Properties of the Absolute Value

Five basic properties of the absolute value are as follows:

1. $|ab| = |a| \cdot |b|$
2. $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
3. $|a - b| = |b - a|$
4. $-|a| \leq a \leq |a|$
5. $|a + b| \leq |a| + |b|$

For example, Property 1 states that the absolute value of the product of two numbers is equal to the product of the absolute values of the numbers. Property 5 is known as *the triangle inequality*.

EXAMPLE 5 **Properties of Absolute Value**

a. $|(-7) \cdot 3| = |-7| \cdot |3| = 21$

b. $|4 - 2| = |2 - 4| = 2$

c. $|7 - x| = |x - 7|$

d. $\left|\frac{-7}{3}\right| = \frac{|-7|}{|3|} = \frac{7}{3}; \left|\frac{-7}{-3}\right| = \frac{|-7|}{|-3|} = \frac{7}{3}$

e. $\left|\frac{x-3}{-5}\right| = \frac{|x-3|}{|-5|} = \frac{|x-3|}{5}$

f. $-|2| \leq 2 \leq |2|$

g. $|(-2) + 3| = |1| = 1 \leq 5 = 2 + 3 = |-2| + |3|$

Now Work Problem 5 ◁

PROBLEMS 1.4

In Problems 1–10, evaluate the absolute value expression.

1. $|-13|$ **2.** $|2^{-1}|$ **3.** $|8 - 2|$

4. $|(-3 - 5)/2|$ **5.** $|2(-\frac{7}{2})|$ **6.** $|3 - 5| - |5 - 3|$

7. $|x| < 4$ **8.** $|x| < 10$ **9.** $|3 - \sqrt{10}|$

10. $|\sqrt{5} - 2|$

11. Using the absolute-value symbol, express each fact.
(a) x is less than 3 units from 7.
(b) x differs from 2 by less than 3.
(c) x is no more than 5 units from 7.
(d) The distance between 7 and x is 4.
(e) $x + 4$ is less than 2 units from 0.
(f) x is between -3 and 3, but is not equal to 3 or -3.
(g) $x < -6$ or $x > 6$.
(h) The number x of hours that a machine will operate efficiently differs from 105 by less than 3.
(i) The average monthly income x (in dollars) of a family differs from 850 by less than 100.

12. Use absolute-value notation to indicate that $f(x)$ and L differ by less than ϵ.

13. Use absolute-value notation to indicate that the prices p_1 and p_2 of two products may differ by no more than 9 (dollars).

14. Find all values of x such that $|x - \mu| < 3\sigma$.

In Problems 15–36, solve the given equation or inequality.

15. $|x| = 7$ **16.** $|-x| = 2$ **17.** $\left|\frac{x}{5}\right| = 7$

18. $\left|\frac{5}{x}\right| = 12$ **19.** $|x - 5| = 9$ **20.** $|4 + 3x| = 6$

21. $|5x - 2| = 0$ **22.** $|7x + 3| = x$ **23.** $|7 - 4x| = 5$

24. $|5 - 3x| = 7$ **25.** $|x| < M$ for $M > 0$

26. $|-x| < 3$ **27.** $\left|\frac{x}{4}\right| > 2$ **28.** $\left|\frac{x}{3}\right| > \frac{1}{2}$

29. $|x + 7| < 3$ **30.** $|2x - 17| < -4$ **31.** $\left|x - \frac{1}{2}\right| > \frac{1}{2}$

32. $|1 - 3x| > 2$ **33.** $|5 - 8x| \leq 1$ **34.** $|3x - 2| \geq 0$

35. $\left|\frac{3x - 8}{2}\right| \geq 4$ **36.** $\left|\frac{x - 7}{3}\right| \leq 5$

In Problems 37–38, express the statement using absolute-value notation.

37. In a science experiment, the measurement of a distance d is 35.2 m, and is accurate to ± 20 cm.

35. $\sum_{k=1}^{6}((k-1)0.5+2.3)$

36. $\sum_{k=1}^{34}((k-1)10+5)$

37. $\sum_{k=1}^{10}100(1/2)^{k-1}$

38. $\sum_{k=1}^{10}50(1.07)^{k-1}$

39. $\sum_{k=1}^{10}50(1.07)^{1-k}$

40. $\sum_{k=1}^{7}5\cdot 2^k$

In Problems 41–46, find the infinite sums, if possible, or state why this cannot be done.

41. $\sum_{k=1}^{\infty}3\left(\frac{1}{2}\right)^{k-1}$

42. $\sum_{i=0}^{\infty}\left(\frac{1}{3}\right)^i$

43. $\sum_{k=1}^{\infty}\frac{1}{2}(17)^{k-1}$

44. $\sum_{k=1}^{\infty}\frac{2}{3}(1.5)^{k-1}$

45. $\sum_{k=1}^{\infty}50(1.05)^{1-k}$

46. $\sum_{j=1}^{\infty}75(1.09)^{1-j}$

47. **Inventory** Every thirty days a grocery store stocks 90 cans of elephant noodle soup and, rather surprisingly, sells 3 cans each day. Describe the inventory levels of elephant noodle soup at the end of each day, as a sequence, and determine the inventory level 19 days after restocking.

48. **Inventory** If a corner store has 95 previously viewed DVD movies for sale today and manages to sell 6 each day, write the first seven terms of the store's daily inventory sequence for the DVDs. How many DVDs will the store have on hand after 10 days?

49. **Checking Account** A checking account, which earns no interest, contains \$125.00 and is forgotten. It is nevertheless subject to a \$5.00 per month service charge. The account is remembered after 9 months. How much does it then contain?

50. **Savings Account** A savings account, which earns interest at a rate of 5% compounded annually, contains \$125.00 and is forgotten. It is remembered 9 years later. How much does it then contain?

51. **Population Change** A town with a population of 50,000 in 2009 is growing at the rate of 8% per year. In other words, at the end of each year the population is 1.08 times the population at the end of the preceding year. Describe the population sequence and determine what the population will be at the end of 2020, if this rate of growth is maintained.

52. **Population Change** Each year 5% of the inhabitants of a rural area move to the city. If the current population is 24,000, and this rate of decrease continues, give a formula for the population k years from now.

53. **Revenue** Current daily revenue at a campus burger restaurant is \$12,000. Over the next seven days revenue is expected to increase by \$1000 each day as students return for the fall semester. What is the projected total revenue for the eight days for which we have projected data?

54. **Revenue** A car dealership's finance department is going to receive payments of \$300 per month for the next 60 months to pay for Bart's car. The kth such payment has a present value of $\$300(1.01)^{-k}$. The sum of the present values of all 60 payments must equal the selling price of the car. Write an expression for the selling price of the car and evaluate it using your calculator.

55. **Future Value** Six years from now, Nicole will need a new tractor for her farm. Starting next month, she is going to put \$100 in the bank each month to save for the inevitable purchase. Six years from now the kth bank deposit will be worth $\$100(1.005)^{72-k}$ (due to compounded interest). Write a formula for the accumulated amount of money from her 72 bank deposits. Use your calculator to determine how much Nicole will have available towards her tractor purchase.

56. **Future Value** Lisa has just turned seven years old. She would like to save some money each month, starting next month, so that on her 21st birthday she will have \$1000 in her bank account. Marge told her that with current interest rates her kth deposit will be worth, on her 21st birthday, $(1.004)^{168-k}$ times the deposited amount. Lisa wants to deposit the same amount each month. Write a formula for the amount Lisa needs to deposit each month to meet her goal. Use your calculator to evaluate the required amount.

57. **Perpetuity** Brad's will includes an endowment to Dalhousie University that is to provide each year after his death, forever, a \$500 prize for the top student in the business mathematics class, MATH 1115. Brad's estate can make an investment at 5% compounded annually to pay for this endowment. Adapt the solution of Example 11 to determine how much this endowment will cost Brad's estate.

58. **Perpetuity** Rework Problem 57 under the assumption that Brad's estate can make an investment at 10% compounded annually.

59. The Fibonacci sequence given in (7) is defined recursively using addition. Is it an arithmetic sequence? Explain.

60. The factorial sequence given in (6) is defined recursively using multiplication. Is it a geometric sequence? Explain.

61. The recursive definition for an arithmetic sequence (b_k) called for starting with a number a and adding a fixed number d to each term to get the next term. Similarly, the recursive definition for a geometric sequence (c_k) called for starting with a number a and multiplying each term by a fixed number r to get the next term. If instead of addition or multiplication we use *exponentiation*, we get two other classes of recursively defined sequences:

$$d_1 = a \text{ and, for each positive integer } k,\ d_{k+1} = (d_k)^p$$

for fixed real numbers a and p and

$$e_1 = a \text{ and, for each positive integer } k,\ e_{k+1} = b^{e_k}$$

for fixed real numbers a and b. To get an idea of how sequences can grow in size, take each of the parameters a, d, r, p, and b that have appeared in these definitions to be the number 2 and write the first five terms of each of the arithmetic sequence (b_k), the geometric sequence (c_k), and the sequences (d_k) and (e_k) defined above.

EXAMPLE 7 **Genetics**

Factorials occur frequently in probability theory.

Suppose two black guinea pigs are bred and produce exactly five offspring. Under certain conditions, it can be shown that the probability P that exactly r of the offspring will be brown and the others black is a function of r, $P = P(r)$, where

$$P(r) = \frac{5!\left(\frac{1}{4}\right)^r\left(\frac{3}{4}\right)^{5-r}}{r!(5-r)!} \qquad r = 0, 1, 2, \ldots, 5$$

The letter P in $P = P(r)$ is used in two ways. On the right side, P represents the function rule. On the left side, P represents the dependent variable. The domain of P is all integers from 0 to 5, inclusive. Find the probability that exactly three guinea pigs will be brown.

Solution: We want to find $P(3)$. We have

$$P(3) = \frac{5!\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2}{3!2!} = \frac{120\left(\frac{1}{64}\right)\left(\frac{9}{16}\right)}{6(2)} = \frac{45}{512}$$

Now Work Problem 35 ◁

PROBLEMS 2.2

In Problems 1–4, determine whether the given function is a polynomial function.

1. $f(x) = x^2 - x^4 + 4$ **2.** $f(x) = \dfrac{x^3 + 7x - 3}{3}$

3. $g(x) = \dfrac{1}{x^2 + 2x + 1}$ **4.** $g(x) = 2^{-3}x^3$

In Problems 5–8, determine whether the given function is a rational function.

5. $f(x) = \dfrac{x^2 + x}{x^3 + 4}$ **6.** $f(x) = \dfrac{3}{2x + 1}$

7. $g(x) = \begin{cases} 1 & \text{if } x < 5 \\ 4 & \text{if } x \geq 5 \end{cases}$ **8.** $g(x) = 4x^{-4}$

In Problems 9–12, find the domain of each function.

9. $k(z) = 26$ **10.** $f(x) = \sqrt{\pi}$

11. $f(x) = \begin{cases} 5x & \text{if } x > 1 \\ 4 & \text{if } x \leq 1 \end{cases}$ **12.** $f(x) = \begin{cases} 4 & \text{if } x = 3 \\ x^2 & \text{if } 1 \leq x < 3 \end{cases}$

In Problems 13–16, state (a) the degree and (b) the leading coefficient of the given polynomial function.

13. $F(x) = 7x^3 - 2x^2 + 6$ **14.** $g(x) = 9x^2 + 2x + 1$

15. $f(x) = \dfrac{1}{\pi} - 3x^5 + 2x^6 + x^7$

16. $f(x) = 9$

In Problems 17–22, find the function values for each function.

17. $f(x) = 8$; $f(2)$, $f(t + 8)$, $f(-\sqrt{17})$

18. $g(x) = |x - 3|$; $g(10)$, $g(3)$, $g(-3)$

19. $F(t) = \begin{cases} 2 & \text{if } t > 1 \\ 0 & \text{if } t = 1 \\ -1 & \text{if } t < 1 \end{cases}$;
$F(12)$, $F(-\sqrt{3})$, $F(1)$, $F\left(\dfrac{18}{5}\right)$

20. $f(x) = \begin{cases} 4 & \text{if } x \geq 0 \\ 3 & \text{if } x < 0 \end{cases}$;
$f(3)$, $f(-4)$, $f(0)$

21. $G(x) = \begin{cases} x - 1 & \text{if } x \geq 3 \\ 3 - x^2 & \text{if } x < 3 \end{cases}$;
$G(8)$, $G(3)$, $G(-1)$, $G(1)$

22. $F(\theta) = \begin{cases} 2\theta - 5 & \text{if } \theta < 2 \\ \theta^2 - 3\theta + 1 & \text{if } \theta > 2 \end{cases}$;
$F(3)$, $F(-3)$, $F(2)$

In Problems 23–28, determine the value of each expression.

23. $6!$ **24.** $(3 - 3)!$ **25.** $(4 - 2)!$

26. $6! \cdot 2!$ **27.** $\dfrac{n!}{(n-1)!}$ **28.** $\dfrac{8!}{5!(8-5)!}$

29. **Subway Ride** A return subway ride ticket within the city costs \$2.50. Write the cost of a return ticket as a function of a passenger's income. What kind of function is this?

30. **Geometry** A rectangular prism has length three more than its width and height one less than twice the width. Write the volume of the rectangular prism as a function of the width. What kind of function is this?

31. **Cost Function** In manufacturing a component for a machine, the initial cost of a die is \$850 and all other additional costs are \$3 per unit produced. (a) Express the total cost C (in dollars) as a linear function of the number q of units produced. (b) How many units are produced if the total cost is \$1600?

32. **Investment** If a principal of P dollars is invested at a simple annual interest rate of r for t years, express the total accumulated amount of the principal and interest as a function of t. Is your result a linear function of t?

33. **Sales** To encourage large group sales, a theater charges two rates. If your group is less than 12, each ticket costs \$9.50. If your group is 12 or more, each ticket costs \$8.75. Write a case-defined function to represent the cost of buying n tickets.

34. **Factorials** The business mathematics class has elected a grievance committee of five to complain to the faculty about the introduction of factorial notation into the course. They decide that they will be more effective if they label themselves as members A, G, M, N, and S, where member A will lobby faculty with

40. Which of the graphs in Figure 2.26 represent one-to-one functions of x?

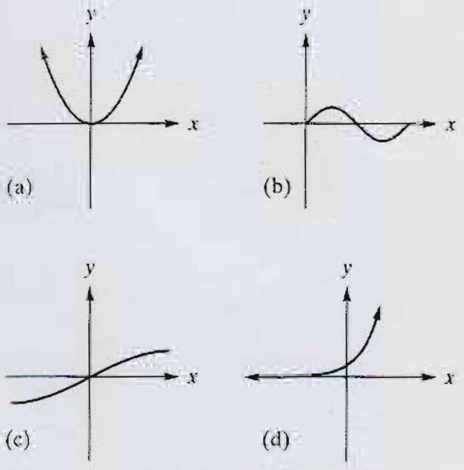

FIGURE 2.26 Diagram for Problem 40.

41. Debt Payments Allison has charged \$9200 on her credit cards. She plans to pay them off at the rate of \$325 per month. Write an equation to represent the amount she owes, excluding any finance charges, after she has made x payments and identify the intercepts, explaining their financial significance if any.

42. Pricing To encourage an even flow of customers, a restaurant varies the price of an item throughout the day. From 6:00 P.M. to 8:00 P.M., customers pay full price. At lunch, from 10:30 A.M. until 2:30 P.M., customers pay half price. From 2:30 P.M. until 4:30 P.M., customers get a dollar off the lunch price. From 4:30 P.M. until 6:00 P.M., customers get \$5.00 off the dinner price. From 8:00 P.M. until closing time at 10:00 P.M., customers get \$5.00 off the dinner price. Graph the case-defined function that represents the cost of an item throughout the day for a dinner price of \$18.

43. Supply Schedule Given the following supply schedule (see Example 6 of Section 2.1), plot each quantity–price pair by choosing the horizontal axis for the possible quantities. Approximate the points in between the data by connecting the data points with a smooth curve. The result is a *supply curve*. From the graph, determine the relationship between price and supply. (That is, as price increases, what happens to the quantity supplied?) Is price per unit a function of quantity supplied?

Quantity Supplied per Week, q	Price per Unit, p
30	\$10
100	20
150	30
190	40
210	50

44. Demand Schedule The following table is called a *demand schedule*. It indicates the quantities of brand X that consumers will demand (that is, purchase) each week at certain prices per unit (in dollars). Plot each quantity–price pair by choosing the vertical axis for the possible prices. Connect the points with a smooth curve. In this way, we approximate points in between the given data. The result is called a *demand curve*. From the graph, determine the relationship between the price of brand X and the amount that will be demanded. (That is, as price decreases, what happens to the quantity demanded?) Is price per unit a function of quantity demanded?

Quantity Demanded, q	Price per Unit, p
5	\$20
10	10
20	5
25	4

45. Inventory Sketch the graph of

$$y = f(x) = \begin{cases} -100x + 1000 & \text{if } 0 \le x < 7 \\ -100x + 1700 & \text{if } 7 \le x < 14 \\ -100x + 2400 & \text{if } 14 \le x < 21 \end{cases}$$

A function such as this might describe the inventory y of a company at time x.

46. Psychology In a psychological experiment on visual information, a subject briefly viewed an array of letters and was then asked to recall as many letters as possible from the array. The procedure was repeated several times. Suppose that y is the average number of letters recalled from arrays with x letters. The graph of the results approximately fits the graph of

$$y = f(x) = \begin{cases} x & \text{if } 0 \le x \le 4 \\ \frac{1}{2}x + 2 & \text{if } 4 < x \le 5 \\ 4.5 & \text{if } 5 < x \le 12 \end{cases}$$

Plot this function.[5]

In Problems 47–50, use a graphing calculator to find all real roots, if any, of the given equation. Round answers to two decimal places.

47. $5x^3 + 7x = 3$

48. $x^2(x - 3) = 2x^4 - 1$

49. $(9x + 3.1)^2 = 7.4 - 4x^2$

50. $(x - 2)^3 = x^2 - 3$

In Problems 51–54, use a graphing calculator to find all x-intercepts of the graph of the given function. Round answers to two decimal places.

51. $f(x) = x^3 + 5x + 7$

52. $f(x) = 2x^4 - 1.5x^3 + 2$

53. $g(x) = x^4 - 1.7x^2 + 2x$

54. $g(x) = \sqrt{3}x^5 - 4x^2 + 1$

[5] Adapted from G. R. Loftus and E. F. Loftus, *Human Memory: The Processing of Information* (New York: Lawrence Erlbaum Associates, Inc., distributed by the Halsted Press, Division of John Wiley & Sons, Inc., 1976).

We have

$$\begin{aligned} r &= pq \\ &= (1000 - 2q)q \\ r &= 1000q - 2q^2 \end{aligned}$$

Note that r is a quadratic function of q, with $a = -2, b = 1000$, and $c = 0$. Since $a < 0$ (the parabola opens downward), r is maximum at the vertex (q, r), where

$$q = -\frac{b}{2a} = -\frac{1000}{2(-2)} = 250$$

The maximum value of r is given by

$$\begin{aligned} r &= 1000(250) - 2(250)^2 \\ &= 250{,}000 - 125{,}000 = 125{,}000 \end{aligned}$$

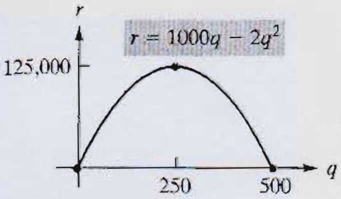

FIGURE 3.25 Graph of revenue function.

Thus, the maximum revenue that the manufacturer can receive is \$125,000, which occurs at a production level of 250 units. Figure 3.25 shows the graph of the revenue function. Only that portion for which $q \geq 0$ and $r \geq 0$ is drawn, since quantity and revenue cannot be negative.

Now Work Problem 29 ◁

PROBLEMS 3.3

In Problems 1–8, state whether the function is quadratic.

1. $f(x) = 5x^2$ **2.** $g(x) = \dfrac{1}{2x^2 - 4}$

3. $g(x) = 7 - 6x$ **4.** $k(v) = 2v^2(2v^2 + 2)$

5. $h(q) = (3 - q)^2$ **6.** $f(t) = 2t(3 - t) + 4t$

7. $f(s) = \dfrac{s^2 - 9}{2}$ **8.** $g(t) = (t^2 - 1)^2$

In Problems 9–12, do not include a graph.

9. (a) For the parabola $y = f(x) = 3x^2 + 5x + 1$, find the vertex. (b) Does the vertex correspond to the highest point or the lowest point on the graph?

10. Repeat Problem 9 if $y = f(x) = 8x^2 + 4x - 1$.

11. For the parabola $y = f(x) = x^2 + x - 6$, find (a) the y-intercept, (b) the x-intercepts, and (c) the vertex.

12. Repeat Problem 11 if $y = f(x) = 5 - x - 3x^2$.

In Problems 13–22, graph each function. Give the vertex and intercepts, and state the range.

13. $y = f(x) = x^2 - 6x + 5$ **14.** $y = f(x) = 9x^2$

15. $y = g(x) = -2x^2 - 6x$ **16.** $y = f(x) = x^2 - 4$

17. $s = h(t) = t^2 + 6t + 9$ **18.** $s = h(t) = 2t^2 + 3t - 2$

19. $y = f(x) = -5 + 3x - 3x^2$

20. $y = H(x) = 1 - x - x^2$

21. $t = f(s) = s^2 - 8s + 14$

22. $t = f(s) = s^2 + 6s + 11$

In Problems 23–26, state whether $f(x)$ has a maximum value or a minimum value, and find that value.

23. $f(x) = 49x^2 - 10x + 17$ **24.** $f(x) = -7x^2 - 2x + 6$

25. $f(x) = 4x - 50 - 0.1x^2$ **26.** $f(x) = x(x + 3) - 12$

In Problems 27 and 28, restrict the quadratic function to those x satisfying $x \geq v$, where v is the x-coordinate of the vertex of the parabola. Determine the inverse of the restricted function. Graph the restricted function and its inverse in the same plane.

27. $f(x) = x^2 - 2x + 4$ **28.** $f(x) = -x^2 + 4x - 3$

29. Revenue The demand function for a manufacturer's product is $p = f(q) = 100 - 10q$, where p is the price (in dollars) per unit when q units are demanded (per day). Find the level of production that maximizes the manufacturer's total revenue and determine this revenue.

30. Revenue The demand function for an office supply company's line of plastic rulers is $p = 0.85 - 0.00045q$, where p is the price (in dollars) per unit when q units are demanded (per day) by consumers. Find the level of production that will maximize the manufacturer's total revenue, and determine this revenue.

31. Revenue The demand function for an electronics company's laptop computer line is $p = 2400 - 6q$, where p is the price (in dollars) per unit when q units are demanded (per week) by consumers. Find the level of production that will maximize the manufacturer's total revenue, and determine this revenue.

32. Marketing A marketing firm estimates that n months after the introduction of a client's new product, $f(n)$ thousand households will use it, where

$$f(n) = \frac{10}{9}n(12 - n), \quad 0 \leq n \leq 12$$

Estimate the maximum number of households that will use the product.

33. Profit The daily profit for the garden department of a store from the sale of trees is given by $P(x) = -x^2 + 18x + 144$, where x is the number of trees sold. Find the function's vertex and intercepts, and graph the function.

34. Psychology A prediction made by early psychology relating the magnitude of a stimulus, x, to the magnitude of a response, y, is expressed by the equation $y = kx^2$, where k is solution of the given system is

$$x = 4 - 2r - s$$
$$y = r$$
$$z = s$$

where r and s can be any real numbers. Each assignment of values to r and s results in a solution of the given system, so there are infinitely many solutions. For example, letting $r = 1$ and $s = 2$ gives the particular solution $x = 0, y = 1$, and $z = 2$. As in the last example, there is nothing special about the names of the parameters. In particular, since $y = r$ and $z = s$, we could consider y and z to be the two parameters.

Now Work Problem 23 ◁

PROBLEMS 3.4

In Problems 1–24, solve the systems algebraically.

1. $\begin{cases} x + 4y = 3 \\ 3x - 2y = -5 \end{cases}$

2. $\begin{cases} 4x + 2y = 9 \\ 5y - 4x = 5 \end{cases}$

3. $\begin{cases} 2x + 3y = 1 \\ x + 2y = 0 \end{cases}$

4. $\begin{cases} 2x - y = 1 \\ -x + 2y = 7 \end{cases}$

5. $\begin{cases} u + v = 5 \\ u - v = 7 \end{cases}$

6. $\begin{cases} 2p + q = 16 \\ 3p + 3q = 33 \end{cases}$

7. $\begin{cases} x - 2y = -7 \\ 5x + 3y = -9 \end{cases}$

8. $\begin{cases} 4x + 12y = 12 \\ 2x + 4y = 12 \end{cases}$

9. $\begin{cases} 4x - 3y - 2 = 3x - 7y \\ x + 5y - 2 = y + 4 \end{cases}$

10. $\begin{cases} 5x + 7y + 2 = 9y - 4x + 6 \\ \frac{21}{2}x - \frac{4}{3}y - \frac{11}{4} = \frac{3}{2}x + \frac{2}{3}y + \frac{5}{4} \end{cases}$

11. $\begin{cases} \frac{2}{3}x + \frac{1}{2}y = 2 \\ \frac{3}{8}x + \frac{5}{6}y = -\frac{11}{2} \end{cases}$

12. $\begin{cases} \frac{1}{2}z - \frac{1}{4}w = \frac{1}{6} \\ \frac{1}{2}z + \frac{1}{4}w = \frac{1}{6} \end{cases}$

13. $\begin{cases} 2p + 3q = 5 \\ 10p + 15q = 25 \end{cases}$

14. $\begin{cases} 5x - 3y = 2 \\ -10x + 6y = 4 \end{cases}$

15. $\begin{cases} 2x + y + 6z = 3 \\ x - y + 4z = 1 \\ 3x + 2y - 2z = 2 \end{cases}$

16. $\begin{cases} x + y + z = -1 \\ 3x + y + z = 1 \\ 4x - 2y + 2z = 0 \end{cases}$

17. $\begin{cases} x + 4y + 3z = 10 \\ 4x + 2y - 2z = -2 \\ 3x - y + z = 11 \end{cases}$

18. $\begin{cases} x + 2y + z = 4 \\ 2x - 4y - 5z = 26 \\ 2x + 3y + z = 10 \end{cases}$

19. $\begin{cases} x - 2z = 1 \\ y + z = 3 \end{cases}$

20. $\begin{cases} 2y + 3z = 1 \\ 3x - 4z = 0 \end{cases}$

21. $\begin{cases} x - y + 2z = 0 \\ 2x + y - z = 0 \\ x + 2y - 3z = 0 \end{cases}$

22. $\begin{cases} x - 2y - z = 0 \\ 2x - 4y - 2z = 0 \\ -x + 2y + z = 0 \end{cases}$

23. $\begin{cases} x - 3y + z = 5 \\ -2x + 6y - 2z = -10 \end{cases}$

24. $\begin{cases} 5x + y + z = 17 \\ 4x + y + z = 14 \end{cases}$

25. Mixture A chemical manufacturer wishes to fill an order for 800 gallons of a 25% acid solution. Solutions of 20% and 35% are in stock. How many gallons of each solution must be mixed to fill the order?

26. Mixture A gardener has two fertilizers that contain different concentrations of nitrogen. One is 3% nitrogen and the other is 11% nitrogen. How many pounds of each should she mix to obtain 20 pounds of a 9% concentration?

27. Fabric A textile mill produces fabric made from different fibers. From cotton, polyester, and nylon, the owners want to produce a fabric blend that will cost \$3.25 per pound to make. The cost per pound of these fibers is \$4.00, \$3.00, and \$2.00, respectively. The amount of nylon is to be the same as the amount of polyester. How much of each fiber will be in the final fabric?

28. Taxes A company has taxable income of \$758,000. The federal tax is 35% of that portion left after the state tax has been paid. The state tax is 15% of that portion left after the federal tax has been paid. Find the federal and state taxes.

29. Airplane Speed An airplane travels 900 mi in 2 h, 55 min, with the aid of a tailwind. It takes 3 h, 26 min, for the return trip, flying against the same wind. Find the speed of the airplane in still air and the speed of the wind.

30. Speed of Raft On a trip on a raft, it took $\frac{1}{2}$ hour to travel 10 miles downstream. The return trip took $\frac{3}{4}$ hour. Find the speed of the raft in still water and the speed of the current.

31. Furniture Sales A manufacturer of dining-room sets produces two styles: early American and contemporary. From past experience, management has determined that 20% more of the early American styles can be sold than the contemporary styles. A profit of \$250 is made on each early American set sold, whereas a profit of \$350 is made on each contemporary set. If, in the forthcoming year, management desires a total profit of \$130,000, how many units of each style must be sold?

32. Survey National Surveys was awarded a contract to perform a product-rating survey for Crispy Crackers. A total of