Subtraction Strategies: Chunking

Compiled by: Theodore M. Savich

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Transcript

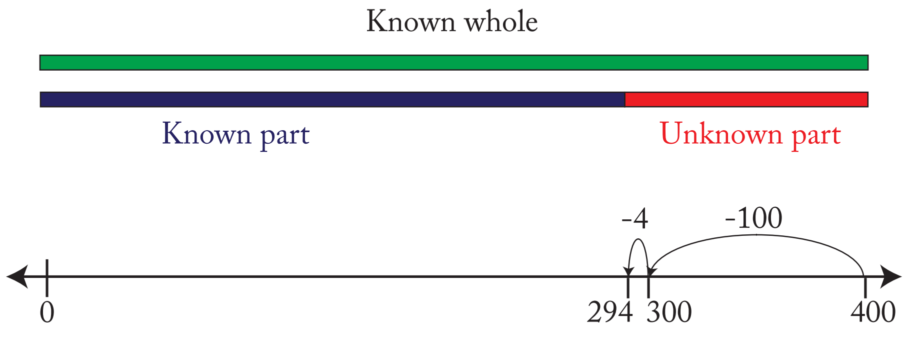
Video from Carpenter et al. (1999). Strategy descriptions and examples adapted from Hackenberg (2025).

- **Teacher:**"One summer T.J. saved $400. At the end of the summer she spent $294 on a new bike. How much money did T.J. have then?"
- **Student:** "400 takeaway 200 is 200. I just put the 4 on the side right now. So then 200 takeaway 90 is 110. So then 110 takeaway 4 is 106.
- **Teacher:** "So how much money did she have left?"
- **Student:** "106."
- **Teacher:** "Nice job."

Here is the notation below to show what the student did:

$$\begin{aligned} 400 - 200 &= 200 \\ 200 - 90 &= 110 \\ 110 - 4 &= 106 \end{aligned}$$

c.) Chunk back from the known whole to the known part



answer: 100 + 4 = 104

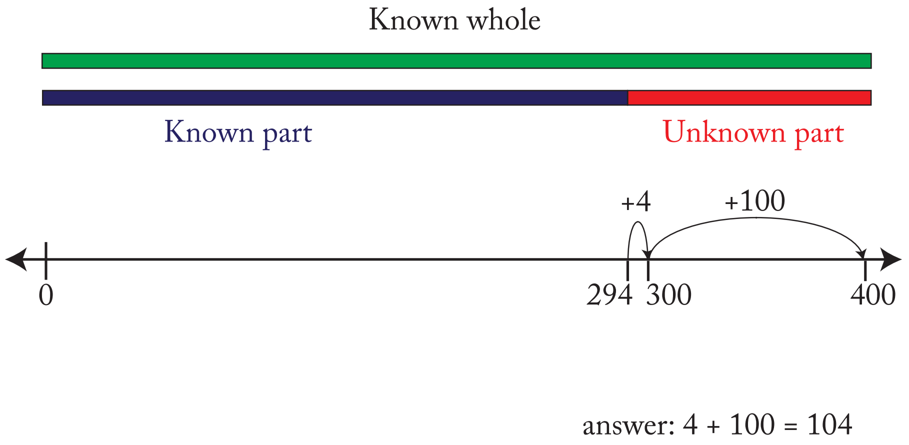
However, this is only one of three structurally different ways that chunking can show up in subtraction.

(a) **Chunking backwards (by known part)** The student starts at the known whole and subtracts backwards by the known part. They arrive at the unknown part.

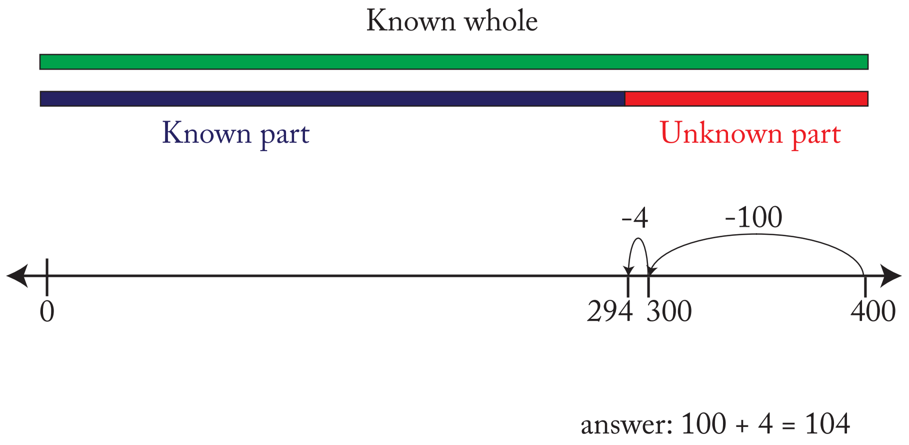
(b) **Chunking forwards** The student subtracts the known whole from the known part.

(c) **Chunking backwards (to the known part)** The student starts at the known whole and subtracts backwards until they reach the known part.

b.) Chunking forwards (from known part to known whole)



c.) Chunk back from the known whole to the known part



Description of Strategy

- Subtract the subtrahend (known part) from the minuend (known whole) by breaking the subtrahend into bases and ones and subtracting in strategic chunks.
- Start from the subtrahend and add strategic chunks to reach the minuend, summing the chunks to find the difference.
- Start at the minuend and subtract strategic chunks until you reach the subtrahend, summing the chunks to find the difference.

Automaton Type - only one type of chunking is analyzed here

Finite State Automaton (FSA) with Counters: Counters are used to manage the sequential subtraction:

- **BaseCounter:** Counts the number of base chunks to subtract.
- **OneCounter:** Counts the number of ones to subtract.
- **Difference:** Accumulates the running difference.

Formal Description of the Automaton

We define the automaton as the tuple

$$M = (Q,\, \Sigma,\, \delta,\, q_{0/accept},\, F,\, C)$$

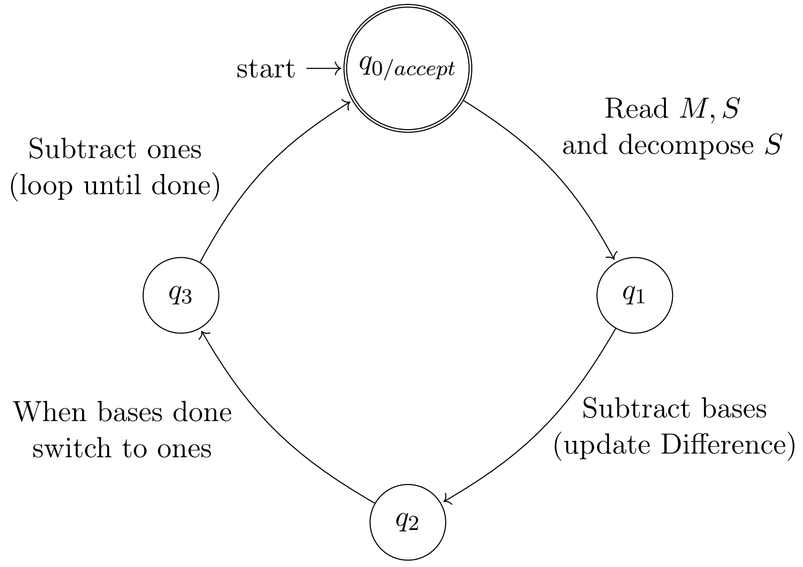
where:

- $Q = \{q_{0/accept},\, q_1,\, q_2,\, q_3\}$ is the set of states.
- $\Sigma = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is the input alphabet (representing the digits of the minuend M and subtrahend S).
- $q_{0/accept}$ is the start state, which is also the accept state.
- $F = \{q_{0/accept}\}$ is the set of accepting states.
- $C = \{\text{BaseCounter, OneCounter, Difference}\}$ is the set of counters.

The transition function δ is defined as follows:

1. $\delta(q_{0/accept},\, \text{“}M, S\text{”}) =$
   $(q_1,\, \text{initialize: set Difference} \leftarrow M,\ \text{Decompose } S \text{ into BaseCounter and OneCounter}).$
2. $\delta(q_1,\, \varepsilon) =$
   $(q_2,\, \text{while BaseCounter} > 0 :\ \text{Difference} \leftarrow \text{Difference}-(\text{base chunk}),\ \text{decrement BaseCounter}).$
3. $\delta(q_2,\, \varepsilon) =$
   $(q_3,\, \text{when BaseCounter} = 0).$
4. $\delta(q_3,\, \varepsilon) =$
   $(q_3,\, \text{while OneCounter} > 0 :\ \text{Difference} \leftarrow \text{Difference}-(\text{ones chunk}),\ \text{decrement OneCounter}).$
5. $\delta(q_3,\, \varepsilon) =$
   $(q_{0/accept},\, \text{when OneCounter} = 0 :\ \text{output Difference}).$

Automaton Diagram for Chunking by Bases and Ones (Forwards or Backwards)



HTML Implementation

```html
<!DOCTYPE html>
<html>
<head>
    <title>Subtraction Strategies: Chunking</title>
    <style>
        body { font-family: sans-serif; }
        #diagramSubChunkingSVG { border: 1px solid #d3d3d3; }
        #outputContainer { margin-top: 20px; }
        fieldset { margin: 15px 0; border: 1px solid #ccc; padding: 10px;}
        legend { font-weight: bold; }
        label { margin-right: 10px; }
        /* Number line styles */
        .number-line-tick { stroke: black; stroke-width: 1; }
        .number-line-break { stroke: black; stroke-width: 1; } /* Solid for zig-zag */
        .number-line-label { font-size: 12px; text-anchor: middle; }
        .jump-arrow {
            fill: none; /* <-- Ensure no fill for the arc */
            stroke-width: 2;
         }
        .jump-arrow-head {
            stroke-width: 2;
            fill: none; /* <-- Ensure no fill for the arrow head */
         }
        .jump-label { font-size: 12px; text-anchor: middle; }
        .stopping-point { fill: red; stroke: black; stroke-width: 1; }
        .number-line-arrow { fill: black; stroke: black;}
        /* Colors for strategies */
        .strategy-a { stroke: darkred; } /* Backwards by part */
        .strategy-b { stroke: darkgreen; } /* Forwards */
        .strategy-c { stroke: darkblue; } /* Backwards to part */
    </style>
</head>
<body>

<h1>Subtraction Strategies: Chunking</h1>

<div>
    <label for="chunkMinuend">Minuend (Whole):</label>
    <input type="number" id="chunkMinuend" value="400">
</div>
<div>
    <label for="chunkSubtrahend">Subtrahend (Part):</label>
    <input type="number" id="chunkSubtrahend" value="294">
</div>

<fieldset>
    <legend>Choose Chunking Strategy:</legend>
    <input type="radio" id="strategyA" name="chunkingStrategy" value="A" checked>
    <label for="strategyA">A: Backwards (by Known Part)</label><br>
    <input type="radio" id="strategyB" name="chunkingStrategy" value="B">
    <label for="strategyB">B: Forwards (from Known Part)</label><br>
    <input type="radio" id="strategyC" name="chunkingStrategy" value="C">
    <label for="strategyC">C: Backwards (to Known Part)</label><br>
</fieldset>


<button onclick="runSubtractionChunkingAutomaton()">Calculate and Visualize</button>

<div id="outputContainer">
    <h2>Explanation:</h2>
    <div id="subChunkingOutput">
        <!-- Text output will be displayed here -->
    </div>
</div>

<h2>Diagram:</h2>
<svg id="diagramSubChunkingSVG" width="700" height="350"></svg>

<script>
document.addEventListener('DOMContentLoaded', function() {
    const outputElement = document.getElementById('subChunkingOutput');
    const minuendInput = document.getElementById('chunkMinuend');
    const subtrahendInput = document.getElementById('chunkSubtrahend');
    const diagramSVG = document.getElementById('diagramSubChunkingSVG');
    const strategyRadios = document.getElementsByName('chunkingStrategy');

    // --- All Helper SVG Drawing Functions Defined Here --- (Keep from previous version) ---
    function createText(svg, x, y, textContent, className = 'number-line-label') {
        const text = document.createElementNS("http://www.w3.org/2000/svg", 'text');
        text.setAttribute('x', x);
        text.setAttribute('y', y);
        text.setAttribute('class', className);
        text.setAttribute('text-anchor', 'middle');
        text.textContent = textContent;
        svg.appendChild(text);
    }

    function drawTick(svg, x, y, size) {
        const tick = document.createElementNS('http://www.w3.org/2000/svg', 'line');
        tick.setAttribute('x1', x);
        tick.setAttribute('y1', y - size / 2);
        tick.setAttribute('x2', x);
        tick.setAttribute('y2', y + size / 2);
        tick.setAttribute('class', 'number-line-tick');
        svg.appendChild(tick);
    }

     function drawScaleBreakSymbol(svg, x, y) {
        const breakOffset = 4;
        const breakHeight = 8;
        const breakLine1 = document.createElementNS('http://www.w3.org/2000/svg', 'line');
        breakLine1.setAttribute('x1', x - breakOffset);
        breakLine1.setAttribute('y1', y - breakHeight);
        breakLine1.setAttribute('x2', x + breakOffset);
        breakLine1.setAttribute('y2', y + breakHeight);
        breakLine1.setAttribute('class', 'number-line-break');
        svg.appendChild(breakLine1);
        const breakLine2 = document.createElementNS('http://www.w3.org/2000/svg', 'line');
        breakLine2.setAttribute('x1', x + breakOffset);
        breakLine2.setAttribute('y1', y - breakHeight);
        breakLine2.setAttribute('x2', x - breakOffset);
        breakLine2.setAttribute('y2', y + breakHeight);
        breakLine2.setAttribute('class', 'number-line-break');
        svg.appendChild(breakLine2);
    }

    function createJumpArrow(svg, x1, y1, x2, y2, jumpArcHeight, direction = 'forward', colorClass = 'strategy-b', arrowSize = 5) {
        const path = document.createElementNS('http://www.w3.org/2000/svg', 'path');
        const cx = (x1 + x2) / 2;
        const cy = y1 - jumpArcHeight;
        path.setAttribute('d', `M ${x1} ${y1} Q ${cx} ${cy} ${x2} ${y1}`);
        path.setAttribute('class', `jump-arrow ${colorClass}`); // Apply strategy color to arc stroke
        path.setAttribute('fill', 'none'); // Explicitly set fill to none to prevent filling
        svg.appendChild(path);

        const arrowHead = document.createElementNS('http://www.w3.org/2000/svg', 'path');
        const dx = x2 - cx;
        const dy = y1 - cy;
        const angleRad = Math.atan2(dy, dx);
        let angleDeg = angleRad * (180 / Math.PI);
        arrowHead.setAttribute('class', `jump-arrow-head ${colorClass}`); // Apply strategy color to head fill/stroke

        if (direction === 'forward') {
            angleDeg += 180;
            arrowHead.setAttribute('d', `M 0 0 L ${arrowSize} ${arrowSize/2} L ${arrowSize} ${-arrowSize/2} Z`);
        } else { // backward
            arrowHead.setAttribute('d', `M 0 0 L ${-arrowSize} ${arrowSize/2} L ${-arrowSize} ${-arrowSize/2} Z`);
        }
        arrowHead.setAttribute('transform', `translate(${x2}, ${y1}) rotate(${angleDeg})`);
        svg.appendChild(arrowHead);
    }


     function drawStoppingPoint(svg, x, y, labelText, size = 5) {
            const circle = document.createElementNS('http://www.w3.org/2000/svg', 'circle');
            circle.setAttribute('cx', x);
            circle.setAttribute('cy', y);
            circle.setAttribute('r', size);
            circle.setAttribute('class', 'stopping-point');
            svg.appendChild(circle);

            // Use the provided y parameter instead of numberLineY
            if (labelText) {
                createText(svg, x, y + labelOffsetBase * 1.5, labelText, 'number-line-label');
            }
        }
    // --- End Helper Functions ---

    // --- Main Automaton Function ---
    window.runSubtractionChunkingAutomaton = function() {
        try {
            const minuend = parseInt(minuendInput.value); // M (Whole)
            const subtrahend = parseInt(subtrahendInput.value); // S (Known Part)
            let selectedStrategy = 'A'; // Default
            for (const radio of strategyRadios) {
                if (radio.checked) {
                    selectedStrategy = radio.value;
                    break;
                }
            }

            if (isNaN(minuend) || isNaN(subtrahend)) {
                outputElement.textContent = 'Please enter valid numbers for Minuend and Subtrahend';
                diagramSVG.innerHTML = '';
                return;
            }
            if (subtrahend > minuend && selectedStrategy !== 'B') {
                 outputElement.textContent = 'Subtrahend cannot be greater than Minuend for strategies A and C.';
                 diagramSVG.innerHTML = '';
                 return;
            }


            let output = `<h2>Subtraction Chunking (Strategy ${selectedStrategy})</h2>\n\n`;
            output += `<p><strong>Problem:</strong> ${minuend} - ${subtrahend}</p>\n\n`;

            const chunkSteps = [];
            let finalDifference = 0;
            let currentVal = 0;
            let targetVal = 0;
            let direction = 'backward'; // Default for A and C
            let startPoint = minuend;
            let endPoint = 0; // Will be calculated
            let totalChunkSum = 0; // For strategies B and C

             let stepCounter = 1; // Initialize step counter


            // --- Logic based on Selected Strategy ---
            switch (selectedStrategy) {
                //==========================================
            case 'A': // Chunking Backwards (by Known Part) M - S = ?
            //=========================================
                output += `Strategy A: Start at ${minuend}, subtract chunks of ${subtrahend}.\n`;
                currentVal = minuend;
                targetVal = minuend - subtrahend;
                startPoint = minuend;

                let tensToSubtract = Math.floor(subtrahend / 10) * 10;
                let onesToSubtract = subtrahend % 10;


                // Subtract Tens Chunk
                if (tensToSubtract > 0) {
                    output += `<p>Step ${stepCounter++}: Subtract tens chunk</p>\n`;
                    chunkSteps.push({ from: currentVal, to: currentVal - tensToSubtract, label: `-${tensToSubtract}` });
                    output += `<p>${currentVal} - ${tensToSubtract} = ${currentVal - tensToSubtract}</p>\n`;
                    currentVal -= tensToSubtract;
                }

                // Subtract Ones Chunks Strategically
                if (onesToSubtract > 0) {
                    output += `<p>Step ${stepCounter++}: Subtract ones chunk(s)</p>\n`;
                    while (onesToSubtract > 0) {
                        let onesToPreviousTen = currentVal % 10;
                        if (onesToPreviousTen === 0 && onesToSubtract > 0) onesToPreviousTen = 10;

                        let chunk = Math.min(onesToSubtract, onesToPreviousTen);
                         if (chunk === 0 && onesToSubtract > 0) chunk = onesToSubtract;
                         if (chunk === 0) break;

                         chunkSteps.push({ from: currentVal, to: currentVal - chunk, label: `-${chunk}` });
                         output += `<p>${currentVal} - ${chunk} = ${currentVal - chunk}`;
                         if (chunk === onesToPreviousTen && chunk !== onesToSubtract && (currentVal - chunk) % 10 === 0) output += ` (Making previous ten)`;
                         output += `</p>\n`;
                         currentVal -= chunk;
                         onesToSubtract -= chunk;
                    }
                }

                finalDifference = currentVal;
                endPoint = finalDifference;
                output += `\n<p><strong>Result (Final Position):</strong> ${finalDifference}</p>`;
                break;

            //=====================================================
            case ’B’: // Chunking Forwards (from Known Part) S + ? = M
            //=====================================================
                output += ‘Strategy B: Start at ${subtrahend}, add chunks to reach ${minuend}.\n‘;
                currentVal = subtrahend;
                targetVal = minuend;
                startPoint = subtrahend;
                endPoint = minuend;
                direction = ’forward’;
                totalChunkSum = 0;

                while (currentVal < targetVal) {
                    output += ‘<p>Step ${stepCounter++}: Add chunk</p>\n‘;
                    let diff = targetVal - currentVal;
                    let chunk = 0;
                    let explanation = ’’;

                    let onesToNextTen = (10 - (currentVal % 10)) % 10;
                    if (onesToNextTen > 0 && onesToNextTen <= diff) {
                        chunk = onesToNextTen;
                        explanation = ’(Making␣the␣next␣ten)’;
                    } else {
                        let tensToNextHundred = (100 - (currentVal % 100)) % 100;
                        if (currentVal % 10 === 0 && tensToNextHundred > 0 && tensToNextHundred <= diff) {
                            chunk = tensToNextHundred;
                            explanation = ’(Making␣the␣next␣hundred)’;
                        } else {
                            if (diff >= 100) chunk = Math.floor(diff / 100) * 100;
                            else if (diff >= 10) chunk = Math.floor(diff / 10) * 10;
                            else chunk = diff;
                        }
                    }
                    if (chunk <= 0) { chunk = diff; explanation = ’’; };

                    chunkSteps.push({ from: currentVal, to: currentVal + chunk, label: ‘+${chunk}‘ });
                    output += ‘<p>${currentVal} + ${chunk} = ${currentVal + chunk} ${explanation}</p>\n‘;
                    currentVal += chunk;
                    totalChunkSum += chunk;
                }

                finalDifference = totalChunkSum;
                 output += ‘\n<p><strong>Result (Sum of Chunks):</strong> ${finalDifference}</p>‘;
                break;

            //=========================================================
            case ’C’: // Chunking Backwards (to Known Part) M - ? = S (REVISED LOGIC)
            //=========================================================
                output += ‘Strategy C: Start at ${minuend}, subtract chunks to reach ${subtrahend}.\n‘;
                currentVal = minuend;
                targetVal = subtrahend;
                startPoint = minuend;
                endPoint = subtrahend;
                direction = 'backward';
                totalChunkSum = 0;

                while (currentVal > targetVal) {
                    output += `<p>Step ${stepCounter++}: Subtract chunk</p>\n`;
                    let diff = currentVal - targetVal;
                    let chunk = 0;
                    let explanation = '';

                    // Priority 1: Subtract ones chunk to land on a ten?
                    let onesToPreviousTen = currentVal % 10;
                    // Only do this if it doesn't overshoot the target AND makes sense
                    if (onesToPreviousTen > 0 && onesToPreviousTen <= diff) {
                        chunk = onesToPreviousTen;
                        explanation = '(Making␣previous␣ten)';
                    } else {
                        // Priority 2: Subtract tens chunk to land on a hundred?
                        let tensToPreviousHundred = currentVal % 100;
                         // Only do this if at a multiple of 10, it doesn't overshoot, and makes sense
                        if (currentVal % 10 === 0 && tensToPreviousHundred > 0 && tensToPreviousHundred <= diff) {
                            chunk = tensToPreviousHundred;
                            explanation = '(Making␣previous␣hundred)';
                        } else {
                            // Priority 3: Subtract largest power of 10 chunk possible without overshooting
                            if (diff >= 100) {
                                chunk = Math.floor(diff / 100) * 100; // Largest hundreds chunk <= diff
                            } else if (diff >= 10) {
                                chunk = Math.floor(diff / 10) * 10; // Largest tens chunk <= diff
                            } else {
                                chunk = diff; // Subtract remaining ones if < 10
                            }
                        }
                    }

                    // Final check to ensure chunk doesn't overshoot
                    chunk = Math.min(chunk, diff);
                    // Ensure positive chunk if difference exists
                    if (chunk <= 0 && diff > 0) { chunk = diff; explanation = ''; };

                    if (chunk === 0) break; // Safety exit if no chunk calculated

                    chunkSteps.push({ from: currentVal, to: currentVal - chunk, label: `-${chunk}` });
                    output += `<p>${currentVal} - ${chunk} = ${currentVal - chunk} ${explanation}</p>\n`;
                    currentVal -= chunk;
                    totalChunkSum += chunk;
                }

                 finalDifference = totalChunkSum;
                 output += ‘\n<p><strong>Result (Sum of Chunks):</strong> ${finalDifference}</p>‘;
                break;
            //=========================================================
        }


        outputElement.innerHTML = output;
        typesetMath();

        // --- Draw Number Line Diagram ---
        let allValues = [startPoint, endPoint];
        chunkSteps.forEach(step => { allValues.push(step.from); allValues.push(step.to); });
        let diagramMin = Math.min(...allValues);
        let diagramMax = Math.max(...allValues);

        drawNumberLineDiagram(diagramSVG,
            startPoint, endPoint,
            diagramMin, diagramMax,
            chunkSteps, direction, selectedStrategy);


    } catch (error) {
        console.error("Error␣in␣runSubtractionChunkingAutomaton:", error);
        outputElement.textContent = ‘Error: ${error.message}‘;
    }
};

function drawNumberLineDiagram(svg, startValue, endValue, diagramMin, diagramMax, chunkSteps, direction, strategy) {
    if (!svg || typeof svg.setAttribute !== ’function’) { console.error("Invalid␣SVG␣element..."); return; }
    svg.innerHTML = ’’;

    const svgWidth = parseFloat(svg.getAttribute(’width’));
    const svgHeight = parseFloat(svg.getAttribute(’height’));
    const startX = 50;
    const endX = svgWidth - 50;
    const numberLineY = svgHeight / 2 + 30;
    const tickHeight = 10;
    const labelOffsetBase = 20;
    const jumpHeightLarge = 60;
    const jumpHeightSmall = 40;
    const jumpLabelOffset = 15;
    const arrowSize = 5;
    const scaleBreakThreshold = 40;

    // Calculate scale and handle potential break
    let displayRangeStart = diagramMin;
```

```
        let scaleStartX = startX;
        let drawScaleBreak = false;

        if (diagramMin > scaleBreakThreshold) {
            displayRangeStart = diagramMin - 10;
            scaleStartX = startX + 30;
            drawScaleBreak = true;
            drawScaleBreakSymbol(svg, scaleStartX - 15, numberLineY);
            drawTick(svg, startX, numberLineY, tickHeight);
            createText(svg, startX, numberLineY + labelOffsetBase, ’0’, ’number-line-label’);
        } else {
            displayRangeStart = 0;
            drawTick(svg, startX, numberLineY, tickHeight);
            createText(svg, startX, numberLineY + labelOffsetBase, ’0’, ’number-line-label’);
        }

        const displayRangeEnd = diagramMax + 10;
        const displayRange = Math.max(displayRangeEnd - displayRangeStart, 1);
        const scale = (endX - scaleStartX) / displayRange;

        // Function to convert value to X coordinate
        function valueToX(value) {
            if (value < displayRangeStart && drawScaleBreak) { return scaleStartX - 10; }
            const scaledValue = scaleStartX + (value - displayRangeStart) * scale;
            return Math.max(scaleStartX, Math.min(scaledValue, endX));
        }

        // Draw the main visible segment of the number line
         const mainLineStartX = valueToX(displayRangeStart);
         const mainLineEndX = valueToX(displayRangeEnd);
         const numberLine = document.createElementNS(’http://www.w3.org/2000/svg’, ’line’);
         numberLine.setAttribute(’x1’, mainLineStartX);
         numberLine.setAttribute(’y1’, numberLineY);
         numberLine.setAttribute(’x2’, mainLineEndX);
         numberLine.setAttribute(’y2’, numberLineY);
         numberLine.setAttribute(’class’, ’number-line-tick’);
         svg.appendChild(numberLine);

         // Add arrowhead to the right end
         const mainArrowHead = document.createElementNS(’http://www.w3.org/2000/svg’, ’path’);
         mainArrowHead.setAttribute(’d’, ‘M ${mainLineEndX - arrowSize} ${numberLineY - arrowSize/2} L ${mainLineEndX} ${numberLineY} L ${mainLineEndX - arrowSize} ${numberLineY + arrowSize/2} Z‘);
         mainArrowHead.setAttribute(’class’, ’number-line-arrow’);
         svg.appendChild(mainArrowHead);


        // Draw Ticks and Labels
        function drawTickAndLabel(value, index) {
            const x = valueToX(value);
            if (x < scaleStartX - 5 && value !== 0) return;

            drawTick(svg, x, numberLineY, tickHeight); // Pass svg
            const labelOffset = labelOffsetBase * (index % 2 === 0 ? 1 : -1.5); // Stagger
            createText(svg, x, numberLineY + labelOffset, value.toString(), 'number-line-label'); // Pass svg
        }

        // Draw ticks for start, end, and all intermediate points
        let allPoints = new Set([startValue, endValue, ...chunkSteps.map(s => s.to), ...chunkSteps.map(s => s.from)]);
        let sortedPoints = Array.from(allPoints).sort((a, b) => a - b);
        let pointIndexMap = {};
        let currentIndex = 0;
        sortedPoints.forEach(point => {
            if (point >= displayRangeStart || (point === 0 && !drawScaleBreak)) {
                if (!(point < displayRangeStart && drawScaleBreak)){
                    pointIndexMap[point] = currentIndex++;
                    drawTickAndLabel(point, pointIndexMap[point]);
                }
            }
        });


        // Draw chunk jumps
        let strategyColorClass = `strategy-${strategy.toLowerCase()}`;
        chunkSteps.forEach((step, index) => {
            const x1 = valueToX(step.from);
            const x2 = valueToX(step.to);
             if (x1 > endX || x2 > endX || x1 < scaleStartX || x2 < scaleStartX || x1 == x2 ) return;

            const isLargeChunk = Math.abs(step.to - step.from) >= 10;
            const currentJumpHeight = isLargeChunk ? jumpHeightLarge : jumpHeightSmall;
            const staggerOffset = index % 2 === 0 ? 0 : currentJumpHeight * 0.4;

            createJumpArrow(svg, x1, numberLineY, x2, numberLineY, currentJumpHeight + staggerOffset, direction, strategyColorClass, arrowSize); // Pass arrowSize
            createText(svg, (x1 + x2) / 2, numberLineY - (currentJumpHeight + staggerOffset) - jumpLabelOffset, step.label, `jump-label ${strategyColorClass}`);
        });

        // Start point marker
         if (valueToX(startValue) >= scaleStartX) {
            drawStoppingPoint(svg, valueToX(startValue), numberLineY, 'Start', labelOffsetBase); // Pass labelOffsetBase
         }
    }

    function typesetMath() { /* Placeholder */ }

    // Initial run on page load
    runSubtractionChunkingAutomaton();

});
</script>

</body>
</html>
```

References

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction* [Includes supplementary material: Children's mathematics: Cognitively guided instruction – videotape logs]. Heinemann; The National Council of Teachers of Mathematics, Inc.

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