HW5 Solution key

1) $l = 3.1\text{m}$ $\quad a_y = 5\,\text{m/s}^2$ $\quad S.R = 19 \Rightarrow \dfrac{\delta_{\text{steeringwheel}}}{\delta_{\text{tire}}} = 19$

$V_{\text{characteristic}} = \sqrt{l/UG}$ for U.S car

$V = [80\ 120]$

For V_{char} to lie in velocity range, what is UG?

$$80 < V_{\text{char}} < 120$$

$$80 < \sqrt{l/UG} < 120$$

$$\frac{80}{3.6} < \sqrt{l/UG} < \frac{120}{3.6}$$

$$\frac{\left(\frac{80}{3.6}\right)^2}{l} < \frac{1}{UG} < \frac{\left(\frac{120}{3.6}\right)^2}{l}$$

$$\frac{l}{\left(\frac{120}{3.6}\right)^2} < UG < \frac{l}{\left(\frac{80}{3.6}\right)^2}$$

$$\boxed{\begin{array}{l} 2.79 \times 10^{-3} < UG < 6.2775 \times 10^{-3} \text{ rad} \\ 0.159 \text{ deg} < UG \quad 0.36 \text{ deg} \end{array}}$$

Steering sensitivity is given by yaw velocity response to steering input

$$\frac{\dot{\psi}}{\delta} = \frac{\frac{V_x}{R}}{\delta_{ACK} + UG \cdot a_y} = \frac{V}{l + UG \cdot V^2}$$

Steering sensitivity is maximized ~~at~~ at a velocity

when $\frac{d\dot{\psi}}{d\delta} = 0$ it happens at $V = V_{char}$

$$\left.\frac{\dot{\psi}}{\delta}\right|_{V=V_{char}} = \frac{V_{char}}{2l}$$

At $V = 120$ kmph

$$\left.\frac{\dot{\psi}}{\delta}\right|_{120} = \frac{120}{3.6 \times 2 \times 3.1} = 5.38 \text{ sec}^{-1}$$

or Substitute values of 80, 120 in $\frac{\dot{\psi}}{\delta}$ and find maximum value

$$\frac{\dot{\psi}}{\delta} = \frac{V}{l + UG \cdot V^2} = 5.38 s^{-1} \text{ at}$$ $V = 120$ kmph

$$\frac{\dot{\psi}}{\delta} = 3.59 s^{-1} \text{ at}$$ $V = 80$ kmph

$\delta_{sw} = (19)(\delta_{tire})$

$= 19\left(\delta_{ackermann} + UG \cdot a_y\right)$

$a_y = \frac{v^2}{R}$ Given $a_y = 5\, m/s^2$

As v varies, R varies to maintain lateral acceleration

We can make plot across range of V_{char}

$$R = \frac{V_{char}^2}{a_y}$$

$$\delta_{sw} = 19\left(\delta_{ack} + UG \cdot \frac{V_{char}^2}{R}\right)$$

δ_{sw} vs V_{char} can be plotted

or $V = 80$ kmph

$$a_y = \frac{v^2}{R} = \frac{\left(\frac{80}{3.6}\right)^2}{R} = 5$$

$$R = 98.77\ m$$

$$\delta_{tire} = \frac{l}{R} + UG \cdot a_y = \frac{3.1}{98.77} + 6.2775 \times 10^{-3} \times 5 =$$

0.0627 rad | 3.59°

$$\boxed{\delta_{SW} = 68.33^\circ \mid 1.19 \text{ rad}}$$

Similarly at $V = 120$ kmph

$$a_y = \frac{v^2}{R} \Rightarrow R = 222.2 \text{ m}$$

$$\delta_{tire} = \frac{3.1}{222.2} + 2.79 \times 10^{-3} \times 5$$

$$= 0.0279 \text{ rad} \mid 1.59^\circ$$

$$\boxed{\delta_{SW} = 30.3^\circ \mid 0.53 \text{ rad}}$$

2) mass = 120 slugs = $120 \times 14.593 = 1751.2$ kg

$$C_f = C_r = 350 \times 4.4822 \times \frac{180}{\pi} = 89202.48 \text{ N/rad}$$

$V = 30$ mph $= 13.41$ m/s $\qquad l = 9$ ft $= 2.7432$ m

$R = 320$ ft $= 97.536$ m

2)

	60% Front	50% front	40% front
	$a = 3.6$ ft, $b = 5.4$ ft	$a = 4.5$ ft, $b = 4.5$ ft	$a = 5.4$ ft, $b = 3.6$ ft
Y_β $-(C_{\alpha f} + C_{\alpha r})$	-1.78×10^5 N/rad	Same	Same
Y_r $\dfrac{bC_{\alpha r} - aC_{\alpha f}}{V_x}$	$3649.1 \dfrac{NS}{rad}$	0	-3649.1 NS/rad
Y_δ $C_{\alpha f}$	89202.4 N/rad	Same	Same
N_β $(bC_{\alpha r} - aC_{\alpha F})$	$48940 \dfrac{Nm}{rad}$	0	$-48940 \dfrac{Nm}{rad}$
N_r $-\dfrac{(b^2 C_{\alpha r} + a^2 C_{\alpha F})}{V_x}$	$-26027 \dfrac{Nm\text{-}s}{rad}$	$-25026.16 \dfrac{Nms}{rad}$	Same as 60% front
N_δ $aC_{\alpha F}$	$97880 \dfrac{Nm}{rad}$	122350 Nm/rad	146820 Nm/rad

$$UG = \frac{m}{l}\left(\frac{b^2 C_{\alpha r} - a\, C_{\alpha F}}{C_{\alpha F} \cdot C_{\alpha r}}\right) \qquad k = \frac{UG}{l}$$

		60% Front	50% Front	40% front
a)	UG	$0.00392 \frac{rad}{m/s^2}$	0	$-0.00392 \frac{rad}{m/s^2}$
b)	K	$0.00143 \frac{rad}{m/s^2}$	0	$-0.00143 \frac{rad}{m/s^2}$
c)	$V_{cr} = \sqrt{\frac{-1}{K}}$	doesnt exist	doesnt exist	26.43 m/s
d)	NSP $\frac{l\, C_{\alpha r}}{C_{\alpha F} + C_{\alpha r}}$	1.3716 m	Same	Same
e)	SM $\left(\frac{NSP - a}{l}\right)$	0.1	0	-0.1

E) $N_\beta \cdot \beta + N_r \cdot r$ vs V_x

$r = \frac{V_x}{R}$ $\quad N_R = f(V_x) = \frac{-(a^2 C_f + b^2 C_{\alpha r})}{V_x}$

$\beta = \frac{a}{l} \frac{m v^2}{R \cdot C_{\alpha r}} - b/R$

F) Control moment

$\delta_{ack} + UG \cdot a_y$

$N_\delta \cdot \delta$ ~~vs~~ $\underbrace{N_\beta \cdot \beta + N_r \cdot r}_{\text{Stabilizing moment}}$