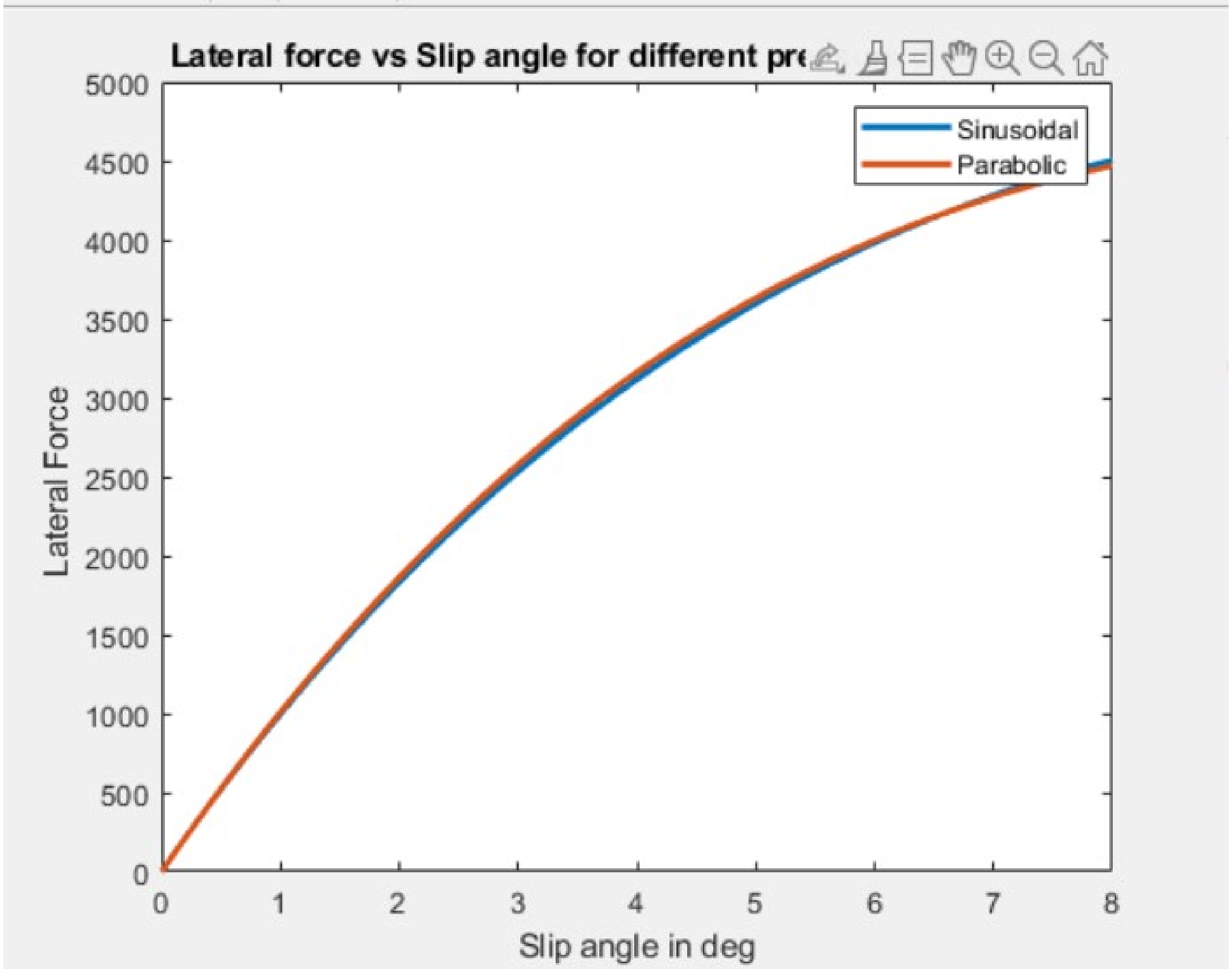


Figure 3 Lateral force vs slip angle for various distributions (graphical analysis)

```
%% Analytical model

Fz=5200; % Normal load in N
a=5e-2; % half of contact patch length in m
ls=0.0775; % lateral slip
C=1250e4; % stiffness per unit lenght
mu=0.91; % Coefficient of friction
theta=mu*pi*Fz/(4*a*C); % paramter 1
K=theta*C;% paramter 2
k2=pi/(2*a);
xs=sqrt(24*a^2*(1-(2*a*ls/(pi*theta)))/pi^2); % Sliding zone's initiation point
Fy=0.5*C*ls*xs^2+K*(1+cos(pi*xs/(2*a)))/(pi/(2*a)); % lateral force for the particular
```

```
%% Analytical model parabolic vs Sinusoidal distribution

alpha=linspace(1*pi/180,8*pi/180,100); % range of slip angles from 1 to 8 deg
ls=tan(alpha); % lateral slip
theta1=2*a^2*C/(3*mu*Fz); % Parameter for Parabolic distribution
xs=sqrt(24*a^2*(1-(2*a*ls/(pi*theta)))/pi^2); % Sliding zone's initiation point for
sinusoidal distribution
xs1=2*a*(1-theta1*ls);% Sliding zone's initiation point for Parabolic distrbution
Fy2=zeros(1,length(xs)); % Initialization array for lateral force for sinusoidal
profile
Fy3=zeros(1,length(xs1));% Initialization array for lateral force for Parabolic
profile
for i=1:length(xs)
    Fy2(i)=0.5*C*ls(i)*xs(i)^2+K*(1+cos(pi*xs(i)/(2*a)))/(pi/(2*a)); % Sinusoidal
Lateral force
    Fy3(i)=mu*Fz*(3*theta1*ls(i)-3*(theta1*ls(i))^2+(theta1*ls(i))^3); % Parabolic
lateral force
end
figure(1)
plot(alpha*180/pi,Fy2,'LineWidth',2)
hold on
plot(alpha*180/pi,Fy3,'LineWidth',2)
xlabel('Slip angle in deg')
ylabel('Lateral Force ')
title('Lateral force vs Slip angle for different pressure distributions')
legend('Sinusoidal', 'Parabolic')

%% Graphical Solution
X=0:2*a/100:2*a; % distance along contact patch in m
ls1=0.0775; % given lateral slip value
defl=ls1*X; % Sticking region's deflection
defs=theta*sin(pi*X/(2*a));% Sliding region's deflection
def=defs-defl; % difference in deflection
figure(2) % Plot of defelction profiles for sticking and sliding regions
plot(X,defl,X,defs,'LineWidth',2)
xlabel('Position on contact patch in m');
ylabel('deflection in m')
title('Deflection vs position')
legend('Sticking region profile', 'Sliding region profile')
theta1=2*a^2*C/(3*mu*Fz);
% lateral force calculation
f1=@(x) C*ls*x;f2=@(x) K*sin(pi*x/(2*a)); % integral functions

q1=integral(f1,0,0.067);q2=integral(f2,0.067,2*a); % calculating lateral force since
from graphical analysis
% we have transition point at 6.7 cm
Fy_int=q1+q2; % lateral force
```

fr

```
%% Part 2 for graphical analysis

SA=linspace(0*pi/180,8*pi/180,100); % array for slip angles
ls_g=tan(SA); % lateral slip
def_lin=zeros(length(ls_g),length(X)); % deflection for sticking region
Fy_int2=zeros(length(ls_g),1); % Lateral force for the distribution
Fy3_int=zeros(length(ls_g),1); % Lateral force for the distribution
for i=2:length(ls_g) % Neglecting the first element due to index error
    def_lin(i,:)=ls_g(i)*X; % Sticking region profile
    a1=find(defs-def_lin(i,:)<0); % finding indices which has deflection more from
sinusoidal distrbution
    Fy_int2(i)=integral(@(x) C*ls_g(i)*x,0,X(a1(1)))+integral(f2,X(a1(1)),2*a); %
calculating the lateral force for a particular slip value
    Fy3_int(i)=mu*Fz*(3*theta1*ls_g(i)-3*(theta1*ls_g(i))^2+(theta1*ls_g(i))^3); %
calculating lateral force for parabolic distribution
    figure(3)
    plot(X,def_lin(i,:),'LineWidth',2)
    hold on
end
plot(X,defs,'LineWidth',2)
xlabel('Position on contact patch in m');
ylabel('deflection in m')
title('Deflection vs position')
% figure for graphical analysis of sinusoidal distribution and parabolic
% distribution
figure(4)
plot(SA*180/pi,Fy_int2(:,1),'LineWidth',2)
hold on
plot(SA*180/pi,Fy3_int,'LineWidth',2)
xlabel('Slip angle in deg')
ylabel('Lateral Force ')
title('Lateral force vs Slip angle for different pressure distributions')
legend('Sinusoidal', 'Parabolic')
```

Problem 2:

In order to derive and expand a non-dimensional tire model, we need to know the cornering stiffness and the friction coefficient at corresponding normal loads. The friction coefficient μ and cornering stiffness C_α are functions of F_z. Before going into the subparts, we will first approximate μ and C_α with a polynomial. The specified data set for this problem contained only 5 normal loads and we can analyze the data to determine the μ and C_α at each applied load (please refer to HW2 solution key for further details). To approximate C_α at unknown loads, a quadratic polynomial was fitted to the calculated C_α v/s F_z data. Similarly, to approximate μ at unknown normal loads, a linear polynomial fit was performed. The complete code for this problem is attached in the Appendix.

The plot for C_α vs F_Z, and μ vs F_Z are given below:

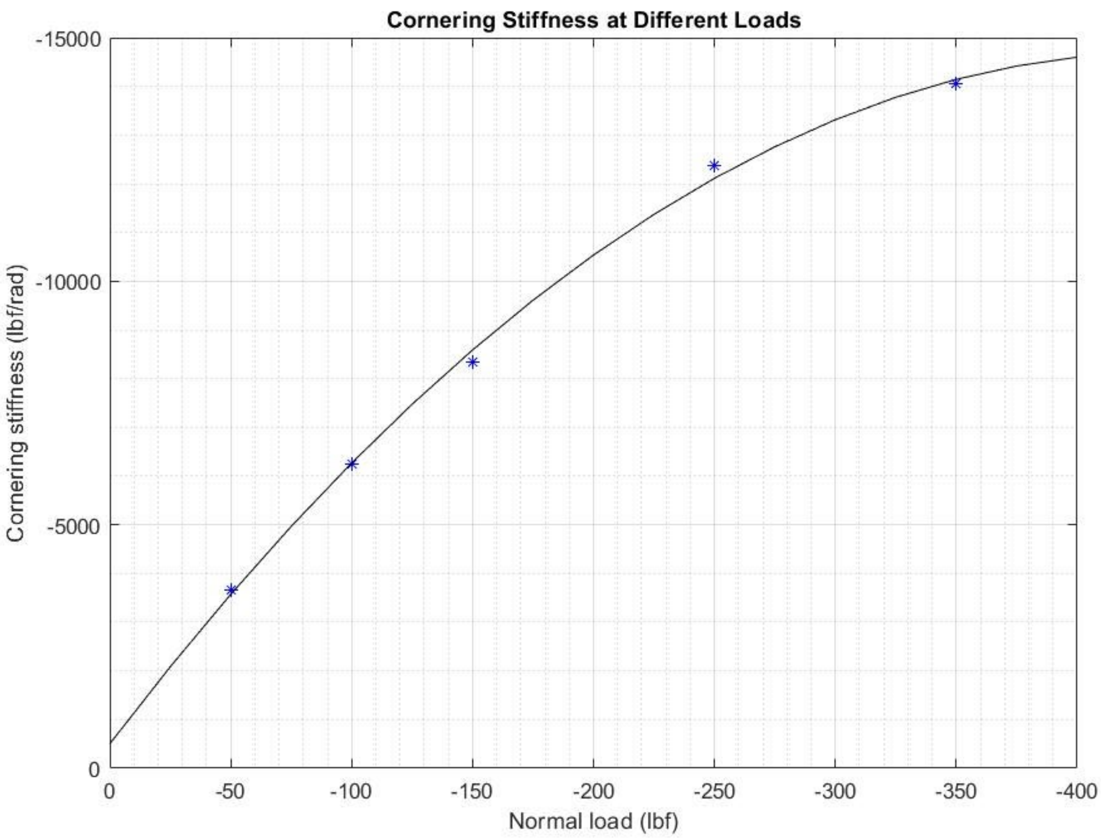


Figure 2: Functional representation of cornering stiffness

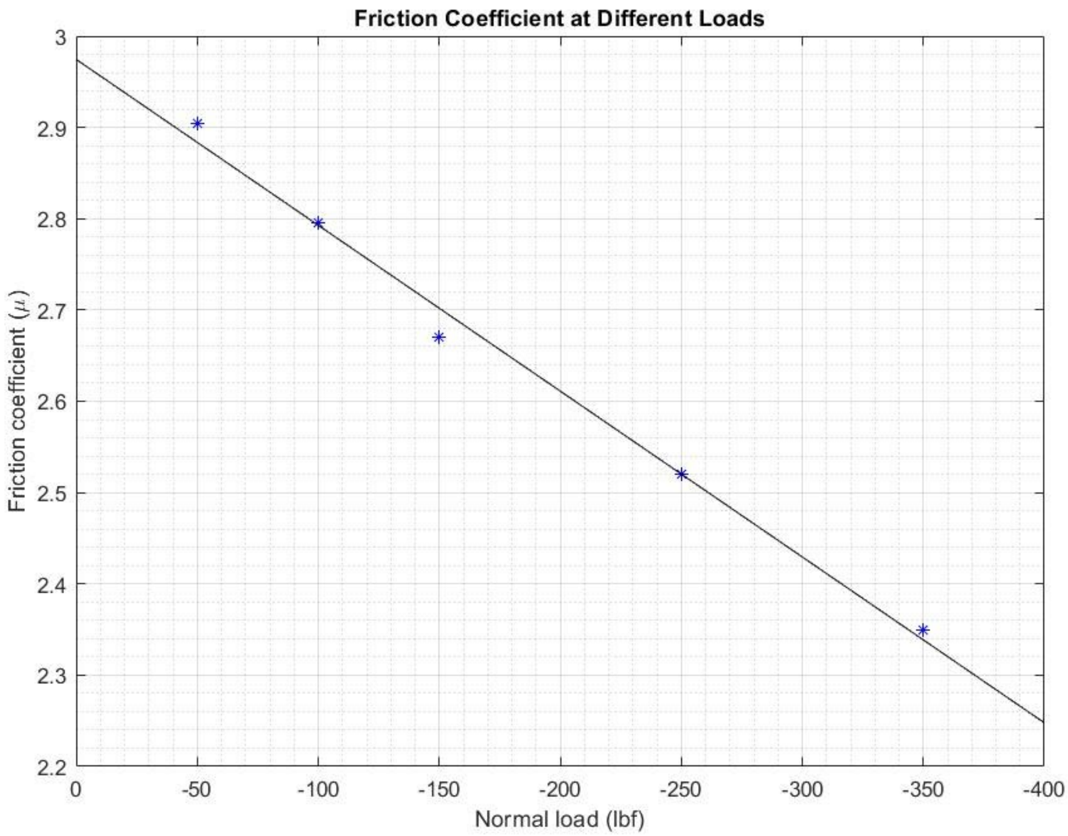


Figure 2: Functional representation of friction coefficient.

Part1: Non-dimensional lateral force v/s non-dimensional slip angle.

This part is simply asking to convert the measured tire data into non-dimensional form. To get the non-dimensional tire data, we need to apply the following transform to the measured lateral forces and slip angles (in radians):

$$\bar{F} = \frac{F_y}{\mu F z}, \quad \bar{\alpha} = \frac{C_\alpha \tan(\alpha)}{\mu F_z}$$

```
%% Problem 1: Non-dimensional data (FYbar and SAbar)
%Separating desired FY and Slip Angles from entire dataset
FY50 = FY_lb(iiFZ50); SArad50 = SA_rad(iiFZ50);
FY100 = FY_lb(iiFZ100); SArad100 = SA_rad(iiFZ100);
FY150 = FY_lb(iiFZ150); SArad150 = SA_rad(iiFZ150);
FY250 = FY_lb(iiFZ250); SArad250 = SA_rad(iiFZ250);
FY350 = FY_lb(iiFZ350); SArad350 = SA_rad(iiFZ350);

%Find indices for left turns only
ii50left=find(FY50<=0);
ii100left=find(FY100<=0);
ii150left=find(FY150<=0);
ii250left=find(FY250<=0);
ii350left=find(FY350<=0);
```

```
%Calculate non-dimensional Fy from measured data
Fy350bar=FY350(ii350left)/(mu(1)*targetFZlist(1));
Fy250bar=FY250(ii250left)/(mu(2)*targetFZlist(2));
Fy150bar=FY150(ii150left)/(mu(3)*targetFZlist(3));
Fy100bar=FY100(ii100left)/(mu(4)*targetFZlist(4));
Fy50bar=FY50(ii50left)/(mu(5)*targetFZlist(5));

%Calculate non-dimensional slip angles from measured data
SA350bar=csrad(1)*tan(SArad350(ii350left))/(mu(1)*targetFZlist(1));
SA250bar=csrad(2)*tan(SArad250(ii250left))/(mu(2)*targetFZlist(2));
SA150bar=csrad(3)*tan(SArad150(ii150left))/(mu(3)*targetFZlist(3));
SA100bar=csrad(4)*tan(SArad100(ii100left))/(mu(4)*targetFZlist(4));
SA50bar=csrad(5)*tan(SArad50(ii50left))/(mu(5)*targetFZlist(5));
```

The plot in figure (4) shows the non-dimensional force against the non-dimensional slip angle.

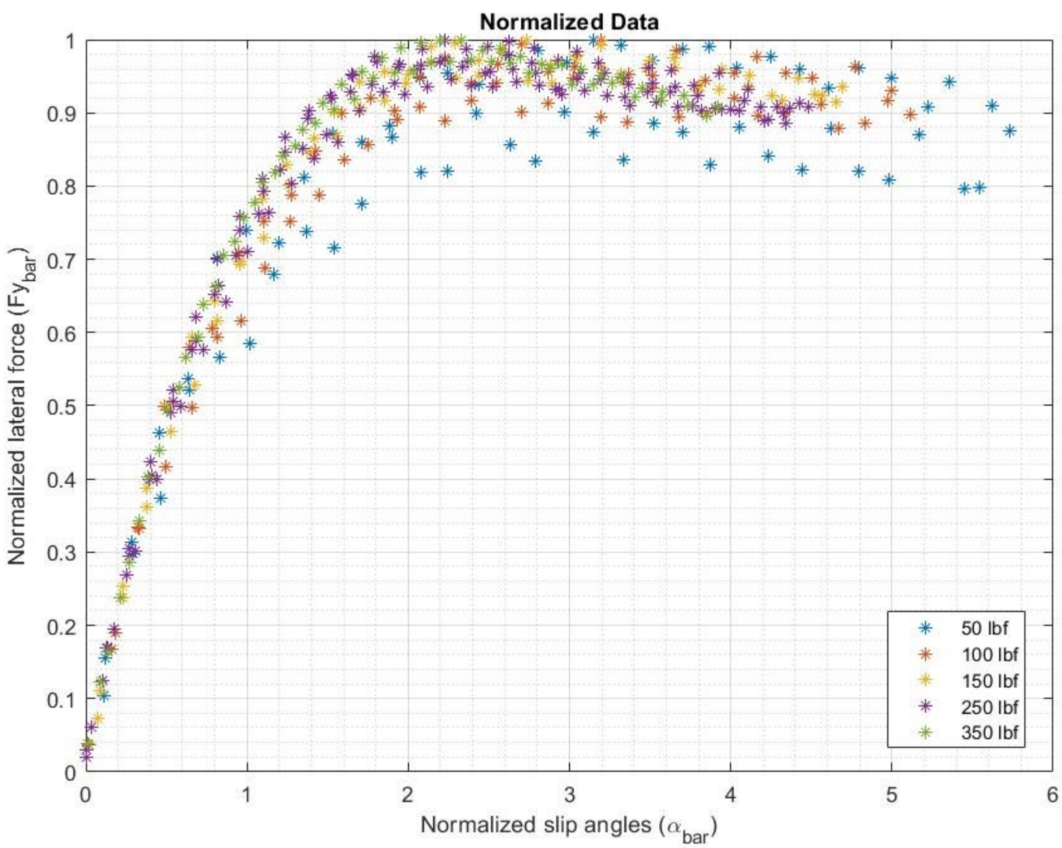


Figure 3:Non-dimensional lateral force v/s non-dimensional slip angle

Part2: Non-dimensional tire model with provided coefficient.

The non-dimensional tire model is essentially the magic formula, given as:

$$Y = D * \sin\left[C * \arctan\{B * x - E * (B * x - \arctan(B * x))\}\right]$$

where:

1. Output Y is the non-dimensional lateral force, $\bar{F_y}$
2. Input x is the non-dimensional slip angle, $\bar{\alpha}$

Given in the problem:

1. D=1.0000
2. C=1.6667
3. B=0.6000
4. E=0.2000

Therefore, we simply need to generate a range of $\bar{\alpha}$ and feed it into the magic formula to get $\bar{F_y}$.

```
%% Problem 2: Tire model with given coefficients
%Given coefficients
Bq=0.6000;
Cq=1.6667;
Dq=1.0000;
Eq=0.2000;

x_input=0:0.01:6;       %Generating inputs for the tire model (a range of non-
dimensional slip angles)
y_output=Dq*sin(Cq*atan(Bq*x_input - Eq*(Bq*x_input-atan(Bq*x_input))));
%Output from tire model (non-dimensional lateral force)
```

The output plot is shown in figure (5).

Part3: Determining the coefficients of the magic formula.

To get the coefficients of the magic formula, we need to analyze the non-dimensional data shown in figure (4) as a whole set. The coefficients are obtained as follows:

1. D is obtained from the peak value of the non-dimensional tire data. For our data, $D = 1.000$

2. $y_s = 0.8751$ can be obtained from the $\bar{F_y}$ value corresponding to the maximum $\bar{\alpha}$. Using this y_s, C was determined as $C = \frac{2}{\pi}\sin^{-1}\left(\frac{y_s}{D}\right)$ = 1.3216.

3. The slope of the function at the origin is $BCD = 0.9905$. Here on, B is obtained as $B = BCD/(CD)$ = 0.7495

4. Finally, $E = \frac{Bx_m - \tan(\frac{C\pi}{2})}{Bx_m - \tan^{-1}(Bx_m)}$ where x_m is the distance of the peak value along the x axis from the origin = -0.4676

5. Whether or not the coefficients are determined correctly can be verified as $y_s = D\,sin\left(\frac{C\pi}{2}\right) = 0.8751$.

Now, we can substitute these coefficients in the tire model and provide it with a range of $\bar{\alpha}$ as input (we can use the same input that was generated for part 2). We will get $\bar{F_y}$ from our tire model.

```
%% Problem 3: Calculate coefficients from the data
%Converting all non-dimensional data to a single vector
Fybar=[Fy50bar', Fy100bar', Fy150bar', Fy250bar', Fy350bar'];
SAbar=[SA50bar', SA100bar', SA150bar', SA250bar', SA350bar'];

D=max(Fybar);
ys=Fybar(find(SAbar==max(SAbar)));
C=2-(2/pi)*asin(ys/D);
iislope=find(SAbar<=0.2);    % Finding indices of data close to origin
BCD=polyfit(SAbar(iislope),Fybar(iislope),1);    % Slope around origin
B=BCD(1)/(C*D);
xm=SAbar(find(Fybar==max(Fybar)));  % This returns an array of 4 different values
(because there is noise in data).
xm=xm(3);         % After evaluating the plots with different values of xm, I picked
the third data.
E=(B*xm-tan(pi/(2*C)))/(B*xm-atan(B*xm));

checkys=D*sin(C*pi/2);          % To check if all calculations are correct
Fybar_calc=D*sin(C*atan(B*x_input - E*(B*x_input-atan(B*x_input))));       %
Generating output from magic tire model with calculated coefficients
```

We can now plot the $\bar{F}_y$ obtained from the tire model with provided coefficients (part 2) along with the $\bar{F}_y$ from our calculated tire model (part 3) and the non-dimensional data from part 1. We observe the below trend (figure 5):

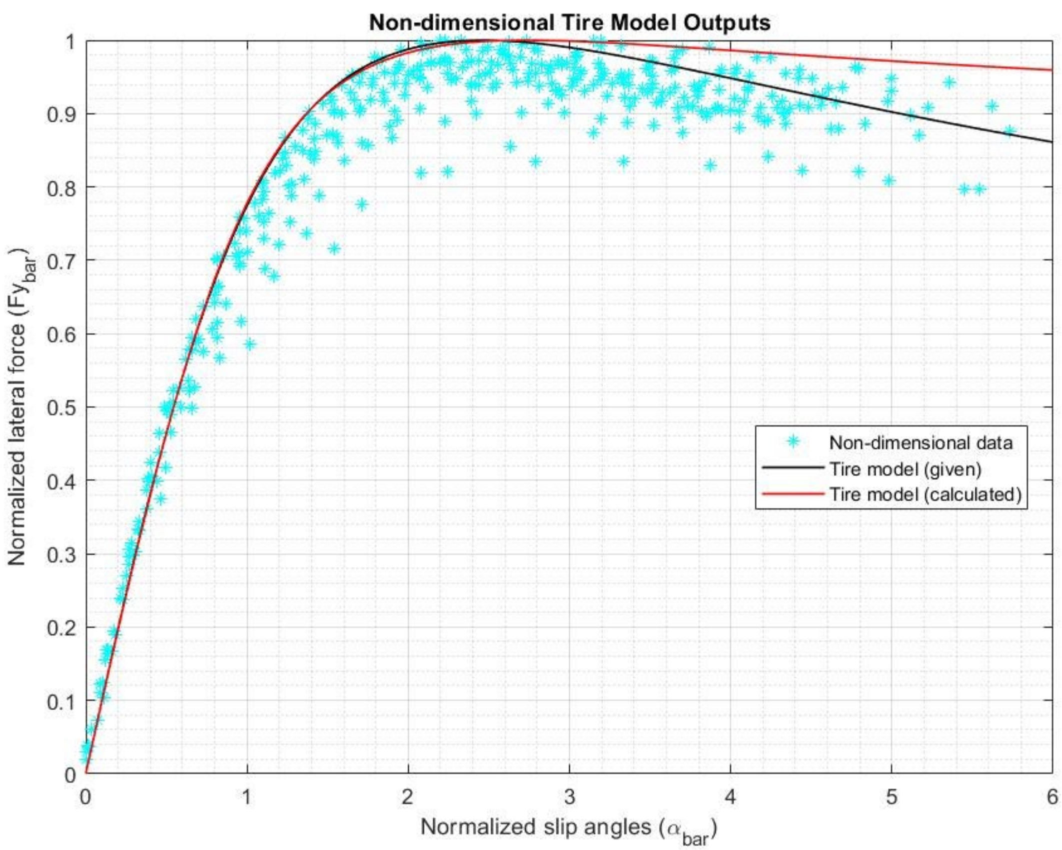


Figure 5: The non-dimensional lateral force obtained from the tire model with 2 different sets of coefficients.

Part 4: Expanding the non-dimensional tire model.

In this part, we are essentially verifying if the calculated tire model is able to predict proper (with dimensions) lateral forces for an applied normal load. Therefore, it is intuitive that here, we are expanding the tire model for the normal loads that were tested in the dataset (50lb, 100lbf, 150 lbf, 250lbf and 350lbf) since we have the measured lateral forces at these normal loads already.

We have already generated a range of $\bar{\alpha}$ ('x_input' in the MATLAB code) in part 2 and we have used this input in part 3 to get the non-dimensional lateral force $\bar{F}_y$ from our tire model. We also have the cornering stiffness C_α and friction coefficient μ at any desired load from figures (2) and (3). So, to expand the non-dimensional tire-model inputs and outputs, $\bar{\alpha}$ and $\bar{F}_y$, we simply need to apply the reverse transforms from part 1, as shown below:

$$F_y = \bar{F} * \mu F_z$$

$$\alpha = \tan^{-1}(\frac{\mu F_z \bar{\alpha}}{C_\alpha})$$

```
%% Problem 4: Expanding non-dimensional tire model at known normal loads
%Expanding lateral force
Fy50exp=Fybar_calc*mu(5)*targetFZlist(5);
Fy100exp=Fybar_calc*mu(4)*targetFZlist(4);
Fy150exp=Fybar_calc*mu(3)*targetFZlist(3);
Fy250exp=Fybar_calc*mu(2)*targetFZlist(2);
Fy350exp=Fybar_calc*mu(1)*targetFZlist(1);

%Expanding slip angles
SA50exp=atan(x_input*mu(5)*targetFZlist(5)/csrad(5));
SA100exp=atan(x_input*mu(4)*targetFZlist(4)/csrad(4));
SA150exp=atan(x_input*mu(3)*targetFZlist(3)/csrad(3));
SA250exp=atan(x_input*mu(2)*targetFZlist(2)/csrad(2));
SA350exp=atan(x_input*mu(1)*targetFZlist(1)/csrad(1));
```

The plot for this part is shown in figure (6).

Part 5: Expanding the non-dimensional tire model for unknown normal loads.

We can expand the non-dimensional tire model for any normal load if we know the corresponding friction coefficient and cornering stiffness. For this part, we can read out the μ and C_α values (for the normal loads -200lbf, -300lbf and -400lbf) from figure (2) and (3) and apply the same transform explained in part 4.

```
%% Problem 5: Expanding non-dimensional tire model at unknown normal loads
%Fetching friction coefficient at unknown loads
mu200=muapprox(find(unknownFZlist==-200))
mu300=muapprox(find(unknownFZlist==-300))
mu400=muapprox(find(unknownFZlist==-400))

%Fetching cornering stiffnesses at unknown loads
cs200=csapprox(find(unknownFZlist==-200))
cs300=csapprox(find(unknownFZlist==-300))
cs400=csapprox(find(unknownFZlist==-400))

%Expanding lateral force
```

```
Fy200exp=Fybar_calc*mu200*(-200);
Fy300exp=Fybar_calc*mu300*(-300);
Fy400exp=Fybar_calc*mu400*(-400);

%Expanding slip angles
SA200exp=atan(x_input*mu200*(-200)/cs200);
SA300exp=atan(x_input*mu300*(-300)/cs300);
SA400exp=atan(x_input*mu400*(-400)/cs400);
```

Figure (6) shows the tire model expanded for known normal loads (part 4) as well as for unknown loads (part 5).

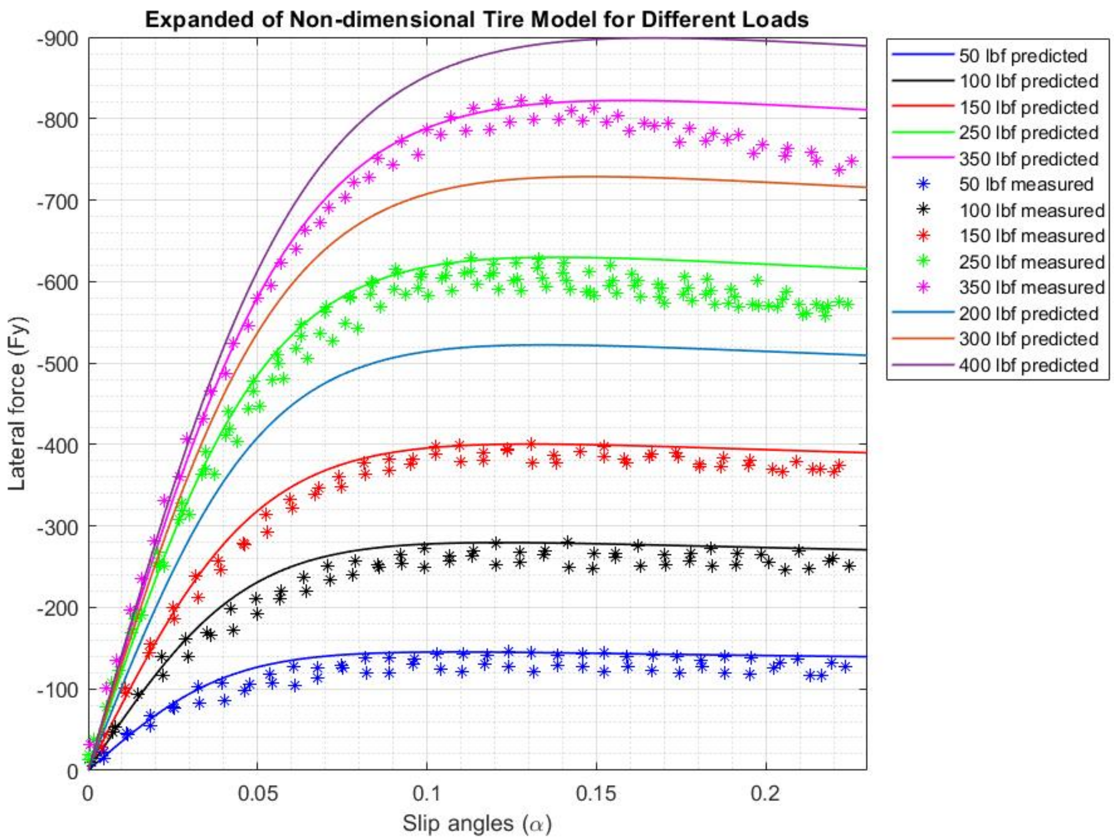


Figure 6: Prediction of lateral force for 200, 300 and 400lb normal load along with the actual tested normal loads

Appendix:

```
clc; clear all; close all;

%% Data: Loading and Conversion
load('tireF15.mat');
FZ_lb=0.22481*FZ;
FY_lb=0.22481*FY;
V_mph=0.621371*V;
```

```
P_psi=0.145038*P;
SA_rad=deg2rad(SA);

%% Identification of indices at target loads
targetFZ = 50*round(FZ_lb/50); % rounded all loads to the nearest 50 lbf
targetFZlist = unique(targetFZ); % removes all repeats; should be a 6x1
vector, for the 6 different normal loads

% Target values for each normal load
iiFZ350 = find(targetFZ == targetFZlist(1) & round(V_mph) == 25 & round(IA)
== 0 & round(P_psi) == 12);
iiFZ300 = find(targetFZ == targetFZlist(2) & round(V_mph) == 25 & round(IA)
== 0 & round(P_psi) == 12); % ends up being empty set; ...remove from
targetFZlist
iiFZ250 = find(targetFZ == targetFZlist(3) & round(V_mph) == 25 & round(IA)
== 0 & round(P_psi) == 12);
iiFZ150 = find(targetFZ == targetFZlist(4) & round(V_mph) == 25 & round(IA)
== 0 & round(P_psi) == 12);
iiFZ100 = find(targetFZ == targetFZlist(5) & round(V_mph) == 25 & round(IA)
== 0 & round(P_psi) == 12);
iiFZ50 = find(targetFZ == targetFZlist(6) & round(V_mph) == 25 & round(IA) ==
0 & round(P_psi) == 12);
targetFZlist(2) = []; % now we have five different normal loads

%% Determining cornering stiffnesses
% Limit slip angle to between -1 and 1 deg for linear part
slope1cs = find(targetFZ == targetFZlist(1) & round(V_mph) == 25 & round(IA)
== 0 & round(P_psi) == 12 & abs(SA) < 1);
slope2cs = find(targetFZ == targetFZlist(2) & round(V_mph) == 25 & round(IA)
== 0 & round(P_psi) == 12 & abs(SA) < 1);
slope3cs = find(targetFZ == targetFZlist(3) & round(V_mph) == 25 & round(IA)
== 0 & round(P_psi) == 12 & abs(SA) < 1);
slope4cs = find(targetFZ == targetFZlist(4) & round(V_mph) == 25 & round(IA)
== 0 & round(P_psi) == 12 & abs(SA) < 1);
slope5cs = find(targetFZ == targetFZlist(5) & round(V_mph) == 25 & round(IA)
== 0 & round(P_psi) == 12 & abs(SA) < 1);

%Calculate cornering stiffness (lbf/deg)
cs1deg = polyfit(SA(slope1cs),FY_lb(slope1cs),1);
cs2deg = polyfit(SA(slope2cs),FY_lb(slope2cs),1);
cs3deg = polyfit(SA(slope3cs),FY_lb(slope3cs),1);
cs4deg = polyfit(SA(slope4cs),FY_lb(slope4cs),1);
cs5deg = polyfit(SA(slope5cs),FY_lb(slope5cs),1);
csdeg = [cs1deg(1), cs2deg(1), cs3deg(1), cs4deg(1), cs5deg(1)];

%Calculate cornering stiffness (lbf/rad)from data. You can also simply ...
%convert lbf/deg to lbf/rad
cs1rad = polyfit(SA_rad(slope1cs),FY_lb(slope1cs),1);
cs2rad = polyfit(SA_rad(slope2cs),FY_lb(slope2cs),1);
cs3rad = polyfit(SA_rad(slope3cs),FY_lb(slope3cs),1);
cs4rad = polyfit(SA_rad(slope4cs),FY_lb(slope4cs),1);
cs5rad = polyfit(SA_rad(slope5cs),FY_lb(slope5cs),1);
csrad = [cs1rad(1), cs2rad(1), cs3rad(1), cs4rad(1), cs5rad(1)];

%Approximate cornering stiffness with second order polynomial
```

```
unknownFZlist=-400:25:0;        % generating a set of new target FZs so that
we can plot the fitted polynomial throught the Cs calculated from the real
tire data and analyze
csp=polyfit(targetFZlist,csrad,2);
csapprox=polyval(csp,unknownFZlist);

%% Determining friction coefficient
%Calculate friction coefficient for target loads in the tire data
fyPeak = [min(FY_lb(iiFZ350)); min(FY_lb(iiFZ250)); min(FY_lb(iiFZ150));
min(FY_lb(iiFZ100)); min(FY_lb(iiFZ50))];
mu = abs(fyPeak./targetFZlist);

%Approximate friction coefficient with a first order polynomial
mup = polyfit(targetFZlist,mu,1);
muapprox = polyval(mup,unknownFZlist);

%% Problem 1: Non-dimensional data (FYbar and SAbar)
%Separating desired FY and Slip Angles from entire dataset
FY50 = FY_lb(iiFZ50); SArad50 = SA_rad(iiFZ50);
FY100 = FY_lb(iiFZ100); SArad100 = SA_rad(iiFZ100);
FY150 = FY_lb(iiFZ150); SArad150 = SA_rad(iiFZ150);
FY250 = FY_lb(iiFZ250); SArad250 = SA_rad(iiFZ250);
FY350 = FY_lb(iiFZ350); SArad350 = SA_rad(iiFZ350);

%Find indices for left turns only
ii50left=find(FY50<=0);
ii100left=find(FY100<=0);
ii150left=find(FY150<=0);
ii250left=find(FY250<=0);
ii350left=find(FY350<=0);

%Calculate non-dimensional Fy from measured data
Fy350bar=FY350(ii350left)/(mu(1)*targetFZlist(1));
Fy250bar=FY250(ii250left)/(mu(2)*targetFZlist(2));
Fy150bar=FY150(ii150left)/(mu(3)*targetFZlist(3));
Fy100bar=FY100(ii100left)/(mu(4)*targetFZlist(4));
Fy50bar=FY50(ii50left)/(mu(5)*targetFZlist(5));

%Calculate non-dimensional slip angles from measured data
SA350bar=csrad(1)*tan(SArad350(ii350left))/(mu(1)*targetFZlist(1));
SA250bar=csrad(2)*tan(SArad250(ii250left))/(mu(2)*targetFZlist(2));
SA150bar=csrad(3)*tan(SArad150(ii150left))/(mu(3)*targetFZlist(3));
SA100bar=csrad(4)*tan(SArad100(ii100left))/(mu(4)*targetFZlist(4));
SA50bar=csrad(5)*tan(SArad50(ii50left))/(mu(5)*targetFZlist(5));

%% Problem 2: Tire model with given coefficients
%Given coefficients
Bq=0.6000;
Cq=1.6667;
Dq=1.0000;
Eq=0.2000;

x_input=0:0.01:6;       %Generating inputs for the tire model (a range of
non-dimensional slip angles)
y_output=Dq*sin(Cq*atan(Bq*x_input - Eq*(Bq*x_input-atan(Bq*x_input))));
%Output from tire model (non-dimensional lateral force)
```

```
%% Problem 3: Calculate coefficients from the data
%Converting all non-dimensional data to a single vector
Fybar=[Fy50bar', Fy100bar', Fy150bar', Fy250bar', Fy350bar'];
SAbar=[SA50bar', SA100bar', SA150bar', SA250bar', SA350bar'];

D=max(Fybar);
ys=Fybar(find(SAbar==max(SAbar)));
C=2-(2/pi)*asin(ys/D);
iislope=find(SAbar<=0.2);                         % Finding indices of data
close to origin
BCD=polyfit(SAbar(iislope),Fybar(iislope),1);     % Slope around origin
B=BCD(1)/(C*D);
xm=SAbar(find(Fybar==max(Fybar)));                % This returns an array of 4
different values (because there is noise in data).
xm=xm(3);                                         % After evaluating the plots
with different values of xm, I picked the third data.
E=(B*xm-tan(pi/(2*C)))/(B*xm-atan(B*xm));

checkys=D*sin(C*pi/2);                            % To check if all
calculations are correct
Fybar_calc=D*sin(C*atan(B*x_input - E*(B*x_input-atan(B*x_input))));        %
Generating output from magic tire model with calculated coefficients

%% Problem 4: Expanding non-dimensional tire model at known normal loads
%Expanding lateral force
Fy50exp=Fybar_calc*mu(5)*targetFZlist(5);
Fy100exp=Fybar_calc*mu(4)*targetFZlist(4);
Fy150exp=Fybar_calc*mu(3)*targetFZlist(3);
Fy250exp=Fybar_calc*mu(2)*targetFZlist(2);
Fy350exp=Fybar_calc*mu(1)*targetFZlist(1);

%Expanding slip angles
SA50exp=atan(x_input*mu(5)*targetFZlist(5)/csrad(5));
SA100exp=atan(x_input*mu(4)*targetFZlist(4)/csrad(4));
SA150exp=atan(x_input*mu(3)*targetFZlist(3)/csrad(3));
SA250exp=atan(x_input*mu(2)*targetFZlist(2)/csrad(2));
SA350exp=atan(x_input*mu(1)*targetFZlist(1)/csrad(1));

%% Problem 5: Expanding non-dimensional tire model at unknown normal loads
%Fetching friction coefficient at unknown loads
mu200=muapprox(find(unknownFZlist==-200))
mu300=muapprox(find(unknownFZlist==-300))
mu400=muapprox(find(unknownFZlist==-400))

%Fetching cornering stiffnesses at unknown loads
cs200=csapprox(find(unknownFZlist==-200))
cs300=csapprox(find(unknownFZlist==-300))
cs400=csapprox(find(unknownFZlist==-400))

%Expanding lateral force
Fy200exp=Fybar_calc*mu200*(-200);
Fy300exp=Fybar_calc*mu300*(-300);
Fy400exp=Fybar_calc*mu400*(-400);

%Expanding slip angles
SA200exp=atan(x_input*mu200*(-200)/cs200);
SA300exp=atan(x_input*mu300*(-300)/cs300);
```

```
SA400exp=atan(x_input*mu400*(-400)/cs400);

%% Plots
fig=1;
figure(fig);
fig=fig+1;
clf;
plot(SArad50(ii50left),FY50(ii50left),'*');
hold on;
plot(SArad100(ii100left),FY100(ii100left),'*');
plot(SArad150(ii150left),FY150(ii150left),'*');
plot(SArad250(ii250left),FY250(ii250left),'*');
plot(SArad350(ii350left),FY350(ii350left),'*');
grid on; grid minor;
xlabel('Slip angles (rad)')
ylabel('Lateral froce (lbf)');
legend('50 lbf','100 lbf','150 lbf','250 lbf','350 lbf','Location','Best')
set(gca,'YDir','reverse')

figure(fig);
fig=fig+1;
clf;
plot(targetFZlist,csrad,'b*')
hold on;
plot(unknownFZlist,csapprox,'k')
grid on, grid minor;
xlabel('Normal load (lbf)')
ylabel('Cornering stiffness (lbf/rad)')
title('Cornering Stiffness at Different Loads')
set(gca,'XDir','reverse','YDir','reverse')

figure(fig);
fig=fig+1;
clf;
plot(targetFZlist,mu,'b*')
hold on;
plot(unknownFZlist,muapprox,'k')
grid on, grid minor;
xlabel('Normal load (lbf)')
ylabel('Friction coefficient (\mu)')
title('Friction Coefficient at Different Loads')
set(gca,'XDir','reverse')

figure(fig);
fig=fig+1;
clf;
plot(SA50bar,Fy50bar,'*')
hold on;
plot(SA100bar,Fy100bar,'*')
plot(SA150bar,Fy150bar,'*')
plot(SA250bar,Fy250bar,'*')
plot(SA350bar,Fy350bar,'*')
grid on, grid minor;
xlabel('Normalized slip angles (\alpha_{bar})')
ylabel('Normalized lateral force (Fy_{bar})')
title('Normalized Data')
legend('50 lbf','100 lbf','150 lbf','250 lbf','350 lbf','Location','Best')
```

```
set(gca,'XLim',[0,6])

figure(fig);
fig=fig+1;
clf;
plot(SAbar,Fybar,'c*')
hold on;
plot(x_input,y_output,'k','LineWidth',1)
plot(x_input,Fybar_calc,'r','LineWidth',1)
grid on, grid minor;
xlabel('Normalized slip angles (\alpha_{bar})')
ylabel('Normalized lateral force (Fy_{bar})')
title('Non-dimensional Tire Model Outputs')
legend('Non-dimensional data','Tire model (given)','Tire model
(calculated)','Location','Best')
set(gca,'XLim',[0,6])

figure(fig);
fig=fig+1;
clf;
plot(SA50exp,Fy50exp,'b','LineWidth',1)
hold on;
plot(SA100exp,Fy100exp,'k','LineWidth',1)
plot(SA150exp,Fy150exp,'r','LineWidth',1)
plot(SA250exp,Fy250exp,'g','LineWidth',1)
plot(SA350exp,Fy350exp,'m','LineWidth',1)
plot(SArad50(ii50left),FY50(ii50left),'b*');
plot(SArad100(ii100left),FY100(ii100left),'k*');
plot(SArad150(ii150left),FY150(ii150left),'r*');
plot(SArad250(ii250left),FY250(ii250left),'g*');
plot(SArad350(ii350left),FY350(ii350left),'m*');
plot(SA200exp,Fy200exp,'Color', '#0072BD','LineWidth',1)
plot(SA300exp,Fy300exp,'Color', '#D95319','LineWidth',1)
plot(SA400exp,Fy400exp,'Color', '#7E2F8E','LineWidth',1)
grid on, grid minor;
xlabel('Slip angles (\alpha)')
ylabel('Lateral force (Fy)')
title('Expanded of Non-dimensional Tire Model for Different Loads')
legend('50 lbf predicted','100 lbf predicted','150 lbf predicted','250 lbf
predicted','350 lbf predicted','50 lbf measured','100 lbf measured','150 lbf
measured','250 lbf measured','350 lbf measured','200 lbf predicted','300 lbf
predicted','400 lbf predicted','Location','BestOutside')
set(gca,'YDir','reverse','XLim',[0,0.23])
```