1 Chapter 1, differential equations

Differential equations(DEs) are equations in the form of

$$y'' + 2ty' + sin(t)y = ln(t)$$

Basically, an equation that has one or more deratives in it.

we can use the following conventions to write the derative:

$$\begin{aligned} y' &= y \\ y'(x) &= y(x) \\ y'(t) &= y(t) \\ \frac{dy}{dt} &= y(t) \end{aligned}$$

differential equations have an order. the order is the highest derative. Also note that the n'th derative for n¿2 is written like $y^{(n)}$ an example would be the following: $ay^{(5)} + by^{(3)} + y'' + 2y = y$, where the order is 5.

Differential equations can be linear or not. to be linear, a function has to be in the following form:

$$ay'' + by' = cy$$

where a,b,c can be any sin, cos, tan, ln, etc of any variable. As long as the y part is linear, meaning no ln(y), sin(y), y^2, etc

2 Solving DEs

First, we have to determine the type of DE. There are lots, but here are the basic ones:

2.1 seperable DEs

If we can split the DE in the following form:

$$(ay + by)\frac{dy}{dx} = (at + bt)$$

Basically, if you can seperate the y's and the t's to different sides. The strategy is then to just integrate both sides to y or t (depending on the side)

2.2 Linear DEs

If the equation is or can be written in the standard form

$$\frac{dy}{dt} + p(t)y = g(t)$$

Then we can multiply both sides by $u(t)$, where it is the following:

$$u(t) = e^{\int p(t)dt}\text{which should deliver}u(t)\frac{dy}{dt} + u(t)p(t)y = u(t)g(t)$$

if done correctly, then

$$u(t)\frac{dy}{dt} + u(t)p(t)y = [p(t)y]'$$

then integrate the right side to t, and bring u(t) y back. like this:

$$u(t)y = \int u(t)g(t)dt$$

then make it an explicit function of y if asked, and determine what c is, if you have a starting value.