Addition Strategies: Chunking by Bases and Ones

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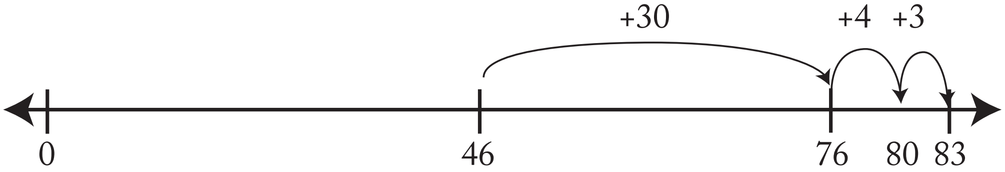
Transcript

Strategy descriptions and examples adapted from Hackenberg (2025). Problem: Max has 46 comic books. For his birthday, his father gives him 37 more comic books. How many comic books does Max have now?

Dionne's solution: "He has 46. Then 37 more. [She writes down 46, 76.] That's the 30. And then 7 more. Well, 4 more makes 80, and then I only need to do 3 more, 83."



Notation Representing Sarah's Solution:

$$\begin{aligned} 46 + 37 &= \square \\ 46 + 30 &= 76 \\ 76 + 4 &= 80 \\ 80 + 3 &= 83 \end{aligned}$$

Description of Strategy:

Objective: Begin with one number. Then, break the other number down into bases and units. In COBO, you count on each base individually - then the ones. With Chunking, instead of adding each base individually, add them in well-chosen, larger groups. Likewise, combine the units in groups rather than one by one—though there are instances when adding a single base or unit makes strategic sense. The overall goal is to create larger, intentional groupings, and it's important to clarify why each grouping is considered strategic. Usually, the goal with chunking on ones is to make a base first, then you can chunk on the rest of the ones. Usually when chunking on the bases, the goal is to make a base-of-bases first (so, in base ten, the goal would be to try and make one hundred), because then you can chunk on the rest of the bases (and ones) all at once.

Description of Strategy

- **Objective:** Similar to COBO but add bases and ones in larger, strategic chunks.
- **Example:** $46 + 37$
  - Start at 46.

  - Add all tens at once: $46 + 30 = 76$.
  - Add ones strategically: $76 + 4 = 80$, then $80 + 3 = 83$.

Automaton Type

Finite State Automaton (FSA) with basic arithmetic capability.

Formal Description of the Automaton

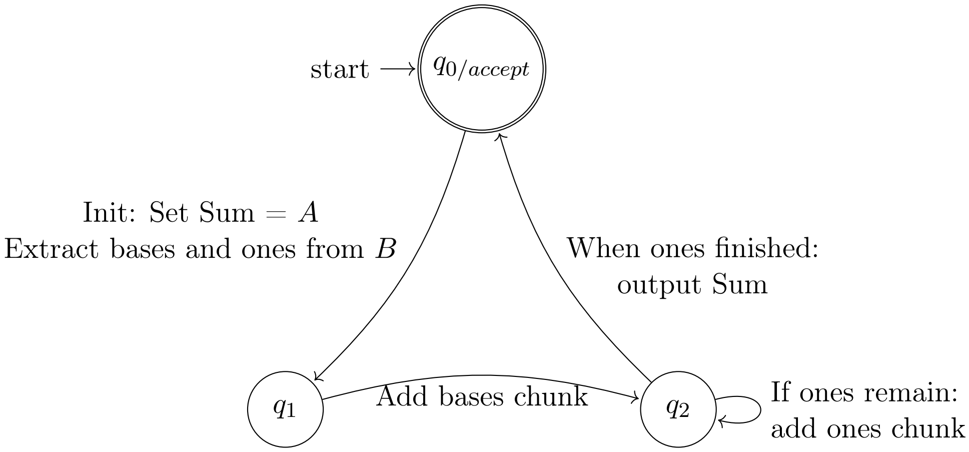
We define the automaton as the tuple

$$M = (Q,\, \Sigma,\, \delta,\, q_{0/accept},\, F)$$

where:

- $Q = \{q_{0/accept},\, q_1,\, q_2\}$ is the set of states.
- $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +\}$ is the input alphabet.
- $q_{0/accept}$ is the start state, which is also the accept state.
- $F = \{q_{0/accept}\}$ is the set of accepting states.
- The transition function δ is defined as:
  1. $\delta(q_{0/accept},\, \text{“}A, B\text{”}) = q_1$ with the action: set Sum $\leftarrow A$ and extract the base and ones chunks from B.
  2. $\delta(q_1,\, \varepsilon) = q_2$ with the action: update Sum $\leftarrow$ Sum+ (the bases chunk from B).
  3. $\delta(q_2,\, \varepsilon) = q_2$ with the action: if ones remain, add a strategic ones chunk to Sum (loop as needed).
  4. $\delta(q_2,\, \varepsilon) = q_{0/accept}$ with the action: when ones are finished, output Sum.

Automaton Diagram for Chunking by Bases and Ones



HTML Implementation

```html
<!DOCTYPE html>
<html>
<head>
    <title>Addition Strategies: Chunking by Bases and Ones</title>
    <style>
        body { font-family: sans-serif; }
        #diagramChunkingSVG { border: 1px solid #d3d3d3; }
        #outputContainer { margin-top: 20px; }
        .number-line-tick { stroke: black; stroke-width: 1; }
        .number-line-break { stroke: black; stroke-width: 1; stroke-dasharray: 5 5;} /* For scale break */
        .number-line-label { font-size: 12px; text-anchor: middle; } /* Centered labels */
        .jump-arrow { fill: none; stroke: green; stroke-width: 2; } /* Changed color */
        .jump-arrow-head { fill: green; stroke: green; } /* Changed color */
        .jump-label { font-size: 12px; text-anchor: middle; fill: green; } /* Changed color */
        .stopping-point { fill: red; stroke: black; stroke-width: 1; }
         /* Number line arrowhead */
         .number-line-arrow { fill: black; stroke: black;}
    </style>
</head>
<body>

<h1>Addition Strategies: Chunking by Bases and Then Ones</h1>

<div>
    <label for="chunkingAddend1">Addend 1:</label>
    <input type="number" id="chunkingAddend1" value="46">
</div>
<div>
    <label for="chunkingAddend2">Addend 2:</label>
    <input type="number" id="chunkingAddend2" value="37">
</div>

<button onclick="runChunkingAutomaton()">Calculate and Visualize</button>

<div id="outputContainer">
    <h2>Explanation:</h2>
    <div id="chunkingOutput">
        <!-- Text output will be displayed here -->
    </div>
</div>

<h2>Diagram:</h2>
<svg id="diagramChunkingSVG" width="700" height="350"></svg>

<script>
document.addEventListener('DOMContentLoaded', function() {
    const outputElement = document.getElementById('chunkingOutput');
    const chunkingAddend1Input = document.getElementById('chunkingAddend1');
    const chunkingAddend2Input = document.getElementById('chunkingAddend2');
    const diagramChunkingSVG = document.getElementById('diagramChunkingSVG');

    if (!outputElement || !diagramChunkingSVG) {
        console.warn('Element␣chunkingOutput␣or␣diagramChunkingSVG␣not␣found');
        return;
    }

    window.runChunkingAutomaton = function() {
        try {
            const addend1 = parseInt(chunkingAddend1Input.value);
            const addend2 = parseInt(chunkingAddend2Input.value);
            if (isNaN(addend1) || isNaN(addend2)) {
                outputElement.textContent = 'Please␣enter␣valid␣numbers␣for␣both␣addends';
                return;
            }

            let output = `<h2>Chunking by Bases and Ones (Flexible)</h2>\n\n`;
            output += `<p><strong>Problem:</strong> ${addend1} + ${addend2}</p>\n\n`;

            let tensToAddTotal = Math.floor(addend2 / 10) * 10;
            let onesToAddTotal = addend2 % 10;

            output += `Step 1: Split ${addend2} into ${tensToAddTotal} (tens) + ${onesToAddTotal} (ones)\n\n`;

            let currentSum = addend1;
            const chunkSteps = [];
            let stepCounter = 2;

            // --- Strategy Decision: Add Ones First or Tens First? ---
            const addOnesFirstDecision = Math.random() < 0.3; // 30% chance to add ones first (if possible)
            let onesAddedFirst = false;

            if (addOnesFirstDecision && onesToAddTotal > 0) {
                // Try adding ones first to make the next ten
                const onesToNextTenInitial = (10 - (currentSum % 10)) % 10;
                if (onesToNextTenInitial > 0 && onesToAddTotal >= onesToNextTenInitial) {
                     output += `Step ${stepCounter}: Add ones chunk first to make a ten\n`;
                     stepCounter++;
                     chunkSteps.push({
                        from: currentSum,
                        to: currentSum + onesToNextTenInitial,
                        label: `+${onesToNextTenInitial}`
                    });
                    output += `<p>${currentSum} + ${onesToNextTenInitial} = ${currentSum + onesToNextTenInitial} (Making the next ten)</p>\n`;
                    currentSum += onesToNextTenInitial;
                    onesToAddTotal -= onesToNextTenInitial;
                    onesAddedFirst = true; // Flag that we adjusted ones already
                    output += '\n';
                }
            }

            // --- Tens Chunking (Potentially after adding some ones) ---
            if (tensToAddTotal > 0) {
                output += ‘Step ${stepCounter}: Add tens chunk(s)\n‘;
                stepCounter++;

                while (tensToAddTotal > 0) {
                    // Calculate tens needed to reach the *next* hundred
                    let amountToNextHundred = (currentSum % 100 === 0) ? 0 : 100 - (currentSum % 100);
                    let tensToNextHundred = Math.floor(amountToNextHundred / 10) * 10;

                    let tensChunk = 0;

                    if (tensToNextHundred > 0 && tensToAddTotal >= tensToNextHundred) {
                        // Option 1: Chunk exactly to the next hundred
                        tensChunk = tensToNextHundred;
                         output += ‘<p>${currentSum} + ${tensChunk} = ${currentSum + tensChunk} (Making the next hundred)</p>\n‘;
                    } else {
                        // Option 2: Add remaining tens, or a smaller "honest" chunk if large amount remains
                        if (tensToAddTotal <= 30 || Math.random() < 0.6) { // More likely to add all if 30 or less, or 60% chance otherwise
                           tensChunk = tensToAddTotal; // Add all remaining tens
                            output += ‘<p>${currentSum} + ${tensChunk} = ${currentSum + tensChunk}</p>\n‘;
                        } else {
                            // Add a smaller "honest" chunk (e.g., 10, 20, or 30) - more random choices possible here
                             tensChunk = (Math.floor(Math.random() * 3) + 1) * 10; // Randomly 10, 20, or 30
                             tensChunk = Math.min(tensChunk, tensToAddTotal); // Don’t add more than available
                             output += ‘<p>${currentSum} + ${tensChunk} = ${currentSum + tensChunk}</p>\n‘;
                        }
                    }

                    if (tensChunk > 0) {
                         chunkSteps.push({
                            from: currentSum,
                            to: currentSum + tensChunk,
                            label: ‘+${tensChunk}‘
                        });
                        currentSum += tensChunk;
                        tensToAddTotal -= tensChunk;
                    } else {
                         // Safety break if something went wrong
                         break;
                    }
                }
                output += ’\n’;
            }

            // --- Remaining Ones Chunking (If not added first or some left over) ---
```

```
        if (onesToAddTotal > 0) {
             output += ‘Step ${stepCounter}: Add remaining ones chunk(s)\n‘;

            // Strategic ones (make next ten) - might happen again if tens landed awkwardly
            const onesToNextTen = (10 - (currentSum % 10)) % 10;

            if (onesToNextTen > 0 && onesToAddTotal >= onesToNextTen) {
                // Chunk 1: Reach the next ten
                chunkSteps.push({
                    from: currentSum,
                    to: currentSum + onesToNextTen,
                    label: ‘+${onesToNextTen}‘
                });
                output += ‘<p>${currentSum} + ${onesToNextTen} = ${currentSum + onesToNextTen} (Making the next ten)</p>\n‘;
                currentSum += onesToNextTen;
                onesToAddTotal -= onesToNextTen;

                // Chunk 2: Add the rest
                if (onesToAddTotal > 0) {
                     chunkSteps.push({
                        from: currentSum,
                        to: currentSum + onesToAddTotal,
                        label: ‘+${onesToAddTotal}‘
                    });
                    output += ‘<p>${currentSum} + ${onesToAddTotal} = ${currentSum + onesToAddTotal}</p>\n‘;
                    currentSum += onesToAddTotal;
                    onesToAddTotal = 0;
                }
            } else if (onesToAddTotal > 0) {
                // Add all remaining ones
                chunkSteps.push({
                    from: currentSum,
                    to: currentSum + onesToAddTotal,
                    label: ‘+${onesToAddTotal}‘
                });
                output += ‘<p>${currentSum} + ${onesToAddTotal} = ${currentSum + onesToAddTotal}</p>\n‘;
                currentSum += onesToAddTotal;
                onesToAddTotal = 0;
            }
             output += ’\n’;
        }


        output += ‘Result: ${addend1} + ${addend2} = ${currentSum}‘;
        outputElement.innerHTML = output;
        typesetMath();

        drawChunkingNumberLineDiagram(’diagramChunkingSVG’, addend1, addend2, chunkSteps, currentSum);

        } catch (error) {
            outputElement.textContent = ‘Error: ${error.message}‘;
        }
    };

    // drawChunkingNumberLineDiagram function remains the same
    // ... (Keep the FULL drawChunkingNumberLineDiagram function and its helpers from previous responses) ...
     function drawChunkingNumberLineDiagram(svgId, addend1, addend2, chunkSteps, finalSum) {
        const svg = document.getElementById(svgId);
        if (!svg) return;
        svg.innerHTML = ’’;

        const svgWidth = parseFloat(svg.getAttribute(’width’));
        const svgHeight = parseFloat(svg.getAttribute(’height’));
        const startX = 50;
        const endX = svgWidth - 50;
        const numberLineY = svgHeight / 2 + 30; // Lower number line slightly
        const tickHeight = 10;
        const labelOffsetBase = 20;
        const jumpHeightLarge = 60; // Increased height for larger jumps
        const jumpHeightSmall = 40; // Height for smaller jumps (ones chunks)
        const jumpLabelOffset = 15;
        const arrowSize = 5;
        const scaleBreakThreshold = 40; // Adjust if needed

        // Draw Number Line & 0 Tick
        const numberLine = document.createElementNS(’http://www.w3.org/2000/svg’, ’line’);
        numberLine.setAttribute(’x1’, startX);
        numberLine.setAttribute(’y1’, numberLineY);
        numberLine.setAttribute(’x2’, endX);
        numberLine.setAttribute(’y2’, numberLineY);
        numberLine.setAttribute(’class’, ’number-line-tick’);
        svg.appendChild(numberLine);

        const zeroTick = document.createElementNS(’http://www.w3.org/2000/svg’, ’line’);
        zeroTick.setAttribute(’x1’, startX);
        zeroTick.setAttribute(’y1’, numberLineY - tickHeight / 2);
        zeroTick.setAttribute(’x2’, startX);
        zeroTick.setAttribute(’y2’, numberLineY + tickHeight / 2);
        zeroTick.setAttribute(’class’, ’number-line-tick’);
        svg.appendChild(zeroTick);
        createText(svg, startX, numberLineY + labelOffsetBase, ’0’, ’number-line-label’);

        // Calculate scale and handle potential break
        let displayRangeStart = 0;
        let scaleStartX = startX;
        let drawScaleBreak = false;

        // Determine the actual min and max values shown *after* the break
        let minValAfterBreak = addend1;
        let maxValAfterBreak = finalSum;
        chunkSteps.forEach(step => {
```

```
        minValAfterBreak = Math.min(minValAfterBreak, step.from, step.to);
        maxValAfterBreak = Math.max(maxValAfterBreak, step.from, step.to);
    });


    if (addend1 > scaleBreakThreshold) {
        displayRangeStart = minValAfterBreak - 10; // Start range slightly before min value shown after break
        scaleStartX = startX + 30; // Leave space for break symbol
        drawScaleBreak = true;
        drawScaleBreakSymbol(svg, scaleStartX - 15, numberLineY); // Draw break symbol
    } else {
        displayRangeStart = 0; // Start from 0 if no break
    }

    const displayRangeEnd = maxValAfterBreak + 10; // End range slightly after max value shown
    const displayRange = Math.max(displayRangeEnd - displayRangeStart, 1); // Avoid division by zero if range is 0
    const scale = (endX - scaleStartX) / displayRange;

    // Function to convert value to X coordinate based on scale
    function valueToX(value) {
         if (value < displayRangeStart && drawScaleBreak) {
             // Values before the effective start are compressed near the break symbol
             return scaleStartX - 10; // Place them just before the break starts visually
         }
          // Ensure values stay within the visible range after the break starts
         const scaledValue = scaleStartX + (value - displayRangeStart) * scale;
         return Math.min(scaledValue, endX); // Cap at endX
    }

    // Draw Ticks and Labels for relevant points
    function drawTickAndLabel(value, index) {
        const x = valueToX(value);
         if (x < scaleStartX - 5 && value !== 0) return; // Don't draw ticks in compressed area unless it's 0 or very close to break

        const tick = document.createElementNS('http://www.w3.org/2000/svg', 'line');
        tick.setAttribute('x1', x);
        tick.setAttribute('y1', numberLineY - tickHeight / 2);
        tick.setAttribute('x2', x);
        tick.setAttribute('y2', numberLineY + tickHeight / 2);
        tick.setAttribute('class', 'number-line-tick');
        svg.appendChild(tick);
        const labelOffset = labelOffsetBase * (index % 2 === 0 ? 1 : -1.5);
        createText(svg, x, numberLineY + labelOffset, value.toString(), 'number-line-label');
    }

    drawTickAndLabel(addend1, 0); // Starting addend
    let lastToValue = addend1;

```

```
        // Draw chunk jumps
        chunkSteps.forEach((step, index) => {
            const x1 = valueToX(step.from);
            const x2 = valueToX(step.to);
             // Check if both start and end points are significantly beyond the SVG width
             if(x1 >= endX - 1 && x2 >= endX - 1) return;

            // Determine jump height based on chunk size (e.g., tens vs ones)
            const isLargeChunk = Math.abs(step.to - step.from) >= 10; // Define what constitutes a "large" chunk
            const currentJumpHeight = isLargeChunk ? jumpHeightLarge : jumpHeightSmall;
            const staggerOffset = index % 2 === 0 ? 0 : currentJumpHeight * 0.5; // Stagger jump height slightly

            createJumpArrow(svg, x1, numberLineY, x2, numberLineY, currentJumpHeight + staggerOffset);
            createText(svg, (x1 + x2) / 2, numberLineY - (currentJumpHeight + staggerOffset) - jumpLabelOffset, step.label, ’jump-label’);
            drawTickAndLabel(step.to, index + 1);
            lastToValue = step.to;
        });

        // Ensure final sum tick is drawn if it wasn’t the last ’to’ value and is within range
        if (finalSum !== lastToValue && valueToX(finalSum) <= endX) {
            drawTickAndLabel(finalSum, chunkSteps.length + 1);
        }

         // Add arrowhead to the right end of the visible number line segment
        const endLineX = valueToX(displayRangeEnd); // Use the calculated end based on scaling
        const mainArrowHead = document.createElementNS(’http://www.w3.org/2000/svg’, ’path’);
        mainArrowHead.setAttribute(’d’, ‘M ${endLineX - arrowSize} ${numberLineY - arrowSize/2} L ${endLineX} ${numberLineY} L ${endLineX - arrowSize} ${numberLineY + arrowSize/2} Z‘);
        mainArrowHead.setAttribute(’class’, ’number-line-arrow’);
        svg.appendChild(mainArrowHead);

        // Start point marker
        drawStoppingPoint(svg, valueToX(addend1), numberLineY, ’Start’);


        // --- Helper SVG drawing functions --- (Keep these the same) ---
         function createText(svg, x, y, textContent, className) {
            const text = document.createElementNS(’http://www.w3.org/2000/svg’, ’text’);
            text.setAttribute(’x’, x);
            text.setAttribute(’y’, y);
            text.setAttribute(’class’, className);
            text.setAttribute(’text-anchor’, ’middle’); // Keep middle align for labels
            text.setAttribute(’font-size’, ’12px’);
            text.textContent = textContent;
            svg.appendChild(text);
        }
```

```

         function drawScaleBreakSymbol(svg, x, y) {
            const breakOffset = 4; // How far apart the lines are
            const breakHeight = 8; // How tall the zig-zag is
            const breakLine1 = document.createElementNS(’http://www.w3.org/2000/svg’, ’line’);
            breakLine1.setAttribute(’x1’, x - breakOffset);
            breakLine1.setAttribute(’y1’, y - breakHeight);
            breakLine1.setAttribute(’x2’, x + breakOffset);
            breakLine1.setAttribute(’y2’, y + breakHeight);
            breakLine1.setAttribute(’class’, ’number-line-break’);
            svg.appendChild(breakLine1);
             const breakLine2 = document.createElementNS(’http://www.w3.org/2000/svg’, ’line’);
            breakLine2.setAttribute(’x1’, x + breakOffset); // Swapped x1/x2
            breakLine2.setAttribute(’y1’, y - breakHeight);
            breakLine2.setAttribute(’x2’, x - breakOffset); // Swapped x1/x2
            breakLine2.setAttribute(’y2’, y + breakHeight);
            breakLine2.setAttribute(’class’, ’number-line-break’);
            svg.appendChild(breakLine2);
        }

        function createJumpArrow(svg, x1, y1, x2, y2, jumpArcHeight) {
            const path = document.createElementNS(’http://www.w3.org/2000/svg’, ’path’);
            const cx = (x1 + x2) / 2;
            const cy = y1 - jumpArcHeight; // Arc is above the line
            path.setAttribute(’d’, ‘M ${x1} ${y1} Q ${cx} ${cy} ${x2} ${y1}‘);
            path.setAttribute(’class’, ’jump-arrow’);
            svg.appendChild(path);

            // Arrowhead
            const jumpArrowHead = document.createElementNS(’http://www.w3.org/2000/svg’, ’path’);
            const dx = x2 - cx; // Approx direction vector
            const dy = y1 - cy;
            const angleRad = Math.atan2(dy, dx);
            const angleDeg = angleRad * (180 / Math.PI);
            jumpArrowHead.setAttribute(’class’, ’jump-arrow-head’);
            jumpArrowHead.setAttribute(’d’, ‘M 0 0 L ${arrowSize} ${arrowSize/2} L ${arrowSize} ${-arrowSize/2} Z‘);
            jumpArrowHead.setAttribute(’transform’, ‘translate(${x2}, ${y1}) rotate(${angleDeg + 180})‘);
            svg.appendChild(jumpArrowHead);
        }

        function drawStoppingPoint(svg, x, y, labelText, labelOffsetBase = 20, index = 0) {
            const circle = document.createElementNS(’http://www.w3.org/2000/svg’, ’circle’);
            circle.setAttribute(’cx’, x);
            circle.setAttribute(’cy’, y);
            circle.setAttribute(’r’, 4);
            circle.setAttribute(’class’, ’stopping-point’);
            svg.appendChild(circle);

            // Use the provided y parameter instead of numberLineY
            if (labelText) {
                // Add staggering based on index to prevent overlap with large values
                const labelOffset = labelOffsetBase * (index % 2 === 0 ? 1.5 : -1.8);
                createText(svg, x, y + labelOffset, labelText, 'number-line-label');
            }
        }
    }

    function typesetMath() {
        // Placeholder
    }

});
</script>

</body>
<!-- New button for viewing PDF documentation -->
<button onclick="openPdfViewer()">Want to learn more about this strategy? Click here.</button>

<script>
    function openPdfViewer() {
        // Opens the PDF documentation for the strategy.
        window.open('../SAR_ADD_CHUNKING.pdf', '_blank');
    }
</script>
</html>
```

References

Hackenberg, A. (2025). *Course notes* [Unpublished course notes].