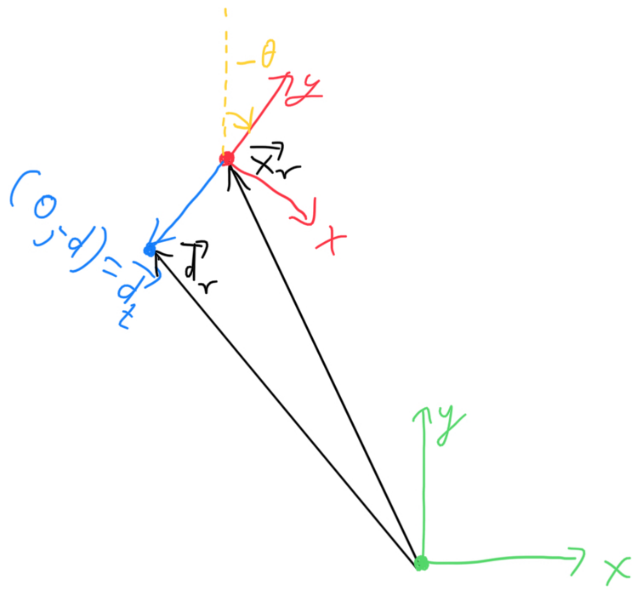
1 Trajectory Geometry



$$\vec{d}_r = R_\theta \vec{d}_t + \vec{x}_r$$

In this diagram, the red axes represents the target's coordinate system. The green axes represent the robot's coordinate system. Points in robot's coordinate system are represented with subscript r, ex. $\vec{d_r}$ and points in target coordinate system are subscript t, so $\vec{d_t}$.

The point we wish to navigate to is a distance d in front of the target. So from the point of view of the target, the coordinate of the target point $\vec{d_t} = (0, d)$. First, we wish to find what are the coordinates of this point are from the point of view of the robot, drawn in the diagram as $\vec{d_r}$.

From CV, we know the rotation $-\theta$ of the target (and its coordinate system) relative to the robot. We also know the displacement $\vec{x}_r$ of the target's origin relative to the robot origin in robot coordinates.

Now we can construct a function $F(\vec{d_t}) \to \vec{d_r}$ that takes points in target coordinates to robot coordinates. Since points transform in the opposite direction as coordinate systems, we must apply a rotation of $+\theta$ first, then add the displacement. Therefore,

$$\vec{d_r} = F(\vec{d_t}) \tag{1}$$

$$\vec{d_r} = \boldsymbol{R_\theta}\vec{d_t} + \vec{x}_r \tag{2}$$

$$\vec{d_r} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 \\ -d \end{pmatrix} + \vec{x}_r \tag{3}$$

Expanding this is left as an exercise for the reader. I don't remember if CV gives you the rotation of points or rotation of coordinate axes. This will flip the sign of θ, but should be easy to figure out.