Multiplication Strategies: Doubling

Theodore M. Savich

March 9, 2025

Doubling

Description of Strategy:

- **Objective:** Use doubling to quickly reach the total number of items by doubling group sizes or totals.
- **Method:** Double the number of items (and the number of groups) repeatedly until reaching or surpassing the target total, then adjust as needed.

Automaton Type:

Finite State Automaton with Registers (Counters): Counters are used to track the current total and the number of groups.

Formal Description of the Automaton

We define the automaton as the tuple

$$M = (Q, \Sigma, \delta, q_{0/accept}, F, V)$$

where:

- $Q = \{q_{0/accept}, q_{\text{double}}, q_{\text{check}}, q_{\text{adjust}}\}$ is the set of states. Here, $q_{0/accept}$ serves as both the start and accept state.
- Σ is the input alphabet (used to initialize the problem parameters).
- $F = \{q_{0/accept}\}$ is the set of accepting states.
- $V = \{\text{CurrentTotal (CT), CurrentGroups (CG), GroupSize (S), TotalGroups (N)}\}$ is the set of registers.

The key transitions are as follows:

1. **Initialization:** From $q_{0/accept}$, on reading the input values (with S and N), initialize $CT \leftarrow S$ and $CG \leftarrow 1$, then transition to q_{double}.
2. **Doubling:** In q_{double}, repeatedly double both CT and CG (i.e., update $CT \leftarrow 2 \times CT$ and $CG \leftarrow 2 \times CG$) until $CG \geq N$.
3. **Checking:** In q_{check}, if $CG = N$ then the target total is reached, and the automaton transitions to the accept state. If $CG > N$, transition to q_{adjust} to fine-tune CT.
4. **Adjustment:** In q_{adjust}, adjust CT appropriately (e.g., subtract the excess) before outputting the final total.

Automaton Diagram for Doubling