Reinforcement Learning IV

Lecture 22

Learning strategy

Reinforcement Learning

Knowledge of **Environment**	Model-based (planning)	Model-free
No knowledge Must learn from experience		**Monte Carlo Learning**
Perfect knowledge Known MDP	**Dynamic Programming** Policy iteration Value iteration	

Reinforcement Learning

Learning strategy

Knowledge of Environment

	Model-based (planning)	Model-free
No knowledge Must learn from experience	**Reinforcement Learning** Simulation-based search value/policy → take actions → experience → learn a model (involves supervised learning techniques) → model → plan (using the model) → value/policy	Monte Carlo Learning Temporal Difference Learning π → Policy evaluation (prediction) → v v → Policy improvement (optimization) → π
Perfect knowledge Known MDP	**Dynamic Programming** Policy iteration Value iteration π → Policy evaluation (prediction) → v v → Policy improvement (optimization) → π	**This class**: 1. How to **compute optimal policies** for known MDPs? 2. How to extend this to the case **without full knowledge** of MDPs (model-free learning)

From Dynamic Programming to true RL

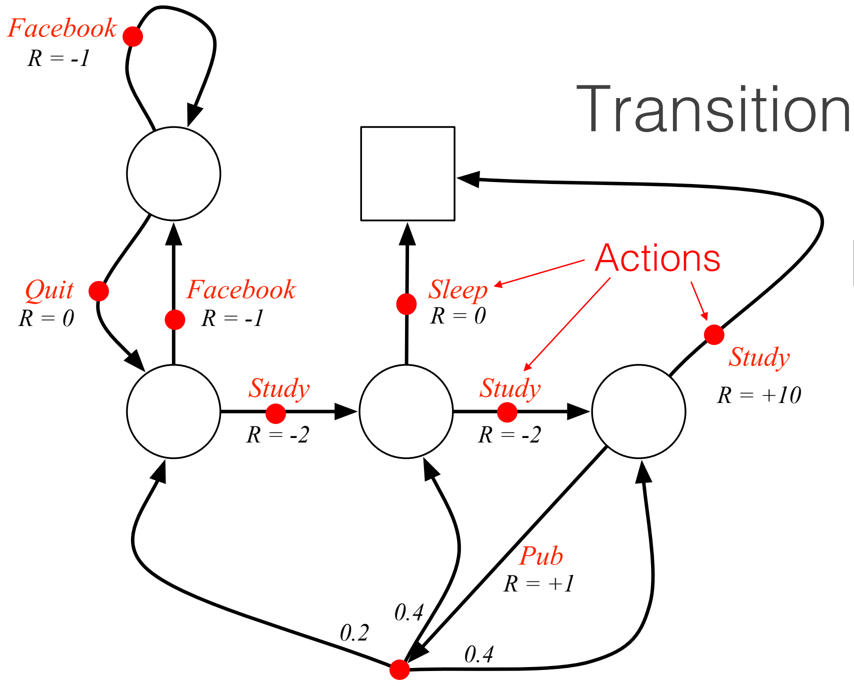
We assume a **fully known MDP environment**
(Markov Decision Process)

1. What returns will a policy yield?	**Policy evaluation**
2. How can we find a better policy?	**Policy improvement**
3. How do we find the best policy?	**Policy iteration**
4. How do we find the best policy faster?	**Value iteration**
5. Are there other approaches?	**Generalized Policy Iteration**
What if we don't have a fully known MDP?	**Monte Carlo Methods**

Dynamic Programming

Markov Decision Process

Components:
State space S
Transition probabilities, P
Rewards, R
Discount rate, γ
Actions, A



Returns (Expected future rewards)

(discount factor weights the the future)

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} \ldots$$

Policy (how we choose actions)

(can be stochastic or deterministic)

$$\pi(a|s) = P(a|s)$$

State value function

(expected return from state s, and following policy π)

$$v_\pi(s) = E[G_t|s]$$

$$v_\pi(s) = E[R_s^a + \gamma v_\pi(s')|s]$$

Action value function

(expected return from state s, taking action a, and following policy π)

$$q_\pi(s,a) = E[G_t|s,a]$$

$$q_\pi(s,a) = E[R_s^a + \gamma q_\pi(s',a')|s,a]$$

David Silver, UCL, 2015

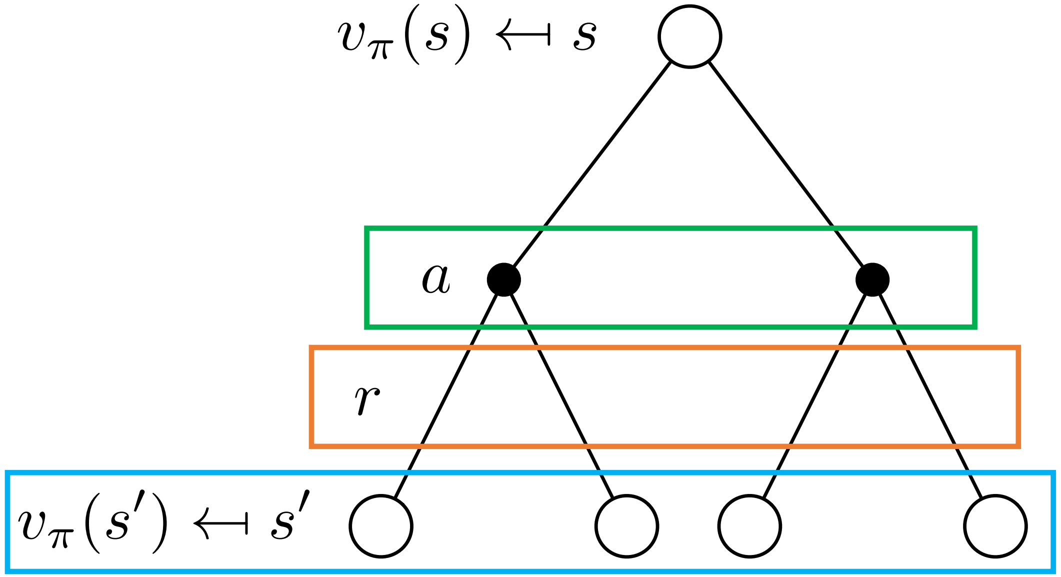
Bellman Expectation Equations for the state value function

(expected return from state s, and following policy π)

$$v_\pi(s) = E[G_t|s]$$

$$v_\pi(s) = E[R_s^a + \gamma v_\pi(s')|s]$$

$$R_s^a = E[r_{t+1}|S_t = s, A_t = a]$$



Expectation over the possible actions

Expectation over the rewards

(based on state and choice of action)

Expectation over the next possible states

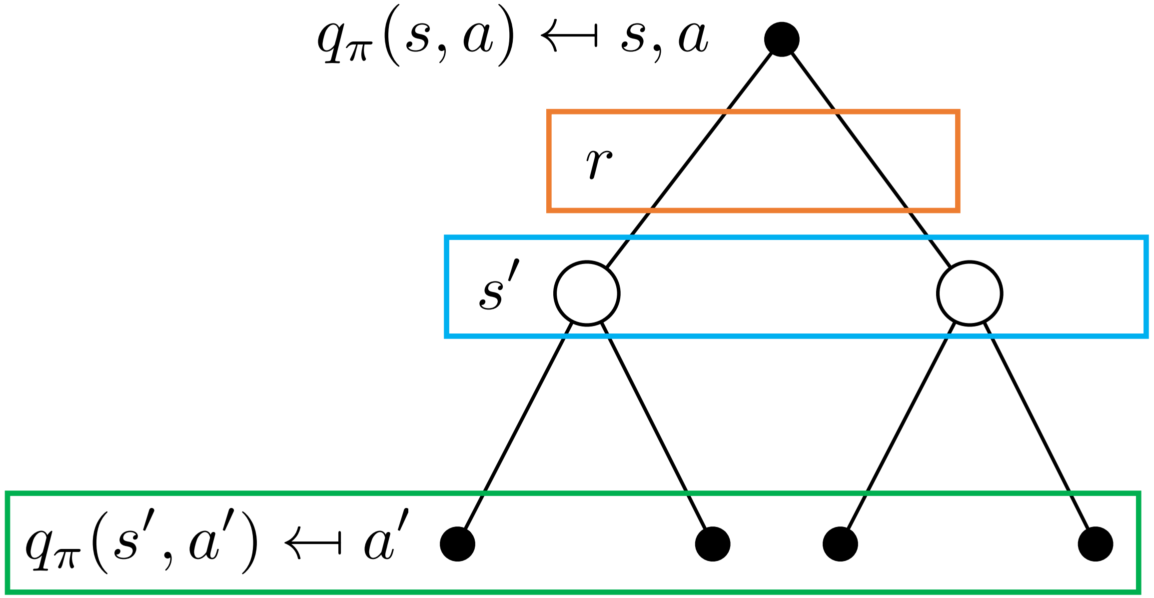
$$v_\pi(s) = \sum_a \pi(a|s)\left(R_s^a + \gamma \sum_{s'} P_{ss'}^a v_\pi(s')\right)$$

Bellman Expectation Equations for the action value function

(expected return from state s, taking action a, then following policy π)

$$q_\pi(s,a) = E[G_t|s,a]$$
$$q_\pi(s,a) = E[R_s^a + \gamma q_\pi(s',a')|s,a]$$

$$R_s^a = E[r_{t+1}|S_t = s, A_t = a]$$



Expectation over the rewards

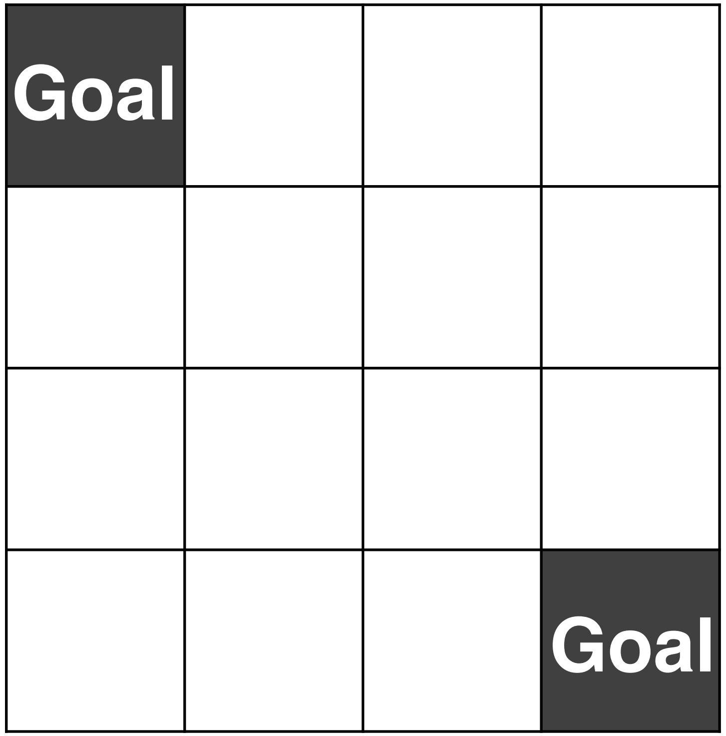
(based on state and choice of action)

Expectation over the next possible states

Expectation over the possible actions

$$q_\pi(s,a) = R_s^a + \gamma \sum_{s'} P_{ss'}^a \sum_{a'} \pi(a'|s') q_\pi(s',a')$$

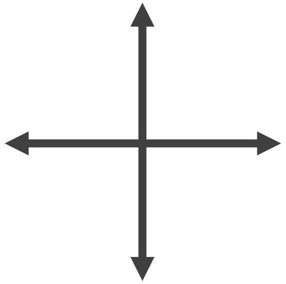
Running example: Gridworld



16 states and 2 terminal states labeled “goal”

Valid actions:

(unless there is a wall)



Reward:

-1 for all transitions

(until the terminal state has been reached)

Note: actions that would take the agent off the board are not allowed

Sutton and Barto, 2018

1. Policy Evaluation

Input: policy $\pi(a|s)$

Output: value function $v_\pi(s)$

(unknown)

1. Select a policy function to evaluate (estimate the value function)
2. Start with a guess of the value function, v_0 (often all zeros)
3. **Iteratively** apply the Bellman Expectation Equation to "backup" the values until they converge on the actual value function for the policy, v_π

$$v_0 \rightarrow v_1 \rightarrow \cdots \rightarrow v_\pi$$

Adapted from David Silver, 2015

1. Policy Evaluation
in Gridworld

$$v_0(s)$$

(initialization)

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

Policy: $$\pi(a|s) = \frac{1}{N_{\text{valid\_actions}}}$$

Randomly go in any valid direction

Value function initialization:

$v_0(s) = 0$ (all zeros)

We estimate the value function that corresponds to the policy: $v_\pi(s)$

1. Policy Evaluation in Gridworld

Policy: $\pi(a|s) = 1/N_{\text{valid\_actions}}$
(randomly go in any direction)

$v_0(s)$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

Bellman Expectation Equation:

$$v_{k+1}(s) = \sum_a \pi(a|s) \left(R_s^a + \gamma \sum_{s'} P_{ss'}^a v_k(s') \right)$$

In Gridworld: $\frac{1}{N_a}$ $\quad -1 \quad$ 1 (once you pick an action there's no uncertainty as to which state you'll transition to)

$$v_{k+1}(s) = \sum_a \frac{1}{N_a} \left(-1 + \sum_{s'} v_k(s') \right) = -1 + \sum_a \frac{1}{N_a} \sum_{s'} v_k(s') \qquad = -1 + \sum_a \frac{1}{N_a} v_k(s')$$

Each action leads to only one state, so the sum over states is not needed

Average of the value of the N_a neighboring states

1. Policy Evaluation
in Gridworld

$$v_{k+1}(s) = -1 + \sum_a \frac{1}{N_a} v_k(s')$$

$$v_1 = -1 + \sum_a \frac{1}{4} v_k(s') \quad = -1$$

$v_0(s)$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

One neighborhood
in $v_0(s)$

	0		
0		0	
	0		

$v_1(s)$

0	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	0

$v_0(s)$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

$v_1(s)$

0	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	0

$v_2(s)$

0	-1.7	-2	-2
-1.7	-2	-2	-2
-2	-2	-2	-1.7
-2	-2	-1.7	0

$v_3(s)$

0	-2.4	-2.9	-3.0
-2.4	-2.9	-3.0	-2.9
-2.9	-3.0	-2.9	-2.4
-3.0	-2.9	-2.4	0

$v_{10}(s)$

0	-6.1	-8.4	-9.0
-6.1	-7.7	-8.4	-8.4
-8.4	-8.4	-7.7	-6.1
-9.0	-8.4	-6.1	0

$v_\infty(s) = v_\pi(s)$

0	-14	-20	-22
-14	-18	-20	-20
-20	-20	-18	-14
-22	-20	-14	0

We've found the value function (expected returns) from our random movement policy

1. Policy Evaluation

in Gridworld

2. Policy Improvement

Input: policy $\pi(a|s)$

Output: better policy $\pi'(a|s)$

Definition of better: has greater or equal expected return in all states:
$v_{\pi'}(s) \geq v_\pi(s)$ for all states

1. Select a policy function to improve
2. Evaluate the value function (our last discussion)
3. **Greedily** select a new policy, π', that chooses actions that maximize value

$$\pi'(s) = \text{greedy}(s)$$

(i.e. pick the action that brings us to the state with highest value)

Adapted from David Silver, 2015

2. Policy Improvement

Input: policy $\pi(a|s)$

Output: better policy $\pi'(a|s)$

How do we do this: $\pi'(s) = \text{greedy}(s)$

i.e. pick the **action** that brings us to the state with **highest value**

We can use the state value function to help us choose the right action:

$$\pi'(s) = \arg\max_a q_\pi(s, a)$$

Reminder:

Action value function

(expected return from state s, taking action a, and following policy π)

$$q_\pi(s, a) = E[G_t | s, a]$$

Adapted from David Silver, 2015

2. Policy Improvement
in Gridworld

$v_0(s)$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

$v_\infty(s) = v_\pi(s)$

0	-14	-20	-22
-14	-18	-20	-20
-20	-20	-18	-14
-22	-20	-14	

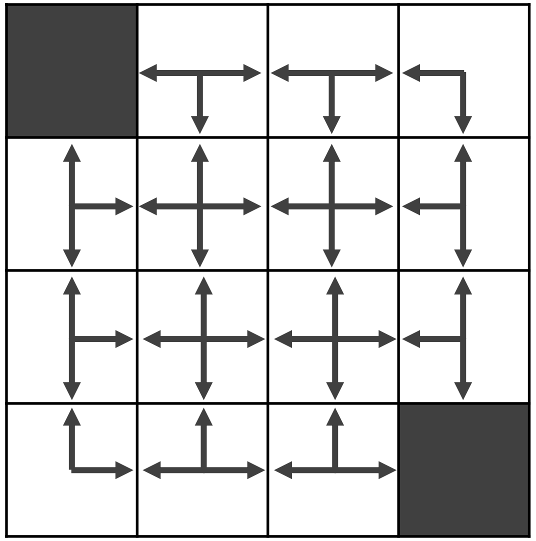
Value function:

In this case, $q_\pi(s, \pi(s)) = v_\pi(s)$ since each action leads to only one state

Initial policy:

$\pi(a|s) =$ randomly go in any valid direction

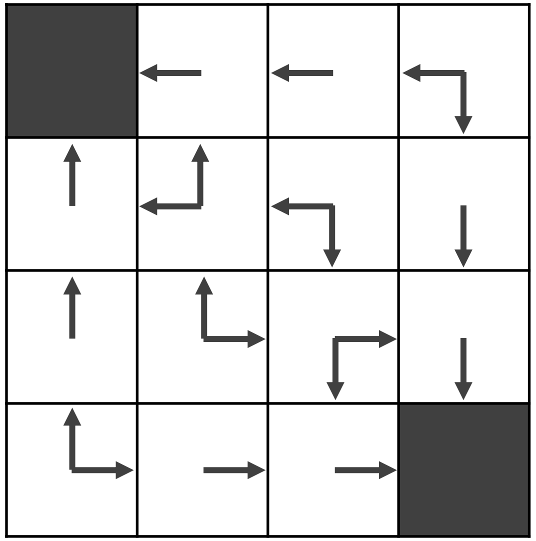
$\pi(s)$



Improved policy:

(in this case this is an optimal policy)

$\pi'(s)$



3. Policy Iteration

Input: policy $\pi(a|s)$

Output: **best** policy $\pi^*(a|s)$

Best in the sense that: $v_{\pi^*}(s) \geq v_\pi(s)$ for all states and for all **policies**

Policy **Evaluation**

$v \rightarrow v_\pi$

$\pi \rightleftarrows v$

$\text{greedy}(v_\pi) \rightarrow \pi'$

Policy **Improvement**

⋮

This process will converge onto the optimal functions

$\pi^* \rightleftarrows v^*$

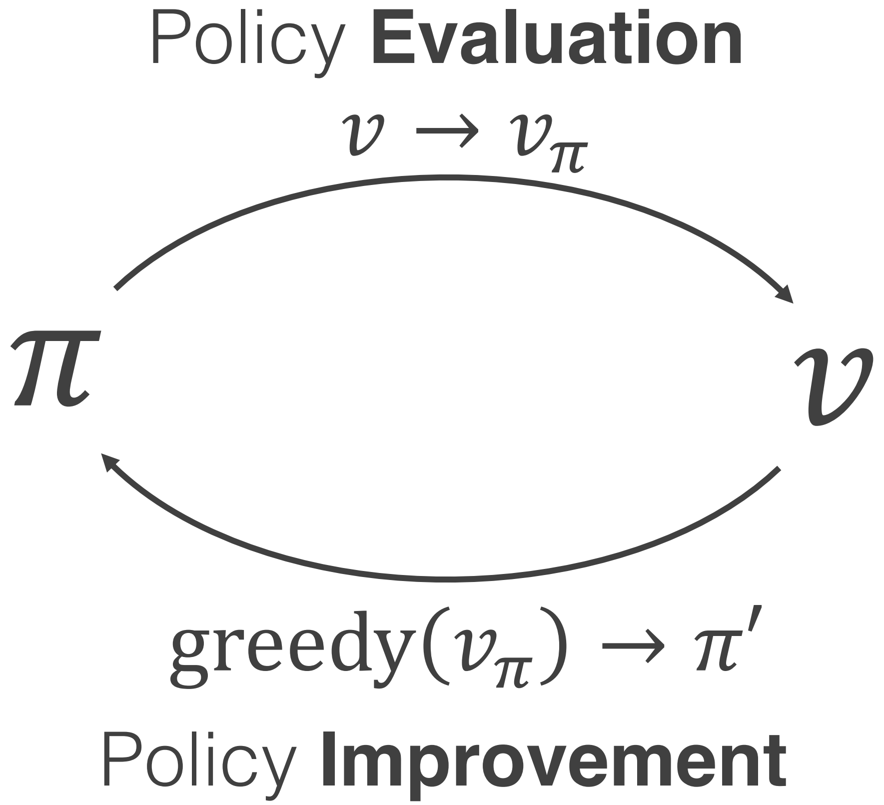
$$\pi_0 \xrightarrow{E} v_{\pi_0} \xrightarrow{I} \pi_1 \xrightarrow{E} v_{\pi_1} \rightarrow \cdots \xrightarrow{I} \pi^* \xrightarrow{E} v^*$$

Adapted from David Silver, 2015 and Sutton and Barto, 1998

3. Policy Iteration

Input: policy $\pi(a|s)$

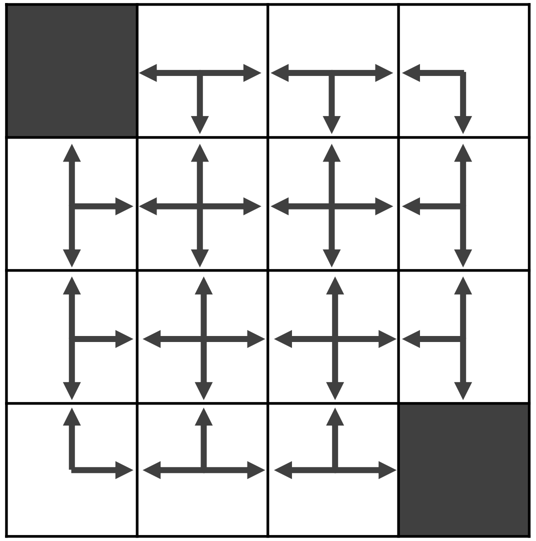
Output: **best** policy $\pi^*(a|s)$



1. **Policy Evaluation**: estimate v_π
   Iterative policy evaluation
   Note: This is VERY slow
2. **Policy Improvement**: generate $\pi' \geq \pi$
   Greedy policy improvement
3. Iterate 1 and 2 until convergence

Adapted from David Silver, 2015 and Sutton and Barto, 1998

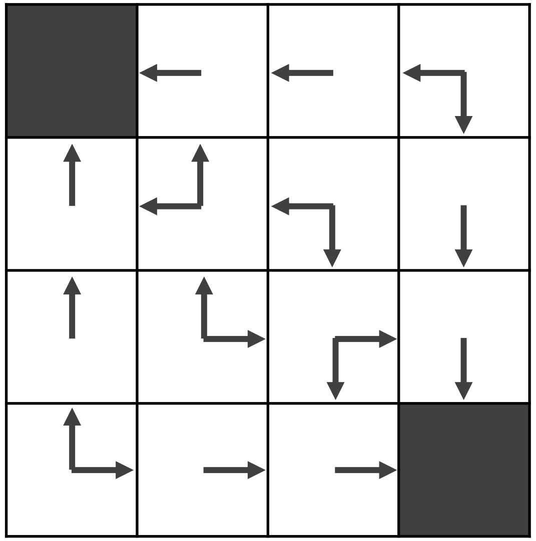
1 $\pi_0(s)$



Policy Evaluation

Policy Improvement

3 $\pi_1(s) = \pi^*(s)$



Policy Evaluation

$v_{\pi^*}(s)$

$v_0(s)$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

2 $v_\infty(s) \rightarrow v_{\pi_0}(s)$

0	-14	-20	-22
-14	-18	-20	-20
-20	-20	-18	-14
-22	-20	-14	

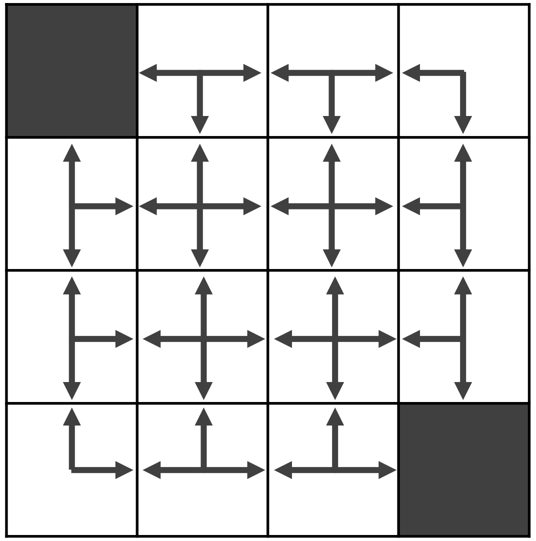
$v_0(s)$

0	-14	-20	-22
-14	-18	-20	-20
-20	-20	-18	-14
-22	-20	-14	

4 $v_\infty(s) \rightarrow v_{\pi_0}(s)$

0	-1	-2	-3
-1	-2	-3	-2
-2	-3	-2	-1
-3	-2	-1	0

$\pi_0(s)$



$v_0(s)$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

$v_1(s)$

0	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	0

What if we stopped after one sweep. This is…

4. Value Iteration

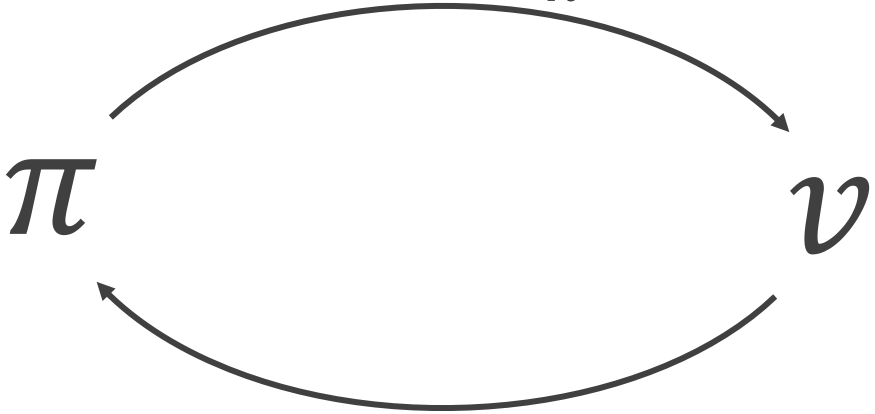
4. Value Iteration

Input: policy $\pi(a|s)$

Output: **best** policy $\pi^*(a|s)$

Policy **Evaluation**

$v \to v_\pi$



$\text{greedy}(v_\pi) \to \pi'$

Policy **Improvement**

1. **Policy Evaluation**: estimate v_π
   One-sweep of policy evaluation
2. **Policy Improvement**: generate $\pi' \geq \pi$
   Greedy policy improvement
3. Iterate 1 and 2 until convergence

Adapted from David Silver, 2015 and Sutton and Barto, 1998

$v_0(s)$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

$v_1(s)$

0	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	0

$v_2(s)$

0	-1.7	-2	-2
-1.7	-2	-2	-2
-2	-2	-2	-1.7
-2	-2	-1.7	0

$v_3(s)$

0	-2.4	-2.9	-3.0
-2.4	-2.9	-3.0	-2.9
-2.9	-3.0	-2.9	-2.4
-3.0	-2.9	-2.4	0

$v_{10}(s)$

0	-6.1	-8.4	-9.0
-6.1	-7.7	-8.4	-8.4
-8.4	-8.4	-7.7	-6.1
-9.0	-8.4	-6.1	0

$v_\infty(s) = v_\pi(s)$

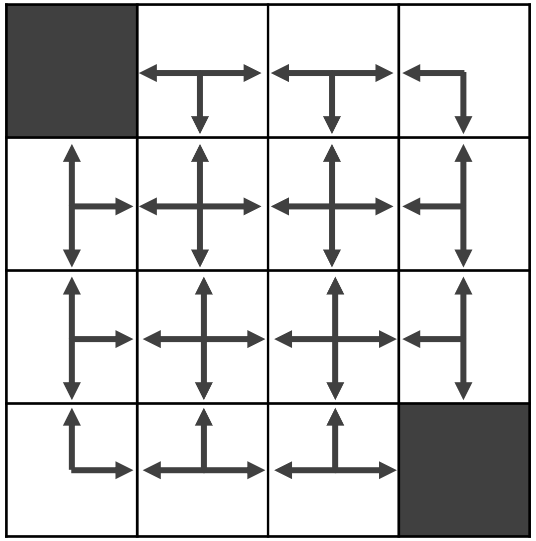
0	-14	-20	-22
-14	-18	-20	-20
-20	-20	-18	-14
-22	-20	-14	

So far, we've run policy evaluation all the way to convergence (**this is slow**)

1 $v_0(s)$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

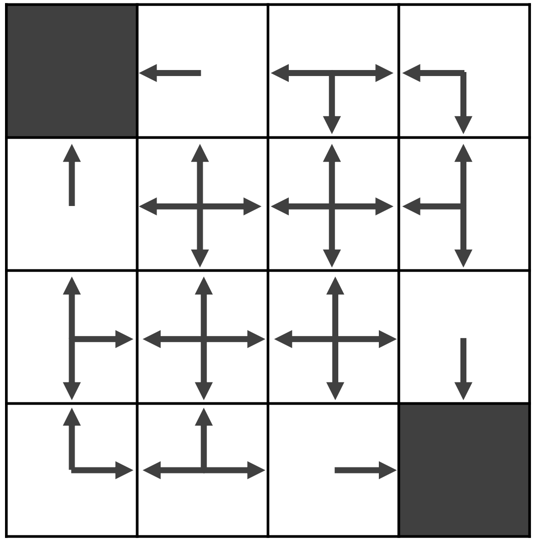
2 $\pi_0(s)$



3 $v_1(s)$

0	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	-1
-1	-1	-1	0

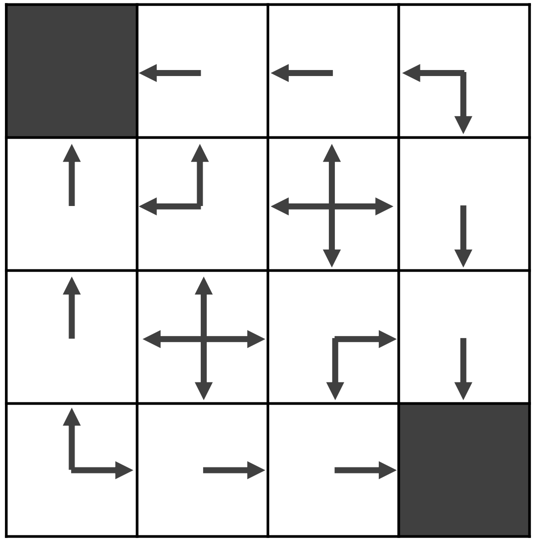
4 $\pi_1(s)$



5 $v_2(s)$

0	-1	-2	-2
-1	-2	-2	-2
-2	-2	-2	-1
-2	-2	-1	0

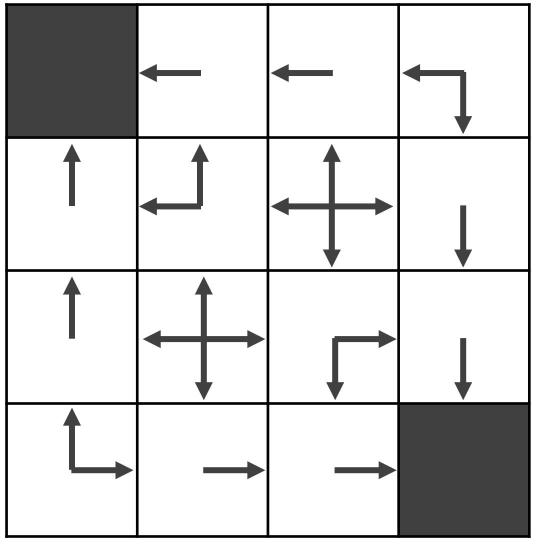
6 $\pi_2(s)$



7 $v_3(s) = v_{\pi^*}(s)$

0	-1	-2	-3
-1	-2	-3	-2
-2	-3	-2	-1
-3	-2	-1	0

8 $\pi_3(s) = \pi^*(s)$



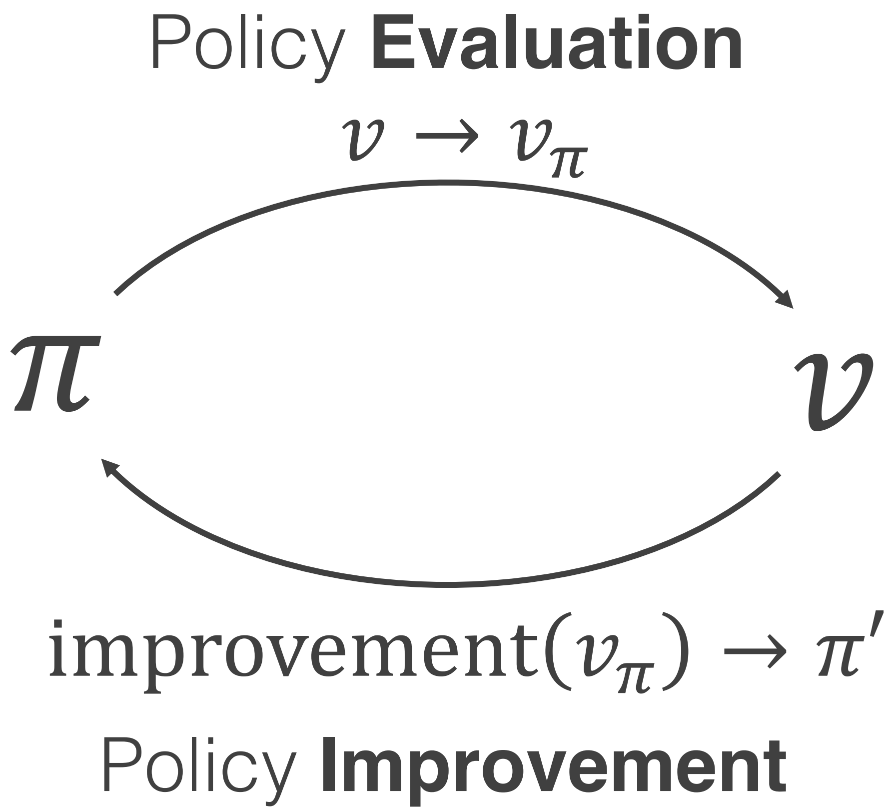
Demo

https://cs.stanford.edu/people/karpathy/reinforcejs/gridworld_dp.html

5. Generalized Policy Iteration

Input: policy $\pi(a|s)$

Output: **best** policy $\pi^*(a|s)$



1. **Policy Evaluation**: estimate v_π
   Any policy evaluation algorithm
2. **Policy Improvement**: generate $\pi' \geq \pi$
   Any policy improvement algorithm
3. Iterate 1 and 2 until convergence

Adapted from David Silver, 2015 and Sutton and Barto, 1998

So far, we've assumed full knowledge of the environment (MDP)

What if we DO NOT assume full knowledge of the environment (MDP)

This means we have to learn by experience:

true reinforcement learning

6. Monte Carlo Policy Evaluation

For **state** values

Input:	policy	$\pi(a\|s)$
Output:	state value	$v_\pi(s)$

1. Select a policy function to evaluate (estimate the value function)
2. Start with a guess of the value function, v_0 (often all zeros)
3. Repeat forever:
   - A. Generate an episode (take actions until a terminal state)
   - B. Save the returns following the first occurrence of each state
   - C. Assign $\text{AVG}(\text{Returns}(s)) \rightarrow \hat{v}_\pi(s)$

Sutton and Barto, 1998

6. Monte Carlo Policy Evaluation

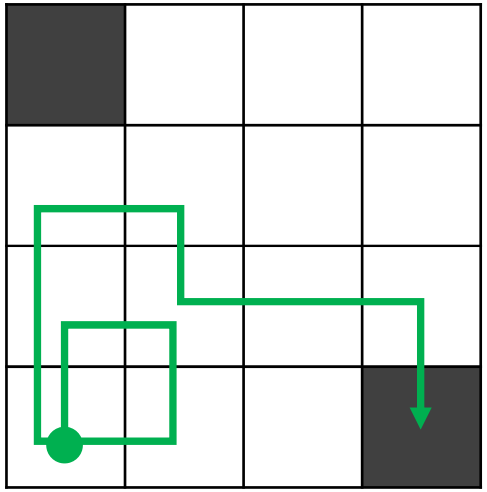
For **state** values

"First Visit"

For each state, we store the running returns seen **after** the first visit to that state

Episode 1

Total Reward: -11



Episode 1 **returns** after the first visit of each state

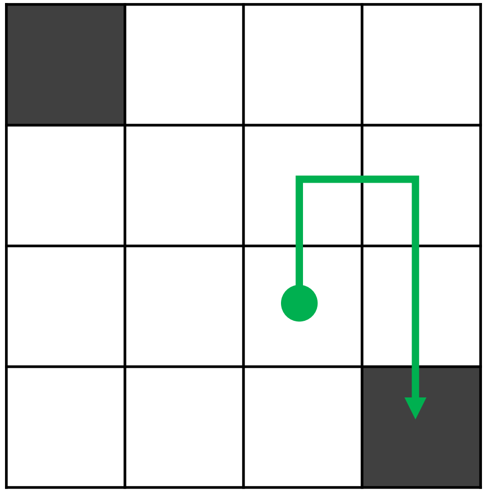
$G^{(1)}$

-5	-4		
-10	-9	-2	-1
-11	-8		

Discount rate: $\gamma = 1$

Episode 2

Total Reward: -4



Episode 2 **returns** from the first visit of each state

$G^{(2)}$

		-3	-2
		-4	-1

Discount rate: $\gamma = 1$

$v_0(s)$

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

The value function is the **running average** of the returns after the visit to that state, averaged over episodes

(only average over episodes when state is visited)

$v_1(s)$

0	0	0	0
-5	-4	0	0
-10	-9	-2	-1
-11	-8	0	0

v_1 is just the first visit returns, $G^{(1)}$

v_2 is the average first visit returns, $G^{(1)}$ and $G^{(2)}$, for those states visited

$v_2(s)$

0	0	0	0
-5	-4	-3	-2
-10	-9	-3	-1
-11	-8	0	0

Which value function?

The **state value function** doesn't tell us directly about actions

If we don't have a model, to pick a policy we need **action values**

Which value function?

Greedy policy improvement over $v(s)$ **requires a model of the MDP**

$$\pi'(s) = \underset{a}{\operatorname{argmax}}\, R_s^a + P_{ss'}^a v(s')$$

Greedy policy improvement over $q(s,a)$ is **model-free**

$$\pi'(s) = \underset{a}{\operatorname{argmax}}\, q(s,a)$$

And the two value functions are related: $v_\pi(s) = \sum_a \pi(a|s) q_\pi(s,a)$

David Silver, UCL, 2015

6. Monte Carlo Policy Evaluation

For **action** values

Input: policy $\pi(a|s)$

Output: action value $q_\pi(s,a)$

1. Select a policy function to evaluate (estimate its value function)
2. Start with a guess of the action value function, q_0 (often all zeros)
3. Repeat forever:
   - A. Generate an episode (take actions until a terminal state)
   - B. Save returns following first occurrence of each state **& action**
   - C. Assign $\mathrm{AVG}(\mathrm{Returns}(\mathrm{s},\mathrm{a})) \rightarrow \hat{q}_\pi(s,a)$

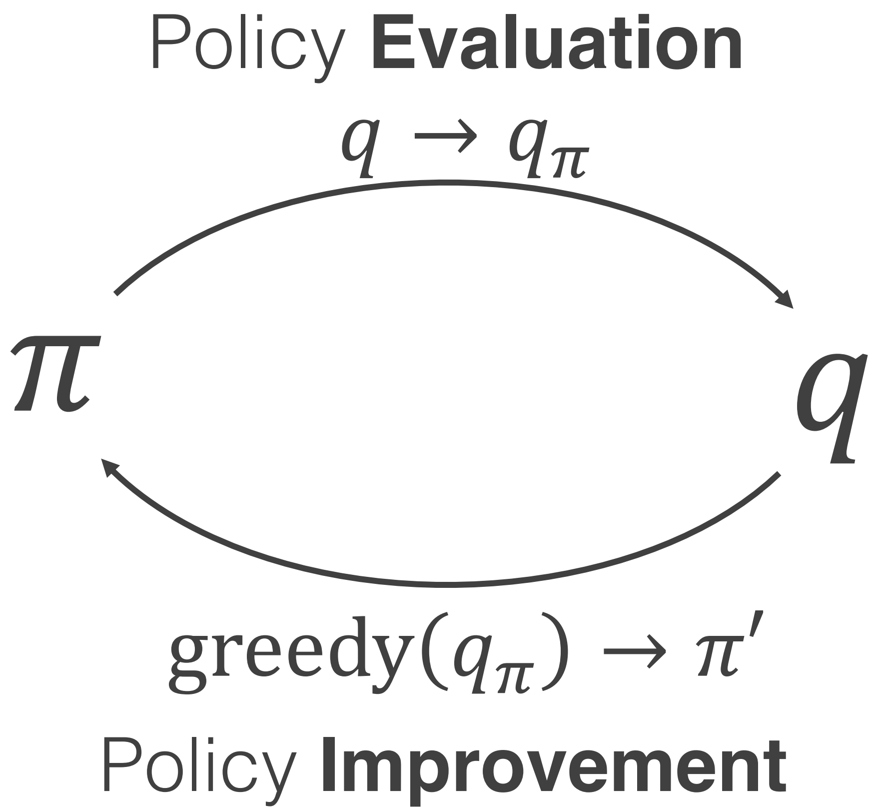
Sutton and Barto, 1998

7. Monte Carlo Control

(policy iteration)

Input: policy $\pi(a|s)$

Output: **best policy** $\pi^*(a|s)$



1. **Policy Evaluation**: estimate q_π
   Monte Carlo action policy evaluation
2. **Policy Improvement**: generate $\pi' \geq \pi$
   Greedy policy improvement
3. Iterate 1 and 2 until convergence

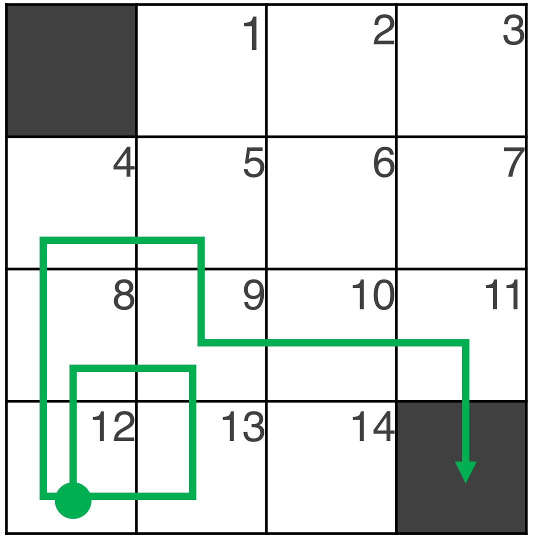
Sutton and Barto, 1998

7. Monte Carlo Control

"First Visit" (of state AND action) is recorded

Episode 1

Total Reward: -11



← State labels

1 MC Policy Evaluation

Episode 1 **returns** after the first visit of each state

-5	**-4**		
-10	**-9**	**-2**	**-1**
-11	**-8**		

Discount rate: $\gamma = 1$

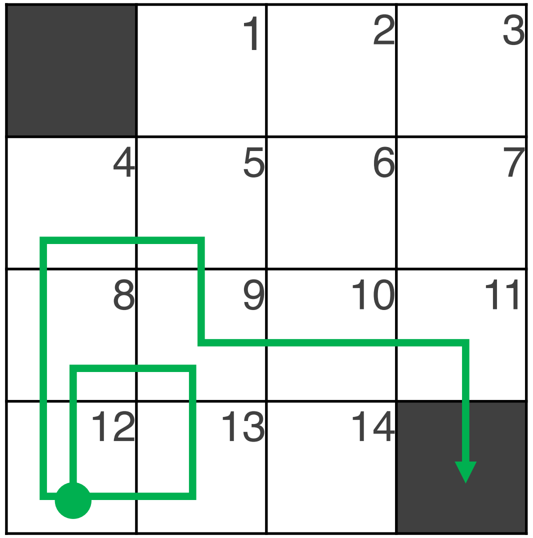
$q_\pi(s, a)$

Action (a): ↑ → ← ↓

State (s)	↑	→	←	↓
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				

7. Monte Carlo Control

"First Visit" (of state AND action) is recorded



State labels

Episode 1

Total Reward: -11

Episode 1 **returns** after the first visit of each state

	1	2	3
4: **-5**	5: **-4**	6	7
8: **-10**	9: **-9**	10: **-2**	11: **-1**
12: **-11**	13: **-8**	14	

Discount rate: $\gamma = 1$

1. MC Policy Evaluation
2. MC Policy Improvement

$$\pi'(s) = \underset{a}{\operatorname{argmax}}\, q_\pi(s,a)$$

Typically this is set to be ϵ-greedy to better learn $q(s,a)$

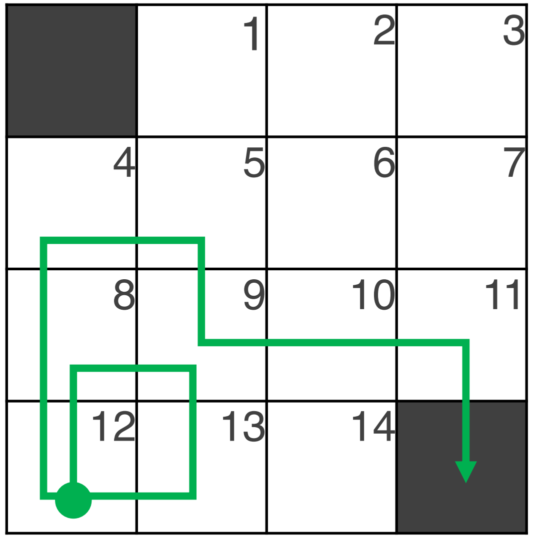
$q_\pi(s,a)$

Invalid action

Action (a):

State (s)	↑	→	←	↓
1				
2				
3				
4		**-5**		
5				**-4**
6				
7				
8	**-6**	**-10**		
9		**-3**		**-9**
10		**-2**		
11				**-1**
12	**-11**			
13			**-8**	
14				

Episode 1



	1	2	3
4 -5	5 -4	6	7
8 -10	9 -9	10 -2	11 -1
12 -11	13 -8	14	

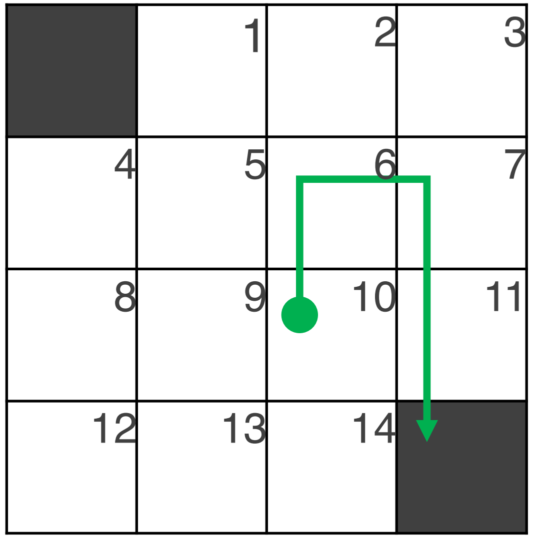
Returns from episode 1

Action (a): ↑ → ← ↓

$q_\pi(s,a)$

State (s)	↑	→	←	↓
1				
2				
3				
4		**-5**		
5				**-4**
6				
7				
8	**-6**	**-10**		
9		**-3**		**-9**
10		**-2**		
11				**-1**
12	**-11**			
13			**-8**	
14				

Episode 2



	1	2	3
4	5	6 -3	7 -2
8	9	10 -4	11 -1
12	13	14	

Returns from episode 2

Action (a): ↑ → ← ↓

$q_\pi(s,a)$

State (s)	↑	→	←	↓
1				
2				
3				
4		**-5**		
5				**-4**
6		**-3**		
7				**-2**
8	**-6**	**-10**		
9		**-3**		**-9**
10	**-4**	**-2**		
11				**-1**
12	**-11**			
13			**-8**	
14				

$q_{\pi^*}(s, a)$

State (s) \ Action (a)	↑	→	←	↓
1		-3	-1	-3
2		-4	-2	-3
3			-3	-3
4	-1	-3		-3
5	-2	-4	-2	-4
6	-3	-3	-3	-3
7	-4		-4	-2
8	-2	-4		-4
9	-3	-3	-3	-3
10	-4	-2	-4	-2
11	-3		-3	-1
12	-3	-3		
13	-4	-2	-4	
14	-3	-1	-3	

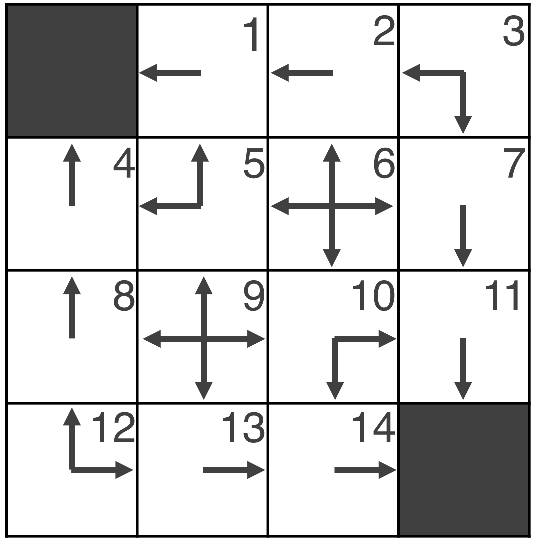
If we're in state 4, take the up action

If we know the optimal action value function, we also have our optimal policy

$v_{\pi^*}(s)$

0	-1 [1]	-2 [2]	-3 [3]
-1 [4]	-2 [5]	-3 [6]	-2 [7]
-2 [8]	-3 [9]	-2 [10]	-1 [11]
-3 [12]	-2 [13]	-1 [14]	0

$\pi^*(s)$



Extensions

Monte Carlo methods require that we finish each episode before updating
Solution: **Temporal Difference** (TD) methods

What if we want to learn about one policy while following or observing another?
Solution: **Off-policy learning** instead of on-policy learning

What if our state space has too many states that we can't build a table of values?
Solution: **Value function approximation** (involving supervised learning techniques)

How can we simulate what the environment might output for next states and rewards?
Solution: **Model-based learning**: simulate the environment and plan ahead

From Dynamic Programming to true RL

We assume a **fully known MDP environment**
(Markov Decision Process) **Sutton & Barto, Chapter 3**

1. What returns will a policy yield? — **Policy evaluation**
2. How can we find a better policy? — **Policy improvement**
3. How do we find the best policy? — **Policy iteration**
4. How do we find the best policy faster? — **Value iteration**
5. Are there other approaches? — **Generalized Policy Iteration**

Sutton & Barto, Chapter 4

What if we don't have a fully known MDP? — **Monte Carlo Methods**
Sutton & Barto, Chapter 5

Reinforcement Learning

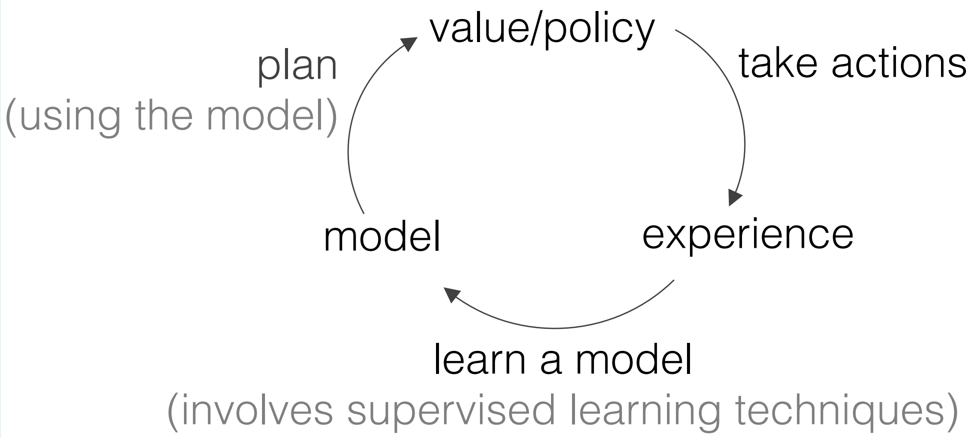
Learning strategy

	Model-based (planning)	Model-free
No knowledge Must learn from experience	Simulation-based search	**Monte Carlo Learning** Temporal Difference Learning
Perfect knowledge Known MDP	**Dynamic Programming** Policy iteration Value iteration	

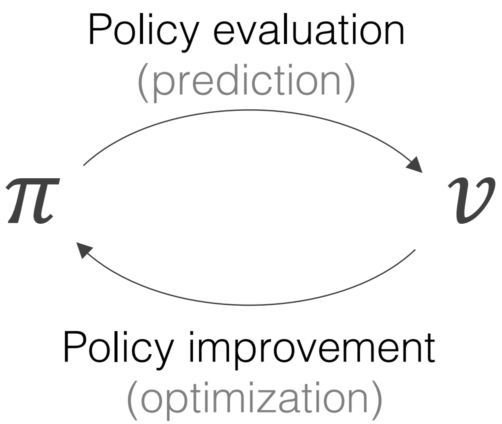
Knowledge of **Environment**

Reinforcement Learning (No knowledge row: Model-based and Model-free)

Simulation-based search



Monte Carlo Learning
Temporal Difference Learning



Dynamic Programming
Policy iteration
Value iteration