

$$(i). \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

Proof. Suppose $n = 1$, we have $1^2 = \frac{1(1+1)(2+1)}{6}$

Assume that for some positive integer k , $\sum_{i=0}^k i = \frac{k(k+1)(2k+1)}{6}$

$$\begin{aligned} \sum_{i=0}^{k+1} i^2 &= \sum_{i=0}^k i^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \\ &= (k+1) \left[\frac{k(2k+1) + 6(k+1)}{6} \right] = (k+1) \left[\frac{2k^3 + 7k + 6}{6} \right] \\ &= \frac{(k+1)(k+2)(2k+3)}{6}, \end{aligned}$$

giving us the desired result. By the Principle of Mathematical Induction,

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

□