MAE 190 – Design of Machine Elements

Winter 2024

Midterm

Name: ____________________   Student ID: ____________________

Exam Instructions:

- The exam must be completed individually. You are not allowed to communicate with your classmates and/or any other outside source.
- Open book exam. No cell phones, cameras, MATLAB, etc.
- Please show all work to receive partial credit and circle final answers.
- If you feel that there is any missing/inconsistent information, please make your own assumption (explain it), and continue your work.
- Budget your time to complete all three problems. Do not overspend time on one problem.

1. The state of stress acting at a critical point on the seat frame of an automobile during a crash test is shown in the figure.

a) Does **ASTM 40 gray cast iron** withstand this load? Use ***Brittle Coulomb Mohr*** and ***Modified Mohr*** theories to justify your answer numerically.

b) If ASTM 40 gray cast iron cannot withstand this load, suggest another type of gray cast iron for the seat frame under the shown loading. Explain your choice using any applicable brittle failure theory of your choice. (10 points)

ASTM 40 Gray Cast Iron: $S_{ut} = 42.5$ Ksi

$S_{uc} = 140$ Ksi

a)

$\sigma_x = 44$ Ksi $\quad \sigma_y = -6$ Ksi $\quad \tau_{xy} = 14$ Ksi

Principle Stress:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{44+(-6)}{2} \pm \sqrt{\left(\frac{44-(-6)}{2}\right)^2 + 14^2}$$

$\sigma_{1,2} = 47.65$ Ksi, -9.65 Ksi

Modified Mohr: $\quad \sigma_A > 0 > \sigma_B \quad \& \quad \left|\frac{\sigma_B}{\sigma_A}\right| \leq 1 \quad \rightarrow \sigma_A = \frac{S_{ut}}{n}$

$$n_{MM} = \frac{S_{ut}}{\sigma_A} = \frac{42.7}{47.65} = 0.892$$

Brittle Cuolomb Mohr: $\quad \frac{\sigma_A}{S_{ut}} + \frac{\sigma_B}{S_{uc}} = \frac{1}{n}$

$$n = \left[\frac{\sigma_A}{S_{ut}} + \frac{\sigma_B}{S_{uc}}\right]^{-1} = \left(\frac{47.65}{42.5} - \frac{(-9.65)}{140}\right)^{-1} = 0.840$$

ASTM 40 will not withstand load

b) Modified Mohr: $\quad \sigma_A > 0 > \sigma_B \quad \left|\frac{\sigma_B}{\sigma_A}\right| \leq 1 \quad n = \frac{S_{ut}}{\sigma_A}$

ASTM 50: $n_{MM} = \frac{52.5}{47.65} = 1.102$

ASTM 50 or 60 will widthstand load because $S_{ut} > \sigma_a$

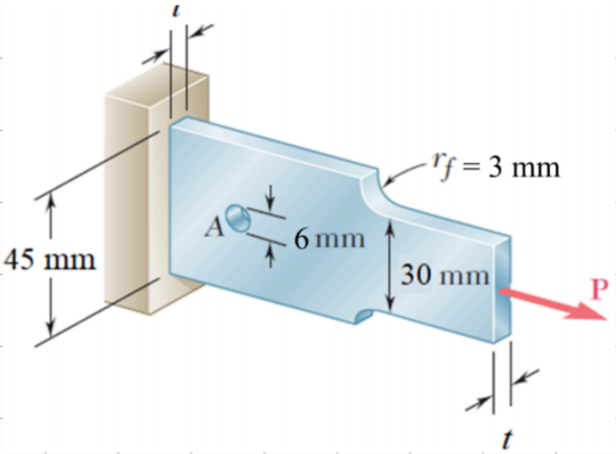
2. A 5-mm-thick plate is **machined** from 1040 Cold-Drawn steel. It is subjected to a completely reversed axial load ***P***, varying between **12 kN** in **compression** and **12 kN** in **tension** at room temperature. The plate has a hole with a **6-mm diameter**, as shown. The **fillet radius** is $\boldsymbol{r_f = 3}$ **mm,** and the plate thickness is $\boldsymbol{t = 5}$ **mm.**

a) Find the **endurance limit** for the link based on a reliability of 95%.

b) Find the **fatigue factor of safety** for the plate based on considering the hole as the critical location.

c) Find the **fatigue factor of safety** for the plate based on considering the shoulder fillet as the critical location. Which section (hole or fillet) is the critical location? (20 points)

1040 Cold Drawn : $S_{ut} = 590\,MPa$ (85Ksi)



a) $S_e = K_a \cdot K_b \cdot K_c \cdot K_d \cdot K_e \cdot S_e'$

$S_{ut} \leq 200\,Ksi\ (1400\,MPa)$

$S_e' = 0.5\,S_{ut} = 0.5(590\,MPa) = 295\,MPa$

$K_a = a\,S_{ut}^{\,b}$ $a = 4.51$ $b = -0.265$ $K_a = 4.51\,(590\,MPa)^{-0.265} = 0.832$ 10th edi.

or $a = 3.04$ $b = -0.217$ $K_a = 3.04\,(590\,MPa)^{-0.217} = 0.761$ 11th edi.

$K_b = 1$ axial load

$K_c = 0.85$ axial loading

$K_d = 1$ room temp.

$K_e = 0.868$ reliability factor 95%

$K_f = 1$

10th : $S_e = 0.832 \cdot 1 \cdot 0.85 \cdot 1 \cdot 0.868 \cdot 1 \cdot 295\,MPa = 180.99\,MPa$

11th : $S_e = 0.761 \cdot 1 \cdot 0.85 \cdot 1 \cdot 0.868 \cdot 1\ 295\,MPa = 165.71\,MPa$

b) Table A-15-1

$\frac{d}{w} = \frac{6mm}{45mm} = 0.13\bar{3}$

$K_t = 2.65$

10th edi.

from fig 6-20 & example 6-6 : $q = 0.84$ for $r = 3.0\,mm$

alternitively: $\sqrt{a} = 0.246 - 3.08(10^{-3})S_{ut} + 1.51(10^{-5})S_{ut}^2 - 2.67(10^{8})S_{ut}^3$

$\sqrt{r} = \sqrt{0.003} = 0.0548$

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\frac{0.0769}{0.0548}} = 0.416$$

$K_f = 1 + q(K_t - 1) = 1 + 0.84(2.65 - 1) \approx 2.386$ or $K_f = 1 + 0.416(2.65 - 1) = 1.68$

$K_f = 2.386$ $\qquad$ $K_f = 1.68$

Hole normal stress: $\sigma_0 = \frac{F}{A} = \frac{12000N}{(5mm)(45mm - 6mm)} = 61.538MPa$

$\sigma_{max} = K_f \cdot \sigma_0 = 2.386(61.5MPa) = 146.83\ MPa$ or $\sigma_{max} = 103.32MPa$

$\sigma_{max} = 147\ MPa$

factor of safety $n = \frac{S_e}{\sigma_{max}} = \frac{180.99}{146.83} = 1.23$

or

$n = \frac{180.99}{103.32} = 1.75$

c) Table A-15-5

$\frac{r}{d} = \frac{3mm}{30mm} = 0.1$ $\qquad$ $\frac{D}{d} = \frac{45mm}{30mm} = 1.5$

$K_t = 2.15$ $\qquad$ $q = 0.84$ or $q = 0.416$

$K_f = 1 + q(K_t - 1) = 1 + 0.85(2.15 - 1) = 1.966$

$K_f = 1.966$ $\qquad$ or $K_f = 1.478$ (if using $q = 0.84$)

Fillet normal stress: $\sigma_0 = \frac{F}{A} = \frac{12000N}{5mm\ (30mm)} = 80MPa$

$\sigma_{max} = 1.966(80MPa) = 157.28\ MPa$ $\qquad$ or $\sigma_{max} = 118.27\ MPa$

factor of safety $n = \frac{S_e}{\sigma_{max}} = \frac{180.99}{157.28} = 1.15$

or $\qquad n = \frac{S_e}{\sigma_{max}} = \frac{180.99}{118.27} = 1.53$ (if using $q = 0.84$)

The fillet is the critical location as $n_{fillet} < n_{Hole}$

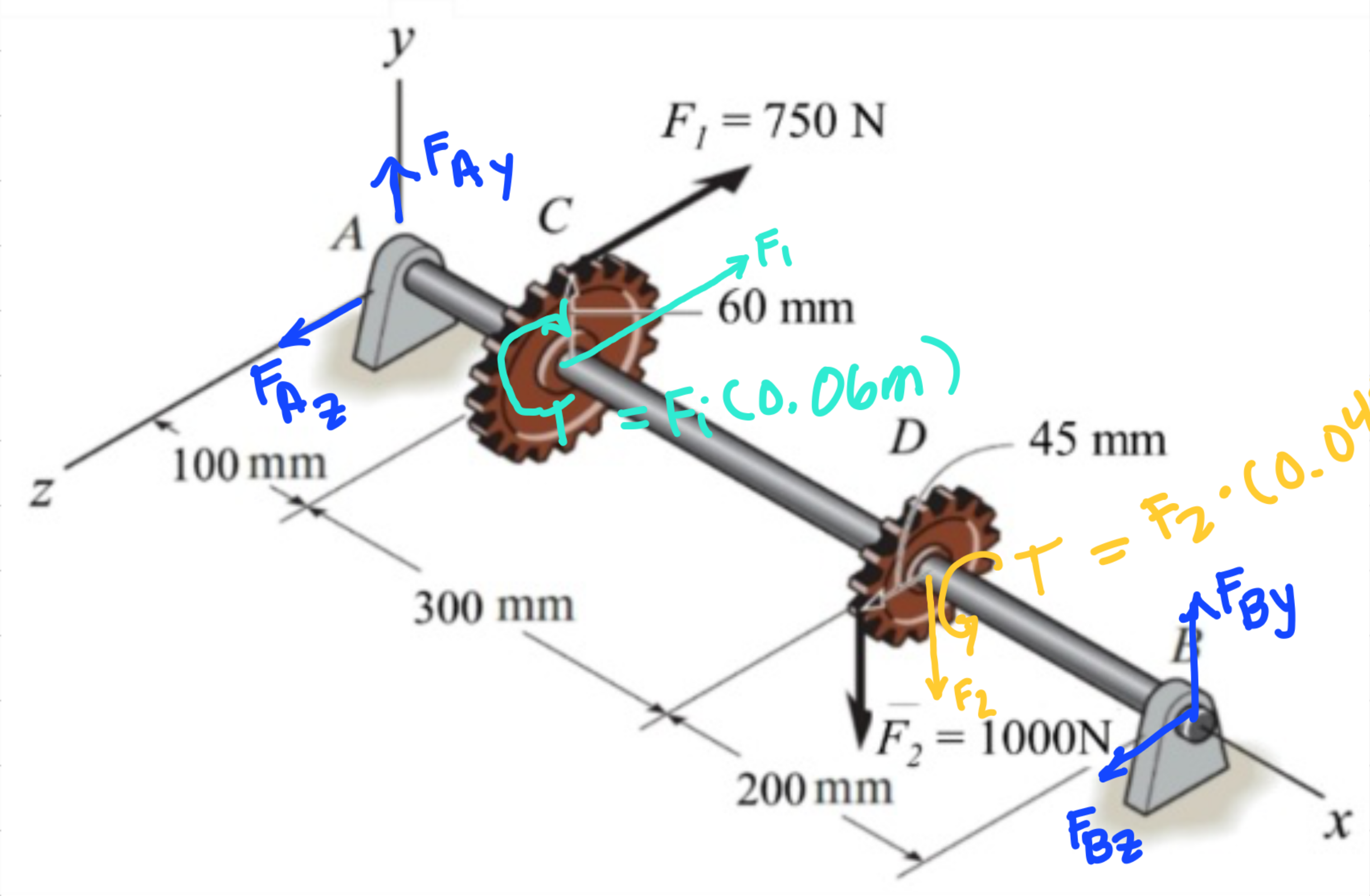
3. The power transmission shaft *AB*, which carries two spur gears of different radii, is shown in the figure. The shaft has a **20 mm diameter** and is made of **wrought aluminum 5052 (H36)**. It is supported by journal bearings at *A* and *B* that only exert radial force components (i.e., *y* and *z* directions). The horizontal force $\boldsymbol{F_1}$ = 750 N, and the vertical force $\boldsymbol{F_2}$ = 1000 N are applied to gears welded to the shaft, as shown.

a) Find the maximum bending moment, *M*, and the transmitted torque, T_x, in the shaft.

b) Determine the minimum factor of safety in the shaft (at the critical section) based on the maximum distortion energy (Von Mises) theory of failure.

Hint: Please use table A-24 (b) for the material properties of wrought aluminum 5052 – H36.

(20 points)



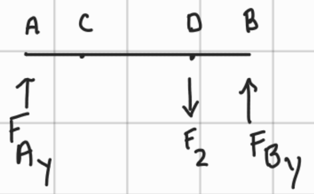
a) Table A-24 : 5056 H36 → $S_y = 234$ MPa

Torques:

$T_1 = -45$N

$T_2 = 45$N

Forces:



$\Sigma F_y = 0 = F_{Ay} + F_{By} - F_2$

$F_{Ay} = F_2 - F_{By}$

$\Sigma M_{Az} = 0 = F_{By}(0.600m) - F_2(0.4m) = 400N$

$F_{By} = \frac{F_2(0.4)}{0.600m} = \frac{400N \cdot m}{0.6m} = 667N$

$F_{Ay} = F_2 - F_{By} = 1000N - 667N$

$F_{Ay} = 333N$

$$\Sigma F_z = F_{Az} + F_{Bz} - F_1$$

$$\Sigma M_{Ay} = 0 = F_1(0.1m) \quad - F_{Bz}(0.600m)$$

$$F_{Bz} = \frac{F_1(0.1m)}{(0.6m)} = \frac{75N \cdot m}{(0.6m)} = 125N$$

$$F_{Az} = F_1 - F_{Bz} = 750N - 125N$$

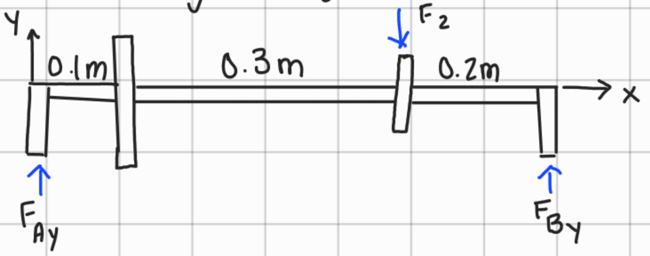
$$F_{Az} = 625N$$

note:

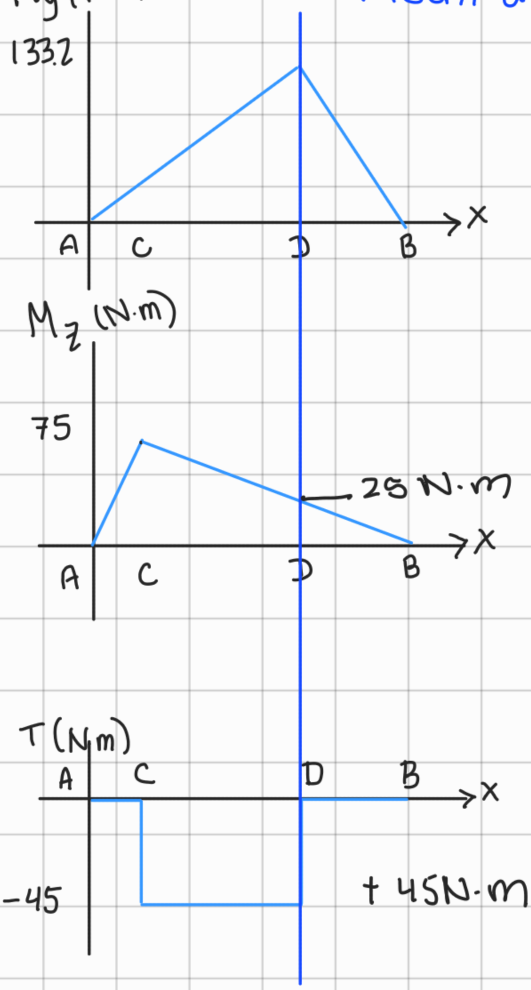
$F_{Ax} = F_{Bx} = 0$

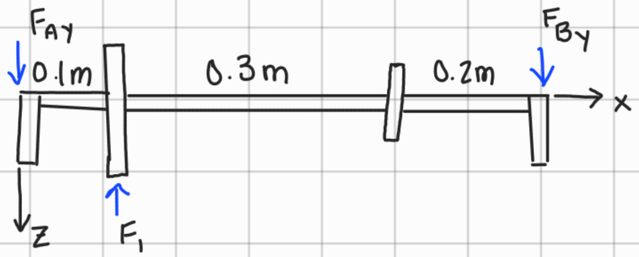
$F_B = (0\hat{i} + 333\hat{j} + 125\hat{k})$ $\quad$ $F_A = (0\hat{i} + 667\hat{j} + 625\hat{k})$

Bending Diagrams

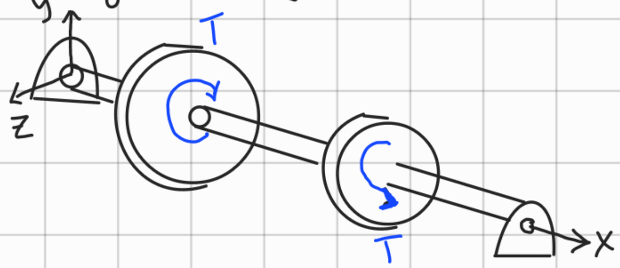


Critical Point at D





Torque Diagram



+ 45N·m also acceptable

Critical Point at D:

$M_y = 133.32 N \cdot m$

$M_z = 25 N \cdot m$

Max Bending Moment:

$$M_{max} = \sqrt{M_y^2 + M_z^2} = \sqrt{(133.32 N \cdot m)^2 + (25 N \cdot m)^2}$$

$$M_{max} = 135.66 N \cdot m$$

Transmitted Torque: $T_c = -45 N \cdot m$

b) $\sigma_x = \frac{M_{max} \cdot c}{I}$ $\tau_{xy} = \frac{T \cdot r}{J}$ c = diameter of shaft = 0.01m

$$I = \frac{1}{4}\pi r^4 = \frac{\pi}{4}(0.01m)^4 = 7.85 \times 10^{-9}\ m^4$$

$$J = \frac{1}{2}\pi r^4 = \frac{\pi}{2}(0.01m)^4 = 1.57 \times 10^{-9}\ m^4$$

$$\sigma_x = \frac{(135.66\ N \cdot m)(0.01m)}{(7.85 \times 10^{-9}\ m^4)} = 172.81\ MPa$$

$$\tau_{xy} = \frac{(-45\ N \cdot m)(0.01m)}{(1.57 \times 10^{-9}\ m^4)} = -28.66\ MPa$$

Von Mises Stress:

$$\sigma' = \sqrt{\sigma_x^2 - \sigma_x \sigma_y + \sigma_y^2 + 3\tau_{xy}^2} = \sqrt{172.81^2 + 3(-28.66)^2}$$

$$\sigma' = 179.8\ MPa$$

$$n = \frac{S_y}{\sigma'} = \frac{234}{179.8} = 1.30144$$

$$n = 1.3$$