3.1-1

Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of Θ-notation, prove that $max(f(n), g(n)) = \Theta(f(n) + g(n))$

if $f(n)$ and $g(n)$ be asymptotically nonnegative functions:

$$f(n) \leq max(f(n), g(n))$$
$$g(n) \leq max(f(n), g(n))$$
$$f(n) + g(n) \leq 2 * max(f(n), g(n))$$
$$1/2(f(n) + g(n)) \leq max(f(n), g(n)) \leq f(n) + g(n)$$

, so $max(f(n), g(n)) = \Theta(f(n) + g(n))$

3.1-2

Show that for any real constants a and b, where $b > 0$,

$$(n + a)^b = \Theta(n^b)$$

The fastest growing part is n^b, so $c_1 * n^b \geq (n + a)^b \geq c_2 * n^b$, for some c_1 and $c_2 = a^b$. So,

$$(n + a)^b = \Theta(n^b)$$

3.1-3

Explane why the statement, "The running time of algorithm A is at least $O(n^2)$," is meaningless.

O- show the upper bound of time. "At least" means lower bound. This concepts contradict each other.

3.1-4

Is $2^{n+1} = O(2^n)$? Is $2^{2n} = O(2^n)$?

$$2^{n+1} = 2 * 2^n$$

, so

$$2 * 2^n \leq c * 2^n$$

, for $c \geq 2$ So $2^{n+1} = O(2^n)$

$$2^{2n} = 2^n * 2^n$$
$$2^n * 2^n \leq c * 2^n$$

2^n is unbounded function, so there is no c bigger than 2^n for all n. So $2^{2n} \neq O(2^n)$

3.1-5

Prove Theorem 3.1.

For any two function $f(n)$ and $g(n)$, we have $f(n) = \Theta(g(n))$ if and only if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.

if $g(n)$ is upper bound and lower bound function than $g(n)$ is tight bound. So if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$ then $f(n) = \Theta(g(n))$.

Let us assume $f(n) = \Theta(g(n))$ and there is same n_k $f(n_k) > O(g(n_k))$. But in this case $\Theta(g(n))$ is not a tight bound. Contradiction. Same with $f(n_k) < \Omega(g(n_k))$

3.1-6

Prove that the running time of an algorithm is $\Theta(g(n))$ if and only if its worst-case running time is $O(g(n))$ and its best-case running time is $\Omega(g(n))$

From Theorem 3.1

3.1-7

Prove that $o(g(n)) \cap \omega(g(n))$ is the empty set.

$$f(n) = o(g(n)) : c * g(n) > f(n)$$

, there is no c that $c * g(n) \leq f(n)$

$$f(n) = \omega(g(n)) : c * g(n) < f(n)$$

, there is no c that $c * g(n) \geq f(n)$

Contradiction.

3.1-8

We can extend out notation to the case of two parameters n and m that can go to infinity independently at different rates. For a given function $g(n, m)$, we denote by $O(g(n, m))$ the set of functions

$O(g(n, m)) = \Big\{$f(n,m), there exist positive c, n_0, and m_0 such that $0 \leq f(n, m) \leq cg(n, m)$ for all $n \geq n_0$ o

Give corresponding difenition for $\Omega(g(m, n))$ and $\Theta(g(m, n))$

$\Omega(g(n, m)) = \Big\{$f(n,m), there exist positive c, n_0, and m_0 such that $cg(n, m) \leq f(n, m)$ for all $n \geq n_0$ or m

$\Theta(g(n, m)) = \Big\{$f(n,m), there exist positive c, n_0, and m_0 such that $c_1 g(n, m) \leq f(n, m) \leq c_2 g(n, m)$ for al