

# A new algorithm for repairing non-manifold surfaces

Yaoping Fei, Songqiao Chen, Dan Su, Jianping Luo, Min Li\*  
School of Information Science and Engineering, Central South University,  
Changsha, 410083, China  
limin@mail.csu.edu.cn

**Abstract**—In the process of solid modeling, non-manifold polygon surfaces may be met frequently. However, most graphics algorithms applied on polygon surfaces require that polygon surfaces must be manifold. Generally, non-manifold surfaces can be transferred into manifold surfaces with similar geometric appearance. The transfer operations include modifying non-manifold surface edges and non-manifold surface vertices. However, new non-manifold may be produced when being transferred from non-manifold surface in mesh modeling. To avoid of generating non-manifold, we propose a new method of non-manifold transferring algorithm. Compared with existed non-manifold surface algorithm, this new repairing algorithm would not produce new non-manifold surfaces after transferring and can be operated and used directly and conveniently.

**Keywords**—*surface modeling; non-manifold, repairing algorithm*

## I. INTRODUCTION

Polygon surfaces are usually used for describing three-dimensional geometrical models, and such models cannot be used only for producing pictures and animations but also in CAD system, scientific visualization, and medical treatment imaging system. In the process of solid modeling, non-manifold polygon surfaces may be met frequently. Firstly, it is inevitable for many modeling technologies to generate non-manifold surfaces. Secondly, in some cases, it is necessary for users to generate some non-manifold surfaces, because non-manifold surfaces have more complicated topological relations than manifold surfaces[1,2]. However, most graphics algorithms applied on polygon surfaces require that polygon surfaces must be manifold, particularly for some surface subdivision algorithms and initial mesh must be an effective and stable 2D manifold structure, such as Doo-Sahin subdivision algorithm[3] and Zorin subdivision algorithm[4]. When such algorithms are applied on non-manifold surfaces, errors may occur or endless loops may be sunk. Except subdivision algorithms, other operations include: compression algorithm of polygon surface data[5] and simplified algorithm of polygon surface[6,7], whose operated surfaces are manifold.

At present, feasible methods for solving such problems include: (1) modify existing graphics algorithm, so that it can treat non-manifold surfaces; (2) find out reasons for the appearance of non-manifold during modeling to avoid non-manifold surfaces as possible; (3) transfer non-manifold surfaces into a manifold surface which is similar in geometric appearance, and then use the above algorithm. As for the first algorithm, researches at present include non-manifold subdivision algorithm proposed by Zorin D[8], non-manifold

compression algorithm proposed by Guezic et al[9], and non-manifold simplified algorithm proposed by Hoppe et al[10]. Through analyzing such algorithms, we find that: firstly, such algorithms are complicated; secondly, they are related to specific application. For example: some surface simplified algorithms allow non-manifold structure, but more “errors” may be generated when algorithms are applied on such manifold structure. However, non-manifold structure is unable to be used for most surface subdivision algorithms. Therefore, there are completely different solutions for different application backgrounds. The second solution is only limited in theory, because, in most cases, it is very difficult to find reasons for generating non-manifold structure. Therefore, transferring generated non-manifold structure into manifold structure with geometric appearance is the simplest and most effective method among the above three methods.

It sounds that transferring generated non-manifold structure into manifold structure which is similar to non-manifold structure in geometric appearance is simple, but the following conditions may be found through serious analysis: actually generated non-manifold structures may be exceedingly strange with many reasons, so that it is impossible to find a method to transfer all possible non-manifold structures and it may be extremely complicated. Therefore, most researches focus on how to modify non-manifold edges and non-manifold vertex on surfaces effectively to form manifold surfaces.

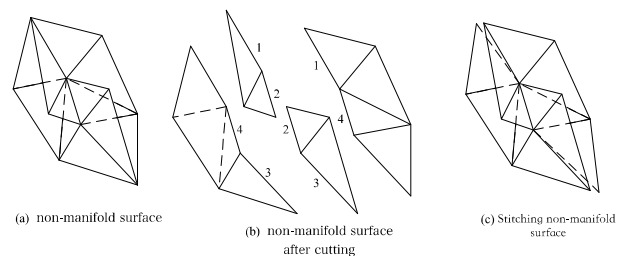


Fig.1 Existing stitch operation may generate non-manifold results (digit presents edge match)

As for repairing algorithm of non-manifold edges, there are many researches at home and abroad: GBarequet and S.Kumar proposed a kind of algorithm to repair boundary description (B.reps) errors of CAD model[11], including cracks in models, some degeneracy, holes, and overlaps etc. Rossignac and Cardoze et al raised MatchMaker algorithm[12], and the greatest strength of such algorithm is reducing the number of vertex copy operation to the minimum when non-manifold surfaces are transferred into manifold surfaces. A.Guezic et al[13] put forward a kind of cutting stitch

operation: cut non-manifold edges at first to form manifold structures with boundaries, and then sew up boundaries. With such method, new non-manifold edges may be generated after suture operation, as shown in Fig.1.

In the paper, a kind of new non-manifold surface repairing algorithm is proposed. With the use of such algorithm, we can gain not only a manifold structure divided on topology but also a manifold structure connected on topology, which is named as a kind of manifold structure whose “appearance likes non-manifold structure”. Results acquired from such algorithm are beneficial supplement to existing researches, which can meet modelers’ demands better.

## II. RELEVANT CONCEPTS

In order to define entities, the concept of 2-manifold is brought in the graphics. When each point on an object surface possesses a sufficiently small neighborhood, such neighborhood and disc on the panel shall have the same structure. If the surface is non-closed (without boundaries) and when each point on an object surface possesses a sufficiently small neighborhood, such neighborhood and semi-disc on the panel shall have the same structure. If there are the above-mentioned properties on the surface, it shall possess 2-manifold. Points on body surfaces cannot be described on computers, two conditions with 2-manifold are given from definitions of vertex, edges, and surfaces, which are as follows:

(2) Each edge shall only have two shared surfaces;

(3) The edge and surface bordering upon a vertex can only compose a “ring” rounding the vertex.

With the above two conditions, 2-manifold of surface is guaranteed. When an edge  $e$  is shared by two surfaces, such edge is called as non-manifold edge, as shown in Fig. 2(a). When all edges and surfaces bordering upon the vertex  $v$  compose more than two topological “rings”, such vertex are non-manifold vertex, as shown in Fig. 2(b). When and only when all edges and vertexes are manifold, surfaces of a grid structure may be manifold. Or else, surfaces are non-manifold [15, 16, 17].

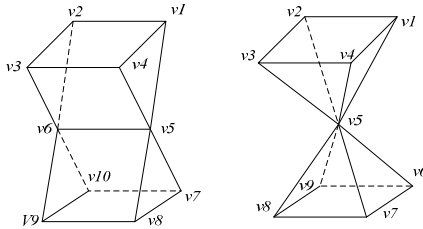


Fig.2 Non-manifold surfaces

## III. RESEARCH ON TRANSFERRING OF NON-MANIFOLD EDGES

Find non-manifold edges (different data structures correspond to different algorithms) according to definitions and features of non-manifold edges (when an edge  $e$  is shared by

two surfaces, such edge is a non-manifold edge) [18] to complete marks on non-manifold edges. Cut non-manifold edges, as well as suture operation. Specific operations are as follows:

### A. Cutting operation

Cutting operation is strictly defined in graphics. A. Guezic [13] has put forward two different cutting methods: partial cutting operation and overall cutting operation. The former only operates surfaces where endpoints of partial non-manifold edges and such vertex border, while the latter operates all endpoints and surfaces at a time. Processes of such two methods are different, but they can gain the same cutting results. When there are many non-manifold edges on surfaces, we shall use overall cutting operation. On the contrary, we use partial cutting operation. In addition, overall cutting operation also impliedly includes processing on non-manifold points, automatically conducting “Multiplication” operation on non-manifold points in the division process of opposite angles. However, as for partial cutting operation, “Multiplication” operation shall be conducted on non-manifold vertexes additionally.

1. Cut all non-manifold edges, namely insert a new vertex  $v$  on non-manifold edges, generally in the middle of non-manifold edges.

2. As for two vertexes of any non-manifold edges  $u$  and  $w$ : firstly, calculate surfaces  $u^*$  bordering upon  $u$  and then divide  $u^*$  according to a kind of “accessible relationship”. If we divide a subset, neighboring surface  $f_a$  and surface  $f_b$  are “accessible” when and only when such two surfaces share a side edge bordering upon  $u$ . Conduct Multiplication operation on vertex  $u$  for  $c$  times according to the division number of subsets.

3. When “Multiplication” operation of all vertexes is completed, cutting operation is completed.

When “Multiplication” operation of all vertexes is completed, cutting operation is completed.

Non-manifold edges  $[v5, v6]$  are shown in the Fig. 3(a). Firstly, subdivide such non-manifold edges and insert vertex  $v$  in the middle to form two edges  $[v5, v]$  and  $[v, v6]$ , as shown in Fig. 3(b). As for vertex  $v5$ , the neighboring surface set  $v5^*$  includes  $\{\{v5, v, v6, v3, v4\}, \{v5, v, v6, v7, v8\}, \{v5, v4, v1\}, \{v5, v, v6, v2, v1\}, \{v5, v, v6, v10, v9\}, \text{ and } \{v5, v9, v8\}\}$ , and neighboring side edges include  $[v5, v4]$ ,  $[v5, v1]$ ,  $[v5, v9]$ , and  $[v5, v8]$ . In accordance with such side edges, the surface set  $v5^*$  is divided into two subsets, namely  $\{\{v5, v, v6, v3, v4\}, \{v5, v, v6, v2, v1\}, \{v5, v4, v1\}\}$  and  $\{\{v5, v, v6, v7, v8\}, \{v5, v, v6, v10, v9\}, \{v5, v9, v8\}\}$ . Therefore, it is necessary to conduct two “Multiplication” operations on  $v5$ , which are  $v5$  and  $v11$ . As for newly inserted vertex  $v$ , its neighboring surface set  $v^*$  is  $\{\{v5, v, v6, v3, v4\}, \{v5, v, v6, v7, v8\}, \{v5, v, v6, v2, v1\}, \text{ and } \{v5, v, v6, v10, v9\}\}$ . There are not neighboring side edges, so that  $v^*$  is divided into four subsets and four “Multiplication” operations are conducted on the vertex  $v$ , namely  $v12, v13, v14$ , and  $v15$ . Conditions of  $v6$  are similar to that of  $v5$ . Two “Multiplication” operations are conducted on

$v_6$ , namely  $v_6$  and  $v_{16}$ . Results after cutting are shown in Fig. 3(c). Such algorithm is applied to general conditions, as shown in Fig. 4.

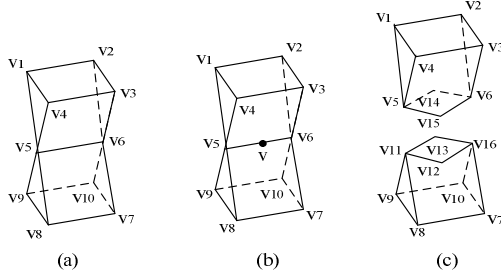


Fig.3 Examples of Cutting Operation

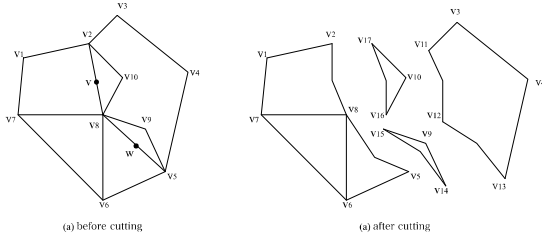


Fig.4 General Cutting Operation

Upon completion of cutting operation, all edges on surfaces are regular, including regular inner edges and regular boundary edges, and non-manifold surfaces have been transferred into manifold surfaces. However, such operation may generate “cracks” or “holes”, so that suture operation is needed to eliminate such “errors”.

#### B. Stitch operation

In brief, stitch operation is effectively suturing boundary edges generated for cutting, so as to gain a closed entity. Existing suture operation may generate new non-manifold edges. We put forward a kind of new stitch algorithm after researching various stitch strategies. Such algorithm is divided into two steps: firstly, stitch connected surfaces after cutting to form closed connector, and such suture operation will not generate new non-manifold edges [19]; secondly, bond connectors based on Boolean operation[22].

Specific processes of the algorithm: firstly, select surfaces whose boundary edges are generated through cutting, as shown in Fig. 6.

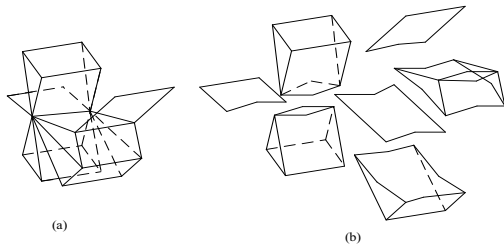


Fig.5 Cutting of Non-manifold Bodies

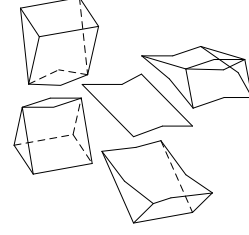


Fig.6 Surfaces except Suspension Surface in the Fig. 5(a)

Doing it like this can separate influences of hanging surfaces on stitch; conduct boundary stitch on same surfaces as per conditions for the first step of stitch operation, namely stitching surfaces that can be stitched. Record various combined manifold items and combined boundaries. In addition, surfaces that can be stitched into manifold items will not participate in boundary stitch operations, as shown in Fig. 7.

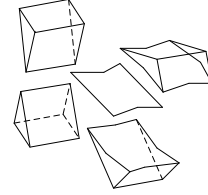


Fig.7 Stitching Surfaces along the Same Surfaces in Fig.6

Secondly, stitch remain parts from selected surface components in advance and at most one stitch operation can be conducted on boundary edges of such surface components. Processes are as follows: firstly, judge whether edge  $\{v_0, v_1\}$  and edge  $\{v_0', v_1'\}$  are boundary edges generated on the same non-manifold edge through cutting operation; secondly, confirm whether such two boundary edges belong to different connection components. If yes, stitch those two edges and mark them, and then manifold items and remain surface components are generated, because boundary edges of surface components are only stitched for only once, namely one edge will not correspond with one surface (non-manifold edges) and all boundary edges of surface components are gained through cutting original edges. Therefore, such surface components can be stitched into a closed object and “holes” will not be generated, as shown in Fig. 8. In addition, only one stitch operation is conducted on surface components acquired from cutting of the same edge.

There may be some surface components except hanging surface, upon completion of the above two steps of suture operation. In addition, stitch operation cannot be conducted under the condition of not destroying other manifold items, as shown in Fig. 8. As for such surface components, our methods are: generating surfaces on such surface components along boundary edges to form a closed manifold item. Gather and operate such newly generated manifold items and manifold items generated after different surface components of above-mentioned stitches, to gain new manifold items, as shown in Fig. 9.

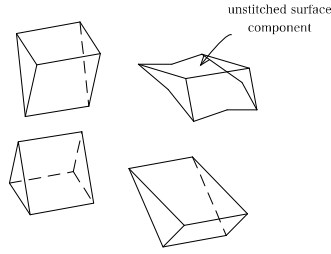


Fig.8 A stitch operation is conducted on surface components in Fig.7 along different surface components

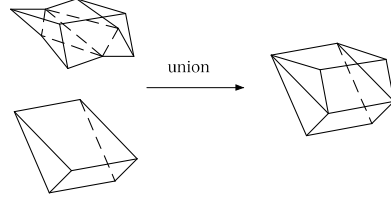


Fig.9 Treatment for surface components not sewed in Fig.8

So far, we have stitched surface components except hanging surface into sub-manifold items. As for the treatment of hanging surface, we shall adopt two strategies: (1) eliminate the hanging surface. It is simple and practicable, but it may influence items' original geometric appearance; (2) we transfer the hanging surface into an object with similar geometric appearance. For example, transfer planes into platforms with small height, and transfer lines into pipes etc. Like this, we can "stitch" all surface components gained from cutting of original objects. In order to keep original geometric appearance of the whole object, it is necessary to expand all non-manifold places of original objects into line pipes and to gather and operate generated sub-manifold items in proper order, and we can gain final manifold items and items which are similar to geometric appearance of original items, as shown in Fig.10.

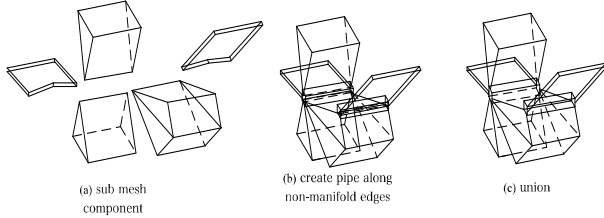


Fig.10 Stitch results cut in Fig.6

It is known from above analysis that stitch operation methods repairing on non-manifold items can generated a manifold item in the stitch process, which has similar geometric appearance with original items. "Bind" individual connection components based on Boolean operation [16,22], to avoid generating new non-manifold structure in stitch process.

#### IV. RESEARCH ON TRANSFERRING OF NON-MANIFOLD POINTS

Repairing operation of non-manifold vertex is simpler than that of non-manifold edges. Find non-manifold vertex, segment non-manifold vertex, and bind them. Cutting of non-manifold vertex is a special case of non-manifold edges, so that they will not be stated here. Binding operation will be discussed mainly.

Binding operation shall be conducted on non-manifold vertex. Scholars at home and abroad have proposed many operations, and separated sub-manifold items are connected through creating pipes [13, 19]. As shown in Fig.11, build pipes in three sub-manifold items.

It can be seen that the number of connected sub-manifold items is less than 6, which can be completed with the use of pipe-building operation. When the number of connected sub-manifold items is more than 6, it is necessary to build pentahedron, hexahedron, and even polyhedron to complete connection operation. In addition, it is necessary to judge directions on surfaces of manifold items in the connection process, and connection operation can be completed only when geometrical information of connectors is the same. Because of the above operations, it seems that such method connecting several sub-manifold bodies is very complicated.

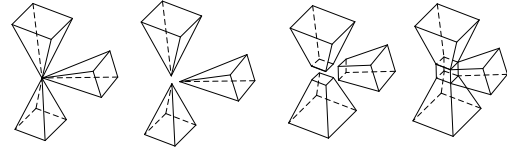


Fig.11 Pipe-building Operations among Three Sub-manifold Bodies

In the following, we will discuss a kind of effective method. No matter how many sub-connectors will be connected, such method can connect sub-connectors effectively with the use of uniform strategies, as shown in Fig 12.

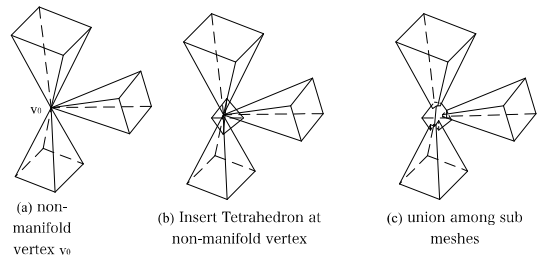


Fig.12 Connect Sub-connectors with the Use of Boolean Operation

"Bind" individual connection components based on Boolean operation [16,22]. If there are more connectable sub-manifold items, they can be connected through gathering and operating other sub-manifold items. Such algorithm is simple and clear, and non-manifold results will not be generated. What's more, there are uniform algorithm steps for repairing

of complicated non-manifold vertices, and additional computation and judges are not needed.

## CONCLUSION

During modeling researches of graphics, it is frequent to meet with non-manifold surfaces. On one hand, it is inevitable that some basement operations may generate non-manifold surfaces; on the other hand, with emerging of virtual reality and computer animation, our requirements to objects' validity have been reduced greatly. Some modeling users prefer using non-manifold surfaces, because they can express more complicated topological relations, with stronger description. However, current graphics algorithms in quantity, such as compression algorithm, simplified algorithm, and subdivision algorithm etc., can only be applied on manifold surfaces, so that how to make generated non-manifold surfaces apply the above algorithms safely forms the starting point for this paper to solve problems. In this paper, algorithms on transferring non-manifold surfaces are researched, and algorithms on transferring non-manifold edges and non-manifold points are discussed specifically. A kind of new non-manifold transferring method is proposed, and new non-manifold structures will not be generated after transferring non-manifold surfaces into non-manifold bodies with the use of such method, so as to guarantee transferring direction of non-manifold items. In addition, generated non-manifold surfaces can be transferred into manifold surfaces whose geometric appearance is similar to that of original entity. We give the following statements before end of this paper.

Some applications shall be related to specific grid data structures. Specific data structures are not proposed in this paper. We know that expression of grid data structures is different from manifold bodies of grid, and only grid manifold bodies are discussed in this paper. Therefore, specific data structures are unnecessary. Different data structures may generate different time complexities, so that it is necessary for readers to consider more proper data structures according to specific conditions (vertex-based data structure, surface-based data structure, and edge-based data structure).

Discusses in this paper focus on oriented 2-manifold modeling; if 2-manifold includes Möbius band [20], it is non-oriented. There are researches on non-oriented 2-manifold modeling in literatures. For example, Guibas et al [21] proposed that edge algebraic model and square-edge structure can treat oriented and non-oriented 2-manifold. As for non-manifold surface transferring algorithms in this paper, they cannot be used in grid models containing Möbius band, which shall be researched further.

This paper has provided beneficial supplements for 2-manifold grid modeling system. Applying non-manifold surface transferring algorithms in this paper is extremely useful to operations where effective 2-manifold manifold

structures (such as subdivision algorithm) are needed during modeling. It provides powerful tools for building a 2-manifold grid modeling system with topologic robustness.

## REFERENCES

- [1] L. Zeng, Y.J.Liu, "Q-Complex: Efficient non-manifold boundary representation with inclusion topology", *Computer-Aided Design*, 2012, 44(11), pp. 1115-1126.
- [2] S. Martin, J.P. Watson, "Non-manifold surface reconstruction from high-dimensional point cloud data", *Computational Geometry*, 2011, 44(8), pp. 427-441.
- [3] S. Doo, M. Sabin, "Analysis of the behaviour of recursive division surfaces near extraordinary points", *Computer Aided Design*, 1978, 10(6), pp.356-360.
- [4] D. Zorin, ed., *Subdivision for modeling and animation*, ACM SIGGRAPH'2000 Course Notes no.23, 2000.
- [5] G. Taubin, W.P. Horn, F. Lazarus, et al. "Gemnet coding and VRML", *Proceedings of the IEEE*, 1998,86(6), pp.1228-1243
- [6] A. Varshney, "Hierarchical geometric approximations", University of North Carolina, Chapel Hill, 1994.
- [7] A. Guezic, "locally tolerance polygonal surface simplification", *IEEE Transactions on Visualization and Computer Graphics*, 1999,5(2), pp.178-199.
- [8] L.X. Ying, D.Zorin, "Nonmanifold subdivision", Technical Report, Computer Science Dept, New York University, 2001.
- [9] P.G. Andre, B. Frank, T. Gabrial, et al., "Efficient compression of non-manifold polygonal meshes", *IEEE Visualization*, 1999, 10, pp.73-80.
- [10] P.Jovan, H.Hugues, "Progressive simplicial complexes", *Proceedings of SIGGRAPH 97*, ISBN 0-9791-896-7. Held in Los Angeles. California, 1997, 8, pp.217-224.
- [11] G Barequet and S.Kumar, "Repairing CAD Models", *Proceedings of Visualization'97*, pp. 363-370, Oct.1997.
- [12] J.Rossignac and D.Cardoze, "Matchmaker: manifold breps for non-manifold r-Sets", *Proceedings of fifth Symposium on Solid Modeling and Applications (SMA'99)*, pp.31-41, June 1999.
- [13] A. Guezic, F. Lazarus, "Cutting and stitching: converting sets of polygons to manifold surfaces", *IEEE Transactions of Visualization and Computer Graphics*, 2001, 7(2), pp.136-150.
- [14] M. Ni, L. Wu, *Computer graphics*, Beijing University Press, 1999.
- [15] A.A.G. Requicha, H.B.Voelcker, "Solid modeling: current status and research directions", *IEEE Computer Graphics and Applications*, 1983, 3(7), pp.25-37
- [16] M. Mantyla, "Boolean operations on 2-manifolds through vertex neighborhood classification", *ACM Transactions on Graphics*, 1986, 5, pp. 1-29.
- [17] Y. Luo, G. Lukacs, "A boundary representation for form features and non-manifold solid object", *Symposium on Solid Modeling Foundations and CAD/CAM Applications*, 1991, pp. 45-60.
- [18] Y. P. Fei, S.Q. Chen, M. Li. "On testing topological validity for manifold modeling systems", *Journal of Computer-Aided Design and Computer Graphics*, 2011, 23 (8), pp.1337-1348.
- [19] M. Sun, Y. P. Fei. "Efficient conversion of non-manifold vertex to a manifold", *Computer Engineering and Applications*, 2007, 43(28), pp.65-69.
- [20] J. L. Gross and T. W. Tucker, "Topological graph theory", Wiley Interscience, NY, 1987.
- [21] L. Guibas and J. Stolfi, "Primitives for the manipulation of general subdivision and computation of Voronoi diagrams", *ACM Transactions on Graphics*, 1985, 4, pp.74-123.
- [22] M. Chen, X. Y. Chen, K. Tang and et al., "Efficient boolean operation on manifold mesh surfaces", *Computer-Aided Design & Applications*, 2010, 7(3), pp.405-415.