Problem 1

Table A–24 Mechanical Properties of Three Non-Steel Metals

(a) Typical Properties of Gray Cast Iron

ASTM Number	Tensile Strength S_{ut}, kpsi	Compressive Strength S_{uc}, kpsi	Shear Modulus of Rupture S_{su}, kpsi	Modulus of Elasticity, Mpsi: Tension†	Modulus of Elasticity, Mpsi: Torsion	Endurance Limit* S_e, kpsi	Brinell Hardness H_B	Fatigue Stress-Concentration Factor K_f
20	22	83	26	9.6–14	3.9–5.6	10	156	1.00
25	26	97	32	11.5–14.8	4.6–6.0	11.5	174	1.05
30	31	109	40	13–16.4	5.2–6.6	14	201	1.10
35	36.5	124	48.5	14.5–17.2	5.8–6.9	16	212	1.15
40	42.5	140	57	16–20	6.4–7.8	18.5	235	1.25
50	52.5	164	73	18.8–22.8	7.2–8.0	21.5	262	1.35
60	62.5	187.5	88.5	20.4–23.5	7.8–8.5	24.5	302	1.50

*Polished or machined specimens.

†The modulus of elasticity of cast iron in compression corresponds closely to the upper value in the range given for tension and is a more constant value than that for tension.

Note: The American Society for Testing and Materials (ASTM) numbering system for gray cast iron is such that the numbers correspond to the *minimum tensile strength* in kpsi. Thus an ASTM No. 20 cast iron has a minimum tensile strength of 20 kpsi. Note particularly that the tabulations are *typical* of several heats.

$S_{ut} = 26$ ksi $\quad S_{uc} = 97$ ksi

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

a) $\sigma_x = 30$ ksi

$\sigma_y = 10$ ksi

$\tau_{xy} = \emptyset$ ksi

$$\Rightarrow \sigma_{1,2} = \frac{30\text{ ksi} + 10\text{ ksi}}{2} \pm \sqrt{\left(\frac{30\text{ ksi} - 10\text{ ksi}}{2}\right)^2 + \emptyset\text{ ksi}^2}$$

$$= 30\text{ ksi},\ 10\text{ ksi}$$

Brittle Coulomb-Mohr: $\dfrac{\sigma_1}{S_{ut}} - \dfrac{\sigma_3}{S_{uc}} = \dfrac{1}{n}$

$$\frac{30\text{ ksi}}{26\text{ ksi}} - \frac{\emptyset\text{ ksi}}{97\text{ ksi}} = \frac{1}{n} \Rightarrow \boxed{n = 0.867}$$

Modified Mohr: $\sigma_A \geq \sigma_B \geq \emptyset \rightarrow \sigma_A = \dfrac{S_{ut}}{n}$

a) continued:

$$30\text{ ksi} = \frac{26\text{ ksi}}{n} \Rightarrow \boxed{n = 0.867}$$

b) $\sigma_x = -20\text{ ksi}$
$\sigma_y = 20\text{ ksi}$
$\tau_{xy} = \varnothing\text{ ksi}$

$$\Rightarrow \sigma_{1,2} = \frac{-20\text{ ksi} + 20\text{ ksi}}{2} \pm \sqrt{\left(\frac{-20\text{ ksi} - 20\text{ ksi}}{2}\right)^2 + \varnothing\text{ ksi}^2}$$

$$= 20\text{ ksi}, -20\text{ ksi}$$

Brittle Coulomb-Mohr: $\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} = \frac{1}{n}$

$$\frac{20\text{ ksi}}{26\text{ ksi}} - \frac{-20\text{ ksi}}{97\text{ ksi}} = \frac{1}{n} \Rightarrow \boxed{n = 1.03}$$

Modified Mohr: $\sigma_A \geq \varnothing \geq \sigma_B$ and $\left|\frac{\sigma_B}{\sigma_A}\right| \leq 1 \Rightarrow \sigma_A = \frac{S_{ut}}{n}$

$$20\text{ ksi} = \frac{26\text{ ksi}}{n} \Rightarrow \boxed{n = 1.3}$$

c) $\sigma_x = 15\text{ ksi}$
$\sigma_y = \varnothing\text{ ksi}$
$\tau_{xy} = -20\text{ ksi}$

$$\Rightarrow \sigma_{1,2} = \frac{15\text{ ksi} + \varnothing\text{ ksi}}{2} \pm \sqrt{\left(\frac{15\text{ ksi} - \varnothing\text{ ksi}}{2}\right)^2 + (-20\text{ ksi})^2}$$

$$= 28.86\text{ ksi}, -13.86\text{ ksi}$$

Brittle Coulomb-Mohr: $\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} = \frac{1}{n}$

$$\frac{28.86\text{ ksi}}{26\text{ ksi}} - \frac{-13.86\text{ ksi}}{97\text{ ksi}} = \frac{1}{n} \Rightarrow \boxed{n = 0.798}$$

Problem 1 (continued)

Modified Mohr: $\sigma_A \geq 0 \geq \sigma_B$ and $\left|\frac{\sigma_B}{\sigma_A}\right| \leq 1 \Rightarrow \sigma_A = \frac{S_{ut}}{n}$

$$28.86 \text{ ksi} = \frac{26 \text{ ksi}}{n} \Rightarrow \boxed{n = 0.900}$$

d) $\sigma_x = -10$ ksi
$\sigma_y = 30$ ksi
$\tau_{xy} = -10$ ksi

$$\Rightarrow \sigma_{1,2} = \frac{30 \text{ ksi} - 10 \text{ ksi}}{2} \pm \sqrt{\left(\frac{30 \text{ ksi} + 10 \text{ ksi}}{2}\right)^2 + (-10 \text{ ksi})^2}$$

$$= 32.36 \text{ ksi}, -12.36 \text{ ksi}$$

Brittle Coulomb-Mohr: $\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} = \frac{1}{n}$

$$\frac{32.36 \text{ ksi}}{26 \text{ ksi}} - \frac{-12.36 \text{ ksi}}{97 \text{ ksi}} = \frac{1}{n} \Rightarrow \boxed{n = 0.728}$$

Modified Mohr: $\sigma_A \geq 0 \geq \sigma_B$ and $\left|\frac{\sigma_B}{\sigma_A}\right| \leq 1 \Rightarrow \sigma_A = \frac{S_{ut}}{n}$

$$32.36 \text{ ksi} = \frac{26 \text{ ksi}}{n} \Rightarrow \boxed{n = 0.803}$$

e) $\sigma_x = 25$ ksi
$\sigma_y = 25$ ksi
$\tau_{xy} = 20$ ksi

$$\Rightarrow \sigma_{1,2} = \frac{25 \text{ ksi} + 25 \text{ ksi}}{2} \pm \sqrt{\left(\frac{25 \text{ ksi} - 25 \text{ ksi}}{2}\right)^2 + (20 \text{ ksi})^2}$$

$$= 45 \text{ ksi}, 5 \text{ ksi}$$

Brittle Coulomb-Mohr: $\frac{\sigma_1}{S_{ut}} - \frac{\sigma_3}{S_{uc}} = \frac{1}{n}$

Problem 1 (continued)

$$\frac{45\text{ ksi}}{26\text{ksi}} - \frac{\emptyset\text{ ksi}}{97\text{ksi}} = \frac{1}{n} \Rightarrow \boxed{n = 0.578}$$

Modified Mohr: $\sigma_A \geq \emptyset \geq \sigma_B$ and $\left|\frac{\sigma_B}{\sigma_A}\right| \leq 1 \Rightarrow \sigma_A = \frac{S_{ut}}{n}$

$$45\text{ ksi} = \frac{26\text{ ksi}}{n} \Rightarrow \boxed{n = 0.578}$$

Problem 2

4340 $\quad S_{ut} = 260$ ksi

$S_e' = 100$ ksi $\qquad S_{ut} > 200$ ksi

$k_a = (39.9)(260 \text{ ksi})^{-0.995}$

$= 0.1578$

$$k_a = aS_{ut}^b \qquad (6\text{–}19)$$

Table 6–2
Parameters for Marin Surface Modification Factor, Eq. (6–19)

Surface Finish	Factor a S_{ut}, kpsi	Factor a S_{ut}, MPa	Exponent b
Ground	1.34	1.58	−0.085
Machined or cold-drawn	2.70	4.51	−0.265
Hot-rolled	14.4	57.7	−0.718
As-forged	39.9	272.	−0.995

From C.J. Noll and C. Lipson, "Allowable Working Stresses," *Society for Experimental Stress Analysis*, vol. 3, no. 2, 1946 p. 29. Reproduced by O.J. Horger (ed.) *Metals Engineering Design ASME Handbook*, McGraw-Hill, New York. Copyright © 1953 by The McGraw-Hill Companies, Inc. Reprinted by permission.

k_b not considered since same d for both, k_c, k_d, k_e not considered since identical for both shafts (loading, temp, reliability)

$S_e = k_a S_e' = 0.1578(100 \text{ ksi}) = \underline{\underline{15.78 \text{ ksi}}}$

1040 $\quad S_{ut} = 113$ ksi

$S_e' = 0.5\, S_{ut}$ since $S_{ut} < 200$ ksi

$S_e' = 56.5$ ksi

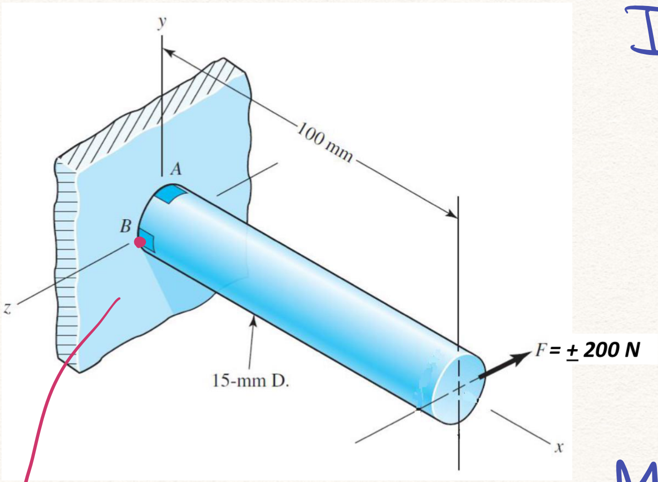
$k_a = (39.9)(113)^{-0.995} = 0.3615$

$S_e = k_a S_e' = (0.3615)(56.5 \text{ ksi}) = \underline{\underline{20.43 \text{ ksi}}}$

The plain carbon steel exhibits higher endurance strength as k_a is more detrimental to as-forged materials with higher S_{ut}. Thus it is <u>not</u> advantageous to use the high-strength steel.

Problem 3 23, HW 3 #1

From Table A-20: $S_{ut} = 590$ MPa, $S_y = 490$ MPa



critcal point B since max bending moment from cantilever loading

$I = \frac{1}{4}\pi r^4 = \frac{1}{4}\pi\left(\frac{0.015m}{2}\right)^4$
$= 2.485 \times 10^{-9}\ m^4$

$J = \frac{1}{2}\pi r^4 = \frac{1}{2}\pi\left(\frac{0.015m}{2}\right)^4$
$= 4.970 \times 10^{-9}\ m^4$

$M = F \times d = (200N)(0.1m) = 20$ Nm

$\sigma_x = \frac{(M_y)(c)}{I_y} = \frac{(20\ Nm)(0.0075m)}{2.485\times10^{-9}\ m^4}$
$= 60.4$ MPa

Table 6-2: cold drawn

$k_a = (4.51)(590\ MPa)^{-0.265} = 0.832$

for non-rotating round beam in bending, need d_{eff}

$d_{eff} = 0.370(d) = 0.370(0.015m) = 0.00555\ m$
$= 5.55\ mm$

$k_b = 1.24 d^{-0.107}$ $\quad 2.79 \le d \le 51$ mm
$= 1.24(5.55)^{-0.107} = 1.032$

$k_c = 1$ (only bending) $\quad k_e = 0.85$
$k_d = 1$ $\quad k_f = 1$

Problem 3 (continued)

$S_e' = 0.5\,S_{ut} = 0.5(590\text{ MPa}) = 295\text{ MPa}$

$\hookrightarrow S_{ut} \leq 1400\text{ MPa}$

$S_e = k_a k_b k_c k_d k_e k_f S_e'$

$= (0.832)(1.032)(1)(0.85)(1)(295\text{ MPa}) = 215.3\text{ MPa}$

$$n_f = \frac{S_e}{\sigma_{rev}} = \frac{215.3\text{ MPa}}{60.4\text{ MPa}} = \boxed{3.56\text{, so infinite life achieved } (>1)}$$

$$n_y = \frac{S_y}{\sigma_{max}} = \frac{490\text{ MPa}}{60.4\text{ MPa}} = \boxed{8.11 = n_y}$$

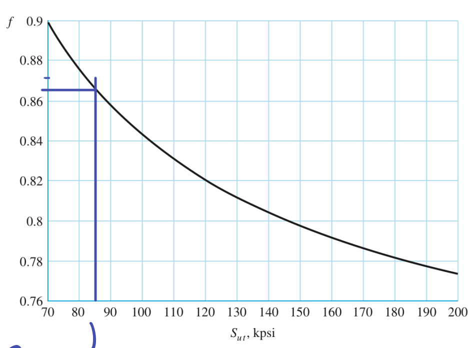
$\hookrightarrow$ yield factor of safety

Alternatively, number of cycles to failure for high-cycle fatigue:

$$N = \left(\frac{\sigma_{rev}}{a}\right)^{1/b}, \quad f \approx 0.865$$

Figure 6–18

Fatigue strength fraction, f, of S_{ut} at 10^3 cycles for $S_e = S_e' = 0.5S_{ut}$ at 10^6 cycles.



$590\text{ MPa} = 85.7\text{ ksi}$

$$a = \frac{(f \cdot S_{ut})^2}{S_e} = \frac{(0.865 \cdot 590\text{ MPa})^2}{215.3\text{ MPa}}$$

$= 1209.7$

$$b = -\frac{1}{3}\log\left(\frac{(f)(S_{ut})}{S_e}\right)$$

$$= -\frac{1}{3}\log\left(\frac{(0.865)(590\text{ MPa})}{215.3\text{ MPa}}\right)$$

$= -0.125$

$$N = \left(\frac{60.4\text{ MPa}}{1209.7}\right)^{-\frac{1}{0.125}}$$

$= \underline{2.59 \times 10^9\text{ cycles}}$

Problem 3 (continued)

a) (continued) because $N = 2.59 \times 10^9 > 10^6$ cycles for infinite life, it will achieve infinite life under these conditions.

b) $M = (1200\,N)(0.1\,m) = 120\,Nm$

$$\sigma_x = \frac{(M_y)(C)}{I} = \frac{(120\,Nm)(0.0075\,m)}{2.485 \times 10^{-9}\,m^4} = 362.2\ MPa$$

Se will remain the same from part a), thus

$$n_f = \frac{S_e}{\sigma_{rev}} = \frac{215.3\,MPa}{362.2\,MPa}$$ $\boxed{= 0.594 < 1, \text{ so infinite life not achieved.}}$

$$n_y = \frac{S_y}{\sigma_{max}} = \frac{490\,MPa}{362.2\,MPa} = 1.35$$ so yielding will not occur (good to check)

For number of cycles to failure:

$N = \left(\frac{\sigma_{rev}}{a}\right)^{1/b}$, using same a and b from part a)

$$N = \left(\frac{362.2\,MPa}{1209.7}\right)^{-1/0.125}$$ $\boxed{= 15{,}500 \text{ cycles} < 10^6 \text{ for infinite life!}}$

Problem 4

1084 Steel $S_{ut} = 551.41 \text{ MPa}$

$S_e' = 0.5 S_{ut} = 0.5(551.41 \text{ MPa}) = 275.7 \text{ MPa}$
↳ for $S_{ut} < 1400 \text{ MPa}$

$k_a = (1.58)(551.41)^{-0.085}$ (for ground finish)
$= 0.924$

$k_b = 0.879(d)^{-0.107}$ (for $0.11 \le d \le 2$ in)
$= 0.879(2)^{-0.107}$
$= 0.8162$

$k_c = 0.59$ (for torsion)

$k_d = 1$ (room temp)

$k_e = 0.702$ (for 99.99% reliability)

$k_f = 1$ (When not specified, $= 1$)

$S_e = k_a k_b k_c k_d k_e k_f S_e'$
$= (0.924)(0.8162)(0.59)(1)(0.702)(1)(275.7 \text{ MPa})$
$= \boxed{86.12 \text{ MPa}}$

Low Carbon Steel $S_{ut} = 447.48 \text{ MPa}$

$S_e' = 0.5 S_{ut} = 0.5(447.48 \text{ MPa}) = 223.74 \text{ MPa}$

$k_a = 4.51(447.48)^{-0.265} = 0.895$ (for machined finish)

Problem 4 (continued)

$k_b = 1$ (axial loading only)

$k_c = 0.85$ (axial loading only)

$k_d = 0.995$ ($500^\circ F$)

$k_e = 0.868$ (for 95% reliability)

$k_f = 1$ (when not specified, $= 1$)

$$S_e = k_a k_b k_c k_d k_e k_f S_e'$$
$$= (0.895)(1)(0.85)(0.995)(0.868)(1)(223.74\ \text{MPa})$$
$$= \boxed{147.00\ \text{MPa}}$$