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ABSTRACT

In this work, we demonstrate the feasibility of fully-Lagrangian finite element simulations of the mechanics of three-dimensional penetration environments. The key enabling component is a robust library informed with state-of-the-art algorithms for mesh healing and optimization, which is repeatedly used during the simulations to eliminate deformation-induced mesh distortion and to maintain the quality of the numerical solution. The computational strategy effectively avoids the need to resort to artifacts such as element deletion or conversion to meshless particles which have been proposed to eliminate the issue of mesh distortion. The effectiveness of the computational strategy is demonstrated in a simulation of deep oblique penetration of a spherical-nosed steel rod on an aluminum target.

1 INTRODUCTION

Lagrangian finite element formulations first showed their potential for the simulation of ballistic penetration in the work of Camacho and Ortiz (Camacho and Ortiz, 1997), which was restricted to the axisymmetric case. It has since been recognized that a key element enabling this modeling paradigm is the possibility to frequently and adaptively reconstruct the computational mesh during the simulations as a means of eliminating deformation-induced mesh distortion and increasing the fidelity of the simulation results. Despite some attempts to extend this modeling strategy to full three–dimensional situations such as arise in oblique penetration (Bessette et al., 2003), the promise of the fully-lagrangian finite element approach of becoming a widely adopted formulation for the simulation of this type of complex phenomena with high-fidelity has yet to materialize. The main obstacle hindering this technology has been the fundamental robustness issues associated with mesh (re)generation for general domains of arbitrarily-evolving geometry and topology.

A number of strategies have been devised to address the problem of mesh distortion in Lagrangian finite element simulations. Instead of insisting in remeshing the deforming problem domain, the severely distorted elements can either be eliminated (Bessette et al., 2003) or converted to meshless particles (Johnson and Stryk, 2003; Beissel and Johnson, 2002).

In this work, we demonstrate the feasibility of fully-Lagrangian finite element simulations of penetration environments. Instead of attempting to redo the mesh from scratch at each remeshing operation, we propose to modify the mesh incrementally by taking advantage of the software library HealMesh[1]. This package provides functionality for mesh healing and optimization and is informed with the latest advances in geometrical and topological mesh optimization as well as with local refinement and coarsening algorithms. The incremental character of the adaptive mesh strategy not only reduces the amount of computational work associated with the remeshing operation but also increases the robustness of the overall approach. An additional advantage of the local mesh optimization strategy over the global mesh change is that it facilitates the operation of transferring the mechanical fields from the old to the new mesh at each remeshing step.

In the following we first describe the mesh optimization algorithms followed by a demonstration of their enabling capability in the simulation of a penetration environment.

2 MESH HEALING AND OPTIMIZATION ALGORITHMS

The HealMesh library implements a suite of hill-climbing methods for mesh optimization. That is, each method sweeps over the mesh, investigating local changes. If the proposed transformation locally improves the quality of the mesh, then the change is accepted. The local changes are comprised of geometric transformations (moving nodes) and topological transformations (changing the connectivities). This is complemented by refinement and coarsening capabilities. Refinement is done with edge splitting; coarsening with edge collapse.

A feature-based representation of the boundary may be used in the above algorithms. For a 3-D mesh, the boundary is represented as a triangle mesh that has surface features, edge features, and corner features. Sharp edges and corners are preserved as nodes are moved and elements are modified.

The following algorithms available in the HealMesh library are found to be instrumental in the simulation of penetration environments:

Algebraic quality metrics. These metrics are functions of the Jacobian matrix of the mapping from the equilateral simplex to the physical simplex. They are differentiable and sensitive to all types of degeneracies.

[1] HealMesh©Parasim Inc. http://alma.para-sim.com/healmesh/

Geometric optimization of node positions. A node is moved to optimize the quality of the incident simplices. Boundary nodes are moved along the feature-based description of the boundary.

Topological optimization. Local transformations (edge flips in 2-D, edge removal and face removal in 3-D) are applied in a hill-climbing strategy.

Refinement. A set of elements can be refined based on a maximum allowed edge length function while preserving the conformity of the mesh.

Coarsening. The mesh is coarsened by removing elements through edge collapse. The mesh can be coarsened by collapsing edges for a set of elements or by specifying a minimum allowed edge length function.

2.1 Simplex quality measures

The choice of adequate mesh quality metrics is critical for the successful operation of the mesh optimization algorithms. Geometric quality metrics are a function of the geometry of the simplex. For example, the maximum or minimum dihedral angle (angle between faces) or the aspect ratio (ratio of inscribed to circumscribed sphere) may be used as quality metrics. In HealMesh, algebraic quality metrics are used instead. These are functions of the Jacobian matrix of the affine map that transforms the reference (equilateral) simplex to the physical simplex. In N-D space, the $N \times N$ Jacobian matrix S has information about the volume, shape and orientation of the element.

In particular, we have used the *condition number metric* and the *mean ratio metric*. In 3-D, the condition number metric is $\kappa(S) = \frac{|S|}{|S^{-1}|}$. The mean ratio metric is $\eta(S) = \frac{|S|^2}{3\det(S)^{2/3}}$. S becomes singular when the element volume vanishes. The metrics measure the distance from singular matrices. They are unity for the equilateral tetrahedron and are singular for tetrahedron of zero volume.

These two algebraic quality metrics have a number of desirable properties (Knupp, 2001), (Freitag and Knupp, 2002). They are sensitive to all types of degeneracies. By contrast, the dihedral angle quality metric is not. Their definition is dimension independent. (In HealMesh the dimension is a template parameter.) Finally, they have continuous derivatives. This enables optimization methods that use the gradient and Hessian.

Because of their singularities, these algebraic metrics cannot be used to optimize meshes with inverted elements. Even for good quality meshes, the optimization algorithm may assess the quality of inverted elements in trying to improve the mesh. To address this problem, we implemented the condition number and mean ratio metrics presented in (Escobar et al., 2003). They modified the metrics to be defined for inverted elements. They can then be used for mesh un-tangling as well as mesh optimization.

2.2 Geometric optimization

In geometric optimization, the nodes of the mesh are moved to improve the quality of the mesh while the topology remains fixed. The user chooses the movable set of nodes. They may choose all or a subset of the interior or boundary nodes. The user may use the condition number or mean ration metrics or supply their own quality metric.

In HealMesh several hill climbing methods for sweeping over the nodes of the mesh are available. One can sweep over all movable nodes or one can sweep over movable nodes that have an adjacent tetrahedron with poor quality. Since these hill climbing methods apply local changes to the mesh, they rapidly improve deformations that are local in nature. However, it may take many iterations to converge if the nodes are widely redistributed.

First consider optimizing the position of an interior node v. The elements adjacent to v form a complex. Node v is moved to optimize the ℓ_2 norm of the quality metrics of the simplices in the complex. The ℓ_2 norm is used because it is differentiable. (The ℓ_∞ norm is not.) This enables the use of a quasi-Newton method (BFGS) (Press et al., 2002), which is far more efficient than methods that do not use gradient information.

For impact/penetration problems, the greatest deformation occurs at the boundary. It is not possible to arrange the interior nodes to obtain a quality mesh if the boundary nodes have poor geometry. HealMesh also provides the ability to apply geometric optimization to boundary nodes. When moving a boundary node the shape of the object must be preserved in some sense. The primary approach is to use a feature-based description of the boundary. A node which is a corner feature may not be moved. A node which is on an edge feature, may only be moved along that edge. Otherwise, a node may be moved along the boundary surface, except that it may not cross edge features. In addition, these operations are applied in a way to minimize the violation of mass conservation.

2.3 Topological optimization

In 3-D, local topological transformations replace a set of tetrahedra with a different set that fills the same domain but with an improved quality. (On the boundary they may fill only approximately the same domain.) For topological optimization, HealMesh implements the algorithms presented in (Shewchuk, 1997). First consider simple transformations. An *m-n* flip replaces m tetrahedra with n tetrahedra. For example, a 2-3 flip removes a face and adds a new edge. The inverse is a 3-2 flip. Two boundary faces are replaced in a 2-2 flip.

The composite topological operations of *edge removal* and *multi-face removal* are also implemented. They are more general than simple flips and may provide additional opportunities for elimination of bad-quality elements. Edge removal and multi-face removal operations can be represented by sequences of 2-3, 3-2, 2-2 and 4-4 flips. Although a given composite operation may improve the mesh, some of its individual flips may temporarily reduce the quality. (They may even invert elements.) Thus using such composite operations is likely to produce better results than using only simple topological changes.

We apply local topological transformations in a hill-climbing method. We use this to find a (local) maximum quality mesh. One might repeatedly sweep over all the edges and faces, applying edge and face removal. However this is very inefficient. A better approach is to keep track of the edges and faces upon which local topological changes could possibly improve the mesh. Thus we maintain a set of active tetrahedra. Tetrahedra are added to the active set upon insertion in the mesh. Tetrahedra are removed from the set when they are removed from the mesh or when it is determined that edge or face removal on its edges and faces will not improve the mesh.

2.4 Refinement and coarsening

Mesh refinement is accomplished through edge splitting. In order to help preserve mesh quality during refinement, Rivara's longest edge propagation path algorithm (Rivara, 1997) is used. If an element is targeted for refinement, its longest edge will be split. However, only mutual longest edges are allowed to be split. This means that adjacent elements may need to be (recursively) refined before the targeted element is refined.

The mesh is coarsened by removing elements through edge collapse. Since collapsing edges may reduce quality of the incident simplices, there are user-defined parameters for controlling the quality of the mesh while performing coarsening.

One can refine/coarsen a set of elements or refine/coarsen the whole mesh based on a maximum/minimum allowed edge length function. A feature-based description of the boundary may be used in these algorithms. In splitting an edge that is registered as an edge feature, the resulting two edges are registered as edge features and the midpoint node is placed on the curve that describes the edge feature. Collapsing certain edges may be disallowed because doing so would alter the character of the boundary.

3 SIMULATION EXAMPLE

In this section, we present a simulation of a penetration environment demonstrating the enabling character of the mesh healing and optimization strategy briefly described above.

The simulation corresponds to the oblique impact of a spherical-nose steel projectile on an aluminum target at a velocity of $800 m/sec$. The radius of the nose is 3.8 mm and the length of the projectile is 31 mm. The angle of impact is 30 degrees with respect to the normal to the target. The target is a block of aluminum with a square-shaped base with a side length of 38 mm and a height of 25 mm.

The simulation is conducted with our Lagrangian finite element solver which takes into account plastic deformations, contact and friction and thermo-mechanical coupling. Initially the target is meshed with 9356 quadratic tetrahedra, while the initial mesh of the bullet has 11798 elements of the same type. The initial mesh is shown in Figure 1. The plastic response of the materials is described by recourse to an isotropic large-deformation plasticity model including power-law hardening and rate-dependency:

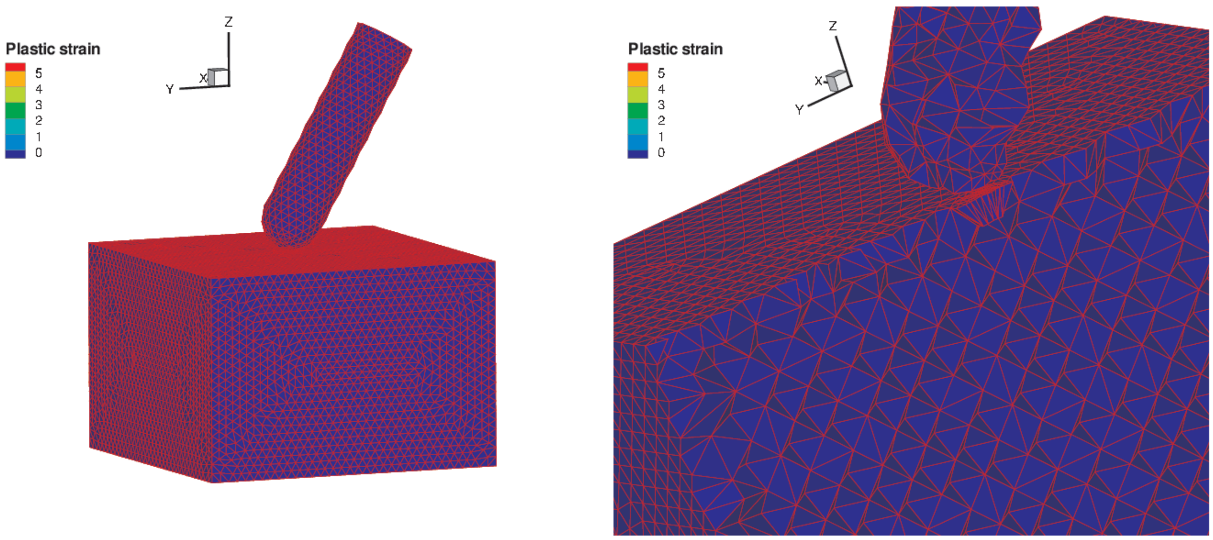
$$\sigma^y = \sigma_0^y \left\{ \left(1 - \frac{T}{T_\mathrm{m}}\right)^{\eta_T} \left(1 + \frac{\varepsilon^\mathrm{pl}}{\varepsilon_0}\right)^{\frac{1}{\eta_\varepsilon}} + \left[\left(1 + \frac{\dot{\varepsilon}^\mathrm{pl}}{\dot{\varepsilon}_0}\right)^{\frac{1}{\eta_{\dot{\varepsilon}}}} - 1 \right] \right\} \tag{1}$$

where σ^y is the effective flow stress, σ_0^y is the initial von Mises yield stress, T_m is the melting temperature, η_T is the thermal exponent, η_ε is the strain hardening exponent, ε^pl is the equivalent plastic strain, ε_0 is the reference plastic strain, $\eta_{\dot{\varepsilon}}$ is the strain rate exponent, $\dot{\varepsilon}^\mathrm{pl}$ is the plastic strain rate and $\dot{\varepsilon}_0$ is the reference plastic strain rate. The material model parameters for steel and aluminum used in the calculation are reported in Table 1. The equations of motion are integrated in time using a standard second-order accurate central-difference explicit scheme.

It is widely known that this type of simulations escapes the possibilities of a barebone fully-Lagrangian simulation, which would soon fail due to excessive mesh distortion. The inability of previously existing three-dimensional adaptive remeshing procedures of coping with the mesh quality restitution demands of this problem is equally generally acknowledged.

The HealMesh library has been integrated into the mechanics solver through its application program interface (API) without much effort. Mesh healing and optimization consisting of a sequence of *geometrical, topological, refinement, geometrical* optimization steps is effected after every 250 integration time steps in the simulation followed by a transfer of the nodal and state fields. It is found that this combination of operations effectively eliminates bad-quality elements resulting from the deformation throughout the simulation in a robust manner.

Snapshots of the simulation at different times in the penetration process are shown in Figure 2. The last snapshot corresponds to time t= 24 μs after 75000 integration steps and 300 mesh healing operations. It can be visually observed in the figures that the elements of the mesh in the target surrounding the projectile remain of good quality throughout the simulation, notwithstanding the extremely large plastic deformations which are in excess of 500%. It can also be observed that, due to the local character of the remeshing operations, the quality of the solution is preserved throughout the simulation despite the large number of nodal and state field transfers from the old to the new mesh. From the demonstration simulation presented, it can be ascertained that the proposed mesh healing and optimization strategy establishes the feasibility

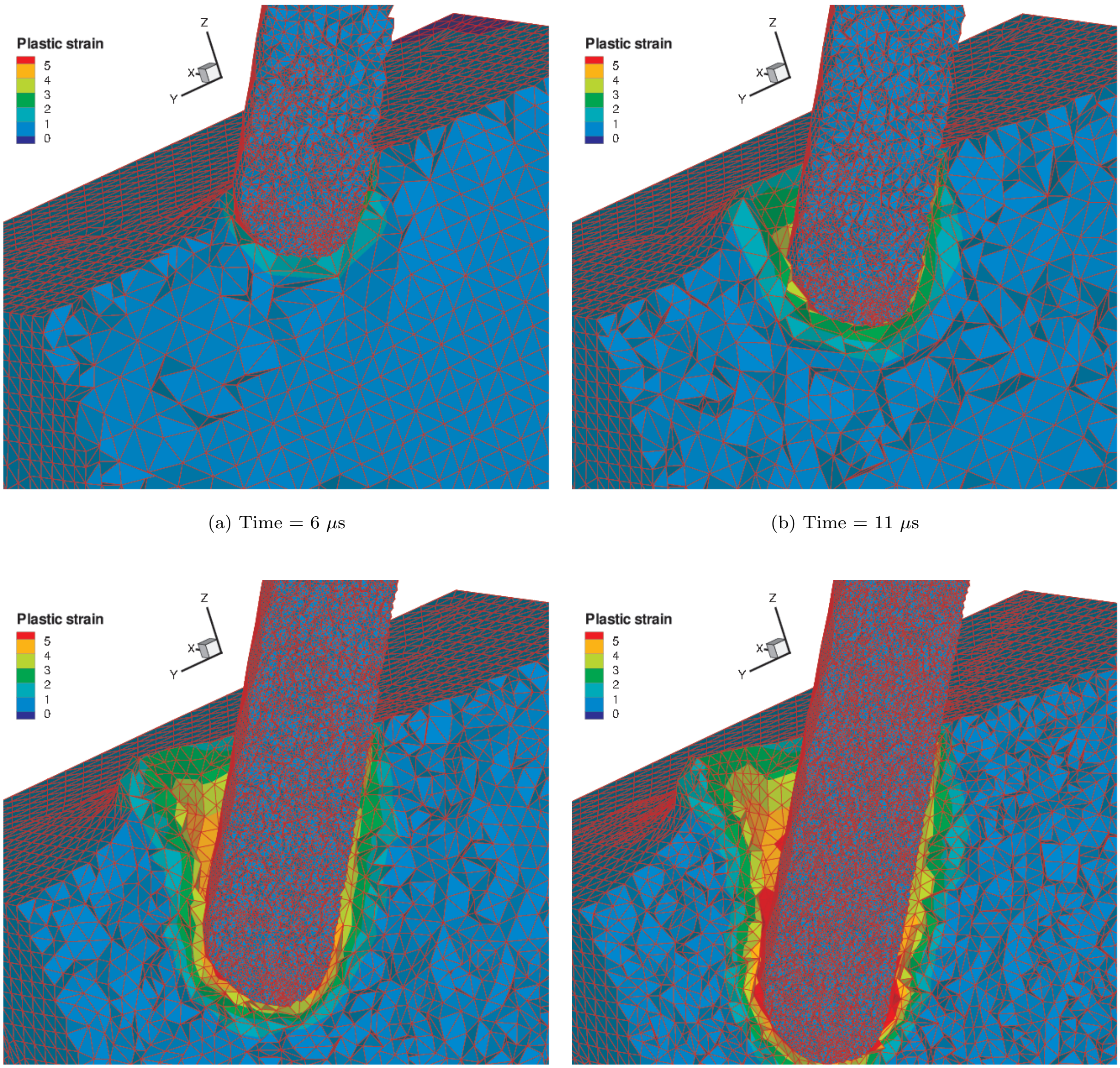


(a) (b)

Figure 1: Simulation of oblique impact of a spherical-nose steel penetrator on an aluminum target - a) initial 3D-mesh - b) cut and zoom of the initial 3D-mesh.

Table 1: Material properties for the penetration test.

Material	Density [kg·m^{-3}]	Young mod. [N·mm^{-2}]	Poisson coef.	σ_0^y N·mm^{-2}	T_m [T]	η_T	η_ε	ε_0	$\eta_{\dot\varepsilon}$	$\dot\varepsilon_0$ [s^{-1}]
Steel	7800	200000	0.3	1000	1793	0.75	3	0.005	10	0.2
Aluminum	2700	130000	0.34	90	1793	0.75	1	0.005	10	10



(a) Time = 6 μs

(b) Time = 11 μs

(c) Time = 19 μs

(d) Time = 24 μs

Figure 2: Simulation of oblique impact of a spherical-nose steel penetrator on an aluminum target Snapshots at times - a) 6 - b) 11 -c) 19 and d) 24 μs showing contours of accumulated plastic strain and the evolving optimized meshes.

and renews the expectations on the fully-Lagrangian finite element method as a plausible computational strategy for the simulation of complex penetration environments. Future work will focus on verification of the computational framework against previously proposed models and on experimental validation.

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