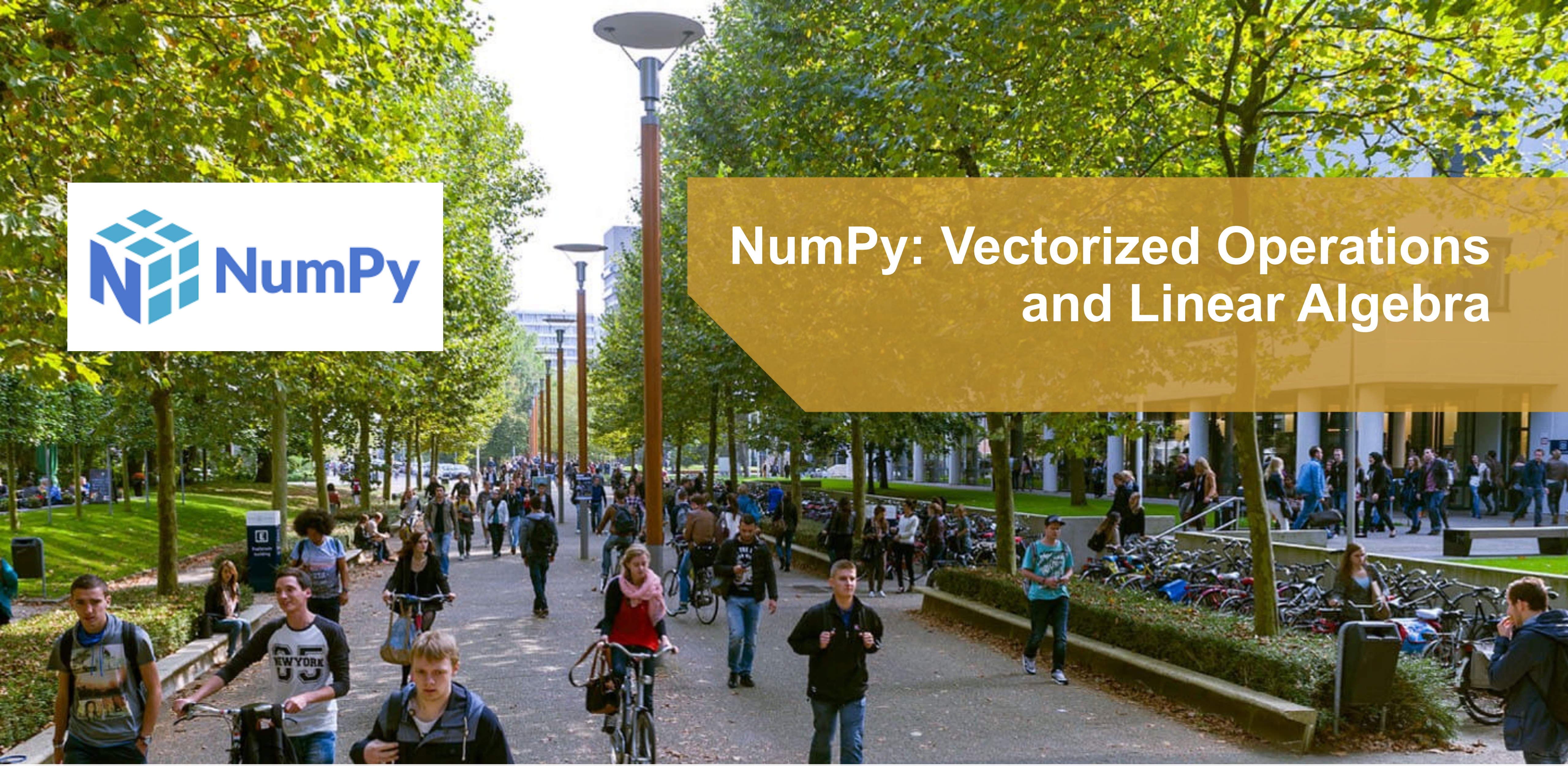
NumPy: Vectorized Operations and Linear Algebra



What is a vectorized operation in NumPy?

❖ A vectorized operation is an operation that can be evaluated on all elements of an array at once instead of looping over individual entries.

NumPy Array

```
a = numpy.random.random((100000))
b = 2
c = a + b
```

Python List

```
a = numpy.random.random((100000)).tolist()
b = 2
c = []
for aa in a:
    c.append(aa + b)
```

What is a vectorized operation in NumPy?

- A vectorized operation is an operation that can be evaluated on all elements of an array at once instead of looping over individual entries.
- The main advantage of vectorized operation is speed in computation. Example: adding a number to an array:

NumPy Array

```
a = numpy.random.random((100000))
b = 2
c = a + b
```

```
1.65 ms ± 50.4 µs per loop (mean ± std.
```

Python List

```
a = numpy.random.random((100000)).tolist()
b = 2
c = []
for aa in a:
    c.append(aa + b)
```

```
28.2 ms ± 1.8 ms per loop (mean ± std. dev.
```

All NumPy functions are implemented as vectorized operations.

Vectorized Operations in NumPy

There are in general three types of vectorized operations in NumPy:

1) Vectorized Transformation: that refers to vectorized operations that are applied to an array and produce an array of the same shape.
    a) Simple scalar transformations
    b) Boolean operations
    c) Mathematical functions

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2) Vectorized Dimension Reduction: that reduces the dimensionality of the input array.

Questions?

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2) Vectorized Dimension Reduction: that reduces the dimensionality of the input array.
3) Vectorized Linear Algebra Operations

Linear Algebra Operations in NumPy

- Element-wise operations
  - On arrays with the same shape

$$A^{n \times m} + B^{n \times m} = C^{n \times m}$$

$$A^{n \times m} - B^{n \times m} = D^{n \times m}$$

$$A^{n \times m} * B^{n \times m} = E^{n \times m}$$

$$A^{n \times m} / B^{n \times m} = F^{n \times m}$$

Linear Algebra Operations in NumPy

- Element-wise operations
  - On arrays with the same shape
  - On arrays with different shape (using broadcasting)

But what is broadcasting?

Broadcasting in Numpy

- Broadcasting enables NumPy to handle arrays with different shapes during vectorized operations.
- Broadcasting conditions (being "broadcastable"):
  - Array dimensions must be equal, or
  - for non-equal dimensions, either of them must be 1.

A (3d array): 5 x 4 x 3
B (3d array): 5 x 4 x 3 ✓

A (3d array): 1 x 4 x 3
B (3d array): 5 x 4 x 3 ✓

A (3d array): 5 x 4 x 3
B (2d array): 4 x 3 ?

A (3d array): 5 x 4 x 3
B (3d array): 4 x 5 x 3 ✗

A (3d array): 5 x 1 x 3
B (3d array): 1 x 4 x 1 ✓

A (3d array): 5 x 4 x 3
B (2d array): 5 x 4 ?

Broadcasting in Numpy

- Broadcasting is performed in five steps:
  - i. **Shift:** If the number of dimensions is different, shift the array with smaller dimension to the right.
  - ii. **Match:** Add 1 to the left so that the arrays have the same number of dimensions.
  - iii. **Check:** Check if they are broadcastable: if yes continue to the next step.
  - iv. **Stretch:** “copy” the single dimensions to match dimensions between the arrays.
  - v. **Perform** the element-wise operation.

A (3d array): 5 x 4 x 3
B (2d array): 4 x 3

Shift →

A (3d array): 5 x 4 x 3
B (2d array): 4 x 3

Match →

A (3d array): 5 x 4 x 3
B (3d array): 1 x 4 x 3

Check ✓

Stretch ↓

A (3d array): 5 x 4 x 3
B (3d array): 5 x 4 x 3

Broadcasting in Numpy

❖ Broadcasting is performed in five steps:

i. **Shift:** If the number of dimensions is different, shift the array with smaller dimension to the right.

ii. **Match:** Add 1 to the left so that the arrays have the same number of dimensions.

iii. **Check:** Check if they are broadcastable: if yes continue to the next step.

iv. **Stretch:** “copy”* all the single dimensions to match dimensions between the arrays.

v. **Perform** the element-wise operation.

A (3d array): 5 x 4 x 3
B (2d array): 5 x 4

Shift →

A (3d array): 5 x 4 x 3
B (2d array):   5 x 4

Match →

A (3d array): 5 x 4 x 3
B (3d array): 1 x 5 x 4

Check ✗

* In the stretch step, no actual copy occurs in the memory, it can be considered a virtual copy. This is why we call it stretch and not copy.

Questions?

Linear Algebra Operations in NumPy

- Element-wise operations
  - On arrays with the same shape
  - On arrays with different shape (using broadcasting)
- Dot product
  - Between two vectors

Dot Product Between Two Vectors

❖ The dot product is the sum of the products of the corresponding entries of two vectors.

$$\begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} = a_1 b_1 + a_2 b_2 + \cdots + a_n b_n$$

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = 1 \times 4 + 2 \times 5 + 3 \times 6 = 32$$

Linear Algebra Operations in NumPy

- Element-wise operations
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- Dot product
  - Between two vectors
  - Between two matrices

Matrix Dot Product

$$\boldsymbol{A}^{\boldsymbol{n}\times \mathrm{m}} \cdot \boldsymbol{B}^{\boldsymbol{m}\times\boldsymbol{p}} = \boldsymbol{C}^{\boldsymbol{n}\times\boldsymbol{p}}$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mp} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1p} \\ c_{21} & c_{22} & \cdots & c_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{np} \end{bmatrix}$$

$$a_{11}b_{11} + a_{12}b_{21} + \cdots + a_{1m}b_{m1} = c_{11}$$

Matrix Dot Product

$$\boldsymbol{A}^{\boldsymbol{n}\times\mathrm{m}} \cdot \boldsymbol{B}^{\boldsymbol{m}\times\boldsymbol{p}} = \boldsymbol{C}^{\boldsymbol{n}\times\boldsymbol{p}}$$

$$\begin{bmatrix} \boxed{a_{11} & a_{12} & \cdots & a_{1m}} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mp} \end{bmatrix} = \begin{bmatrix} c_{11} & \boxed{c_{12}} & \cdots & c_{1p} \\ c_{21} & c_{22} & \cdots & c_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ c_{n1} & c_{n2} & \cdots & c_{np} \end{bmatrix}$$

$$a_{11}b_{12} + a_{12}b_{22} + \cdots + a_{1m}b_{m2} = c_{12}$$

Matrix Dot Product

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\5 & 6\end{bmatrix}=\begin{bmatrix}1\times1+2\times3+3\times5 & 1\times2+2\times4+3\times6\\4\times1+5\times3+6\times5 & 4\times2+5\times4+6\times6\end{bmatrix}=\begin{bmatrix}22 & 28\\49 & 64\end{bmatrix}$$

```
A = numpy.array([[1,2,3],[4,5,6]])
B = numpy.array([[1,2],[3,4],[5,6]])
C = A.dot(B)
print(C)
```

```
[[22 28]
 [49 64]]
```

Questions?

Linear Algebra Operations in NumPy

- Element-wise operations
  - On arrays with the same shape
  - On arrays with different shape (using broadcasting)
- Dot product
  - Between two vectors
  - Between two matrices
- Matrix transpose

Matrix Transpose

- The transpose of a matrix (or a vector) switches the row and column indices of the matrix $\boldsymbol{A}$, often denoted by $\boldsymbol{A}^{\boldsymbol{T}}$.

```
A = numpy.array([[1,2],[3,4],[5,6]])
print(A)
print('#################')
print(A.T)
```

```
[[1 2]
 [3 4]
 [5 6]]
#################
[[1 3 5]
 [2 4 6]]
```

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- Matrix inversion

Matrix Inversion

- The inverse matrix of a square matrix is the inverse element of that matrix with respect to the dot product operation.

- Conditions for an invertible matrix:
  - It must be a square matrix
  - It must be a non-singular matrix, i.e., $\det(\boldsymbol{A}) \neq \mathbf{0}$.

- Important properties:
  - $(\boldsymbol{A}^{-1})^{-1} = \boldsymbol{A}$
  - $\boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{I}$
  - $(\boldsymbol{A}^{T})^{-1} = (\boldsymbol{A}^{-1})^{T}$

```
from numpy.linalg import inv
A = numpy.random.uniform(0,1,(3,3))
print(inv(A))
```

```
[[-0.92714039  0.11653039  1.41232812]
 [-0.40393881  1.46465429 -0.11835523]
 [ 2.1522805  -0.64040583 -0.23933963]]
```

Thanks!