algebra

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1. Write the quadratic equation in x whose roots are 2 and -5.

2. (a) If α and β are zeros of the quadratic polynomial $f(x) = x^2 - x - 4$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta} - \alpha\beta$.

   (b) If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then find the value of k.

3. (a) If $3\sin A = 1$, then find the value of $\sec A$.

   (b) Show that: $\frac{1+\cot^2\theta}{1+\tan^2\theta} = \cot^2\theta$.

4. Simplify :
$$\csc^2 60° \sin^2 30° - \sec^2 60°$$

5. If $\tan\theta + \cot\theta = \frac{4\sqrt{3}}{3}$, then find the value of $\tan^2\theta + \cot^2\theta$.

6. Divide the polynomial $f(x) = 5x^3 + 10x^2 - 30x - 15$ by the polynomial $g(x) = x^2 + 1 + x$ and hence, find the quotient and the remainder.

7. (a) Prove:

$$\frac{1}{(\cot A)(\sec A) - \cot A} - \csc A = \csc A - \frac{1}{(\cot A)(\sec A) + \cot A}$$

   (b) Prove:
$$\sin^6 A + 3\sin^2 A \cos^2 A = 1 - \cos^6 A$$

8. (a) One of the root of the quadratic equation $2x^2 - 8x - k = 0$ is $\frac{5}{2}$. Find the value of k, Also find the root.

   (b) Using quadratic formula, solve the following equation for x:

$$abx^2 + (b^2 - ac)x - bc = 0$$

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9. With vertices A,B and C of a triangle ABC as centers, arcs are drawn with radii 2 cm each as/ shown in the figure. If AB = 6 cm, BC = 8 cm and AC = 10 cm, find the area of the shaded region.
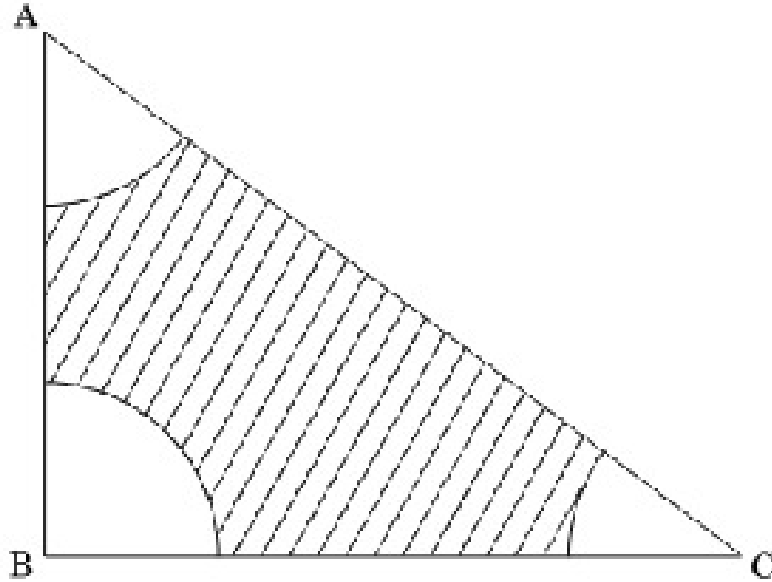
Figure 1:

10. Water is being pumped out through a circular pipe whose internal diameter is 8 cm. If the rate of flow of water is 80 cm/s, then how many liters of water is being pumped out through this pipe in one hour ?

11. (a) A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 18 minutes for the angle of depression to change from 30° to 60°, how soon after this will the car reach the tower ?

   (b) A girl on a ship standing on a wooden platform, which is 50 m above water level, observes the angle of elevation of a top of a hill as 30° and the angle of depression of the base of the hill as 60°. Calculate the distance of the hill from the platform and the height of the hill.