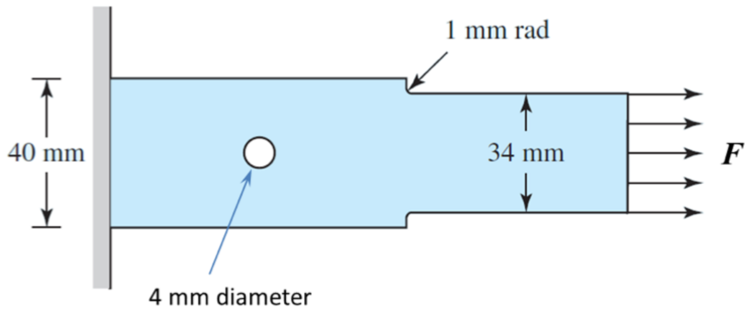
1 - A connecting link (shown below) is machined from 1050 CD steel. It is subjected to an axial load ***F***, which fluctuates between **2kN** in **compression** to **6 kN** in **tension**. As a designer, you decide that the minimum required fatigue factor of safety for this part is $n_f = 2$.

a) Determine the **minimum plate thickness** needed to meet the selected safety factor requirement (n_f = 2). Please note that you have to determine the critical location for the link. Use any appropriate criteria of fatigue of your choice.

b) Determine the **Yielding Factor of Safety** based on the thickness you find part (a) and compare it with the selected fatigue factor of safety (n_f = 2).



Geometry: for fillet

$$\frac{D}{d} = \frac{40}{34} = 1.176, \quad \frac{r}{d} = \frac{1}{34} = 0.03 \quad \Rightarrow \quad K_t \simeq 2.5, \quad q \simeq 0.75 \quad \Rightarrow \quad K_f = 1 + q(K_t - 1), \quad K_f = 2.12$$

for hole

$$\frac{d}{w} = \frac{4}{40} = 0.1 \quad \Rightarrow \quad K_t = 2.7, \quad q = 0.85 \quad \Rightarrow \quad K_f = 1 + q(K_t - 1) = 2.45$$

Material & strength   1050 CD : $S_{ut} = 690\,MPa$, $S_y = 580\,MPa$

S_e :

$$k_a = 4.51(690)^{-0.265} = 0.798$$ (Note: k_a is differt if you use 11th edition parameters)

$$k_b = 1$$ (only axial load)

$$k_c = 0.85$$ (axial load)

$$k_d = k_e = k_f = 1$$ (Note: you may select different k_e based on desired reliability)

$$\Rightarrow S_e = k_a k_b k_c k_d k_e k_f \left(\tfrac{1}{2} S_{ut}\right) = 234\,MPa$$

Stress & factor of safety

For hole : $$\sigma_{min} = (K_f)_{hole} \frac{F_{min}}{Area} = (K_f)_{hole} \frac{-2 \times 10^3\,N}{[(40-4) \times 10^{-3}] \cdot t}$$

$$\sigma_{max} = (K_f)_{hole} \frac{F_{max}}{Area} = (K_f)_{hole} \frac{6 \times 10^3 N}{[(40-4)\times 10^{-3}] \times t}$$

$$\sigma_m = \frac{\sigma_{min} + \sigma_{max}}{2} = K_f \frac{2 \times 10^3}{[(40-4)\times 10^{-3}] \times t} \; ; \; \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = K_f \frac{4 \times 10^3}{[(40-4)\times 10^{-3}] \times t}$$

$$\Rightarrow \sigma_a = 2.45 \left(\frac{4 \times 10^3}{36 \times 10^{-3} t} \right) = \frac{2.72 \times 10^5}{t}$$

using Goodman: $n_f = \dfrac{1}{\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}}}$

$$\Rightarrow \sigma_m = 2.45 \left(\frac{2 \times 10^3}{36 \times 10^{-3} t} \right) = \frac{1.36 \times 10^5}{t}$$

$$\Longrightarrow 2 = \frac{1}{\dfrac{0.272}{234t} + \dfrac{0.136}{690t}} \Longrightarrow t = \underline{0.00271\,m} = \underline{\underline{2.71\,mm}}$$

---

for fillet: $\sigma_{min} = (K_f)_{fillet} \dfrac{F_{min}}{Area} = (K_f)_{fillet} \dfrac{-2 \times 10^3 N}{(34 \times 10^{-3}) \times t}$

$$\sigma_{max} = K_{f\,fillet} \frac{F_{max}}{Area} = K_{f\,fillet} \frac{6 \times 10^3 N}{(34 \times 10^{-3}) \times t}$$

$$\sigma_m = \frac{\sigma_{min} + \sigma_{max}}{2} = K_f \frac{2 \times 10^3}{(34 \times 10^{-3}) \times t} = \qquad \sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2} = K_f \frac{4 \times 10^3}{(34 \times 10^{-3}) \times t} =$$

$$\sigma_a = 2.12 \left(\frac{4 \times 10^3}{(34 \times 10^{-3}) t} \right) = \frac{2.49 \times 10^5}{t}$$

using Goodman: $n_f = \dfrac{1}{\dfrac{\sigma_a}{S_e} + \dfrac{\sigma_m}{S_{ut}}}$

$$\sigma_m = 2.12 \left(\frac{2 \times 10^3}{(34 \times 10^{-3}) t} \right) = \frac{1.25 \times 10^5}{t}$$

$$\Longrightarrow 2 = \frac{1}{\dfrac{0.249}{234t} + \dfrac{0.125}{690t}} \Longrightarrow t = \underline{0.00249\,m} = 2.49\,mm$$

the <u>Larger t</u> Controls the design, so $\boxed{t = 2.71\ mm}$

b) $n_y = \dfrac{S_y}{\sigma_a + \sigma_m} = \dfrac{580}{100.4 + 50.2}$

$n_y > n_f$ is desired in design

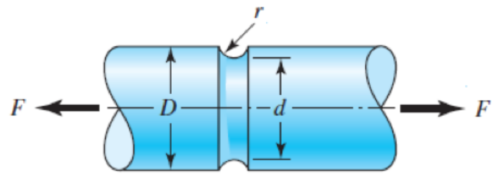
$$\boxed{n_y = 3.85 > 2 = n_f}$$ ✓

2. A grade 60 gray cast iron grooved round rod (shown below) is subjected to an axial load fluctuating between **300 N** in **compression** to **1500 N in tension**. The larger diameter is D = 23 mm, the smaller diameter is d = 20 mm, and groove radius is r = 1.5 mm.

a) Find the fatigue factor of safety for axial load fluctuating between **300 N** in **compression** to **1500 N in tension**.

b) Find fatigue factor of safety for axial load fluctuating between **300 N** in **tension** to **1500 N in compression** and compare it with part (a) results.

Tip: The endurance limit of cast iron is readily available from table A-24 (k_a and k_b already included, use $k_c = 0.9$)



4) grade 60 gray cast Iron

From A-24

$S_{ut} = 62.5 \text{ ksi} = 431 \text{ MPa}$

$S_{uc} = 187.5 \text{ ksi} = 1293 \text{ MPa}$

$S_e'(k_a)(k_b) = 24.5 \text{ ksi} \Longrightarrow S_e = k_a k_b \underset{\text{axial}}{k_c} = (0.9)(24.5 \text{ ksi}) = 22.05 \text{ ksi} = \underline{152 \text{ MPa}}$

$$\left.\begin{array}{l} F_{min} = -300 \text{ N} \\ F_{max} = 1500 \text{ N} \end{array}\right\} \quad \begin{array}{l} F_m = 600 \text{ N} \\ F_a = 900 \text{ N} \end{array} \qquad r = \frac{F_a}{F_m} = \frac{\sigma_a}{\sigma_m} = 1.5$$

fluctuating tensile:

$$S_a = \frac{r S_{ut} + S_e}{2}\left[-1 + \sqrt{1 + \frac{4r S_{ut} S_e}{(r S_{ut} + S_e)^2}}\right] = \frac{(1.5)(431) + 152}{2}\left\{-1 + \sqrt{1 + \frac{(6)(431)(152)}{(1.5 \times 431 + 152)^2}}\right\}$$

$$= 399\left\{-1 + \sqrt{1 + \frac{3.93 \times 10^5}{6.38 \times 10^5}}\right\} = 108.3 \text{ MPa} = 15.7 \text{ ksi}$$

From A-15-13 : $\frac{D}{d} = \frac{23}{20} = 1.15$ & $\frac{r}{d} = \frac{1.5}{20} = 0.075 \Longrightarrow K_t \simeq 2.3$

$$K_f = 1 + q(K_t - 1) = 1 + 0.2(1.3) = 1.26$$

$$A_{section} = \pi\left(\frac{d}{2}\right)^2 = \pi(0.01)^2 = 3.14 \times 10^{-4} \text{ m}^2$$

$$\sigma_m = K_f \frac{F_m}{A} = 1.26 \frac{6 \times 10^2}{3.14 \times 10^{-4}} = 2.41 \text{ MPa}$$

$$\sigma_a = K_f \frac{F_a}{A} = 1.26 \frac{9 \times 10^2}{3.14 \times 10^{-4}} = 3.61 \text{ MPa}$$

$$\Longrightarrow n = \frac{S_a}{\sigma_a} = \frac{108.3}{3.61} = \boxed{30.0}$$

b) Geometry is similar to part (a)

$$\left.\begin{aligned} \frac{D}{d} &= \frac{23}{20} = 1.15 \\ \frac{r}{d} &= \frac{1.5}{20} = 0.075 \end{aligned}\right\} \xRightarrow{A-15-13} K_t \simeq 2.3$$

$$\Longrightarrow K_f = 1 + q(K_t - 1) = 1 + 0.2(1.3) = 1.26$$

Loading $F_m = -600\ N$, $F_a = 900\ N$ $\Longrightarrow r = \frac{900}{-600} = -1.5$

$$\sigma_m = K_f \frac{F_m}{A} = 1.26 \frac{-6 \times 10^2}{3.14 \times 10^{-4}} = -2.41 \quad MPa$$

$$\sigma_a = K_f \frac{F_a}{A} = 1.26 \frac{9 \times 10^2}{3.14 \times 10^{-4}} = 3.61 \quad MPa$$

This time we use fluctuating compressive formula to find S_a

$$S_a = \frac{S_e}{1 - \frac{1}{r}\left(\frac{S_e}{S_{ut}} - 1\right)} \xRightarrow{\text{Plugging in}} S_a = \frac{152}{1 - \frac{1}{-1.5}\left(\frac{152}{431} - 1\right)}$$

$$S_a = 267.4\ MPa$$

$$n = \frac{S_a}{\sigma_a} = \frac{267.4\ MPa}{3.61\ MPa} = 74.1$$

The brittle part will show higher factor of safety in compressive fluctuations compared to fluctuating load with mean tensile stress (Part a)
That said, the load seems to be very small for the part, as the very high safety factors suggest.

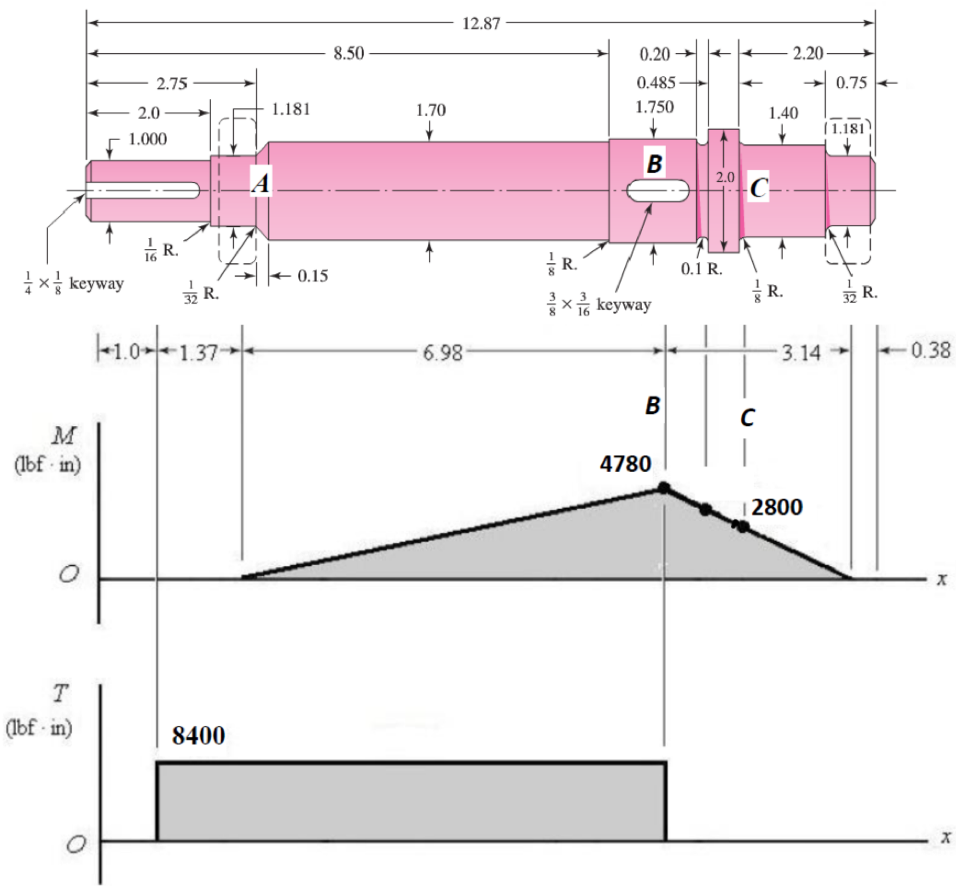
3. The shaft in the figure is **machined** from AISI 1018 CD steel. The load analysis results (***M*** and ***T*** diagrams) are mapped under the corresponding points of the shaft layout. Due to the shaft rotation, the bending stress will be completely reversed, while the torsional stress will be steady.

a) Determine the **fatigue factor of safety** at **the end-milled keyway** at ***B*** (with $r/d = 0.02$), using **DE-Gerber** criteria. (all dimensions in the shaft figure are in inches).

b) Determine the **fatigue factor of safety** at the **shoulder fillet** at ***C*** (with **fillet radius** $r = 1/8$ in.), using **DE- ASME** criteria. Note that at shoulder ***C***, the larger diameter is $D = 2$ *in.*, and the smaller diameter is $d = 1.4$ in (as shown in the shaft layout).

All **dimensions** are in ***inches***.

Reminder: **All keyway and shoulder fillet radii follow r/d = 0.02.**



1018 CD

(From Table A-20)

$S_{ut} =$ 64 ksi (440 MPa) $\Longrightarrow S_e' = \frac{S_{ut}}{2} = 32$ ksi

$S_y =$ 54 ksi (370 MPa)

a)

for edition 11th $k_a = 0.811$

Surface Factor

Table 6-2 $\Longrightarrow$ $\left.\begin{array}{l} a = 2.7 \\ b = -0.265 \end{array}\right\}$ $k_a = 2.7(64)^{-0.265} = 0.897$

Size Factor

Eq. 6.20 $\Longrightarrow$ $k_b = \left(\dfrac{\overset{1.75''}{d}}{0.3}\right)^{-0.107} = 0.828$

$k_c = 1$ (bending), $k_d = 1$ (room temp), $k_e = k_f = 1$

Selecting $k_e < 1$ is ok!

$$S_e = \underbrace{(0.897)}_{k_a}\underbrace{(0.828)}_{k_b}(1)(\overset{1}{k_e})\underbrace{(32)}_{S_e'} = 23.76 \text{ ksi}$$

Loading @ B

$T_m = 8400$ (lb.in)

$M_a = 4780$ (lb.in)

Geometry

for keyway @ B $\xrightarrow[\text{Table 7.1}]{r/d = 0.02}$ $K_t = 2.14$, $K_{ts} = 3$

$d = 1.75''$ $\xrightarrow{r/d = 0.02}$ $\left.\begin{array}{l} r = 0.035'' \\ S_{ut} = 64 \text{ ksi} \end{array}\right\}$ $\begin{array}{l} q \simeq 0.65 \\ q_s \simeq 0.7 \end{array}$

(Figure 6-20 & 6-21)

$K_f = 1 + q(K_t - 1) = 1 + 0.65(2.14 - 1) = 1.74$

$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.7(3 - 1) = 2.4$

using DE - Gerber

$Se = 23760$ Psi $\quad d = 1.75$

$S_{ut} = 64000$ Psi

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}$$

where

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2} = \sqrt{4(K_f M_a)^2} = \sqrt{4[(1.74)(4780)]^2} = 16\,634 \text{ psi}$$

$$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2} = \sqrt{3(k_f T_m)^2} = \sqrt{3[(2.4)(8400)]^2} = 34918 \text{ Psi}$$

M_m & $T_a = 0$

$$\frac{1}{n} = \frac{8\,(16634)}{\pi(1.75)^3(23760)}\left\{1 + \left[1 + \left(\frac{2\,(34918)(23760)}{(16634)(64000)}\right)^2\right]^{1/2}\right\}$$

$$\frac{1}{n} = 0.332 \left\{1 + \left[1 + (1.55)^2\right]^{1/2}\right\} = 0.947$$

$\Longrightarrow n = 1.05$ barely safe !!

$n \simeq 0.98$ with 11th edition numbers

we may have $n<1$, by choosing k_e (reliability factor) Less Than 1

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b) Shoulder @ C

size factor (k_b) is sligthly different, as $d = 1.4''$

$\Longrightarrow$ this also changes Se a little.

Size Factor Eq. 6.20 $\Longrightarrow k_b = \left(\frac{d}{0.3}\right)^{-0.107} = 0.848$

1.4

0.811 in 11th ed.

$Se = \underbrace{(0.897)}_{k_a}\underbrace{(0.848)}_{k_b}\underbrace{(1)}_{k_c}(\overset{1}{k_e})\underbrace{(32)}_{S_e'} = \underline{24.33}$ ksi

(22 ksi with the 11th edition numbers)

Loading @ C

$T_m = 0$ (lb.in)

$M_a = 2800$ (lb.in)

Geometry for shoulder @ B

$$\frac{r}{d} = \frac{1/8}{1.4} = 0.089$$

$$\frac{D}{d} = \frac{2}{1.4} = 1.43$$

$\Longrightarrow$ from A-15-9 $\underline{K_t = 1.7}$

$$\left.\begin{array}{l} r_f = \frac{1''}{8} = 0.125'' \\ S_{ut} = 64 \text{ ksi} \end{array}\right\} \underline{q \simeq 0.78}$$

approximate (0.75 to 0.8 acceptable)

from Fig 6-20

$$K_f = 1 + q(K_t - 1) = 1 + 0.78(1.7 - 1) = \underline{\underline{1.5}}$$

approximate

q_s & K_{ts} are not important as we don't have τ (shear stress) @ shoulder B

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using ASME Criteria

$$\frac{1}{n} = \frac{16}{\pi d^3}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{\cancel{K_{fs} T_a}}{S_e}\right)^2 + 4\left(\frac{\cancel{K_f M_m}}{S_y}\right)^2 + 3\left(\frac{\cancel{K_{fs} T_m}}{S_y}\right)^2\right]^{1/2}$$

0 0 0

$$\frac{1}{n} = \frac{16}{\pi d^3}\left\{4\left(\frac{K_f M_a}{S_e}\right)^2\right\}^{1/2} = \frac{16}{\pi d^3}\left\{2\left(\frac{K_f M_a}{S_e}\right)\right\} = \frac{16}{\pi (1.4)^3}\left\{2\,\frac{1.5 \; 2800}{24330}\right\}$$

$$\frac{1}{n} = 0.641 \Longrightarrow n = 1.56 \text{ safe}$$

($n \simeq 1.4$ with the 11th edition numbers)

note: you may use ($k_e < 1$), reliability factor!