CS101 Advanced Engineering Mathematics (I)

工程數學(一)

[Guidelines]

- All the homework in this course will involve solving advanced engineering mathematics problems (differential equations in particular) by hand and computer.
- While discussion with other classmates is allowed, you MUST work independently to generate your own solutions to the problems.
- Python programming will be used for plotting solutions. You should reference the Python Tutorial (課程教學影片) for detail information.
- For each homework, you must submit a **written report (書面報告)**.

[General Instructions]

To get a good grading in homework assignments, you are advised to do the following:

- Do not copy other classmate's works! (請遵守學術倫理，嚴禁抄襲)
- Provide correct answers in details. (詳細推導過程與標明正確答案)
- Prepare your written reports in good quality (使用 Template 檔並書寫工整).
- Meet the deadline! Late homework will **not** be collected. (按時繳交，逾時不候)

指導教授：張元翔

Homework Assignment 2

Higher-Order Differential Equations and Systems

Deadline: 1 / 06 / 2022 (星期五)

(期末考前一週繳交至電學 603 計算機視覺研究室)

[Problems 1 ~ 7] (35% 每題 5 分)

Please answer the following:

(a) Solve the following *Differential Equation* (DE) or *Initial-Value Problem* (IVP). (手寫推導)

(b) Use Python programming to plot the solution curves. The interval I is the given range for the x-data in the plots. The plots must be carefully ***labeled***, ***titled***, and ***copyright*** for full credits.

1. $y'' + 2y' + 10y = 0,\ y(0) = 2,\ y'(0) = -1$
   (For plotting, let I : [0, 1])

2. $y'' + 2y' + 10y = 0,\ y(0) = 1,\ y'(0) = 0$
   (For plotting, let I : [0, 2π])

3. $y''' - y'' + 2y = 0$
   (For plotting: let I : [-2π, 2π] and $c_1 = c_2 = c_3 = 1$)

4. $y'' - 16y = e^{-4x},\ y(0) = 1,\ y'(0) = 0$
   (For plotting, let I : [0, 1] and $c_1 = c_2 = 1$)

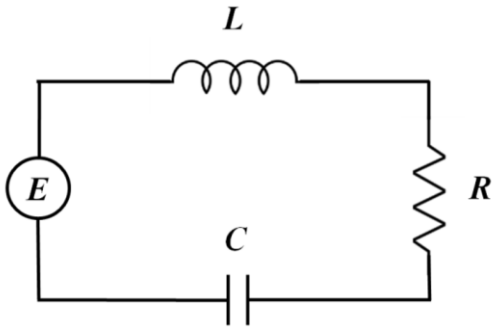
5. $y'' + 4y = \cos(2x),\ y(0) = 1,\ y'(0) = 0$
   (For plotting, let I : [0, 4π])

6. $y'' + y = \cos^2 x,\ y(0) = 4/3,\ y'(0) = -1$
   (For plotting, let I : [0, 4π]).

7. $x^2y'' - 2xy' + 2y = x^3e^x$
   (For plotting, let I : [0, 1] and $c_1 = c_2 = 1$)

[Problems 8 ~ 9] (30%，每題 15 分)

8. Consider the following *LRC* Series Circuit, where L = 1 H, R = 20 Ω, C = 0.001 F, and $E(t) = 10\sin(60t)$ V.



Please answer the following:

(a) Assume the initial charge is 0 and the initial current is 0. Determine the charge $q(t)$ and the current $i(t)$. (手寫推導)

(b) Give the two plots for the charge and the current, respectively (for t = 0 ~ 2 seconds). The plots must be carefully ***labeled***, ***titled***, and ***copyright*** for full credits.

(c) 試用文字敘述所觀察到的電路運作情形。

9. Following Problem 8, if L = 1 H, R = 0 Ω (短路), C = 0.001 F, and $E(t)$ = 0V.

(a) Assume the initial charge is 50 (coulomb) and the initial current is 0. Determine the charge $q(t)$ and the current $i(t)$. (手寫推導)

(b) Give the two plots for the charge and the current, respectively (for t = 0 ~ 2 seconds). The plots must be carefully ***labeled***, ***titled***, and ***copyright*** for full credits.

(c) 試用文字敘述所觀察到的電路運作情形。

[Problems 10 ~ 11] (35%)

10. Please answer the following: (15%)

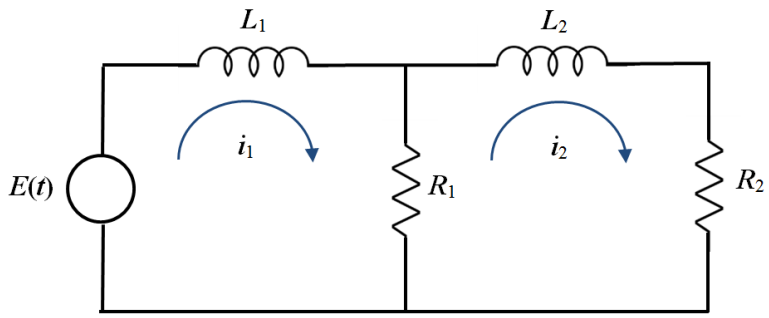
(a) Solve the following *System of differential equations*:

$$\begin{aligned} \frac{dx}{dt} &= -y + t, \\ \frac{dy}{dt} &= x - t \end{aligned}$$

(手寫推導)

(b) Use Python programming to plot the two solution curves for $x(t)$ and $y(t)$ in one plot for comparison (assuming $c_1 = c_2 = 1$). The plots must be carefully ***labeled***, ***titled***, and ***copyright*** for full credits.

11. Given the following circuit: (20%)



The ***system of differential equations*** for the currents $i_1(t)$ and $i_2(t)$ in the electrical network is given as follows:

$$\begin{cases} L_1 \dfrac{di_1}{dt} + R_1(i_1 - i_2) = E(t) \\ R_1(i_2 - i_1) + L_2 \dfrac{di_2}{dt} + R_2 i_2 = 0 \end{cases}$$

(a) If $L_1 = L_2 = 10\text{H}$, $R_1 = 20\Omega$, $R_2 = 30\Omega$, and $E(t) = 20\text{V}$ and initial current $i_1(0) = 0$, $i_2(0) = 0$, solve the *system of differential equations*. (手寫推導)

(b) Use Python programming to plot the currents i_1、i_2 in the same plot and show your result ($t = 0 \sim 10$ second). The plots must be carefully ***labeled***, ***titled***, and ***copyright*** for full credits.

(c) 試用文字敘述所觀察到的電路運作情形。