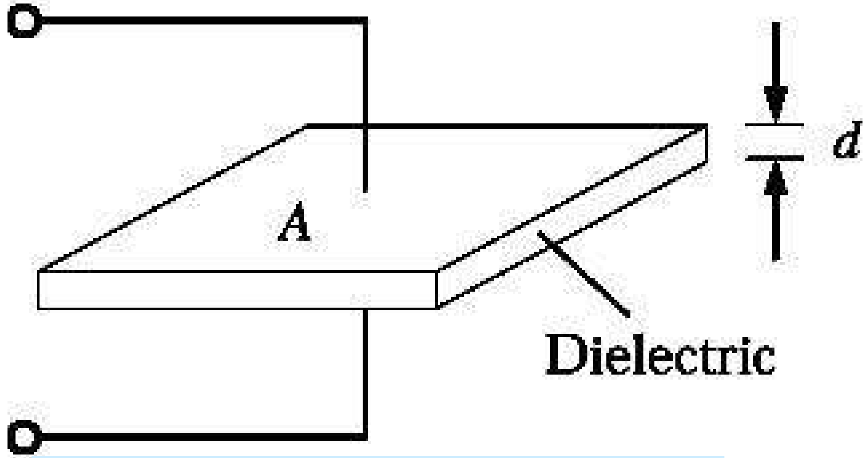
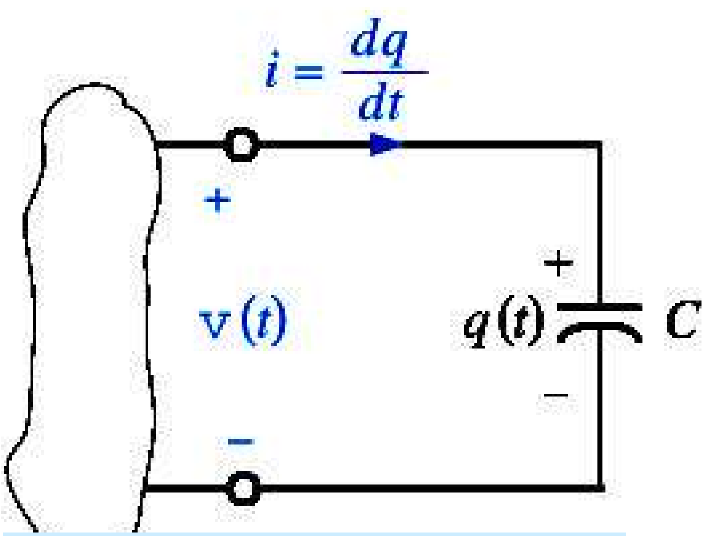
CONDENSADORES E BOBINES

DISPOSITIVOS PASSIVOS DE ARMAZENAMENTO DE ENERGIA

CONDENSADOR



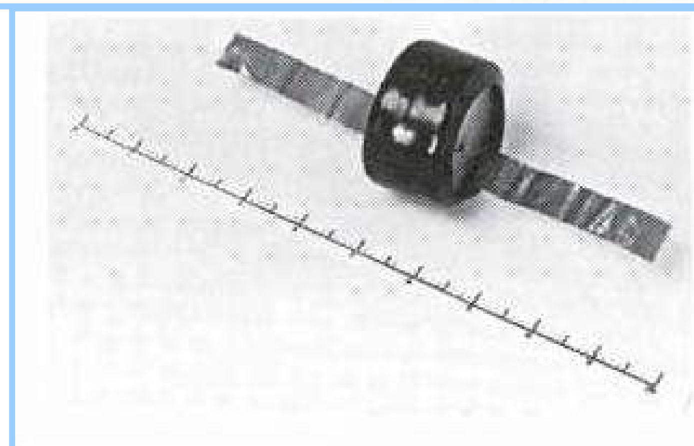
Basic parallel-plates capacitor



CIRCUIT REPRESENTATION

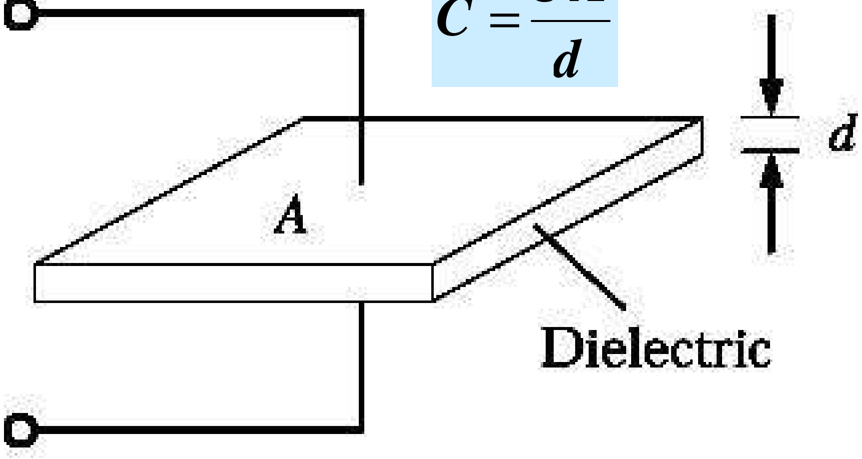
CONVENÇÃO PASSIVA PARA A CORRENTE

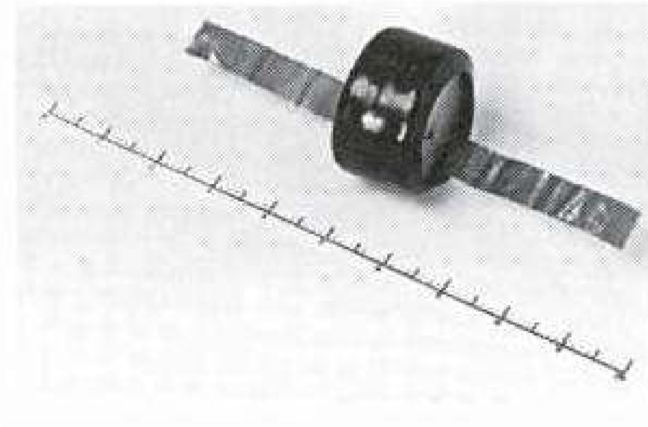
Typical Capacitors



A 55-F, 500-J capacitor and a 10-in ruler for comparison.

$$C = \frac{\varepsilon A}{d}$$





A 55-F, 500-J capacitor and a 10-in ruler

ε Constante dielectrica do material

$$55F = \frac{8.85\times10^{-12} A}{1.016\times10^{-4}} \Rightarrow A = 6.3141\times10^{8} m^{2}$$

Valores de capacidade são baixos.Microfarads é típico
Circuitos integrados é ainda mais baixo … pico farads é típico

Condensadores obedecem
À Lei de Coulomb :

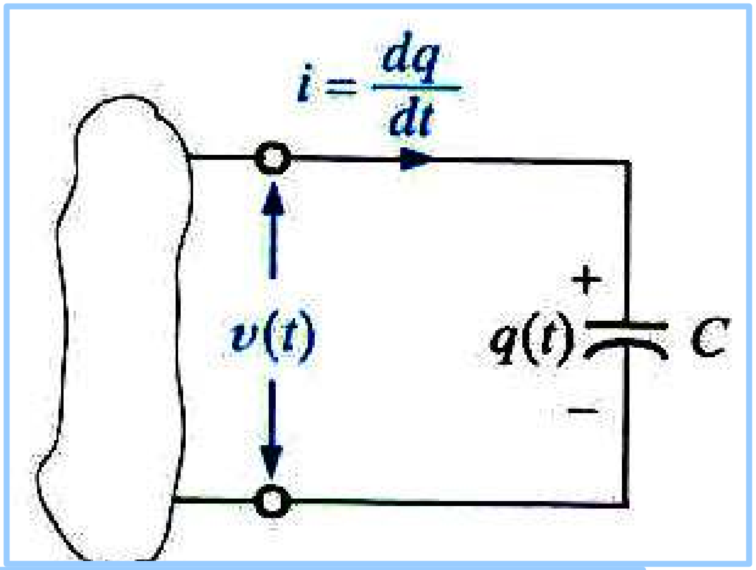
$$Q = CV_C$$

C é a capacidade do condensador

$$\frac{\text{carga}}{\text{tensão el.}}$$

Um Farad(F) é a capacidade de um dispositivo
Que armazena um coulomb de carga a um volt.

$$\text{Farad} = \frac{\text{Coulomb}}{\text{Volt}}$$



Representação do elem. circuito

EXAMPLE

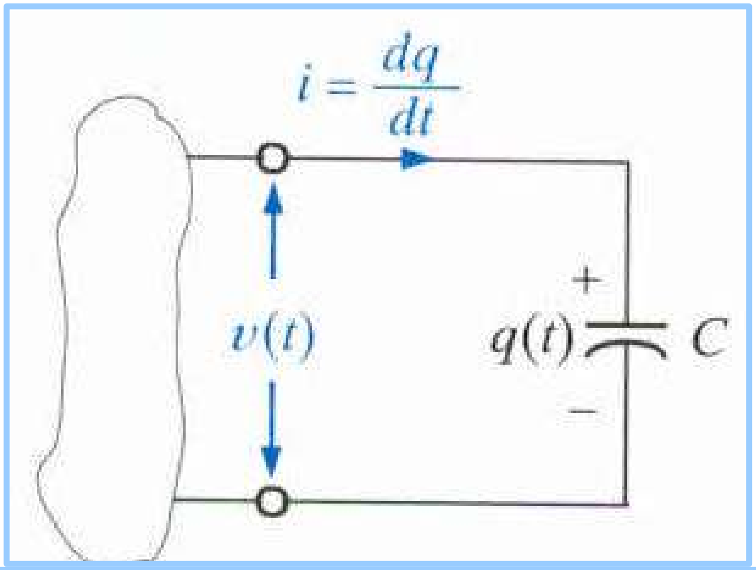
Queda de tensão de um cond. De 2 micro
Farads que armazene 10mC de carga

$$V_C = \frac{1}{C}Q = \frac{1}{2*10^{-6}} 10*10^{-3} = 5000 \quad \text{V}$$

Capacitance in Farads, charge in Coulombs
result in voltage in Volts

CONDENSADORES PODEM SER PERIGOSOS!!!!!!!!!

Condensadores apenas armazenam e libertam carga.
NÃO “CRI AM” CARGA.



Linear capacitor circuit representation

$$i(t) = C\frac{dv}{dt}(t)$$

$$Q_C = CV_C$$

LEI DA CAPACIDADE!

Se a tensão varia a carga varia e portanto há uma corrente Que flui……….

Podemos exprimir a tensão em função Da corrente.

$$V_C(t) = \frac{1}{C}Q = \frac{1}{C}\int_{-\infty}^{t} i_C(x)dx$$

Forma integral .

Uma consequência é que ...

$$V_C(t-) = V_C(t+);\ \forall t$$

Tensão aos terminais do condensador TEM QUE SER CONTÍNUA.

… Ou a corrente em função da tensão Aos seus terminais

$$i_C = \frac{dQ}{dt} = C\frac{dV_C}{dt}$$

Forma diferencial da lei da capacitância

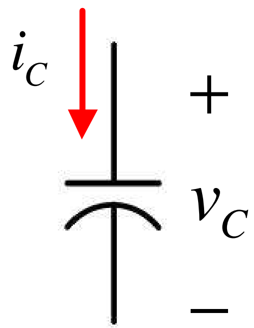
E daí ??

$$V_C = Const \Rightarrow i_C = 0$$

Comportamento estacionário ou DC

Se a tensão é constante o condensador COMPORTA-SE COMO UM CIRCUITO ABERTO.

Condensador como elemento de circuito



$$i_C(t) = C\frac{dv_c}{dt}(t)$$

$$i_R = \frac{1}{R}v_R$$

$$v_R = Ri_R$$

Ohm's Law

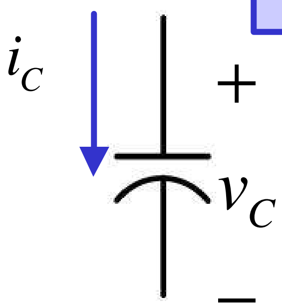
$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(x)dx$$

$$\int_{-\infty}^{t} = \int_{-\infty}^{t_0} + \int_{t_0}^{t}$$

$$v_C(t) = \overbrace{\boxed{\frac{1}{C}\int_{-\infty}^{t_0} i_C(x)dx}}^{v_c(t_0)} + \frac{1}{C}\int_{t_0}^{t} i_C(x)dx$$

$$v_C(t) = v_C(t_0) + \frac{1}{C}\int_{t_0}^{t} i_C(x)dx$$

Condensador como armazenador de energia



Potência instantânea **W**

$$p_C(t) = v_C(t)\, i_C(t)$$

$$i_C(t) = C\frac{dv_c}{dt}(t)$$

$$p_C(t) = C v_C(t)\frac{dv_c}{dt}$$

$$\boldsymbol{p_C(t) = C\frac{d}{dt}\left(\frac{1}{2}v_C^2(t)\right)}$$

Energia é integral da potência

$$\boldsymbol{w_C(t_2,t_1) = \int_{t_1}^{t_2} p_C(x)dx}$$

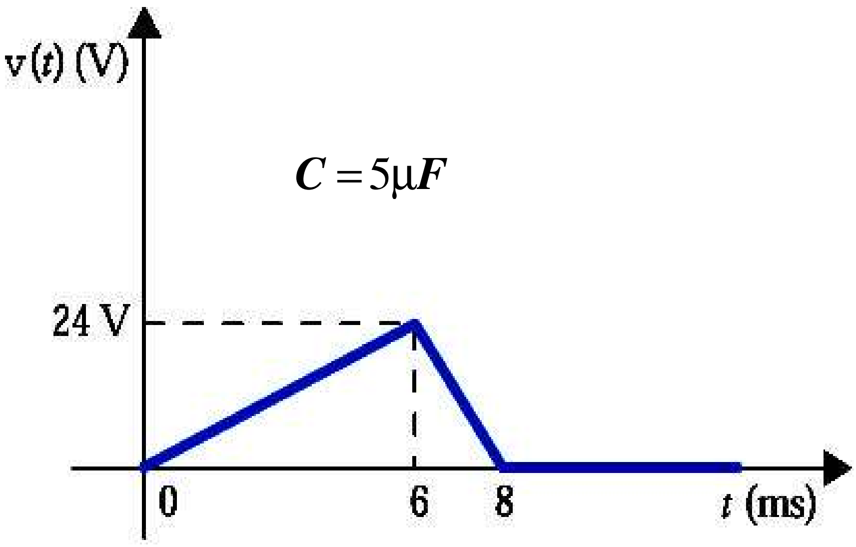
$$\boldsymbol{p_C(t) = \frac{1}{C}\frac{d}{dt}\left(\frac{1}{2}q_c^2(t)\right)}$$

Se t1 é menos infinito temos “energia armazenada em t2.”

Se ambos os limites são infinitos temos A energia total armazenada.

$$\boldsymbol{w_C(t_2,t_1) = \frac{1}{2}Cv_C^2(t_2) - \frac{1}{2}Cv_C^2(t_1)}$$

$$\boldsymbol{w_C(t_2,t_1) = \frac{1}{C}q_C^2(t_2) - \frac{1}{C}q_C^2(t_1)}$$



Exemplo

Energia armazenada de 0 - 6 msec

$$w_C(0,6) = \frac{1}{2} C v_C^2(6) - \frac{1}{2} C v_C^2(0)$$

$$w_C(0,6) = \frac{1}{2} 5 * 10^{-6} [F] * (24)^2 [V^2]$$

Carga armazenada aos 3msec

$$q_C(3) = C v_C(3)$$

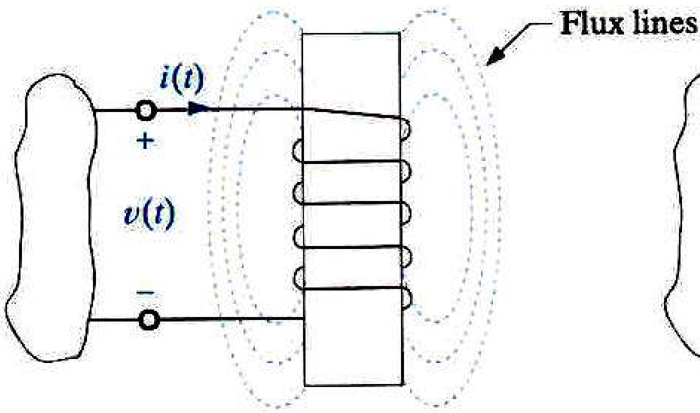
$$q_C(3) = 5 * 10^{-6} [F] * 12 [V] = 60 \mu C$$

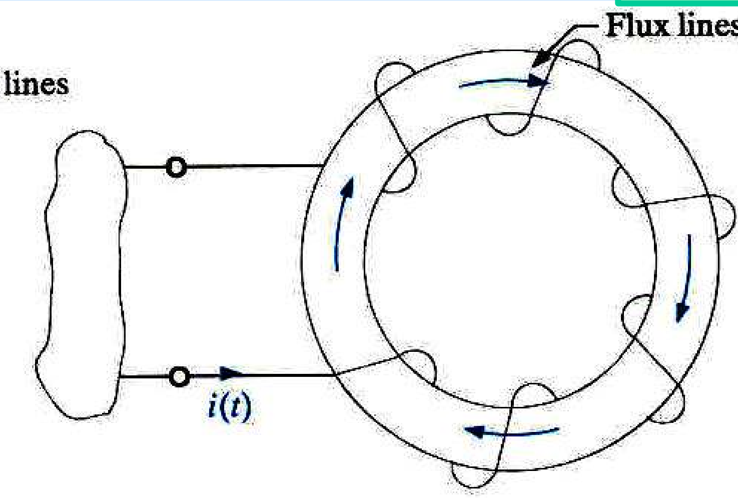
“energia total armazenada?” ....

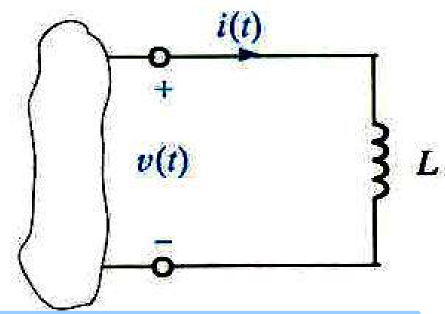
“carga total armazenada?” ...

BOBINES

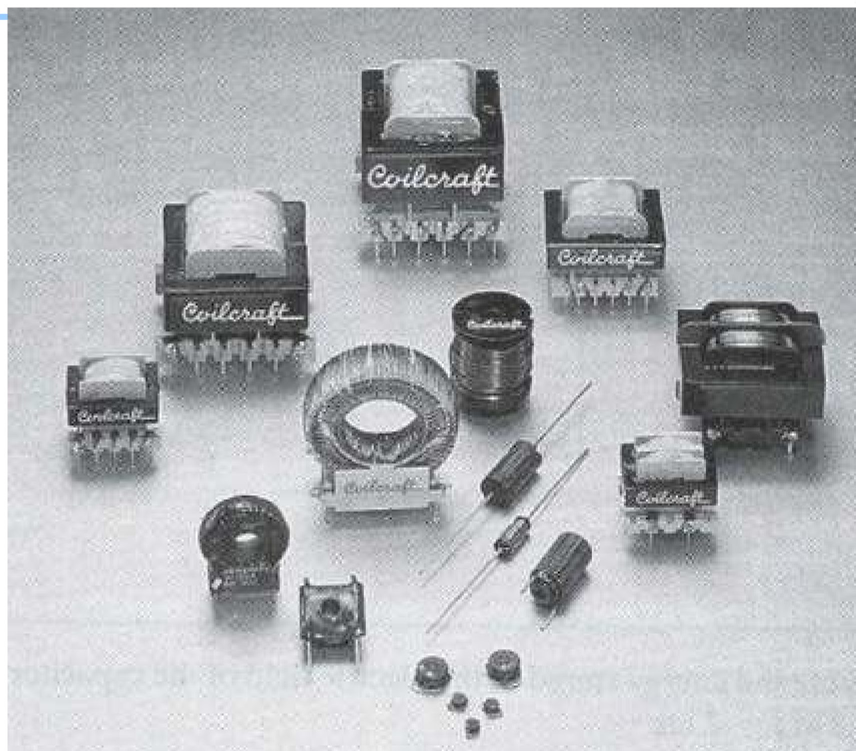
ATENÇÃO À CONVENÇÃO PARA TENSÃO E CORRENTE







SIMBOLO DA BOBINE



Um fluxo variável no tempo Induz um campo electro--magnético, surgindo uma Tensão (induzida) aos Terminais da bobine.

FLUXO MAGNÉTICO VARIÁVEL NO TEMPO INDUZ UMA TENSÃO NA BOBINE

$$v_L = \frac{d\phi}{dt}$$

Lei da indução

Para uma bobine "normal" (linear), o fluxo é Proporcional à corrente.

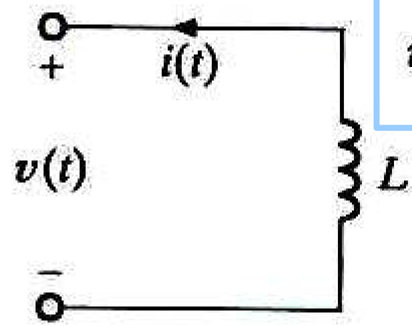
$$\phi = Li_L \Rightarrow \quad v_L = L\frac{di_L}{dt}$$

A CONSTANTE DE PROPORCIONALIDADE L CHAMA--SE INDUTÂNCIA

A INDUTÂNCIA MEDE-SE EM HENRY

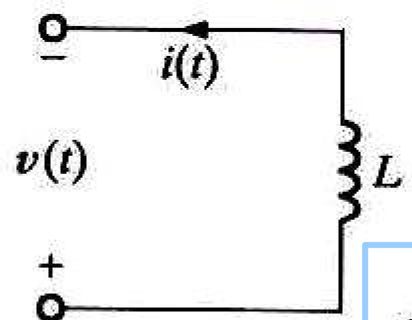
$$\text{HENRY} = \frac{\text{Volt}}{\text{Amp}/\text{sec}}$$

Write the i-v relationship for the following inductors.



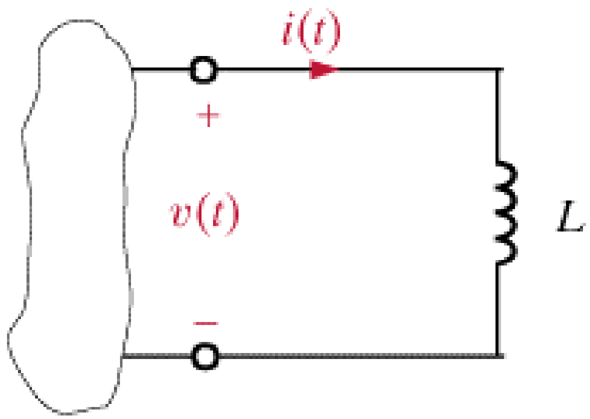
(a)

$$v(t) = -L\frac{di(t)}{dt}$$



(b)

$$v(t) = L\frac{di(t)}{dt}$$



$$v_L = L\frac{di_L}{dt}$$ Differential form of induction law

$$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(x)dx$$ Integral form of induction law

$$i_L(t) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} v_L(x)dx;\ \ t \geq t_0$$

Conseq. Directa da forma integral…

$$i_L(t-) = i_L(t+);\quad \forall t$$ A corrente tem que ser contínua…

Outra consequência ….

$$i_L = Const. \Rightarrow v_L = 0$$ Comportamento DC

Potência e energia armazenada

$$p_L(t) = v_L(t)i_L(t)$$ W

$$p_L(t) = L\frac{di_L}{dt}(t)i_L(t) = \frac{d}{dt}\left(\frac{1}{2}Li_L^2(t)\right)$$

$$w_L(t_2,t_1) = \int_{t_1}^{t_2}\frac{d}{dt}\left(\frac{1}{2}Li_L^2(x)\right)dx$$ J

Current in Amps, Inductance in Henrys
yield energy in Joules

$$w(t_2,t_1) = \frac{1}{2}Li_L^2(t_2) - \frac{1}{2}Li_L^2(t_1)$$

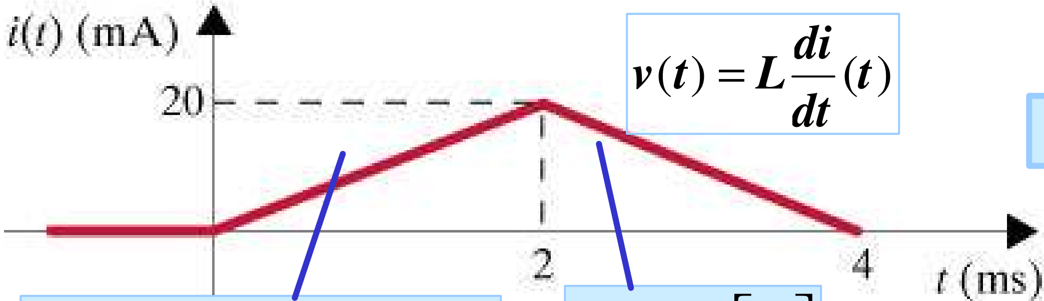
Energy stored on the interval
Can be positive or negative

$$w_L(t) = \frac{1}{2}Li_L^2(t)$$

"Energy stored at time t"
Must be non-negative.
Passive element!!!

LEARNING EXAMPLE

L=10mH. FIND THE VOLTAGE



$$v(t) = L\frac{di}{dt}(t)$$

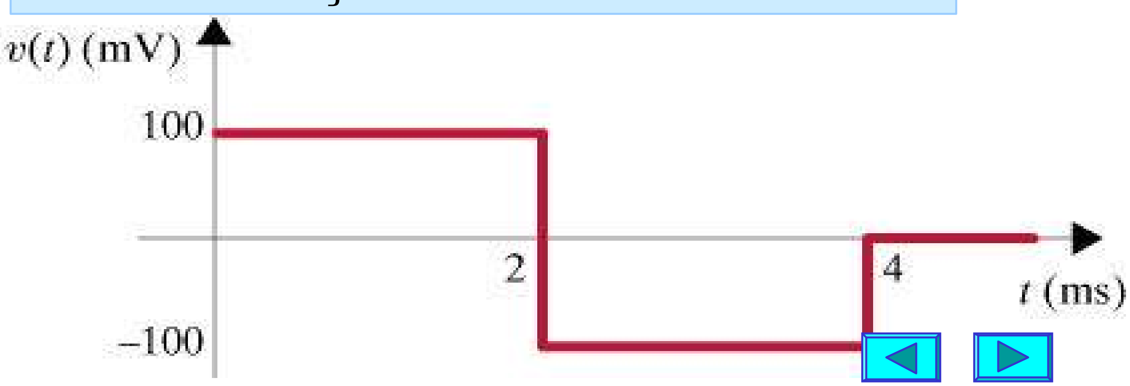
$$m = \frac{20\times10^{-3}A}{2\times10^{-3}s} = 10\left[\frac{A}{s}\right]$$

$$m = -10\left[\frac{A}{s}\right]$$

THE DERIVATIVE OF A STRAIGHT LINE IS ITS SLOPE

$$\frac{di}{dt} = \begin{cases} 10(A/s) & 0 \le t \le 2ms \\ -10(A/s) & 2 < t \le 4ms \\ 0 & elsewhere \end{cases}$$

$$\left.\begin{array}{l} \frac{di}{dt}(t) = 10(A/s) \\ L = 10\times10^{-3}H \end{array}\right\} \Rightarrow v(t) = 100\times10^{-3}V = 100mV$$



ENERGY STORED BETWEEN 2 AND 4 ms

$$w(4,2) = \frac{1}{2}Li_L^2(4) - \frac{1}{2}Li_L^2(2)$$

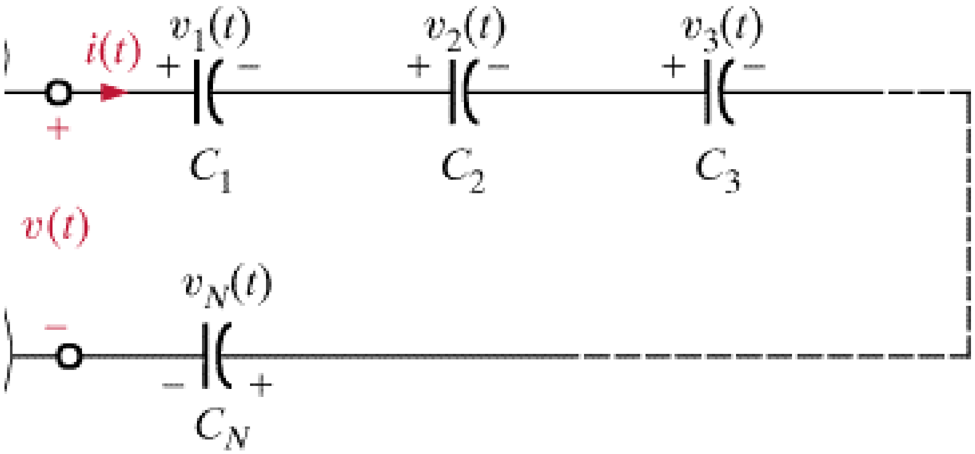
$$w(4,2) = 0 - 0.5*10*10^{-3}(20*10^{-3})^2 \text{ J}$$

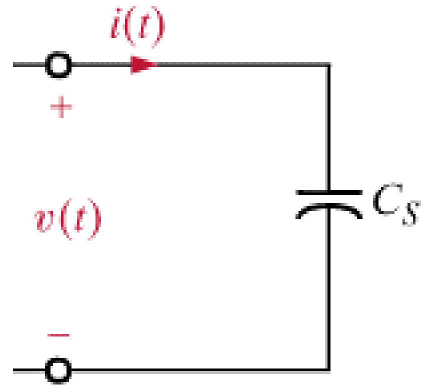
THE VALUE IS NEGATIVE BECAUSE THE INDUCTOR IS SUPPLYING ENERGY PREVIOUSLY STORED

The Dual Relationship for Capacitors and Inductors

Capacitor		Inductor
$i(t) = C\dfrac{dv(t)}{dt}$		$v(t) = L\dfrac{di(t)}{dt}$
$v(t) = \dfrac{1}{C}\displaystyle\int_{t_0}^{t} i(x)\,dx + v(t_0)$		$i(t) = \dfrac{1}{L}\displaystyle\int_{t_0}^{t} v(x)\,dx + i(t_0)$
$p(t) = Cv(t)\dfrac{dv(t)}{dt}$	$C \rightarrow L$	$p(t) = Li(t)\dfrac{di(t)}{dt}$
	$v \rightarrow i$	
$w(t) = \frac{1}{2}Cv(t)^2$	$i \rightarrow v$	$w(t) = \frac{1}{2}Li^2(t)$

Condensadores em série





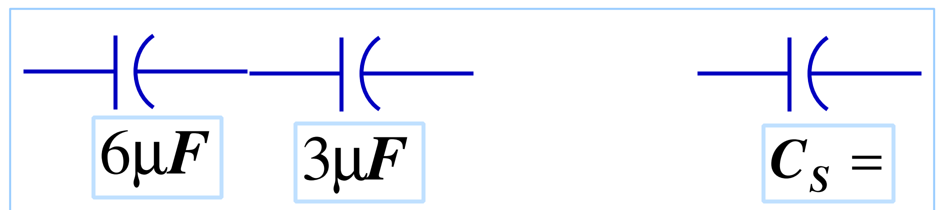
$$v(t) = v_1(t) + v_2(t) + v_3(t) + \cdots + v_N(t)$$

$$C_s = \frac{C_1 C_2}{C_1 + C_2}$$

Combinação serie de 2 condensadores

$$v_i(t) = \frac{1}{C_i}\int_{t_0}^{t} i(t)\,dt + v_i(t_0)$$

$$v(t) = \left(\sum_{i=1}^{N}\frac{1}{C_i}\right)\int_{t_0}^{t} i(t)\,dt + \sum_{i=1}^{N} v_i(t_0)$$



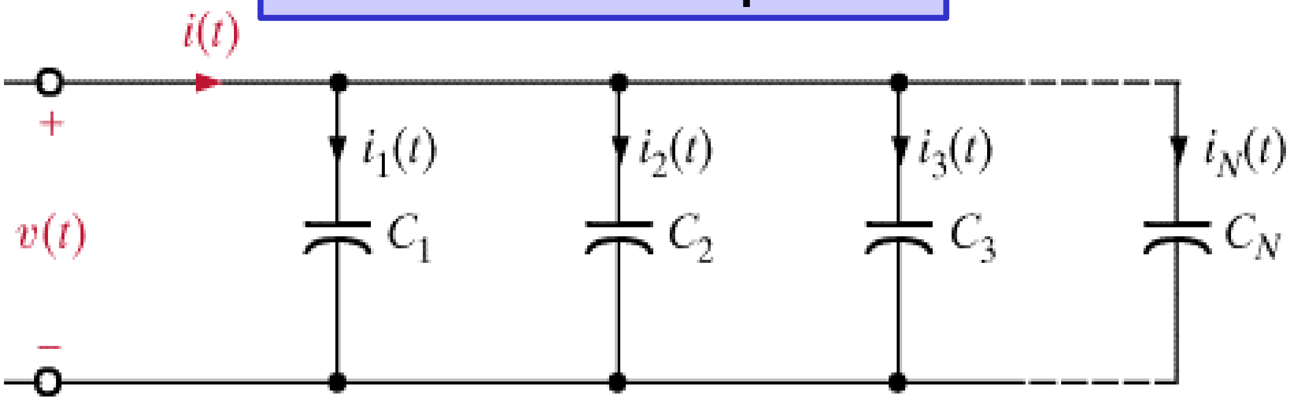
$2\mu\, F$

$$\frac{1}{C_S} = \sum_{i=1}^{N}\frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}$$

Semelhante a resist. Em paralelo

$$v(t_0) = \sum_{i=1}^{N} v_i(t_0)$$

Condensadores em paralelo



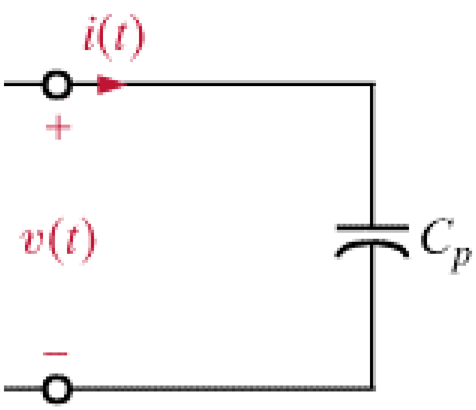
$$i(t) = i_1(t) + i_2(t) + i_3(t) + \cdots + i_N(t)$$

$$i_k(t) = C_k \frac{dv}{dt}(t)$$

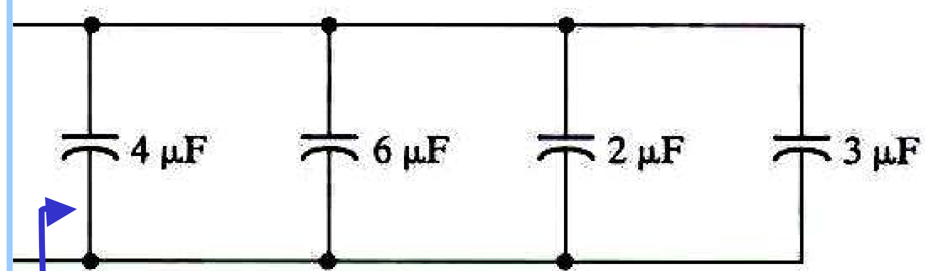
$$i(t) = C_1 \frac{dv(t)}{dt} + C_2 \frac{dv(t)}{dt} + C_3 \frac{dv(t)}{dt} + \cdots + C_N \frac{dv(t)}{dt}$$

$$= \left(\sum_{i=1}^{N} C_i \right) \frac{dv(t)}{dt}$$

$$C_p = C_1 + C_2 + C_3 + \cdots + C_N$$

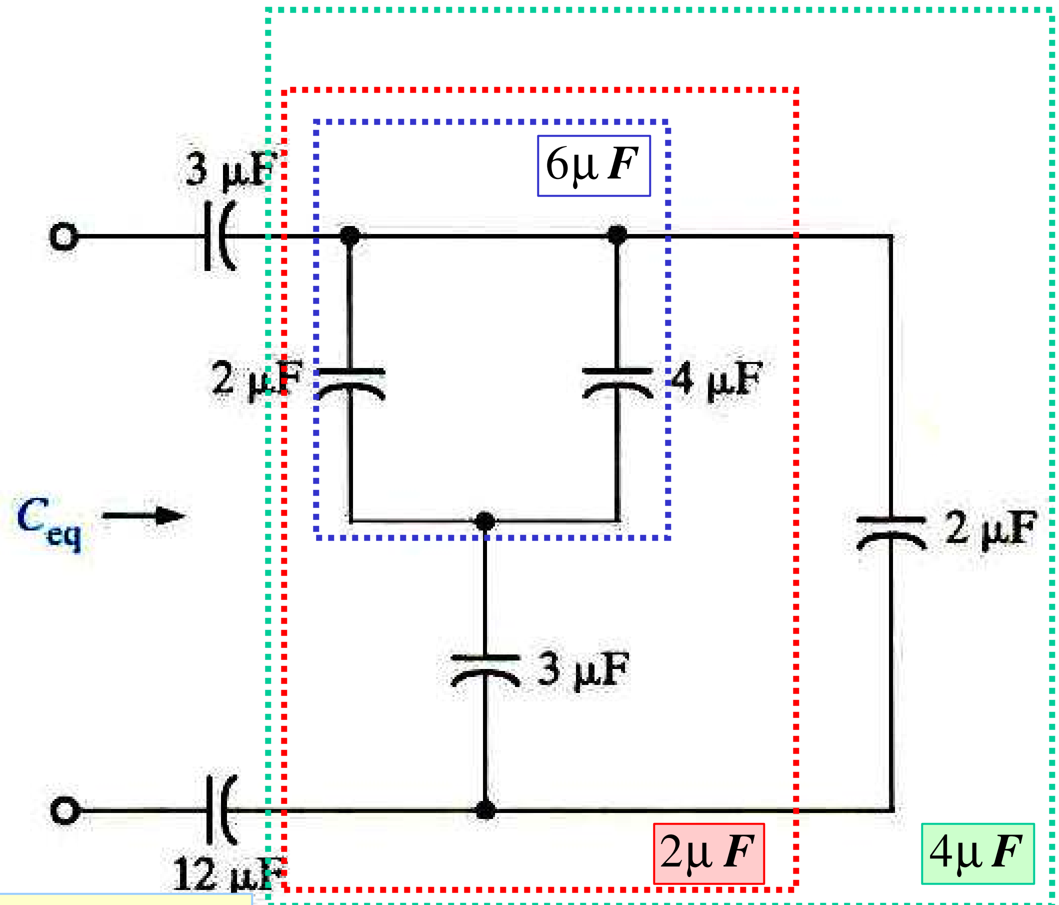


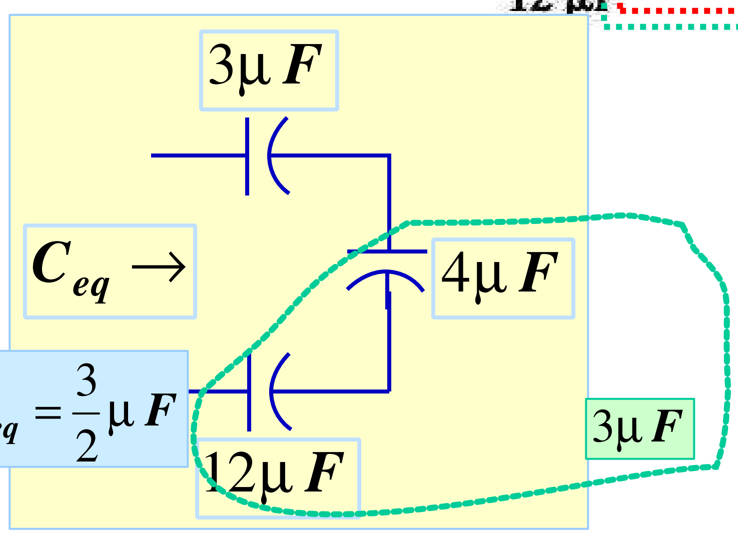
EXEMPLO



$$C_P = 4+6+2+3 = 15\mu F$$

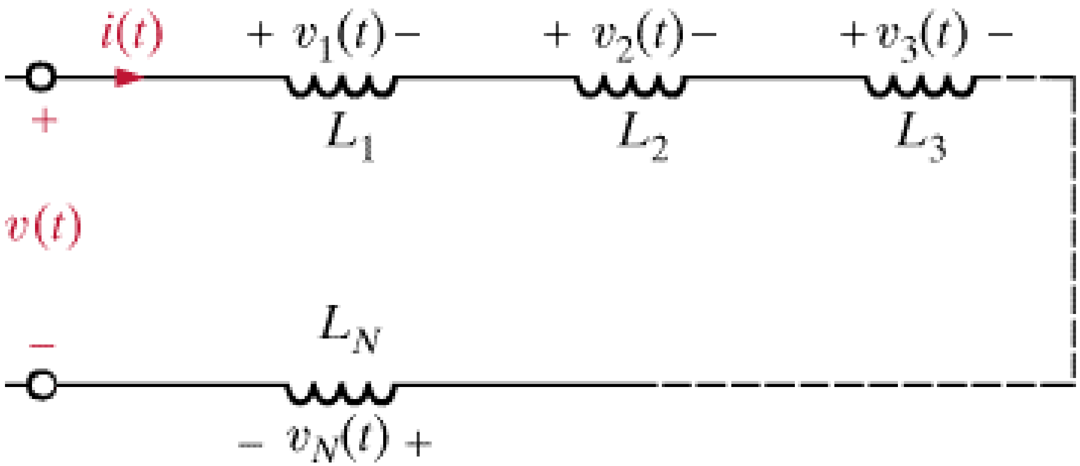
EXEMPLO

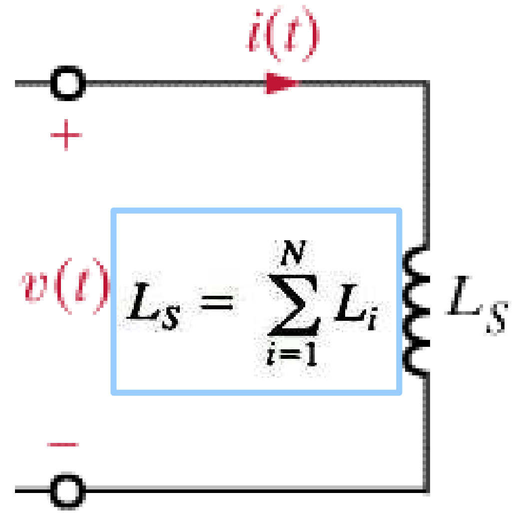




$$C_{eq} = \frac{3}{2}\mu\, F$$

SERIES INDUCTORS





$$v(t) = v_1(t) + v_2(t) + v_3(t) + \cdots + v_N(t)$$

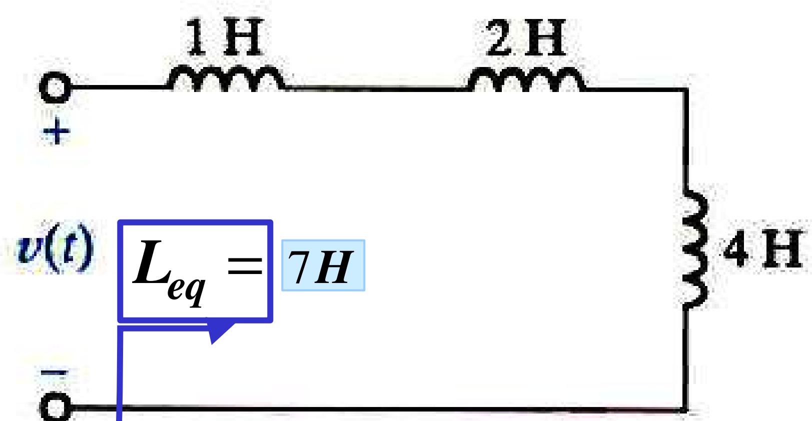
$$v_k(t) = L_k \frac{di}{dt}(t)$$

$$v(t) = L_1 \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + L_3 \frac{di(t)}{dt} + \cdots + L_N \frac{di(t)}{dt} = \left(\sum_{i=1}^{N} L_i \right) \frac{di(t)}{dt}$$

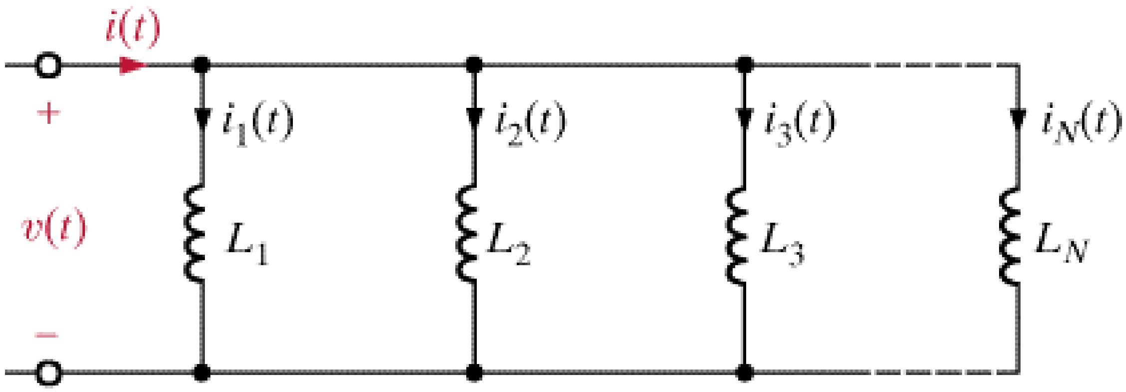
$$L_S = \sum_{i=1}^{N} L_i$$

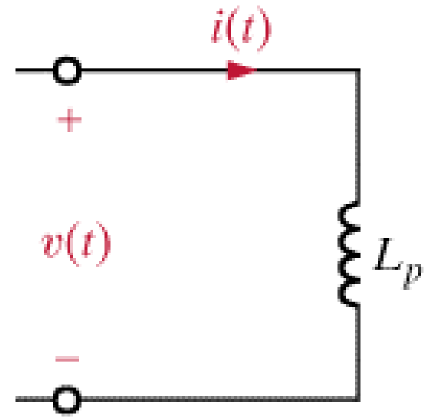
$$v(t) = L_S \frac{di}{dt}(t)$$

LEARNING EXAMPLE



PARALLEL INDUCTORS





$$i(t) = i_1(t) + i_2(t) + i_3(t) + \cdots + i_N(t)$$

$$\boxed{i_j(t) = \frac{1}{L_j}\int_{t_0}^{t} v(x)\,dx + i_j(t_0)}$$

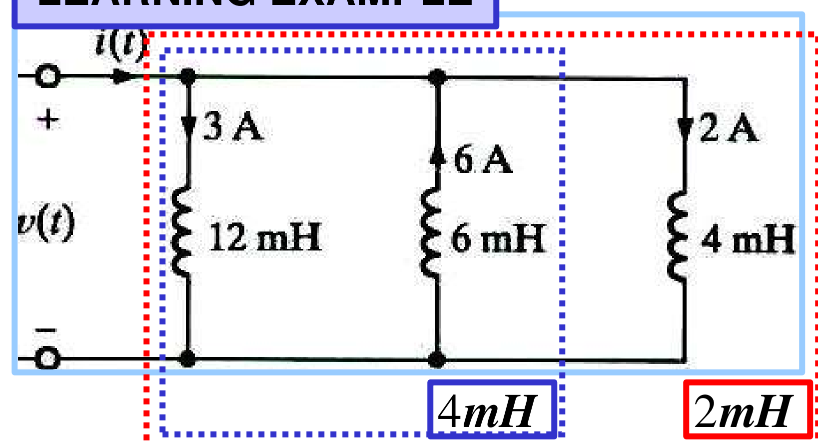
$$\boxed{i(t)} = \boxed{\frac{1}{L_P}\int_{t_0}^{t} v(x)\,dx + i(t_0)}$$

$$i(t) = \left(\sum_{j=1}^{N}\frac{1}{L_j}\right)\int_{t_0}^{t} v(x)\,dx + \sum_{j=1}^{N} i_j(t_0)$$

$$\boxed{\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}}$$

$$\boxed{i(t_0) = \sum_{j=1}^{N} i_j(t_0)}$$

LEARNING EXAMPLE



$$\boxed{i(t_0) = 3A - 6A + 2A = -1A}$$

INDUCTORS COMBINE LIKE RESISTORS
CAPACITORS COMBINE LIKE CONDUCTANCES

SUMMARY

- The important (dual) relationships for capacitors and inductors are as follows:

$$q = Cv$$

$$i(t) = C\frac{dv(t)}{dt} \qquad v(t) = L\frac{di(t)}{dt}$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(x)\,dx \qquad i(t) = \frac{1}{L}\int_{-\infty}^{t} v(x)\,dx$$

$$p(t) = Cv(t)\frac{dv(t)}{dt} \qquad p(t) = Li(t)\frac{di(t)}{dt}$$

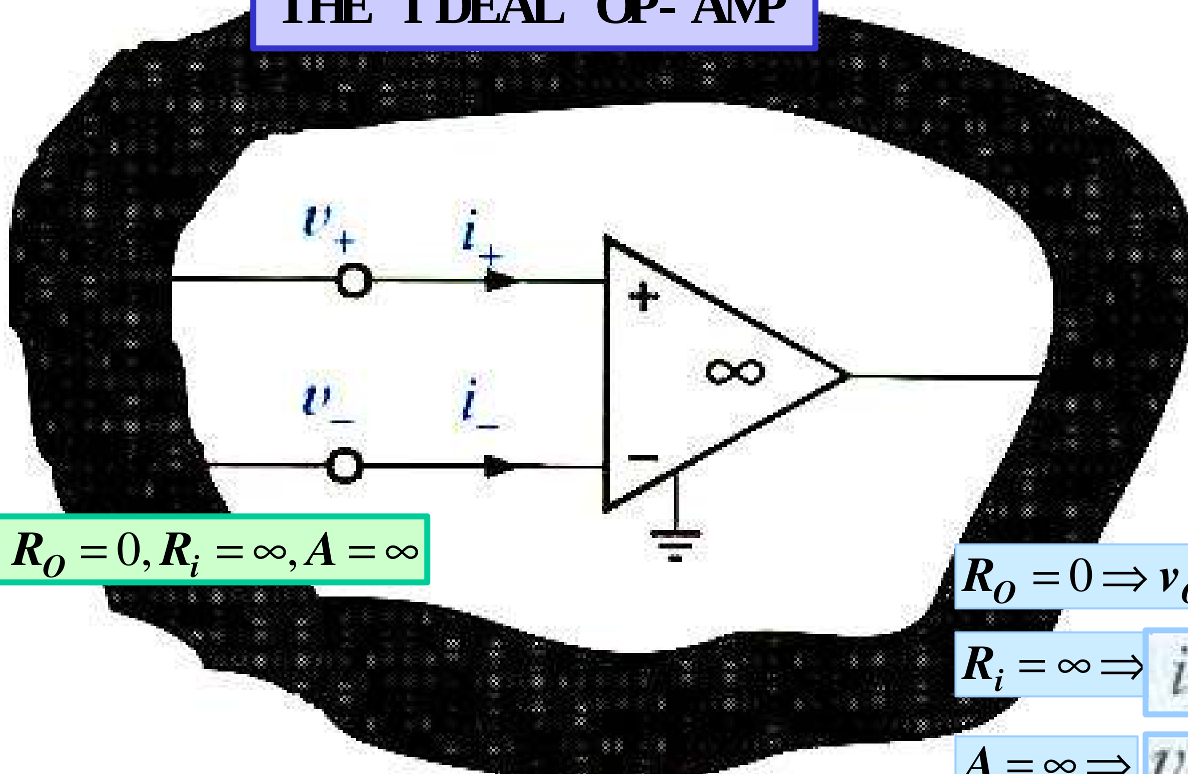
$$W_C(t) = 1/2Cv^2(t) \qquad W_L(t) = 1/2Li^2(t)$$

- The passive sign convention is used with capacitors and inductors.
- In dc steady state a capacitor looks like an open circuit and an inductor looks like a short circuit.
- Leakage resistance is present in practical capacitors and inductors.
- When capacitors are interconnected, their equivalent capacitance is determined as follows: Capacitors in series combine like resistors in parallel and capacitors in parallel combine like resistors in series.
- When inductors are interconnected, their equivalent inductance is determined as follows: Inductors in series combine like resistors in series and inductors in parallel combine like resistors in parallel.
- *RC* operational amplifier circuits can be used to differentiate or integrate an electrical signal.

RC OPERATIONAL AMPLIFIER CIRCUITS

INTRODUCES TWO VERY IMPORTANT PRACTICAL CIRCUITS BASED ON OPERATIONAL AMPLIFIERS

THE IDEAL OP-AMP



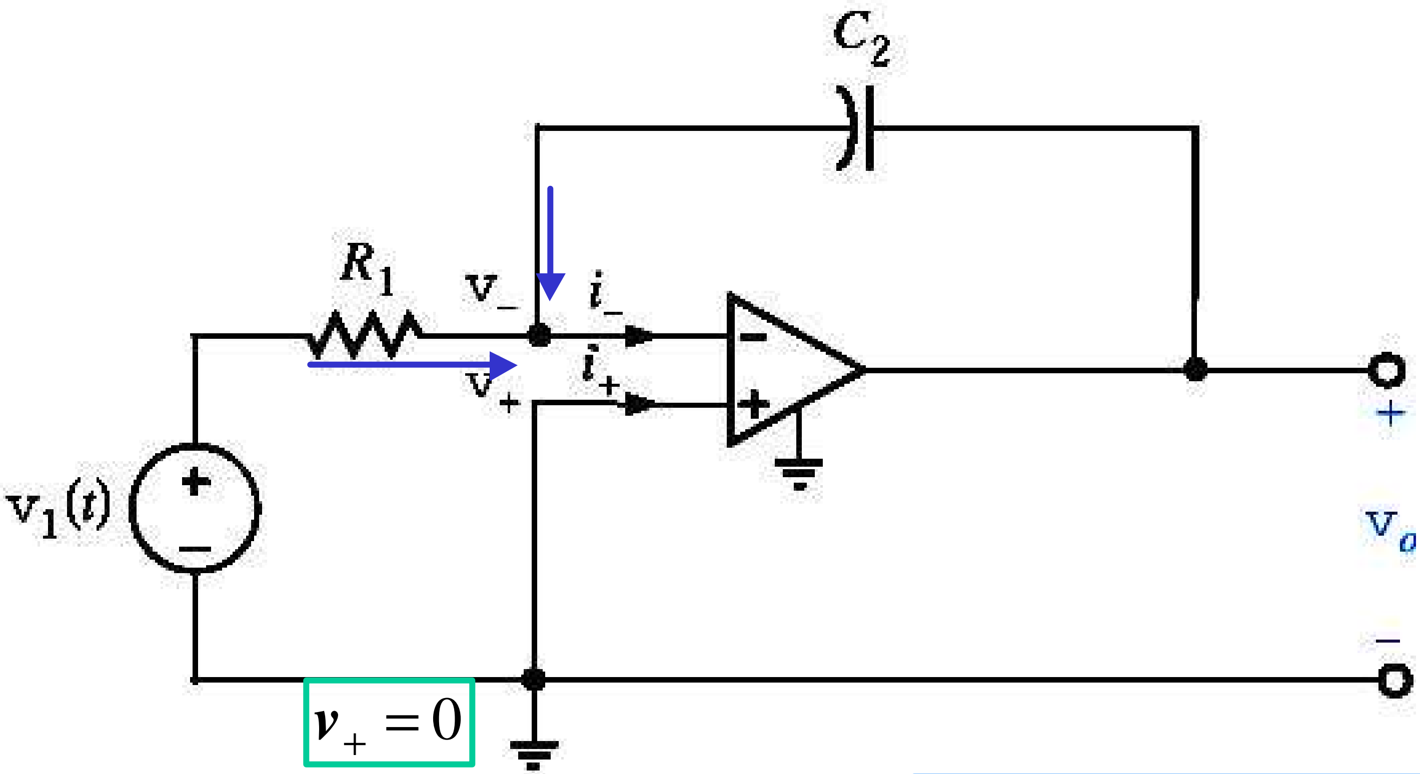
IDEAL $\Rightarrow R_O = 0, R_i = \infty, A = \infty$

$R_O = 0 \Rightarrow v_O = A(v_+ - v_-)$

$R_i = \infty \Rightarrow i_+ = i_- = 0$

$A = \infty \Rightarrow v_+ = v_-$

RC OPERATIONAL AMPLIFIER CIRCUITS -THE INTEGRATOR



$v_+ = 0$

$$\frac{v_1 - v_-}{R_1} + C_2 \frac{d}{dt}(v_o - v_-) = i_-$$

IDEAL OP-AMP ASSUMPTIONS

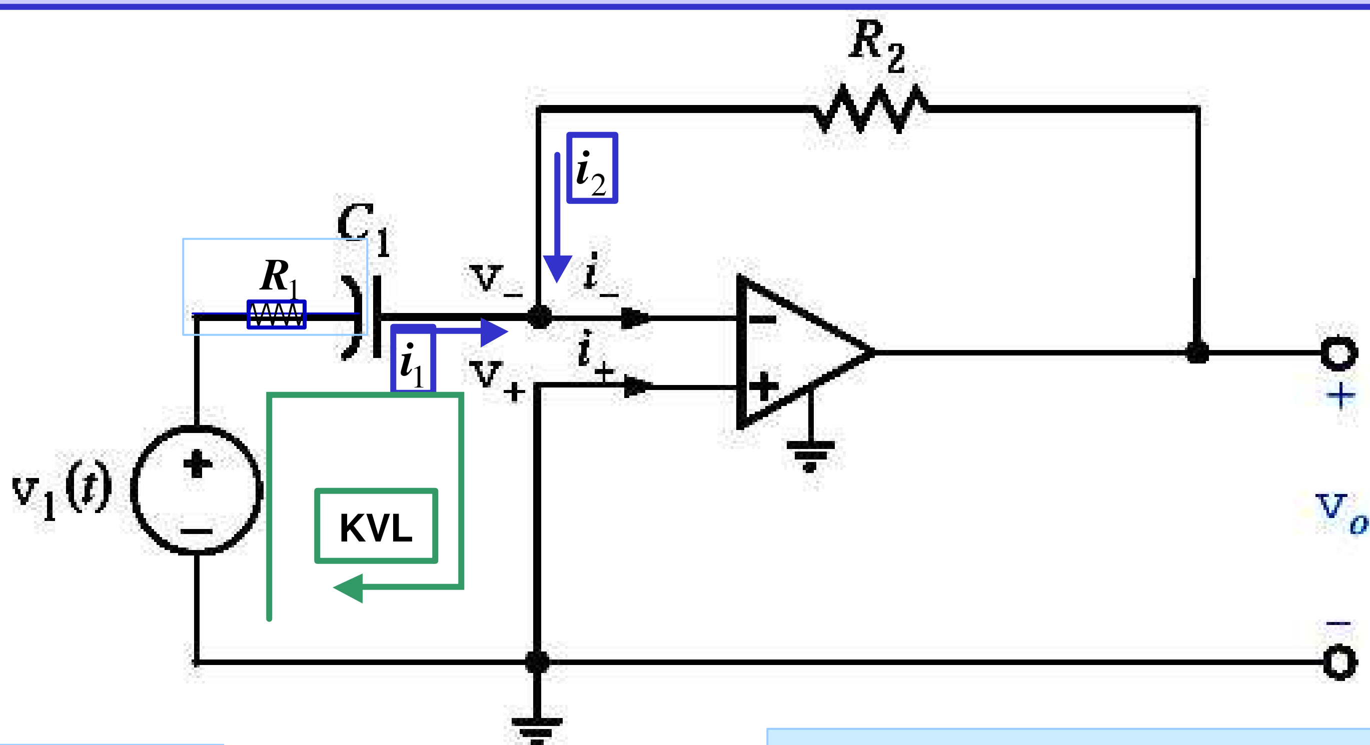
$v_- = v_+ \quad (A = \infty)$

$i_- = 0 \quad (R_i = \infty)$

$$\frac{v_1}{R_1} = -C_2 \frac{dv_o}{dt}$$

$$v_o(t) = \frac{-1}{R_1 C_2} \int_{-\infty}^{t} v_1(x)\,dx$$
$$= \frac{-1}{R_1 C_2} \int_{0}^{t} v_1(x)\,dx + v_o(0)$$

RC OPERATIONAL AMPLIFIER CIRCUITS - THE DIFFERENTIATOR



$v_+ = 0$

KCL@v_- : $i_1 + i_2 = i_-$

IDEAL OP-AMP ASSUMPTIONS

$v_- = v_+ \quad (A = \infty)$

$i_- = 0 \quad (R_i = \infty)$

$$i_1 + \frac{v_O}{R_2} = 0$$

$$v_1(t) = R_1 i_1 + \frac{1}{C_1}\int_{-\infty}^{t} i_1(x)dx$$

DIFFERENTIATE

$$R_1 C_1 \frac{di_1}{dt} + i_1 = C_1 \frac{dv_1}{dt}(t)$$

replace i_1 in terms of v_o $(i_1 = -\frac{v_o}{R_2})$

$$R_1 C_1 \frac{dv_o}{dt} + v_o = -R_2 C_1 \frac{dv_1}{dt}(t)$$

**IF R1 COULD BE SET TO ZERO WE WOULD HAVE AN <u>IDEAL</u> DIFFERENTIATOR.
IN PRACTICE AN IDEAL DIFFERENTIATOR AMPLIFIES ELECTRIC NOISE AND DOES NOT OPERATE.
THE RESISTOR INTRODUCES A FILTERING ACTION. ITS VALUE IS KEPT AS SMALL AS POSSIBLE TO APPROXIMATE A DIFFERENTIATOR**