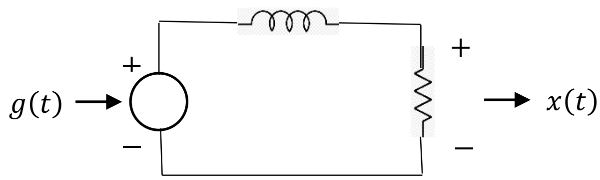
Example 2.15 blz 94_95 Analytisch (Integreren_Primitiveren).docx

[met correctie]

RL-kring (Fig. 2.25)



We de mogen de integratiegrensen $-\infty \;\; tot \;\; \infty$ in de convolutieintegraal $\int_{-\infty}^{\infty} g(\tau)h(t-\tau)d\tau$ vervangen door $0 \;\; tot \;\; t$ omdat het systeem voldoet aan:

i. LTI
ii. input $g(t) = u(t) - u(t-4)$ werkt alleen voor $t > 0$; $g(t) = 0 \; voor \; t < 0$!
iii. causaal systeem (gevolg komt na de oorzaak; output komt na input)

$x(t)$	$= g(t)$	$\times$	$h(t)$	$= \int_0^t g(\tau)h(t-\tau)d\tau$	$= \int_0^t h(\tau)g(t-\tau)d\tau$
$output$	$= \; input$	$\times$	$ImpulseResponse$	$= Convolutie(g,h)$	$= Convolutie(h,g)$
$(response)$					

$t = parameter$

$\tau = lopende \; integratie\_variabele$

dummy variabelen!

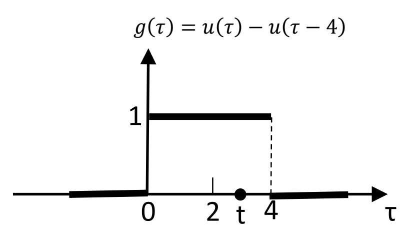
$g(t) = u(t) - u(t-4)$

$h(t) = e^{-t}u(t)$

2 Scenario's: óf $0 < t < 4$ óf $t > 4$;

<u>1</u> $0 < t < 4$
<u>2</u> $t > 4$

<u>1</u>: $0 < t < 4$:



$$x'(t) = \int_0^t g(\tau)h(t-\tau)d\tau =$$

$$= \int_0^t \big(u(\tau) - u(\tau-4)\big)\, e^{-(t-\tau)}u(t-\tau)d\tau =$$

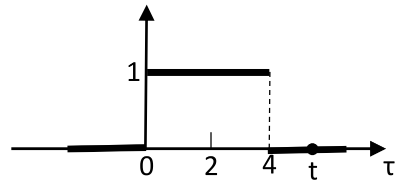
$$= \int_0^t 1.e^{-(t-\tau)}.1d\tau = \int_0^t e^{-(t-\tau)}\, d\tau =$$

($u(t-\tau) = 1, voor \; 0 < \tau < t$ $\;\; en \;\; e^{-t}$ = constant tijdens het integreren)

$$= e^{-t}\int_0^t e^{\tau}\, d\tau = \; e^{-t}[e^{\tau}]_0^t = e^{-t}[e^t - 1] = 1 - e^{-t}$$

2. $t > 4$

$$x'(t) = \int_0^t \big(u(\tau) - u(\tau-4)\big)\, e^{-(t-\tau)} d\tau =$$



$$= \int_0^4 1.e^{-(t-\tau)} d\tau + \int_4^t 0.e^{-(t-\tau)} d\tau =$$

$$= \int_0^4 e^{-(t-\tau)} d\tau = e^{-t} \int_0^4 e^{\tau}\, d\tau$$

$$= e^{-t}[e^{\tau}]_0^4 = e^{-t}[e^4 - 1]$$

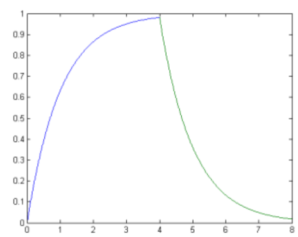
Resume:

$$output\ \ x(t) \quad = 1 - e^{-t} \quad , \quad 0 < t < 4$$
$$= e^{-t}[e^4 - 1]\ , \quad t > 4$$

Controle:

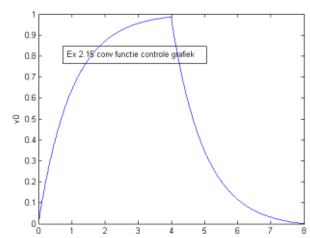
- Grafiek van: $x(t)=1-e^{-t}$, $0<t<4$
  $= e^{-t}[e^4-1]$, $t>4$

  Ex_2_15_grafiek_controle.m



- Numeriek (conv functie):

  Ex_2_15_conv_functie_controle_grafiek.m



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