* Problem 1

1.1. We know the following:

Aerodynamic drag force, $F_D = \frac{1}{2} \rho v^2 C_D A_f$ — (1.1.1)

Power eq^n, $F_A \cdot v = P_w - vF_D$ — (1.1.2)

P_{engine} @ wheels.

At terminal vel., acc. $= 0 \Rightarrow F_A = 0$

$\Rightarrow P_w - vF_D = 0 \Rightarrow \boxed{F_D = \frac{P_w}{v}}$ — (1.1.3)

1.2. $P_w = \eta_d \cdot P_e$ — (1.2.1)

Power eq^n, $F_A \cdot v = P_w - vF_D$ — (1.2.2)

At terminal vel., acc $= 0 \Rightarrow F_A = 0$

$\Rightarrow P_w = vF_D = v\left[\frac{1}{2} \rho v^2 C_D A_f\right]$

$$\boxed{C_D A_f = \frac{2 P_w}{\rho v^3}}$$ — (1.2.3)

Acc. to (Wang, 2001), $A_f \approx$ 79% to 84% of (width x height)

Taking average value, $A_f = \left(\frac{79+84}{2}\right)$% of (width x height)

$A_f = \frac{81.5}{100} \times$ width x height — (1.2.4)

$\therefore C_D = \frac{\text{value of (1.2.3)}}{A_f}$

1.3. Power eq^n, $F_A \cdot v = P_w - vF_D$ — (1.3.1)

At terminal vel., $acc = 0 \Rightarrow F_A = 0$

$\Rightarrow P_w - vF_D = 0$

$\Rightarrow P_w - v[1/2 \rho v^2 C_D A_f] = 0$

$\Rightarrow \boxed{C_D = \frac{2P_w}{\rho v^3 A_f}}$ — (1.3.2)

1.4. Power eq^n, $F_A \cdot v = P_w - vF_D$ — (1.4.1)

At terminal vel., $acc = 0 \Rightarrow F_A = 0$

$\Rightarrow P_w - vF_D = 0$ — (1.4.2)

For achieving same speed "v"

$P_{w_{M}} - vF_{D_M} = P_{w_{K}} - vF_{D_K}$

M: Mercedes K: Koenigsegg

$\therefore \boxed{P_{w_M} = P_{w_K} - vF_{D_K} + vF_{D_M}}$ — (1.4.3)

* Problem 2

2.C. $a_y = \frac{V_{corner}^2}{R} \Rightarrow \boxed{V_{corner} = \left|\sqrt{R \cdot a_y}\right|}$ — (2.C.1)

2.E. $\boxed{S_T = \frac{(V_{i_{brake}})^2 - (V_{i_{acc}})^2 + 2|a_{brake}| \cdot L}{2(a_{acc} + |a_{brake}|)}}$ — (2.E.1)

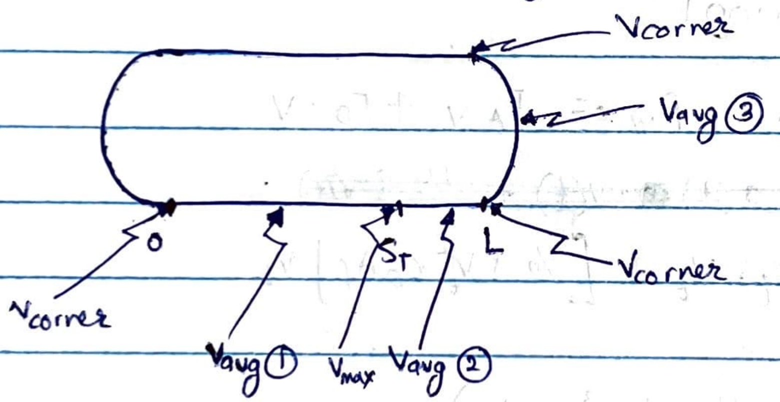
2.D. max. vel. is at transition point S_T.

$$v_{max}^2 = v_{corner}^2 + 2 \cdot a_{acc} \cdot S_T$$

$$\therefore \boxed{V_{max} = \left| \sqrt{V_{corner}^2 + 2 \cdot a_{acc} \cdot S_T} \right|} \quad -(2.D.1)$$

2.B. For const. acc., $V_{avg} = \dfrac{V_f + V_i}{2}$ $\quad -(2.B.1)$

The track has 3 segments:



$V_{avg}① = \dfrac{V_{max} + V_{corner}}{2}$

$V_{avg}② = \dfrac{V_{corner} + V_{max}}{2}$

$V_{avg}③ = V_{corner}$

Note: since the track is symmetrical, avg. vel. in other half is same

$$V_{avg} = \frac{V_{avg}① + V_{avg}② + V_{avg}③}{3} = \frac{2\left(\frac{V_{max} + V_{corner}}{2}\right) + V_{corner}}{3}$$

$$\therefore \boxed{V_{avg} = \frac{V_{max} + 2V_{corner}}{3}} \quad -(2.B.2)$$

2.A. Since we solved this problem for optimal breaking performance, it is evident that this is the highest avg. vel. possible

$$\Rightarrow \boxed{t_{min} = d / V_{avg}} \quad -(2.A.1)$$

* Problem 3

3.C. $a_y = \frac{V_{corner}^2}{R} \rightarrow \boxed{V_{corner} = \left|\sqrt{R \cdot a_y}\right|}$ — (3.C.1)

3.E. ~~Let Δt = 0.001~~ Forward Acceleration is variable:

Start at the begining of straight,

$$\begin{cases} t(0) = 0 \\ V(0) = V_{corner} \\ S(0) = 0 \\ \Delta t = 0.001 \end{cases}$$

Power eq^n. $P_w = F_A V + F_D \cdot V$

$\cancel{\Rightarrow P_w = m \cdot a(t) \cdot v(t) + \frac{1}{2} \rho v^2(t)}$

$\Rightarrow P_w = m \cdot a_t \cdot V_t + \left[\frac{1}{2} \rho V_t^2 C_D A_f\right] V_t$

$$\Rightarrow a_t = \frac{P_w - \left(\frac{1}{2} \rho V_t^3 C_D A_f\right)}{m \cdot V_t} \quad \text{— (3.E.1)}$$

Euler forward:

$$\begin{bmatrix} V \\ S \end{bmatrix}_{t+1} = \begin{bmatrix} V \\ S \end{bmatrix}_t + \Delta t \cdot \begin{bmatrix} a_t \\ V_t \end{bmatrix} \quad \text{— (3.E.2)}$$

solve over $S = [0; L]$

Meanwhile, braking has constant acceleration:

$$(V_{brake})_t^2 = (V_{corner})^2 + 2\,|a_{brake}|\, S_t \quad \text{— (3.E.3)}$$

After each iteration, check if the forward acc. velocity $(V_{i_{acc}})_t$ is equal to the braking velocity ~~of the~~ $(V_{i_{brake}})_t$ corresponding to distance S_t as given by eq^n (3·E·2) and (3·E·3) respectively.

The point where these velocities match is the transition point, and hence the Iterations can be stopped.

The corresponding point in S_t from eq^n (3·E·2) gives the distance "S_T" of transition point from start of the straight.

3·D· The max. vel. can be retrieved based on the index of S_T in the S_t array. The velocity in V_t array corresponding to that index is V_{max}.

3·B· As in case of 2·B·, we have 3 segments,

(i) V_{avg} ① can be computed by taking the mean of V_t array ; $V_{avg}① = \frac{\sum_{t=0}^{n} V_t}{n}$

(ii) $V_{avg}② = \frac{V_{corner} + V_{max}}{2}$

(iii) $V_{avg}③ = V_{corner}$

$$\therefore \boxed{V_{avg} = \frac{\text{mean}(V_t) + \frac{V_{corner} + V_{max}}{2} + V_{corner}}{3}} \quad \text{—(3·B·1)}$$

3·A· As in case of 2·A·,

$$\boxed{t_{min} = d / V_{avg}} \quad \text{—(3·A·1)}$$

* Problem 4

4·1· Solved programatically [add V_{wind} to vehicle vel. to account for the headwind]

4·2· Solved manually by looking at the plot.

4·3·

(a) Drag coeff.

$$\boxed{C_D = \frac{2 \cdot m \cdot \beta \cdot \tan^{-1}(\beta)}{V_0 \cdot T \cdot \rho \cdot A_f}} \quad \text{—(4·3·1)}$$

substitute $V_0 = (V_0 + V_{wind})$ to account for headwind.

(b) Rolling resistance,

$$\boxed{R_x = \frac{V_0 \cdot m \cdot \tan^{-1}(\beta)}{\beta T}} \quad \text{—(4·3·2)}$$

substitute $V_0 = (V_0 + V_{wind})$ to account for headwind