Toetsen van Hypothesen.

84 $\bar{X} = 1002$ $\quad n = 100$ $\quad \sigma = 10$ $\quad \alpha = 0.05$

(a) $H_0 = \mu = 1000$

$H_1 = \mu \neq 1000$

Teststatistic:

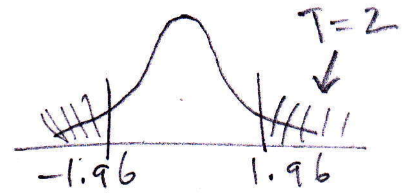
$$T = \frac{\sqrt{n}\,(\bar{x} - \mu_0)}{\sigma}$$

$$\Rightarrow \frac{\sqrt{100}\,(1002 - 1000)}{10} = 2$$

Verwerp H_0 als:

$T \leq z_{0.025}$ OF $T \geq z_{0.975}$

$z_{0.025} = -1.96$ $\quad z_{0.975} = 1.96$



Concl: Verwerp H_0, want $2 > 1.96$

(b) Onderscheidingsvermogen $1 - \beta = 0.90$

$\Rightarrow \beta = 0.10$

$$n \geq \frac{\sigma^2 \left(z_{\frac{\alpha}{2}} + z_{\beta} \right)^2}{(\mu_0 - \mu_1)^2}$$

$z_{\frac{\alpha}{2}} = z_{0.025}$ $\quad z_{\beta} = z_{0.10}$ → zoek inverse waarden

$$n \geq \frac{100\,(-1.96 - 1.28)^2}{(1000 - 1002)^2}$$

$n \geq 262.44$ (rond af naar boven)

$n \geq 263$

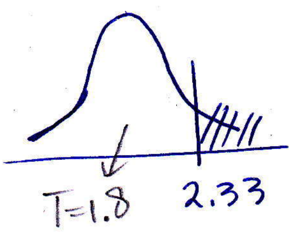
(85) In de klas voorgedaan.

(86) $n = 9 \qquad \sigma = \frac{4}{12} \qquad \alpha = 0.01 \qquad \bar{x} = 2.2$

(a) $H_0 : \mu \leq 2$

$H_1 : \mu > 2$

$$T = \frac{\sqrt{9}\,(2.2 - 2)}{0.333} = 1.8 \qquad z_\alpha = 2.33$$



$T < 2.33$ dus H_0 wordt niet verworpen.

De deskundigen krijgen gelijk.

(b) $P(T \notin K \mid \mu = 2.25)$

$$= P(T < 2.33) = P\left(\frac{\sqrt{n}\,(\mu_1 - \mu_0)}{\sigma} < 2.33 \;\middle|\; \mu = 2.25\right)$$

$$= P\left(U < 2.33 - \frac{\sqrt{9}\,(2.25 - 2)}{0.333}\right)$$

$$= P(U < 0.08) = 0.5319$$

(c) Eenzijdige toets : $n \geq \frac{\sigma^2 (z_\alpha + z_\beta)^2}{(\mu_0 - \mu_1)^2}$

$$n \geq \frac{0.111\,(-2.33 - 2.33)^2}{(2 - 2.25)^2}$$

$$n \geq 38.57$$

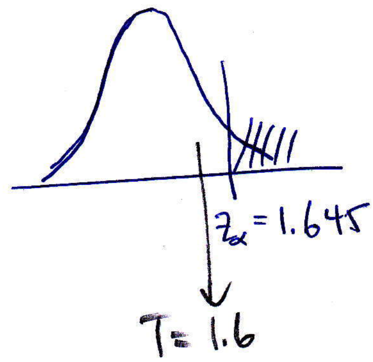
minimaal 39.

87 $\bar{X} = 270$ $\sigma = 50$ $n = 16$ $\alpha = 0.05$

$H_0 : \mu \leq 250$

$H_1 : \mu > 250$

$$T = \frac{\sqrt{16}\,(270 - 250)}{50} = 1.6$$



z_α voor rechtseenzijdig: 1.645

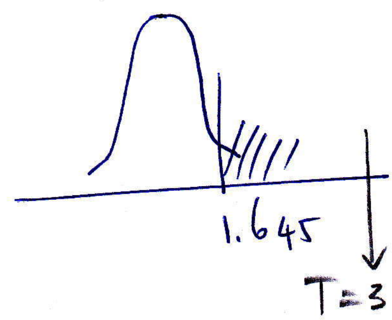
$T < 1.645$ dus verwerp H_0 Niet.

88 $n = 25$ $\bar{X} = 61$ $\sigma = 15$ $\alpha = 0.05$

a $H_0 : \mu \leq 52$

$H_1 : \mu > 52$

$$T = \frac{\sqrt{25}\,(61 - 52)}{15} = 3$$



z_α voor RechtsEenzijdig: 1.645

$3 > 1.645$ dus H_0 wordt verworpen.

91 $\quad n=5 \quad \bar{X}$ $\Big\}$ zelf berekenen. $\quad H_0 : \mu \le 23$

$\alpha = 0.1 \quad s^2$ $\quad H : \mu > 23$

$$\bar{x} = \frac{26+28+22+23+29}{5} = 25.6$$

$$s^2 = \frac{1}{4}\left(\sum x_i^2 - n\cdot\bar{x}^2\right) = \frac{1}{4}\left(3314 - 3276.8\right)$$

$$= 9.3$$

$$s = \sqrt{9.3} = 3.0496$$

Teststatistic: $T = \dfrac{\sqrt{n}\,(\bar{x}-\mu)}{s}$ (zoek in de T tabel).

$$= \frac{\sqrt{5}\,(25.6-23)}{3.0496}$$

$$= 1.91$$

$t_4(0.10) = 1.533$

$T > t_4(0.10)$

$\Rightarrow 1.91 \geq 1.533$ dus verwerp de hypothese! dat $H_0 \le 23$

(92) $s=5$ $n=30$ $\alpha=0.05$ $df=29$ (48)

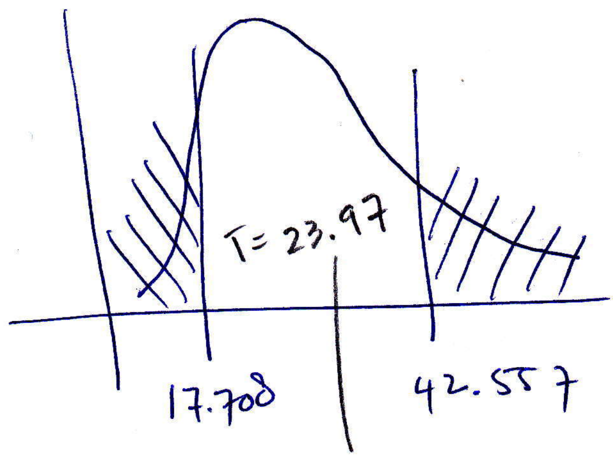
$H_0: \sigma^2 = (5.5)^2$

$H_0: \sigma^2 \neq (5.5)^2$

$$T = \frac{(n-1)s^2}{\sigma^2} = \frac{5^2(30-1)}{(5.5)^2} = 23.97$$

$\chi^2_{29}(0.05) = 42.557$

$\chi^2_{29}(1-0.05) = 17.708$



Verwerp Ho niet by $\alpha = 0.05$

Er is geen bewijs dat $\sigma^2 \neq (5.5)^2$