# Multilayer stream graphs

## JFLI Workshop - 2019

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# Introduction

**Graphs** : interactions (edges) between individuals (nodes).

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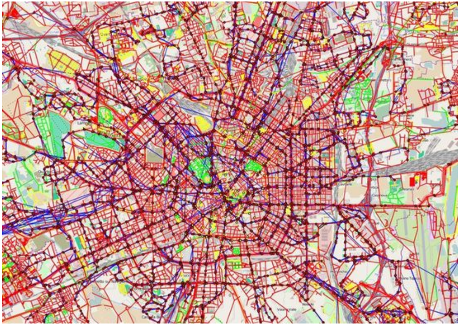

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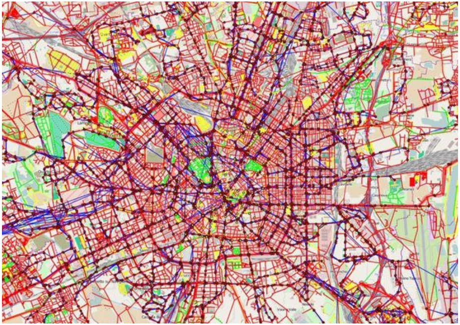

⇒ More complex systems : How to deal with...

- different types of nodes and/or different types of links ?
- time-dependence ?

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    - applicable
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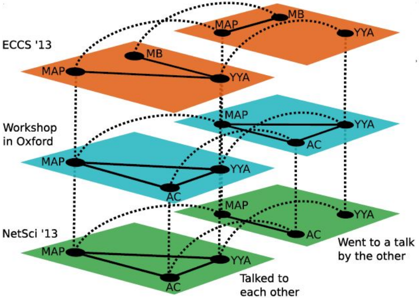
- generalizing the first two existing notions :
  - multilayer graphs
  - stream graphs

# Multilayer graphs : complex structures

$$M = (V_M, E_M, V, L)$$

$V = \{MAP, MB, YYA, AC\}$
$L = \{$conferences, relationship types$\}$
$V_M = \{$(MAP/ECCS'13/Talk to each other), etc $\}$
$E_M = \{$Edges between elements of $V_M\}$

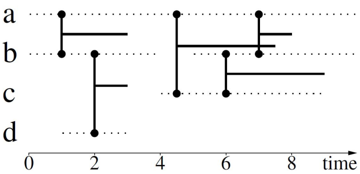



$$\left.\begin{array}{l}\text{Different types of interaction}\\ \text{Different types of nodes}\end{array}\right\} \rightarrow \textbf{More complex structures}$$

# Stream Graphs : time-dependence

$$S = (T, V, W, E)$$

$T = [0, 10]$
$V = \{a, b, c, d\}$
$W = \{(t, u) | u \text{ appears at } t\}$
$E = \{(t, (u, v)) | (u, v) \text{ appears at } t\}$

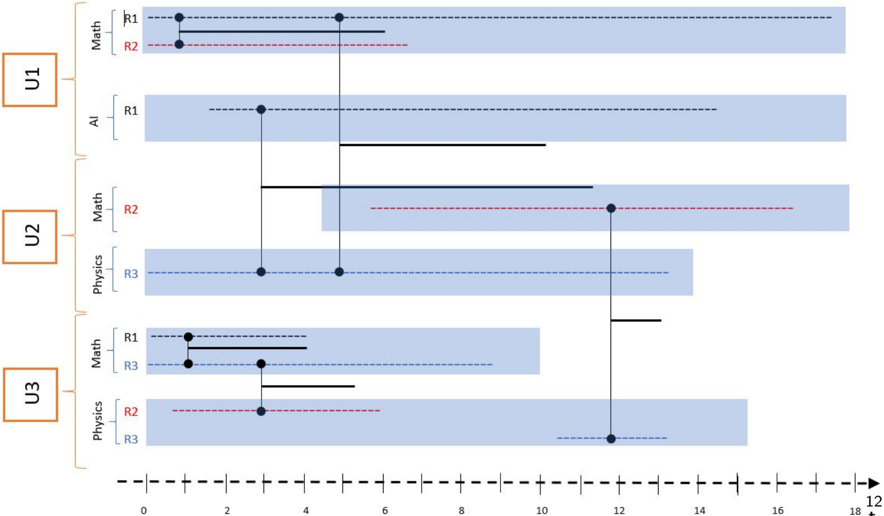



- nodes can appear or disappear in function of continuous time
- links can appear or disappear in function of continuous time

$\rightarrow$ **Model interactions over time**

# The multilayer stream graph

$$G = (T, T_M, V, W_M, E_M, \mathcal{L})$$



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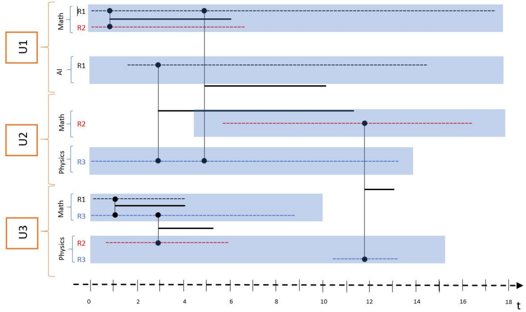


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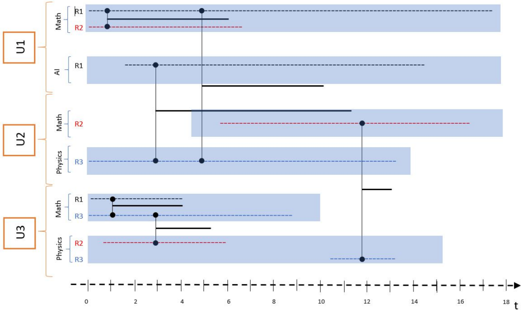


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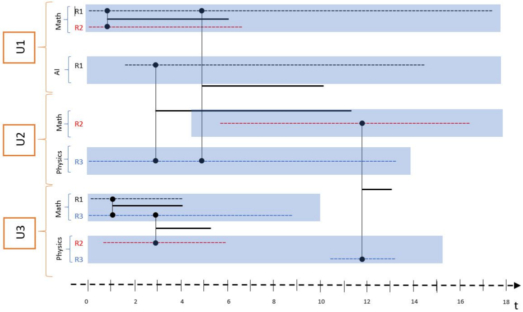


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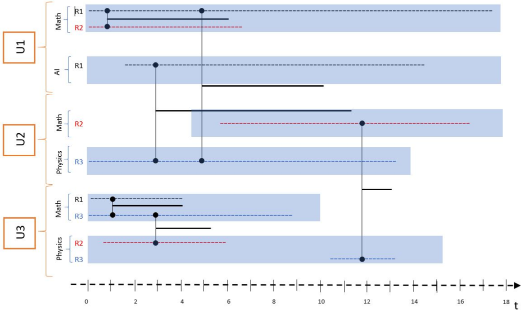


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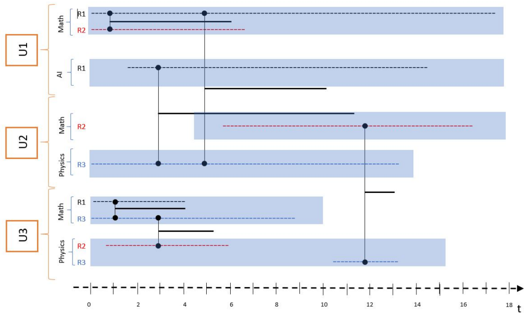


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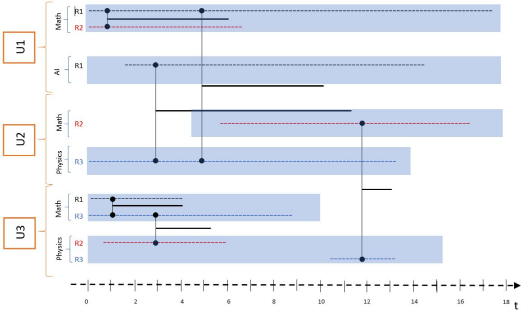


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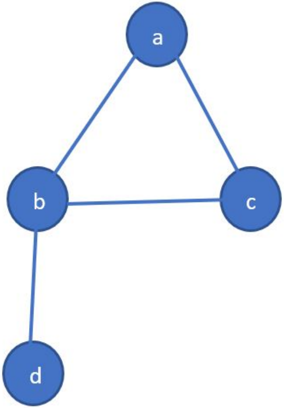
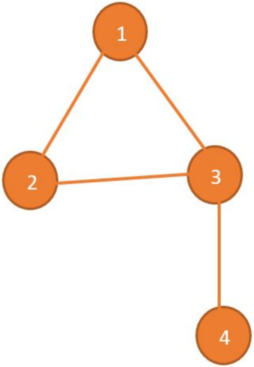
$E_M = \{(t, (R1, U1, Math), (R2, U2, Math) | t \in [1, 6]), \dots\}$

# Isomorphism

$G_1$ and $G_2$ are **isomorphic** if we can find a bijection which map the nodes of $G1$ to the nodes of $G2$



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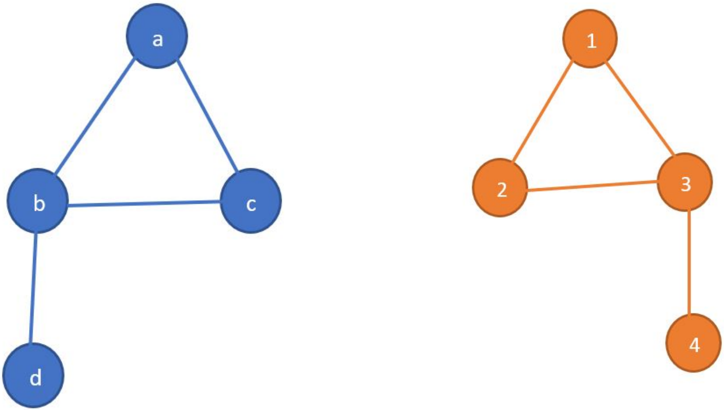


$$a \rightarrow 2, b \rightarrow 3, c \rightarrow 1, d \rightarrow 4$$

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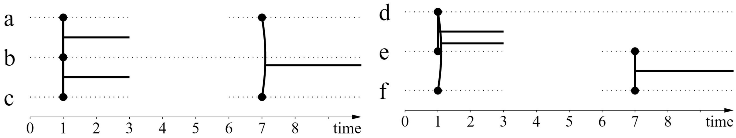

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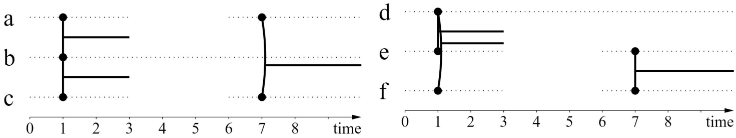


$a \to f$, $b \to d$, $c \to e$

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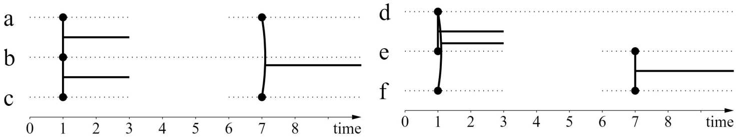


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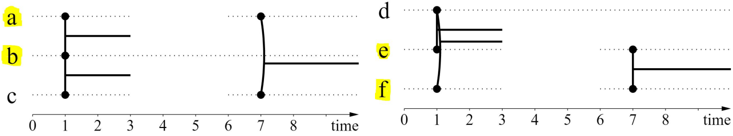

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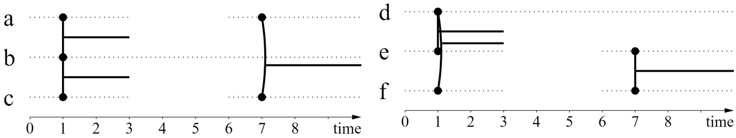


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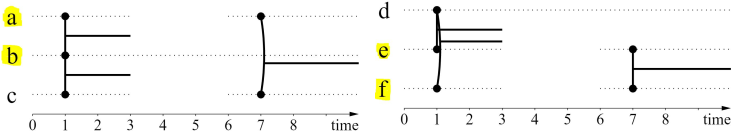
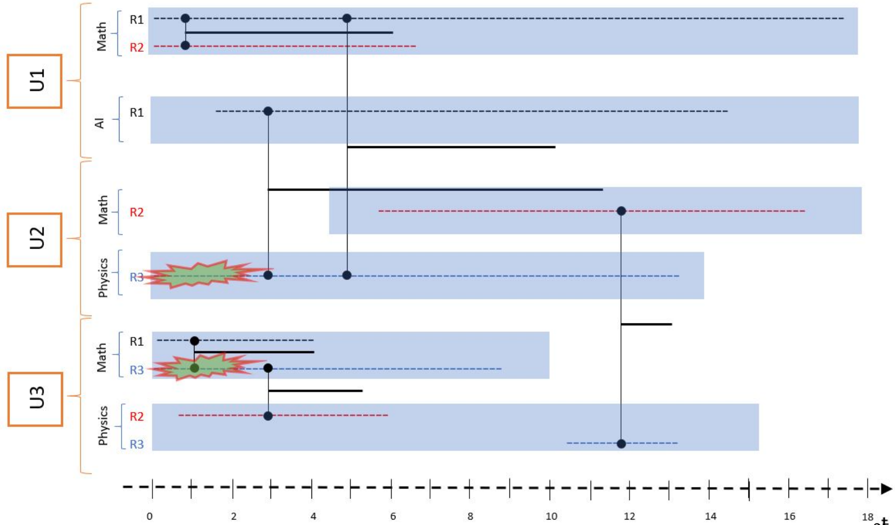



There is no bijection that respects layer structure.

# Possibilities : diffusion

Find the "decisive" times and and nodes/links/layer : stop a diffusion, find a weakness in the network...



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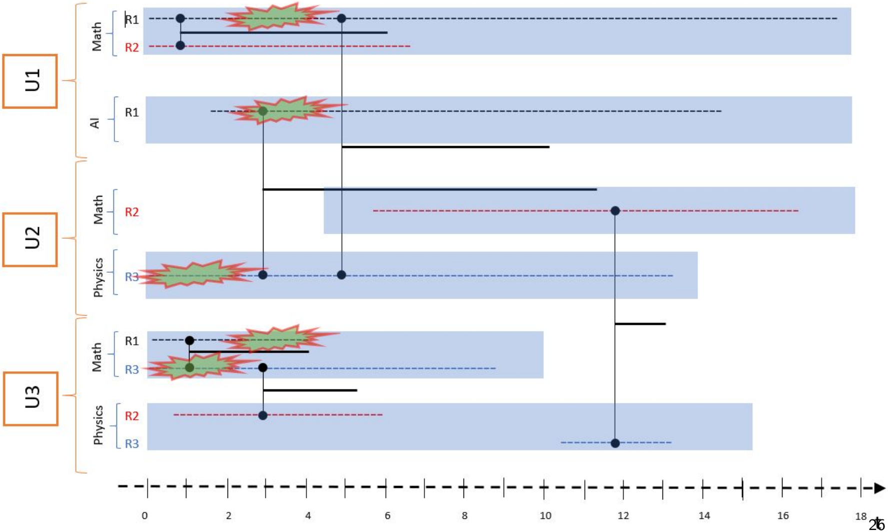

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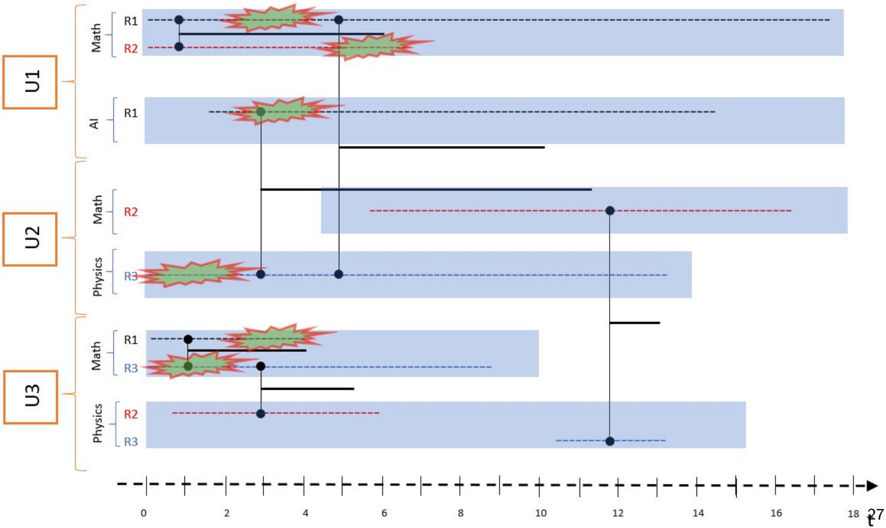
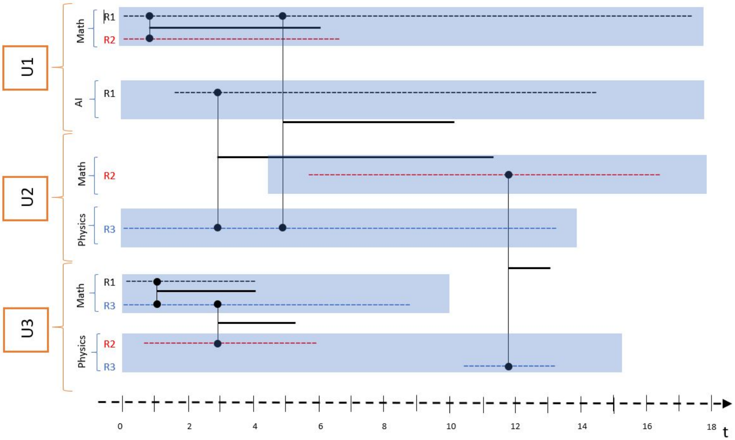
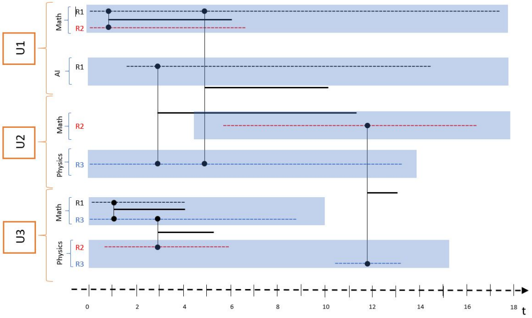


# Possibilities : influence

Measure "influence" : find a "precursory", find which parameter are "important" and which are not (ex : most of researchers who won the price P worked in dept D in university U at time T) ...



# Thank you !




- **Multilayer stream graphs** are graphs with nodes and edges of different types and can change with time
- What we have done : formal object $\mathbf{G} = (\mathbf{T}, \mathbf{T_M}, \mathbf{V}, \mathbf{W_M}, \mathbf{E_M}, \mathcal{L})$
- What we are doing : computational object, visualization
- Next step : **measure of "influence"**
- Tracks for the future : **study of diffusion**