Subtraction Strategies: Sliding to Make Bases

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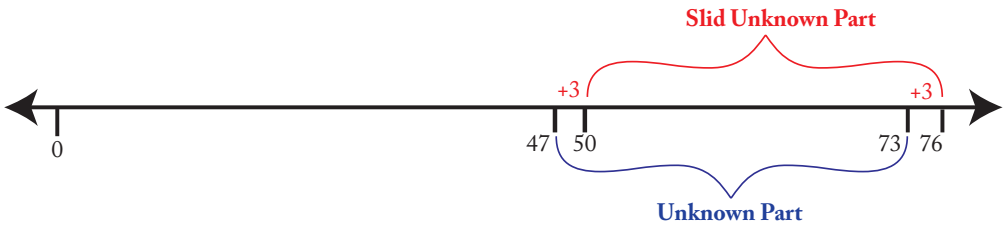
Transcript

Strategy descriptions and examples adapted from Hackenberg (2025). This is not based on a CGI video. I fake a student example.

- Teacher: John had 73 pieces of halloween candy. He gave 47 pieces to his friend. How many pieces of candy does John have left?
- Student: I can pretend I gave away 50 pieces and also pretend I had three more than I did. So that's like 76-50, which is 26.

Notation Representing Rita's Solution:

$$\begin{aligned} 73 - 47 &= \square \\ 73 + 3 &= 76 \\ 47 + 3 &= 50 \\ 73 - 47 &= 76 - 50 \\ &= 26 \end{aligned}$$



In the sliding strategy, you adjust both the number you're subtracting from (the whole) and the number being subtracted (the part) by the same amount. The goal is to shift the subtrahend into a 'friendly' number (usually a multiple of a base). By doing this, the difference between the adjusted values remains identical to the original difference, simplifying the subtraction process.

Description of Strategy

- **Objective:** Adjust both the minuend (known whole) and subtrahend (known part) by the same amount to make the subtraction easier, keeping the difference the same.

Automaton Type

Finite State Automaton (FSA): Adjustments are made consistently and can be tracked without additional memory.

Formal Description of the Automaton

We define the automaton as the tuple

$$M = (Q, \Sigma, \delta, q_{0/accept}, F)$$

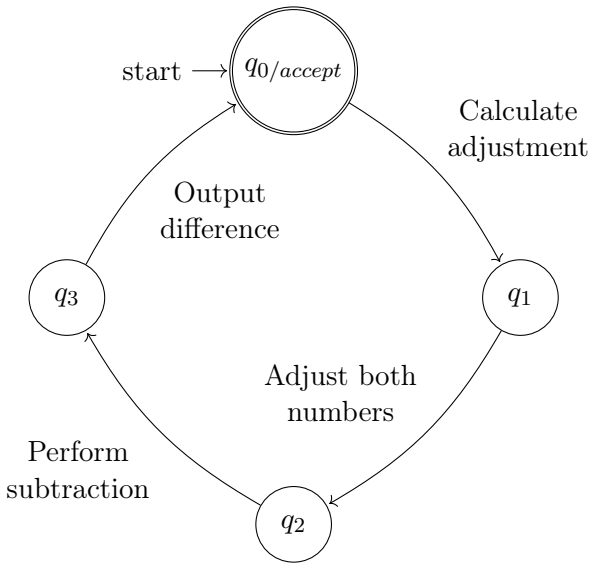
where:

- $Q = \{q_{0/accept}, q_1, q_2, q_3\}$ is the set of states.
- $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is the input alphabet (representing the digits of the minuend M and subtrahend S).
- $q_{0/accept}$ is the start state, which is also the accept state.
- $F = \{q_{0/accept}\}$ is the set of accepting states.

The transition function δ is defined as follows:

1. $\delta(q_{0/accept}, \text{“}M, S\text{”}) = q_1$ (Calculate the adjustment needed to make the subtrahend a base multiple.)
2. $\delta(q_1, \varepsilon) = q_2$ (Adjust both the minuend and subtrahend by the same amount.)
3. $\delta(q_2, \varepsilon) = q_3$ (Perform the subtraction on the adjusted numbers.)
4. $\delta(q_3, \varepsilon) = q_{0/accept}$ (Output the final difference.)

Automaton Diagram for Sliding to Make Bases



HTML Implementation

```
<!DOCTYPE html>
<html>
<head>
    <title>Subtraction Strategies: Sliding to Make Bases</title>
    <style>
        /* Global styles */
        body {
            font-family: 'Segoe UI', Tahoma, Geneva, Verdana, sans-serif;
            line-height: 1.6;
            color: #333;
            max-width: 1200px;
            margin: 0 auto;
            padding: 20px;
            background-color: #f9f9f9;
        }

        h1, h2 {
            color: #2c3e50;
            margin-top: 20px;
        }

        input, button {
            padding: 8px 12px;
            margin: 5px 0;
            border: 1px solid #ddd;
            border-radius: 4px;
            font-size: 14px;
        }

        button {
            background-color: #4CAF50;
            color: white;
            border: none;
            cursor: pointer;
            transition: background-color 0.3s;
        }

        button:hover {
            background-color: #45a049;
        }

        /* SVG container styles */
        #diagramSlidingSVG {
            border: 1px solid #d3d3d3;
            background-color: white;
            box-shadow: 0 2px 5px rgba(0,0,0,0.1);
            border-radius: 5px;
            margin: 15px 0;
        }

        #outputContainer {
            margin-top: 20px;
```

```
            padding: 15px;
            background-color: white;
            border-radius: 5px;
            box-shadow: 0 2px 5px rgba(0,0,0,0.1);
        }

        /* Number line styles */
        .number-line-tick {
            stroke: #555;
            stroke-width: 1;
        }

        .number-line-break {
            stroke: #555;
            stroke-width: 1;
        }

        .number-line-label {
            font-size: 12px;
            text-anchor: middle;
            fill: #444;
        }

        .original-marker {
            fill: #3498db; /* Blue */
            font-weight: bold;
        }

        .adjusted-marker {
            fill: #2ecc71; /* Green */
            font-weight: bold;
        }

        .slide-arrow {
            fill: none;
            stroke: #e67e22; /* Orange */
            stroke-width: 2;
            filter: drop-shadow(0px 1px 1px rgba(0,0,0,0.2));
        }

        .slide-arrow-head {
            fill: #e67e22;
            stroke: #e67e22;
        }

        .slide-label {
            font-size: 11px;
            fill: #e67e22;
            text-anchor: middle;
            font-weight: bold;
        }

        .label-box {
            fill: rgba(255, 255, 255, 0.8);
```

```
            stroke: #ddd;
            stroke-width: 1;
            rx: 3;
            ry: 3;
        }

        .difference-bracket {
            stroke: #e74c3c; /* Red */
            stroke-width: 2;
            fill: none;
        }

        .difference-label {
            font-size: 13px;
            fill: #e74c3c;
            text-anchor: middle;
            font-weight: bold;
        }

        .number-line-arrow {
            fill: #555;
            stroke: #555;
        }

        /* Responsive adjustments */
        @media (max-width: 768px) {
            body {
                padding: 10px;
            }

            h1 {
                font-size: 1.5em;
            }
        }
    </style>
</head>
<body>

<h1>Subtraction Strategies: Sliding to Make Bases</h1>

<div>
    <label for="slideMinuend">Minuend:</label>
    <input type="number" id="slideMinuend" value="73">
</div>
<div>
    <label for="slideSubtrahend">Subtrahend:</label>
    <input type="number" id="slideSubtrahend" value="47">
</div>

<button onclick="runSlidingAutomaton()">Calculate and Visualize</button>

<div id="outputContainer">
    <h2>Explanation:</h2>
    <div id="slidingOutput">
```

```
        <!-- Text output will be displayed here -->
    </div>
</div>

<h2>Diagram:</h2>
<svg id="diagramSlidingSVG" width="700" height="300"></svg>

<!-- New button for viewing PDF documentation -->
<button onclick="openPdfViewer()">Want to learn more about this strategy? Click here.</
    button>

<script>
    function openPdfViewer() {
        // Opens the PDF documentation for the strategy.
        window.open('../SAR_SUB_Sliding.pdf', '_blank');
    }
</script>

<script>
document.addEventListener('DOMContentLoaded', function() {
    const outputElement = document.getElementById('slidingOutput');
    const minuendInput = document.getElementById('slideMinuend');
    const subtrahendInput = document.getElementById('slideSubtrahend');
    const diagramSVG = document.getElementById('diagramSlidingSVG');

    // --- Helper SVG Functions ---
     function createText(svg, x, y, textContent, className = 'number-line-label') {
        const text = document.createElementNS("http://www.w3.org/2000/svg", 'text');
        text.setAttribute('x', x);
        text.setAttribute('y', y);
        text.setAttribute('class', className);
        text.setAttribute('text-anchor', 'middle');
        text.textContent = textContent;
        svg.appendChild(text);
    }

    function drawTick(svg, x, y, size, colorClass = '') { // Added colorClass option
        const tick = document.createElementNS('http://www.w3.org/2000/svg', 'line');
        tick.setAttribute('x1', x);
        tick.setAttribute('y1', y - size / 2);
        tick.setAttribute('x2', x);
        tick.setAttribute('y2', y + size / 2);
        tick.setAttribute('class', `number-line-tick ${colorClass}`.trim()); // Apply
            color class if provided
        tick.setAttribute('stroke', colorClass ? 'currentColor' : 'black'); // Use CSS
            color or default black
        svg.appendChild(tick);
    }

     function drawScaleBreakSymbol(svg, x, y) {
        const breakOffset = 4;
        const breakHeight = 8;
        const breakLine1 = document.createElementNS('http://www.w3.org/2000/svg', 'line');
```

```
        breakLine1.setAttribute('x1', x - breakOffset); breakLine1.setAttribute('y1', y - breakHeight);
        breakLine1.setAttribute('x2', x + breakOffset); breakLine1.setAttribute('y2', y + breakHeight);
        breakLine1.setAttribute('class', 'number-line-break'); svg.appendChild(breakLine1);
        const breakLine2 = document.createElementNS('http://www.w3.org/2000/svg', 'line');
        breakLine2.setAttribute('x1', x + breakOffset); breakLine2.setAttribute('y1', y - breakHeight);
        breakLine2.setAttribute('x2', x - breakOffset); breakLine2.setAttribute('y2', y + breakHeight);
        breakLine2.setAttribute('class', 'number-line-break'); svg.appendChild(breakLine2);
    }

    function createStraightArrow(svg, x1, y1, x2, y2, arrowClass = 'slide-arrow', headClass = 'slide-arrow-head', arrowSize = 5) {
        const line = document.createElementNS("http://www.w3.org/2000/svg", 'line');
        line.setAttribute('x1', x1); line.setAttribute('y1', y1);
        line.setAttribute('x2', x2); line.setAttribute('y2', y2);
        line.setAttribute('class', arrowClass);
        svg.appendChild(line);

        // Arrowhead pointing right assumed for slide
        const arrowHead = document.createElementNS("http://www.w3.org/2000/svg", 'path');
        arrowHead.setAttribute('d', `M ${x2 - arrowSize} ${y2 - arrowSize/2} L ${x2} ${y2} L ${x2 - arrowSize} ${y2 + arrowSize/2} Z`);
        arrowHead.setAttribute('class', headClass);
        svg.appendChild(arrowHead);
    }

    function drawDifferenceBracket(svg, x1, x2, y, label, colorClass = 'difference-') {
        const bracketHeight = 10;
        const path = document.createElementNS("http://www.w3.org/2000/svg", 'path');
        path.setAttribute('d', `M ${x1} ${y - bracketHeight} L ${x1} ${y} L ${x2} ${y} L ${x2} ${y - bracketHeight}`);
        path.setAttribute('class', `${colorClass}bracket`);
        svg.appendChild(path);
        createText(svg, (x1 + x2) / 2, y + 15, label, `${colorClass}label`);
    }

    function drawStoppingPoint(svg, x, y, labelText, labelOffsetBase = 20, index = 0) {
        const circle = document.createElementNS('http://www.w3.org/2000/svg', 'circle');
        circle.setAttribute('cx', x);
        circle.setAttribute('cy', y);
        circle.setAttribute('r', 4);
        circle.setAttribute('class', 'stopping-point');
        svg.appendChild(circle);

        // Use the provided y parameter instead of numberLineY
        if (labelText) {
            // Add staggering based on index to prevent overlap with large values
            const labelOffset = labelOffsetBase * (index % 2 === 0 ? 1.5 : -1.8);
```

```
                createText(svg, x, y + labelOffset, labelText, 'number-line-label');
            }
        }
    // --- End Helper Functions ---


    // --- Main Sliding Automaton Function ---
    window.runSlidingAutomaton = function() {
        try {
            const minuend = parseInt(minuendInput.value);
            const subtrahend = parseInt(subtrahendInput.value);

            if (isNaN(minuend) || isNaN(subtrahend)) {
                outputElement.textContent = 'Please enter valid numbers for Minuend and Subtrahend';
                diagramSVG.innerHTML = ''; return;
            }
             if (subtrahend > minuend) {
                 outputElement.textContent = 'Subtrahend cannot be greater than Minuend.';
                 diagramSVG.innerHTML = ''; return;
             }

            let output = `<h2>Sliding to Make Bases</h2>\n\n`;
            output += `<p><strong>Problem:</strong> ${minuend} - ${subtrahend}</p>\n\n`;

            // Calculate adjustment (usually round subtrahend UP)
            // For better learning, we'll always "slide" using one of two approaches:
            // 1. If subtrahend is not a multiple of 10, adjust to make it one
            // 2. If subtrahend is already a multiple of 10, we'll slide by +5 to demonstrate the invariance

            let adjustment;
            let slideReason;

            if (subtrahend % 10 === 0) {
                // Even though subtrahend is already a multiple of 10, we'll slide by +5
                // to demonstrate that sliding works regardless of the amount
                adjustment = 5;
                slideReason = `Even though ${subtrahend} is already a multiple of 10, we'll slide by +${adjustment} to demonstrate the technique.`;
            } else {
                adjustment = (10 - (subtrahend % 10)) % 10;
                slideReason = `Calculate adjustment to make ${subtrahend} a multiple of 10.`;
            }

            const adjustedMinuend = minuend + adjustment;
            const adjustedSubtrahend = subtrahend + adjustment;
            const difference = adjustedMinuend - adjustedSubtrahend; // Should equal minuend - subtrahend

            output += `Step 1: ${slideReason}\n`;
            output += `<p>Adjustment = +${adjustment}</p>\n`;
            output += `Step 2: Adjust (slide) both numbers by +${adjustment}.\n`
```

```
            output += `<p>New Minuend: ${minuend} + ${adjustment} = ${adjustedMinuend}</p>\n`;
            output += `<p>New Subtrahend: ${subtrahend} + ${adjustment} = ${adjustedSubtrahend}</p>\n`;
            output += `Step 3: Subtract adjusted numbers.\n`;
            output += `<p>${adjustedMinuend} - ${adjustedSubtrahend} = ${difference}</p>\n\n`;


            output += `<strong>Result:</strong> ${difference}`;
            outputElement.innerHTML = output;
            typesetMath();

            // Draw Diagram
            drawSlidingNumberLine(diagramSVG, minuend, subtrahend, adjustedMinuend, adjustedSubtrahend, adjustment, difference);

        } catch (error) {
            console.error("Error in runSlidingAutomaton:", error);
            outputElement.textContent = `Error: ${error.message}`;
        }
    };

    function drawSlidingNumberLine(svg, M, S, M_adj, S_adj, adj, diff) {
        if (!svg || typeof svg.setAttribute !== 'function') { console.error("Invalid SVG element..."); return;}
        svg.innerHTML = '';

        const svgWidth = parseFloat(svg.getAttribute('width'));
        const svgHeight = parseFloat(svg.getAttribute('height'));
        const startX = 50;
        const endX = svgWidth - 50;
        const numberLineY = svgHeight * 0.6; // Position number line lower
        const tickHeight = 12; // Slightly larger ticks
        const labelOffsetY = 20; // Offset for labels below line
        const slideArrowY = numberLineY - 40; // Y position for slide arrows
        const diffBracketY = numberLineY + 45; // Increased spacing for difference bracket
        const arrowSize = 6; // Slightly larger arrows
        const scaleBreakThreshold = 40;

        // Title for the diagram at the top
        createLabelWithBackground(svg, 20, 20, "Number Line Visualization of Sliding Strategy", "diagram-label");

        // Determine range for scaling
        let diagramMin = Math.min(0, S);
        let diagramMax = M_adj; // Need to show the adjusted minuend

        // Calculate scale and handle potential break
        let displayRangeStart = diagramMin;
        let scaleStartX = startX;
        let drawScaleBreak = false;

```

```
␣␣␣␣␣␣␣␣␣␣if␣(diagramMin␣>␣scaleBreakThreshold)␣{␣//␣Break␣logic␣focuses␣on␣start
␣␣␣␣␣␣␣␣␣␣␣␣␣␣displayRangeStart␣=␣diagramMin␣-␣10;
␣␣␣␣␣␣␣␣␣␣␣␣␣␣scaleStartX␣=␣startX␣+␣30;
␣␣␣␣␣␣␣␣␣␣␣␣␣␣drawScaleBreak␣=␣true;
␣␣␣␣␣␣␣␣␣␣␣␣␣␣drawScaleBreakSymbol(svg,␣scaleStartX␣-␣15,␣numberLineY);
␣␣␣␣␣␣␣␣␣␣␣␣␣␣drawTick(svg,␣startX,␣numberLineY,␣tickHeight);
␣␣␣␣␣␣␣␣␣␣␣␣␣␣createLabelWithBackground(svg,␣startX,␣numberLineY␣+␣labelOffsetY,␣'0',␣'
    number-line-label');
␣␣␣␣␣␣␣␣␣␣}␣else␣{
␣␣␣␣␣␣␣␣␣␣␣␣␣␣displayRangeStart␣=␣0;␣//␣Include␣0
␣␣␣␣␣␣␣␣␣␣␣␣␣␣drawTick(svg,␣startX,␣numberLineY,␣tickHeight);
␣␣␣␣␣␣␣␣␣␣␣␣␣␣createLabelWithBackground(svg,␣startX,␣numberLineY␣+␣labelOffsetY,␣'0',␣'
    number-line-label');
␣␣␣␣␣␣␣␣␣␣}

␣␣␣␣␣␣␣␣␣␣const␣displayRangeEnd␣=␣diagramMax␣+␣10;
␣␣␣␣␣␣␣␣␣␣const␣displayRange␣=␣Math.max(displayRangeEnd␣-␣displayRangeStart,␣1);
␣␣␣␣␣␣␣␣␣␣const␣scale␣=␣(endX␣-␣scaleStartX)␣/␣displayRange;

␣␣␣␣␣␣␣␣␣␣//␣Function␣to␣convert␣value␣to␣X␣coordinate
␣␣␣␣␣␣␣␣␣␣function␣valueToX(value)␣{
␣␣␣␣␣␣␣␣␣␣␣␣␣␣if␣(value␣<␣displayRangeStart␣&&␣drawScaleBreak)␣{␣return␣scaleStartX␣-␣10;␣
    }
␣␣␣␣␣␣␣␣␣␣␣␣␣␣const␣scaledValue␣=␣scaleStartX␣+␣(value␣-␣displayRangeStart)␣*␣scale;
␣␣␣␣␣␣␣␣␣␣␣␣␣␣return␣Math.max(scaleStartX,␣Math.min(scaledValue,␣endX));
␣␣␣␣␣␣␣␣␣␣}

␣␣␣␣␣␣␣␣␣␣//␣Draw␣main␣line␣segment
␣␣␣␣␣␣␣␣␣␣const␣mainLineStartX␣=␣valueToX(displayRangeStart);
␣␣␣␣␣␣␣␣␣␣const␣mainLineEndX␣=␣valueToX(displayRangeEnd);
␣␣␣␣␣␣␣␣␣␣const␣numberLine␣=␣document.createElementNS('http://www.w3.org/2000/svg', 'line
    ');
        numberLine.setAttribute('x1', mainLineStartX); numberLine.setAttribute('y1',
            numberLineY);
        numberLine.setAttribute('x2', mainLineEndX); numberLine.setAttribute('y2',
            numberLineY);
        numberLine.setAttribute('class', 'number-line-tick'); svg.appendChild(numberLine)
            ;

        // Add arrowhead
        const mainArrowHead = document.createElementNS('http://www.w3.org/2000/svg', '
            path');
        mainArrowHead.setAttribute('d', `M ${mainLineEndX - arrowSize} ${numberLineY -
            arrowSize/2} L ${mainLineEndX} ${numberLineY} L ${mainLineEndX - arrowSize} $
            {numberLineY + arrowSize/2} Z`);
        mainArrowHead.setAttribute('class', 'number-line-arrow'); svg.appendChild(
            mainArrowHead);

        // Draw key markers based on values
        const keyValues = [];
        for (let i = Math.ceil(displayRangeStart / 10) * 10; i <= displayRangeEnd; i +=
            10) {
            if (i !== S && i !== M && i !== S_adj && i !== M_adj) { // Don't draw if it's
                already one of our special points
```

```
            keyValues.push(i);
        }
    }

    // Draw key value markers (multiples of 10)
    keyValues.forEach(val => {
        const x = valueToX(val);
        // Only draw if not too close to other markers
        if (!isNearSpecialPoint(val, [S, M, S_adj, M_adj], 5)) {
            drawTick(svg, x, numberLineY, tickHeight * 0.7); // Smaller ticks for
                regular values
            createText(svg, x, numberLineY + labelOffsetY, val.toString(), 'number-
                line-label');
        }
    });

    // Helper function to check if a value is near any special point
    function isNearSpecialPoint(val, specialPoints, threshold) {
        return specialPoints.some(sp => Math.abs(val - sp) < threshold);
    }

    // Mark Original Points (Blue) with background boxes to prevent overlap
    const xS = valueToX(S);
    const xM = valueToX(M);

    drawTick(svg, xS, numberLineY, tickHeight * 1.2, 'original-marker'); // Larger
        ticks for important points
    createLabelWithBackground(svg, xS, numberLineY + labelOffsetY, S.toString(), '
        original-marker');

    drawTick(svg, xM, numberLineY, tickHeight * 1.2, 'original-marker');
    createLabelWithBackground(svg, xM, numberLineY + labelOffsetY, M.toString(), '
        original-marker');

    // Section label for original values
    createLabelWithBackground(svg, 20, numberLineY - 70, "Original␣Values", "diagram-
        label");

    if (adj > 0) { // Only draw adjusted points and arrows if there was a slide
        // Section label for adjusted values
        createLabelWithBackground(svg, 20, numberLineY - 10, "Adjusted␣Values␣(+" +
            adj + ")", "diagram-label");

        // Mark Adjusted Points (Green) with background boxes
        const xS_adj = valueToX(S_adj);
        const xM_adj = valueToX(M_adj);

        drawTick(svg, xS_adj, numberLineY, tickHeight * 1.2, 'adjusted-marker');
        // Position the label with offset to avoid overlap
        createLabelWithBackground(svg, xS_adj, numberLineY + labelOffsetY + 20, S_adj
            .toString(), 'adjusted-marker');

        drawTick(svg, xM_adj, numberLineY, tickHeight * 1.2, 'adjusted-marker');
```

```
            createLabelWithBackground(svg, xM_adj, numberLineY + labelOffsetY + 20, M_adj
                .toString(), 'adjusted-marker');

            // Draw Slide Arrows (Orange) with varied positioning to avoid overlap
            // First arrow (for subtrahend)
            createStraightArrow(svg, xS, slideArrowY, xS_adj, slideArrowY);
            createLabelWithBackground(svg, (xS + xS_adj) / 2, slideArrowY - 15, `+${adj
                }`, 'slide-label');

            // Second arrow (for minuend) - offset slightly to avoid overlap if points
                are close
            const arrowYOffset = (Math.abs(xM - xS) < 50) ? -15 : 0;
            createStraightArrow(svg, xM, slideArrowY + arrowYOffset, xM_adj, slideArrowY
                + arrowYOffset);
            createLabelWithBackground(svg, (xM + xM_adj) / 2, slideArrowY + arrowYOffset
                - 15, `+${adj}`, 'slide-label');

            // Draw Difference Bracket (Red) below adjusted points
            drawDifferenceBracket(svg, xS_adj, xM_adj, diffBracketY, `Difference = ${diff
                }`);
        } else {
            // Draw Difference Bracket (Red) below original points if no slide
            drawDifferenceBracket(svg, xS, xM, diffBracketY, `Difference = ${diff}`);
        }
    }

    // Helper function to create a label with a background box for better readability
    function createLabelWithBackground(svg, x, y, text, className) {
        // First create text element to measure its size
        const textElem = document.createElementNS("http://www.w3.org/2000/svg", 'text');
        textElem.setAttribute('x', x);
        textElem.setAttribute('y', y);
        textElem.setAttribute('class', className);
        textElem.setAttribute('text-anchor', 'middle');
        textElem.textContent = text;
        svg.appendChild(textElem);

        // Get the bounding box
        const bbox = textElem.getBBox();

        // Create the background rectangle
        const padding = 3;
        const rect = document.createElementNS("http://www.w3.org/2000/svg", 'rect');
        rect.setAttribute('x', bbox.x - padding);
        rect.setAttribute('y', bbox.y - padding);
        rect.setAttribute('width', bbox.width + (padding * 2));
        rect.setAttribute('height', bbox.height + (padding * 2));
        rect.setAttribute('class', 'label-box');

        // Insert rectangle before text so it appears behind
        svg.insertBefore(rect, textElem);

        return textElem;
    }
```

```
    function typesetMath() {
        // Placeholder function to prevent errors
        console.log("typesetMath called - no operation performed.");
    }
});
</script>

</body>
</html>
```

References

Hackenberg, A. (2025). *Course notes* [Unpublished course notes].