Seminars on Process Systems Engineering
Aalto University, September 25, 2025

# Predictive control and feedback equilibrium seeking for sustainable water resource recovery

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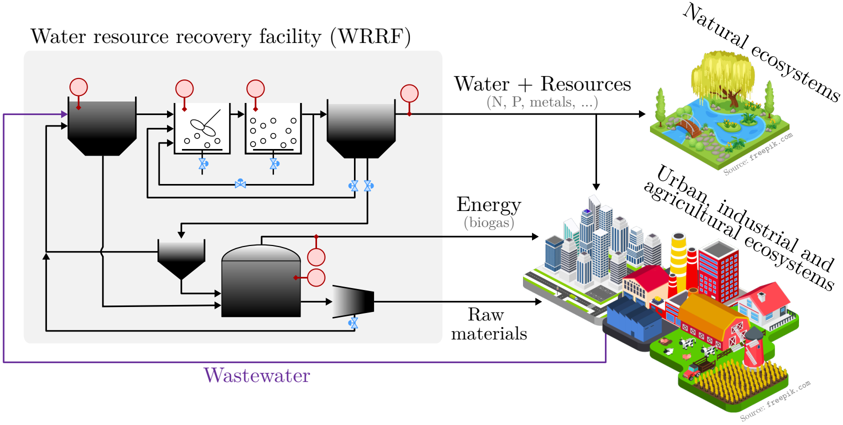
# Predictive control, operating WWTPs as self-sufficient WRRFs

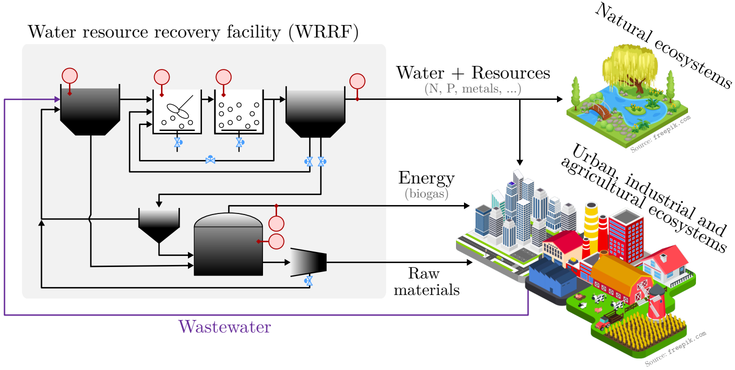
**Paradigm shift:** Wastewater as a sustainable source of water, energy, and raw materials



# (Proposal A)
# Predictive control for transitioning WWTPs into self-sufficient WRRFs

# Predictive control, operating WWTPs as self-sufficient WRRFs

We consider the task of operating a **biological wastewater treatment plant** (WWTP, secondary treatment) as a **water resource recovery facility** (WRRF)



# Predictive control, operating WWTPs as self-sufficient WRRFs

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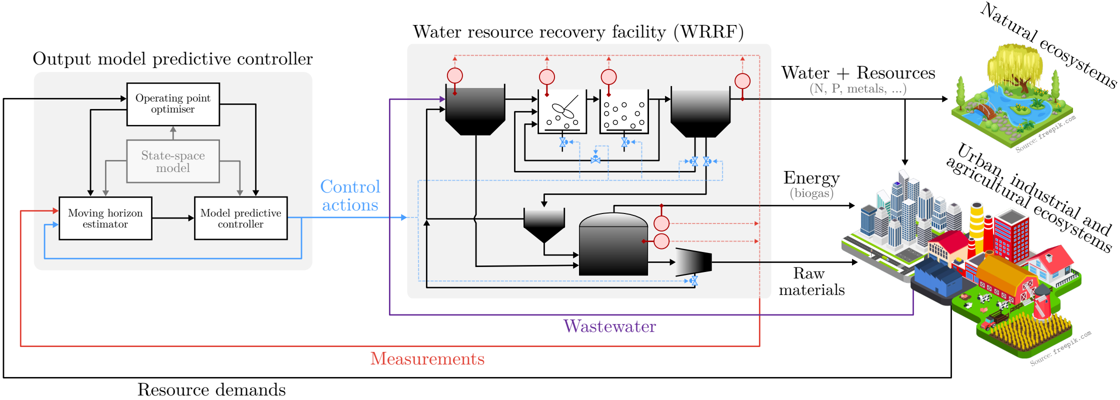
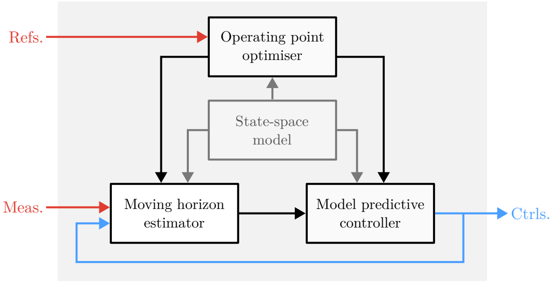


# Predictive control, general architecture and specific configuration

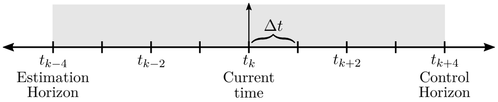
Output model predictive controller




We design a **model-based** output-feedback controller

$$\Sigma := \begin{cases} \frac{d}{dt}x(t) = f(x(t), w(t), u(t)) & \text{(dynamics)} \\ y(t) = g(x(t), w(t), u(t)) & \text{(measurements)} \\ z(t) = h(x(t), w(t), u(t)) & \text{(performance)} \end{cases}$$

which autonomously operates the plant in cycles



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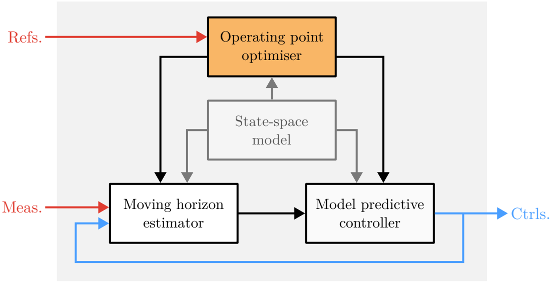


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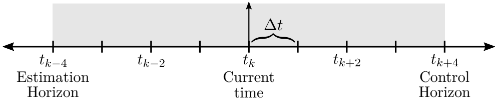


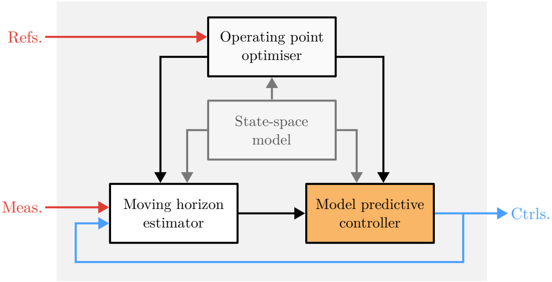

Operating point optimizer | OPO(·)

$$\begin{aligned} \underset{x_k,\ u_k}{\text{minimize}} \quad & \left\| \begin{bmatrix} W_{z|\text{ref}} & \\ & W_{u|\text{ref}} \end{bmatrix} \begin{bmatrix} z_k - \bar{z}_k^{\text{ref}} \\ u_k - \bar{u}_k^{\text{ref}} \end{bmatrix} \right\|_2^2 \\ \text{subject to} \quad & 0 = f(x_k, \bar{w}_k^{\text{ref}}, u_k) \\ & z_k \in \mathcal{Z}_{\text{ref}}, \quad x_k \in \mathcal{X}_{\text{ref}}, \quad u_k \in \mathcal{U}_{\text{ref}} \end{aligned}$$

$$\Sigma^{\delta|k} := \begin{cases} zx^{\delta}[n] = A_k x^{\delta}[n] + B_{w|k} w^{\delta}[n] + B_{u|k} u^{\delta}[n] \\ y^{\delta}[n] = C_k x^{\delta}[n] + D_{w|k} w^{\delta}[n] \end{cases}$$

# Predictive control, general architecture and specific configuration

Output model predictive controller

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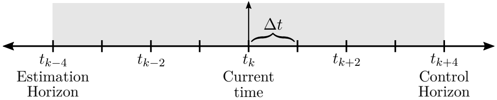



## Model predictive controller | MPC($\cdot$)

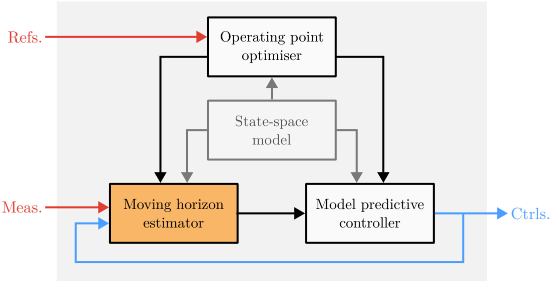
$$\underset{u^\delta[\cdot],\ x^\delta[\cdot]}{\text{minimize}} \quad \sum_{n=0}^{N_c-1} \left\| \begin{bmatrix} W_{x|n} & \\ & W_{u|n} \end{bmatrix} \begin{bmatrix} x^\delta[n] \\ u^\delta[n] \end{bmatrix} \right\|_2^2 + \left\| W_{x|N_c} x^\delta[N_c] \right\|_2^2$$

$$\text{subject to} \quad \Sigma^{\delta|k} \text{ with } w^\delta[n] = \hat{w}^\delta_{N_e-1} \text{ and } x^\delta[0] = \hat{x}^\delta_{N_e}$$

$$x^\delta_{\mathrm{lb}} \preceq x^\delta[n] \preceq x^\delta_{\mathrm{ub}}, \quad u^\delta_{\mathrm{lb}} \preceq u^\delta[n] \preceq u^\delta_{\mathrm{ub}}$$

# Predictive control, general architecture and specific configuration

## Output model predictive controller

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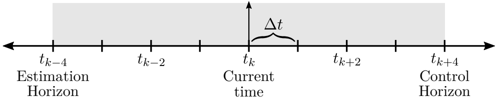



### Moving horizon estimator | MHE$(\cdot)$

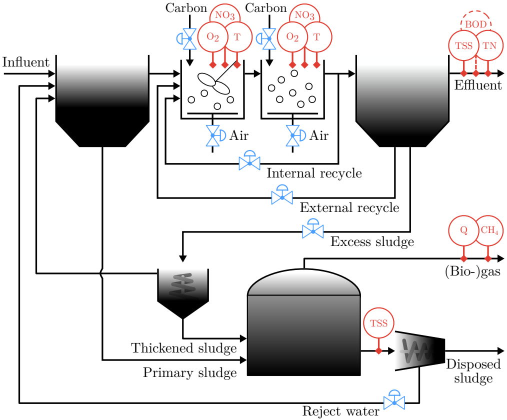
$$\underset{w^\delta[\cdot],\ x^\delta[\cdot]}{\text{minimize}} \quad \sum_{n=0}^{N_e-1} \left\| \begin{bmatrix} W_{y|n} & \\ & W_{w|n} \end{bmatrix} \begin{bmatrix} y^\delta[n] - y_n^{\mathrm{data}|\delta} \\ w^\delta[n] - w_n^{\mathrm{data}|\delta} \end{bmatrix} \right\|_2^2 + \left\| W_{y|N_e} \left( y^\delta[N_e] - y_{N_e}^{\mathrm{data}|\delta} \right) \right\|_2^2$$

$$\text{subject to} \quad \Sigma^{\delta|k} \text{ with } u^\delta[n] = u_n^{\mathrm{data}|\delta}$$

$$x_{\mathrm{lb}}^\delta \preceq x^\delta[n] \preceq x_{\mathrm{ub}}^\delta, \quad w_{\mathrm{lb}}^\delta \preceq w^\delta[n] \preceq w_{\mathrm{ub}}^\delta.$$

# Predictive control, experimental study

We consider the task of operating a **biological wastewater treatment plant** (WWTP, secondary treatment) as a **water resource recovery facility** (WRRF)



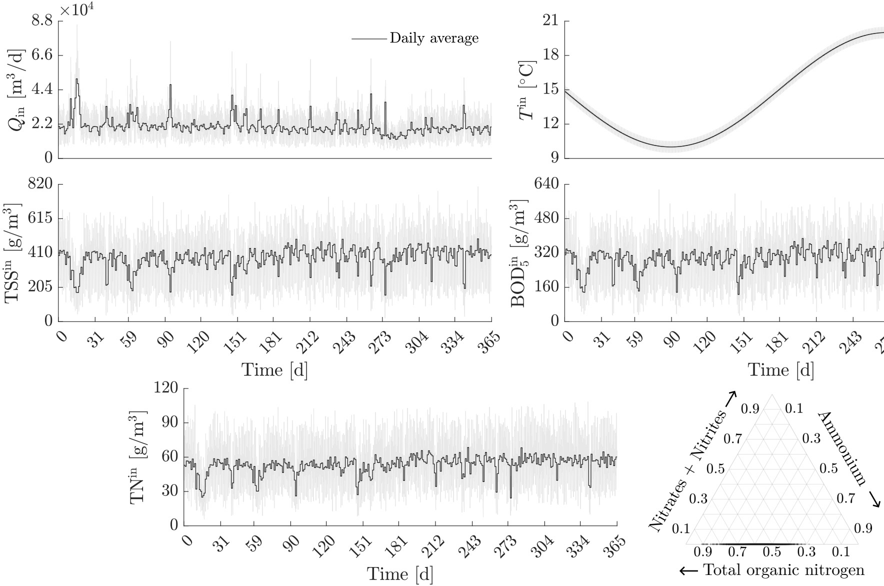

- **Primary objective:**
  on demand, produce effluent water of specific quality

| Water class | Biochemical profile | | |
|---|---|---|---|
| | TSS | BOD | TN |
| **A** | $\leq 30$ g/m$^3$ | $\leq 10$ g/m$^3$ | $\leq 15$ g/m$^3$ |
| **B** | $\leq 30$ g/m$^3$ | $\leq 15$ g/m$^3$ | $\leq 30$ g/m$^3$ |
| **C** | $\leq 30$ g/m$^3$ | $\leq 20$ g/m$^3$ | $\leq 45$ g/m$^3$ |

- **Secondary objective:**
  produce biogas to ensure nonpositive energy cost index,

$$\mathrm{ECI} = \mathrm{AE} + \mathrm{PE} + \mathrm{ME} - \eta_E \mathrm{MP} + \max(0, \mathrm{HE} - \eta_H \mathrm{MP})$$

# Predictive control, experimental study (cont.)



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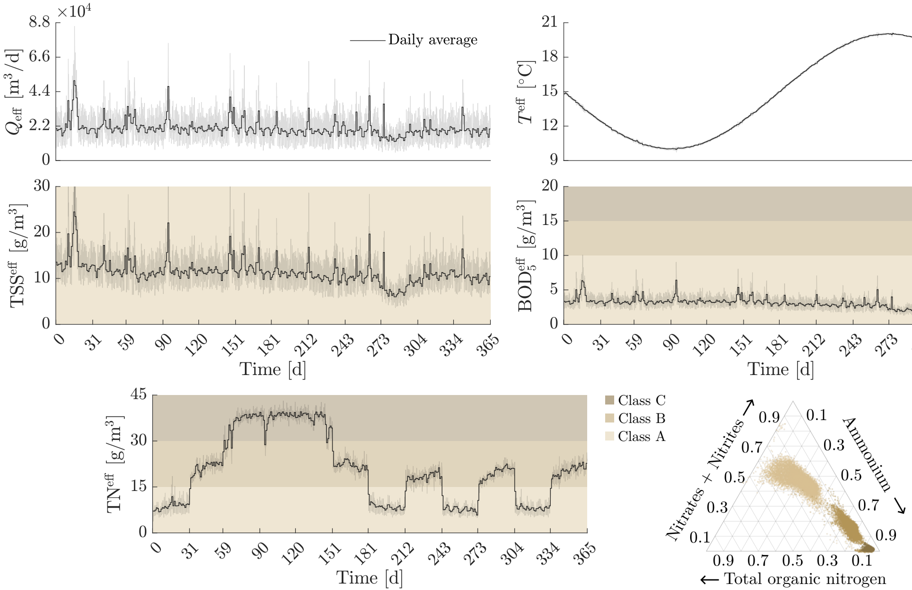

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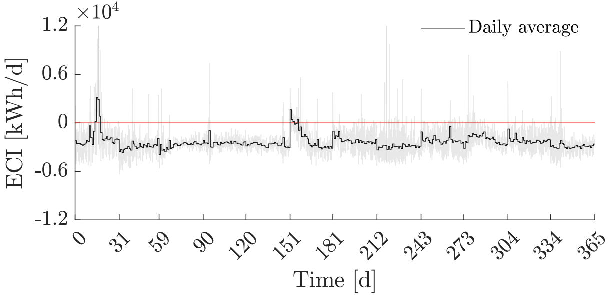



**Summary**

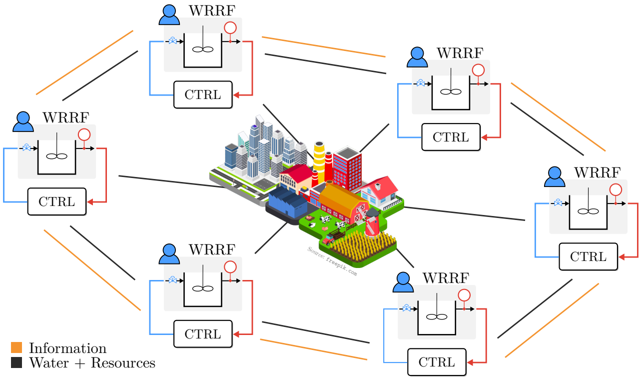
Total production: 83 GWh

Energy output: $2385 \pm 1312$ kWh

The controller renders the WRRF energetically self-sufficient (on average)

# (Proposal B)
# Equilibrium-seeking for (noncooperative) multi-agent WRRF networks

# Equilibrium-seeking, control in (noncooperative) WRRF networks




**Goal:**

⇝ Optimal control in multi-agent settings

**Challenges:**

⇝ Large-scale network of subsystems

⇝ Noncooperative decision-making agents

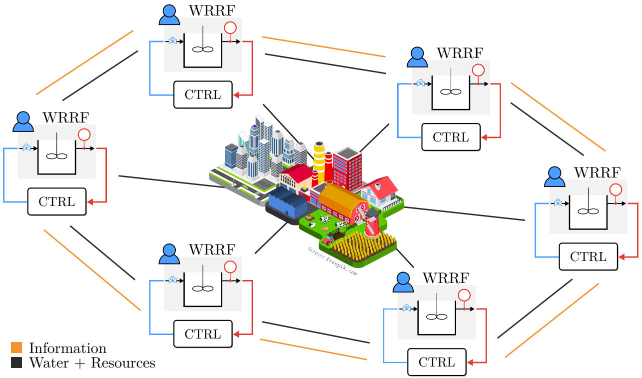
⇝ Subsystems are only partially observed

⇝ Information exchange often asymmetric

Feedback controllers (**control theory**)
+
Equilibrium-seeking (**game theory**)

We propose a systematic approach to **seeking feedback equilibrium strategies** in $N_P$-players noncooperative dynamic games

# Equilibrium-seeking, control in (noncooperative) WRRF networks

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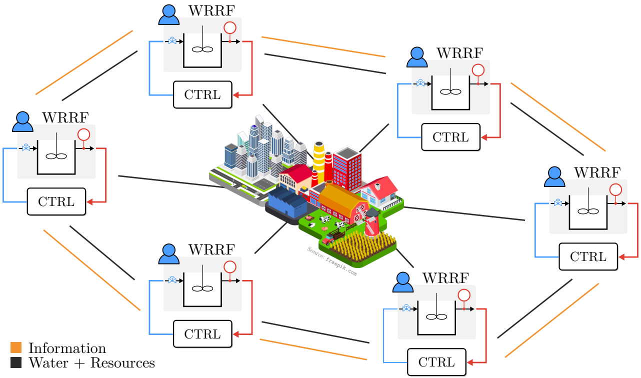


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# Equilibrium-seeking, generalized feedback Nash equilibrium problems

**Generalized feedback Nash equilibrium problem**

Find $K^\star = ({K^p}^\star)_{p=1}^{N_P}$ such that $K^\star \in \mathbf{fix}(\mathrm{BR}^1 \times \cdots \times \mathrm{BR}^{N_P})$

– The *best-response map* $\mathrm{BR}^p(K^{-p})$ is the solution set of –

$$
\begin{aligned}
\underset{K^p}{\text{minimize}} \quad & \mathrm{E} \sum_{n=0}^{\infty} \left\| \begin{bmatrix} W_x^p & \\ & W_u^p \end{bmatrix} \begin{bmatrix} x[n] \\ u^p[n] \end{bmatrix} \right\|_2^2 \\
\text{subject to} \quad & x[n+1] = Ax[n] + B_w w[n] + \textstyle\sum_p B_u^p u[n] \\
& u^p[n] = K^p(Cx[n] + D_w w[n]) \\
& \begin{bmatrix} G_x & G_u \\ & e_p^\top \otimes G_{u^p} \end{bmatrix} \begin{bmatrix} x[n] \\ u[n] \end{bmatrix} \preceq 1, \quad K^p[n] \in \mathcal{S}^p[n]
\end{aligned}
$$

**Equilibrium-seeking methods**

(How to solve a GFNE problem?)

– At each $k \in \mathbb{N}$, policies are updated –

$$
\begin{aligned}
K_{k+1/2}^p &:= T^p(K_k^p,\ K_k^{-p}) \\
K_{k+1}^p &:= R(K_{k+1/2})
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The policy update operator

$R \circ (T^1 \times \cdots \times T^{N_P})$

depends on BR and its properties

- ✖ $\mathrm{BR}^p$ cannot be solved numerically (in general)
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- ✖ $\mathcal{S}^p$ nonconvex for many relevant problems

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# Equilibrium-seeking, generalized feedback Nash equilibrium problems

**System-level generalized feedback Nash equilibrium problem**

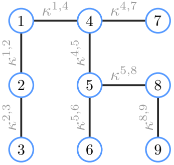
$$\text{Find } K^\star = \left(\Phi_{uy}^{p^\star} - \Phi_{ux}^{p^\star}(\Phi_{xx}^\star)^{-1}\Phi_{xy}^\star\right)_{p=1}^{N_P} \text{ such that } (\Phi_{ux}^\star, \Phi_{uy}^\star) \in \mathbf{fix}\left(\mathrm{BR}_\Phi^1 \times \cdots \times \mathrm{BR}_\Phi^{N_P}\right)$$

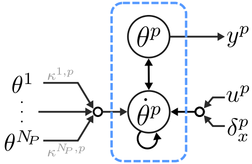
– The *system-level best-response maps* $\mathrm{BR}_\Phi^p(\Phi_{ux}^{-p}, \Phi_{uy}^{-p})$ is the solution set of the problem –

$$\begin{aligned}
\underset{\Phi_{ux}^p, \Phi_{uy}^p}{\text{minimize}} \quad & \sum_{n=0}^{\infty} \left\| \begin{bmatrix} W_x^p & \\ & W_u^p \end{bmatrix} \begin{bmatrix} \Phi_{xx}[n] & \Phi_{xy}[n] \\ \Phi_{ux}^p[n] & \Phi_{uy}^p[n] \end{bmatrix} \begin{bmatrix} B_w \\ D_w \end{bmatrix} \right\|_F^2 \\
\text{subject to} \quad & \begin{bmatrix} zI - A & -B_u^1 & \cdots & -B_u^{N_P} \end{bmatrix} \begin{bmatrix} \Phi_{xx} & \Phi_{xy} \\ \Phi_{ux} & \Phi_{uy} \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix}, \quad \begin{bmatrix} \Phi_{xx} & \Phi_{xy} \\ \Phi_{ux} & \Phi_{uy} \end{bmatrix} \begin{bmatrix} zI - A \\ -C \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix}^\top \\
& \left\| \left( \begin{bmatrix} G_x & G_u \\ & e_p^\top \otimes G_{u^p} \end{bmatrix}_{i,:} \begin{bmatrix} \Phi_{xx} & \Phi_{xy} \\ \Phi_{ux} & \Phi_{uy} \end{bmatrix} \begin{bmatrix} B_w \\ D_w \end{bmatrix} \right)^\top \right\|_{\ell_2} \leq \frac{1}{Q(\eta)}, \quad \begin{bmatrix} \Phi_{xx}[n] & \Phi_{xy}[n] \\ \Phi_{ux}^p[n] & \Phi_{uy}^p[n] \end{bmatrix} \in \mathcal{S}_\Phi^p[n]
\end{aligned}$$

- ☑ $\mathrm{BR}_\Phi^p$ amenable to numerical solution (w/ FIR assumption)
- ☑ existence (and uniqueness) of GFNE easy to establish
- ☑ $\mathcal{S}_\Phi^p$ convex for many relevant problems

# Example, power-grid with capacity constraints



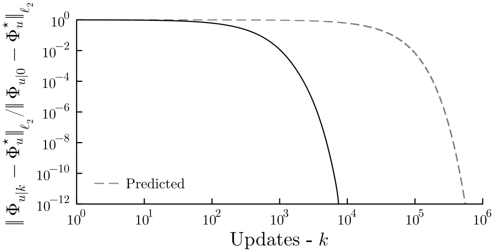




Continuous-time dynamics (each node):

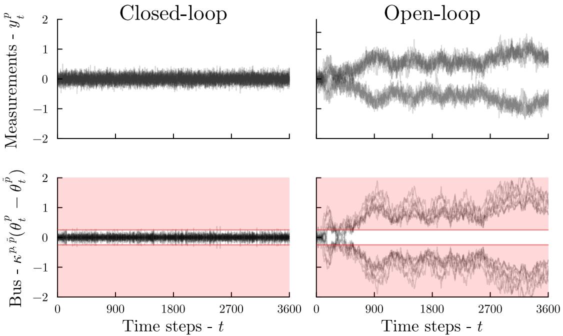
$$m^p \ddot{\theta}^p + d^p \dot{\theta}^p = -\sum_{\tilde{p}\in\mathcal{P}} \kappa^{p,\tilde{p}}(\theta^p - \theta^{\tilde{p}}) + u^p + \delta_x^p$$

Link capacity constraint (each node):

$$-5 \le \kappa^{p,\tilde{p}}(\theta^p - \theta^{\tilde{p}}) \le 5, \quad \forall t \in \mathbb{N},\ \tilde{p} \in \mathcal{P}$$




Convergence to GFNE (albeit slow) while ensuring constraint satisfaction



[tiominho.github.io]

# Thank you!

**Questions?**