Formule blad Signalen & Systemen (4 pagina's) – Formuleblad inleveren.

Ch3. The Laplace transform

Two sided Laplace transform: $\mathcal{L}[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$

Properties of the Laplace transform:

Property	x(t)	X(s)
1. Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X(s) + a_2X_2(s)$
2. Scaling	$x(at)$	$\frac{1}{a}X\left(\frac{s}{a}\right), \quad a > 0$
3. Time shifting	$x(t-a)u(t-a)$	$e^{-as}X(s)$
4. Frequency shifting	$e^{-at}x(t)$	$X(s+a)$
5. Time differentiation	$\frac{dx}{dt}$	$sX(s) - x(0^-)$
2nd derivative	$\frac{d^2x}{dt^2}$	$s^2X(s) - sx(0^-) - x'(0^-)$
*n*th derivative	$\frac{d^nx}{dt^n}$	$s^nX(s) - s^{n-1}x(0^-) - s^{n-2}x'(0^-) - \cdots - x^{(n-1)}(0^-)$
6. Time convolution	$x(t) * h(t)$	$X(s)H(s)$
7. Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi j}X_1(s) * X_2(s)$
8. Time integration	$\int_0^t x(t)dt$	$\frac{1}{s}X(s)$
9. Frequency integration	$\frac{x(t)}{t}$	$\int_s^{\infty} X(\lambda)d\lambda$
10. Frequency Differentiation	$tx(t)$	$-\frac{d}{ds}X(s)$
Multiplication by t^n	$t^nx(t)$	$(-1)^n\frac{d^n}{ds^n}X(s)$
11. Time periodicity	$x(t) = x(t+nT)$	$\frac{X_1(s)}{1-e^{-sT}}$

Ch4. Fourier series

1. $x(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$

2. $x(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega_0 t + \phi_n)$

3. $x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t})$

4. $a_0 = \frac{1}{T}\int_0^T x(t)dt$

5. $a_n = \frac{2}{T}\int_0^T x(t)\cos n\omega_0 t\, dt$

6. $b_n = \frac{2}{T}\int_0^T x(t)\sin n\omega_0 t\, dt$

7. $A_n = \sqrt{a_n^2 + b_n^2}, \quad \phi_n = -tan^{-1}\frac{b_n}{a_n}$

8. $C_n = \frac{1}{T}\int_0^T x(t)e^{-jn\omega_0 t}dt$

Ch5. Fourier transform

Fourier transform of *x(t)*: $X(\omega) = \mathcal{F}[x(t)] = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

Properties of Fourier transform:

Property	x(t)	X(ω)
1. Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X(\omega) + a_2X_2(\omega)$
2. Scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$
3. Time shift	$x(t-a)$	$e^{-j\omega a}X(\omega)$
4. Frequency shift	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$
5. Time differentiation	$\frac{dx}{dt}$	$j\omega X(\omega)$
*n*th derivative	$\frac{d^nx}{dt^n}$	$(j\omega)^nX(\omega)$
6.Frequency differentiation	$(-jt)^nx(t)$	$\frac{d^n}{d\omega^n}X(\omega)$
7. Time integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$
8. Time reversal	*x(-t)*	$X(-\omega)$
9. Time convolution	$x_1(t) * x_2(t)$	$X_1(\omega)X_2(\omega)$
10. Frequency convolution	$x_1(t)x_2(t)$	$\frac{1}{2\pi}X_1(\omega) * X_2(\omega)$
11. Parseval's relation	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2d\omega$

App. A: Mathematical formulas

A.2 TRIGONOMETRIC IDENTITIES

$$\sin(-x) = -\sin x$$
$$\cos(-x) = \cos x$$
$$\sin(x \pm 90^\circ) = \pm\cos x$$
$$\cos(x \pm 90^\circ) = \mp\sin x$$
$$\sin(x \pm 180^\circ) = -\sin x$$
$$\cos(x \pm 180^\circ) = -\cos x$$
$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$
$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{1}{\tan x}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{(law of sines)}$$

$$a^2 = b^2 + c^2 - 2bc\cos A \quad \text{(law of cosines)}$$

$$\frac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)} = \frac{a-b}{a+b} \quad \text{(law of tangents)}$$

$$2\sin x\sin y = \cos(x-y) - \cos(x+y)$$
$$2\sin x\cos y = \sin(x+y) - \sin(x-y)$$
$$2\cos x\cos y = \cos(x+y) - \cos(x-y)$$

$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\sin^2 x = \frac{1}{2}(1-\cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1+\cos 2x)$$

A.3 HYPERBOLIC FUNCTIONS

$$\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$$

$$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$$

$$\operatorname{csc}\mathrm{h}x = \frac{1}{\sinh x}$$

$$\sec \mathrm{h}x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\coth x = \frac{1}{\tanh x}$$

$$\sinh(x \pm y) = \sinh x\cosh y \pm \cosh x\sinh y$$

$$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$$

$$\tanh(x \pm y) = \frac{\sinh(x \pm y)}{\cosh(x \pm y)}$$

A.5 INDEFINITE INTEGRALS

If $U = U(x)$, $V = V(x)$, and a = constant,

$$\int UdV = UV - \int VdU \quad \text{(integration by parts)}$$

$$\int U^n dU = \frac{U^{n+1}}{n+1} + C, \quad n \neq 1$$

$$\int a^U dU = \frac{a^U}{\ln a} + C, \quad a > 0, a \neq 1$$

$$\int \ln xdx = x\ln x - x + C$$

$$\int \sin axdx = -\frac{1}{a}\cos ax + C$$

$$\int \cos axdx = \frac{1}{a}\sin ax + C$$

$$\int \sin^2 axdx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$$

$$\int x^2 \sin axdx = \frac{1}{a^3}\left(2ax\sin ax + 2\cos ax - a^2x^2\cos ax\right) + C$$

$$\int x^2 \cos axdx = \frac{1}{a^3}\left(2ax\cos ax - 2\sin ax + a^2x^2\sin ax\right) + C$$

$$\int e^{ax}\sin bxdx = \frac{e^{ax}}{a^2+b^2}\left(a\sin bx - b\cos bx\right) + C$$

$$\int e^{ax}\cos bxdx = \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right) + C$$

$$\int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1) + C$$

$$\int x^2e^{ax}dx = \frac{e^{ax}}{a^3}\left(a^2x^2 - 2ax + 2\right) + C$$

$$\int \cos^2 axdx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$$

$$\int x\sin axdx = \frac{1}{a^2}\left(\sin ax - ax\cos ax\right) + C$$

$$\int x\cos axdx = \frac{1}{a^2}\left(\cos ax + ax\sin ax\right) + C$$

$$\int \sin ax \sin bx dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$\int \sin ax \cos bx dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$\int \cos ax \cos bx dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

$$\int \frac{dx}{\left(a^2+x^2\right)^2} = \frac{1}{2a^2}\left(\frac{x}{x^2+a^2} + \frac{1}{a}\tan^{-1}\frac{x}{a}\right) + C$$

$$\int \frac{x^2 dx}{a^2+x^2} = x - a\tan^{-1}\frac{x}{a} + C$$

A.6 DEFINITE INTEGRALS

If m and n are integers,

$$\int_0^{2\pi} \sin ax \, dx = 0 \qquad \int_0^{2\pi} \cos ax \, dx = 0$$

$$\int_0^{2\pi} \sin mx \sin nx dx = \int_{-\pi}^{\pi} \sin mx \sin nx dx = \begin{cases} 0, & m \neq n \\ \pi, & m \neq n \end{cases}$$

$$\int_0^{\pi} \sin^2 ax \, dx = \int_0^{\pi} \cos^2 ax dx = \frac{\pi}{2}$$

$$\int_0^{\infty} \frac{\sin ax}{x} dx = \begin{cases} \frac{\pi}{2}, & a > 0 \\ 0, & a = 0 \\ -\frac{\pi}{2}, & a < 0 \end{cases}$$

$$\int_0^{\pi} \sin mx \sin nx dx = \int_0^{\pi} \cos mx \cos nx dx = 0, \quad m \neq n$$

$$\int_0^{\pi} \sin mx \cos nx dx = \begin{cases} 0, & m+n = \text{even} \\ \frac{2m}{m^2-n^2}, & m+n = \text{odd} \end{cases}$$

$$\int_0^{\infty} \frac{\sin^2 x}{x} dx = \frac{\pi}{2}$$

$$\int_0^{\infty} \frac{x \sin bx}{x^2+a^2} dx = \frac{\pi}{2} e^{-ab}, \quad a > 0, b > 0$$

$$\int_0^{\infty} \frac{\cos bx}{x^2+a^2} dx = \frac{\pi}{2a} e^{-ab}, \quad a > 0, b > 0$$

$$\int_0^{\infty} \sin cx dx = \int_0^{\infty} \sin c^2 x dx = \frac{1}{2}$$

$$\int_0^{\pi} \sin^2 nx dx = \int_0^{\pi} \sin^2 x dx = \int_0^{\pi} \cos^2 nx dx = \int_0^{\pi} \cos^2 x dx = \frac{\pi}{2}, \quad n = \text{an integer}$$

$$\int_{-\infty}^{\infty} e^{\pm j2\pi t x} dx = \delta(t)$$

$$\int_0^{\pi} \sin mx \sin nx dx = \int_0^{\pi} \cos mx \cos nx dx = 0, \quad m \neq n, m, n \text{ integers}$$

$$\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$$

$$\int_0^{\pi} \sin mx \cos nx dx = \begin{cases} \frac{2m}{m^2-n^2}, & m+n = \text{odd} \\ 0, & m+n = \text{even} \end{cases}$$

$$\int_0^{\infty} e^{-a^2x^2} dx = \frac{\sqrt{\pi}}{2a}, \quad a > 0$$

$$\int_0^{\infty} x^{2n+1} e^{-ax^2} dx = \frac{n!}{2a^{n+1}}, \quad a > 0$$

$$\int_0^{\infty} x^{2n} e^{-ax^2} dx = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}}$$