1 Chapter 1, differential equations

Differential equations(DEs) are equations in the form of

$$y'' + 2ty' + sin(t)y = ln(t)$$

Basically, an equation that has one or more deratives in it.

we can use the following conventions to write the derative:

$$\begin{aligned} y' &= y \\ y'(x) &= y(x) \\ y'(t) &= y(t) \\ \frac{dy}{dt} &= y(t) \end{aligned}$$

DEs have an order, and the order is the highest derative. Also note that the n'th derative for n¿2 is written like $y^{(n)}$ an example would be the following: $ay^{(5)} + by^{(3)} + y'' + 2y = y$, where the order is 5.

Differential equations can be linear or not. to be linear, a function has to be in the following form:

$$ay'' + by' = cy$$

where a,b,c can be any sin, cos, tan, ln, etc of any variable. As long as the y part is linear, meaning no ln(y), sin(y), y^2, etc

2 Solving DEs

First, we have to determine the type of DE. There are lots, but here are the basic ones:

2.1 Seperable DEs

If we can split the DE in the following form:

$$(ay + by)\frac{dy}{dx} = (at + bt)$$

Basically, if you can seperate the y's and the t's to different sides. The strategy is then to just integrate both sides to y or t (depending on the side)

2.2 Linear DEs

If the equation is or can be written in the standard form

$$\frac{dy}{dt} + p(t)y = g(t)$$

Then we can multiply both sides by $u(t)$, where u(t) is the following:

$$u(t) = e^{\int p(t)dt}$$

Which should deliver

$$u(t)\frac{dy}{dt} + u(t)p(t)y = u(t)g(t)$$

If done correctly, then the following holds true:

$$u(t)\frac{dy}{dt} + u(t)p(t)y = [p(t)y]'$$

The next step is to integrate the right side to t, and bring u(t) y back. like this:

$$u(t)y = \int u(t)g(t)dt$$

The final step is to make it an explicit function of y if asked, and to determine what C is if given a starting value.

2.3 Homogenous DEs

If the equation can be written in the form of

$$\frac{dy}{dx} = f(\frac{y}{x})$$

This is most often done by factoring out an x^n We then substitute every $\frac{y}{x}$ with **v**, and **y'** with **v' x + v** We then make it into a seperable DE with the Vs on one side, and the Xs on the other, integrate both sides and then substitute the v back with $\frac{y}{x}$

2.4 Exact DEs

A DE is exact if we can put it in the following form:

$$Mdx + Ndy = 0$$
$$\text{where } \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

When that's done, *NOTE, ASK JOSH FOR CORRECT NOTES*