Final Exam (Chinmay Samak)

Solution 1 :

1.5 $\delta_{sw} = G \cdot \delta \Rightarrow \delta = \delta_{sw} / G$ (rad)

$\delta = \delta_A + \cancel{K G a_y}$ $\xrightarrow{}$ 0 (neutral steer) $= \delta_A \Rightarrow \tan\delta = \frac{l}{R}$

$\Rightarrow R = \left|\frac{l}{\tan\delta}\right| = \left|\frac{l}{\tan(\delta_{sw}/G)}\right|$ (m)

1.3 Stabilizing moment : $M = \cancel{N_\beta} \cdot \beta + N_r \cdot r \stackrel{!}{=} 3200$ lb.ft

0 (neutral steer)

(Nm) $\quad r \to -12°/s$

$\Rightarrow$ Yaw damping derivative : $N_r = M / r$ (N ms / rad)

1.4 $\cancel{N_\beta} = a C_f - b C_r \stackrel{!}{=} 0$ (neutral steer)

$N_r = \frac{a^2 C_f + b^2 C_r}{V}$

(soln 1.3)

$$\Rightarrow \begin{bmatrix} a & -b \\ a^2/V & b^2/V \end{bmatrix} \begin{bmatrix} C_f \\ C_r \end{bmatrix} = \begin{bmatrix} 0 \\ N_r \end{bmatrix}$$ ← solve system of eqns

(N/rad)

1.2 Control moment derivative : $N_\delta = -a C_f$ (Nm/ rad)

1.1 Static stability derivative: $N_\beta = a C_f - b C_r \stackrel{!}{=} 0$ (neutral steer)

(Nm/ rad)

Table of Contents

Clear Workspace

```
close all;
clear;
clc;
```

Given Data

```
l = 10/3.281; % Wheelbase (m)
a = 0.6*l; % Length of CG from front axle (m) for 40% front load
b = 0.4*l; % Length of CG from rear axle (m) for 40% front load
v = 66/3.281; % Velocity (m/s)
d = deg2rad(-40); % Steering wheel angle (rad)
G = 16; % Steering ratio
M = 3200*1.3558; % Stabilizing moment (Nm)
r = deg2rad(-12); % Yaw rate (rad/s)
```

1.5

```
R = abs(l/tan(d/G));
fprintf('Ans 1.5: The path radius (R) is %f m\n\n',R)
```

Ans 1.5: The path radius (R) is 69.807271 m

1.3

```
Nr = M/r;
fprintf('Ans 1.3: The yaw damping derivative (Nr) is %f Nms/rad\n\n',Nr)
```

Ans 1.3: The yaw damping derivative (Nr) is -20715.098097 Nms/rad

1.4

```
C = [a,-b;(a^2/v),(b^2/v)]\[0;Nr];
Cf = C(1);
Cr = C(2);
```

```
fprintf('Ans 1.4: The front (Cf) and rear (Cr) cornering stiffnesses are %f
 and %f N/rad\n\n',Cf,Cr)
```

Ans 1.4: The front (Cf) and rear (Cr) cornering stiffnesses are -74762.860542 and -112144.290813 N/rad

1.2

```
Nd = -a*Cf;
fprintf('Ans 1.2: The control moment derivative (Nd) is %f Nm/rad\n\n',Nd)
```

Ans 1.2: The control moment derivative (Nd) is 136719.647440 Nm/rad

1.1

```
NB = a*Cf - b*Cr;
fprintf('Ans 1.1: The static stability derivative (NB) is %f Nm/rad\n\n',NB)
```

Ans 1.1: The static stability derivative (NB) is 0.000000 Nm/rad

Published with MATLAB® R2022a

Solution 2:

$V = 35$ mph

$= 35/2.237 = 15.646$ m/s

$a = -10$ ft/sec^2

$= -10/3.281 = -3.048$ m/s^2

$\theta = 30^\circ$

$= 30 \times (\pi/180) = 0.5236$ rad

$\dot{\theta} = 10^\circ/s$

$= 10 \times (\pi/180) = 0.1745$ rad/s

$b = 5$ ft

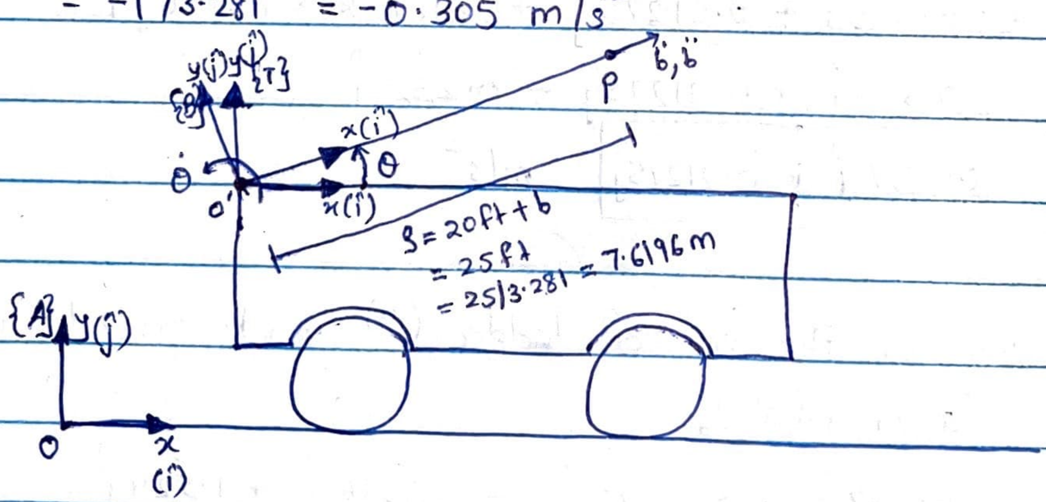
$= 5/3.281 = 1.524$ m

$\dot{b} = 2$ ft/s

$= 2/3.281 = 0.6096$ m/s

$\ddot{b} = -1$ ft/s^2

$= -1/3.281 = -0.305$ m/s^2



By transport theorem

$$(a^P)_A = (a^{o'})_A + (a^P)_B + \alpha_{B/A} \times \rho + 2\omega_{B/A} \times (v^P)_B + \omega_{B/A} \times (\omega_{B/A} \times \rho)$$

inertial acc (wrt ground) | relative acc (truck wrt inertial) | body acc | tangential acc | Coriolis acc | centripetal acc

With respect to the established notations,

$(a^P)_B = \ddot{b} = -0.305\,\hat{i}$ m/s²

$\alpha_{B/A} = 0\,\hat{k}$ rad/s²

$\rho = 7.6196\,\hat{i}$ m

$\omega_{B/A} = \dot{\theta} = 0.1745\,\hat{k}$ rad/s

$(v^P)_B = \dot{b} = 0.6096\,\hat{i}$ m/s

$(a^{o'})_A = a\cos\theta\,\hat{i} - a\sin\theta\,\hat{j} = -3.048\cos(30^\circ)\,\hat{i} + 3.048\sin(30^\circ)\,\hat{j}$

$= -2.6396\,\hat{i} + 1.524\,\hat{j}$ m/s²

2(a) Acceleration of the end of ladder (P) w.r.t. truck is

$(a^P)_T = (a^P)_A - (a^{o'})_A$

$= (a^P)_B + \cancel{\alpha_{B/A}}^{\,0} \times \rho + 2\,\omega_{B/A} \times (v^P)_B + \omega_{B/A} \times (\omega_{B/A} \times \rho)$

$= -0.305\,\hat{i} + 2\left[0.1745\,\hat{k} \times 0.6096\,\hat{i}\right] + 0.1745\,\hat{k} \times \left[0.1745\,\hat{k} \times 7.6196\,\hat{i}\right]$

$= -0.305\,\hat{i} + 0.21275\,\hat{j} + \left[0.1745\,\hat{k} \times 1.3296\,\hat{j}\right]$

$= -0.305\,\hat{i} + 0.21275\,\hat{j} - 0.232\,\hat{i}$

$\Rightarrow \boxed{(a^P)_T = -0.537\,\hat{i} + 0.21275\,\hat{j}}$ m/s²

2(b) Acceleration of the end of ladder (P) w.r.t. ground is

$(a^P)_A = (a^P)_T + (a^{o'})_A$

$= -0.537\,\hat{i} + 0.21275\,\hat{j} - 2.6396\,\hat{i} + 1.524\,\hat{j}$

$\Rightarrow \boxed{(a^P)_A = -3.1766\,\hat{i} + 1.73675\,\hat{j}}$ m/s²

Solution 3A:

3A.A $\delta_A = \tan^{-1}\left(\frac{l}{R}\right)$ (rad)

3A.B $a_y = \frac{V^2}{R}$ (m/s^2)

3A.C $\dot{\psi} = r = \frac{V}{R}$ (rad/s)

3A.D, 3A.E
$F_y = m\, a_y$ (N)

$\sum F_y : F_{y_f} + F_{y_r} - F_s - F_y = 0$ (N)

$\sum M_z : F_{y_r} \cdot b - F_{y_f} \cdot a + F_s \cdot (a+d) = 0$ (Nm)

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ -a & b & (a+d) \end{bmatrix} \begin{bmatrix} F_{yf} \\ F_{yr} \\ F_{s+F_y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ -a & b \end{bmatrix} \begin{bmatrix} F_{yf} \\ F_{yr} \end{bmatrix} = \begin{bmatrix} F_s + F_y \\ -F_s(a+d) \end{bmatrix}$$ ← solve system of equations

3A.F, 3A.G
$$C_f = 2\left(0.2196\,(\underbrace{0.5\times0.4}_{\text{front load distrib}}\times \underbrace{W}_{\text{lb}}) - 7.35e\text{-}6\,(\underbrace{0.5\times0.4}_{\text{front load distrb}}\times \underbrace{W}_{\text{lb}})^2\right) \times \underbrace{4.448}_{\text{lb}\to\text{N}} \times \underbrace{\frac{180}{\pi}}_{\text{deg}^{-1}\to\text{rad}^{-1}}$$
(N/rad)

$$C_r = 2\left(0.2196\,(0.5\times0.6\times W) - 7.35e\text{-}6\,(0.5\times0.6\times W)^2\right) \times 4.448 \times \frac{180}{\pi}$$
(N/rad)

$\alpha_f = F_{y_f} / C_f$ (rad)

$\alpha_r = F_{yr} / C_r$ (rad)

3A.H $M_z = a\,F_{yf} - b\,F_{yr}$ (Nm)

3A.I $\beta = \frac{b}{R} - \alpha_r$ (rad)

3A.J $\delta = \delta_A + \alpha_f - \alpha_r$ (rad)

Table of Contents

Clear Workspace

```
close all;
clear;
clc;
```

Given Data

```
w = 4e4*4.448; % Weight (N)
m = w/9.81; % Mass (kg)
l = 15/3.281; % Wheelbase (m)
a = 0.6*l; % Length of CG from front axle (m) for 40% front load
b = 0.4*l; % Length of CG from rear axle (m) for 40% front load
d = 4/3.281; % Length of front axle from snowplow (m)
v = 50/2.237; % Velocity (m/s)
R = 500/3.281; % Turning radius
Fs = 1000*4.448; % Snow force (N)
```

3A.A

```
dA = atan2(l,R);
fprintf('Ans 3A.A: The Ackerman steering angle (dA) is %f rad\n\n',dA)
```

Ans 3A.A: The Ackerman steering angle (dA) is 0.029991 rad

3A.B

```
ay = v^2/R;
fprintf('Ans 3A.B: The lateral acceleration (ay) is %f m/s^2\n\n',ay)
```

Ans 3A.B: The lateral acceleration (ay) is 3.278267 m/s^2

3A.C

```
r = v/R;
fprintf('Ans 3A.C: The yaw rate (r) is %f rad/s\n\n',r)
```

Ans 3A.C: The yaw rate (r) is 0.146670 rad/s

3A.D

```
Fy = m*ay;
Fyfr = [1,1;-a,b]\[(Fs+Fy);-Fs*(a+d)];
Fyf = Fyfr(1);
fprintf('Ans 3A.D: The front lateral force (Fyf) is %f N\n\n',Fyf)
```

Ans 3A.D: The front lateral force (Fyf) is 29416.771056 N

3A.E

```
Fyr = Fyfr(2);
fprintf('Ans 3A.E: The rear lateral force (Fyr) is %f N\n\n',Fyr)
```

Ans 3A.E: The rear lateral force (Fyr) is 34487.823251 N

3A.F

```
Cf = 2*(0.2196*(0.5*0.4*(w/4.448)) -
 (7.35e-6*(0.5*0.4*(w/4.448))^2))*4.448*(180/pi);
af = Fyf/Cf;
fprintf('Ans 3A.F: The front slip angle (af) is %f rad\n\n',af)
```

Ans 3A.F: The front slip angle (af) is 0.044864 rad

3A.G

```
Cr = 2*(0.2196*(0.5*0.6*(w/4.448)) -
 (7.35e-6*(0.5*0.6*(w/4.448))^2))*4.448*(180/pi);
ar = Fyr/Cr;
fprintf('Ans 3A.G: The rear slip angle (ar) is %f rad\n\n',ar)
```

Ans 3A.G: The rear slip angle (ar) is 0.042911 rad

3A.H

```
Mz = a*Fyf - b*Fyr;
fprintf('Ans 3A.H: The total yaw moment (Mz) is %f Nm\n\n',Mz)
```

Ans 3A.H: The total yaw moment (Mz) is 17623.895154 Nm

3A.I

```
B = b/R - ar;
fprintf('Ans 3A.I: The vehicle sideslip angle (B) is %f rad\n\n',B)
```

Ans 3A.I: The vehicle sideslip angle (B) is -0.030911 rad

3A.J

```
d = dA + af - ar;
fprintf('Ans 3A.J: The steering angle (d) is %f rad\n\n',d)
```

Ans 3A.J: The steering angle (d) is 0.031944 rad

Published with MATLAB® R2022a

Solution 3B :

3B·A $\delta_A = \tan^{-1}\left(\frac{l}{R}\right)$ (rad)

3B·B $a_y = v^2/R$ (m/s^2)

3B·C $\dot{\psi} = r = V/R$ (rad/s)

3B·D, 3B·E $F_y = m a_y$ (N)

$\sum F_y : F_{y_f} + F_{y_r} - F_S - F_y = 0$ (N)

$\sum M_z : F_{y_r} \cdot b - F_{y_f} \cdot a + F_S (a+d) = 0$ (Nm)

$$\begin{bmatrix} 1 & 1 \\ -a & b \end{bmatrix} \begin{bmatrix} F_{yf} \\ F_{yr} \end{bmatrix} = \begin{bmatrix} F_S + F_y \\ -F_S(a+d) \end{bmatrix}$$ ← solve system of equations

3B·F, 3B·G

$S_f = 0.4\, w/2$ (N)

$F_{z_{fl}} = S_f - \frac{m a_y h}{t}\, 0.45$ (N)

$F_{z_{fr}} = S_f + \frac{m a_y h}{t}\, 0.45$ (N)

$S_r = 0.6\, w/2$ (N)

$F_{z_{rl}} = S_r - \frac{m a_y h}{t}\, 0.55$ (N)

$F_{z_{rr}} = S_r + \frac{m a_y h}{t}\, 0.55$ (N)

$C_{fl} = \left[0.2196 \cdot (F_{z_{fl}}/4.448) - 7.35e{-6}\,(F_{z_{fl}}/4.448)^2\right] 4.448 \times \frac{180}{\pi}$ (N/rad)

$C_{fr} = \left[0.2196 (F_{z_{fr}}/4.448) - 7.35e{-6}\,(F_{z_{fr}}/4.448)^2\right] 4.448 \times \frac{180}{\pi}$ (N/rad)

$C_{rl} = \left[0.2196 (F_{z_{rl}}/4.448) - 7.35e{-6}\,(F_{z_{rl}}/4.448)^2\right] 4.448 \times \frac{180}{\pi}$ (N/rad)

$C_{rr} = \left[0.2196 (F_{z_{rr}}/4.448) - 7.35e{-6}\,(F_{z_{rr}}/4.448)^2\right] 4.448 \times \frac{180}{\pi}$ (N/rad)

$$C_f = C_{fl} + C_{fr} \qquad (N/rad)$$

$$C_r = C_{rl} + C_{rr} \qquad (N/rad)$$

$$\alpha_f = F_{yf} / C_f \qquad (rad)$$

$$\alpha_r = F_{yr} / C_r \qquad (rad)$$

3B·H $$M_z = a F_{yf} - b F_{yr} \qquad (Nm)$$

3B·I $$\beta = \frac{b}{R} - \alpha_r \qquad (rad)$$

3B·J $$\delta = \delta_A + \alpha_f - \alpha_r \qquad (rad)$$

Table of Contents

Clear Workspace

```
close all;
clear;
clc;
```

Given Data

```
w = 4e4*4.448; % Weight (N)
m = w/9.81; % Mass (kg)
l = 15/3.281; % Wheelbase (m)
t = 9/3.281; % Trackwidth (m)
h = 4.7/3.281; % Height of CG (m)
a = 0.6*l; % Length of CG from front axle (m) for 40% front load
b = 0.4*l; % Length of CG from rear axle (m) for 40% front load
d = 4/3.281; % Length of front axle from snowplow (m)
v = 50/2.237; % Velocity (m/s)
R = 500/3.281; % Turning radius
Fs = 1000*4.448; % Snow force (N)
```

3B.A

```
dA = atan2(l,R);
fprintf('Ans 3B.A: The Ackerman steering angle (dA) is %f rad\n\n',dA)
```

Ans 3B.A: The Ackerman steering angle (dA) is 0.029991 rad

3B.B

```
ay = v^2/R;
fprintf('Ans 3B.B: The lateral acceleration (ay) is %f m/s^2\n\n',ay)
```

Ans 3B.B: The lateral acceleration (ay) is 3.278267 m/s^2

3B.C

```
r = v/R;
fprintf('Ans 3B.C: The yaw rate (r) is %f rad/s\n\n',r)
```

Ans 3B.C: The yaw rate (r) is 0.146670 rad/s

3B.D

```
Fy = m*ay;
Fyfr = [1,1;-a,b]\[(Fs+Fy);-Fs*(a+d)];
Fyf = Fyfr(1);
fprintf('Ans 3B.D: The front lateral force (Fyf) is %f N\n\n',Fyf)
```

Ans 3B.D: The front lateral force (Fyf) is 29416.771056 N

3B.E

```
Fyr = Fyfr(2);
fprintf('Ans 3B.E: The rear lateral force (Fyr) is %f N\n\n',Fyr)
```

Ans 3B.E: The rear lateral force (Fyr) is 34487.823251 N

3B.F

```
Sf = (w*0.4)/2; % 40% front (static) load distribution
Fzfl = Sf - ((m*ay*h)/t)*0.45;
Fzfr = Sf + ((m*ay*h)/t)*0.45;
Cfl = (0.2196*((Fzfl/4.448)) - (7.35e-6*((Fzfl/4.448))^2))*4.448*(180/pi);
Cfr = (0.2196*((Fzfr/4.448)) - (7.35e-6*((Fzfr/4.448))^2))*4.448*(180/pi);
Cf = Cfl + Cfr;
af = Fyf/Cf;
fprintf('Ans 3B.F: The front slip angle (af) is %f rad\n\n',af)
```

Ans 3B.F: The front slip angle (af) is 0.047545 rad

3B.G

```
Sr = (w*0.6)/2; % 60% rear (static) load distribution
FZrl = Sr - ((m*ay*h)/t)*0.55;
FZrr = Sr + ((m*ay*h)/t)*0.55;
Crl = (0.2196*((FZrl/4.448)) - (7.35e-6*((FZrl/4.448))^2))*4.448*(180/pi);
Crr = (0.2196*((FZrr/4.448)) - (7.35e-6*((FZrr/4.448))^2))*4.448*(180/pi);
Cr = Crl + Crr;
ar = Fyr/Cr;
```

```
fprintf('Ans 3B.G: The rear slip angle (ar) is %f rad\n\n',ar)
```

Ans 3B.G: The rear slip angle (ar) is 0.046077 rad

3B.H

```
Mz = a*Fyf - b*Fyr;
fprintf('Ans 3B.H: The total yaw moment (Mz) is %f Nm\n\n',Mz)
```

Ans 3B.H: The total yaw moment (Mz) is 17623.895154 Nm

3B.I

```
B = b/R - ar;
fprintf('Ans 3B.I: The vehicle sideslip angle (B) is %f rad\n\n',B)
```

Ans 3B.I: The vehicle sideslip angle (B) is -0.034077 rad

3B.J

```
d = dA + af - ar;
fprintf('Ans 3B.J: The steering angle (d) is %f rad\n\n',d)
```

Ans 3B.J: The steering angle (d) is 0.031459 rad

Published with MATLAB® R2022a

- Comparing Solution 3A and 3B

Parameter	Unit	Value (3A)		Value (3B)
δ_A	rad	0.029991	=	0.029991
a_y	m/s^2	3.278267	=	3.278267
$\dot{\varphi}$	rad/s	0.146670	=	0.146670
F_{yf}	N	29416.771056	=	29416.771056
F_{yr}	N	34487.823251	=	34487.823251
α_f	rad	0.044864	<	0.047545
α_r	rad	0.042911	<	0.046077
M_z	Nm	17623.895154	=	17623.895154
β	rad	-0.030911	>	-0.034077
δ	rad	0.031944	>	0.031459

From the above table it can be clearly seen that without (3A) or with (3B) load transfer taken into consideration, the parameters that change are α_f, α_r, β and δ. α_f and α_r increase as the vehicle has to now react with larger slip angles (in response to the load transfer). β is decreasing in -ve direction (cw) i.e. increasing in magnitude due to load transfer. Finally, δ decreases indicating that the understeering behavior of the vehicle is reduced.

Solution 4 A:

4A·A $LF + RF + LR + RR - W = 0$

$LF + RF - W(W_{Front}) = 0$

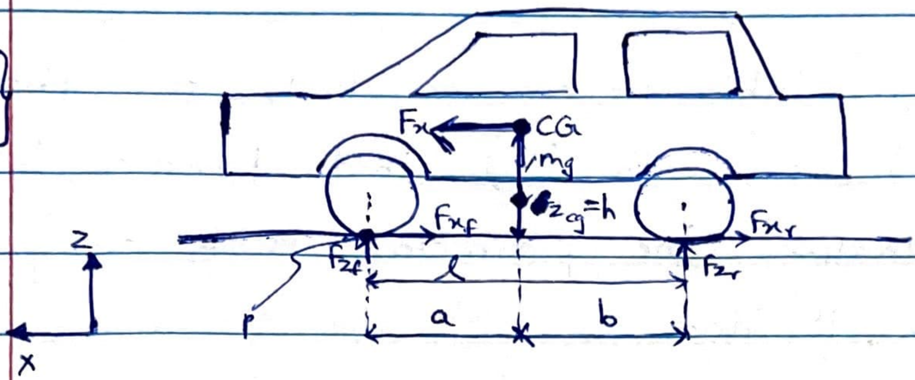
$RF + RR - W(W_{Right}) = 0$

$(LF + RR) - (RF + LR) - D = 0$

4 eqⁿs in 4 unknowns

Solved using MATLAB symbolic toolbox

4A.B
4A.C



Let $z_{cg} = h$

Assume CG @ $\ell/2$ $\Rightarrow$ when $a_x = 0$, $|F_{zr}| = |F_{zf}| = \left|\frac{mg}{2}\right|$

Taking moment about point P (front tire contact) : ($\circlearrowleft$+)

$$0 = -mg\frac{\ell}{2} + |F_{zr}|\ell + m a_x h$$

$$\Rightarrow F_{zr} = \frac{mg}{2} - \frac{m a_x h}{\ell} \qquad \text{(Rear)}$$

$$\Rightarrow \underbrace{F_{zf}}_{\text{Dynamic Wheel Load}} = \underbrace{\frac{mg}{2}}_{\text{Static Wheel Load}} + \underbrace{\frac{m a_x h}{\ell}}_{\text{Load Transfer Due to } a_x} \qquad \text{(Front)}$$

$$\boxed{\underset{(N)}{\Delta W_x} = \frac{m a_x h}{\ell} = \frac{W A_x h}{\ell}} \qquad \begin{cases} W = mg \\ A_x = a_x/g \end{cases}$$

4A.D For all load to be transferred to front from rear in a straight line (i.e. no lateral load transfer).

$F_{zr} = 0$

$$\Rightarrow \frac{mg}{2} - \frac{m a_x h}{l} = 0$$

$$\Rightarrow \frac{\cancel{m}g}{2} = \frac{\cancel{m}a_x h}{l}$$

$$\Rightarrow a_x = \frac{lg}{2h}$$

Generalizing above eq^n specific to $a = \frac{l}{2}$ to a general "a" distance we get:

$$\boxed{\underset{(m/s^2)}{a_x} = \frac{ga}{h}}$$

Table of Contents

Clear Workspace

```
close all;
clear;
clc;
```

4A.A

```
W = 143*14.594*9.81; % Total vehicle weight
WFront = 0.54; % Front weight ratio
WRear = 1-WFront; % Rear weight ratio
WLeft = 0.52; % Left weight ratio
WRight = 1-WLeft; % Right weight ratio
D = 500; % Diagonal weight (N)
syms LF RF LR RR
assume(LF>=0 & RF>=0 & LR>=0 & RR>=0); % Assume all static loads are positive
equation1 = LF+RF+LR+RR-W; % W = LF+RF+LR+RR
equation2 = LF+RF-(W*WFront); % WFront = ((LF+RF)/W)
equation3 = RF+RR-(W*WRight); % WRight = ((RF+RR)/W)
equation4 = (LF+RR)-(RF+LR)-D; % D = (LF+RR)-(RF+LR)
solution = solve(equation1,equation2,equation3,equation4); % Solve 4 equations
 simultaneously
LF = double(solution.LF); % Left front wheel load (N)
RF = double(solution.RF); % Right front wheel load (N)
LR = double(solution.LR); % Left rear wheel load (N)
RR = double(solution.RR); % Right rear wheel load (N)
fprintf('Ans 4A.A: The static wheel loads for given vehicle parameters are LF
 = %.2f N, RF = %.2f N, LR = %.2f N, RR = %.2f N\n\n',LF,RF,LR,RR)
```

```
Ans 4A.A: The static wheel loads for given vehicle parameters are LF = 5857.41
 N, RF = 5197.95 N, LR = 4788.50 N, RR = 4629.04 N
```

4A.B

dWx = (m*ax*h)/l = (W*Ax*h)/l

```
fprintf('Ans 4A.B: Expression "dWx = (m*ax*h)/l = (W*Ax*h)/l" gives
 longitudinal load transfer while acceleration/braking\n\n')
```

```
Ans 4A.B: Expression "dWx = (m*ax*h)/l = (W*Ax*h)/l" gives longitudinal load
 transfer while acceleration/braking
```

4A.C

```
m = 143*14.594; % Vehicle mass (kg)
ax = 0.5*9.81; % Longitudinal (braking) acceleration (m/s^2)
h = 2.3/3.281; % Height of CG (m)
l = 9.45/3.281; % Wheelbase (m)
dWx = (m*ax*h)/l; % Longitudinal load transfer
fprintf('Ans 4A.C: The load transferred from the rear tires to the front tires
 when the vehicle is braking at 0.5g is %f N\n\n',dWx)
```

```
Ans 4A.C: The load transferred from the rear tires to the front tires when the
 vehicle is braking at 0.5g is 2491.411235 N
```

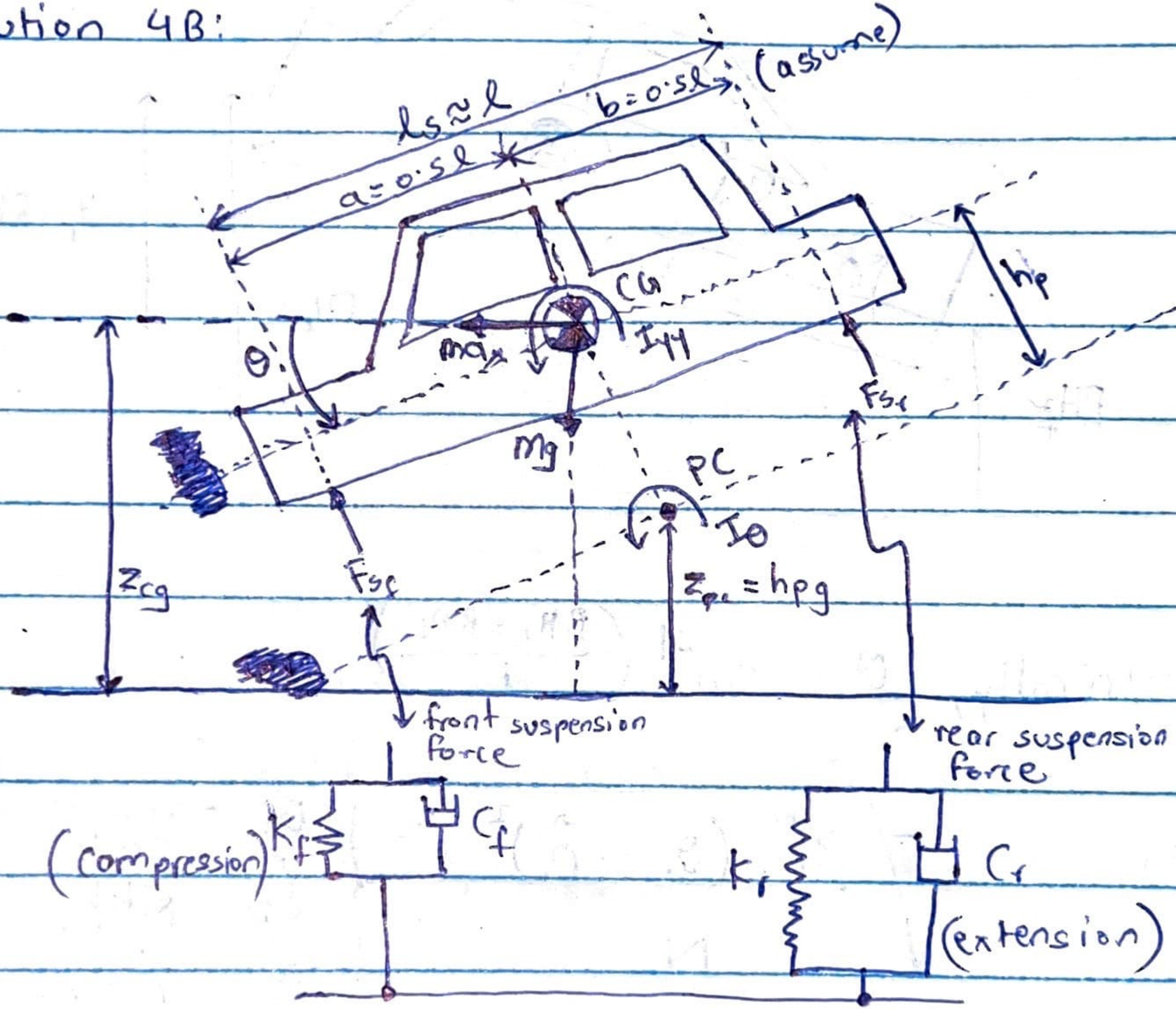
4A.D

```
l = 9.45/3.281; % Wheelbase (m)
g = 9.81; % Acceleration due to gravity (m/s^2)
a = (1-0.54)*l; % Distance of CG from front axle (m)
h = 2.3/3.281; % Height of CG (m)
ax = (g*a)/h;
fprintf('Ans 4A.D: At %f m/s^2 longitudinal acceleration all the load will be
 transferred to front from rear in a straight-line motion\n\n',ax)
```

```
Ans 4A.D: At 18.540900 m/s^2 longitudinal acceleration all the load will be
 transferred to front from rear in a straight-line motion
```

Published with MATLAB® R2022a

Solution 4B:



$l_s \approx l$
$b = 0.5l$
(assume)
$a = 0.5l$
CG
h_p
θ
ma_x
I_{yy}
F_{sr}
mg
PC
I_θ
z_{cg}
F_{sf}
$z_{pc} = h_{pg}$
front suspension force
rear suspension force
K_f
C_f
(compression)
K_r
C_r
(extension)

Taking moment about PC ($\circlearrowleft +$)

$$(I_{yy} + m h_p^2)\ddot{\theta} = \sum M_y$$

$$= m \cdot a_x \cdot h_p \cdot \cos\theta + mg \cdot h_p \cdot \sin\theta - F_{sf} \frac{l_s}{2} + F_{sr} \frac{l_s}{2}$$

$$= m a_x h_p \cos\theta + mgh_p \sin\theta + \frac{l_s}{2}(F_{sr} - F_{sf}) \quad ①$$

Deflection of sprung mass (from undeflected stat)

$$Z_{sf} = -\frac{l_s}{2}\sin\theta \quad \text{(front)} \quad -②$$

$$Z_{sr} = +\frac{l_s}{2}\sin\theta \quad \text{(rear)} \quad -③$$

Velocity of sprung mass (deflection derivative)

Differentiating ② & ③ we get

$$\dot{z}_{sf} = -\frac{l_s}{2}\cos\theta\,\dot{\theta} \quad \text{(front)} \quad -④$$

$$\dot{z}_{sr} = +\frac{l_s}{2}\cos\theta\,\dot{\theta} \quad \text{(rear)} \quad -⑤$$

Dynamic suspension forces (compression +)

From (2), (3), (4), (5) we get

$$F_{sf} = k_f \frac{l_s}{2} \sin\theta + C_f \frac{l_s}{2} \cos\theta\, \dot{\theta} \quad -(6)$$

$$F_{sr} = \underbrace{-k_r \frac{l_s}{2} \sin\theta}_{\text{spring}} - \underbrace{C_r \frac{l_s}{2} \cos\theta\, \dot{\theta}}_{\text{Damper}} \quad -(7)$$

Substituting (6) & (7) in (1) we get

$$(I_{yy} + m h_p^2)\ddot{\theta} = m a_x h_p \cos\theta + m g h_p \sin\theta - \frac{l_s}{2}\left[k_r \frac{l_s}{2}\sin\theta + C_r \frac{l_s}{2}\cos\theta\dot{\theta} + k_f \frac{l_s}{2}\sin\theta + C_f \frac{l_s}{2}\cos\theta\dot{\theta}\right]$$

$$= m a_x h_p \cos\theta + m g h_p \sin\theta - \frac{k_f l_s^2 \sin\theta}{4} - \frac{k_r l_s^2 \sin\theta}{4} - \frac{C_f l_s^2 \cos\theta\dot{\theta}}{4} - \frac{C_r l_s^2 \cos\theta\dot{\theta}}{4}$$

r = resultant

Let $k_{fr} = k_f l_s^2/4$

$k_{rr} = k_r l_s^2/4$

$C_{fr} = C_f l_s^2/4$

$C_{rr} = C_r l_s^2/4$

pitch axis $I_\theta = I_{yy} + m h_p^2$

$$\Rightarrow I_\theta \ddot{\theta} = m a_x h_p \cos\theta + m g h_p \sin\theta - k_{fr}\sin\theta - k_{rr}\sin\theta - C_{fr}\cos\theta\dot{\theta} - C_{rr}\cos\theta\dot{\theta} \quad -(8)$$

Applying small angle approximation on (8) we get

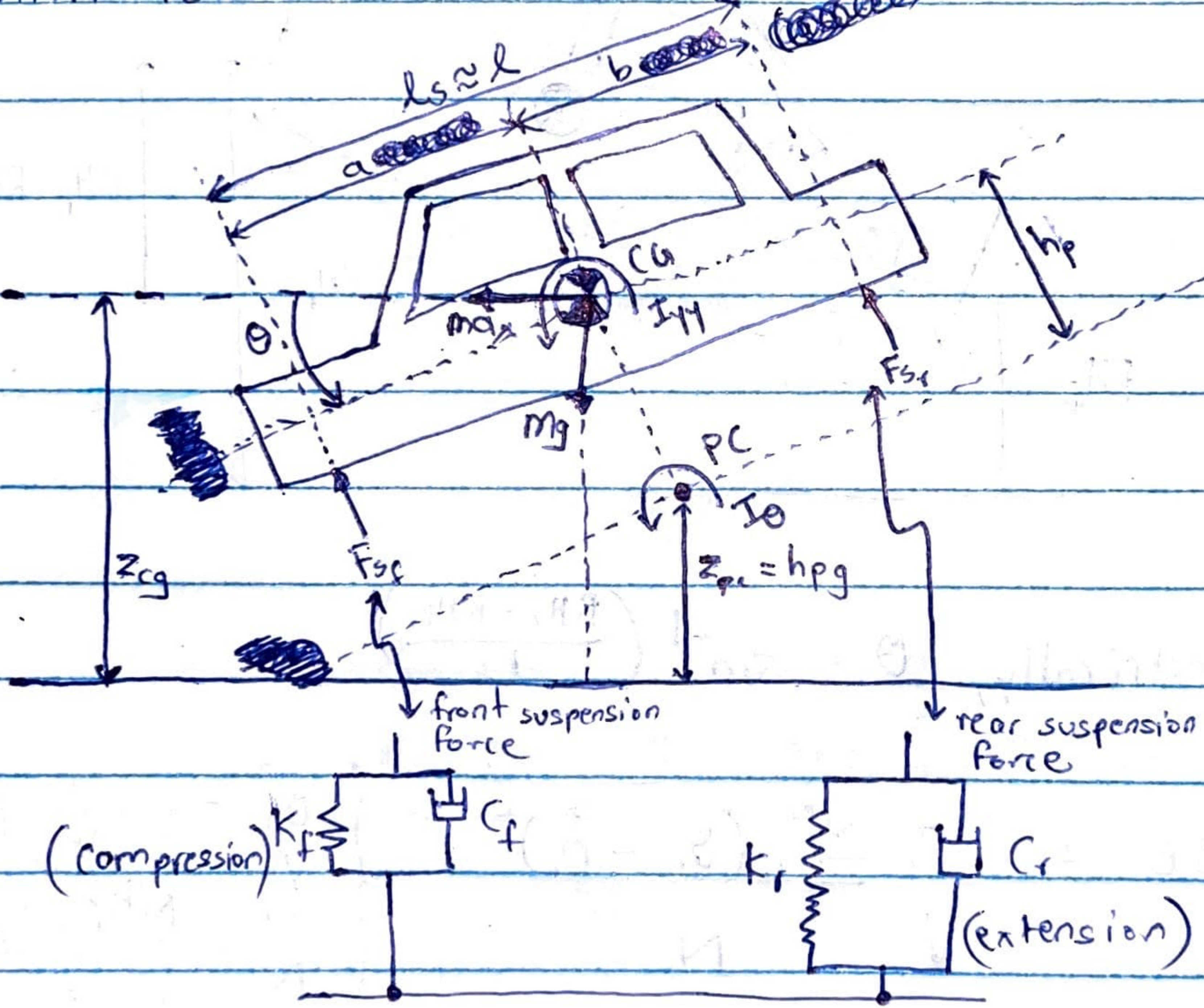
$$I_\theta \ddot{\theta} = m a_x h_p + m g h_p \theta - k_{fr}\theta - k_{rr}\theta - C_{fr}\dot{\theta} - C_{rr}\dot{\theta} \quad -(9)$$

At steady state, $\ddot{\theta} = 0$ & $\dot{\theta} = 0$

$\Rightarrow m a_x h_p + m g h_p \theta - K_{fr} \theta - K_{rr} \theta = 0$

$\Rightarrow \theta (K_{fr} + K_{rr} - m g h_p) = m a_x h_p$

$\Rightarrow \boxed{\theta = \dfrac{m a_x h_p}{(K_{fr} + K_{rr} - m g h_p)}}$ (rad)



ls ≈ l
a
b
CG
Iyy
θ
ma_x
Mg
F_{sr}
hp
PC
Iθ
z_{cg}
F_{sf}
$z_{pc} = hpg$
front suspension force
rear suspension force
(compression) K_f
C_f
K_r
C_r
(extension)

Assuming a & b are not equal to $0.5\, l_s$

Taking moment about PC (↺)

$$(I_{yy} + m h_p^2)\,\ddot{\theta} = \Sigma M_y$$

$$= m a_x h_p \cos\theta + m g h_p \sin\theta - a F_{sf} + b F_{sr} \quad -①$$

Deflection of sprung mass (from undeflected state)

$$Z_{sf} = -a \sin\theta \quad \text{(Front)} \quad -②$$

$$Z_{sr} = b \sin\theta \quad \text{(Rear)} \quad -③$$

Velocity of sprung mass (deflection derivative)

Differentiating ② & ③ wrt time we get

$$\dot{Z}_{sf} = -a \cos\theta\, \dot{\theta} \quad \text{(Front)} \quad -④$$

$$\dot{Z}_{sr} = b \cos\theta\, \dot{\theta} \quad \text{(Rear)} \quad -⑤$$

Dynamic suspension forces (compression +)

From ②, ③, ④, ⑤ we have

$$F_{sf} = \underbrace{K_f\, a \sin\theta}_{\text{spring}} + \underbrace{C_f\, a \cos\theta\, \dot{\theta}}_{\text{damper}} \quad -⑥$$

$$F_{sr} = -K_r\, b \sin\theta - C_r\, b \sin\theta\, \dot{\theta} \quad -⑦$$

Substituting 6 & 7 in 1 we get

$$\underbrace{(I_{YY} + mh_p^2)}_{I_O}\ddot{\theta} = ma_x h_p \cos\theta + mg h_p \sin\theta - K_f a^2 \sin\theta - C_f a^2 \cos\theta\,\dot{\theta} - K_r b^2 \sin\theta - C_r b^2 \cos\theta\,\dot{\theta} \quad -(8)$$

Applying small angle approximation on 8 we get

$$I_O\ddot{\theta} = ma_x h_p + mg h_p\theta - K_f a^2\theta - C_f a^2\dot{\theta} - K_r b^2\theta - C_r b^2\dot{\theta} \quad -(9)$$

At steady state, $\ddot{\theta} = 0$ & $\dot{\theta} = 0$

$$\Rightarrow ma_x h_p + mg h_p\theta - K_f a^2\theta - K_r b^2\theta = 0$$

$$\Rightarrow \theta(K_f a^2 + K_r b^2 - mg h_p) = ma_x h_p$$

$$\Rightarrow \boxed{\theta = \frac{ma_x h_p}{(K_f a^2 + K_r b^2 - mg h_p)}} \quad \text{(rad)}$$

Solution 4C:

4C.A Assumed vehicle parameters from 4B

$h_p = h - h_{pg}$ @ zero roll

w = trackwidth, L = wheelbase

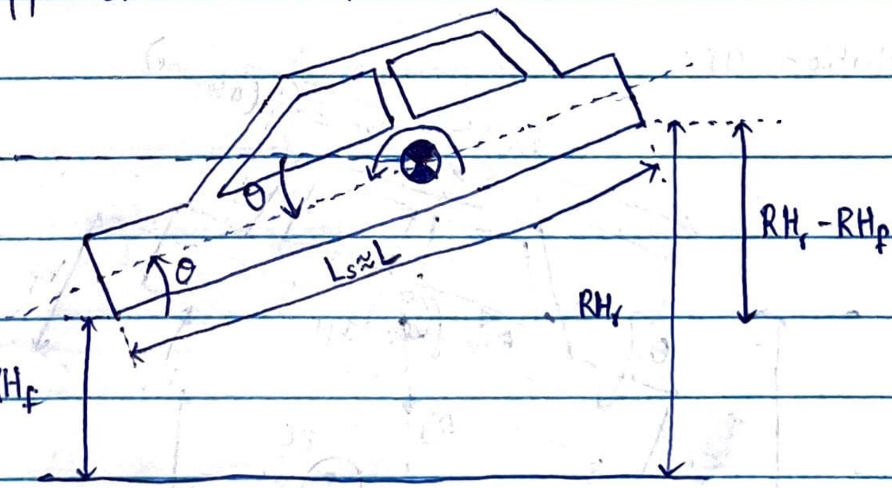
$$I_{yy} = 0.4 \frac{m}{12}(L^2 + w^2)$$

$$I_O = I_{yy} + m h_p^2$$

$$\ddot{\theta} = \frac{1}{I_O}\left[m a_x h_p + m g h_p \theta - K_{fr}\theta - K_{rr}\theta - C_{fr}\dot{\theta} - C_{rr}\dot{\theta}\right]$$

$$\left.\begin{aligned} \dot{\theta}_{t+1} &= \dot{\theta}_t + \ddot{\theta}_t \Delta t \\ \theta_{t+1} &= \theta_t + \dot{\theta}_t \Delta t \end{aligned}\right\} \text{Forward Euler method.}$$

4C.B Similar approach as 4C.A



Geometrically, $\theta = \sin^{-1}\left(\frac{RH_r - RH_f}{L}\right)$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(\theta_i - \hat{\theta}_i)^2}{N}}$$

θ_i = recorded/measured pitch angle

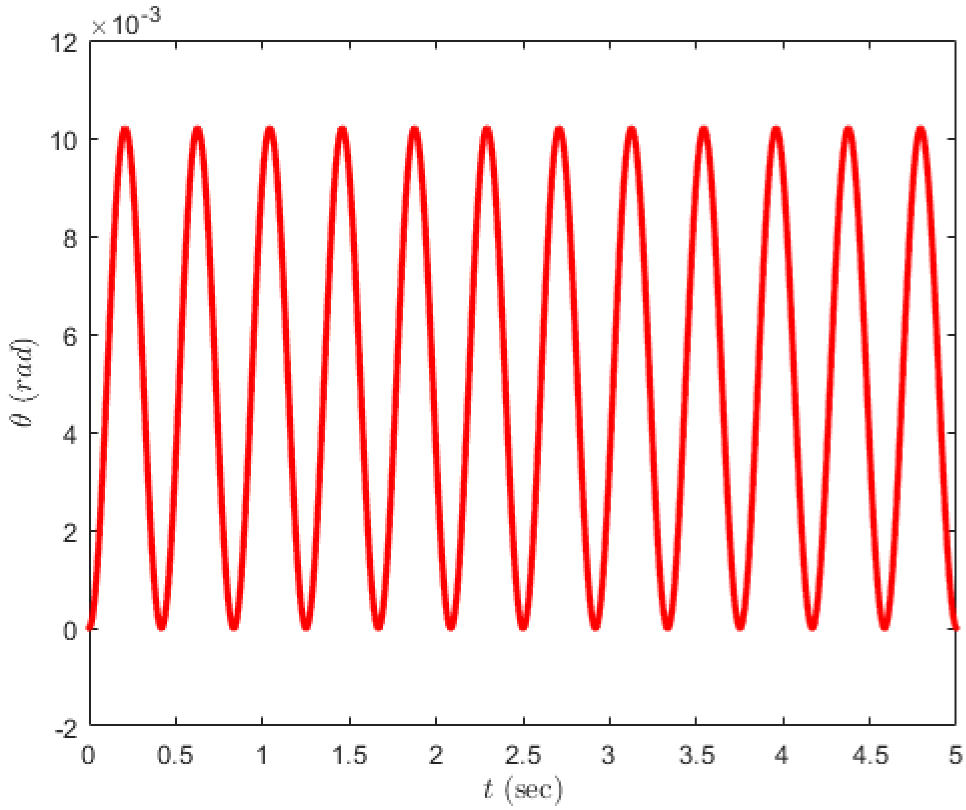
$\hat{\theta}_i$ = pitch angle updated using model developed in 4B

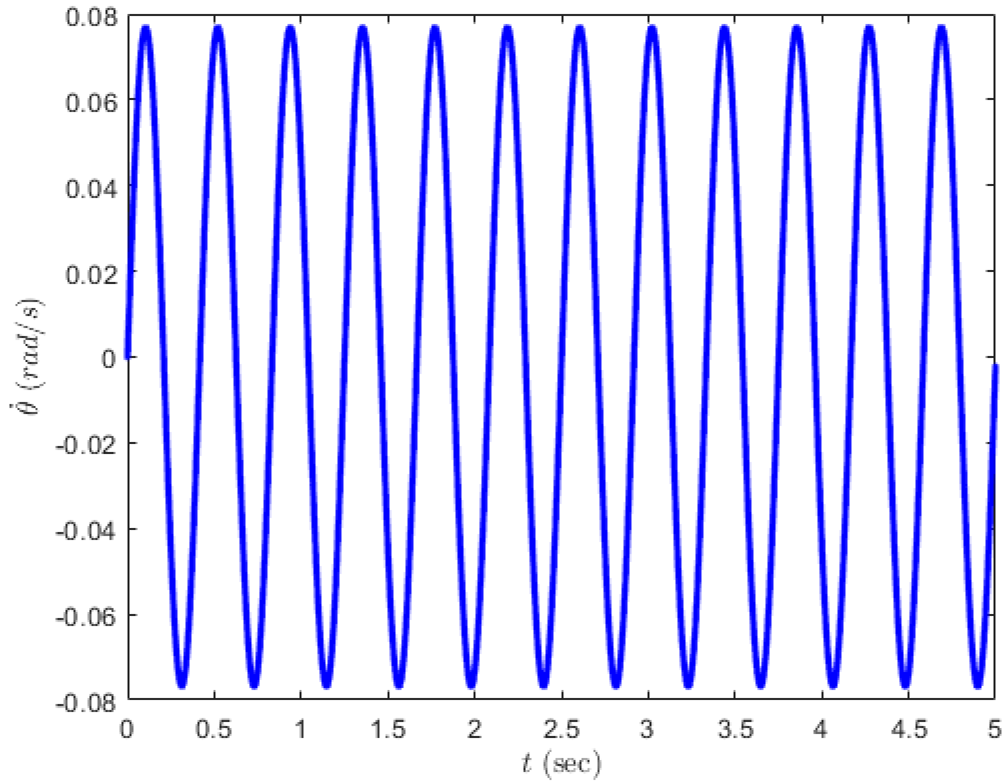
N = number of data points

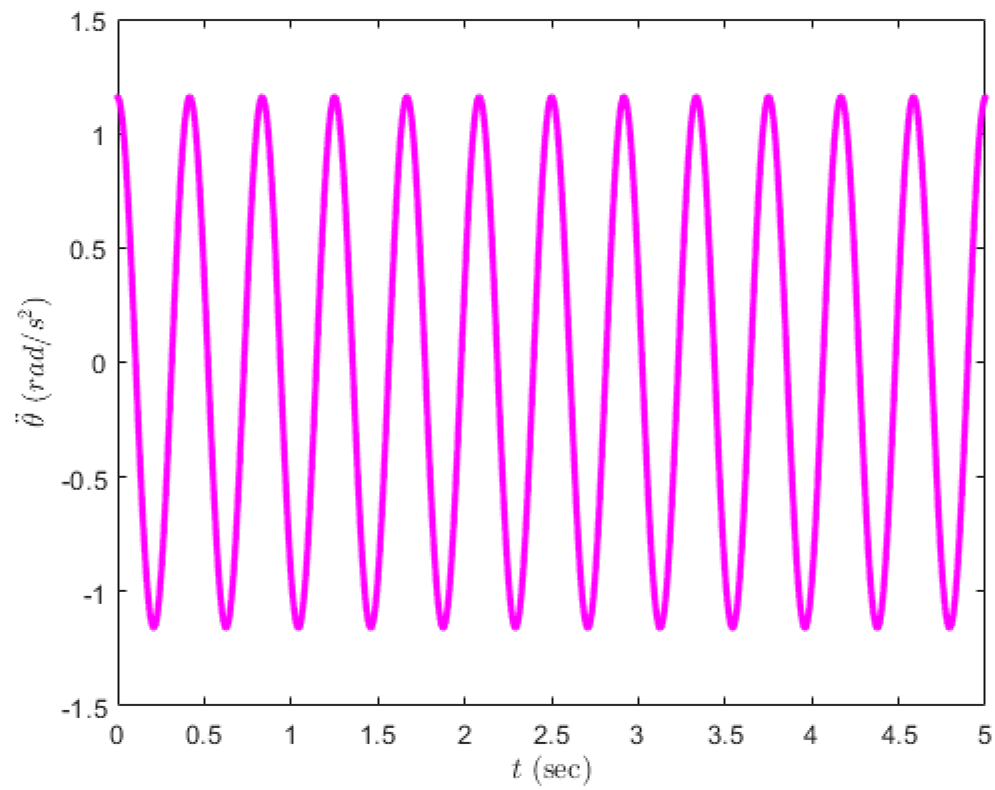
4C.A

```
close all;
clear;
clc;

% Data assumed from 4A
m = 143*14.594; % Vehicle mass (kg)
g = 9.81; % Acceleration due to gravity (m/s^2)
h = 2.3/3.281; % Height of CG (m)
L = 9.45/3.281; % Wheelbase (m)
W = 5.25/3.281; % Trackwidth (m)
a = (1-0.54)*L; % Distance of CG from front axle (m)
b = L-a; % Distance of CG from rear axle (m)

% Data given in 4C
ax = 2; % Longitudinal acceleration (m/s^2) (braking assumed positive in 4B)
hpg = 1.5/3.281; % Height of PC from ground (m)
Kf = 45e3; % Front suspension stiffness (N/m)
Kr = 52e3; % Rear suspension stiffness (N/m)
Cf = 0; % Front suspension damping (Ns/m)
Cr = 0; % Rear suspension damping (Ns/m)

% Intermediate variables
hp = h - hpg; % Height of PC from CG (m)
Iyy = 0.4*((m/12)*(L^2+W^2)); % Pitch MOI about CG (kg-m^2)
Itheta = Iyy + m*hp^2; % Pitch MOI about PC (kg-m^2)

% Simulation settings
ms = 1*m; % Sprung mass assumed to be 100% of total vehicle mass (kg)
theta_sim = [0]; % Initial pitch angle assumption (rad)
theta_dot = [0]; % Initial pitch velocity assumption (rad/s)
dt = 1e-6; % Simulation timestep (s)
i = 1; % Iterator count

% Simulate from i=0 to i=5 (@ 2 m/s^2 braking acceleration)
for t = 0+dt:dt:5
    theta_ddot(i) = 1/Itheta * (ms*ax*hp + ms*g*hp*theta_sim(i) -
 Kf*a^2*theta_sim(i) - Kr*b^2*theta_sim(i) - Cf*a^2*theta_dot(i) -
 Cr*b^2*theta_dot(i));
    theta_dot(i+1) = theta_dot(i) + theta_ddot(i)*dt;
    theta_sim(i+1) = theta_sim(i) + theta_dot(i)*dt;
    i = i+1;
end

% Plot theta vs t
figure()
plot((0:dt:5),theta_sim,'.','color','red')
xlabel('${t}$ (sec)','interpreter','latex')
ylabel('${\theta}$ ($rad$)','interpreter','latex')

% Plot theta_dot vs t
```

```
figure()
plot((0:dt:5),theta_dot,'.','color','blue')
xlabel('${t}$ (sec)','interpreter','latex')
ylabel('${\dot\theta}$ ($rad/s$)','interpreter','latex')

% Plot theta_ddot vs t
figure()
plot((dt:dt:5),theta_ddot,'.','color','magenta')
xlabel('${t}$ (sec)','interpreter','latex')
ylabel('${\ddot\theta}$ ($rad/s^2$)','interpreter','latex')
```




0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
$\dot{\theta}$ (rad/s)
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
t (sec)

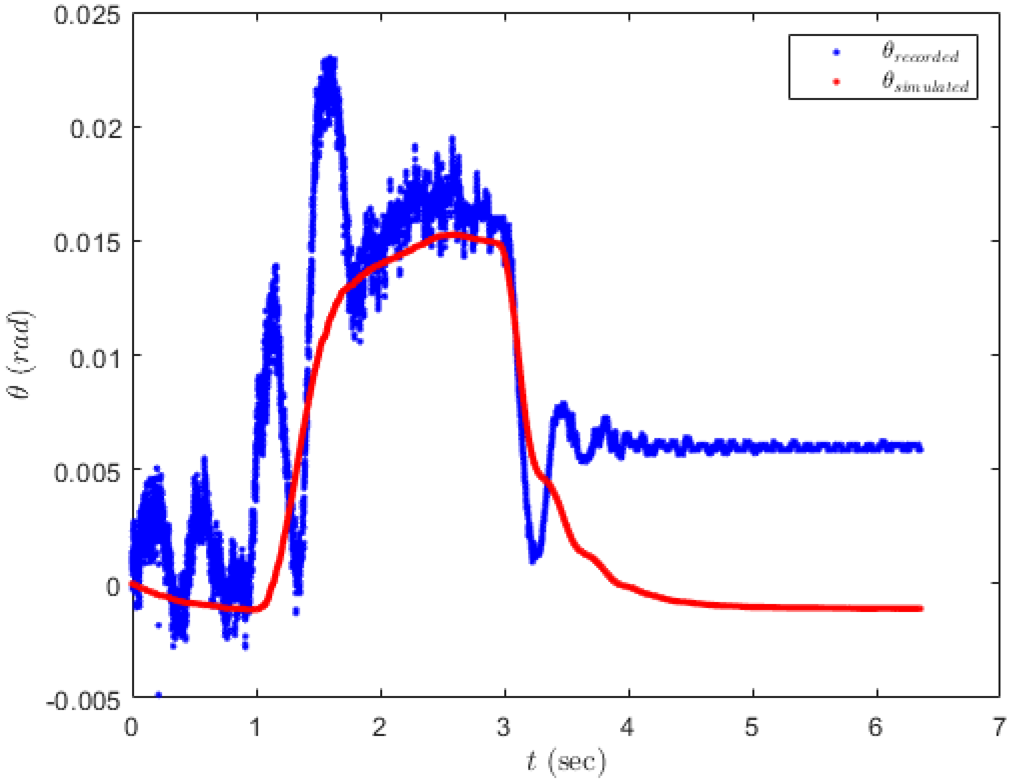

1.5
1
0.5
0
-0.5
-1
-1.5
$\ddot{\theta}$ (rad/s^2)
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
t (sec)

4C.B

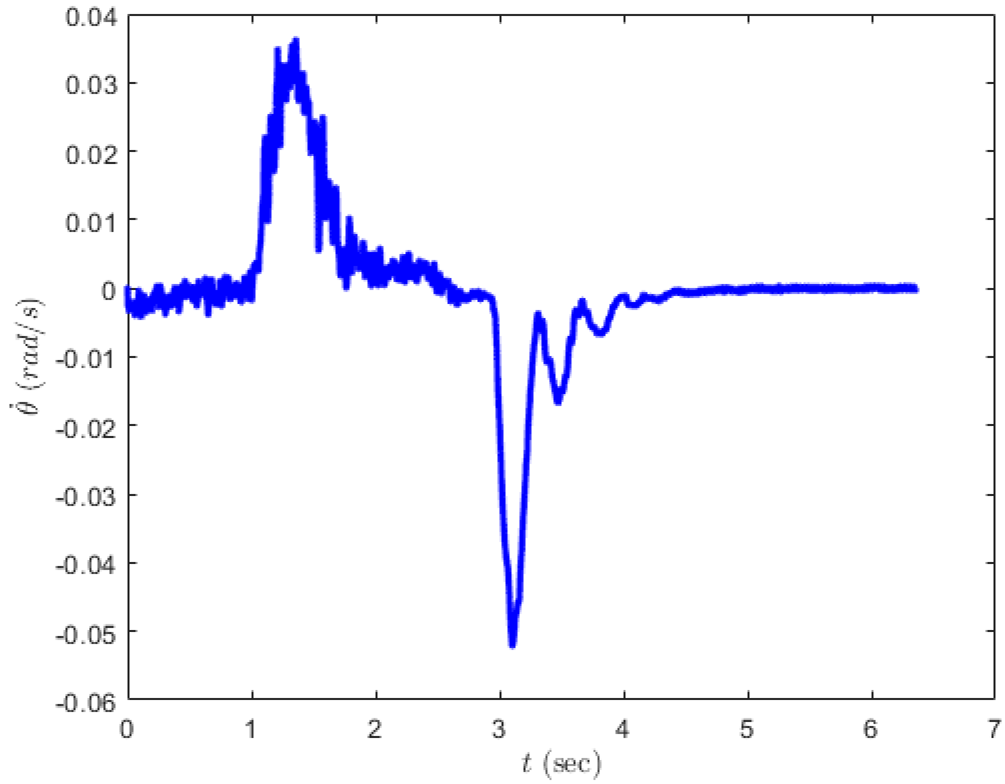
```
clear;
clc;

% Data assumed from 4A
m = 143*14.594; % Vehicle mass (kg)
g = 9.81; % Acceleration due to gravity (m/s^2)
h = 2.3/3.281; % Height of CG (m)
L = 9.45/3.281; % Wheelbase (m)
W = 5.25/3.281; % Trackwidth (m)
a = (1-0.54)*L; % Distance of CG from front axle (m)
b = L-a; % Distance of CG from rear axle (m)

% RECORDED DATA
load('FinalExamData.mat'); % Load given data
theta_rec = asin((Rh_lr-Rh_lf)./L);

% (SUB)OPTIMAL ESTIMATION OF SUSPENSION DAMPING COEFFICIENTS (ASSUMING THEY
 ARE EQUAL)
ax = Acc_x; % Longitudinal acceleration (m/s^2) (braking assumed positive in
 4B)
hpg = 1.5/3.281; % Height of PC from ground (m)
Kf = 45e3; % Front suspension stiffness (N/m)
Kr = 52e3; % Rear suspension stiffness (N/m)
hp = h - hpg; % Height of PC from CG (m)
Iyy = 0.4*((m/12)*(L^2+W^2)); % Pitch MOI about CG (kg-m^2)
Itheta = Iyy + m*hp^2; % Pitch MOI about PC (kg-m^2)
ms = 1*m; % Sprung mass assumed to be 100% of total vehicle mass (kg)
theta_sim = [0]; % Initial pitch angle assumption (rad)
theta_dot = [0]; % Initial pitch velocity assumption (rad/s)
dt = time(2) - time(1); % Simulation timestep (s)
i = 1; % Iterator count
C_optimal = 0;
error_minimum = Inf;
for C = 0:10:1e6
    i = 1;
    for t = 0+dt:dt:time(end)
        theta_ddot(i) = 1/Itheta * (ms*ax(i)*hp + ms*g*hp*theta_sim(i)
 - Kf*a^2*theta_sim(i) - Kr*b^2*theta_sim(i) - C*a^2*theta_dot(i) -
 C*b^2*theta_dot(i));
        theta_dot(i+1) = theta_dot(i) + theta_ddot(i)*dt;
        theta_sim(i+1) = theta_sim(i) + theta_dot(i)*dt;
        i = i+1;
    end
    error = sqrt((sum((theta_rec - theta_sim).^2))/length(theta_rec)); % RMSE
    if error < error_minimum
        error_minimum = error;
        C_optimal = C;
    end
end
fprintf(['Ans 4C.B: The estimated suspension damping coefficients (assumung
 they are equal)\n' ...
```

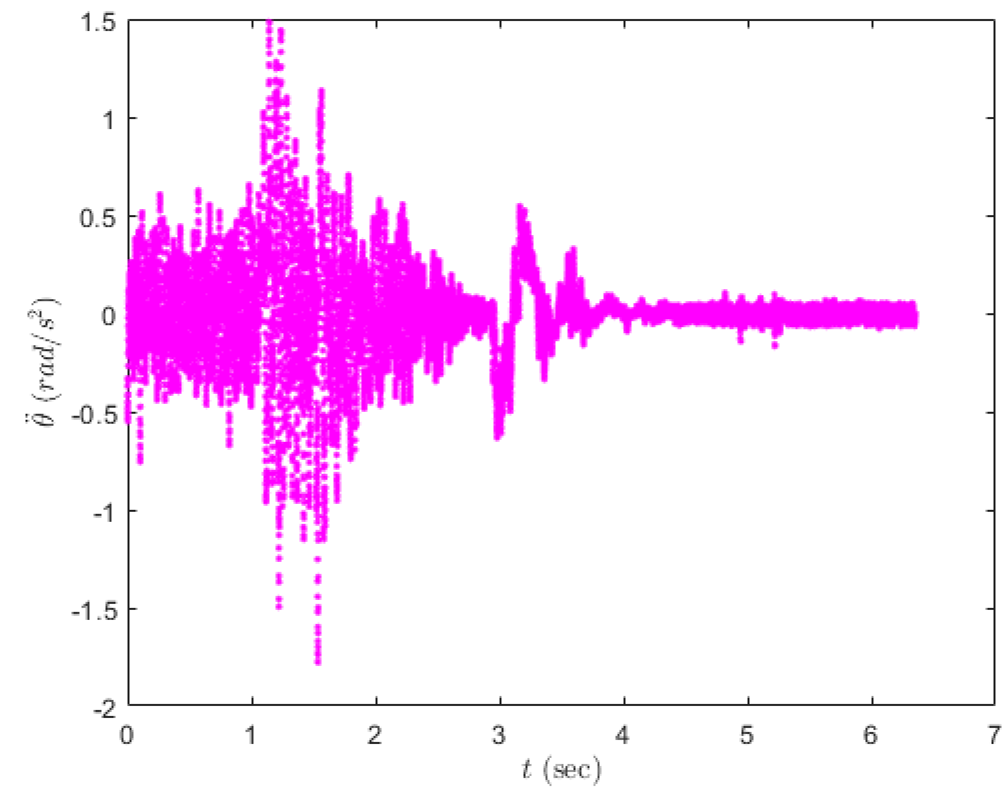
```
        'considering RMSE between given data and simulation results are %f
 Ns/m\n\n'],C_optimal)

% SIMULATION
ax = Acc_x; % Longitudinal acceleration (m/s^2) (braking assumed positive in
 4B)
hpg = 1.5/3.281; % Height of PC from ground (m)
Kf = 45e3; % Front suspension stiffness (N/m)
Kr = 52e3; % Rear suspension stiffness (N/m)
Cf = C_optimal; % Front suspension damping (Ns/m)
Cr = C_optimal; % Rear suspension damping (Ns/m)
hp = h - hpg; % Height of PC from CG (m)
Iyy = 0.4*((m/12)*(L^2+W^2)); % Pitch MOI about CG (kg-m^2)
Itheta = Iyy + m*hp^2; % Pitch MOI about PC (kg-m^2)
ms = 1*m; % Sprung mass assumed to be 100% of total vehicle mass (kg)
theta_sim = [0]; % Initial pitch angle assumption (rad)
theta_dot = [0]; % Initial pitch velocity assumption (rad/s)
dt = time(2) - time(1); % Simulation timestep (s)
i = 1; % Iterator count
for t = 0+dt:dt:time(end)
    theta_ddot(i) = 1/Itheta * (ms*ax(i)*hp + ms*g*hp*theta_sim(i)
 - Kf*a^2*theta_sim(i) - Kr*b^2*theta_sim(i) - Cf*a^2*theta_dot(i) -
 Cr*b^2*theta_dot(i));
    theta_dot(i+1) = theta_dot(i) + theta_ddot(i)*dt;
    theta_sim(i+1) = theta_sim(i) + theta_dot(i)*dt;
    i = i+1;
end

% Plot theta vs t
figure()
plot((0:dt:time(end)),theta_rec,'.','color','blue')
hold on
plot((0:dt:time(end)),theta_sim,'.','color','red')
xlabel('${t}$ (sec)','interpreter','latex')
ylabel('${\theta}$ ($rad$)','interpreter','latex')
legend('${\theta}_{recorded}$','${\theta}_{simulated}$','interpreter','latex')
hold off

% Plot theta_dot vs t
figure()
plot((0:dt:time(end)),theta_dot,'.','color','blue')
xlabel('${t}$ (sec)','interpreter','latex')
ylabel('${\dot\theta}$ ($rad/s$)','interpreter','latex')

% Plot theta_ddot vs t
figure()
plot((dt:dt:time(end)),theta_ddot,'.','color','magenta')
xlabel('${t}$ (sec)','interpreter','latex')
ylabel('${\ddot\theta}$ ($rad/s^2$)','interpreter','latex')
```

Ans 4C.B: The estimated suspension damping coefficients (assumung they are equal)
considering RMSE between given data and simulation results are 18470.000000 Ns/m



$\theta_{recorded}$
$\theta_{simulated}$
0.025
0.02
0.015
0.01
0.005
0
-0.005
θ (rad)
0
1
2
3
4
5
6
7
t (sec)


0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
$\dot{\theta}$ (rad/s)
0
1
2
3
4
5
6
7
t (sec)


1.5
1
0.5
0
-0.5
-1
-1.5
-2
$\ddot{\theta}$ (rad/s^2)
0
1
2
3
4
5
6
7
t (sec)

Published with MATLAB® R2022a