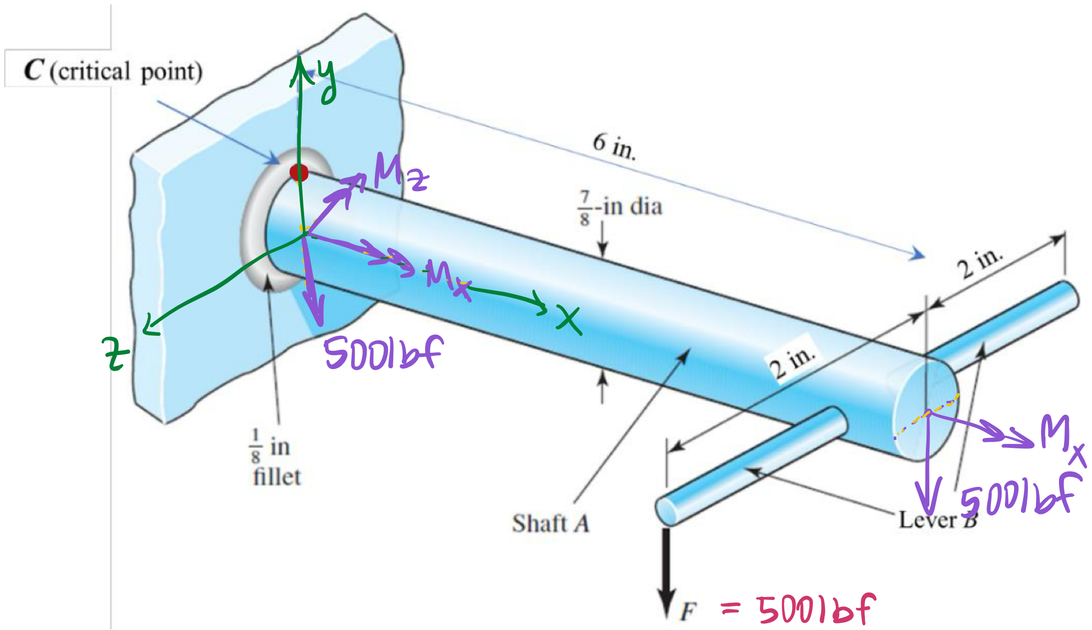
HW 1 Solution

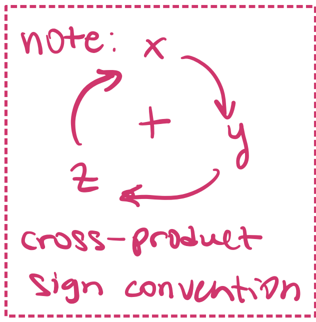
Problem 1

$$r = \tfrac{7}{8}\text{in.}/2 = 0.4375\text{ in}$$



$$I = \tfrac{1}{4}\pi r^4 = \tfrac{1}{4}(\pi)(0.4375\text{in})^4 = 0.0288\text{ in}^4$$

$$J = \tfrac{1}{2}\pi r^4 = \tfrac{1}{2}(\pi)(0.4375\text{in})^4 = 0.0575\text{ in}^4$$



Transfer of Forces

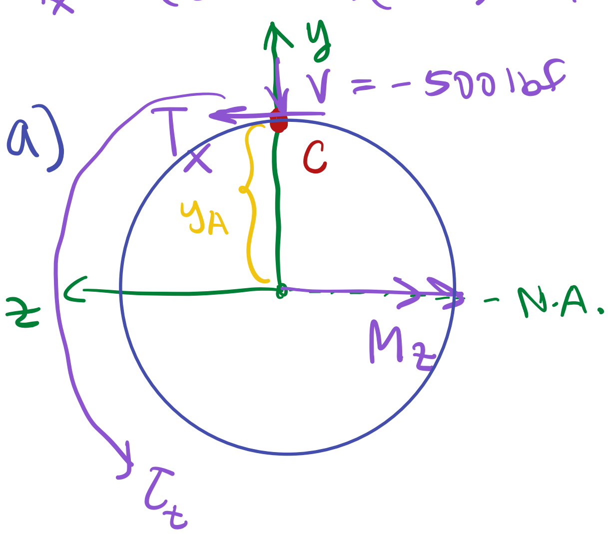
$$V = -F = -500\text{ lbf}$$

$$M_x = F \times r = (500\text{ lbf})(2\text{ in}) = 1000\text{ lb}\cdot\text{in}$$

$$M_z = F \times r = -(500\text{ lbf})(6\text{ in}) = -3000\text{ lb}\cdot\text{in}$$

$$T_x = (500\text{ lbf})(2\text{ in}) = 1000\text{ lb}\cdot\text{in}$$

a)



from cross product of z-moment with y-direction

$$\sigma_x = -\frac{(M_z)(y_A)}{I}$$

$$= -\frac{(-3000\text{ lb}\cdot\text{in})(0.4375\text{ in})}{0.0288\text{ in}^4}$$

$$= 45{,}614\text{ psi} = \boxed{45.6\text{ ksi}}$$

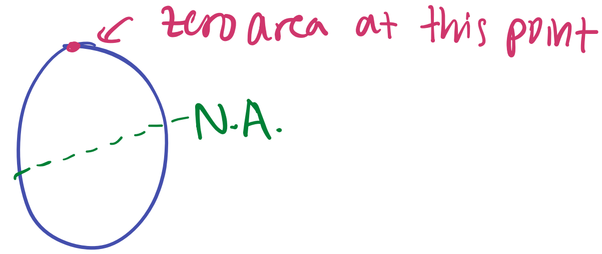
$$\boxed{\sigma_z = \emptyset\text{ ksi}}$$

a) continued:

$$(\tau_{xz})_{shear} = \frac{VQ}{It}$$

$$= \emptyset$$

$$Q = A'y'$$
$$= \emptyset$$



$$(\tau_{xz})_{Torsion} = \frac{T_x r}{J} = \frac{(1000 lb\cdot in)(0.4375 in)}{0.0575 in^4} = 7602 \text{ psi}$$
$$= 7.6 \text{ ksi}$$

$$\tau_{xz} = (\tau_{xz})_{shear} + (\tau_{xz})_{Torsion} = \boxed{7.602 \text{ ksi}}$$

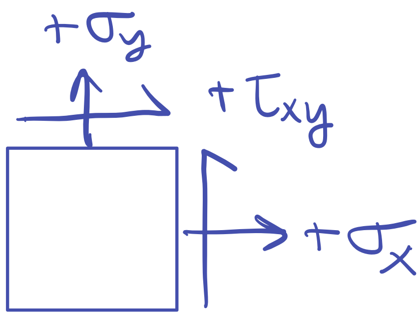
b) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_z}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2}$$

$$\boxed{\Rightarrow \sigma_1 = 46.83 \text{ ksi}, \ \sigma_2 = -1.234 \text{ ksi}}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_z}{2}\right)^2 + \tau_{xz}^2} = \boxed{24.034 \text{ ksi}}$$

2.1

Sign convention:



$\Rightarrow \sigma_x = -20\,\text{MPa}$

$\sigma_y = 280\,\text{MPa}$

$\tau_{xy} = -130\,\text{MPa}$

a) $$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-20\,\text{MPa} + 280\,\text{MPa}}{2} \pm \sqrt{\left(\frac{-20\,\text{MPa} - 280\,\text{MPa}}{2}\right)^2 + (-130\,\text{MPa})^2}$$

$$= 130\,\text{MPa} \pm 198.49\,\text{MPa}$$

$$\Rightarrow \boxed{\sigma_1 = 328.5\,\text{MPa}, \quad \sigma_2 = -68.5\,\text{MPa}}$$

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{328.5\,\text{MPa} - (-68.5\,\text{MPa})}{2}$$

$$\boxed{= 198.5\,\text{MPa}}$$

2.2

a) $\sigma_x = 150\,MPa$, $\sigma_y = 400\,MPa$, $\tau_{xy} = -40\,MPa$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \underbrace{\frac{150+400}{2} \pm \sqrt{\left(\frac{150-400}{2}\right)^2 + (-40)^2}}$$

$$\boxed{\Rightarrow \sigma_1 = 406\,MPa,\quad \sigma_2 = 144\,MPa,\quad \sigma_3 = \emptyset\,MPa}$$

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{406\,MPa - \emptyset\,MPa}{2} = \boxed{203\,MPa}$$

b) MSS: $n = \dfrac{S_y/2}{\tau_{max}}$; note that $\sigma_3 = \emptyset\,MPa$, σ_1, σ_2 same as part a
(Tresca)

Von Mises: $n = \dfrac{S_y}{(\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2)^{1/2}}$

For failure not to occur: $n \geq 1$ so

$$1 \leq \frac{S_y/2}{\tau_{max}} \rightarrow 203\,MPa \leq \frac{S_y}{2} \rightarrow S_y \geq 406\,MPa$$

or Von Mises:

$$1 \leq \frac{S_y}{(150^2 - (150)(400) + (400)^2 + 3(-40)^2)^{1/2}} \rightarrow 1 \leq \frac{S_y}{356.8}$$

$S_y \geq 356.8\,MPa$

1018 CD, 1020 CD, 1030 CD, 1035 CD, 1040 CD, 1045CD, 1050CD, 1060, 1080, 1095

all 4000 series

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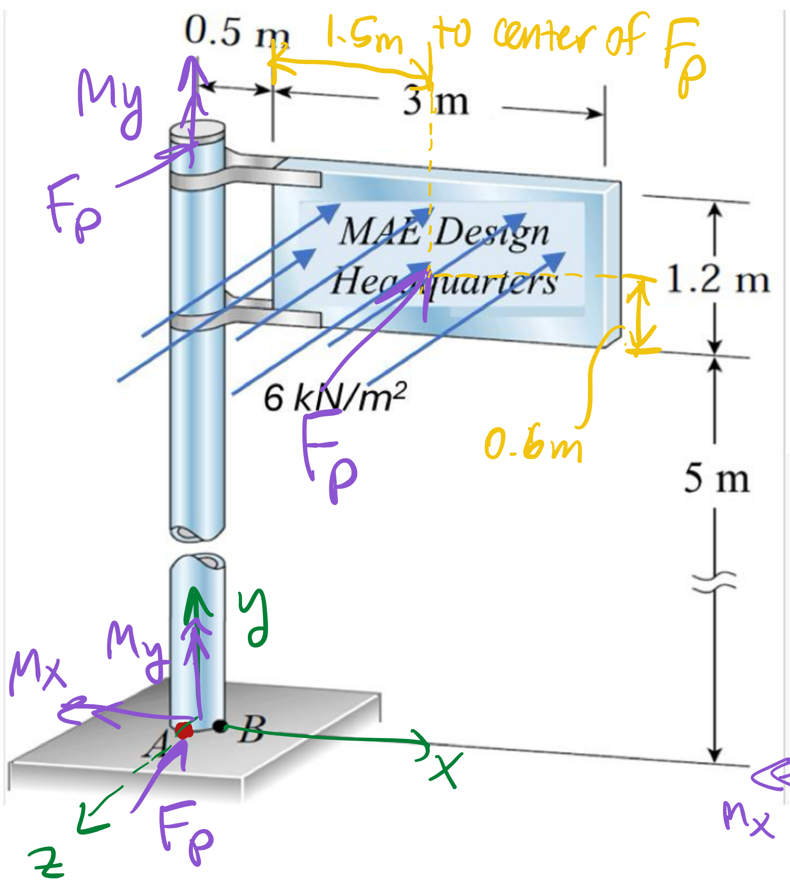
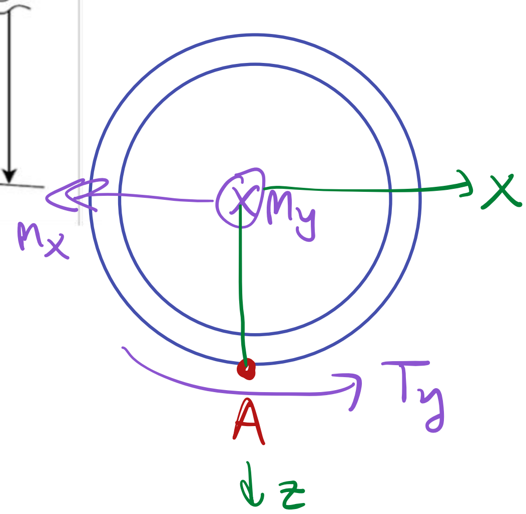
a)

$r_o = 0.11\,m,\ r_i = 0.09\,m$



$I = \frac{\pi}{4}(r_o^4 - r_i^4)$
$= 6.346 \times 10^{-5}\ m^4$

$J = \frac{\pi}{2}(r_o^4 - r_i^4)$
$= 1.269 \times 10^{-4}\ m$



Transfer of Forces

$V_z = F_p = PA = (6\,kN/m^2)(3m \times 1.2m) = -21.6\ kN$

$T_y = F_p(1.5m + 0.5m) = (21.6\,kN)(2m) = 43.2\ kNm$

$M_x = F_p(5m + 0.6m) = (-21.6\,kN)(5.6m) = -120.96\ kNm$

from cross product x and z (see prob. 1 for diagram)

Axial

$\sigma_y = \frac{-M_x r}{I} = \frac{-(-120.96 \times 10^3 N)(0.11m)}{6.346 \times 10^{-5} m^4} = \boxed{209.7\ MPa}$

Shear:

$\tau_{yx} = \frac{T_y r}{J} = \frac{(43.2 \times 10^3 N)(0.11m)}{1.269 \times 10^{-4} m^4} = \boxed{37.4\ MPa}$

b)

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{209.7}{2} \pm \sqrt{\left(\frac{209.7}{2}\right)^2 + (37.4)^2}$$

$$\boxed{\sigma_1 = 216.17 \text{ MPa} \qquad \sigma_2 = -6.47 \text{ MPa}}$$

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{216.17 \text{ MPa} - (-6.47 \text{ MPa})}{2}$$

$$= 111.32 \text{ MPa}$$