# AuE-6600: Dynamic Performance of Vehicles
# Homework #6

**Author:** Tanmay Samak

## Problem 1-A

**Approach:**

I defined the single-track vehicle model in state-space form:

$$X = \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \beta \\ r \end{bmatrix} \text{ where, } A = \begin{bmatrix} -\frac{C_{\alpha_R}+C_{\alpha_F}}{m*v} & \frac{(C_{\alpha_R}*b)-(C_{\alpha_F}*a)}{m*v^2}-1 \\ \frac{(C_{\alpha_R}*b)-(C_{\alpha_F}*a)}{I_z} & -\frac{(C_{\alpha_R}*b^2)+(C_{\alpha_F}*a^2)}{I_z*v} \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{C_{\alpha_F}}{m*v} \\ \frac{C_{\alpha_F}*a}{Iz} \end{bmatrix}$$

I used Euler-forward to update the states throughout the simulation:

$$X_{t+1} = X_t + \dot{X}_t * \Delta t$$

I then calculated the velocity components of the vehicle:

$$v_x = v * \cos(\psi + \beta)$$

$$v_y = v * sin(\psi + \beta)$$

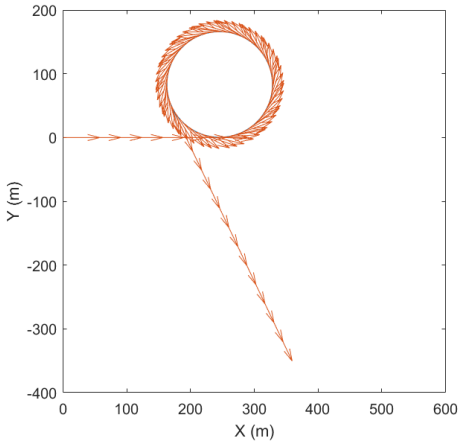
Then, I again used Euler-forward to update the position based on the velocity components:

$$P_{x_{t+1}} = P_{x_t} + v_{x_t} * \Delta t$$

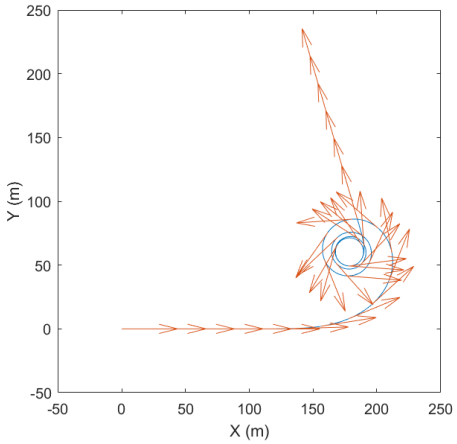
$$P_{y_{t+1}} = P_{y_t} + v_{y_t} * \Delta t$$

**Plots:**

I plot the CG position for all time instants, and the velocity vectors at each 1-second interval (the CG origin is the starting point of each arrow).




Step Maneuver - Trajectory




Fishhook Maneuver - Trajectory

**Inference:**

The step input causes the vehicle to go in a circular trajectory of constant radius, while the fishhook maneuver causes the vehicle to go in circular trajectory of decreasing radii.

## Problem 1-B

**Approach:**

I first calculated the coefficients `a` and `b`, which govern relationship between cornering stiffness and normal load. For this, I solved the linear system of equations (since it was an over-defined system of equations, I used MATLAB's `linsolve` function, which uses QR factorization with column pivoting to solve over-defined system of equations):

```
Ca = 0.4474
Cb = -2.8236e-05
```

Next, I calculated static load over each tire:

$$\begin{cases} S_{fl} = 0.5 * 0.6 * m * 9.81 \\ S_{fr} = 0.5 * 0.6 * m * 9.81 \\ S_{rl} = 0.5 * 0.4 * m * 9.81 \\ S_{rr} = 0.5 * 0.4 * m * 9.81 \end{cases}$$

In the simulation loop (Euler forward), I first calculated lateral acceleration, $a_{y_t} = v * (\dot{\beta_{t-1}} + \dot{\psi_t})$. Next, I calculated the load transfer on each tire:

$$\begin{cases} \mathrm{dW_f} = 0.6 * \dfrac{\mathrm{m} * \mathrm{a_{y_t}} * \mathrm{h}}{\mathrm{w}} \\ \mathrm{dW_r} = 0.4 * \dfrac{\mathrm{m} * \mathrm{a_{y_t}} * \mathrm{h}}{\mathrm{w}} \end{cases}$$

$$\begin{cases} \mathrm{F_{z_{fl}}} = \mathrm{S_{fl}} - \mathrm{dW_f} \\ \mathrm{F_{z_{fr}}} = \mathrm{S_{fr}} + \mathrm{dW_f} \\ \mathrm{F_{z_{rl}}} = \mathrm{S_{rl}} - \mathrm{dW}_r \\ \mathrm{F_{z_{rr}}} = \mathrm{S}_{rr} + \mathrm{dW}_r \end{cases}$$

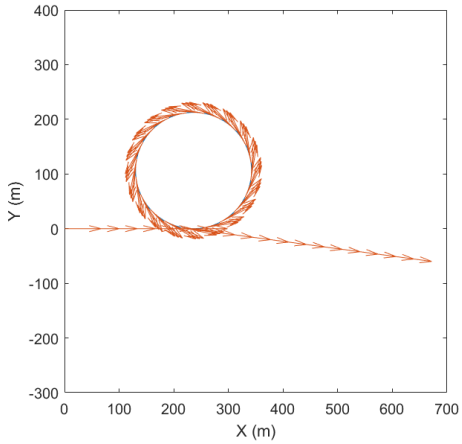
Next, I calculated the cornering stiffness for each tire based on the normal load:

$$\begin{cases} \mathrm{C_{\alpha_{fl}}} = (\mathrm{Ca} * \mathrm{F_{z_{fl}}} + \mathrm{Cb} * \mathrm{F_{z_{fl}}^2}) * (180/\pi) \\ \mathrm{C_{\alpha_{fr}}} = (\mathrm{Ca} * \mathrm{F_{z_{fr}}} + \mathrm{Cb} * \mathrm{F_{z_{fr}}^2}) * (180/\pi) \\ \mathrm{C_{\alpha_{rl}}} = (\mathrm{Ca} * \mathrm{F_{z_{rl}}} + \mathrm{Cb} * \mathrm{F_{z_{rl}}^2}) * (180/\pi) \\ \mathrm{C_{\alpha_{rr}}} = (\mathrm{Ca} * \mathrm{F_{z_{rr}}} + \mathrm{Cb} * \mathrm{F_{z_{rr}}^2}) * (180/\pi) \end{cases}$$

$$\begin{cases} \mathrm{C_{\alpha_f}} = \mathrm{C_{\alpha_{fl}}} + \mathrm{C_{\alpha_{fr}}} \\ \mathrm{C_{\alpha_r}} = \mathrm{C_{\alpha_{rl}}} + \mathrm{C_{\alpha_{rr}}} \end{cases}$$

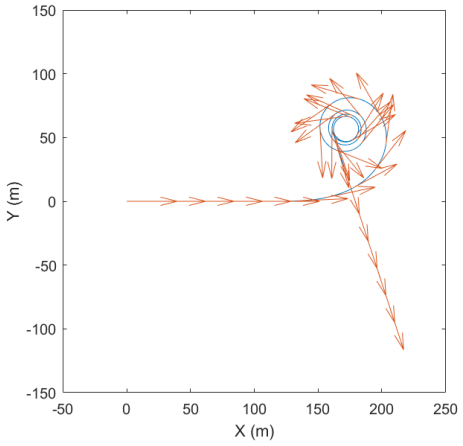
Finally, I simulated the state-space model of the vehicle using Euler forward and calculated velocity and position components (also using Euler forward), as described in Problem 1-A.

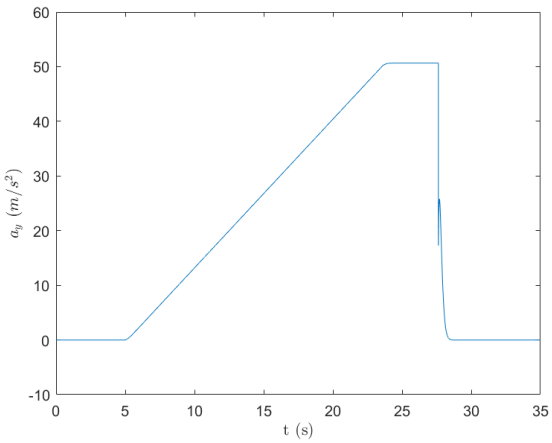
**Plots:**



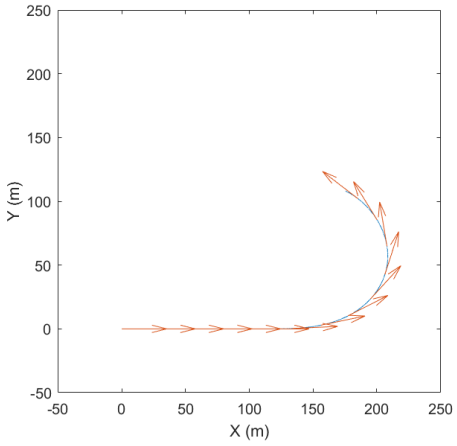

Step Maneuver - Trajectory

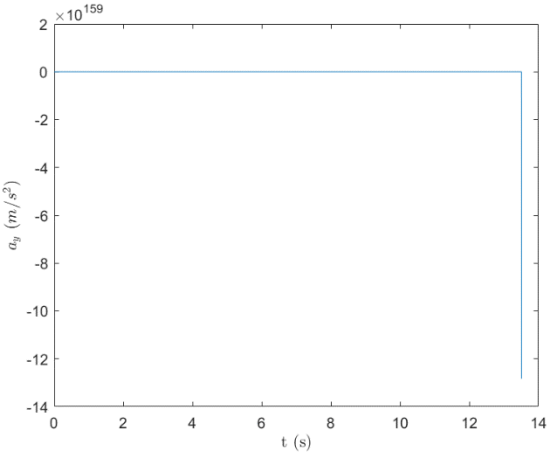
**Fishhook Maneuver without Load Transfer**



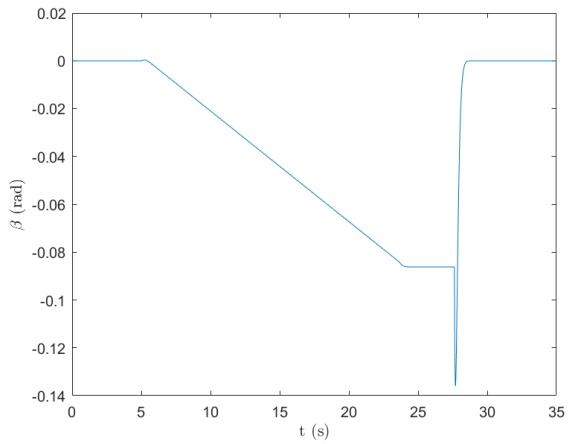



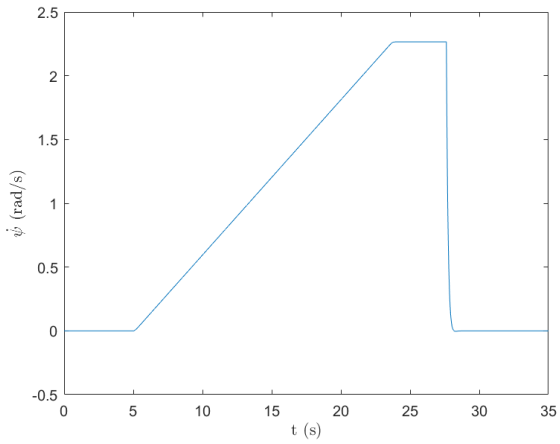

**Fishhook Maneuver with Load Transfer**



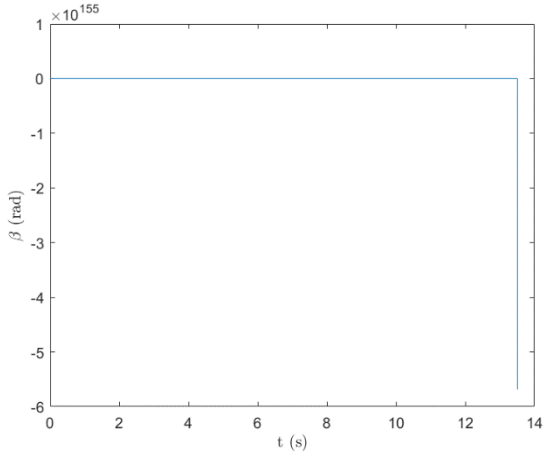



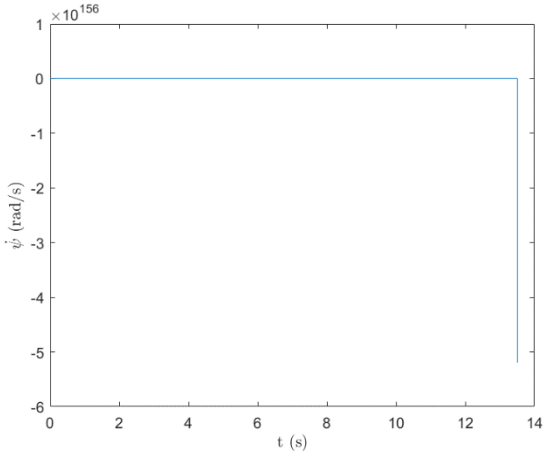
**Fishhook Maneuver without Load Transfer**




**Fishhook Maneuver with Load Transfer**




**Inference:**

The linear tire model blows up after certain lateral acceleration (when accounting load transfer). Since the step maneuver is within the bounds, the entire simulation can be completed. However, for the fishhook maneuver, we can observe that even without load transfer, the maximum lateral acceleration is going close to 50 m/s$^2$, which is too high a value for linear tire model. Hence, after considering load transfer, the values blow up (NaN) after simulating for certain portion of the maneuver.

## Problem 2-A

**Approach:**

Since it was not clearly mentioned, I did this problem considering two cases (i) without load transfer and (ii) with load transfer.

In addition to solution for Problem 1-B, I calculate the slip angles for front and rear tires:

$$\begin{cases} \alpha_f = \delta - \dfrac{\dot{\psi} * a}{v} - \beta \\ \alpha_r = \dfrac{\dot{\psi} * b}{v} - \beta \end{cases}$$

Next, I calculate the lateral force on each tire using the `nonlintire` function:

$$\begin{cases} F_{y_{fl}} = -\text{nonlintire}(\alpha_f, F_{z_{fl}}, v) \\ F_{y_{fr}} = -\text{nonlintire}(\alpha_f, F_{z_{fr}}, v) \\ F_{y_{rl}} = -\text{nonlintire}(\alpha_r, F_{z_{rl}}, v) \\ F_{y_{rr}} = -\text{nonlintire}(\alpha_r, F_{z_{rr}}, v) \end{cases}$$

$$\begin{cases} F_{y_f} = F_{y_{fl}} + F_{y_{fr}} \\ F_{y_r} = F_{y_{rl}} + F_{y_{rr}} \end{cases}$$

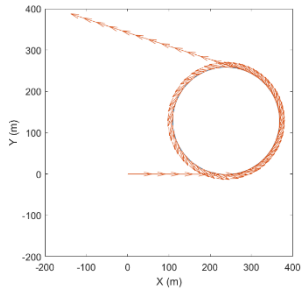
Next, I calculate the state derivative matrix:

$$\dot{X} = \begin{bmatrix} \frac{F_{y_f} + F_{y_r}}{m * v} - \dot{\psi} \\ \frac{F_{y_f} * a - F_{y_r} * b}{Iz} \end{bmatrix}$$

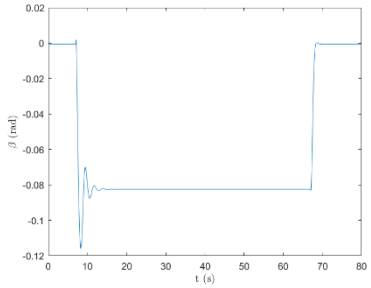
Finally, I simulated the state-space model of the vehicle using Euler forward and calculated velocity and position components (also using Euler forward), as described in Problem 1-A.

**Plots:**

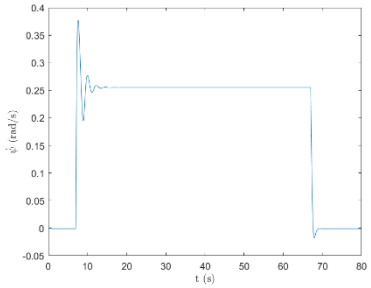
Step Maneuver without Load Transfer:




Trajectory

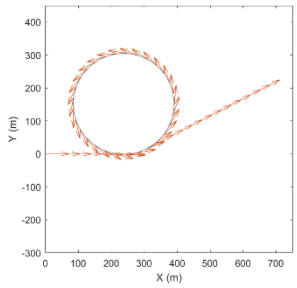



Vehicle Slip Angle

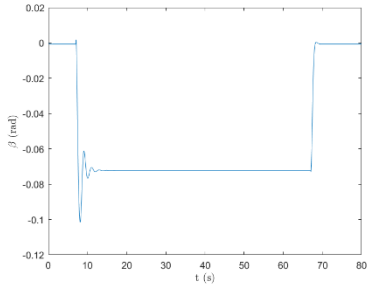



Vehicle Angular Velocity

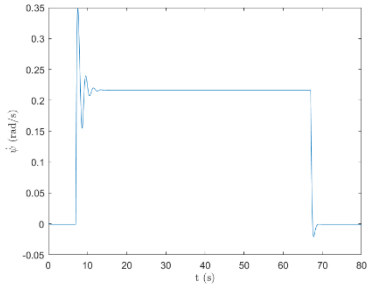
Step Maneuver with Load Transfer:




Trajectory

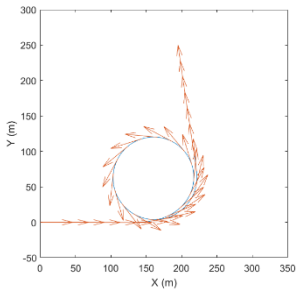



Vehicle Slip Angle

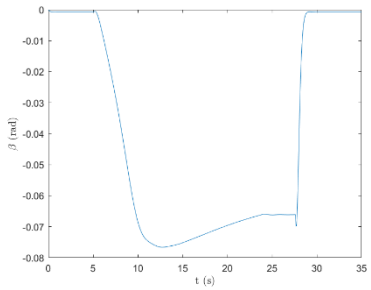



Vehicle Angular Velocity

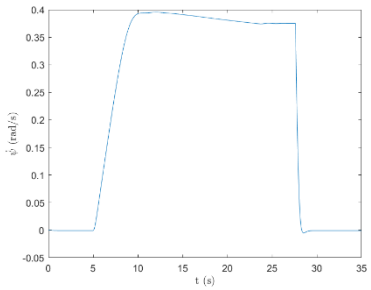
Fishhook Maneuver without Load Transfer:




Trajectory

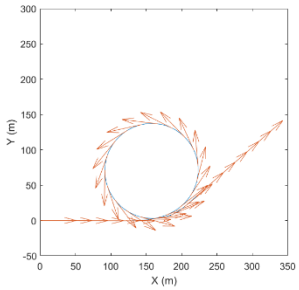



Vehicle Slip Angle

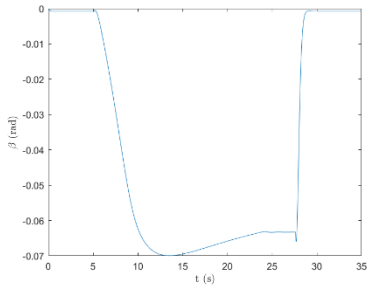



Vehicle Angular Velocity

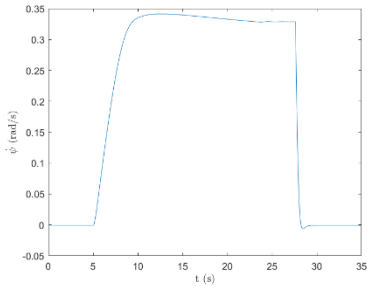
Fishhook Maneuver with Load Transfer:




Trajectory




Vehicle Slip Angle




Vehicle Angular Velocity

**Inference:**

The non-linear model captures the tire dynamics better than the linear model (especially at higher lateral acceleration values). We can observe that the radius of the curves has increased significantly, since the vehicle was sliding outwards at high lateral acceleration values (even more so with load transfer).

## Problem 2-B-(i)

**Approach:**

I followed most of the approach as described in Problem 2-A (without load transfer). I recorded the lateral acceleration values along with steering commands and plotted them. Next, I fit a line to the linear region of the plot and calculate its slope.

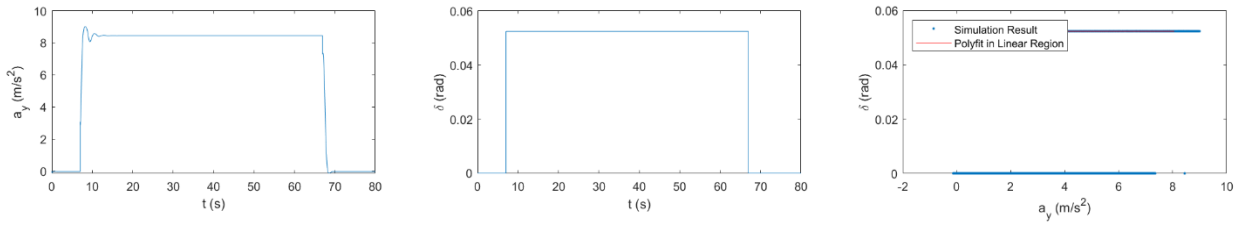
$$UG = polyder(UG_{polyfit})$$

However, for the fishhook maneuver (where radius is changing), we need to subtract the slope of neutral-steer gradient:

$$UG = polyder(UG_{polyfit}) - \frac{l}{v^2}$$

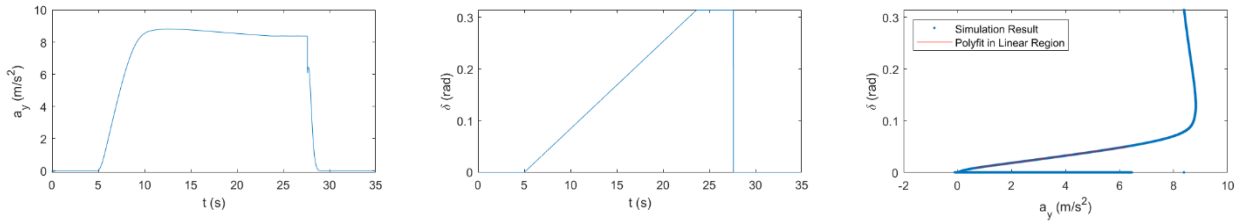
**Plots:**

Step Maneuver without Load Transfer:




```
UG for step maneuver with non-linear tire model, without load transfer is 0.0000 rad-s^2/m
```

Fishhook Maneuver without Load Transfer:




```
UG for fishhook maneuver with non-linear tire model, without load transfer is 0.0018 rad-s^2/m
```

**Inference:**

The vehicle is sublimit understeer, since UG is positive. However, calculating UG for step input does not seem truly possible. This is because UG can be calculated in either of the following cases:

- Vehicle is travelling at a constant radius (similar to step maneuver), but at varying speeds.
- Vehicle is travelling at a constant speed, but at gradually changing radius (similar to fishhook maneuver)

However, since in the case of step maneuver, we were given a constant velocity, the UG calculated may only reflect the UG at the given condition and may or may not be generalizable over the entire sublimit range.

## Problem 2-B-(ii)

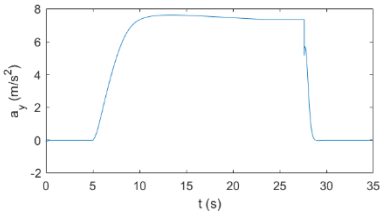
**Approach:**

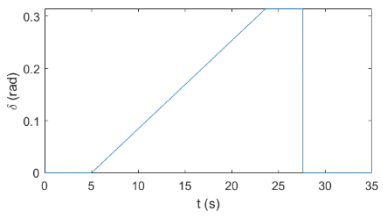
I followed most of the approach as described in Problem 2-B-(i), but now only for fishhook maneuver with load transfer. I recorded the lateral acceleration values along with steering commands and plotted them. Next, I fit a line to the linear region of the plot and calculate its slope. However, since this is for the fishhook maneuver (where radius is changing), we need to subtract the slope of neutral-steer gradient:

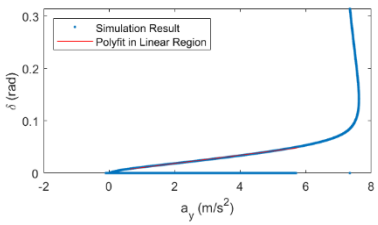
$$UG = polyder\left(UG_{polyfit}\right) - \frac{l}{v^2}$$

**Plots:**

Fishhook Maneuver with Load Transfer:



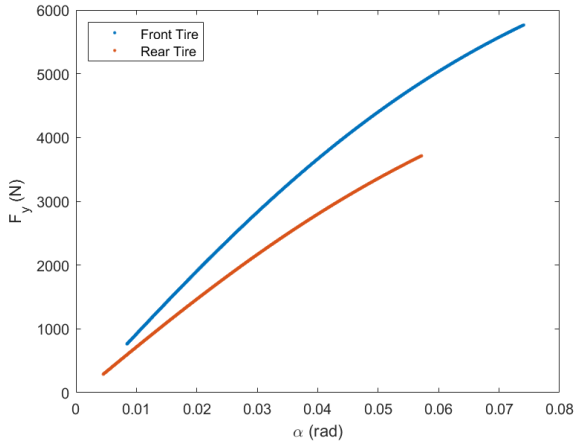




```
UG for fishhook maneuver with non-linear tire model, with load transfer is 0.0025 rad-s^2/m
```

**Inference:**

The vehicle is sublimit understeer, since UG is positive.

For <u>limit behavior</u> I plotted the tire forces in the linear region of $\delta$ vs. $a_y$ plot:




From the above plot, it is evident that rear tire will saturate first (assuming identical tires in front and back). This means that at limit, the vehicle would exhibit oversteering behavior.

## Problem 2-B-(iii)

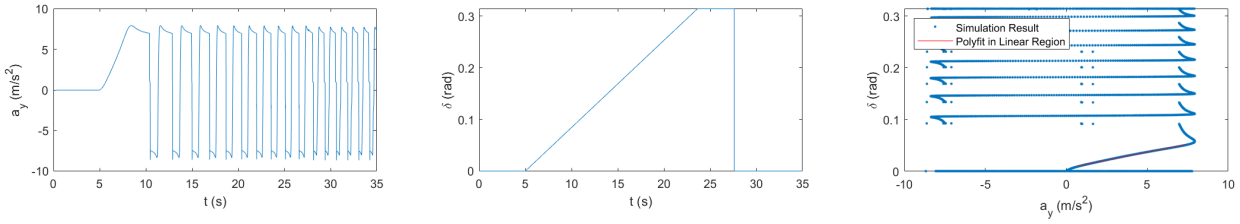
**Approach:**

I followed most of the approach as described in Problem 2-B-(ii), but now with the new roll stiffness distribution. I recorded the lateral acceleration values along with steering commands and plotted them. Next, I fit a line to the linear region of the plot and calculate its slope. However, since this is for the fishhook maneuver (where radius is changing), we need to subtract the slope of neutral-steer gradient:

$$UG = polyder\left(UG_{polyfit}\right) - \frac{l}{v^2}$$

**Plots:**

Fishhook Maneuver with Load Transfer:

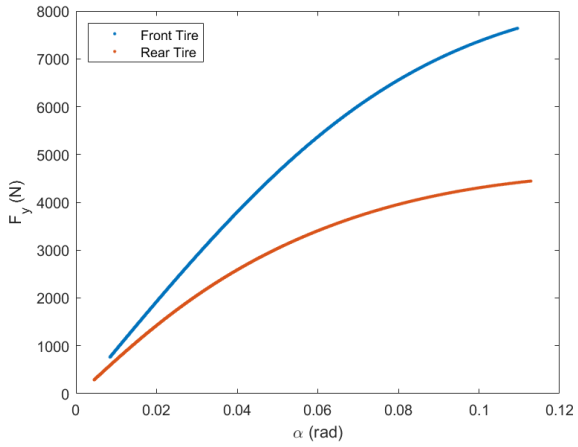



```
UG for fishhook maneuver with non-linear tire model, with load transfer is 0.0007 rad-s^2/m
```

**Inference:**

The vehicle is sublimit understeer, since UG is positive.

For <u>limit behavior</u> I plotted the tire forces in the linear region of $\delta$ vs. $a_y$ plot:




From the above plot, it is evident that rear tire will saturate first (assuming identical tires in front and back). This means that at limit, the vehicle would exhibit oversteering behavior.

Comparing this with plot from Problem 2-B-(ii), we can see that the difference in slopes in this case is more than the difference in slopes in case of Problem 2-B-(ii). This indicates that at limit behavior this vehicle would oversteer more than the vehicle in case of Problem 2-B-(ii).

# AuE-6600 | Dynamic Performance of Vehicles

## Homework 6

**Vehicle Parameters:**

- Mass: 1637 kg
- Yaw inertia: 3326 kg-m^2
- Wheelbase: 2.736 m
- Trackwidth: 1.7 m
- Weight distribution: 60% to front
- Front cornering stiffness: 1500 N/deg per tire (static load)
- Rear cornering stiffness: 1146 N/deg per tire (static load)
- Steering ratio: 15:1 (15 degree at the steering wheel = 1 degree at the front wheel)
- CG height: 2.4 ft

**Maneuver Description:**

In this assignment, you are required to simulate the bicycle model for 2 different maneuvers that are described below. Based on the description, for each maneuver, you should create a vector of inputs for the simulation

1. The roadwheel steering angle is the input to your bicycle model. In the first maneuver, provide a step input for this steering angle. The vehicle is first being driven down a straight line at 74 mph for 7 seconds after which the driver holds the handwheel position constant at 45 degrees for 60 seconds. The handwheel position is then returned to 0 degrees and the maneuver is terminated
2. The second maneuver is called the “fishhook maneuver” where you need to gradually increase the steering angle. To begin this maneuver, the vehicle is first driven in a straight line at 50mph. The driver must attempt to maintain this speed for 5 seconds. After that, the handwheel position is linearly increased from 0 to 270 degrees at a rate of 14.5 degrees per second. The handwheel position is held constant at 270 degrees for 4 seconds, after which the maneuver is concluded. The handwheel is then returned to 0 as a convenience to the driver.

```
% Vehicle parameters
m = 1637; % kg
Iz = 3326; % kg-m^2
l = 2.736; % m
w = 1.7; % m
a = 0.4*l;
b = l-a;
C_alpha_F = 1500; % N/deg
C_alpha_F = C_alpha_F*(180/pi); % N/rad
C_alpha_R = 1146; % N/deg
C_alpha_R = C_alpha_R*(180/pi); % N/rad
SR = 15;
h = 2.4; % ft
h = h/3.281; % m
```

```
% Maneuver description
% Step maneuver
v_step = 74; % mph
v_step = v_step/2.237; % m/s
u_step = [ones(1,7000)*0 ones(1,60000)*(deg2rad(45)/SR) ones(1,13000)*0];
% Fishhook manuever
v_fishhook = 50; % mph
v_fishhook = v_fishhook/2.237; % m/s
u_fishhook = [ones(1,5000)*0 deg2rad(0:(14.5/1000):270)/SR ones(1,4000)*deg2rad(270)/SR ones(1,
```

## Problem 1

**Part A:**

Now let's revisit the HW 5 problem 3, extend the bicycle model developed in HW5 problem 3 to determine the position of the CG as well as the direction of the velocity vector in the world (inertial) coordinate frame. For each maneuver, provide a plot for the trajectory of the vehicle. Clearly indicate the location of CG and the velocity vector, at an interval of 1 second, for the complete maneuver. (2 plots in total)

Assume that the CG is laterally in the middle of the vehicle, i.e., the static load is equally distributed to the left and to the right.

**Part B:**

Now it is time to include the effects of Lateral load transfer on cornering stiffnesses of front and rear axle, perform the simulation. Assume that the roll stiffness distribution is the same as the weight distribution

The relationship between cornering stiffness and normal load is given as:

$$C_\alpha = a * F_z + b * F_z^2$$

where, $F_z$ is normal load in N and $C_\alpha$ is cornering stiffness in N/deg

The following tabular data can be used to find the coefficients 'a' and 'b'.

| Cornering Stiffness (N/deg) | Normal load (N) | Cornering Stiffness (N/deg) | Normal load (N) |
|---|---|---|---|
| 848 | 2200 | 1461 | 4600 |
| 972 | 2600 | 1531 | 5000 |
| 1088 | 3000 | 1593 | 5400 |
| 1195 | 3400 | 1645 | 5800 |
| 1292 | 3800 | 1689 | 6200 |
| 1381 | 4200 | 1723 | 6600 |
| | | 1748 | 7000 |

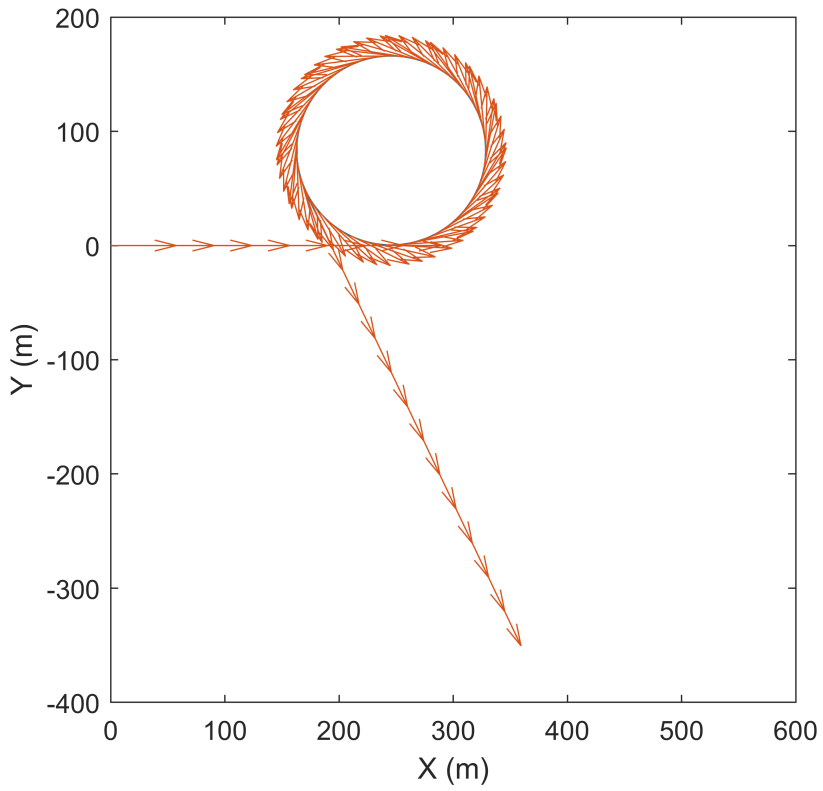
## Solution 1

**Part A**

1. Step Maneuver

```
% Single-Track Model
```

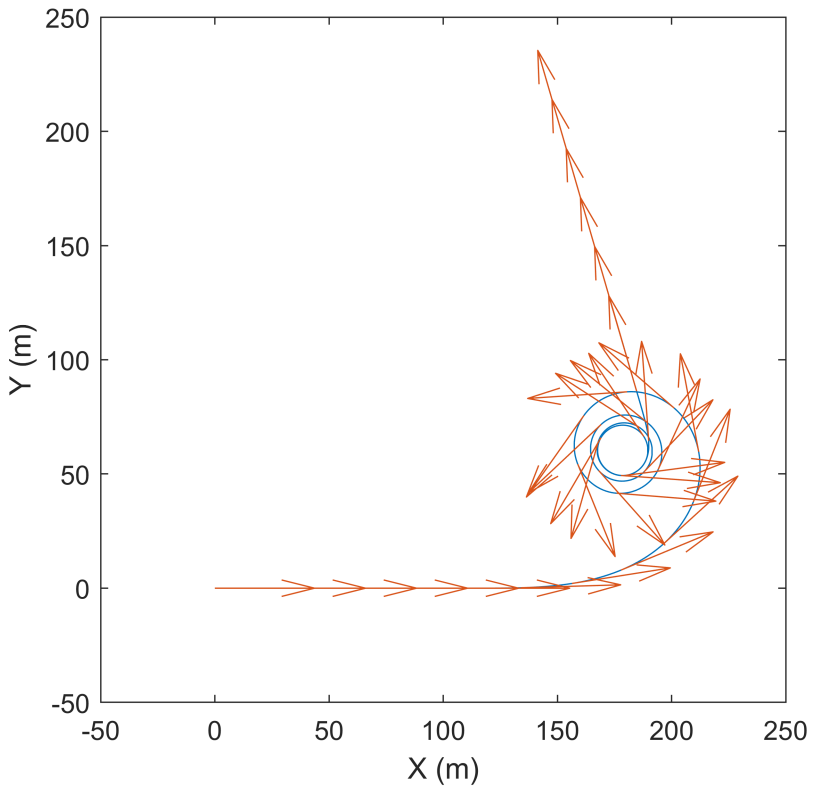
```
a11 = -((C_alpha_R+C_alpha_F)/(m*v_step));
a12 = (((C_alpha_R*b)-(C_alpha_F*a))/(m*v_step^2))-1;
a21 = ((C_alpha_R*b)-(C_alpha_F*a))/(Iz);
a22 = -((C_alpha_R*b^2)+(C_alpha_F*a^2))/(Iz*v_step);
b1 = (C_alpha_F)/(m*v_step);
b2 = (C_alpha_F*a)/(Iz);
A = [a11 a12; a21 a22];
B = [b1; b2];
% Initialization
dt = 0.001; % s
X = [];
X(:,1) = [0;0];
X_dot = [];
Psi = [];
Psi(1) = 0;
Beta = [];
Vx = [];
Vy = [];
Px = [];
Px(1) = 0;
Py = [];
Py(1) = 0;
% Simulation at each millisecond
for t=1:80000
    X_dot(:,t) = A*X(:,t)+B*u_step(t);
    X(:,t+1) = X(:,t) + X_dot(:,t)*dt;
    Beta(t) = X(1,t);
    Psi(t+1) = Psi(t) + X(2,t)*dt;
    Vx(t) = v_step*cos(Psi(t)+Beta(t));
    Vy(t) = v_step*sin(Psi(t)+Beta(t));
    Px(t+1) = Px(t) + Vx(t)*dt;
    Py(t+1) = Py(t) + Vy(t)*dt;
end
% Plot
plot(Px(1:end-1),Py(1:end-1))
hold on;
quiver(Px(1:1000:end-1000),Py(1:1000:end-1000),Vx(1:1000:end),Vy(1:1000:end))
hold off;
xlabel("X (m)")
ylabel("Y (m)")
xlim([0,600])
ylim([-400,200])
pbaspect([1 1 1])
```




2. Fishhook Maneuver

```
% Single-Track Model
a11 = -((C_alpha_R+C_alpha_F)/(m*v_fishhook));
a12 = (((C_alpha_R*b)-(C_alpha_F*a))/(m*v_fishhook^2))-1;
a21 = ((C_alpha_R*b)-(C_alpha_F*a))/(Iz);
a22 = -((C_alpha_R*b^2)+(C_alpha_F*a^2))/(Iz*v_fishhook);
b1 = (C_alpha_F)/(m*v_fishhook);
b2 = (C_alpha_F*a)/(Iz);
A = [a11 a12; a21 a22];
B = [b1; b2];
% Initialization
dt = 0.001; % s
X = [];
X(:,1) = [0;0];
X_dot = [];
Psi = [];
Psi(1) = 0;
Beta = [];
Vx = [];
Vy = [];
Px = [];
Px(1) = 0;
Py = [];
Py(1) = 0;
```

```
% Simulation at each millisecond
for t=1:35000
    X_dot(:,t) = A*X(:,t)+B*u_fishhook(t);
    X(:,t+1) = X(:,t) + X_dot(:,t)*dt;
    Beta(t) = X(1,t);
    Psi(t+1) = Psi(t) + X(2,t)*dt;
    Vx(t) = v_fishhook*cos(Psi(t)+Beta(t));
    Vy(t) = v_fishhook*sin(Psi(t)+Beta(t));
    Px(t+1) = Px(t) + Vx(t)*dt;
    Py(t+1) = Py(t) + Vy(t)*dt;
end
% Plot
plot(Px(1:end-1),Py(1:end-1))
hold on;
quiver(Px(1:1000:end-1000),Py(1:1000:end-1000),Vx(1:1000:end),Vy(1:1000:end))
hold off;
xlabel("X (m)")
ylabel("Y (m)")
xlim([-50,250])
ylim([-50,250])
pbaspect([1 1 1])
```




**Part B**

```
% Table of C_alpha vs F_z
C_alpha = [848 972 1088 1195 1292 1381 1461 1531 1593 1645 1689 1723 1748];
```

```
F_z = [2200 2600 3000 3400 3800 4200 4600 5000 5400 5800 6200 6600 7000];
% Calculate a and b
F = [F_z' (F_z.^2)'];
C = C_alpha';
ab = linsolve(F,C);
Ca = ab(1)
```

```
Ca = 0.4474
```

```
Cb = ab(2)
```

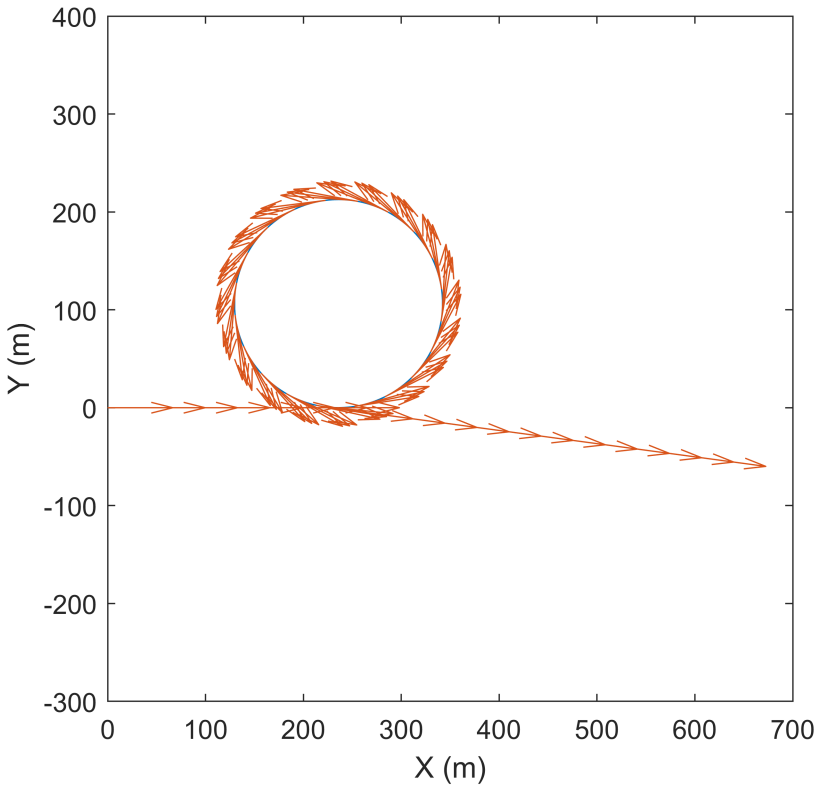
```
Cb = -2.8236e-05
```

```
% Static load
S_fl = 0.5*0.6*m*9.81;
S_fr = 0.5*0.6*m*9.81;
S_rl = 0.5*0.4*m*9.81;
S_rr = 0.5*0.4*m*9.81;
```

1. Step Maneuver

```
% Initialization
dt = 0.001; % s
X = [];
X(:,1) = [0;0];
X_dot = [];
Psi = [];
Psi(1) = 0;
Beta = [];
B_dot = 0;
Vx = [];
Vy = [];
Px = [];
Px(1) = 0;
Py = [];
Py(1) = 0;
a_y = [];
% Simulation at each millisecond
for t=1:80000
    a_y(t) = v_step*(B_dot+X(2,t));
    dW_f = 0.6*(m*a_y(t)*h)/w;
    dW_r = 0.4*(m*a_y(t)*h)/w;
    Fz_fl = S_fl - dW_f;
    Fz_fr = S_fr + dW_f;
    Fz_rl = S_rl - dW_r;
    Fz_rr = S_rr + dW_r;
    C_alpha_fl = (Ca*Fz_fl + Cb*Fz_fl^2)*(180/pi);
    C_alpha_fr = (Ca*Fz_fr + Cb*Fz_fr^2)*(180/pi);
    C_alpha_rl = (Ca*Fz_rl + Cb*Fz_rl^2)*(180/pi);
    C_alpha_rr = (Ca*Fz_rr + Cb*Fz_rr^2)*(180/pi);
    C_alpha_F = C_alpha_fl+C_alpha_fr;
```

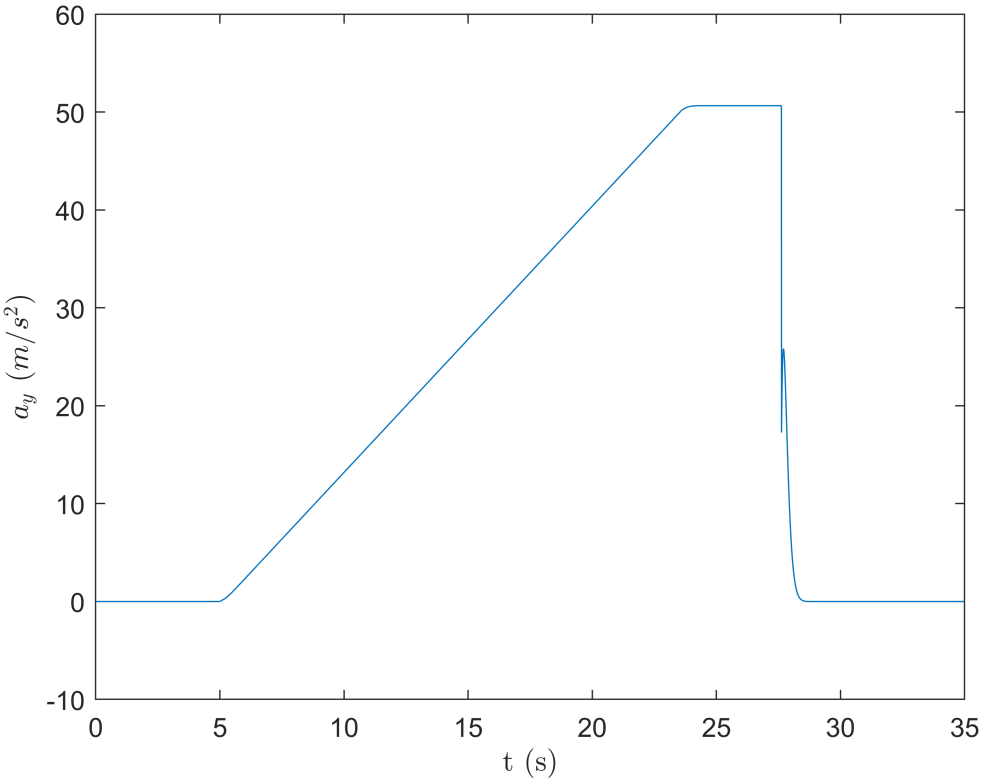
```
    C_alpha_R = C_alpha_rl+C_alpha_rr;
    A = [-((C_alpha_R+C_alpha_F)/(m*v_step)) (((C_alpha_R*b)-(C_alpha_F*a))/(m*v_step^2))-1;
         ((C_alpha_R*b)-(C_alpha_F*a))/(Iz)  -((C_alpha_R*b^2)+(C_alpha_F*a^2))/(Iz*v_step)];
    B = [(C_alpha_F)/(m*v_step);
         (C_alpha_F*a)/(Iz)];
    X_dot(:,t) = A*X(:,t)+B*u_step(t);
    B_dot = X_dot(1,t);
    X(:,t+1) = X(:,t) + X_dot(:,t)*dt;
    Beta(t) = X(1,t);
    Psi(t+1) = Psi(t) + X(2,t)*dt;
    Vx(t) = v_step*cos(Psi(t)+Beta(t));
    Vy(t) = v_step*sin(Psi(t)+Beta(t));
    Px(t+1) = Px(t) + Vx(t)*dt;
    Py(t+1) = Py(t) + Vy(t)*dt;
end
% Plot
plot(Px(1:end-1),Py(1:end-1))
hold on;
quiver(Px(1:1000:end-1000),Py(1:1000:end-1000),Vx(1:1000:end),Vy(1:1000:end))
hold off;
xlabel("X (m)")
ylabel("Y (m)")
xlim([0,700])
ylim([-300,400])
pbaspect([1 1 1])
```



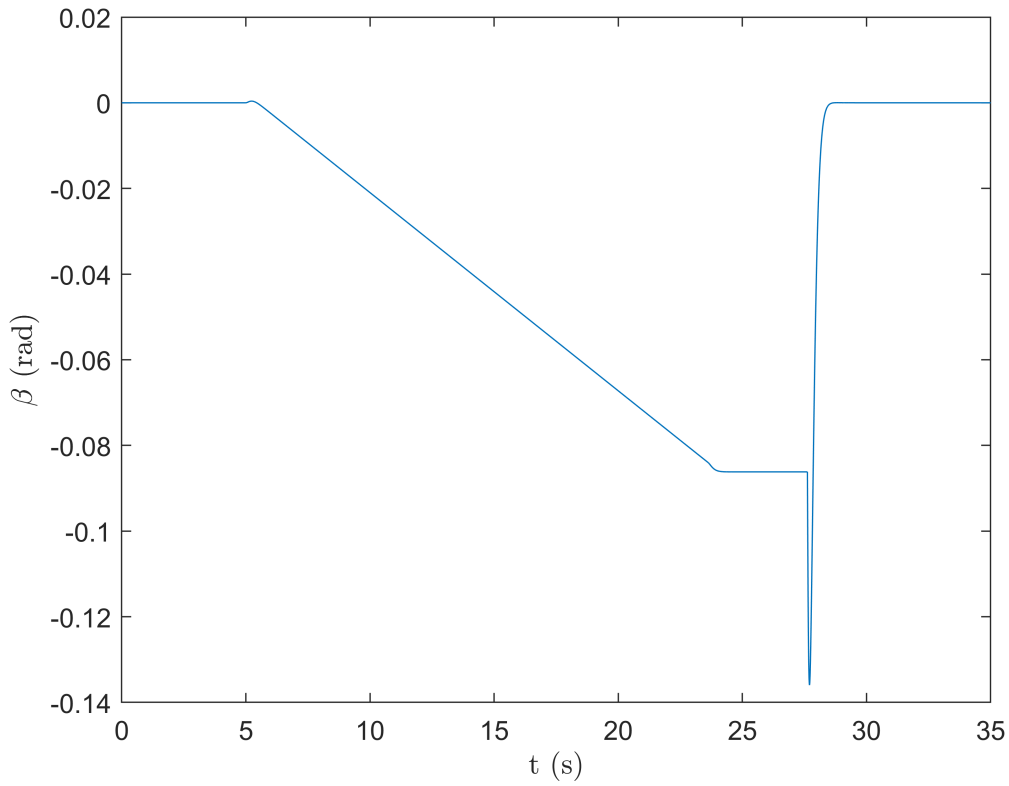
2(a). Fishhook Maneuver without Load Transfer

```
% Initialization
dt = 0.001; % s
X = [];
X(:,1) = [0;0];
X_dot = [];
Psi = [];
Psi(1) = 0;
Beta = [];
B_dot = 0;
Vx = [];
Vy = [];
Px = [];
Px(1) = 0;
Py = [];
Py(1) = 0;
a_y = [];
% Simulation at each millisecond
for t=1:35000
    a_y(t) = v_fishhook*(B_dot+X(2,t));
    dW_f = 0.6*(m*a_y(t)*h)/w;
    dW_r = 0.4*(m*a_y(t)*h)/w;
    Fz_fl = S_fl;
    Fz_fr = S_fr;
    Fz_rl = S_rl;
    Fz_rr = S_rr;
    C_alpha_fl = (Ca*Fz_fl + Cb*Fz_fl^2)*(180/pi);
    C_alpha_fr = (Ca*Fz_fr + Cb*Fz_fr^2)*(180/pi);
    C_alpha_rl = (Ca*Fz_rl + Cb*Fz_rl^2)*(180/pi);
    C_alpha_rr = (Ca*Fz_rr + Cb*Fz_rr^2)*(180/pi);
    C_alpha_F = C_alpha_fl+C_alpha_fr;
    C_alpha_R = C_alpha_rl+C_alpha_rr;
    A = [-((C_alpha_R+C_alpha_F)/(m*v_fishhook)) (((C_alpha_R*b)-(C_alpha_F*a))/(m*v_fishhook^2
         ((C_alpha_R*b)-(C_alpha_F*a))/(Iz)  -((C_alpha_R*b^2)+(C_alpha_F*a^2))/(Iz*v_fishhook)
    B = [(C_alpha_F)/(m*v_fishhook);
         (C_alpha_F*a)/(Iz)];
    X_dot(:,t) = A*X(:,t)+B*u_fishhook(t);
    B_dot = X_dot(1,t);
    X(:,t+1) = X(:,t) + X_dot(:,t)*dt;
    Beta(t) = X(1,t);
    Psi(t+1) = Psi(t) + X(2,t)*dt;
    Vx(t) = v_fishhook*cos(Psi(t)+Beta(t));
    Vy(t) = v_fishhook*sin(Psi(t)+Beta(t));
    Px(t+1) = Px(t) + Vx(t)*dt;
    Py(t+1) = Py(t) + Vy(t)*dt;
end
% Plot
t=0:35000;
plot(t/1000,[0 a_y])
```

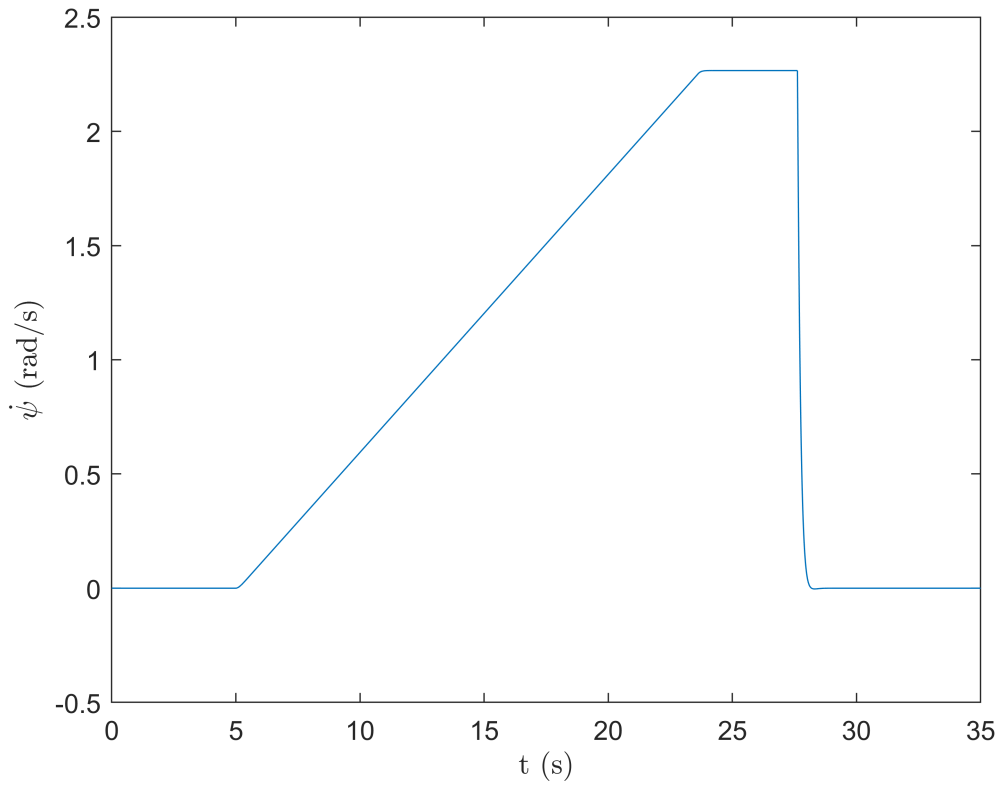
```
xlabel("t (s)",'interpreter','latex')
ylabel("$a_y$ $(m/s^2)$",'interpreter','latex')
```



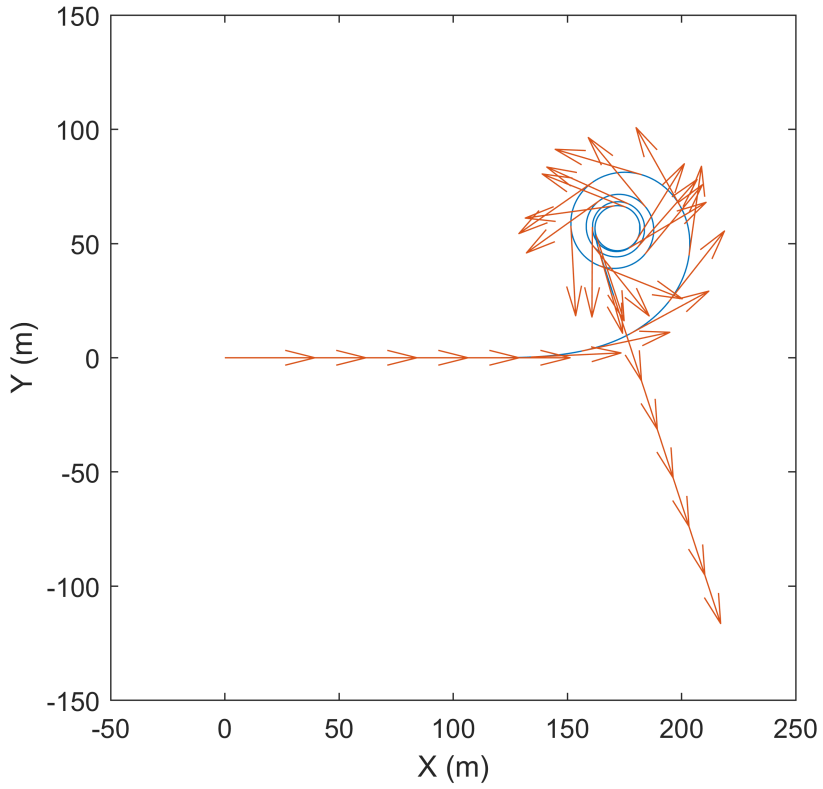

```
t=0:35000;
plot(t/1000,X(1,:))
xlabel("t (s)",'interpreter','latex')
ylabel("$\beta$ (rad)",'interpreter','latex')
```




```
t=0:35000;
plot(t/1000,X(2,:))
xlabel("t (s)",'interpreter','latex')
ylabel("$\dot{\psi}$ (rad/s)",'interpreter','latex')
```



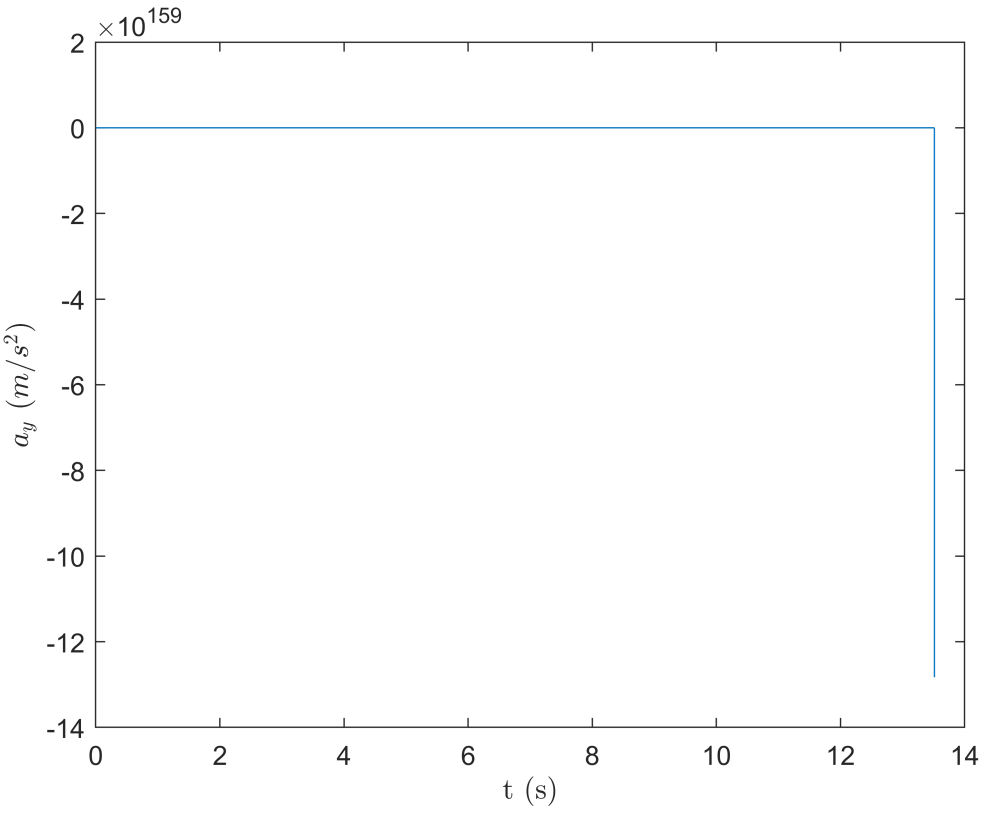

```
plot(Px(1:end-1),Py(1:end-1))
hold on;
quiver(Px(1:1000:end-1000),Py(1:1000:end-1000),Vx(1:1000:end),Vy(1:1000:end))
hold off;
xlabel("X (m)")
ylabel("Y (m)")
xlim([-50,250])
ylim([-150,150])
pbaspect([1 1 1])
```



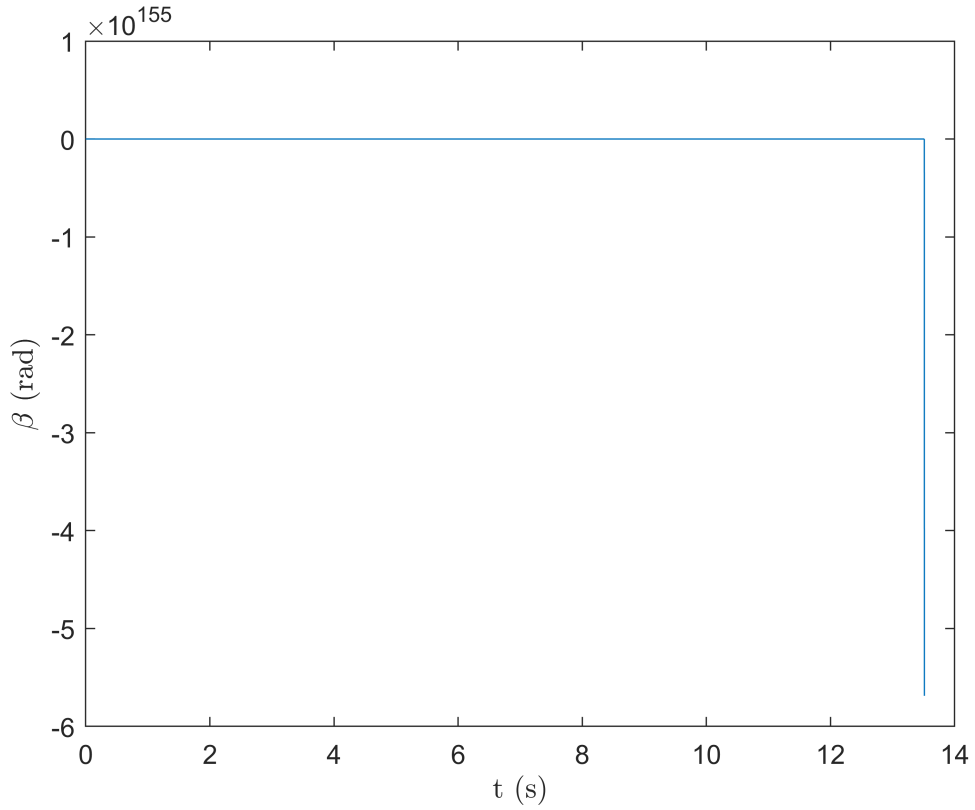

2(b). Fishhook Maneuver with Load Transfer

```
% Initialization
dt = 0.001; % s
X = [];
X(:,1) = [0;0];
X_dot = [];
Psi = [];
Psi(1) = 0;
Beta = [];
B_dot = 0;
Vx = [];
Vy = [];
Px = [];
Px(1) = 0;
Py = [];
Py(1) = 0;
a_y = [];
% Simulation at each millisecond
for t=1:35000
    a_y(t) = v_fishhook*(B_dot+X(2,t));
    dW_f = 0.6*(m*a_y(t)*h)/w;
    dW_r = 0.4*(m*a_y(t)*h)/w;
    Fz_fl = S_fl - dW_f;
    Fz_fr = S_fr + dW_f;
    Fz_rl = S_rl - dW_r;
```

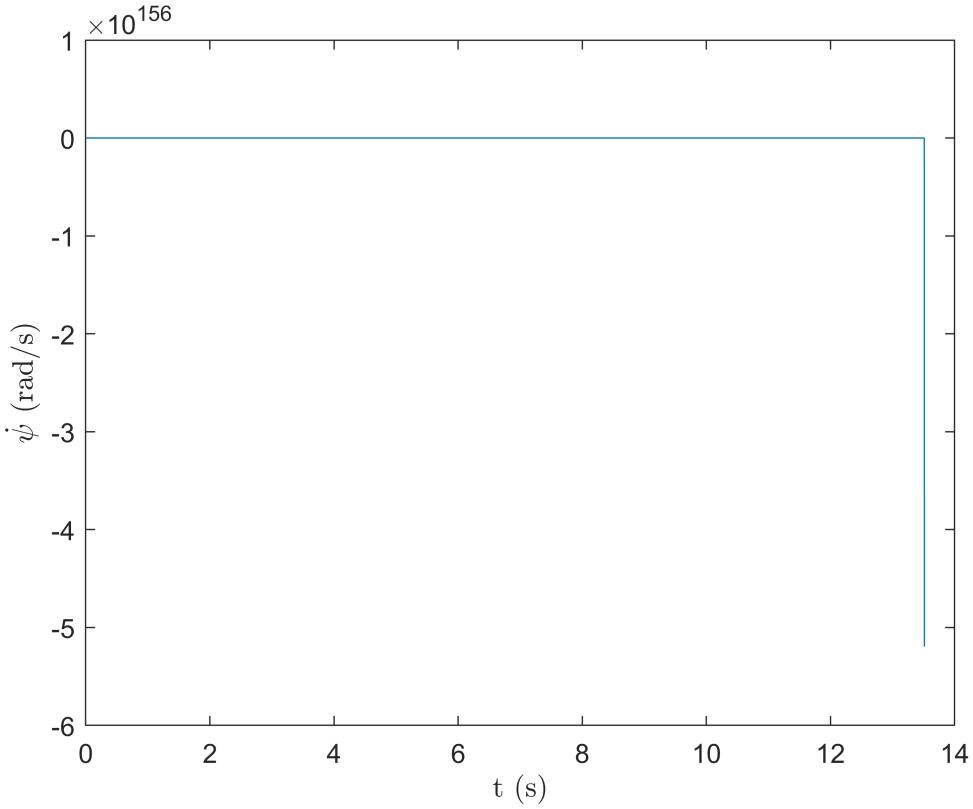
```
    Fz_rr = S_rr + dW_r;
    C_alpha_fl = (Ca*Fz_fl + Cb*Fz_fl^2)*(180/pi);
    C_alpha_fr = (Ca*Fz_fr + Cb*Fz_fr^2)*(180/pi);
    C_alpha_rl = (Ca*Fz_rl + Cb*Fz_rl^2)*(180/pi);
    C_alpha_rr = (Ca*Fz_rr + Cb*Fz_rr^2)*(180/pi);
    C_alpha_F = C_alpha_fl+C_alpha_fr;
    C_alpha_R = C_alpha_rl+C_alpha_rr;
    A = [-((C_alpha_R+C_alpha_F)/(m*v_fishhook)) (((C_alpha_R*b)-(C_alpha_F*a))/(m*v_fishhook^2
         ((C_alpha_R*b)-(C_alpha_F*a))/(Iz)  -((C_alpha_R*b^2)+(C_alpha_F*a^2))/(Iz*v_fishhook)
    B = [(C_alpha_F)/(m*v_fishhook);
         (C_alpha_F*a)/(Iz)];
    X_dot(:,t) = A*X(:,t)+B*u_fishhook(t);
    B_dot = X_dot(1,t);
    X(:,t+1) = X(:,t) + X_dot(:,t)*dt;
    Beta(t) = X(1,t);
    Psi(t+1) = Psi(t) + X(2,t)*dt;
    Vx(t) = v_fishhook*cos(Psi(t)+Beta(t));
    Vy(t) = v_fishhook*sin(Psi(t)+Beta(t));
    Px(t+1) = Px(t) + Vx(t)*dt;
    Py(t+1) = Py(t) + Vy(t)*dt;
end
% Plot
t=0:35000;
plot(t/1000,[0 a_y])
xlabel("t (s)",'interpreter','latex')
ylabel("$a_y$ $(m/s^2)$",'interpreter','latex')
```



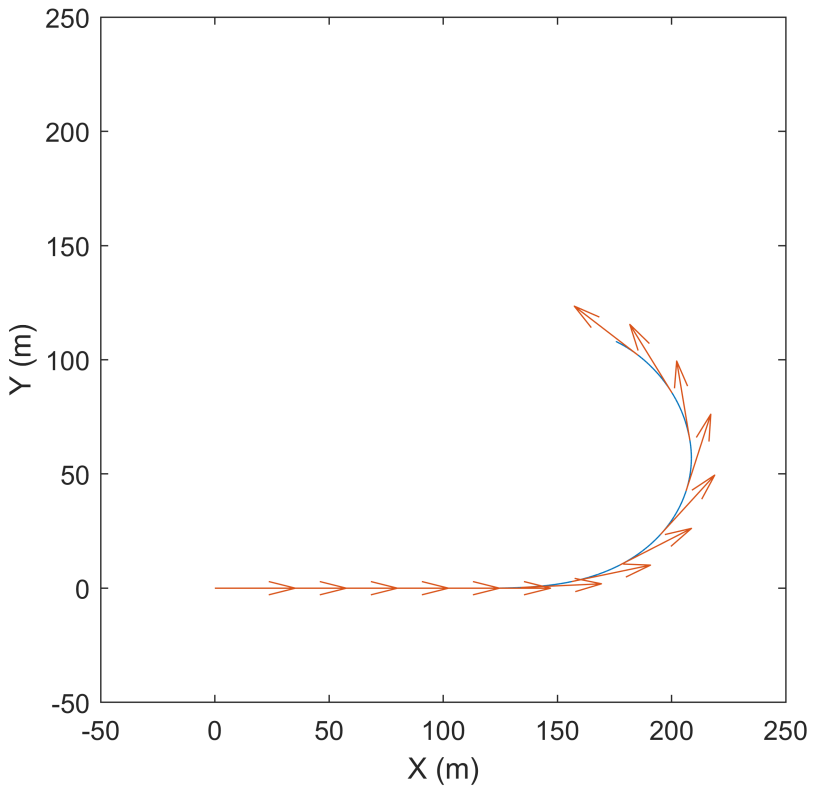
```
t=0:35000;
plot(t/1000,X(1,:))
xlabel("t (s)",'interpreter','latex')
ylabel("$\beta$ (rad)",'interpreter','latex')
```




```
t=0:35000;
plot(t/1000,X(2,:))
xlabel("t (s)",'interpreter','latex')
ylabel("$\dot{\psi}$ (rad/s)",'interpreter','latex')
```




```
plot(Px(1:end-1),Py(1:end-1))
hold on;
quiver(Px(1:1000:end-1000),Py(1:1000:end-1000),Vx(1:1000:end),Vy(1:1000:end))
hold off;
xlabel("X (m)")
ylabel("Y (m)")
xlim([-50,250])
ylim([-50,250])
pbaspect([1 1 1])
```




## Problem 2

In problem 1, the bicycle model was simulated assuming a linear tire model for a given load (of course, it is nonlinear model with respect to Fz). For this part, a non-linear tire model (nonlintire.m) is being provided to you. This tire model accepts an argument in the format $(\alpha, F_z, v)$ where $\alpha$ is the slip angle, $F_z$ is the normal load, and $v$ is the longitudinal velocity of the wheel center, and returns a lateral force, $F_y$.

The given nonlinear tire model produces forces according to the non‐adapted ISO system, i.e., assuming negative cornering stiffness. However, our models are derived assuming positive cornering stiffness, i.e., positive slip angles correspond to positive forces. Therefore, you have to make the tire model compliant with your model.

**Part A:**

Using this non-linear tire model, for each maneuver, simulate the states of the bicycle model and determine the position of CG as well as the direction of the velocity vector in the inertial coordinate frame. Provide plots for the states as well as the trajectory of CG and direction of velocity vector – i.e., total 6 plots

**Part B:**

1. Calculate the Understeer Gradient (UG) of fishhook maneuver and step input without load transfer.

2. From the Fishhook maneuver, determine the sublimit Understeer Gradient (UG) with load transfer. Comment on the limit behavior.

3. Now assume a roll stiffness distribution of 70% rear 30% front and perform the same analysis as in section 2 of Part B. Comment on the results.

**NOTE:** In your report, clearly mention your approach, the equations used, and your observations along with the plots of your result. Attach your MATLAB code in the appendix of the report for reference.

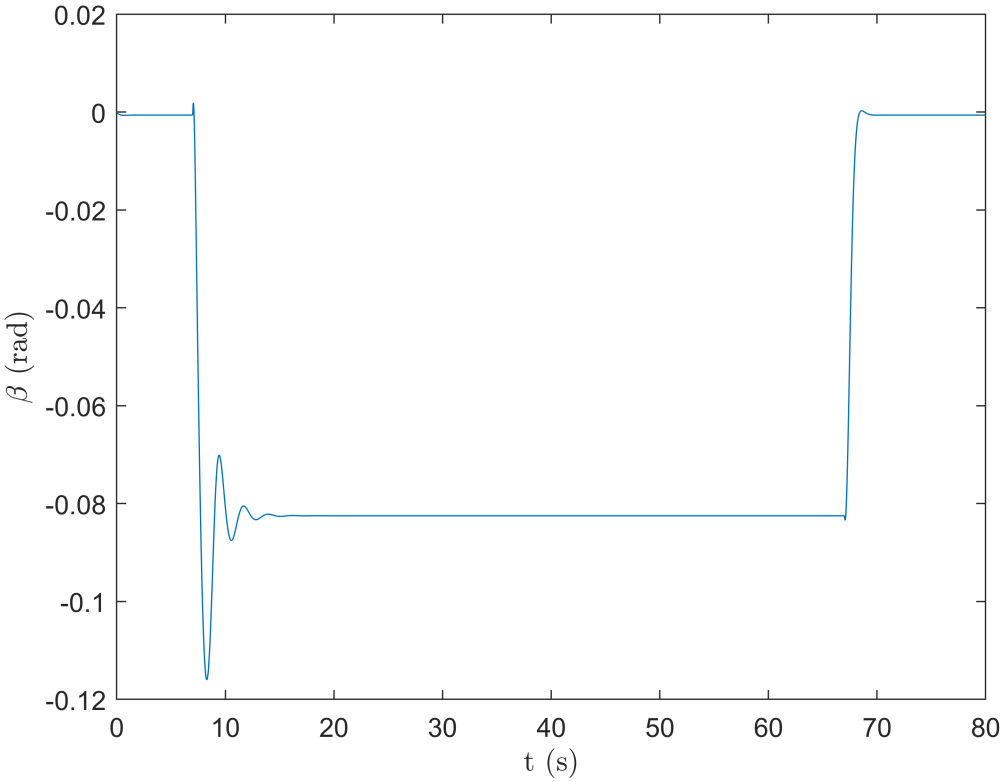
## Solution 2

### Part A

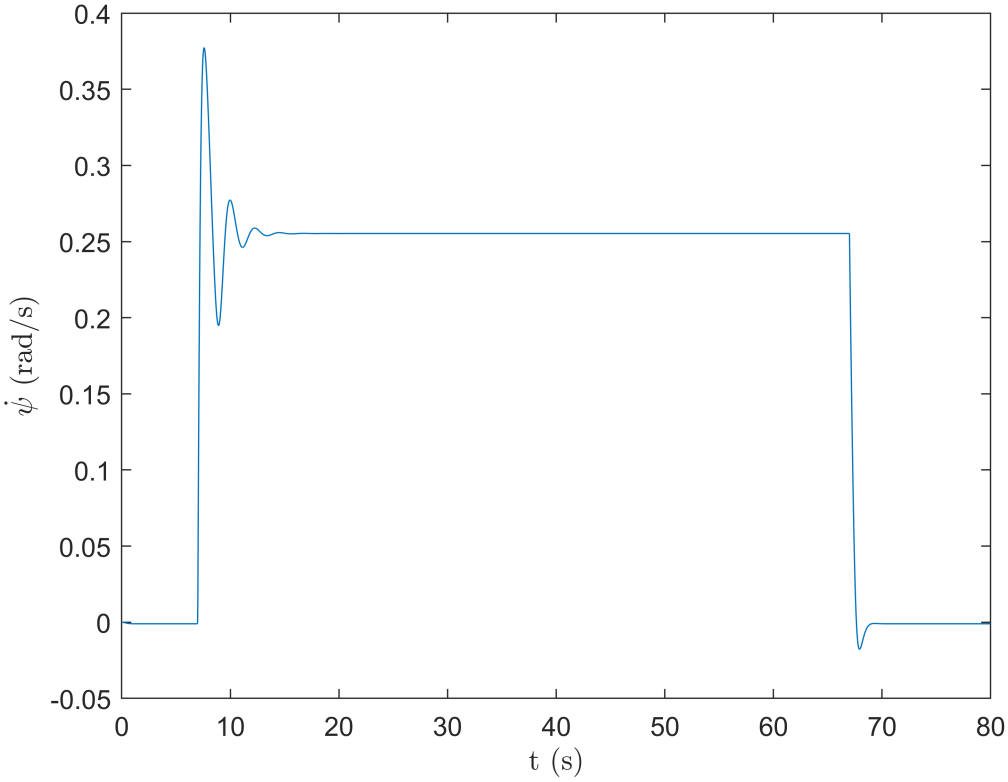
1(a). Step Maneuver without Load Transfer

```
% Initialization
dt = 0.001; % s
X = [];
X(:,1) = [0;0];
X_dot = [];
Psi = [];
Psi(1) = 0;
Beta = [];
B_dot = 0;
Vx = [];
Vy = [];
Px = [];
Px(1) = 0;
Py = [];
Py(1) = 0;
% Simulation at each millisecond
for t=1:80000
    a_y = v_step*(B_dot+X(2,t));
    dW_f = 0.6*(m*a_y*h)/w;
    dW_r = 0.4*(m*a_y*h)/w;
    Fz_fl = S_fl;
    Fz_fr = S_fr;
    Fz_rl = S_rl;
    Fz_rr = S_rr;
    alpha_f = u_step(t) - (X(2,t)*a)/v_step - X(1,t);
    alpha_r = (X(2,t)*b)/v_step - X(1,t);
    Fy_fl = -nonlintire(alpha_f, Fz_fl, v_step);
    Fy_fr = -nonlintire(alpha_f, Fz_fr, v_step);
    Fy_rl = -nonlintire(alpha_r, Fz_rl, v_step);
    Fy_rr = -nonlintire(alpha_r, Fz_rr, v_step);
    Fy_f = Fy_fl+Fy_fr;
    Fy_r = Fy_rl+Fy_rr;
    X_dot(:,t) = [(Fy_f+Fy_r)/(m*v_step) - X(2,t);
                  (Fy_f*a-Fy_r*b)/(Iz)];
    B_dot = X_dot(1,t);
    X(:,t+1) = X(:,t) + X_dot(:,t)*dt;
    Beta(t) = X(1,t);
    Psi(t+1) = Psi(t) + X(2,t)*dt;
    Vx(t) = v_step*cos(Psi(t)+Beta(t));
```

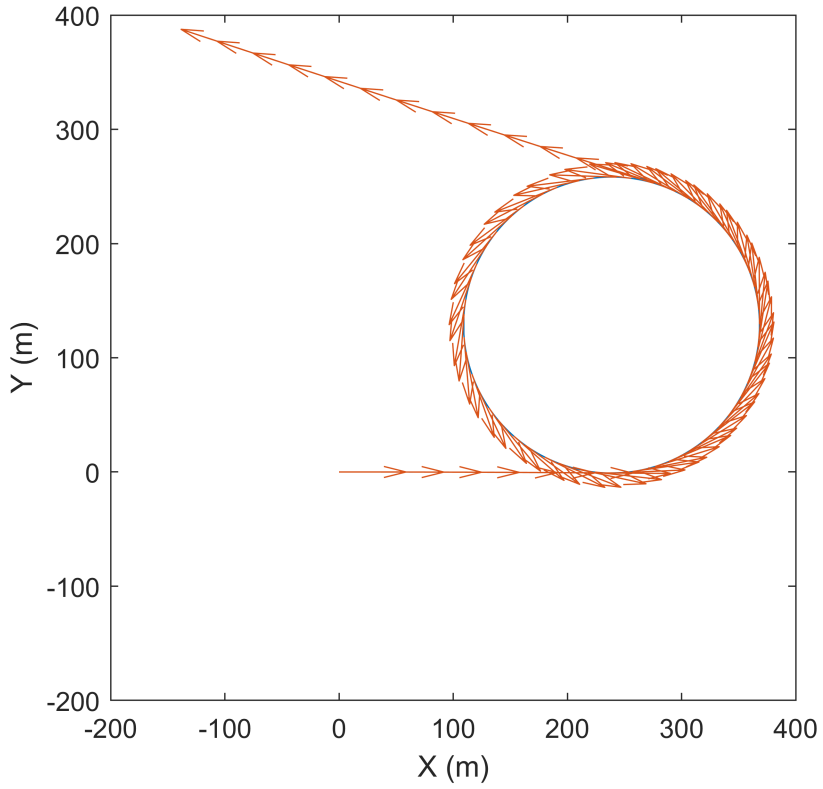
```
    Vy(t) = v_step*sin(Psi(t)+Beta(t));
    Px(t+1) = Px(t) + Vx(t)*dt;
    Py(t+1) = Py(t) + Vy(t)*dt;
end
% Plots
t=0:80000;
plot(t/1000,X(1,:))
xlabel("t (s)",'interpreter','latex')
ylabel("$\beta$ (rad)",'interpreter','latex')
```




```
plot(t/1000,X(2,:))
xlabel("t (s)",'interpreter','latex')
ylabel("$\dot{\psi}$ (rad/s)",'interpreter','latex')
```



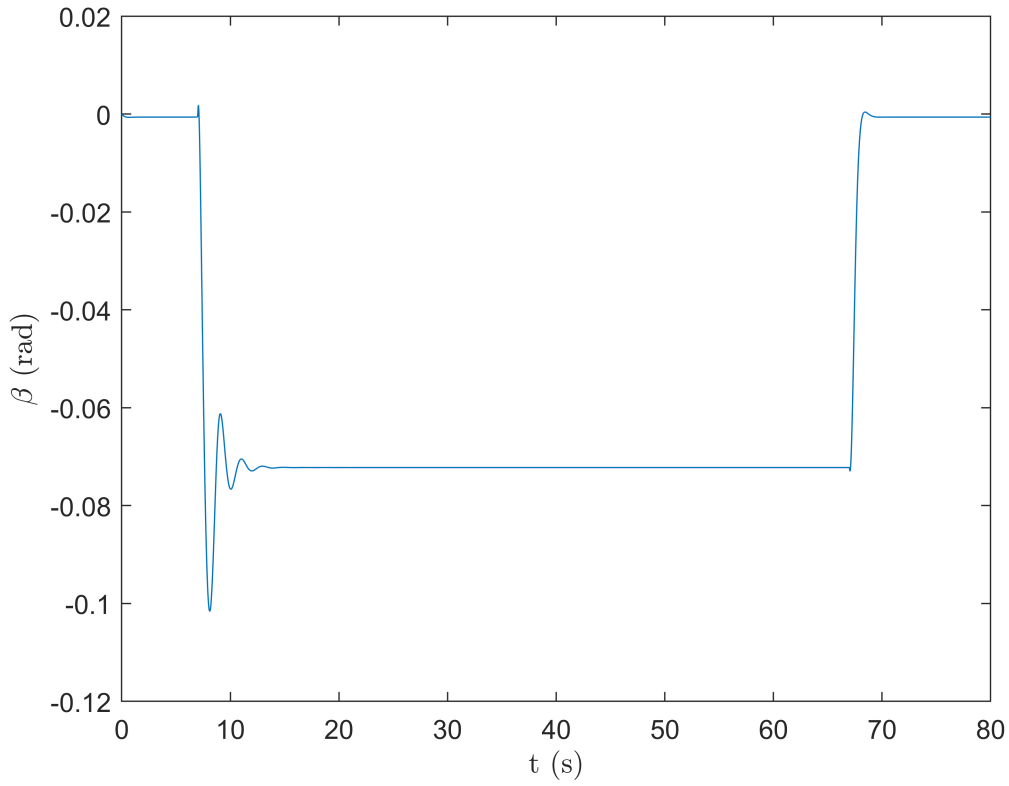

```
plot(Px(1:end-1),Py(1:end-1))
hold on;
quiver(Px(1:1000:end-1000),Py(1:1000:end-1000),Vx(1:1000:end),Vy(1:1000:end))
hold off;
xlabel("X (m)")
ylabel("Y (m)")
xlim([-200,400])
ylim([-200,400])
pbaspect([1 1 1])
```




1(b). Step Maneuver with Load Transfer

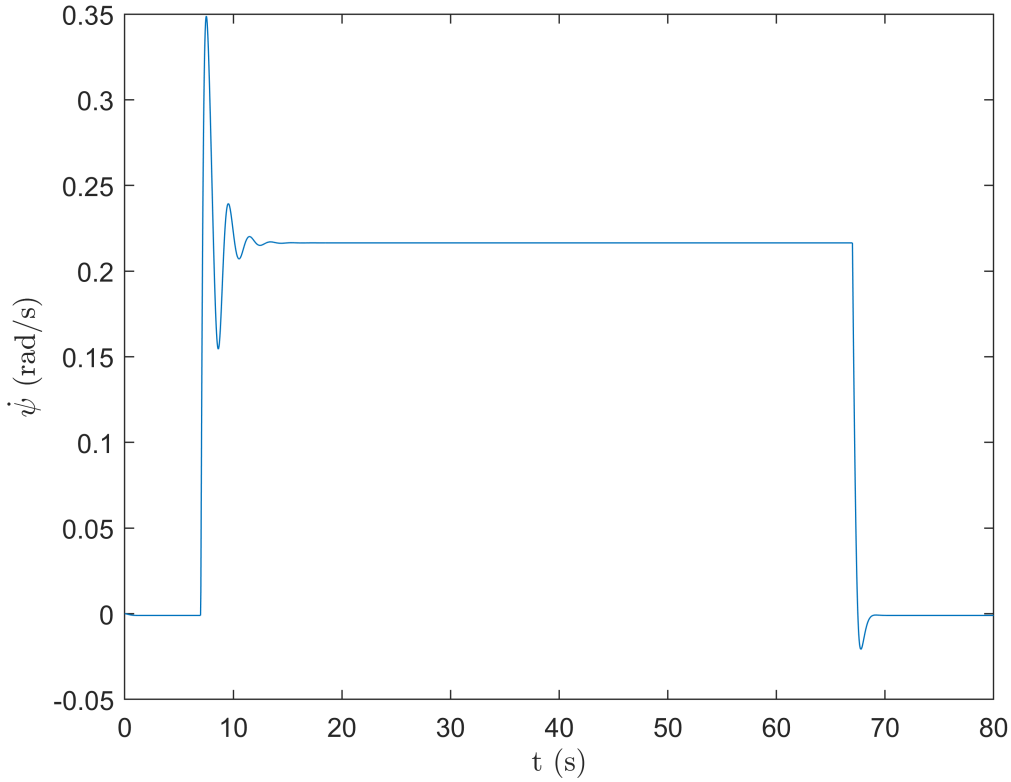
```
% Initialization
dt = 0.001; % s
X = [];
X(:,1) = [0;0];
X_dot = [];
Psi = [];
Psi(1) = 0;
Beta = [];
B_dot = 0;
Vx = [];
Vy = [];
Px = [];
Px(1) = 0;
Py = [];
Py(1) = 0;
% Simulation at each millisecond
for t=1:80000
    a_y = v_step*(B_dot+X(2,t));
    dW_f = 0.6*(m*a_y*h)/w;
    dW_r = 0.4*(m*a_y*h)/w;
    Fz_fl = S_fl - dW_f;
    Fz_fr = S_fr + dW_f;
    Fz_rl = S_rl - dW_r;
```

```
    Fz_rr = S_rr + dW_r;
    alpha_f = u_step(t) - (X(2,t)*a)/v_step - X(1,t);
    alpha_r = (X(2,t)*b)/v_step - X(1,t);
    Fy_fl = -nonlintire(alpha_f, Fz_fl, v_step);
    Fy_fr = -nonlintire(alpha_f, Fz_fr, v_step);
    Fy_rl = -nonlintire(alpha_r, Fz_rl, v_step);
    Fy_rr = -nonlintire(alpha_r, Fz_rr, v_step);
    Fy_f = Fy_fl+Fy_fr;
    Fy_r = Fy_rl+Fy_rr;
    X_dot(:,t) = [(Fy_f+Fy_r)/(m*v_step) - X(2,t);
                  (Fy_f*a-Fy_r*b)/(Iz)];
    B_dot = X_dot(1,t);
    X(:,t+1) = X(:,t) + X_dot(:,t)*dt;
    Beta(t) = X(1,t);
    Psi(t+1) = Psi(t) + X(2,t)*dt;
    Vx(t) = v_step*cos(Psi(t)+Beta(t));
    Vy(t) = v_step*sin(Psi(t)+Beta(t));
    Px(t+1) = Px(t) + Vx(t)*dt;
    Py(t+1) = Py(t) + Vy(t)*dt;
end
% Plots
t=0:80000;
plot(t/1000,X(1,:))
xlabel("t (s)",'interpreter','latex')
ylabel("$\beta$ (rad)",'interpreter','latex')
```

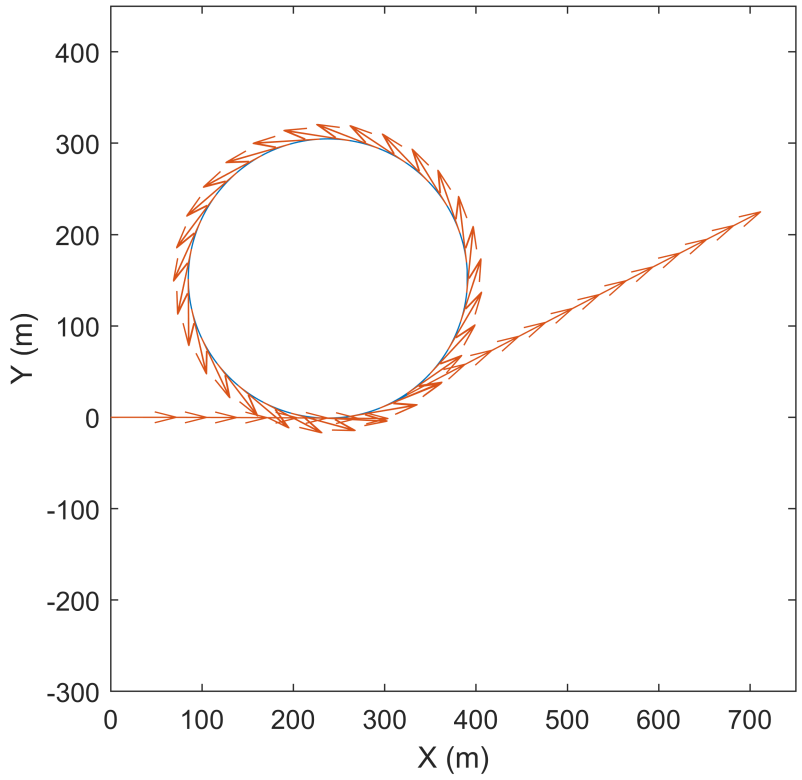



```
plot(t/1000,X(2,:))
```

```
xlabel("t (s)",'interpreter','latex')
ylabel("$\dot{\psi}$ (rad/s)",'interpreter','latex')
```



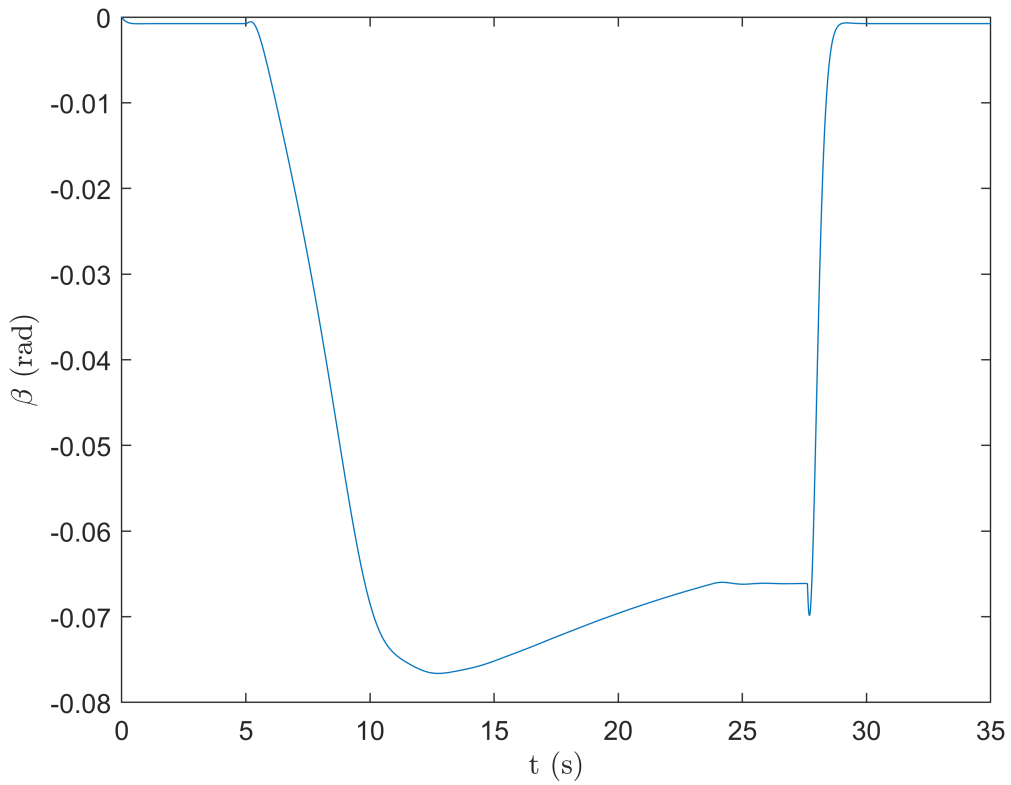

```
plot(Px(1:end-1),Py(1:end-1))
hold on;
quiver(Px(1:1000:end-1000),Py(1:1000:end-1000),Vx(1:1000:end),Vy(1:1000:end))
hold off;
xlabel("X (m)")
ylabel("Y (m)")
xlim([0,750])
ylim([-300,450])
pbaspect([1 1 1])
```




2(a). Fishhook Maneuver without Load Transfer

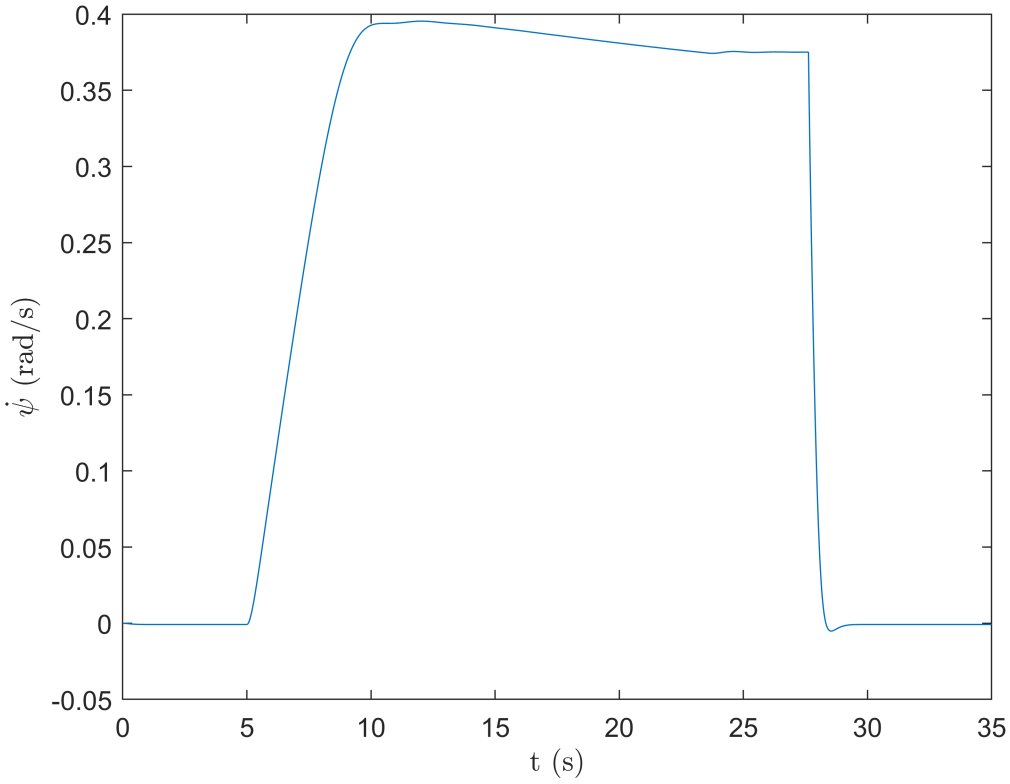
```
% Initialization
dt = 0.001; % s
X = [];
X(:,1) = [0;0];
X_dot = [];
Psi = [];
Psi(1) = 0;
Beta = [];
B_dot = 0;
Vx = [];
Vy = [];
Px = [];
Px(1) = 0;
Py = [];
Py(1) = 0;
% Simulation at each millisecond
for t=1:35000
    a_y = v_fishhook*(B_dot+X(2,t));
    dW_f = 0.6*(m*a_y*h)/w;
    dW_r = 0.4*(m*a_y*h)/w;
    Fz_fl = S_fl;
    Fz_fr = S_fr;
    Fz_rl = S_rl;
    Fz_rr = S_rr;
```

```
    alpha_f = u_fishhook(t) - (X(2,t)*a)/v_fishhook - X(1,t);
    alpha_r = (X(2,t)*b)/v_fishhook - X(1,t);
    Fy_fl = -nonlintire(alpha_f, Fz_fl, v_fishhook);
    Fy_fr = -nonlintire(alpha_f, Fz_fr, v_fishhook);
    Fy_rl = -nonlintire(alpha_r, Fz_rl, v_fishhook);
    Fy_rr = -nonlintire(alpha_r, Fz_rr, v_fishhook);
    Fy_f = Fy_fl+Fy_fr;
    Fy_r = Fy_rl+Fy_rr;
    X_dot(:,t) = [(Fy_f+Fy_r)/(m*v_fishhook) - X(2,t);
                  (Fy_f*a-Fy_r*b)/(Iz)];
    B_dot = X_dot(1,t);
    X(:,t+1) = X(:,t) + X_dot(:,t)*dt;
    Beta(t) = X(1,t);
    Psi(t+1) = Psi(t) + X(2,t)*dt;
    Vx(t) = v_fishhook*cos(Psi(t)+Beta(t));
    Vy(t) = v_fishhook*sin(Psi(t)+Beta(t));
    Px(t+1) = Px(t) + Vx(t)*dt;
    Py(t+1) = Py(t) + Vy(t)*dt;
end
% Plot
t=0:35000;
plot(t/1000,X(1,:))
xlabel("t (s)",'interpreter','latex')
ylabel("$\beta$ (rad)",'interpreter','latex')
```

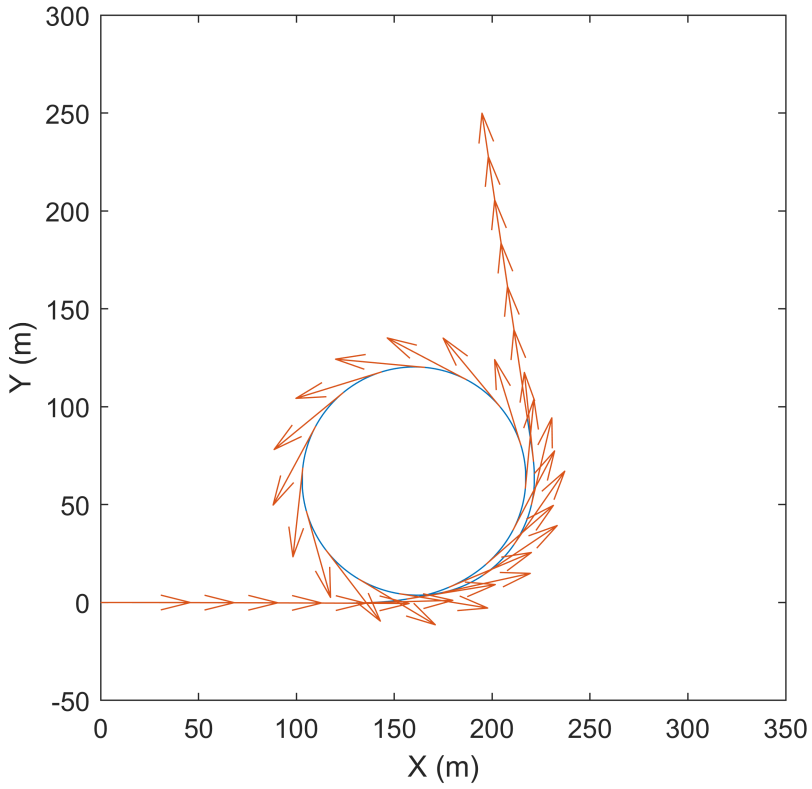



```
t=0:35000;
plot(t/1000,X(2,:))
```

```
xlabel("t (s)",'interpreter','latex')
ylabel("$\dot{\psi}$ (rad/s)",'interpreter','latex')
```



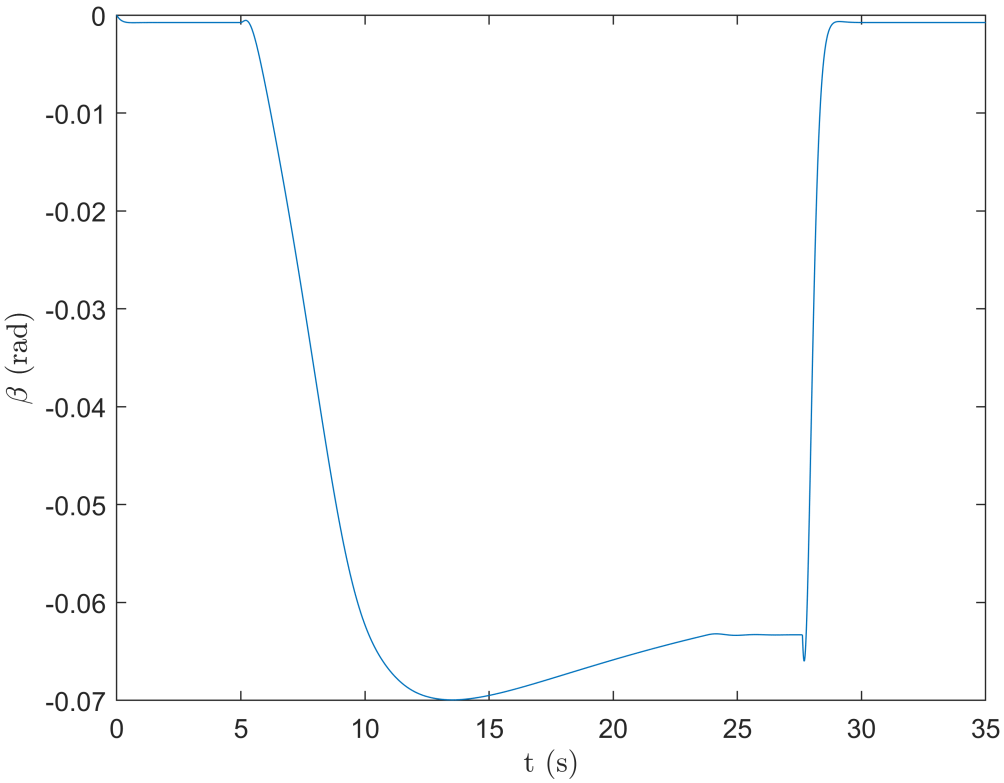

```
plot(Px(1:end-1),Py(1:end-1))
hold on;
quiver(Px(1:1000:end-1000),Py(1:1000:end-1000),Vx(1:1000:end),Vy(1:1000:end))
hold off;
xlabel("X (m)")
ylabel("Y (m)")
xlim([0,350])
ylim([-50,300])
pbaspect([1 1 1])
```




2(b). Fishhook Maneuver with Load Transfer

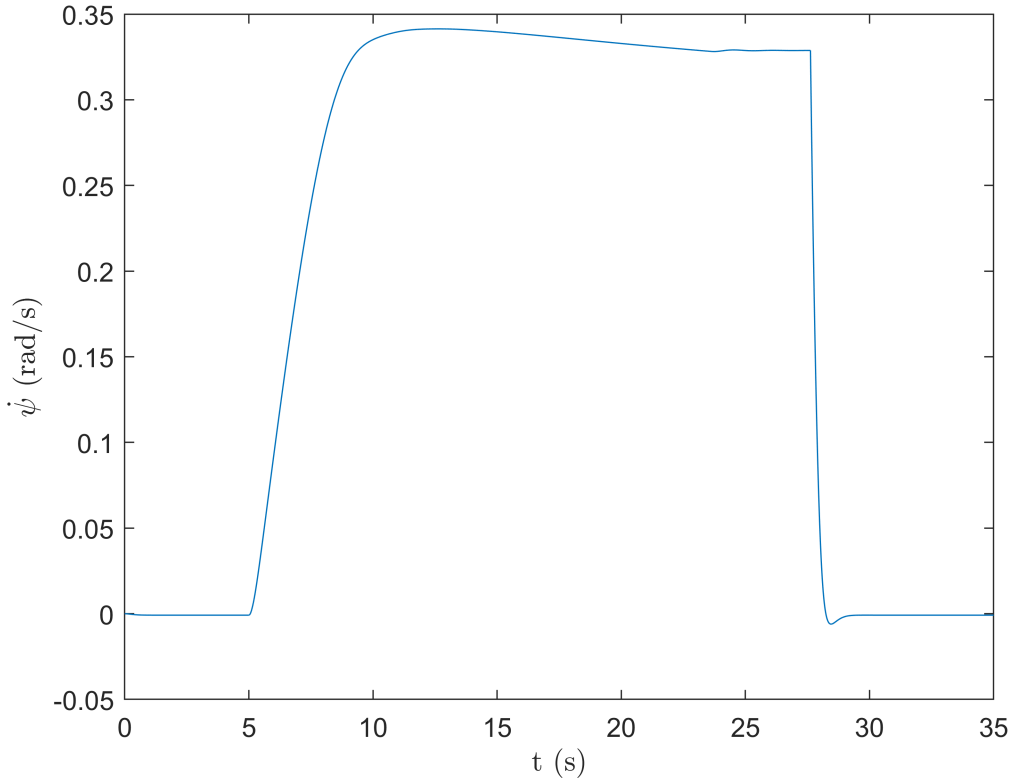
```
% Initialization
dt = 0.001; % s
X = [];
X(:,1) = [0;0];
X_dot = [];
Psi = [];
Psi(1) = 0;
Beta = [];
B_dot = 0;
Vx = [];
Vy = [];
Px = [];
Px(1) = 0;
Py = [];
Py(1) = 0;
% Simulation at each millisecond
for t=1:35000
    a_y = v_fishhook*(B_dot+X(2,t));
    dW_f = 0.6*(m*a_y*h)/w;
    dW_r = 0.4*(m*a_y*h)/w;
    Fz_fl = S_fl - dW_f;
    Fz_fr = S_fr + dW_f;
    Fz_rl = S_rl - dW_r;
    Fz_rr = S_rr + dW_r;
```

```
    alpha_f = u_fishhook(t) - (X(2,t)*a)/v_fishhook - X(1,t);
    alpha_r = (X(2,t)*b)/v_fishhook - X(1,t);
    Fy_fl = -nonlintire(alpha_f, Fz_fl, v_fishhook);
    Fy_fr = -nonlintire(alpha_f, Fz_fr, v_fishhook);
    Fy_rl = -nonlintire(alpha_r, Fz_rl, v_fishhook);
    Fy_rr = -nonlintire(alpha_r, Fz_rr, v_fishhook);
    Fy_f = Fy_fl+Fy_fr;
    Fy_r = Fy_rl+Fy_rr;
    X_dot(:,t) = [(Fy_f+Fy_r)/(m*v_fishhook) - X(2,t);
                  (Fy_f*a-Fy_r*b)/(Iz)];
    B_dot = X_dot(1,t);
    X(:,t+1) = X(:,t) + X_dot(:,t)*dt;
    Beta(t) = X(1,t);
    Psi(t+1) = Psi(t) + X(2,t)*dt;
    Vx(t) = v_fishhook*cos(Psi(t)+Beta(t));
    Vy(t) = v_fishhook*sin(Psi(t)+Beta(t));
    Px(t+1) = Px(t) + Vx(t)*dt;
    Py(t+1) = Py(t) + Vy(t)*dt;
end
% Plot
t=0:35000;
plot(t/1000,X(1,:))
xlabel("t (s)",'interpreter','latex')
ylabel("$\beta$ (rad)",'interpreter','latex')
```

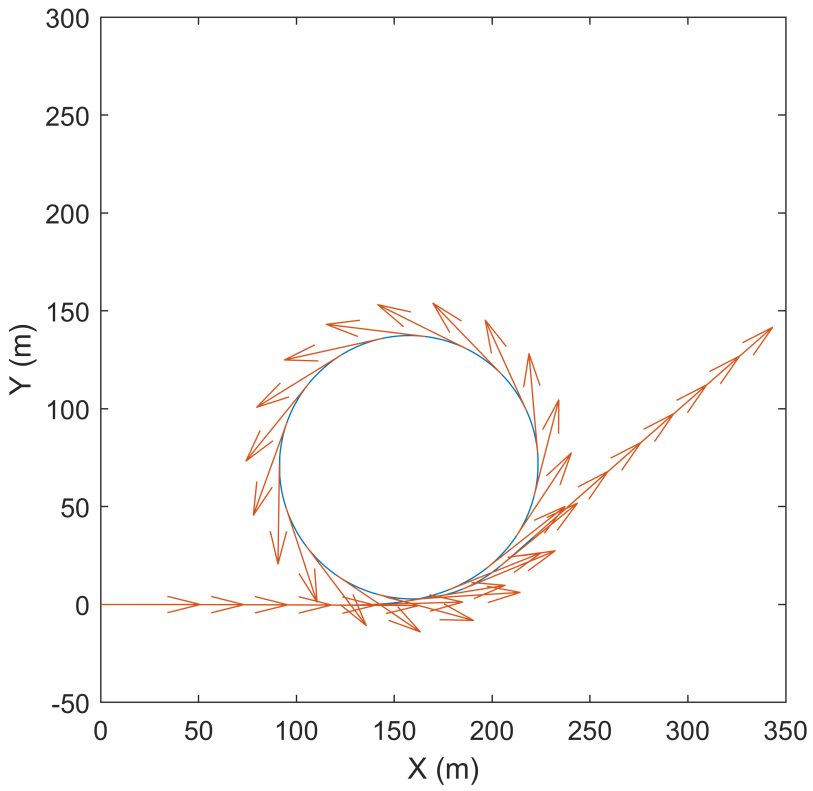



```
t=0:35000;
plot(t/1000,X(2,:))
```

```
xlabel("t (s)",'interpreter','latex')
ylabel("$\dot{\psi}$ (rad/s)",'interpreter','latex')
```




```
plot(Px(1:end-1),Py(1:end-1))
hold on;
quiver(Px(1:1000:end-1000),Py(1:1000:end-1000),Vx(1:1000:end),Vy(1:1000:end))
hold off;
xlabel("X (m)")
ylabel("Y (m)")
xlim([0,350])
ylim([-50,300])
pbaspect([1 1 1])
```



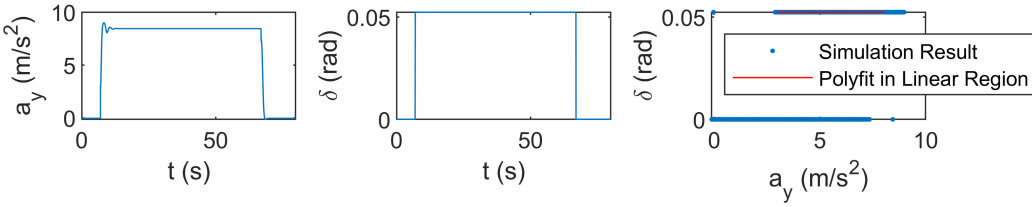

**Part B:**

1. Calculate the Understeer Gradient (UG) of fishhook maneuver and step input without load transfer.

(a) Step Maneuver

```
% Initialization
dt = 0.001; % s
X = [];
X(:,1) = [0;0];
X_dot = [];
Psi = [];
Psi(1) = 0;
Beta = [];
B_dot = 0;
Vx = [];
Vy = [];
Px = [];
Px(1) = 0;
Py = [];
Py(1) = 0;
% Simulation at each millisecond
for t=1:80000
    a_y(t) = v_step*(B_dot+X(2,t));
    dW_f = 0.6*(m*a_y(t)*h)/w;
    dW_r = 0.4*(m*a_y(t)*h)/w;
```

```
    Fz_fl = S_fl;
    Fz_fr = S_fr;
    Fz_rl = S_rl;
    Fz_rr = S_rr;
    alpha_f(t) = u_step(t) - (X(2,t)*a)/v_step - X(1,t);
    alpha_r(t) = (X(2,t)*b)/v_step - X(1,t);
    Fy_fl = -nonlintire(alpha_f(t), Fz_fl, v_step);
    Fy_fr = -nonlintire(alpha_f(t), Fz_fr, v_step);
    Fy_rl = -nonlintire(alpha_r(t), Fz_rl, v_step);
    Fy_rr = -nonlintire(alpha_r(t), Fz_rr, v_step);
    Fy_f = Fy_fl+Fy_fr;
    Fy_r = Fy_rl+Fy_rr;
    X_dot(:,t) = [(Fy_f+Fy_r)/(m*v_step) - X(2,t);
                  (Fy_f*a-Fy_r*b)/(Iz)];
    B_dot = X_dot(1,t);
    X(:,t+1) = X(:,t) + X_dot(:,t)*dt;
    Beta(t) = X(1,t);
    Psi(t+1) = Psi(t) + X(2,t)*dt;
    Vx(t) = v_step*cos(Psi(t)+Beta(t));
    Vy(t) = v_step*sin(Psi(t)+Beta(t));
    Px(t+1) = Px(t) + Vx(t)*dt;
    Py(t+1) = Py(t) + Vy(t)*dt;
end
% Plot
t = 1:80000;
figure()
subplot(1,3,1)
plot(t/1000,a_y)
xlabel("t (s)")
ylabel("a_y (m/s^2)")
pbaspect([2 1 1])
subplot(1,3,2)
plot(t/1000,u_step)
xlabel("t (s)")
ylabel("\delta (rad)")
pbaspect([2 1 1])
subplot(1,3,3)
plot(a_y,u_step,'.')
hold on;
UG_poly = polyfit(a_y(7100:7600), u_step(7100:7600), 1);
plot(a_y(7100:7600),polyval(UG_poly,a_y(7100:7600)),'r');
hold off;
xlabel("a_y (m/s^2)")
ylabel("\delta (rad)")
legend("Simulation Result","Polyfit in Linear Region",'Location','NW')
pbaspect([2 1 1])
```



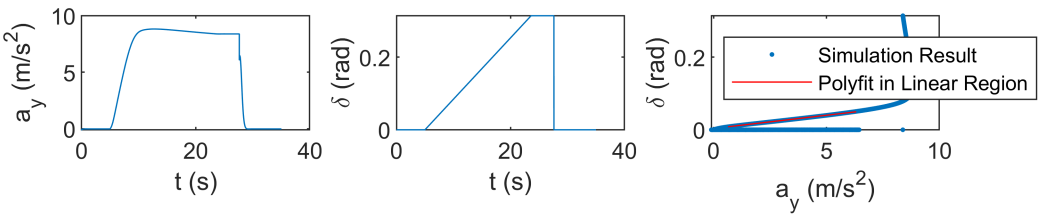

```
UG = (alpha_f(55000)-alpha_r(55000))/a_y(55000);
UG = polyder(UG_poly);
fprintf("UG for step maneuver with non-linear tire model, without load transfer is %.4f rad-s^2
```

```
UG for step maneuver with non-linear tire model, without load transfer is 0.0000 rad-s^2/m
```

(b) Fishhook Maneuver

```
% Initialization
dt = 0.001; % s
X = [];
X(:,1) = [0;0];
X_dot = [];
Psi = [];
Psi(1) = 0;
Beta = [];
B_dot = 0;
Vx = [];
Vy = [];
Px = [];
Px(1) = 0;
Py = [];
Py(1) = 0;
a_y = [];
% Simulation at each millisecond
for t=1:35000
    a_y(t) = v_fishhook*(B_dot+X(2,t));
```

```
    dW_f = 0.6*(m*a_y(t)*h)/w;
    dW_r = 0.4*(m*a_y(t)*h)/w;
    Fz_fl = S_fl;
    Fz_fr = S_fr;
    Fz_rl = S_rl;
    Fz_rr = S_rr;
    alpha_f = u_fishhook(t) - (X(2,t)*a)/v_fishhook - X(1,t);
    alpha_r = (X(2,t)*b)/v_fishhook - X(1,t);
    Fy_fl = -nonlintire(alpha_f, Fz_fl, v_fishhook);
    Fy_fr = -nonlintire(alpha_f, Fz_fr, v_fishhook);
    Fy_rl = -nonlintire(alpha_r, Fz_rl, v_fishhook);
    Fy_rr = -nonlintire(alpha_r, Fz_rr, v_fishhook);
    Fy_f = Fy_fl+Fy_fr;
    Fy_r = Fy_rl+Fy_rr;
    X_dot(:,t) = [(Fy_f+Fy_r)/(m*v_fishhook) - X(2,t);
                  (Fy_f*a-Fy_r*b)/(Iz)];
    B_dot = X_dot(1,t);
    X(:,t+1) = X(:,t) + X_dot(:,t)*dt;
    Beta(t) = X(1,t);
    Psi(t+1) = Psi(t) + X(2,t)*dt;
    Vx(t) = v_fishhook*cos(Psi(t)+Beta(t));
    Vy(t) = v_fishhook*sin(Psi(t)+Beta(t));
    Px(t+1) = Px(t) + Vx(t)*dt;
    Py(t+1) = Py(t) + Vy(t)*dt;
end
% Plot
t = 1:35000;
figure()
subplot(1,3,1)
plot(t/1000,a_y)
xlabel("t (s)")
ylabel("a_y (m/s^2)")
pbaspect([2 1 1])
subplot(1,3,2)
plot(t/1000,u_fishhook)
xlabel("t (s)")
ylabel("\delta (rad)")
pbaspect([2 1 1])
subplot(1,3,3)
plot(a_y,u_fishhook,'.')
hold on;
UG_poly = polyfit(a_y(5500:8000), u_fishhook(5500:8000), 1);
plot(a_y(5500:8000),polyval(UG_poly,a_y(5500:8000)),'r');
hold off;
xlabel("a_y (m/s^2)")
ylabel("\delta (rad)")
legend("Simulation Result","Polyfit in Linear Region",'Location','NW')
pbaspect([2 1 1])
```




```
UG = polyder(UG_poly) - 1/(v_fishhook^2);
fprintf("UG for fishhook maneuver with non-linear tire model, without load transfer is %.4f rad
```

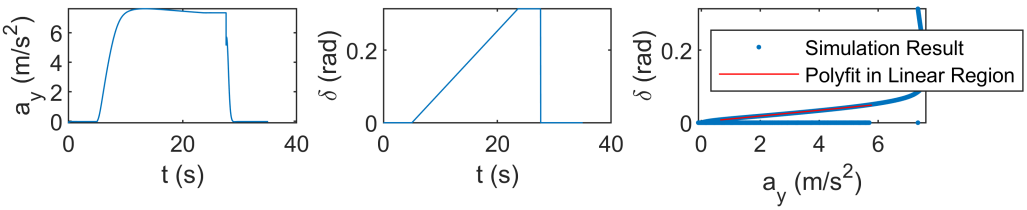
```
UG for fishhook maneuver with non-linear tire model, without load transfer is 0.0018 rad-s^2/m
```

2. From the Fishhook maneuver, determine the sublimit Understeer Gradient (UG) with load transfer. Comment on the limit behavior.

```
% Initialization
dt = 0.001; % s
X = [];
X(:,1) = [0;0];
X_dot = [];
Psi = [];
Psi(1) = 0;
Beta = [];
B_dot = 0;
Vx = [];
Vy = [];
Px = [];
Px(1) = 0;
Py = [];
Py(1) = 0;
a_y = [];
alpha_f = [];
alpha_r = [];
```

```
Fy_f = [];
Fy_r = [];
% Simulation at each millisecond
for t=1:35000
    a_y(t) = v_fishhook*(B_dot+X(2,t));
    dW_f = 0.6*(m*a_y(t)*h)/w;
    dW_r = 0.4*(m*a_y(t)*h)/w;
    Fz_fl = S_fl - dW_f;
    Fz_fr = S_fr + dW_f;
    Fz_rl = S_rl - dW_r;
    Fz_rr = S_rr + dW_r;
    alpha_f(t) = u_fishhook(t) - (X(2,t)*a)/v_fishhook - X(1,t);
    alpha_r(t) = (X(2,t)*b)/v_fishhook - X(1,t);
    Fy_fl = -nonlintire(alpha_f(t), Fz_fl, v_fishhook);
    Fy_fr = -nonlintire(alpha_f(t), Fz_fr, v_fishhook);
    Fy_rl = -nonlintire(alpha_r(t), Fz_rl, v_fishhook);
    Fy_rr = -nonlintire(alpha_r(t), Fz_rr, v_fishhook);
    Fy_f(t) = Fy_fl+Fy_fr;
    Fy_r(t) = Fy_rl+Fy_rr;
    X_dot(:,t) = [(Fy_f(t)+Fy_r(t))/(m*v_fishhook) - X(2,t);
                  (Fy_f(t)*a-Fy_r(t)*b)/(Iz)];
    B_dot = X_dot(1,t);
    X(:,t+1) = X(:,t) + X_dot(:,t)*dt;
    Beta(t) = X(1,t);
    Psi(t+1) = Psi(t) + X(2,t)*dt;
    Vx(t) = v_fishhook*cos(Psi(t)+Beta(t));
    Vy(t) = v_fishhook*sin(Psi(t)+Beta(t));
    Px(t+1) = Px(t) + Vx(t)*dt;
    Py(t+1) = Py(t) + Vy(t)*dt;
end
% Plot
t = 1:35000;
figure()
subplot(1,3,1)
plot(t/1000,a_y)
xlabel("t (s)")
ylabel("a_y (m/s^2)")
pbaspect([2 1 1])
subplot(1,3,2)
plot(t/1000,u_fishhook)
xlabel("t (s)")
ylabel("\delta (rad)")
pbaspect([2 1 1])
subplot(1,3,3)
plot(a_y,u_fishhook,'.')
hold on;
UG_poly = polyfit(a_y(5500:8000), u_fishhook(5500:8000), 1);
plot(a_y(5500:8000),polyval(UG_poly,a_y(5500:8000)),'r');
hold off;
xlabel("a_y (m/s^2)")
```

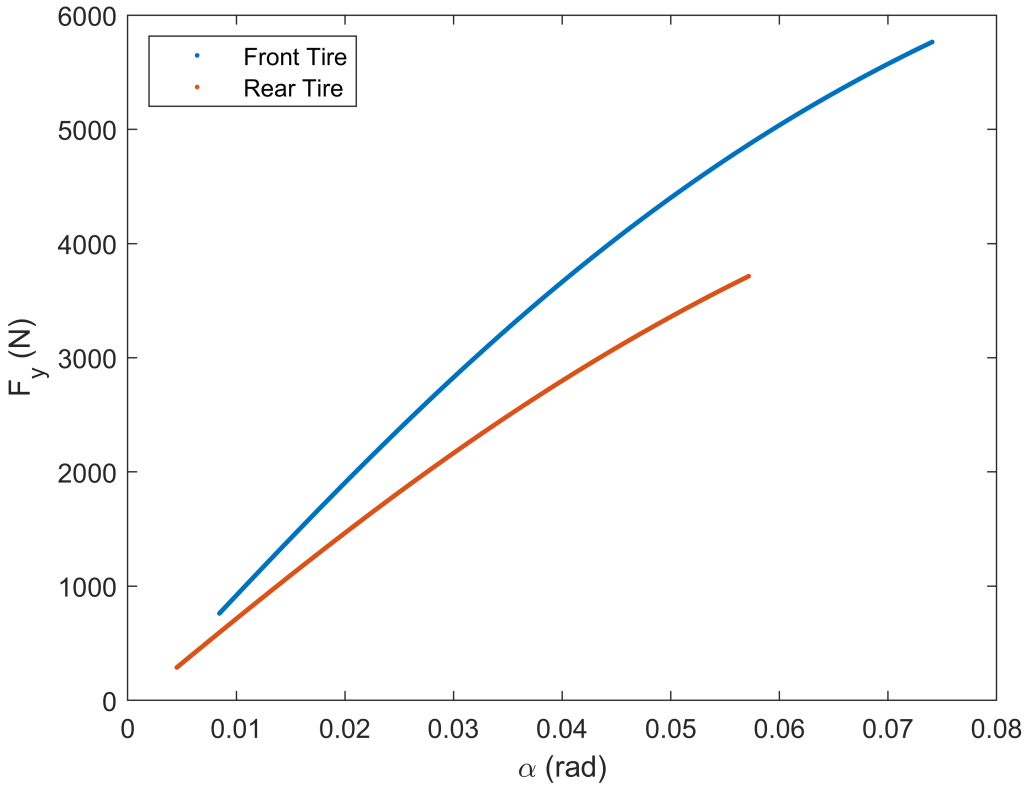
```
ylabel("\delta (rad)")
legend("Simulation Result","Polyfit in Linear Region",'Location','NW')
pbaspect([2 1 1])
```




```
UG = polyder(UG_poly) - 1/(v_fishhook^2);
fprintf("UG for fishhook maneuver with non-linear tire model, with load transfer is %.4f rad-s^
```

```
UG for fishhook maneuver with non-linear tire model, with load transfer is 0.0025 rad-s^2/m
```

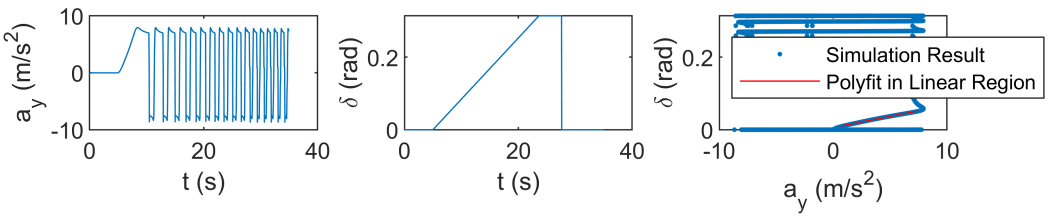
```
figure()
plot(alpha_f(5500:8000),Fy_f(5500:8000),'.')
hold on;
plot(alpha_r(5500:8000),Fy_r(5500:8000),'.')
hold off;
xlabel("\alpha (rad)")
ylabel("F_y (N)")
legend("Front Tire","Rear Tire",'Location','NW')
```




3. Now assume a roll stiffness distribution of 70% rear 30% front and perform the same analysis as in section 2 of Part B. Comment on the results.

```
% Initialization
dt = 0.001; % s
X = [];
X(:,1) = [0;0];
X_dot = [];
Psi = [];
Psi(1) = 0;
Beta = [];
B_dot = 0;
Vx = [];
Vy = [];
Px = [];
Px(1) = 0;
Py = [];
Py(1) = 0;
a_y = [];
alpha_f = [];
alpha_r = [];
Fy_f = [];
Fy_r = [];
% Simulation at each millisecond
for t=1:35000
```

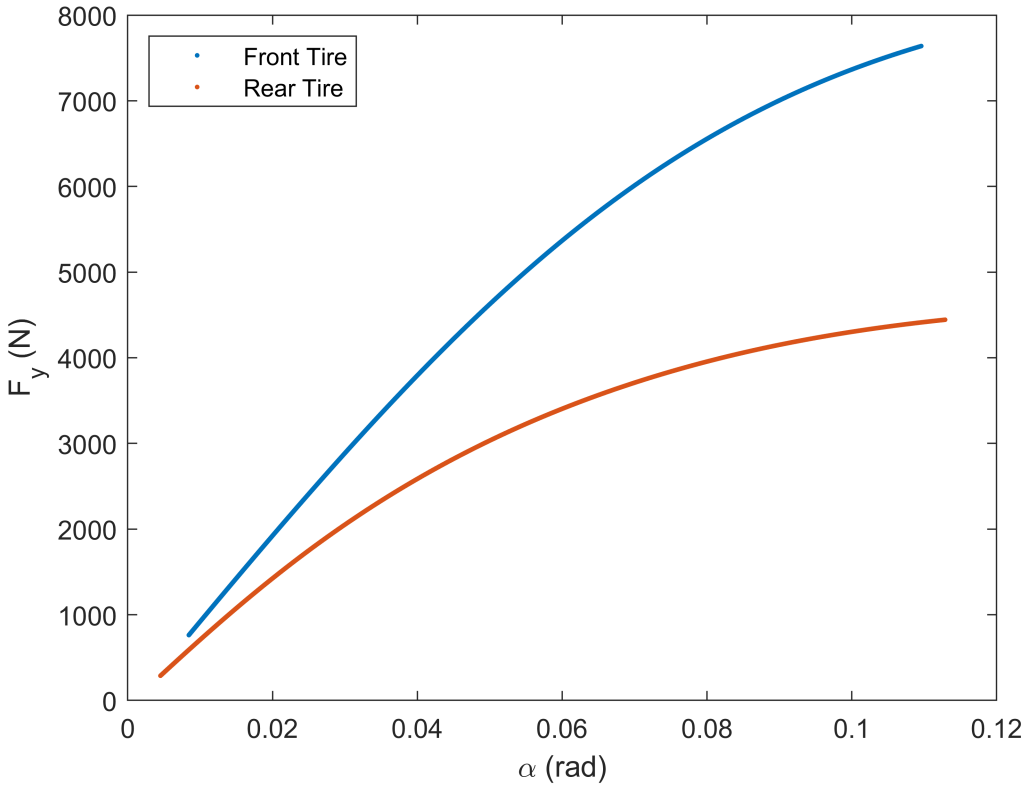
```
    a_y(t) = v_fishhook*(B_dot+X(2,t));
    dW_f = 0.3*(m*a_y(t)*h)/w;
    dW_r = 0.7*(m*a_y(t)*h)/w;
    Fz_fl = S_fl - dW_f;
    Fz_fr = S_fr + dW_f;
    Fz_rl = S_rl - dW_r;
    Fz_rr = S_rr + dW_r;
    alpha_f(t) = u_fishhook(t) - (X(2,t)*a)/v_fishhook - X(1,t);
    alpha_r(t) = (X(2,t)*b)/v_fishhook - X(1,t);
    Fy_fl = -nonlintire(alpha_f(t), Fz_fl, v_fishhook);
    Fy_fr = -nonlintire(alpha_f(t), Fz_fr, v_fishhook);
    Fy_rl = -nonlintire(alpha_r(t), Fz_rl, v_fishhook);
    Fy_rr = -nonlintire(alpha_r(t), Fz_rr, v_fishhook);
    Fy_f(t) = Fy_fl+Fy_fr;
    Fy_r(t) = Fy_rl+Fy_rr;
    X_dot(:,t) = [(Fy_f(t)+Fy_r(t))/(m*v_fishhook) - X(2,t);
                  (Fy_f(t)*a-Fy_r(t)*b)/(Iz)];
    B_dot = X_dot(1,t);
    X(:,t+1) = X(:,t) + X_dot(:,t)*dt;
    Beta(t) = X(1,t);
    Psi(t+1) = Psi(t) + X(2,t)*dt;
    Vx(t) = v_fishhook*cos(Psi(t)+Beta(t));
    Vy(t) = v_fishhook*sin(Psi(t)+Beta(t));
    Px(t+1) = Px(t) + Vx(t)*dt;
    Py(t+1) = Py(t) + Vy(t)*dt;
end
% Plot
t = 1:35000;
figure()
subplot(1,3,1)
plot(t/1000,a_y)
xlabel("t (s)")
ylabel("a_y (m/s^2)")
pbaspect([2 1 1])
subplot(1,3,2)
plot(t/1000,u_fishhook)
xlabel("t (s)")
ylabel("\delta (rad)")
pbaspect([2 1 1])
subplot(1,3,3)
plot(a_y,u_fishhook,'.')
hold on;
UG_poly = polyfit(a_y(5500:8000), u_fishhook(5500:8000), 1);
plot(a_y(5500:8000),polyval(UG_poly,a_y(5500:8000)),'r');
hold off;
xlabel("a_y (m/s^2)")
ylabel("\delta (rad)")
legend("Simulation Result","Polyfit in Linear Region",'Location','NW')
pbaspect([2 1 1])
```




```
UG = polyder(UG_poly) - l/(v_fishhook^2);
fprintf("UG for fishhook maneuver with non-linear tire model, with load transfer is %.4f rad-s^
```

```
UG for fishhook maneuver with non-linear tire model, with load transfer is 0.0007 rad-s^2/m
```

```
figure()
plot(alpha_f(5500:8000),Fy_f(5500:8000),'.')
hold on;
plot(alpha_r(5500:8000),Fy_r(5500:8000),'.')
hold off;
xlabel("\alpha (rad)")
ylabel("F_y (N)")
legend("Front Tire","Rear Tire",'Location','NW')
```


Front Tire
Rear Tire
8000
7000
6000
5000
4000
3000
2000
1000
0
$F_y$ (N)
0
0.02
0.04
0.06
0.08
0.1
0.12
$\alpha$ (rad)