Sol^n 1

1.1 Power equation :

$$F_A \cdot V = P_W - V \cdot F_D$$

F_A — acc. force; V — vel.; P_W — power at wheels; V — vel.; F_D — aero drag force

@ terminal vel. $\Rightarrow acc = 0 \Rightarrow F_A = 0 \Rightarrow F_A \cdot V = 0$

$$\Rightarrow 0 = P_W - v \cdot F_D$$

$$\Rightarrow v F_D = P_W$$

$$\Rightarrow F_D = \frac{P_W}{V}$$

given $\begin{cases} P_W = 230 \text{ hp} = 230 \times 745.6998 = 171510.954 \text{ watts} \\ v = 290 \text{ km/hr} = \frac{290}{3.6} = 80.5556 \text{ m/s} \end{cases}$

$$\Rightarrow F_D = \frac{171510.954}{80.5556} = 2129.1015 \text{ N}$$

Ans: $\boxed{F_D = 2129.1015 \text{ N}}$

1.2 Power effeciency :

$$P_W = \eta_d P_e$$

P_W — Power at wheels; η_d — Driveline efficiency; P_e — Power of engine

$\rightarrow \eta_d = 100 - 12.5\ \%$

Power equation :

$$F_A \cdot V = P_W - V \cdot F_D$$

@ terminal vel $\Rightarrow acc = 0 \Rightarrow F_A = 0 \Rightarrow F_A \cdot V = 0$

$$\Rightarrow P_W = V \cdot F_D$$

Aero drag:

$$F_D = \frac{1}{2} \rho v^2 C_D A_f$$

Aero drag force, Air density, Apparent vel., Drag coeff., Frontal area

$\Rightarrow P_w = v F_D = v \cdot \left(\frac{1}{2} \rho v^2 C_D A_f\right) = \frac{1}{2} \rho v^3 C_D A_f$

$\Rightarrow C_D A_f = \dfrac{2 P_w}{\rho v^3}$

(i) given
$$\begin{cases} P_e = 310 \text{ hp} = 310 \times 745.6998 = 231166.938 \text{ watts} \\ \eta_d = \dfrac{100-12.5}{100} = 0.875 \\ v = 290 \text{ km/hr} = \dfrac{290}{3.6} = 80.5556 \text{ m/s} \\ \rho = 1.225 \text{ kg/m}^3 \text{ (from lecture notes)} \end{cases}$$

$\Rightarrow P_w = 0.875 \times 231166.938 = 202271.0708 \text{ N}$

$\Rightarrow C_D A_f = \dfrac{2 \times 202271.0708}{1.225 \times (80.5556)^3} = 0.6317 \text{ m}^2$

Ans: $\boxed{C_D A_f = 0.6317 \text{ m}^2}$

(ii) given
$$\begin{cases} w = 1750 \text{ mm} = 1.75 \text{ m} \\ h = 1210 \text{ mm} = 1.21 \text{ m} \\ A_f \approx (79-84)\% \text{ of } w \times h \quad \text{(from lecture notes)} \end{cases}$$

$\Rightarrow A_f = \left(\dfrac{79+84}{2}\right)\% \text{ of } w \times h = 81.5\% \text{ of } w \times h$

$= \dfrac{81.5}{100} \times 1.75 \times 1.21 = 1.7258 \text{ m}^2$

$\Rightarrow C_D = \dfrac{C_D \cdot A_f}{A_f} = \dfrac{0.6317}{1.7258} = 0.366$

Ans: $\boxed{C_D = 0.366}$

1.3 Power eq^n @ terminal vel (from sol^n 1.2 and 1.1)

$$P_W = vF_D = v\cdot\left(\tfrac{1}{2}\rho v^2 C_D A_f\right) = \tfrac{1}{2}\rho v^3 C_D A_f$$

$$\therefore \Rightarrow C_D = \frac{2P_W}{\rho v^3 A_f}$$

given
$$\begin{cases} P_W = 1160\ hp = 1160 \times 745.6998 = \cancel{86535768}\ 865011.768 \text{ watts} \\ \rho = 1.225\ kg/m^3 \text{ (lecture notes)} \\ v = 278\ mph = 278 \times 0.447 = 124.266\ m/s \\ A_f = 20.16\ ft^2 = 20.16 \times 0.0929 = 1.8729\ m^2 \end{cases}$$

$$\Rightarrow C_D = \frac{2 \times \cancel{86535768}\ 865011.768}{1.225 \times (124.266)^3 \times 1.8729} = 0.393$$

Ans: $\boxed{C_D = 0.393}$

1.4 Power eq^n @ terminal vel (from sol^n 1.1, 1.2, 1.3)

$$P_W - vF_D = 0$$

$\Rightarrow$ For achieving the same speed (go as fast) "v" = $V_{terminal_k} = v_k$ (let)

$$\underbrace{P_{W_M} - vF_{D_M}}_{\text{M = Mercedes Uhlenhaut Coupe}} = \underbrace{P_{W_K} - vF_{D_K}}_{\text{K = Koenigsegg Agera RS}} = 0$$

$$\Rightarrow P_{W_M} = P_{W_K} - v_K F_{D_K} + v_K F_{D_M}$$

Required power

given $\begin{cases} P_{w_k} = 1160\ hp = 1160 \times 745.6998 = \overset{865011.768}{\cancel{865357.68}}\ \text{watts} \\ v_k = 278\ mph = 278 \times 0.447 = 124.266\ m/s \end{cases}$

Aero drag

$$F_D = \frac{1}{2}\, \rho\, v^2\, C_D\, A_f$$

$$\Rightarrow F_{D_K} = \frac{1}{2} \times 1.225 \times (124.266)^2 \times 0.393 \times 1.8729$$

from sol^n 1.3

$$\Rightarrow F_{D_K} = 6961.7411\ N$$

similarly, $F_{D_M} = \frac{1}{2} \times 1.225 \times (124.266)^2 \times 0.366 \times 1.7258$

from sol^n 1.2 (ii)

$$\Rightarrow F_{D_M} = 5974.2347\ N$$

$$\Rightarrow P_{w_M} = \overset{865011.768}{\cancel{865357.68}} - (124.66 \times 6961.7411) + (124.66 \times 5974.2347)$$

$$= \overset{865011.768 - 867850.645 + 744748.0977}{\cancel{865357.68 - 867850.6455 + 744748.0977}}$$

$$= \cancel{742255.1322\ \text{watts}}\ 741909.2207\ \text{watts}$$

$$\Rightarrow P_{w_M} = \cancel{\frac{742255.1322}{745.998}} = \frac{741909.2207}{745.6998} = 994.92\ hp$$

<u>Ans:</u> $\boxed{P_{w_M} = 994.92\ hp}$

Sol^n 2

2.C $a_y = \frac{V_{corner}^2}{R}$

$\Rightarrow V_{corner} = \left|\sqrt{a_y \cdot R}\right|$

given $\begin{cases} a_y = 1.2\,g = 1.2 \times 9.81 = 11.772\ m/s^2 \\ 2\pi R = 0.35 \text{ miles} = 0.35 \times 1609.344 = 563.2704\ m \end{cases}$

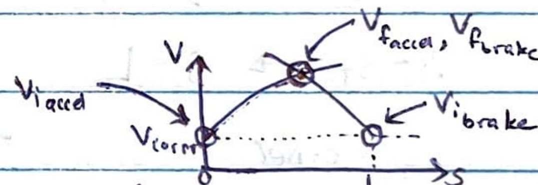
$\Rightarrow R = \frac{563.2704}{3.14} = 179.2945\ m$

$\Rightarrow V_{corner} = \left|\sqrt{11.772 \times \cancel{563.2704}\ 179.2945}\right|$

$= |\pm 45.94|$

$= \cancel{81.4298\ m/s}\ 45.94$

Ans: $\boxed{V_{corner} = 45.94\ m/s}$



2.E From lecture notes,

$$S_T = \frac{(V_{i_{brake}})^2 - (V_{i_{accel}})^2 + 2\,|a_{brake}| \cdot L}{2\,(a_{accel} + |a_{brake}|)}$$

given $\begin{cases} L = 0.4 \text{ miles} = 0.4 \times 1609.344 = 643.7376\ m \\ V_{i_{brake}} = V_{corner} = 45.94\ m/s \\ V_{i_{accel}} = V_{corner} = 45.94\ m/s \\ |a_{brake}| = |-1.2g| = 1.2\,g = 1.2 \times 9.81 = 11.772\ m/s^2 \\ a_{accel} = 0.5g = 0.5 \times 9.81 = 4.905\ m/s^2 \end{cases}$

$$\Rightarrow S_T = \frac{\cancel{(45.94)^2} - \cancel{(45.94)^2} + (2 \times 11.772 \times 643.7376)}{2 \times (4.905 + 11.772)}$$

$= 454.403\ m$

Ans: $\boxed{S_T = 454.403\ m}$

2.D Max. speed achieved on straight is at point S_T

$$V_{max}^2 = V_{corner}^2 + 2 \cdot a_{accel} \cdot S_T \quad \ldots \quad (v^2 = u^2 + 2as)$$

$$\Rightarrow V_{max} = \left| \sqrt{V_{corner}^2 + 2 \cdot a_{accel} \cdot S_T} \right|$$

$$= \left| \sqrt{(45.94)^2 + (2 \times 4.905 \times 454.403)} \right|$$

$$= |\pm 81.044|$$

$$= 81.044 \text{ m/s}$$

Ans: $\boxed{V_{max} = 81.044 \text{ m/s}}$

2.B For const. acc, $V_{avg} = \frac{V_{final} + V_{initial}}{2}$

3 parts: (2 for straight + 1 for corner):

$S = 0 \rightarrow S = S_T \quad \Rightarrow V_1 = \frac{V_{max} + V_{corner}}{2} = \frac{81.044 + 45.94}{2} = 63.492 \text{ m/s}$

$S = S_T \rightarrow S = L \quad \Rightarrow V_2 = \frac{V_{corner} + V_{max}}{2} = \frac{45.94 + 81.044}{2} = 63.492 \text{ m/s}$

Corner $\Rightarrow V_3 = V_{corner}$ (const vel.) $= 45.94 \text{ m/s}$

3 more (same) parts for remainder track (which we can ignore since race circuit is symmetrical).

$$\Rightarrow V_{avg} = \frac{V_1 + V_2 + V_3}{3}$$

$$= \frac{63.492 + 63.492 + 45.94}{3}$$

$$= 57.641 \text{ m/s}$$

Ans: $\boxed{V_{avg} = 57.641 \text{ m/s}}$

2.A Since we solved this problem for maximum average velocity (minimum time) in order to "win" the race:

$$t_{min} = \frac{dist}{V_{avg}}$$

given dist = 1.5 miles = 1.5×1609.344 = 2414.016 m

$\Rightarrow t_{min} = \dfrac{2414.016}{57.641} = 41.8802$ sec

Ans: $\boxed{t_{min} = 41.8802 \text{ sec.}}$

Sol^n 3

3.C $a_y = \dfrac{V_{corner}^2}{R}$

$\Rightarrow V_{corner} = \left| \sqrt{a_y \cdot R} \right|$

Solving programatically in MATLAB, given $a_y = 1.2g$ and πR = 0.35 miles, we get V_{corner} = 45.9419 m/s

Ans: $\boxed{V_{corner} = 45.9419 \text{ m/s}}$

3.E Acceleration on straight is variable

@ begining of straight:

$t(0) = 0$

$v(0) = V_{corner}$

$s(0) = 0$

choose $\Delta t = 0.001$, let $t_i = i \cdot \Delta t$ with $i = 0, 1, 2, \ldots$

Power eq^n : $F_A \cdot V = P_W - V \cdot F_D \Rightarrow P_W = F_A \cdot V + F_D \cdot V$

$$\Rightarrow P_W = m\, a_{accel}(t_i) \cdot V(t_i) + \left[\tfrac{1}{2} \rho\, V^2(t_i)\, C_D A_F\right] V(t_i)$$

$$\Rightarrow a_{accel}(t_i) = \frac{P_W - \left[\tfrac{1}{2} \rho\, V^3(t_i)\, C_D A_F\right]}{m \cdot V(t_i)}$$

$a_{accel}(v)_{(t_i)}$ ← accel. as function of vel

Euler forward: $\quad \leftarrow$ solved in MATLAB

$$\begin{bmatrix} v \\ s \end{bmatrix}_{(t_{i+1})} = \begin{bmatrix} v \\ s \end{bmatrix}_{(t_i)} + \Delta t \cdot \begin{bmatrix} a_{accel}(v) \\ v \end{bmatrix}_{(t_i)}$$

calculate over $s = [0, L]$ i.e. length of straight.

Deceleration (braking) over straight is constant

$$\Rightarrow \left(v_{i_{brake}}\right)^2_{(t_i)} = \left(v_{corner}\right)^2 + 2\,|a_{brake}|\, s_{(t_i)} \quad \ldots \left(v^2 = u^2 + 2as\right)$$

The iteration where velocities from above two equations become equal (very close) is the transition point and the distance corresponding to this is the distance from $s = 0$ to $s = s_T$. Thus, iterations can be stopped.

Ans: $\boxed{s_T = 643.7667\ m}$

3.D Max. speed achieved on straight is at $s = s_T$. Since we stop iterations of Euler forward algorithm in Sol^n 3.E upon reaching $s = s_T$, the last velocity value in the array is max. velocity (solved in MATLAB)

Ans: $\boxed{v_{max} = 72.1287\ m/s}$

3.B Similar to Sol^n 2.B, we have 3 parts:

$s = 0 \rightarrow s = s_T$ (non-uniform accel) $\Rightarrow v_1 = \dfrac{\sum_{i=0}^{n} v_{(t_i)}}{n} \quad (t_i = i \cdot \Delta t)$

$s = s_T \rightarrow s = L$ $\Big\{$ (uniform accel) $\Rightarrow v_2 = \dfrac{v_{corner} + v_{max}}{2}$

Corner (accel $= 0$) $\Rightarrow v_3 = v_{corner}$

v_1 can be ~~sol~~ found by taking arithmatic mean of all elements of velocity array as a result of Sol^n 3.F Euler forward algorithm. V_2 and V_3 can be simply calculated by substituting V_{corner} and V_{max} found in Sol^n 3.C and Sol^n 3.D (solved in MATLAB)

Ans: $\boxed{V_{avg} = 55.4903 \text{ m/s}}$

3.A Since we solved for minimum time to "win" the race

$$t_{min} = \frac{dist}{V_{avg}} \quad \begin{array}{l} \leftarrow \text{given dist} = 1.5 \text{ miles} \\ \leftarrow \text{Sol}^n \text{ 3.B} \end{array}$$

Ans: $\boxed{t_{min} = 43.5034 \text{ sec.}}$

Sol^n 4.

4.1 Solve $V_t = \frac{V_0}{\beta} \tan\left(1 - \frac{t}{T} \tan^{-1}(\beta)\right) + \underbrace{V_{wind}}_{\text{headwind}}$ for $\beta = [1, 1.4, 1.8, 2.2, 2.6, 3.0]$

from $t=0$ to $t = t_{final}$ and compute $\frac{V_t}{V_0}$ and $\frac{t}{T}$ for each value, where $V_0 = 27.8049$ m/s and $T = 122.31$ sec.

Ans: $\boxed{\text{Solved in MATLAB}}$

4.2 Since $V_0 = 27.8049$ m/s and $T = 122.31$ sec (from given data), For each value of velocity 'v' & corresponding timestamp 't' we can find (and plot) $\frac{V_t}{V_0}$ and $\frac{t}{T}$ (solved in MATLAB. Visually observing (or using least squares) we can determine : β that corresponds to real word data.

Ans: $\boxed{\beta = 1.8}$

4.3

(a) Drag coeff:

$$C_D = \frac{2 \cdot m \cdot \beta \cdot \tan^{-1}(\beta)}{(V_0 + V_{wind}) \cdot \rho \cdot A_f}$$

given $m = 78.8$ slugs, $V_0 = 27.8049$ m/s, $V_{wind} = 4.8$ mph,
$\rho = 1.225$ kg/m^3, $A_f = 22.17$ ft^2, $\beta = 1.8$ (sol^n 4.2)

we can compute C_D (solved in MATLAB)

Ans: $\boxed{C_D = 0.4446}$

(b) Rolling resistance

$$R_x = \frac{(V_0 + V_{wind}) m \cdot \tan^{-1}(\beta)}{\beta \cdot T}$$

given $m = 78.8$ slugs, $V_0 = 27.8049$ m/s, $V_{wind} = 4.8$ mph,
$\beta = 1.8$ (sol^n 4.2), $T = 122.31$ sec

we can compute R_x (solved in MATLAB)

Ans: $\boxed{R_x = 178.337 \text{ N}}$