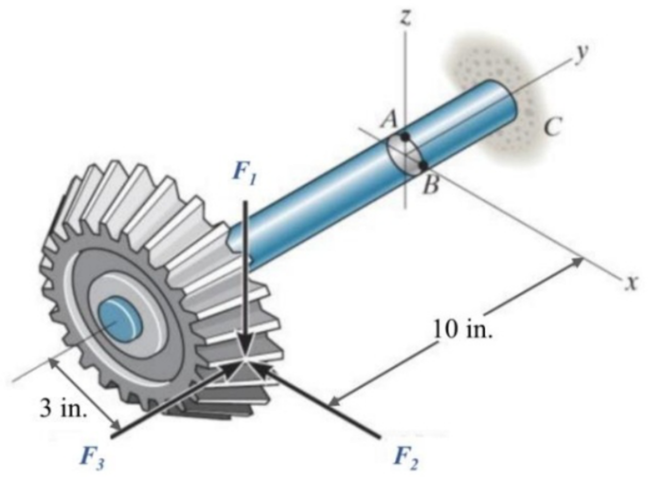
1. Bevel gears are machine components used to transmit power between two intersecting shafts. The bevel gear in the figure is subjected to the forces $F_1 = 2000$ lb, $F_3 = 1500$ lb, while F_2 is negligible. The shaft is made from **1035 HR** (Hot-Rolled) steel and has a diameter of 1 in.

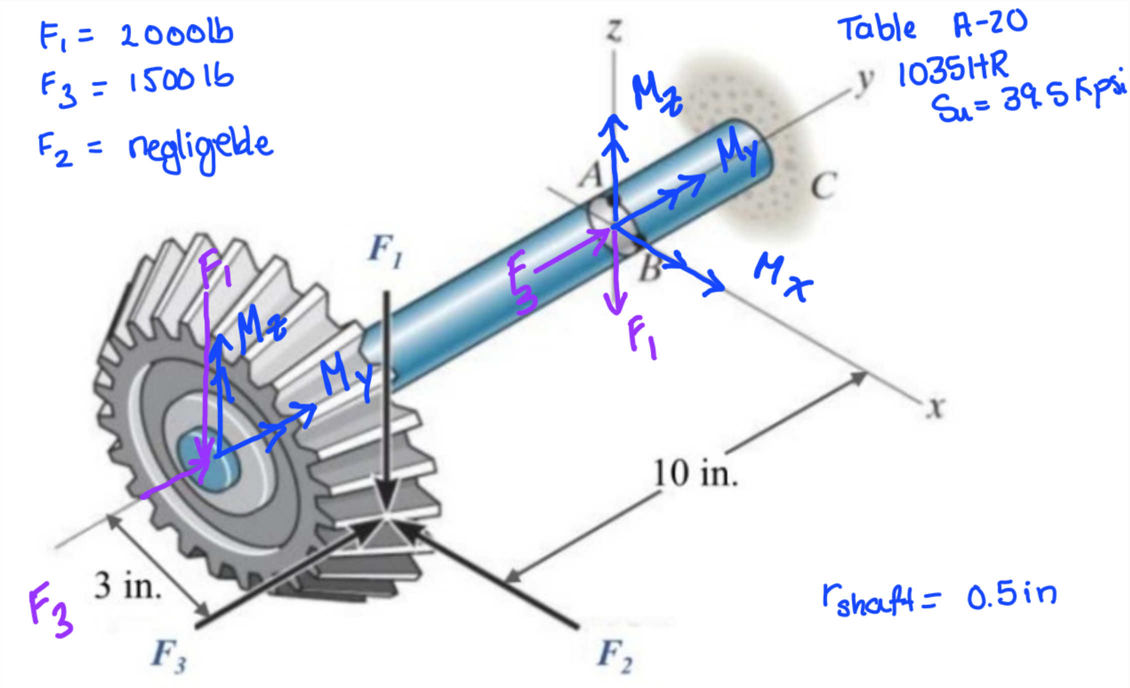


It is suggested that point *B* is a critical point on the shaft and should be analyzed. Determine

(a) the principal, maximum shear stresses, and von Mises stress at point ***B***.

(b) the factor of safety at point ***B*,** based on Distortion Energy (Von Mises) and Max Shear Stress (Tresca) theories.

a)



Note:

cross-product sign convention

1a) $I = \frac{1}{4}\pi r^4 = \frac{1}{4}\pi(0.5\,in)^4 \approx 0.04908\,in^4$

$J = \frac{1}{2}\pi r^4 = \frac{1}{2}\pi(0.5\,in)^4 = 0.09817\,in^4$

Transfer of Forces

$V = -F_1 = -2000\,lbs$

$M_x = F_1 \cdot (10\,in) = 20000\,lb{\cdot}in$

$M_y = F_1 \cdot (3\,in) = 6000\,lb{\cdot}in$

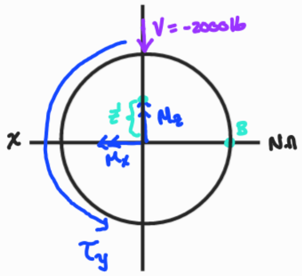
$M_z = F_3 \cdot (3\,in) = 4500\,lb{\cdot}in$

$T_y = M_y = F_1 \cdot (3\,in) = 6000\,lb$

Shear stress

$$\tau_{xy_s} = \frac{VQ}{It} \qquad \text{where } Q = A'z' = \pi(0.5in)^2(0.212in)$$

$$Q = \frac{0.1665 in^3}{2} = 0.8325 in^3$$



$$z' = \frac{4r}{3\pi} = \frac{4(0.5in)}{3\pi} = 0.212 in$$

Shear: $\tau_{xy_s} = \frac{2000lb \cdot (0.08325 in^3)}{(0.0491 in^4)(1 in)} = 3391.91 \text{ psi}$

Torsion: $\tau_{xy_T} = \frac{T_y \cdot r}{J}$

$$\tau_{xy_T} = \frac{6000lb(0.5in)}{(0.0982 in^4)} = 30.5577 \text{ Kpsi}$$

$$\tau_{yx_{total}} = \tau_{xy_s} + \tau_{xy_T} = 3.391 \text{Kpsi} + 30.5 \text{Kpsi} = 33.9487$$

$$\boxed{\tau_{xy} = 33.9 \text{ ksi}}$$

Normal Stress

$$\sigma_y = \frac{F}{A} + \Sigma \frac{Mr}{I}$$

$$\sigma_{y_1} = \frac{F_3}{\pi r^2} = \frac{-1500lb}{\pi(0.5in)^2} = -1909.86 \text{ psi}$$

$$\sigma_{y_2} = \frac{M_z \cdot x}{I} = \frac{-4500 lb \cdot in (0.5in)}{\frac{1}{4}\pi(0.5in)^2} = -45836.6 \text{ Kpsi}$$

$$\sigma_{y_3} = \frac{M_x \cdot z}{I} = \frac{(20000 lb \cdot in)(0)}{(0.049 in^4)} = 0$$

$$\sigma_y = -(1.9 \text{Kpsi} - 45.8 \text{Kpsi}) = -47.7 \text{ Kpsi}$$

$$\boxed{\sigma_y = -47.7 \text{ KPsi}}$$

no σ_x or σ_y by inspection

Principle Stress

$$\sigma_{1,2} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-47.7 + 0}{2} \pm \sqrt{\left(\frac{47.7 - 0}{2}\right)^2 + (33.9)^2}$$

$$\sigma_1 \approx -65 \text{ Ksi}$$

$$\sigma_2 = +17.6 \text{ Ksi}$$

Max Shear Stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{0 - 47.7}{2}\right)^2 + (33.9)^2}$$

$$\tau_{max} \approx 41.449 \text{ Ksi}$$

Von Mises stress factor of safety

$$n = \frac{S_y}{\sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2}}$$

$$n = \frac{39.5 \text{ KPsi}}{\sqrt{(47.7)^2 \; 0 - 0 + 3(33.9)^2}}$$

$$\boxed{n \simeq 0.52}$$

Tresca Criteria factor of Safety

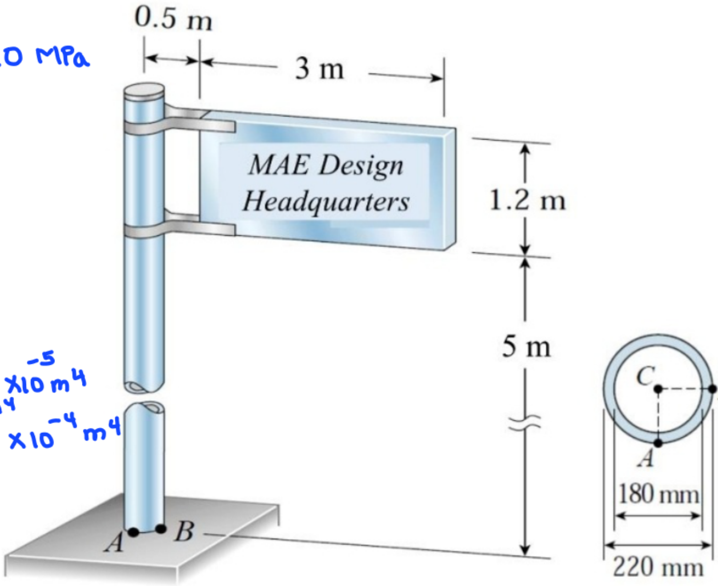
$$n = \frac{S_y/2}{\tau_{max}}$$

$$n = \frac{\left(\frac{39.5}{2} \text{Ksi}\right)}{(41.4 \text{ Ksi}} \approx 0.477$$

2. [We revisit sign post problem from HW 1, this time from design stand point]
The 3 m x 1.2 m sign is supported by a hollow circular post made from 1080 HR. The post has an outer radius of 11cm and an inner radius of 9 cm. The sign is offset 0.5 m from the centerline of the post and its lower edge is 5 m above the ground. We assume point ***A*** is the critical point. Determine the max amount of wind pressure (in *kPa*) that the sign can tolerate to based on
(a) Distortion Energy (Von Mises), and
(b) Max Shear Stress (Tresca) design theories.
(You may assume a factor of safety *n = 1*, i.e., minimum theoretically acceptable)

from table A-20

1080 HR: $S_u = 420$ MPa



$F = P \cdot A$

$I = \frac{\pi}{4}(r_o^4 - r_i^4) = 6.346 \times 10^{-5}\ m^4$

$J = \frac{\pi}{2}(r_o^4 - r_i^4) = 1.269 \times 10^{-4}\ m^4$

$r_i = 0.09\,m$

$r_o = 0.11\,m$

Transfer of forces:

$V_x = -3.6\,P$

$T_z = (3.6P)(2m) = 7.2P$

$M_z = (3.6P)(5.6\,m = -20.16P$

Axial

$$\sigma_z = -\frac{M_y x}{I} = \frac{(-20.16P)(0.11m)}{6.346 \times 10^{-5} m^4} = 3.4945 \times 10^4 P$$

shear

$$\tau_{yz} = \frac{Tc}{J} = \frac{(7.2P)(0.11m)}{1.269 \times 10^{-4}} = 6.241 \times 10^3 P$$

$$\sigma_{1,2} = \sqrt{\frac{34945P + 0}{2} \pm \left(\frac{34945P - 0}{2}\right)^2 + (6241P)^2}$$

$$\sigma_{1,2} = 17472P \pm 18553P$$

$\sigma_1 = 36025P$

$\sigma_2 = -1081P$

$\sigma_3 = 0$

a) Distorsion Energy (von Mises)

$$n = \frac{S_y}{\sigma'} = \frac{420\,MPa}{\sigma_1} = 1$$

$$\sigma' = \left[\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}{2}\right]^{1/2}$$

$$\sigma' = \left[\frac{(36025P + 1081P)^2 + (-1081P)^2 + (36025P)^2)}{2}\right]^{1/2}$$

$= 36577P$

$$\sigma' = \frac{S_y}{n} = \frac{420\,MPa}{1} = 36577P$$

$$P = \frac{420\,MPa}{36577\,Pa} = \boxed{11.482\ kPa}$$

b) Tresca: Max shear

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = 18553P$$

$$n = \frac{S_u/2}{\tau_{max}} = \frac{\left(\frac{420\,MPa}{2}\right)}{18553P} = 1$$

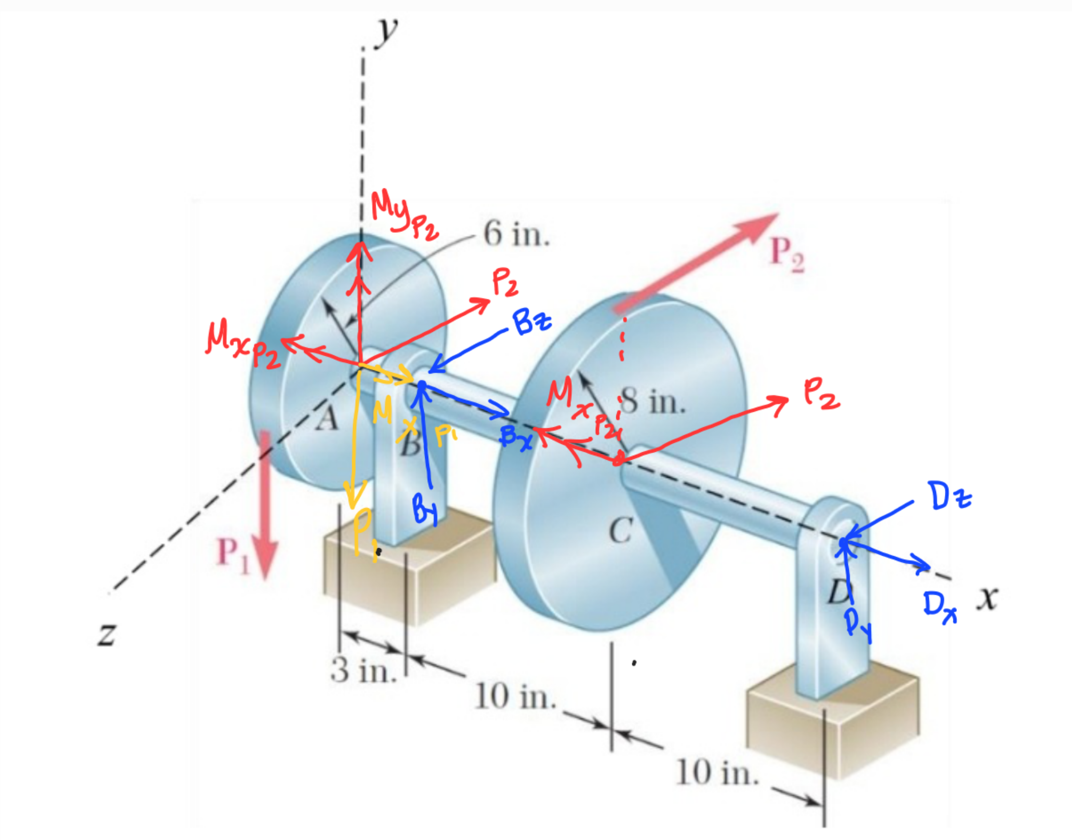
$$\boxed{P = 11.318\ kPa}$$

3. The power transmission shaft is supported by journal bearings at B and D that only exert radial force components (i.e., y and z directions). The shaft is **1 in. diameter** and made of **wrought aluminum 3004 (H34)**. As shown, the vertical force $P_1 = 400$ lb and the horizontal force $P_2 = 300$ lb are applied to gear disks welded to the solid shaft AD. Please use Table A-24 (b) for the material properties of wrought aluminum.

a) Determine the bearing loads (support reactions at ***B*** and ***D***). The bearing loads are crucial for bearing selection from different catalogs.

b) Find the maximum bending moment, M, and the transmitted torque, T_x, in the shaft.

c) Determine the smallest factor of safety in the shaft based on the maximum distortion energy (Von Mises) theory of failure.



Forces

$\Sigma F_y = 0 = B_y + D_y - P_1$ eq.1 $\rightarrow B_y + D_y = P_1 = 400$

$\Sigma F_z = 0 = B_z + D_z - P_2$ eq.2 $\rightarrow B_z + D_z = P_2 = 300$

Transfered forces

$T_{P_2} = Mx_{P_2} = P_2 \cdot (8in) = 300lb \cdot 8in = 2400$ psi

$My_{P_2} = P_2 \cdot (13in) = 300lb \cdot 13in = 3900$ psi

$T_{P_1} = Mx_{P_1} = P_1 \cdot (6in) = 400lb \cdot 6in = 2400$ psi

Moments about origin

$\Sigma M_x = Mx_{P_2} - Mx_{P_1} = 0$

eq.3 $\Sigma M_y = My_{P_2} - B_z \cdot (3in) - D_z \cdot (23in) = 0$

eq.4 $\Sigma M_z = B_y \cdot (3in) + D_y \cdot (23in) = 0$

eq.1 $B_y = P_1 - D_y$

eq.3 $B_y \cdot 3 + D_y \cdot 23 = 0$

$3(P_1 - D_y) + D_y \cdot 23 = 0$

$3P_1 - 3D_y + 23D_y = 0$

$-20D_y = 3P_1 \rightarrow D_y = \frac{3}{20}P_1$

$D_y = \frac{-3}{20} \cdot 400 = -60$

$B_y = 400 - (-60) = 460$

eq.2 $B_z = P_2 - D_z$

eq.4 $0 = My_{P_2} - B_z \cdot 3in - D_z \cdot 23in$

$\hookrightarrow My_{P_2} = 3B_z + 23D_z$

Plug eq. 2 into eq. 4:

$My_{P_2} = 3(P_2 - D_z) + 23 \cdot D_z$

$13P_2 - 3P_2 = D_z(-3 + 23)$

$D_z = \frac{10P_2}{20} = \frac{P_2}{2} = \frac{300}{2}$

$D_z = 150$

Solve for B_z:

$B_z = P_2 - D_z$
$= 300 - 150 = 150$

Reaction Forces:

$B_z = 150\text{lb}$ $D_z = 150\text{ lb}$

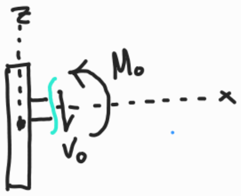
$B_y = 460$ $D_y = -60\text{lb}$

$B_x = B_z = 0$ b/c able to move

b) Maximum Bending Moment & Torque

Bending Diagram (Z)

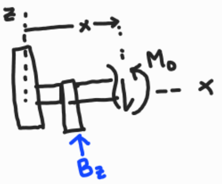
$0 \le x < 3$



$\Sigma M_o = 0 = M_o$

$M_o = 0$

$3 \le x < 13$

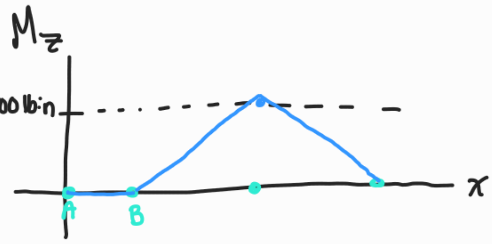


$\Sigma M_o = 0 = M_o - B_y \cdot x$

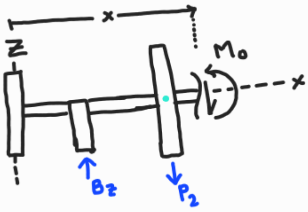
$M_o = B_y (x-3)$

at $x=3$ $M_o = 150 \cdot 0 = 0 \text{ lb}\cdot\text{in}$

at $x=13$ $M_o = 150 \cdot 10 = 1500 \text{ lb}\cdot\text{in}$



$13 \le x \le 23$



$\Sigma M_o = 0 = -B_z(x-3) + P_2(x-13) + M_o$

$M_o = B_z(x-3) - P_2(x-13)$

$x = 13$

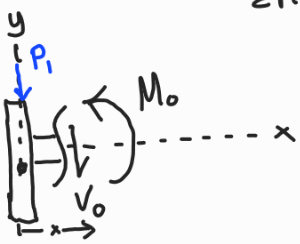
$M_o = 150\ (13-3) = 1500 \text{ lb}\cdot\text{in}$

$x = 23$

$M_o = 150(20) - 300(10) = 0 \text{ lb}\cdot\text{in}$

Bending Diagram (Y)

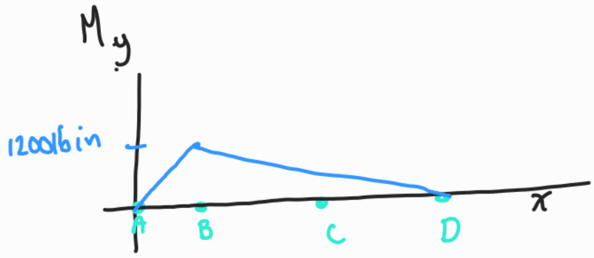
$0 \le x < 3$



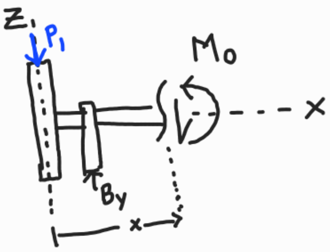
$\Sigma M_o = 0 = P_1 \cdot x + M_o$

$x = 3$

$M_o = P_1 \cdot x = -3 \cdot 400 = -1200 \text{ lb}\cdot\text{in}$



$3 \le x \le 23$



$\Sigma M_o = 0 = P_1 \cdot x - B_y(x-3) + M_o$

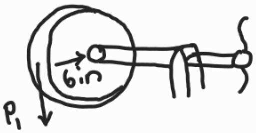
$M_o = B_y(x-3) - P_1 \cdot x$

$x=3$ $M_o = -400(3) = -1200 \text{ lb}\cdot\text{in}$

$x=23$ $M_o = 460(20) - 400(23) = 0$

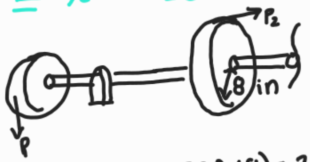
Torque Diagram

$0 \le x < 13$

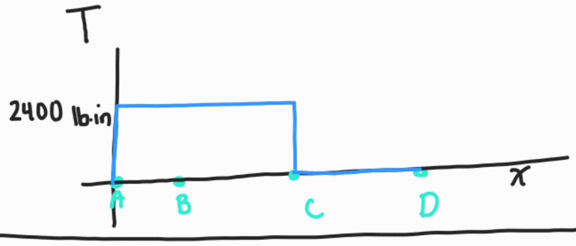


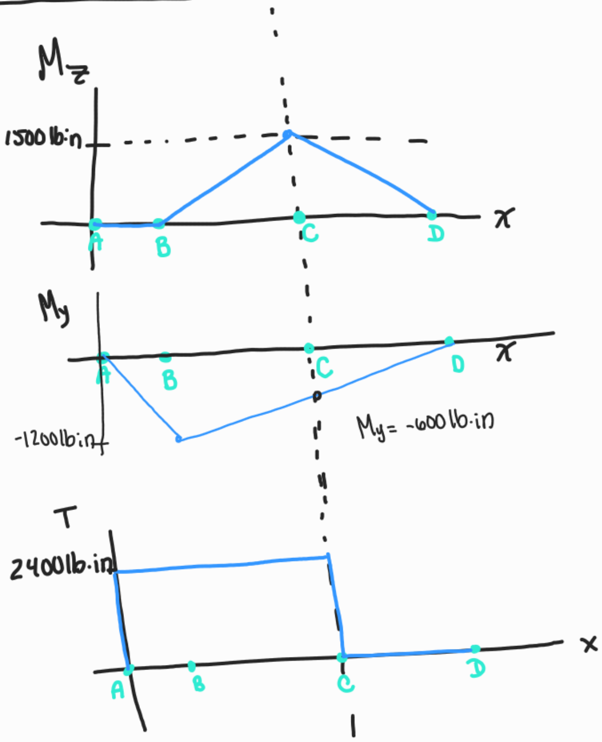
$T = P_1 \cdot 6\text{in} = 400 \cdot 6 = -2400 \text{ lb}\cdot\text{in}$

$13 \le x < 23$



$T = P_2 \cdot 8\text{in} = 300(8) = 2400 \text{ lb}\cdot\text{in}$





Critical Point at C

for M_y @ C : $x = 13$

$M_y = 460(13) - 400(10) = -600 \text{ lb}\cdot\text{in}$

Maximum Bending Moment

$$M_{max} = \sqrt{(600 \text{ lb}\cdot\text{in})^2 + (1500 \text{ lb}\cdot\text{in})^2}$$

$$\boxed{M_{max} = 1615.55 \text{ lb}\cdot\text{in}}$$

Transmitted Torque

$$\boxed{T_c = 2400 \text{ lb}\cdot\text{in}}$$

c) Smallest Factor of safety:

$$\sigma_x = \frac{M_{max} \cdot c}{I} = \frac{(1615.55 \text{ lb}\cdot\text{in})(0.5)}{(0.0491 \text{ in}^4)} = 16.5 \text{ Kpsi}$$

$$I = \frac{1}{4}\pi r^4 = \frac{1}{4}\pi (0.5\text{in})^4 = 0.0491 \text{ in}^4$$

$$J = \frac{1}{2}\pi r^4 = \frac{1}{2}\pi (0.5\text{in})^4 = 0.0982 \text{ in}^4$$

$$\tau_{xy} = \frac{Tr}{J} = \frac{(2400 \text{ lb}\cdot\text{in})(0.5)}{0.0982 \text{ in}^4} = 12.2 \text{ Kpsi}$$

$$\sigma' = \sqrt{\sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2} = \sqrt{(16.5 \text{ Kpsi})^2 + 3(12.2 \text{ KPsi})^2} = 26.81 \text{ Kpsi}$$

Von Mises:

$$n = \frac{S_{ut}}{\sigma'} = \frac{27}{26.81 \text{ Kpsi}} = 1.00709$$

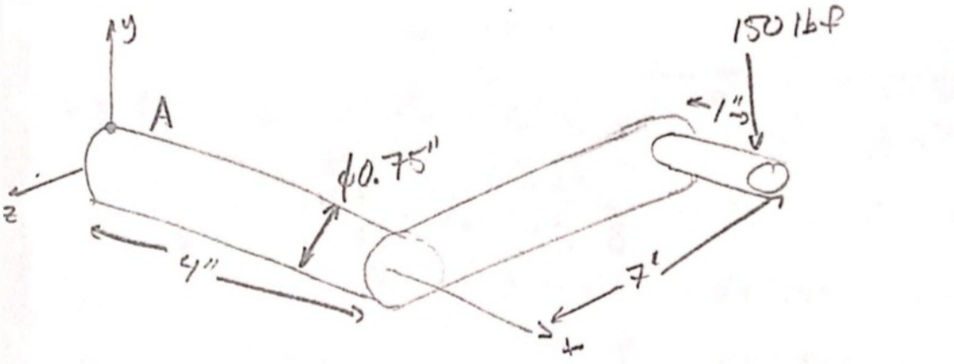
$$\boxed{n = 1.01}$$

4. A handle structure is made from ASTM grade 30 cast iron (refer to Table A-24 for material properties). It is subjected to $\boldsymbol{F}$ = 150 lbf, as shown. We need to analyze the state of stress and safety in rod AB. It is suggested that point A (located on top surface of the shaft, along the y axis, as shown in the figure) is a critical point in the structure. Determine

(a) the principal and max shear stresses at point A located on the surface of the shaft at the *y* axis

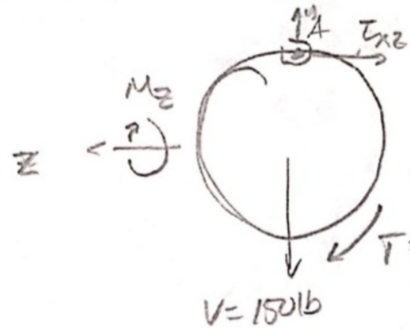
(b) the factor of safety using the Coulomb-Mohr and modified Mohr theories

Note: the distance of force *F* to point *A* along **x** *direction* is: 1" + 4" = 5"



grade 30 cast iron
A-24 $\begin{cases} S_{ut} = 31\text{ ksi} \\ S_{uc} = 109\text{ ksi} \end{cases}$

- Transfer of Forces



$M_z = (5")(150\text{lb}) = \underline{750\text{in-lb}}$

$T = 7"(150\text{lb}) = \underline{1050\text{in-lb}}$

a) principal & max shear

→ no axial load, Q @ pt. A = 0

$\Rightarrow \tau_{xz} = -\dfrac{Tr}{J} = -\dfrac{(1050)(.375)}{\frac{\pi}{2}(.375)^4} = -12.676\text{ ksi}$ ~ +x plane, −z direction $\Rightarrow \tau_{xz} < 0$

$\sigma_x = \dfrac{Mc}{I} = \dfrac{(750)(.375)}{\frac{\pi}{4}(.375)^4} = 18.108\text{ ksi}$ → $\sigma_x > 0$ since A is in tension

$\sigma_{1,2} = \dfrac{18.108+0}{2} \pm \sqrt{\left(\dfrac{18.108-0}{2}\right)^2 + (-12.676)^2} = 9.054 \pm \underline{15.577\text{ ksi}}$ ↳ τ_{max}

$\Rightarrow$ $\boxed{\begin{aligned} \sigma_1 &= 24.631\text{ ksi} \rightarrow \sigma_A \\ \sigma_2 &= -6.523\text{ ksi} \rightarrow \sigma_B \\ \tau_{max} &= 15.577\text{ ksi} \end{aligned}}$

b) Coulomb Mohr $\sigma_A \geq 0 \geq \sigma_B \Rightarrow \dfrac{\sigma_A}{S_{ut}} - \dfrac{\sigma_B}{S_{uc}} = \dfrac{1}{n} \Rightarrow n = \left(\dfrac{24.631}{31} - \dfrac{-6.523}{109}\right)^{-1} = \boxed{1.17}$

Modified Mohr $\sigma_A \geq 0 \geq \sigma_B$ & $|\sigma_B/\sigma_A| \leq 1$

$\Rightarrow \sigma_A = \dfrac{S_{ut}}{n} \Rightarrow \quad n = \dfrac{S_{ut}}{\sigma_A} = \dfrac{31}{24.631} = \boxed{1.26}$