Kumpulan Soal Bentuk Pangkat dan Akar

1 Tentukan nilai dari $\frac{5^{2-n}-(0,2)^n}{5^{1-n}+(0,2)^n}$ ...

Penyelesaian:

$$\frac{5^{2-n}-(0,2)^n}{5^{1-n}+(0,2)^n}$$

$$=\frac{\frac{5^2}{5^n}-\left(\frac{1}{5}\right)^n}{\frac{5}{5^n}+\left(\frac{1}{5}\right)^n} \qquad \text{sifat } a^m : a^n = a^{m-n}$$

$$=\frac{\frac{5^2}{5^n}-\left(5^{-1}\right)^n}{\frac{5}{5^n}+(5^{-1})^n} \qquad \text{sifat } a^{-m} = \frac{1}{a^m}$$

$$=\frac{\frac{5^2}{5^n}-5^{-n}}{\frac{5}{5^n}+5^{-n}}$$

$$=\frac{\frac{25}{5^n}-\frac{1}{5^n}}{\frac{5}{5^n}+\frac{1}{5^n}}$$

$$=\frac{\frac{25-1}{5^n}}{\frac{5+1}{5^n}}$$

$$=\frac{24}{6^{\cancel{n}}}\times\frac{6^{\cancel{n}}}{6}$$

$$=\frac{24}{6}=4.$$

2. Jika $3^{\frac{x}{y}}$ adalah penyederhanaan dari $\sqrt{3\sqrt{9\sqrt{27}}}$, Maka tentukan nilai x + y ...

Penyelesaian:

$$\sqrt{3\sqrt{9\sqrt{27}}}$$

$$=\sqrt{3\sqrt{9\sqrt{3^3}}}$$

$$=\sqrt{3\sqrt{9.3^{\frac{3}{2}}}} \qquad \text{sifat } \sqrt{a^n}=a^{\frac{n}{2}}$$

$$=\sqrt{3\sqrt{3^2.3^{\frac{3}{2}}}}$$

$$=\sqrt{3\sqrt{3^{\frac{7}{2}}}}$$

$$=\sqrt{3.\left(3^{\frac{7}{2}}\right)^{\frac{1}{2}}} \qquad \text{sifat } (a^m)^n = a^{m.n}$$

$$=\sqrt{3.3^{\frac{7}{4}}} \qquad \text{sifat } a^m.a^n = a^{m+n}$$

$$=\sqrt{(3)^{1+\frac{7}{4}}}$$

$$=\sqrt{3^{\frac{11}{4}}}$$

$$=\left(3^{\frac{11}{4}}\right)^{\frac{1}{2}}=3^{\frac{11}{8}}$$

3. Tentukan nilai $\sqrt{\sqrt[3]{16}} \times \sqrt[3]{\frac{1}{4}}$ …

Penyelesaian:

$$\sqrt{\sqrt[3]{16}} \times \sqrt[3]{\frac{1}{4}}$$
$$= \sqrt{\sqrt[3]{2^4}} \times \sqrt[3]{\frac{1}{2^2}}$$
$$= \sqrt{2^{\frac{4}{3}}} \times \sqrt[3]{2^{-2}} \qquad \text{sifat } \sqrt[n]{a^m} = a^{\frac{m}{n}}$$
$$= \left(2^{\frac{4}{3}}\right)^{\frac{1}{2}} \times 2^{-\frac{2}{3}}$$
$$= 2^{\frac{4}{6}} \times 2^{-\frac{2}{3}}$$
$$= 2^{\left(\frac{2}{3}+\left(-\frac{2}{3}\right)\right)}$$
$$= 2^0 = 1 \qquad \text{sifat } a^0 = 1,\ a \neq 0$$

4. Hasil dari $m^{1\frac{2}{3}} : m^{-\frac{1}{4}} \times \left(\frac{1}{m}\right)^{\frac{5}{12}}$ adalah…

Penyelesaian:

$$m^{1\frac{2}{3}} : m^{-\frac{1}{4}} \times \left(\frac{1}{m}\right)^{\frac{5}{12}}$$
$$= m^{\frac{5}{3}} : m^{-\frac{1}{4}} \times (m)^{-\frac{5}{12}} \qquad \text{sifat } \frac{1}{a} = a^{-1}$$
$$= m^{\frac{5}{3}-\left(-\frac{1}{4}\right)} \times m^{-\frac{5}{12}} \qquad \text{sifat } a^m : a^n = a^{m-n}$$
$$= m^{\frac{20}{12}+\frac{3}{12}} \times m^{-\frac{5}{12}} \qquad \text{penyamaan penyebut}$$
$$= m^{\frac{23}{12}} \times m^{-\frac{5}{12}}$$
$$= m^{\frac{23}{12}+\left(-\frac{5}{12}\right)}$$
$$= m^{\frac{18}{12}} = m^{\frac{3}{2}}$$
$$= m.m^{\frac{1}{2}}$$
$$= m\sqrt{m} \qquad \text{sifat } a^{\frac{1}{2}} = \sqrt{a}$$

5. Tentukan nilai dari $\frac{1}{1+a^{p-q}}+\frac{1}{1+a^{q-p}}\cdots$

Penyelesaian:

$$\frac{1}{1+a^{p-q}}+\frac{1}{1+a^{q-p}}$$

$$=\frac{1}{1+\frac{a^p}{a^q}}+\frac{1}{1+\frac{a^q}{a^p}} \qquad \text{sifat } a^{m-n}=\frac{a^m}{a^n}$$

$$=\frac{1}{\frac{a^q}{a^q}+\frac{a^p}{a^q}}+\frac{1}{\frac{a^p}{a^p}+\frac{a^q}{a^p}}$$

$$=\frac{1}{\frac{a^q+a^p}{a^q}}+\frac{1}{\frac{a^p+a^q}{a^p}}$$

$$=\frac{a^q}{a^q+a^p}+\frac{a^p}{a^p+a^q} \qquad \text{sifat } \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc}$$

$$=\frac{a^p+a^q}{a^p+a^q}$$

$$=1$$

Semoga Bermanfaat