"Optimizing PID Control for a 2 DOF Rotational System"

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Abstract:

Proportional-integral-derivative (PID) control is the most popular form of feedback control. They are easy to use and perform well in many conditions while being accurate and inexpensive [1]. In this lab, a 2 Degree of Freedom (DOF) rotational system was used to better understand the efforts involved in PID tuning. This experiment focused on stabilizing a 2 DOF system by identifying unknown parameters, then tuning a PID controller based on a validated LTI model to achieve less than 25% overshoot and a rapid 2% settling time.

Two 1 DOF steps responses were performed to find the physical parameters of the system. To confirm the system parameters, a Linear Time Invariant (LTI) model was created using MATLAB to simulate the step responses from both the 1 DOF systems. The Ziegler Nichols tuning method and MATLAB function pidtune() were used to attempt to find PID parameters for the system that achieved the design requirements listed above.

The PID controller of the 2 DOF system was successfully tuned to achieve an overshoot within our specified range. The results from this experiment can contribute to a deeper understanding of implementing a PID controller on a system with 2 degrees of freedom (DOF) to reduce overshoot to less than 25%. Two methods were described that can help achieve this. In real-world applications, precise PID tuning is crucial for ensuring machinery and processes meet stringent performance benchmarks, such as minimal overshoot and quick settling times. This fine-tuning enhances system stability and efficiency, directly impacting productivity and safety in industries ranging from manufacturing to aerospace.

Introduction:

Today control systems are present in everyday life. They are present in house appliances, offices, and forms of entertainment as well as in transportation systems and manufacturing operations.[2] PID Controllers are the most common control loop with feedback and are incredibly versatile and simple to manipulate. Tuning the 3 parameters of a PID controller —proportional, integral, and derivative gains—can be a challenge but if done correctly a set point can be tracked rejecting disturbances reaching stability. Other characteristics of the system can be achieved by proper tuning such as setting time, overshoot, and steady-state error. Controller tuning encompasses a variety of methods. However, before standardized techniques were developed, tuning often relied on trial and error, with adjustments made until a satisfactory performance was attained [3].

In this experiment, we will focus on studying a rotation 2 DOF system and attempt to develop a PID controller with less than 25% overshoot that reaches a 2% settling time as quickly as possible. To develop a PID control algorithm that meets the desired specifications, it is essential to first accurately model and validate the 2 DOF system's parameters. Following this, implementing two distinct PID tuning strategies and assessing their effectiveness will be carried out. An initial dividing of the 2 DOF system into two separate 1 DOF systems was conducted, using the step responses of each, to estimate the parameters of the 2 DOF system. Subsequently, MATLAB was employed to develop LTI models for each 1 DOF system, allowing the comparison of them with the experimentally measured responses of the 1 DOF systems to validate the estimated system parameters. Furthermore, an analysis of the frequency response of one of the 1 DOF systems was conducted, providing additional validation for some of the parameters. Following the validation of the 2 DOF system parameters, the Ziegler–Nichols PID tuning method was first employed, then, as a secondary method upon the failure of the first, the MATLAB built in function "pidtune()" was used in conjunction with the found system parameters to obtain the optimal PID parameters. The finding of this work was that there was uncertainty with the validity of the found parameters and the LTI model of the 2 DOF system but it was successfully overcome and compensated with the “pidtune()” function in which the obtained PID parameters produced a 12.32% overshoot and fast 2% settling time of 0.186 seconds.

Results:

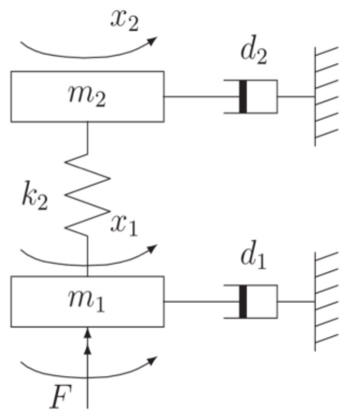

Figure 1: Schematic for the 2 DOF system with the model parameters shown. The damping constants are (d_1) and (d_2) while the spring constant for the torsion rod between disk 1 (m_1) and disk 2 (m_2) is (k_2). (F) is a rotational force enacted directly beneath disk 1. The rotational position of each disk (in counts) measured from encoder 1 and encoder 2 is (x_1) and (x_2) respectively.

The process of identifying the unknown model parameters of the 2 DOF system began by dividing it into two separate 1 DOF systems. We calculated the unknown model parameters for each 1 DOF system and then aggregated these parameters to form the complete model of the 2 DOF system. The lower 1 DOF system included a fixed disk 2, while the upper 1 DOF system required a fixed disk 1. For the lower system, a step input of 0.5V was applied, and its step response was recorded. In the case of the upper system, which lacked a motor, an impulse flick was administered to the top plate, and its impulse response was recorded. Five trials for each system were performed, yielding values for the initial time of the system's response (t_0), the ending time of the system's response (t_n), the initial output value (y_0), the final output value (y_n), the steady-state output value (y_∞), and input voltage (U). Following this, the characteristic parameters of the system such the damped resonance frequency(ω_d), the resonance frequency (ω_n), the damping ratio (β) were obtained using Equation 1- 4 (refer to the appendix) then tabulated in Table 6. After obtaining the characteristic parameters of the system. The final step was to obtain the physical parameters of the system such as the spring constant (k), the moment of inertia (m), and the damping constant (d) using Equation 5-7 (refer to the appendix). The parameters values were calculated for all 5 trials of each experiment and tabulated in Table 4 and Table 5. Experiment of 1 DoF Step response of encoder 1 was used to obtained k, m_1, and d_1.

Experiment of 1 DoF Impulse response of encoder 2 was used to obtain m_2, and d_2 .The mean and the standard deviations of the parameters of the systems were calculated and tabulated in Table 1 to ensure accuracy of the obtained results.

Table 1: Mean and standard deviation of the parameters of the system that were obtained from 5 trials of 1 Degree of Freedom experiment of Encoder 1 and Encoder 2.

	$k\ [\frac{V*s^2}{counts}]$ (Mean $\pm$ STD)	$m_1[\frac{V*s^2}{counts}]$ (Mean $\pm$ STD)	$d_1[\frac{V*s}{counts}]$ (Mean $\pm$ STD)	$m_2[\frac{V*s^2}{counts}]$ (Mean $\pm$ STD)	$d_2[\frac{V*s}{counts}]$ (Mean $\pm$ STD)
1 DoF Step Response of Encoder 1	0.0022 ± 0.00012	$0.15 * 10^{-5} \pm 9.43 * 10^{-8}$	$0.21 * 10^{-4} \pm 2.28 * 10^{-6}$		
1 DoF Impulse Response of Encoder 2	0.0022 ± 0.00012			$3.79 * 10^{-6} \pm 8.65 * 10^{-7}$	$4.69 * 10^{-6} \pm 1.59 * 10^{-6}$

After the physical parameters of the system were calculated. The complete system could be modeled using the given “maelab.m” program. After inputting the parameters into the program, the appropriate options of encoder and DOF were chosen and the average experimental data was inputted. The program provided a transfer function and plotted the experimental data and simulated system.

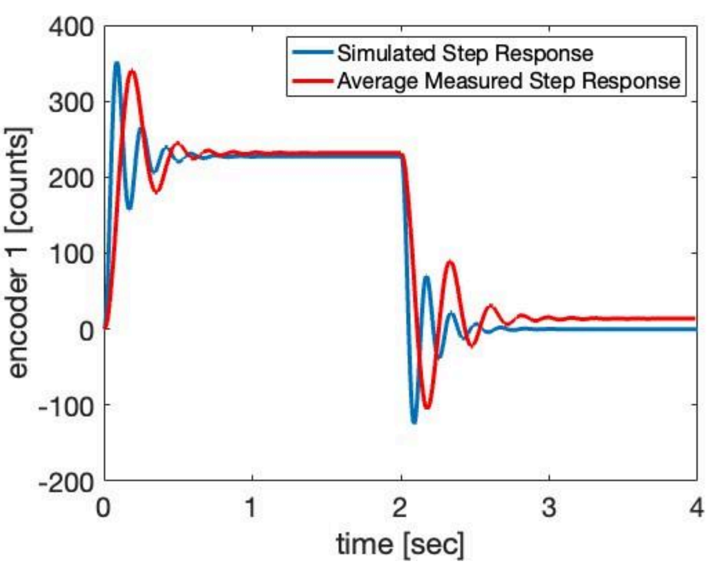

Figure 2: Encoder 1 position over time of simulated step response vs the averaged measured step response of 1 DOF experiment where disk 2 was fixed.

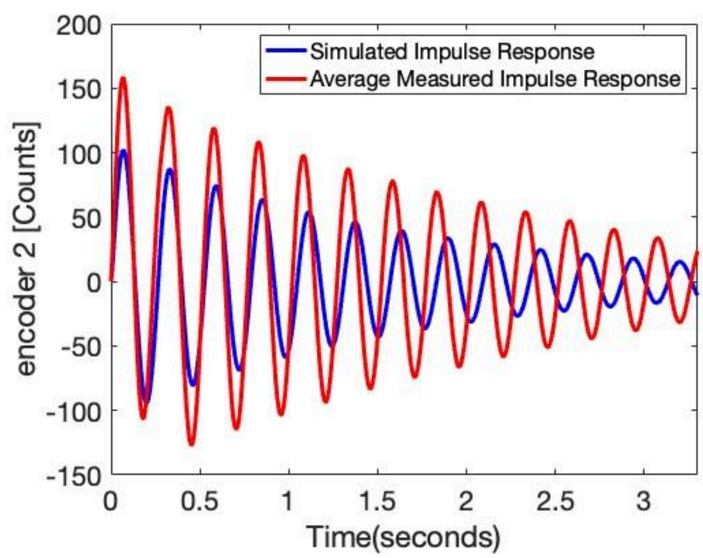

Figure 3: Encoder 2 position over time of simulated impulse response vs the averaged measured impulse response of 1 DOF experiment where disk 1 was fixed and disk 2 was impulse flicked.

To further validate the parameters derived from the simulated 1 DOF system with the encoder 2 plates fixed, an analysis of frequency responses was conducted. An experimental frequency response plot was obtained by subjecting the 1 DOF system to a swept sinusoidal input spanning frequencies from 1 to 5 Hz. Subsequently, a Fast Fourier Transform (FFT) was applied to both the output response and the input sine sweep in MATLAB, alongside additional scaling operations, to generate an experimental frequency response magnitude plot Figure 4. Meanwhile, leveraging parameters identified from previous step response experiments and a simulated Linear Time-Invariant (LTI) model of the 1 DOF system, a theoretical frequency response magnitude plot was produced Figure 5. These plots played a crucial role in illustrating the 1 DOF system's behavior across a spectrum of frequencies and in identifying corresponding resonance frequencies. Notably, Figure 4. identified a resonant frequency around 1.1 Hz, while Figure 5 pinpointed a resonant frequency near 5 Hz.

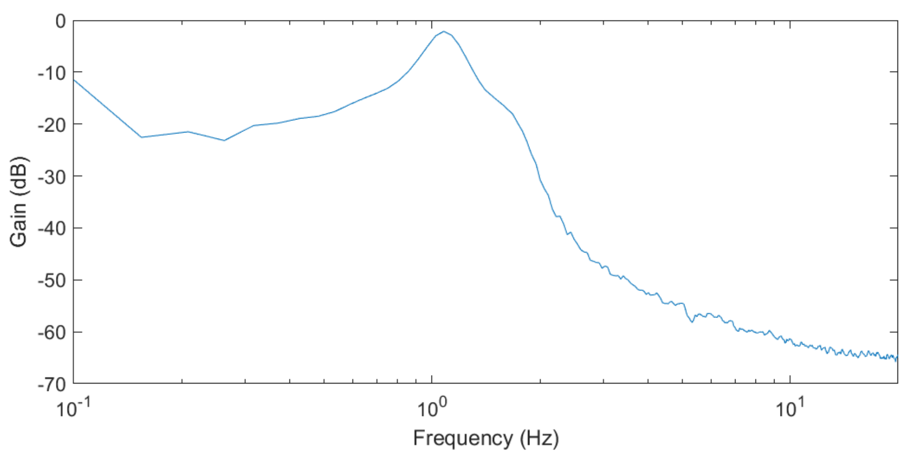

Figure 4: Experimentally found frequency response magnitude plot for sine sweep input into 1 DOF system where the plate corresponding to encoder 2 was fixed.

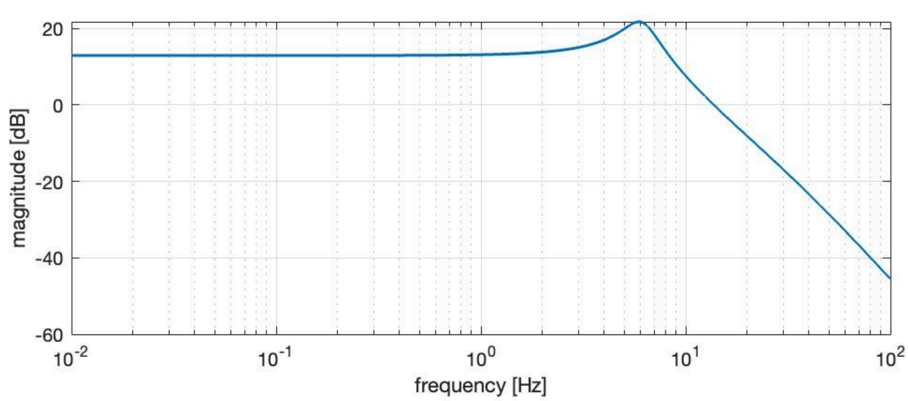

Figure 5: Theoretical frequency response magnitude plot for sine sweep input into 1 DOF system where the plate corresponding to encoder 2 was fixed.

PID tuning:

The approach taken to tune the PID controller was achieved via two methods which included the Ziegler–Nichols tuning method and MATLAB's default function pidtune. The Ziegler-Nichols method was completed by setting the integral (Ki) and derivative (Kd) to zero and starting with a low proportional (Kp) value on the controller via the ECP software. Gradually, Kp was increased in small increments until controlled oscillations were observed in response to a 0.5v step input. The Kp value at which these oscillations occurred was reported as the ultimate gain (Ku) which was determined to be 0.22. To determine the ultimate period (Pu) of these oscillations, the average values of three consecutive peaks were calculated which was determined to be 0.267. These values were used to find Kp, Ki, and Kd using the Ziegler-Nichols method as outlined in Table 7 (Refer to the appendix). Table 2 outlines the found Kp, Ki, and Kd values which are 0.129000, 0.133500, and 0.033375 respectively. However, the Ki value was altered as it fell outside of the provided limits and was set to the maximum allowable value of 0.0199 when applied to the ECP software. Testing a step response resulted in an extremely unstable response and had to be shut off.

The second method used MATLAB's default function "pidtune". The obtained values for the 2 degrees of freedom (DOF) of the entire system were utilized to find the transfer function as seen in Table 1. Once the plant transfer function was obtained the dynamics of the DC motor and hardware were combined to obtain the transfer function of the complete system. This transfer function was then inputted into the pidtune function, which designs a PID controller for a given model. The found parameters of this approach are outlined in Table 2, where pidtune yielded optimal parameters to be kp = 0.027100, ki = 0.03330, and kd = 0.00551. Applying these values resulted in a response that achieved the desired design criteria. The overshoot of this PID controller was recorded to be 12.32% and a settling time of 0.1860 seconds.

Table 2. Illustrates the two PID tuning methods and their respective system parameters.

PID tuning method	Ultimate Gain (Ku)	Oscillation Period (Tu)	Gain Factor (Kp)	Integral Factor (Ki)	Derivative Factor (Kd)
Ziegler–Nichols	0.22	0.267	0.129000	0.133500	0.033375
pidtune function	N/A	N/A	0.027100	0.03330	0.00551

To evaluate the design of the controller, a sensitivity analysis was performed by testing 2 alternative mass setups. Table 2 outlines the PID controller found via pidtune and the overshoot and settling time of each respective setup. Figure 6 depicts the step response of the two alternative mass configurations compared to the baseline. In the baseline setup, masses were positioned as close to the center of the plate as possible. In contrast, the first alternate setup moved the bottom plate's masses as far away from the center as possible. This increased the overshoot by 14.04% while maintaining the same settling time. The second alternative mass configuration moved both the top and bottom masses away from the center. This resulted in a further increase in the overshoot by 57.13% and increased the settling time to 0.1950 seconds.

Table 3. Percent overshoot for each of our three mass setups, the change in overshoot compared to the baseline setup, and the settling time to achieve 2%.

Setup	Overshoot	Change in Overshoot	2% Settling time
Baseline	12.32%	N/A	0.1860s
Inertial Setup 2	14.04%	14.04%	0.1860s
Inertial Setup 3	19.35%	57.13%	0.1950s

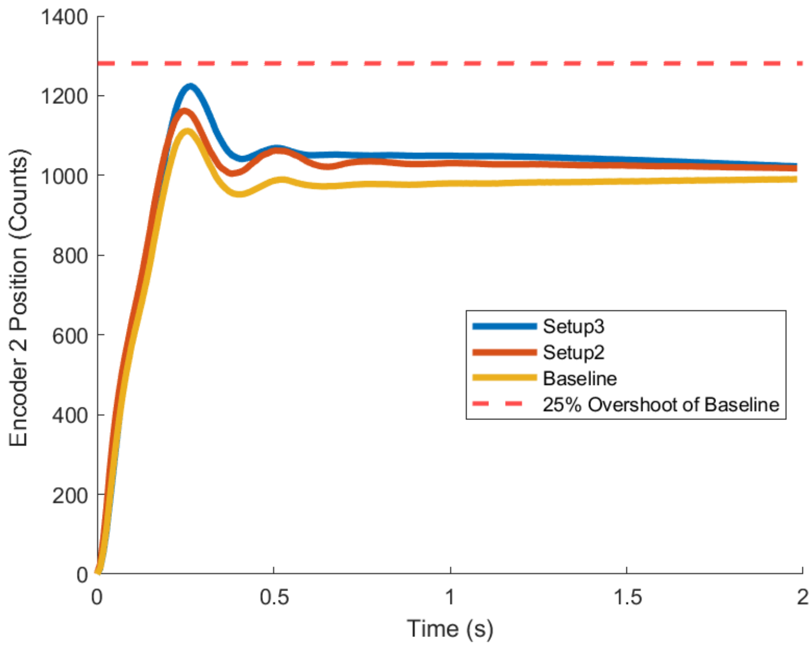

Figure 6. The density graph illustrates the overshoot and how the tuned PID controller behaves as masses are moved to different positions.

Discussion:

Prior to the development of a PID control algorithm, it was vital to obtain the parameters for the 2 DOF system and validate their accuracy. By repeating the experiment multiple times it was possible to quantify the consistency of the data that we were able to obtain. From Table 1**,** it was found that the standard deviation of each parameter was relatively small compared to the mean value which showed that the data were consistent throughout the trial. In order to see the accuracy of the experimentally obtained parameters, a simulation of the system was performed using the parameters using the maelab.m program. From Figure 2 it can be qualitatively observed that the simulated response in Figure 1 follows really closely to the average experimentally obtained data. This shows that the parameters obtained were fairly accurate. It can also be seen in Figure 3**,** that the simulated impulse response of the 1 DF system also closely followed the experimentally obtained data with a relatively small difference in the amplitude of the two signals.

In examining the magnitude of the frequency responses of the 1 DOF system from Figures 2 and 3 it can be noted that the resonant frequencies were within the same order of magnitude. This highlights that the parameters m1, d1, and k2 were accurate as they produced similar experimentally and theoretically found frequency responses, highlighted specifically by the similar resonance frequencies around 5 Hz. The error between the resonant frequencies could be the result of inaccurate LTI modeling of the 1 DOF system

PID Tuning

When attempting to tune the PID controller using the Ziegler-Nichols method, the system was unstable. This can be attributed to control saturation which is when the controller reaches its maximum or minimum limits resulting in an unwanted behavior. The ultimate gain of the PID controller was determined to fall within an acceptable range, but upon applying the formula in Table 7 in the appendix to calculate the derivative value, it yielded a value of 0.033375. This exceeded our specified Kd limit of 0 to 0.02. The inability to incorporate the calculated derivative value could have possibly contributed to the unstable response observed when a disturbance is applied, indicating control saturation is responsible for the behavior. Applying the maximum value of 0.02 resulted in unstable behavior, once again indicating that the controller is saturated. At this lower derivative value, the motor is unable to behave as desired when commanded by the controller.

In contrast to the Ziegler-Nichols method, the built-in MATLAB function pidtune successfully produced a controller that stabilizes the system as desired. With an overshoot of only 12.32%, well below the target of 25%, the disturbance was effectively stabilized. The effectiveness of this approach can be attributed to pidtune's approach to optimizing PID gains to balance response time and stability. It utilized the determined system transfer function as outlined earlier in the experiment to obtain the PID parameters. Additionally, any model errors are mitigated as the transfer function is obtained experimentally which will encompass deviating behaviors. This allows for the implementation in systems that may have slightly different setups allowing for a large range of applications. The PID controller's robustness is evaluated by introducing changes to the system's mass distribution. Although the overshoot may have increased to 14.04% and 19.35% as outlined in Table 3 for mass setup 2 and mass setup 3 respectively, they remain within the acceptable 25% overshoot range. Additionally settling time remained the same for the first mass setup and slightly increased to 0.1950 seconds for the second mass setup. This affirmed the acceptance of the PID controller's performance in varying setups.

Appendix:

Table 4. Calculated parameters of all 5 trials of the system with 1 Degree of Freedom obtained from encoder 1

Trial	$k_2\ [\frac{V*s^2}{counts}]$	$m_1[\frac{V*s^2}{counts}]$	$d_1[\frac{V*s}{counts}]$
1	0.0020	$0.1339 * 10^{-5}$	$0.1759 * 10^{-4}$
2	0.0024	$0.1476 * 10^{-5}$	$0.2090 * 10^{-4}$
3	0.0020	$0.1467 * 10^{-5}$	$0.1880 * 10^{-4}$
4	0.0021	$0.1514 * 10^{-5}$	$0.2371 * 10^{-4}$
5	0.0023	$0.1633 * 10^{-5}$	$0.2263 * 10^{-4}$
Average	0.0022	$0.1486 * 10^{-5}$	$0.2073 * 10^{-4}$
Standard Deviation	0.0001633	$9.43 *10^{-8}$	$2.28*10^{-6}$

Table 5. Calculated parameters of all 5 trials of the system with 1 Degree of Freedom obtained from encoder 2

Trial	$k_2\ [\frac{V*s^2}{counts}]$	$m_2[\frac{V*s^2}{counts}]$	$d_2[\frac{V*s}{counts}]$
1	0.0023	$0.3137 * 10^{-5}$	$0.3533 * 10^{-5}$
2	0.0020	$0.4447 * 10^{-5}$	$0.4522 * 10^{-5}$
3	0.0021	$0.2704 * 10^{-5}$	$0.3095 * 10^{-5}$
4	0.0021	$0.3901 * 10^{-5}$	$0.5167 * 10^{-5}$
5	0.0023	$0.4761 * 10^{-5}$	$0.7126 * 10^{-5}$
Average	0.0022	$3.79 * 10^{-6}$	$4.6886 * 10^{-6}$
Standard deviation	0.00012111	$8.6496 * 10^{-7}$	$1.5869 * 10^{-6}$

Table 6. Calculated parameters of the 2 Degree of Freedom (2 DOF) system.

Wn_1 ($\frac{Rad}{s}$)	Wn_2($\frac{Rad}{s}$)	B_1	B_2	K_2 ($\frac{V}{m}$)
38.1780	24.2116	0.1824	0.0255	0.022

<u>Equation list:</u>

$$\widehat{\omega}_d = 2\pi \frac{n}{t_n - t_0}$$ **Equation 1.**

$$\widehat{\beta\omega}_n = \frac{1}{t_n - t_0} ln(\frac{y_0 - y_\infty}{y_n - y})$$ **Equation 2.**

$$\widehat{\omega}_n = \sqrt{\widehat{\omega}_d^{\,2} + (\widehat{\beta\omega}_n)^2}$$ **Equation 3.**

$$\widehat{\beta} = \frac{\widehat{\beta\omega}_n}{\widehat{\omega}_n}$$ **Equation 4.**

$$k = \frac{U}{y_\infty}$$ **Equation 5.**

$$m = k \times \frac{1}{\widehat{\omega}_n^{\,2}}$$ **Equation 6.**

$$d = k(\frac{2\widehat{\beta}}{\widehat{\omega}_n})$$ **Equation 7.**

Table 7: The table above shows the coefficients necessary to apply to the PID parameters Kp, Ki, and Kd while tuning a PID controller using the Ziegler-Nichols method [4].

Controller	K_p	K_i	K_d
P	$\frac{K_u}{2}$	—	—

PI	$\frac{K_u}{2.2}$	$\frac{P_u}{1.2}$	—
PID	$\frac{K_u}{1.7}$	$\frac{P_u}{2}$	$\frac{P_u}{8}$

Individual Contributions:

Micah: Wrote part of the introduction, did frequency analysis results and discussion, did part of step response results, and appendix

David Deharo : Assisted with ECP software utilization and system tuning. Contributed to abstract composition and provided support for overall writing efforts.

Eric Montejo-Francisco: Worked on the ECP software to collect data outlined throughout the report. Responsible for the sections of the PID tuning portion of the lab including the results and discussion.

Tin Nguyen: Worked on one degree of freedom step response and impulse response experiment analysis. Finding system's parameters. Process experimental data and simulated system based on parameters. Contributed to the write up of the results and Discussion section.

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