

Chapter 1

- 1.1 $(2/3)^{10} = 0.0173$ yd; $6(2/3)^{10} = 0.104$ yd (compared to a total of 5 yd)
 1.3 $5/9$ 1.4 $9/11$ 1.5 $7/12$
 1.6 $11/18$ 1.7 $5/27$ 1.8 $25/36$
 1.9 $6/7$ 1.10 $15/26$ 1.11 $19/28$
 1.13 \$1646.99 1.15 Blank area = 1
 1.16 At $x = 1$: $1/(1+r)$; at $x = 0$: $r/(1+r)$; maximum escape at $x = 0$ is $1/2$.

- 2.1 1 2.2 $1/2$ 2.3 0
 2.4 ∞ 2.5 0 2.6 ∞
 2.7 e^2 2.8 0 2.9 1

- 4.1 $a_n = 1/2^n \rightarrow 0$; $S_n = 1 - 1/2^n \rightarrow 1$; $R_n = 1/2^n \rightarrow 0$
 4.2 $a_n = 1/5^{n-1} \rightarrow 0$; $S_n = (5/4)(1 - 1/5^n) \rightarrow 5/4$; $R_n = 1/(4 \cdot 5^{n-1}) \rightarrow 0$
 4.3 $a_n = (-1/2)^{n-1} \rightarrow 0$; $S_n = (2/3)[1 - (-1/2)^n] \rightarrow 2/3$; $R_n = (2/3)(-1/2)^n \rightarrow 0$
 4.4 $a_n = 1/3^n \rightarrow 0$; $S_n = (1/2)(1 - 1/3^n) \rightarrow 1/2$; $R_n = 1/(2 \cdot 3^n) \rightarrow 0$
 4.5 $a_n = (3/4)^{n-1} \rightarrow 0$; $S_n = 4[1 - (3/4)^n] \rightarrow 4$; $R_n = 4(3/4)^n \rightarrow 0$
 4.6 $a_n = \frac{1}{n(n+1)} \rightarrow 0$; $S_n = 1 - \frac{1}{n+1} \rightarrow 1$; $R_n = \frac{1}{n+1} \rightarrow 0$
 4.7 $a_n = (-1)^{n+1} \left(\frac{1}{n} + \frac{1}{n+1} \right) \rightarrow 0$; $S_n = 1 + \frac{(-1)^{n+1}}{n+1} \rightarrow 1$; $R_n = \frac{(-1)^n}{n+1} \rightarrow 0$

- 5.1 D 5.2 Test further 5.3 Test further
 5.4 D 5.5 D 5.6 Test further
 5.7 Test further 5.8 Test further
 5.9 D 5.10 D

- 6.5 (a) D 6.5 (b) D

Note: In the following answers, $I = \int_0^\infty a_n dn$; ρ = test ratio.

- 6.7 D, $I = \infty$ 6.8 D, $I = \infty$ 6.9 C, $I = 0$
 6.10 C, $I = \pi/6$ 6.11 C, $I = 0$ 6.12 C, $I = 0$
 6.13 D, $I = \infty$ 6.14 D, $I = \infty$ 6.18 D, $\rho = 2$
 6.19 C, $\rho = 3/4$ 6.20 C, $\rho = 0$ 6.21 D, $\rho = 5/4$
 6.22 C, $\rho = 0$ 6.23 D, $\rho = \infty$ 6.24 D, $\rho = 9/8$
 6.25 C, $\rho = 0$ 6.26 C, $\rho = (e/3)^3$ 6.27 D, $\rho = 100$
 6.28 C, $\rho = 4/27$ 6.29 D, $\rho = 2$ 6.31 D, cf. $\sum n^{-1}$
 6.32 D, cf. $\sum n^{-1}$ 6.33 C, cf. $\sum 2^{-n}$ 6.34 C, cf. $\sum n^{-2}$
 6.35 C, cf. $\sum n^{-2}$ 6.36 D, cf. $\sum n^{-1/2}$

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|-------|-------|-------|
| 7.1 C | 7.2 D | 7.3 C |
| 7.5 C | 7.6 D | 7.7 C |
| 7.4 C | 7.8 C | |
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| 9.1 D, cf. $\sum n^{-1}$ | 9.2 D, $a_n \not\rightarrow 0$ |
| 9.3 C, $I = 0$ | 9.4 D, $I = \infty$, or cf. $\sum n^{-1}$ |
| 9.5 C, cf. $\sum n^{-2}$ | 9.6 C, $\rho = 1/4$ |
| 9.7 D, $\rho = 4/3$ | 9.8 C, $\rho = 1/5$ |
| 9.9 D, $\rho = e$ | 9.10 D, $a_n \not\rightarrow 0$ |
| 9.11 D, $I = \infty$, or cf. $\sum n^{-1}$ | 9.12 C, cf. $\sum n^{-2}$ |
| 9.13 C, $I = 0$, or cf. $\sum n^{-2}$ | 9.14 C, alt. ser. |
| 9.15 D, $\rho = \infty$, $a_n \not\rightarrow 0$ | 9.16 C, cf. $\sum n^{-2}$ |
| 9.17 C, $\rho = 1/27$ | 9.18 C, alt. ser. |
| 9.19 C | 9.20 C |
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- | | | |
|----------------------|------------|-------------|
| 9.21 C, $\rho = 1/2$ | 9.22 (a) C | (b) D |
| | | (c) $k > e$ |
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- | | | |
|---|---------------------------|-------------------------------|
| 10.1 $ x < 1$ | 10.2 $ x < 3/2$ | 10.3 $ x \leq 1$ |
| 10.4 $ x \leq \sqrt{2}$ | 10.5 All x | 10.6 All x |
| 10.7 $-1 \leq x < 1$ | 10.8 $-1 < x \leq 1$ | 10.9 $ x < 1$ |
| 10.10 $ x \leq 1$ | 10.11 $-5 \leq x < 5$ | 10.12 $ x < 1/2$ |
| 10.13 $-1 < x \leq 1$ | 10.14 $ x < 3$ | 10.15 $-1 < x < 5$ |
| 10.16 $-1 < x < 3$ | 10.17 $-2 < x \leq 0$ | 10.18 $-3/4 \leq x \leq -1/4$ |
| 10.19 $ x < 3$ | 10.20 All x | 10.21 $0 \leq x \leq 1$ |
| 10.22 No x | 10.23 $x > 2$ or $x < -4$ | 10.24 $ x < \sqrt{5}/2$ |
| 10.25 $n\pi - \pi/6 < x < n\pi + \pi/6$ | | |
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- 13.4 $\binom{-1/2}{0} = 1; \binom{-1/2}{n} = \frac{(-1)^n(2n-1)!!}{(2n)!!}$
- Answers to part (b), Problems 5 to 19:
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|--|--|
| 13.5 $-\sum_1^{\infty} \frac{x^{n+2}}{n}$ | 13.6 $\sum_0^{\infty} \binom{1/2}{n} x^{n+1}$ (see Example 2) |
| 13.7 $\sum_0^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$ | 13.8 $\sum_0^{\infty} \binom{-1/2}{n} (-x^2)^n$ (see Problem 13.4) |
| 13.9 $1 + 2 \sum_1^{\infty} x^n$ | 13.10 $\sum_0^{\infty} \frac{(-1)^n x^{4n+2}}{(2n+1)!}$ |
| 13.11 $\sum_0^{\infty} \frac{(-1)^n x^n}{(2n+1)!}$ | 13.12 $\sum_0^{\infty} \frac{(-1)^n x^{4n+1}}{(2n)!(4n+1)}$ |
| 13.13 $\sum_0^{\infty} \frac{(-1)^n x^{2n+1}}{n!(2n+1)}$ | 13.14 $\sum_0^{\infty} \frac{x^{2n+1}}{2n+1}$ |
| 13.15 $\sum_0^{\infty} \binom{-1/2}{n} (-1)^n \frac{x^{2n+1}}{2n+1}$ | |
| 13.16 $\sum_0^{\infty} \frac{x^{2n}}{(2n)!}$ | 13.17 $2 \sum_{\text{oddn}}^{\infty} \frac{x^n}{n}$ |
| 13.18 $\sum_0^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!}$ | 13.19 $\sum_0^{\infty} \binom{-1/2}{n} \frac{x^{2n+1}}{2n+1}$ |
| 13.20 $x + x^2 + x^3/3 - x^5/30 - x^6/90 \dots$ | |
| 13.21 $x^2 + 2x^4/3 + 17x^6/45 \dots$ | |
| 13.22 $1 + 2x + 5x^2/2 + 8x^3/3 + 65x^4/24 \dots$ | |
| 13.23 $1 - x + x^3 - x^4 + x^6 \dots$ | |

- 13.24 $1 + x^2/2! + 5x^4/4! + 61x^6/6! \dots$
 13.25 $1 - x + x^2/3 - x^4/45 \dots$
 13.26 $1 + x^2/4 + 7x^4/96 + 139x^6/5760 \dots$
 13.27 $1 + x + x^2/2 - x^4/8 - x^5/15 \dots$
 13.28 $x - x^2/2 + x^3/6 - x^5/12 \dots$
 13.29 $1 + x/2 - 3x^2/8 + 17x^3/48 \dots$
 13.30 $1 - x + x^2/2 - x^3/2 + 3x^4/8 - 3x^5/8 \dots$
 13.31 $1 - x^2/2 - x^3/2 - x^4/4 - x^5/24 \dots$
 13.32 $x + x^2/2 - x^3/6 - x^4/12 \dots$
 13.33 $1 + x^3/6 + x^4/6 + 19x^5/120 + 19x^6/120 \dots$
 13.34 $x - x^2 + x^3 - 13x^4/12 + 5x^5/4 \dots$
 13.35 $1 + x^2/3! + 7x^4/(3 \cdot 5!) + 31x^6/(3 \cdot 7!) \dots$
 13.36 $u^2/2 + u^4/12 + u^6/20 \dots$
 13.37 $-(x^2/2 + x^4/12 + x^6/45 \dots)$
 13.38 $e(1 - x^2/2 + x^4/6 \dots)$
 13.39 $1 - (x - \pi/2)^2/2! + (x - \pi/2)^4/4! \dots$
 13.40 $1 - (x - 1) + (x - 1)^2 - (x - 1)^3 \dots$
 13.41 $e^3[1 + (x - 3) + (x - 3)^2/2! + (x - 3)^3/3! \dots]$
 13.42 $-1 + (x - \pi)^2/2! - (x - \pi)^4/4! \dots$
 13.43 $-[(x - \pi/2) + (x - \pi/2)^3/3 + 2(x - \pi/2)^5/15 \dots]$
 13.44 $5 + (x - 25)/10 - (x - 25)^2/10^3 + (x - 25)^3/(5 \cdot 10^4) \dots$
- 14.6 Error $< (1/2)(0.1)^2 \div (1 - 0.1) < 0.0056$
 14.7 Error $< (3/8)(1/4)^2 \div (1 - \frac{1}{4}) = 1/32$
 14.8 For $x < 0$, error $< (1/64)(1/2)^4 < 0.001$
 For $x > 0$, error $< 0.001 \div (1 - \frac{1}{2}) = 0.002$
 14.9 Term $n + 1$ is $a_{n+1} = \frac{1}{(n+1)(n+2)}$, so $R_n = (n + 2)a_{n+1}$.
 14.10 $S_4 = 0.3052$, error < 0.0021 (cf. $S = 1 - \ln 2 = 0.307$)
- 15.1 $-x^4/24 - x^5/30 \dots \simeq -3.376 \times 10^{-16}$
 15.2 $x^8/3 - 14x^{12}/45 \dots \simeq 1.433 \times 10^{-16}$
 15.3 $x^5/15 - 2x^7/45 \dots \simeq 6.667 \times 10^{-17}$
 15.4 $x^3/3 + 5x^4/6 \dots \simeq 1.430 \times 10^{-11}$
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|---------------------------------------|--|----------|
| 15.5 0 | 15.6 12 | 15.7 10! |
| 15.8 1/2 | 15.9 -1/6 | 15.10 -1 |
| 15.11 4 | 15.12 1/3 | 15.13 -1 |
| 15.14 $t - t^3/3$, error $< 10^{-6}$ | 15.15 $\frac{2}{3}t^{3/2} - \frac{2}{5}t^{5/2}$, error $< \frac{1}{7}10^{-7}$ | |
| 15.16 $e^2 - 1$ | 15.17 $\cos \frac{\pi}{2} = 0$ | |
| 15.18 $\ln 2$ | 15.19 $\sqrt{2}$ | |
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|----------------------|--------------|--------------|
| 15.20 (a) 1/8 | (b) 5e | (c) 9/4 |
| 15.21 (a) 0.397117 | (b) 0.937548 | (c) 1.291286 |
| 15.22 (a) $\pi^4/90$ | (b) 1.202057 | (c) 2.612375 |
| 15.23 (a) 1/2 | (b) 1/6 | (c) 1/3 |
| 15.24 (a) $-\pi$ | (b) 0 | (c) -1 |
| (d) 0 | (e) 0 | (f) 0 |
- 15.27 (a) $1 - \frac{v}{c} = 1.3 \times 10^{-5}$, or $v = 0.999987c$
 (b) $1 - \frac{v}{c} = 5.2 \times 10^{-7}$
 (c) $1 - \frac{v}{c} = 2.1 \times 10^{-10}$
 (d) $1 - \frac{v}{c} = 1.3 \times 10^{-11}$
- 15.28 $mc^2 + \frac{1}{2}mv^2$
 15.29 (a) $F/W = \theta + \theta^3/3 \dots$
 (b) $F/W = x/l + x^3/(2l^3) + 3x^5/(8l^5) \dots$

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|-------|--|--------------|----------------------------|-------------------|------------------------|
| 15.30 | (a) $T = F(5/x + x/40 - x^3/16000 \dots)$ | | | | |
| | (b) $T = \frac{1}{2}(F/\theta)(1 + \theta^2/6 + 7\theta^4/360 \dots)$ | | | | |
| 15.31 | (a) finite | (b) infinite | | | |
| 16.1 | (c) overhang: | 2 | 3 | 10 | |
| | books needed: | 32 | 228 | 2.7×10^8 | |
| 16.4 | C, $\rho = 0$ | 16.5 | D, $a_n \not\rightarrow 0$ | 16.6 | C, cf. $\sum n^{-3/2}$ |
| 16.7 | D, $I = \infty$ | 16.8 | D, cf. $\sum n^{-1}$ | 16.9 | $-1 \leq x < 1$ |
| 16.10 | $ x < 4$ | 16.11 | $ x \leq 1$ | 16.12 | $ x < 5$ |
| 16.13 | $-5 < x \leq 1$ | | | | |
| 16.14 | $1 - x^2/2 + x^3/2 - 5x^4/12 \dots$ | | | | |
| 16.15 | $-x^2/6 - x^4/180 - x^6/2835 \dots$ | | | | |
| 16.16 | $1 - x/2 + 3x^2/8 - 11x^3/48 + 19x^4/128 \dots$ | | | | |
| 16.17 | $1 + x^2/2 + x^4/4 + 7x^6/48 \dots$ | | | | |
| 16.18 | $x - x^3/3 + x^5/5 - x^7/7 \dots$ | | | | |
| 16.19 | $-(x - \pi) + (x - \pi)^3/3! - (x - \pi)^5/5! \dots$ | | | | |
| 16.20 | $2 + (x - 8)/12 - (x - 8)^2/(2^5 \cdot 3^2) + 5(x - 8)^3/(2^8 \cdot 3^4) \dots$ | | | | |
| 16.21 | $e[1 + (x - 1) + (x - 1)^2/2! + (x - 1)^3/3! \dots]$ | | | | |
| 16.22 | $\arctan 1 = \pi/4$ | 16.23 | $1 - (\sin \pi)/\pi = 1$ | | |
| 16.24 | $e^{\ln 3} - 1 = 2$ | 16.25 | -2 | | |
| 16.26 | $-1/3$ | 16.27 | $2/3$ | | |
| 16.28 | 1 | 16.29 | $6!$ | | |
| 16.30 | (b) For $N = 130$, $10.5821 < \zeta(1.1) < 10.5868$ | | | | |
| 16.31 | (a) 10^{430} terms. For $N = 200$, $100.5755 < \zeta(1.01) < 100.5803$ | | | | |
| 16.31 | (b) 2.66×10^{86} terms. For $N = 15$, $1.6905 < S < 1.6952$ | | | | |
| 16.31 | (c) $e^{e^{200}} = 10^{3.1382 \times 10^{86}}$ terms. For $N = 40$, $38.4048 < S < 38.4088$ | | | | |

Chapter 2

	x	y	r	θ
4.1	1	1	$\sqrt{2}$	$\pi/4$
4.2	-1	1	$\sqrt{2}$	$3\pi/4$
4.3	1	$-\sqrt{3}$	2	$-\pi/3$
4.4	$-\sqrt{3}$	1	2	$5\pi/6$
4.5	0	2	2	$\pi/2$
4.6	0	-4	4	$-\pi/2$
4.7	-1	0	1	π
4.8	3	0	3	0
4.9	-2	2	$2\sqrt{2}$	$3\pi/4$
4.10	2	-2	$2\sqrt{2}$	$-\pi/4$
4.11	$\sqrt{3}$	1	2	$\pi/6$
4.12	-2	$-2\sqrt{3}$	4	$-2\pi/3$
4.13	0	-1	1	$3\pi/2$
4.14	$\sqrt{2}$	$\sqrt{2}$	2	$\pi/4$
4.15	-1	0	1	$-\pi$ or π
4.16	5	0	5	0
4.17	1	-1	$\sqrt{2}$	$-\pi/4$
4.18	0	3	3	$\pi/2$
4.19	4.69	1.71	5	$20^\circ = 0.35$
4.20	-2.39	-6.58	7	$-110^\circ = -1.92$
5.1	1/2	-1/2	$1/\sqrt{2}$	$-\pi/4$
5.2	-1/2	-1/2	$1/\sqrt{2}$	$-3\pi/4$ or $5\pi/4$
5.3	1	0	1	0
5.4	0	2	2	$\pi/2$
5.5	2	$2\sqrt{3}$	4	$\pi/3$
5.6	-1	0	1	π
5.7	7/5	-1/5	$\sqrt{2}$	$-8.13^\circ = -0.14$
5.8	1.6	-2.7	3.14	$-59.3^\circ = -1.04$
5.9	-10.4	22.7	25	$2 = 114.6^\circ$
5.10	-25/17	19/17	$\sqrt{58/17}$	$142.8^\circ = 2.49$
5.11	17	-12	20.8	$-35.2^\circ = -0.615$
5.12	2.65	1.41	3	$28^\circ = 0.49$
5.13	1.55	4.76	5	$2\pi/5$
5.14	1.27	-2.5	2.8	$-1.1 = -63^\circ$
5.15	21/29	-20/29	1	$-43.6^\circ = -0.76$
5.16	1.53	-1.29	2	$-40^\circ = -0.698$
5.17	-7.35	-10.9	13.1	$-124^\circ = -2.16$
5.18	-0.94	-0.36	1	201° or -159° , 3.51 or -2.77

- 5.19 $(2 + 3i)/13; (x - yi)/(x^2 + y^2)$
5.20 $(-5 + 12i)/169; (x^2 - y^2 - 2ixy)/(x^2 + y^2)^2$
5.21 $(1 + i)/6; (x + 1 - iy)/[(x + 1)^2 + y^2]$
5.22 $(1 + 2i)/10; [x - i(y - 1)]/[x^2 + (y - 1)^2]$
5.23 $(-6 - 3i)/5; (1 - x^2 - y^2 + 2iy)/[(1 - x)^2 + y^2]$
5.24 $(-5 - 12i)/13; (x^2 - y^2 + 2ixy)/(x^2 + y^2)$
5.26 1
5.27 $\sqrt{13/2}$
5.28 1
5.29 $5\sqrt{5}$
5.30 $3/2$
5.31 1
5.32 169
5.33 5
5.34 1
5.35 $x = -4, y = 3$
5.36 $x = -1/2, y = 3$
5.37 $x = y = 0$
5.38 $x = -7, y = 2$
5.39 $x = y = \text{any real number}$
5.40 $x = 0, y = 3$
5.41 $x = 1, y = -1$
5.42 $x = -1/7, y = -10/7$
5.43 $(x, y) = (0, 0), \text{ or } (1, 1), \text{ or } (-1, 1)$
5.44 $x = 0, y = -2$
5.45 $x = 0, \text{ any real } y; \text{ or } y = 0, \text{ any real } x$
5.46 $y = -x$
5.47 $(x, y) = (-1, 0), (1/2, \pm\sqrt{3}/2)$
5.48 $x = 36/13, y = 2/13$
5.49 $x = 1/2, y = 0$
5.50 $x = 0, y \geq 0$
5.51 Circle, center at origin, radius = 2
5.52 y axis
5.53 Circle, center at $(1, 0), r = 1$
5.54 Disk, center at $(1, 0), r = 1$
5.55 Line $y = 5/2$
5.56 Positive y axis
5.57 Hyperbola, $x^2 - y^2 = 4$
5.58 Half plane, $x > 2$
5.59 Circle, center at $(0, -3), r = 4$
5.60 Circle, center at $(1, -1), r = 2$
5.61 Half plane, $y < 0$
5.62 Ellipse, foci at $(1, 0)$ and $(-1, 0)$, semi-major axis = 4
5.63 The coordinate axes
5.64 Straight lines, $y = \pm x$
5.65 $v = (4t^2 + 1)^{-1}, a = 4(4t^2 + 1)^{-3/2}$
5.66 Motion around circle $r = 1$, with $v = 2, a = 4$
5.67
5.68
6.2 $D, \rho = \sqrt{2}$
6.3 $C, \rho = 1/\sqrt{2}$
6.4 $D, |a_n| = 1 \not\rightarrow 0$
6.5 D
6.6 C
6.7 $D, \rho = \sqrt{2}$
6.8 $D, |a_n| = 1 \not\rightarrow 0$
6.9 C
6.10 $C, \rho = \sqrt{2}/2$
6.11 $C, \rho = 1/5$
6.12 C
6.13 $C, \rho = \sqrt{2/5}$
7.1 All z
7.2 $|z| < 1$
7.3 All z
7.4 $|z| < 1$
7.5 $|z| < 2$
7.6 $|z| < 1/3$
7.7 All z
7.8 All z
7.9 $|z| < 1$
7.10 $|z| < 1$
7.11 $|z| < 27$
7.12 $|z| < 4$
7.13 $|z - i| < 1$
7.14 $|z - 2i| < 1$
7.15 $|z - (2 - i)| < 2$
7.16 $|z + (i - 3)| < 1/\sqrt{2}$
8.3 See Problem 17.30.

- 9.1 $(1-i)/\sqrt{2}$ 9.2 i 9.3 $-9i$
 9.4 $-e(1+i\sqrt{3})/2$ 9.5 -1 9.6 1
 9.7 $3e^2$ 9.8 $-\sqrt{3}+i$ 9.9 $-2i$
 9.10 -2 9.11 $-1-i$ 9.12 $-2-2i\sqrt{3}$
 9.13 $-4+4i$ 9.14 64 9.15 $2i-4$
 9.16 $-2\sqrt{3}-2i$ 9.17 $-(1+i)/4$ 9.18 1
 9.19 16 9.20 i 9.21 1
 9.22 $-i$ 9.23 $(\sqrt{3}+i)/4$ 9.24 $4i$
 9.25 -1 9.26 $(1+i\sqrt{3})/2$ 9.29 1
 9.30 $e^{\sqrt{3}}$ 9.31 5 9.32 $3e^2$
 9.33 $2e^3$ 9.34 $4/e$ 9.35 21
 9.36 4 9.37 1 9.38 $1/\sqrt{2}$
- 10.1 $1, (-1 \pm i\sqrt{3})/2$ 10.2 $3, 3(-1 \pm i\sqrt{3})/2$
 10.3 $\pm 1, \pm i$ 10.4 $\pm 2, \pm 2i$
 10.5 $\pm 1, (\pm 1 \pm i\sqrt{3})/2$ 10.6 $\pm 2, \pm 1 \pm i\sqrt{3}$
 10.7 $\pm\sqrt{2}, \pm i\sqrt{2}, \pm 1 \pm i$ 10.8 $\pm 1, \pm i, (\pm 1 \pm i)/\sqrt{2}$
 10.9 $1, 0.309 \pm 0.951i, -0.809 \pm 0.588i$
 10.10 $2, 0.618 \pm 1.902i, -1.618 \pm 1.176i$
 10.11 $-2, 1 \pm i\sqrt{3}$ 10.12 $-1, (1 \pm i\sqrt{3})/2$
 10.13 $\pm 1 \pm i$ 10.14 $(\pm 1 \pm i)/\sqrt{2}$
 10.15 $\pm 2i, \pm\sqrt{3} \pm i$ 10.16 $\pm i, (\pm\sqrt{3} \pm i)/2$
 10.17 $-1, 0.809 \pm 0.588i, -0.309 \pm 0.951i$
 10.18 $\pm(1+i)/\sqrt{2}$ 10.19 $-i, (\pm\sqrt{3}+i)/2$
 10.20 $2i, \pm\sqrt{3}-i$ 10.21 $\pm(\sqrt{3}+i)$
 10.22 $r = \sqrt{2}, \theta = 45^\circ + 120^\circ n: 1+i, -1.366+0.366i, 0.366-1.366i$
 10.23 $r = 2, \theta = 30^\circ + 90^\circ n: \pm(\sqrt{3}+i), \pm(1-i\sqrt{3})$
 10.24 $r = 1, \theta = 30^\circ + 45^\circ n:$
 $\pm(\sqrt{3}+i)/2, \pm(1-i\sqrt{3})/2, \pm(0.259+0.966i), \pm(0.966-0.259i)$
 10.25 $r = \sqrt[10]{2}, \theta = 45^\circ + 72^\circ n: 0.758(1+i), -0.487+0.955i,$
 $-1.059-0.168i, -0.168-1.059i, 0.955-0.487i$
 10.26 $r = 1, \theta = 18^\circ + 72^\circ n: i, \pm 0.951+0.309i, \pm 0.588-0.809i$
 10.28 $\cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta$
 $\sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta$
- 11.3 $3(1-i)/\sqrt{2}$ 11.4 -8 11.5 $1+i$ 11.6 $13/5$
 11.7 $3i/5$ 11.8 $-41/9$ 11.9 $4i/3$ 11.10 -1
- 12.20 $\cosh 3z = \cosh^3 z + 3 \cosh z \sinh^2 z, \sinh 3z = 3 \cosh^2 z \sinh z + \sinh^3 z$
 12.22 $\sinh x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}, \cosh x = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$
 12.23 $\cos x, |\cos x|$
 12.24 $\cosh x$
 12.25 $\sin x \cosh y - i \cos x \sinh y, \sqrt{\sin^2 x + \sinh^2 y}$
 12.26 $\cosh 2 \cos 3 - i \sinh 2 \sin 3 = -3.725 - 0.512i, 3.760$
 12.27 $\sin 4 \cosh 3 + i \cos 4 \sinh 3 = -7.62 - 6.55i, 10.05$
 12.28 $\tanh 1 = 0.762$ 12.29 1
 12.30 $-i$ 12.31 $(3+5i\sqrt{3})/8$
 12.32 $-4i/3$ 12.33 $i \tanh 1 = 0.762i$
 12.34 $i \sinh(\pi/2) = 2.301i$ 12.35 $-\cosh 2 = -3.76$
 12.36 $i \cosh 1 = 1.543i$ 12.37 $\cosh \pi$

- 14.1 $1 + i\pi$ 14.2 $-i\pi/2$ or $3\pi i/2$
14.3 $\text{Ln } 2 + i\pi/6$ 14.4 $(1/2) \text{Ln } 2 + 3\pi i/4$
14.5 $\text{Ln } 2 + 5i\pi/4$ 14.6 $-i\pi/4$ or $7\pi i/4$
14.7 $i\pi/2$ 14.8 $-1, (1 \pm i\sqrt{3})/2$
14.9 $e^{-\pi}$ 14.10 $e^{-\pi^2/4}$
14.11 $\cos(\text{Ln } 2) + i \sin(\text{Ln } 2) = 0.769 + 0.639i$
14.12 $-ie^{-\pi/2}$
14.13 $1/e$
14.14 $2e^{-\pi/2}[i \cos(\text{Ln } 2) - \sin(\text{Ln } 2)] = 0.3198i - 0.2657$
14.15 $e^{-\pi \sinh 1} = 0.0249$
14.16 $e^{-\pi/3} = 0.351$
14.17 $\sqrt{2} e^{-3\pi/4} e^{i(\text{Ln } \sqrt{2} + 3\pi/4)} = -0.121 + 0.057i$
14.18 -1 14.19 $-5/4$
14.20 1 14.21 -1
14.22 $-1/2$ 14.23 $e^{\pi/2} = 4.81$

15.1 $\pi/2 + 2n\pi \pm i \text{Ln } (2 + \sqrt{3}) = \pi/2 + 2n\pi \pm 1.317i$
15.2 $\pi/2 + n\pi + (i \text{Ln } 3)/2$
15.3 $i(\pm\pi/3 + 2n\pi)$
15.4 $i(2n\pi + \pi/6), i(2n\pi + 5\pi/6)$
15.5 $\pm [\pi/2 + 2n\pi - i \text{Ln } (3 + \sqrt{8})] = \pm [\pi/2 + 2n\pi - 1.76i]$
15.6 $i(n\pi - \pi/4)$
15.7 $\pi/2 + n\pi - i \text{Ln } (\sqrt{2} - 1) = \pi/2 + n\pi + 0.881i$
15.8 $\pi/2 + 2n\pi \pm i \text{Ln } 3$
15.9 $i(\pi/3 + n\pi)$
15.10 $2n\pi \pm i \text{Ln } 2$
15.11 $i(2n\pi + \pi/4), i(2n\pi + 3\pi/4)$
15.12 $i(2n\pi \pm \pi/6)$
15.13 $i(\pi + 2n\pi)$
15.14 $2n\pi + i \text{Ln } 2, (2n + 1)\pi - i \text{Ln } 2$
15.15 $n\pi + 3\pi/8 + i \text{Ln } 2)/4$
15.16 $(\text{Ln } 2)/4 + i(n\pi + 5\pi/8)$

16.2 Motion around circle $|z| = 5; v = 5\omega, a = 5\omega^2$.
16.3 Motion around circle $|z| = \sqrt{2}; v = \sqrt{2}, a = \sqrt{2}$.
16.4 $v = \sqrt{13}, a = 0$
16.5 $v = |z_1 - z_2|, a = 0$
16.6 (a) Series: $3 - 2i$ (b) Series: $2(1 + i\sqrt{3})$
Parallel: $5 + i$ Parallel: $i\sqrt{3}$
16.7 (a) Series: $1 + 2i$ (b) Series: $5 + 5i$
Parallel: $3(3 - i)/5$ Parallel: $1.6 + 1.2i$
16.8 $[R - i(\omega CR^2 + \omega^3 L^2 C - \omega L)] / [(\omega CR)^2 + (\omega^2 LC - 1)^2]$; this
simplifies to $\frac{L}{RC}$ if $\omega^2 = \frac{1}{LC} \left(1 - \frac{R^2 C}{L}\right)$, that is, at resonance.

16.9 (a) $\omega = \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}$ (b) $\omega = 1/\sqrt{LC}$
16.10 (a) $\omega = -\frac{1}{2RC} \pm \sqrt{\frac{1}{4R^2 C^2} + \frac{1}{LC}}$ (b) $\omega = 1/\sqrt{LC}$
16.12 $(1 + r^4 - 2r^2 \cos \theta)^{-1}$

- 17.1 -1 17.2 $(\sqrt{3} + i)/2$
 17.3 $r = \sqrt{2}, \theta = 45^\circ + 72^\circ n : 1 + i, -0.642 + 1.260i, -1.397 - 0.221i,$
 $-0.221 - 1.397i, 1.260 - 0.642i$
 17.4 $i \cosh 1 = 1.54i$ 17.5 i
 17.6 $-e^{-\pi^2} = -5.17 \times 10^{-5}$ or $-e^{-\pi^2} \cdot e^{\pm 2n\pi^2}$
 17.7 $e^{\pi/2} = 4.81$ or $e^{\pi/2} \cdot e^{\pm 2n\pi}$
 17.8 -1 17.9 $\pi/2 \pm 2n\pi$
 17.10 $\sqrt{3} - 2$ 17.11 i
 17.12 $-1 \pm \sqrt{2}$ 17.13 $x = 0, y = 4$
 17.14 Circle with center $(0, 2)$, radius 1
 17.15 $|z| < 1/e$ 17.16 $y < -2$
 17.26 1 17.27 (c) $e^{-2(x-t)^2}$
 17.28 $1 + \left[\frac{a^2 + b^2}{2ab} \right]^2 \sinh^2 b$ 17.29 $(-1 \pm i\sqrt{3})/2$
 17.30 $e^x \cos x = \sum_{n=0}^{\infty} \frac{x^n 2^{n/2} \cos n\pi/4}{n!}$
 $e^x \sin x = \sum_{n=0}^{\infty} \frac{x^n 2^{n/2} \sin n\pi/4}{n!}$

Chapter 3

$$2.3 \quad \begin{pmatrix} 1 & 0 & -3 \\ 0 & 1 & 5 \end{pmatrix}, \quad x = -3, y = 5)$$

$$2.4 \quad \begin{pmatrix} 1 & 0 & -1/2 & 1/2 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \quad x = (z + 1)/2, y = 1$$

$$2.5 \quad \begin{pmatrix} 1 & 1/2 & -1/2 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \text{no solution}$$

$$2.6 \quad \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -1 & 0 \end{pmatrix}, \quad x = 1, z = y$$

$$2.7 \quad \begin{pmatrix} 1 & 0 & -4 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}, \quad x = -4, y = 3$$

$$2.8 \quad \begin{pmatrix} 1 & -1 & 0 & -11 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad x = y - 11, z = 7$$

$$2.9 \quad \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \text{inconsistent, no solution}$$

$$2.10 \quad \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \text{inconsistent, no solution}$$

$$2.11 \quad \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -3 \end{pmatrix}, \quad x = 2, y = -1, z = -3$$

$$2.12 \quad \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}, \quad x = -2, y = 1, z = 1$$

$$2.13 \quad \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & -2 & 5/2 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad x = -2, y = 2z + 5/2$$

$$2.14 \quad \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \text{inconsistent, no solution}$$

- $$\begin{array}{ll}
2.15 & R = 2 \\
2.17 & R = 2 \\
3.1 & -11 \qquad 3.2 \quad -721 \qquad 3.3 \quad 1 \qquad 3.4 \quad 2140 \\
3.5 & -544 \qquad 3.6 \quad 4 \qquad 3.11 \quad 0 \qquad 3.12 \quad 16 \\
3.16 & A = -(K + ik)/(K - ik), |A| = 1 \\
3.17 & x = \gamma(x' + vt'), t = \gamma(t' + vx'/c^2) \\
3.18 & D = 3b(a + b)(a^2 + ab + b^2), z = 1 \\
& \text{(Also } x = a + 2b, y = a - b; \text{ these were not required.)} \\
4.11 & -3\mathbf{i} + 8\mathbf{j} - 6\mathbf{k}, \mathbf{i} - 10\mathbf{j} + 3\mathbf{k}, 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}. \\
4.12 & \arccos(-1/\sqrt{2}) = 3\pi/4 \\
4.13 & -5/3, -1, \cos\theta = -1/3 \\
4.14 & (a) \arccos(1/3) = 70.5^\circ \\
& (b) \arccos(1/\sqrt{3}) = 54.7^\circ \\
& (c) \arccos\sqrt{2/3} = 35.3^\circ \\
4.15 & (a) (2\mathbf{i} - \mathbf{j} + 2\mathbf{k})/3 \\
& (b) 8\mathbf{i} - 4\mathbf{j} + 8\mathbf{k} \\
& (c) \text{Any combination of } \mathbf{i} + 2\mathbf{j} \text{ and } \mathbf{i} - \mathbf{k}. \\
& (d) \text{Answer to (c) divided by its magnitude.} \\
4.17 & \text{Legs} = \text{any two vectors with dot product} = 0; \\
& \text{hypotenuse} = \text{their sum (or difference).} \\
4.18 & 2\mathbf{i} - 8\mathbf{j} - 3\mathbf{k} \qquad 4.19 \quad \mathbf{i} + \mathbf{j} + \mathbf{k} \\
4.20 & 2\mathbf{i} - 2\mathbf{j} + \mathbf{k} \qquad 4.22 \quad \text{Law of cosines} \\
4.24 & A^2 B^2
\end{array}$$

In the following answers, note that the point and vector used may be *any* point on the line and *any* vector along the line.

- 5.1 $\mathbf{r} = (2, -3) + (4, 3)t$
- 5.2 $3/2$
- 5.3 $\mathbf{r} = (3, 0) + (1, 1)t$
- 5.4 $\mathbf{r} = (1, 0) + (2, 1)t$
- 5.5 $\mathbf{r} = \mathbf{j}t$
- 5.6 $\frac{x-1}{1} = \frac{y+1}{-2} = \frac{z+5}{2}; \quad \mathbf{r} = (1, -1, -5) + (1, -2, 2)t$
- 5.7 $\frac{x-2}{3} = \frac{y-3}{-2} = \frac{z-4}{-6}; \quad \mathbf{r} = (2, 3, 4) + (3, -2, -6)t$
- 5.8 $\frac{x}{3} = \frac{z-4}{-5}, y = -2; \quad \mathbf{r} = (0, -2, 4) + (3, 0, -5)t$
- 5.9 $x = -1, z = 7; \quad \mathbf{r} = -\mathbf{i} + 7\mathbf{k} + \mathbf{j}t$
- 5.10 $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z+1}{6}; \quad \mathbf{r} = (3, 4, -1) + (2, -3, 6)t$
- 5.11 $\frac{x-4}{1} = \frac{z-3}{-2}, y = -1; \quad \mathbf{r} = (4, -1, 3) + (1, 0, -2)t$
- 5.12 $\frac{x-5}{5} = \frac{y+4}{-2} = \frac{z-2}{1}; \quad \mathbf{r} = (5, -4, 2) + (5, -2, 1)t$
- 5.13 $x = 3, \frac{y}{3} = \frac{z+5}{1}; \quad \mathbf{r} = 3\mathbf{i} - 5\mathbf{k} + (-3\mathbf{j} + \mathbf{k})t$
- 5.14 $36x - 3y - 22z = 23$
- 5.15 $5x + 6y + 3z = 0$
- 5.16 $5x - 2y + z = 35$
- 5.17 $3y - z = 5$
- 5.18 $x + 6y + 7z + 5 = 0$
- 5.19 $x + y + 3z + 12 = 0$
- 5.20 $x - 4y - z + 5 = 0$
- 5.21 $\cos \theta = 25 / (7\sqrt{30}) = 0.652, \theta = 49.3^\circ$
- 5.22 $\cos \theta = 2 / \sqrt{6}, \theta = 35.3^\circ$
- 5.23 $\cos \theta = 4 / 21, \theta = 79^\circ$
- 5.24 $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + (\mathbf{j} + 2\mathbf{k})t, d = 2\sqrt{6/5}$
- 5.25 $\mathbf{r} = (1, -2, 0) + (4, 9, -1)t, d = (3\sqrt{3}) / 7$
- 5.26 $\mathbf{r} = (8, 1, 7) + (14, 2, 15)t, d = \sqrt{2/17}$
- 5.27 $y + 2z + 1 = 0$
- 5.28 $4x + 9y - z + 27 = 0$
- 5.29 $2/\sqrt{6}$
- 5.30 1
- 5.31 $5/7$
- 5.32 $10/\sqrt{27}$

5.33 $\sqrt{43/15}$

5.34 $\sqrt{11/10}$

5.35 $\sqrt{5}$

5.36 3

5.37 Intersect at $(1, -3, 4)$

5.38 $\arccos \sqrt{21/22} = 12.3^\circ$

5.39 $t_1 = 1, t_2 = -2$, intersect at $(3, 2, 0)$, $\cos \theta = 5/\sqrt{60}$, $\theta = 49.8^\circ$

5.40 $t_1 = -1, t_2 = 1$, intersect at $(4, -1, 1)$, $\cos \theta = 5/\sqrt{39}$, $\theta = 36.8^\circ$

5.41 $\sqrt{14}$

5.42 $1/\sqrt{5}$

5.43 $20/\sqrt{21}$

5.44 $2/\sqrt{10}$

5.45 $d = \sqrt{2}, t = -1$

6.1 $AB = \begin{pmatrix} -5 & 10 \\ 1 & 24 \end{pmatrix} \quad BA = \begin{pmatrix} -2 & 8 \\ 11 & 21 \end{pmatrix} \quad A + B = \begin{pmatrix} 1 & 3 \\ 3 & 9 \end{pmatrix}$

$A - B = \begin{pmatrix} 5 & -1 \\ 1 & 1 \end{pmatrix} \quad A^2 = \begin{pmatrix} 11 & 8 \\ 16 & 27 \end{pmatrix} \quad B^2 = \begin{pmatrix} 6 & 4 \\ 2 & 18 \end{pmatrix}$

$5A = \begin{pmatrix} 15 & 5 \\ 10 & 25 \end{pmatrix} \quad 3B = \begin{pmatrix} -6 & 6 \\ 3 & 12 \end{pmatrix} \quad \det(5A) = 5^2 \det A$

6.2 $AB = \begin{pmatrix} -2 & -2 \\ 1 & 2 \end{pmatrix} \quad BA = \begin{pmatrix} -6 & 17 \\ -2 & 6 \end{pmatrix} \quad A + B = \begin{pmatrix} 1 & -1 \\ -1 & 5 \end{pmatrix}$

$A - B = \begin{pmatrix} 3 & -9 \\ -1 & 1 \end{pmatrix} \quad A^2 = \begin{pmatrix} 9 & -25 \\ -5 & 14 \end{pmatrix} \quad B^2 = \begin{pmatrix} 1 & 4 \\ 0 & 4 \end{pmatrix}$

$5A = \begin{pmatrix} 10 & -25 \\ -5 & 15 \end{pmatrix} \quad 3B = \begin{pmatrix} -3 & 12 \\ 0 & 6 \end{pmatrix}$

6.3 $AB = \begin{pmatrix} 7 & -1 & 0 \\ 3 & 1 & -1 \\ 3 & 9 & 5 \end{pmatrix} \quad BA = \begin{pmatrix} 4 & -1 & 2 \\ 6 & 3 & 1 \\ 0 & 1 & 6 \end{pmatrix} \quad A + B = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 1 & 1 \\ 3 & 4 & 1 \end{pmatrix}$

$A - B = \begin{pmatrix} 0 & -1 & 2 \\ 3 & -3 & -1 \\ -3 & 6 & 1 \end{pmatrix} \quad A^2 = \begin{pmatrix} 1 & 10 & 4 \\ 0 & 1 & 6 \\ 15 & 0 & 1 \end{pmatrix} \quad B^2 = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 3 & 2 \\ 3 & 1 & -1 \end{pmatrix}$

$5A = \begin{pmatrix} 5 & 0 & 10 \\ 15 & -5 & 0 \\ 0 & 25 & 5 \end{pmatrix} \quad 3B = \begin{pmatrix} 3 & 3 & 0 \\ 0 & 6 & 3 \\ 9 & -3 & 0 \end{pmatrix} \quad \det(5A) = 5^3 \det A$

6.4 $BA = \begin{pmatrix} 12 & 10 & 2 & 12 \\ 0 & 2 & 1 & -9 \\ 4 & 8 & 3 & -17 \end{pmatrix} \quad C^2 = \begin{pmatrix} 5 & 1 & 7 \\ 6 & 5 & 12 \\ -3 & -1 & -2 \end{pmatrix}$

$CB = \begin{pmatrix} 14 & 4 \\ 1 & 19 \\ 1 & -5 \end{pmatrix} \quad C^3 = \begin{pmatrix} 7 & 4 & 20 \\ 20 & 1 & 20 \\ -8 & -2 & -9 \end{pmatrix}$

$C^2B = \begin{pmatrix} 32 & 12 \\ 53 & 7 \\ -13 & -9 \end{pmatrix} \quad CBA = \begin{pmatrix} 36 & 46 & 14 & -36 \\ 40 & 22 & 1 & 91 \\ -8 & -2 & 1 & -29 \end{pmatrix}$

6.5 $AA^T = \begin{pmatrix} 30 & -13 \\ -13 & 30 \end{pmatrix} \quad A^TA = \begin{pmatrix} 8 & 8 & 2 & 2 \\ 8 & 10 & 3 & -7 \\ 2 & 3 & 1 & -4 \\ 2 & -7 & -4 & 41 \end{pmatrix}$

$BB^T = \begin{pmatrix} 20 & -2 & 2 \\ -2 & 2 & 4 \\ 2 & 4 & 10 \end{pmatrix} \quad B^TB = \begin{pmatrix} 14 & 4 \\ 4 & 18 \end{pmatrix}$

$CC^T = \begin{pmatrix} 14 & 1 & 1 \\ 1 & 21 & -6 \\ 1 & -6 & 2 \end{pmatrix} \quad C^TC = \begin{pmatrix} 21 & -2 & -3 \\ -2 & 2 & 5 \\ -3 & 5 & 14 \end{pmatrix}$

$$6.8 \quad 5x^2 + 3y^2 = 30$$

$$6.9 \quad AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad BA = \begin{pmatrix} 22 & 44 \\ -11 & -22 \end{pmatrix}$$

$$6.10 \quad AC = AD = \begin{pmatrix} 11 & 12 \\ 33 & 36 \end{pmatrix}$$

$$6.13 \quad \begin{pmatrix} 5/3 & -3 \\ -1 & 2 \end{pmatrix}$$

$$6.14 \quad \frac{1}{6} \begin{pmatrix} 3 & 1 \\ 0 & -2 \end{pmatrix}$$

$$6.15 \quad -\frac{1}{2} \begin{pmatrix} 4 & 5 & 8 \\ -2 & -2 & -2 \\ 2 & 3 & 4 \end{pmatrix}$$

$$6.16 \quad \frac{1}{8} \begin{pmatrix} -2 & 1 & 1 \\ 6 & -3 & 5 \\ 4 & 2 & 2 \end{pmatrix}$$

$$6.17 \quad A^{-1} = \frac{1}{6} \begin{pmatrix} 2 & 2 & -1 \\ 4 & 4 & -5 \\ 8 & 2 & -4 \end{pmatrix}$$

$$B^{-1} = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 0 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$

$$B^{-1}AB = \begin{pmatrix} 3 & 1 & 2 \\ -2 & -2 & -2 \\ -2 & -1 & 0 \end{pmatrix}$$

$$B^{-1}A^{-1}B = \frac{1}{6} \begin{pmatrix} 2 & 2 & -2 \\ -4 & -4 & -2 \\ 2 & -1 & 4 \end{pmatrix}$$

$$6.19 \quad A^{-1} = \frac{1}{7} \begin{pmatrix} 1 & 2 \\ -3 & -1 \end{pmatrix}, \quad (x, y) = (5, 0)$$

$$6.20 \quad A^{-1} = \frac{1}{7} \begin{pmatrix} -4 & 3 \\ 5 & -2 \end{pmatrix}, \quad (x, y) = (4, -3)$$

$$6.21 \quad A^{-1} = \frac{1}{5} \begin{pmatrix} -1 & 2 & 2 \\ -2 & -1 & 4 \\ 3 & -1 & -1 \end{pmatrix}, \quad (x, y, z) = (-2, 1, 5)$$

$$6.22 \quad A^{-1} = \frac{1}{12} \begin{pmatrix} 4 & 4 & 0 \\ -7 & -1 & 3 \\ 1 & -5 & 3 \end{pmatrix}, \quad (x, y, z) = (1, -1, 2)$$

$$6.30 \quad \sin kA = A \sin k = \begin{pmatrix} 0 & \sin k \\ \sin k & 0 \end{pmatrix}, \quad \cos kA = I \cos k = \begin{pmatrix} \cos k & 0 \\ 0 & \cos k \end{pmatrix},$$

$$e^{kA} = \begin{pmatrix} \cosh k & \sinh k \\ \sinh k & \cosh k \end{pmatrix}, \quad e^{ikA} = \begin{pmatrix} \cos k & i \sin k \\ i \sin k & \cos k \end{pmatrix}$$

$$6.32 \quad e^{i\theta B} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

In the following, L = linear, N = not linear.

$$7.1 \quad N \quad 7.2 \quad L \quad 7.3 \quad N \quad 7.4 \quad L \quad 7.5 \quad L$$

$$7.6 \quad N \quad 7.7 \quad L \quad 7.8 \quad N \quad 7.9 \quad N \quad 7.10 \quad N$$

$$7.11 \quad N \quad 7.12 \quad L \quad 7.13 \quad (a) L \quad (b) L$$

$$7.14 \quad N \quad 7.15 \quad L \quad 7.16 \quad N \quad 7.17 \quad N$$

$$7.22 \quad D = 1, \text{ rotation } \theta = -45^\circ \quad 7.23 \quad D = 1, \text{ rotation } \theta = 210^\circ$$

$$7.24 \quad D = -1, \text{ reflection line } x + y = 0 \quad 7.25 \quad D = -1, \text{ reflection line } y = x\sqrt{2}$$

$$7.26 \quad D = -1, \text{ reflection line } x = 2y \quad 7.27 \quad D = 1, \text{ rotation } \theta = 135^\circ$$

$$7.28 \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}, \quad \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}$$

$$7.29 \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

7.30 $R = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, $S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$; R is a 90° rotation about the z axis; S is a 90° rotation about the x axis.

7.31 From problem 30, $RS = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, $SR = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$;
 RS is a 120° rotation about $\mathbf{i} + \mathbf{j} + \mathbf{k}$; SR is a 120° rotation about $\mathbf{i} - \mathbf{j} + \mathbf{k}$.

7.32 180° rotation about $\mathbf{i} - \mathbf{k}$

7.33 120° rotation about $\mathbf{i} - \mathbf{j} - \mathbf{k}$

7.34 Reflection through the plane $y + z = 0$

7.35 Reflection through the (x, y) plane, and 90° rotation about the z axis.

8.1 In terms of basis $\mathbf{u} = \frac{1}{9}(9, 0, 7)$, $\mathbf{v} = \frac{1}{9}(0, -9, 13)$, the vectors are: $\mathbf{u} - 4\mathbf{v}$, $5\mathbf{u} - 2\mathbf{v}$, $2\mathbf{u} + \mathbf{v}$, $3\mathbf{u} + 6\mathbf{v}$.

8.2 In terms of basis $\mathbf{u} = \frac{1}{3}(3, 0, 5)$, $\mathbf{v} = \frac{1}{3}(0, 3, -2)$, the vectors are: $\mathbf{u} - 2\mathbf{v}$, $\mathbf{u} + \mathbf{v}$, $-2\mathbf{u} + \mathbf{v}$, $3\mathbf{u}$.

8.3 Basis $\mathbf{i}, \mathbf{j}, \mathbf{k}$.

8.4 Basis $\mathbf{i}, \mathbf{j}, \mathbf{k}$.

8.6 $\mathbf{V} = 3\mathbf{A} - \mathbf{B}$

8.7 $\mathbf{V} = \frac{3}{2}(1, -4) + \frac{1}{2}(5, 2)$

8.17 $x = 0$, $y = \frac{3}{2}z$

8.18 $x = -3y$, $z = 2y$

8.19 $x = y = z = w = 0$

8.20 $x = -z$, $y = z$

8.21 $\begin{pmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{pmatrix} = 0$

8.22 $\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix} = 0$

8.23 For $\lambda = 3$, $x = 2y$; for $\lambda = 8$, $y = -2x$

8.24 For $\lambda = 7$, $x = 3y$; for $\lambda = -3$, $y = -3x$

8.25 For $\lambda = 2$: $x = 0$, $y = -3z$; for $\lambda = -3$: $x = -5y$, $z = 3y$;
 for $\lambda = 4$: $z = 3y$, $x = 2y$

8.26 $\mathbf{r} = (3, 1, 0) + (-1, 1, 1)z$

8.27 $\mathbf{r} = (0, 1, 2) + (1, 1, 0)x$

8.28 $\mathbf{r} = (3, 1, 0) + (2, 1, 1)z$

9.3 $A^\dagger = \begin{pmatrix} 1 & 2i & 1 \\ 0 & 2 & 1-i \\ -5i & 0 & 0 \end{pmatrix}$, $A^{-1} = \frac{1}{10} \begin{pmatrix} 0 & 5i-5 & -10i \\ 0 & -5i & 10 \\ -2i & -1-i & 2 \end{pmatrix}$

9.4 $A^\dagger = \begin{pmatrix} 0 & i & 3 \\ -2i & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix}$, $A^{-1} = \frac{1}{6} \begin{pmatrix} 0 & 0 & 2 \\ 0 & 3 & i \\ -6 & 6i & -2 \end{pmatrix}$

9.14 $C^T B A^T$, $C^{-1} M^{-1} C$, H

10.1 (a) $d = 5$ (b) $d = 8$ (c) $d = \sqrt{56}$

10.2 The dimension of the space = the number of basis vectors listed.

One possible basis is given; other bases consist of the same number of independent linear combinations of the vectors given.

(a) $(1, -1, 0, 0)$, $(-2, 0, 5, 1)$

(b) $(1, 0, 0, 5, 0, 1)$, $(0, 1, 0, 0, 6, 4)$, $(0, 0, 1, 0, -3, 0)$

(c) $(1, 0, 0, 0, -3)$, $(0, 2, 0, 0, 1)$, $(0, 0, 1, 0, -1)$, $(0, 0, 0, 1, 4)$

- 10.3 (a) Label the vectors \mathbf{A} , \mathbf{B} , \mathbf{C} , \mathbf{D} . Then $\cos(\mathbf{A}, \mathbf{B}) = \frac{1}{\sqrt{15}}$,
 $\cos(\mathbf{A}, \mathbf{C}) = \frac{\sqrt{2}}{3}$, $\cos(\mathbf{A}, \mathbf{D}) = \frac{3}{\sqrt{23}}$, $\cos(\mathbf{B}, \mathbf{C}) = \frac{2}{3\sqrt{15}}$,
 $\cos(\mathbf{B}, \mathbf{D}) = \sqrt{\frac{17}{690}}$, $\cos(\mathbf{C}, \mathbf{D}) = \frac{\sqrt{21}}{6\sqrt{23}}$.
 (b) $(1, 0, 0, 5, 0, 1)$ and $(0, 0, 1, 0, -3, 0)$
- 10.4 (a) $\mathbf{e}_1 = (0, 1, 0, 0)$, $\mathbf{e}_2 = (1, 0, 0, 0)$, $\mathbf{e}_3 = (0, 0, 3, 4)/5$
 (b) $\mathbf{e}_1 = (0, 0, 0, 1)$, $\mathbf{e}_2 = (1, 0, 0, 0)$, $\mathbf{e}_3 = (0, 1, 1, 0)/\sqrt{2}$
 (c) $\mathbf{e}_1 = (1, 0, 0, 0)$, $\mathbf{e}_2 = (0, 0, 1, 0)$, $\mathbf{e}_3 = (0, 1, 0, 2)/\sqrt{5}$
- 10.5 (a) $\|\mathbf{A}\| = \sqrt{43}$, $\|\mathbf{B}\| = \sqrt{41}$, |Inner product of \mathbf{A} and \mathbf{B} | = $\sqrt{74}$
 (b) $\|\mathbf{A}\| = 7$, $\|\mathbf{B}\| = \sqrt{60}$, |Inner product of \mathbf{A} and \mathbf{B} | = $\sqrt{5}$

11.5 $\theta = 1.1 = 63.4^\circ$

11.11 $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$, not orthogonal

In the following answers, for each eigenvalue, the components of a corresponding eigenvector are listed in parentheses.

- 11.12 4 (1, 1) 11.13 3 (2, 1)
 -1 (3, -2) -2 (-1, 2)
- 11.14 4 (2, -1) 11.15 1 (0, 0, 1)
 -1 (1, 2) -1 (1, -1, 0)
 5 (1, 1, 0)
- 11.16 2 (0, 1, 0) 11.17 7 (1, 0, 1)
 3 (2, 0, 1) 3 (1, 0, -1)
 -2 (1, 0, -2) 3 (0, 1, 0)
- 11.18 4 (2, 1, 3) 11.19 3 (0, 1, -1)
 2 (0, -3, 1) 5 (1, 1, 1)
 -3 (5, -1, -3) -1 (2, -1, -1)
- 11.20 3 (0, -1, 2) 11.21 -1 (-1, 1, 1)
 4 (1, 2, 1) 2 (2, 1, 1)
 -2 (-5, 2, 1) -2 (0, -1, 1)
- 11.22 -4 (-4, 1, 1)
 5 (1, 2, 2)
 -2 (0, -1, 1)
- 11.23 18 (2, 2, -1)
 9 { Any two vectors orthogonal to (2, 2, -1) and to each
 9 { other, for example : (1, -1, 0) and (1, 1, 4)
- 11.24 8 (2, 1, 2)
 -1 { Any two vectors orthogonal to (2, 1, 2) and to each
 -1 { other, for example : (1, 0, -1) and (1, -4, 1)
- 11.25 1 (-1, 1, 1)
 2 (1, 1, 0)
 -2 (1, -1, 2)
- 11.26 4 (1, 1, 1)
 1 { Any two vectors orthogonal to (1, 1, 1) and to each
 1 { other, for example : (1, -1, 0) and (1, 1, -2)
- 11.27 $\mathbf{D} = \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$, $\mathbf{C} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$
- 11.28 $\mathbf{D} = \begin{pmatrix} 1 & 0 \\ 0 & 6 \end{pmatrix}$, $\mathbf{C} = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$

- 11.29 $D = \begin{pmatrix} 11 & 0 \\ 0 & 1 \end{pmatrix}$, $C = \frac{1}{\sqrt{5}} \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}$
- 11.30 $D = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}$, $C = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$
- 11.31 $D = \begin{pmatrix} 5 & 0 \\ 0 & 1 \end{pmatrix}$, $C = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$
- 11.32 $D = \begin{pmatrix} 7 & 0 \\ 0 & 2 \end{pmatrix}$, $C = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$
- 11.41 $\lambda = 1, 3$; $U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$
- 11.42 $\lambda = 1, 4$; $U = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1-i \\ -1-i & 1 \end{pmatrix}$
- 11.43 $\lambda = 2, -3$; $U = \frac{1}{\sqrt{5}} \begin{pmatrix} 2 & -i \\ -i & 2 \end{pmatrix}$
- 11.44 $\lambda = 3, -7$; $U = \frac{1}{5\sqrt{2}} \begin{pmatrix} 5 & -3-4i \\ 3-4i & 5 \end{pmatrix}$
- 11.47 $U = \frac{1}{2} \begin{pmatrix} -1 & i\sqrt{2} & 1 \\ -1 & -i\sqrt{2} & 1 \\ \sqrt{2} & 0 & \sqrt{2} \end{pmatrix}$
- 11.51 Reflection through the plane $3x - 2y - 3z = 0$, no rotation
- 11.52 60° rotation about $-\mathbf{i}\sqrt{2} + \mathbf{k}$ and reflection through the plane $z = x\sqrt{2}$
- 11.53 180° rotation about $\mathbf{i} + \mathbf{j} + \mathbf{k}$
- 11.54 -120° (or 240°) rotation about $\mathbf{i}\sqrt{2} + \mathbf{j}$
- 11.55 Rotation -90° about $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, and reflection through the plane $x - 2y + 2z = 0$
- 11.56 45° rotation about $\mathbf{j} - \mathbf{k}$
- 11.58 $f(M) = \frac{1}{5} \begin{pmatrix} f(1) + 4f(6) & 2f(1) - 2f(6) \\ 2f(1) - 2f(6) & 4f(1) + f(6) \end{pmatrix}$
- $M^4 = \frac{1}{5} \begin{pmatrix} 1 + 4 \cdot 6^4 & 2 - 2 \cdot 6^4 \\ 2 - 2 \cdot 6^4 & 4 + 6^4 \end{pmatrix}$ $M^{10} = \frac{1}{5} \begin{pmatrix} 1 + 4 \cdot 6^{10} & 2 - 2 \cdot 6^{10} \\ 2 - 2 \cdot 6^{10} & 4 + 6^{10} \end{pmatrix}$
- $e^M = \frac{e}{5} \begin{pmatrix} 1 + 4e^5 & 2(1 - e^5) \\ 2(1 - e^5) & 4 + e^5 \end{pmatrix}$
- 11.59 $M^4 = 2^3 \begin{pmatrix} 1 + 2^4 & 1 - 2^4 \\ 1 - 2^4 & 1 + 2^4 \end{pmatrix}$ $M^{10} = 2^3 \begin{pmatrix} 1 + 2^{10} & 1 - 2^{10} \\ 1 - 2^{10} & 1 + 2^{10} \end{pmatrix}$
- $e^M = e^3 \begin{pmatrix} \cosh 1 & -\sinh 1 \\ -\sinh 1 & \cosh 1 \end{pmatrix}$
- 12.2 $3x'^2 - 2y'^2 = 24$
- 12.3 $10x'^2 = 35$
- 12.4 $5x'^2 - 5y'^2 = 8$
- 12.5 $x'^2 + 3y'^2 + 6z'^2 = 14$
- 12.6 $3x'^2 + \sqrt{3}y'^2 - \sqrt{3}z'^2 = 12$
- 12.7 $3x'^2 + 5y'^2 - z'^2 = 60$
- 12.14 $y = x$ with $\omega = \sqrt{k/m}$; $y = -x$ with $\omega = \sqrt{5k/m}$
- 12.15 $y = 2x$ with $\omega = \sqrt{3k/m}$; $x = -2y$ with $\omega = \sqrt{8k/m}$
- 12.16 $y = 2x$ with $\omega = \sqrt{2k/m}$; $x = -2y$ with $\omega = \sqrt{7k/m}$
- 12.17 $x = -2y$ with $\omega = \sqrt{2k/m}$; $3x = 2y$ with $\omega = \sqrt{2k/(3m)}$
- 12.18 $y = x$ with $\omega = \sqrt{2k/m}$; $x = -5y$ with $\omega = \sqrt{16k/(5m)}$
- 12.19 $y = -x$ with $\omega = \sqrt{3k/m}$; $y = 2x$ with $\omega = \sqrt{3k/(2m)}$
- 12.21 $y = 2x$ with $\omega = \sqrt{k/m}$; $x = -2y$ with $\omega = \sqrt{6k/m}$
- 12.22 $y = -x$ with $\omega = \sqrt{2k/m}$; $y = 3x$ with $\omega = \sqrt{2k/(3m)}$
- 12.23 $y = -x$ with $\omega = \sqrt{k/m}$; $y = 2x$ with $\omega = \sqrt{k/(4m)}$

13.5 The 4's group

13.6 The cyclic group

13.7 The 4's group

13.10 If $R = 90^\circ$ rotation, $P =$ reflection through the y axis, and $Q = PR$, then the 8 matrices of the symmetry group of the square are:

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, R^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I,$$

$$R^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = -R, P = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, PR = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = Q,$$

$$PR^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = -P, PR^3 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = -Q,$$

with multiplication table:

	I	R	-I	-R	P	Q	-P	-Q
I	I	R	-I	-R	P	Q	-P	-Q
R	R	-I	-R	I	-Q	P	Q	-P
-I	-I	-R	I	R	-P	-Q	P	Q
-R	-R	I	R	-I	Q	-P	-Q	P
P	P	Q	-P	-Q	I	R	-I	-R
Q	Q	-P	-Q	P	-R	I	R	-I
-P	-P	-Q	P	Q	-I	-R	I	R
-Q	-Q	P	Q	-P	R	-I	-R	I

13.11 The 4 matrices of the symmetry group of the rectangle are

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, P = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = -P, \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I$$

This group is isomorphic to the 4's group.

Class	I	$\pm R$	$-I$	$\pm P$	$\pm Q$
Character	2	0	-2	0	0

13.20 Not a group (no unit element)

13.21 $SO(2)$ is Abelian; $SO(3)$ is not Abelian.

For Problems 2 to 10, we list a possible basis.

14.2 $e^x, x e^x, e^{-x}$, or the three given functions

14.3 $x, \cos x, x \cos x, e^x \cos x$

14.4 $1, x, x^3$

14.5 $1, x + x^3, x^2, x^4, x^5$

14.6 Not a vector space

14.7 $(1 + x^2 + x^4 + x^6), (x + x^3 + x^5 + x^7)$

14.8 $1, x^2, x^4, x^6$

14.9 Not a vector space; the negative of a vector with positive coefficients does not have positive coefficients.

14.10 $(1 + \frac{1}{2}x), (x^2 + \frac{1}{2}x^3), (x^4 + \frac{1}{2}x^5), (x^6 + \frac{1}{2}x^7), (x^8 + \frac{1}{2}x^9),$
 $(x^{10} + \frac{1}{2}x^{11}), (x^{12} + \frac{1}{2}x^{13})$

15.3 (a) $\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z-2}{-2}$, $\mathbf{r} = (4, -1, 2) + (1, -2, -2)t$

(b) $x - 5y + 3z = 0$

(c) $5/7$

(d) $5\sqrt{2}/3$

(e) $\arcsin(19/21) = 64.8^\circ$

15.4 (a) $4x + 2y + 5z = 10$

(b) $\arcsin(2/3) = 41.8^\circ$

(c) $2/\sqrt{5}$

(d) $2x + y - 2z = 5$

(e) $x = \frac{5}{2}, \frac{y}{2} = z, \mathbf{r} = \frac{5}{2}\mathbf{i} + (2\mathbf{j} + \mathbf{k})t$

15.5 (a) $y = 7, \frac{x-2}{3} = \frac{z+1}{4}, \mathbf{r} = (2, 7, -1) + (3, 0, 4)t$

(b) $x - 4y - 9z = 0$

(c) $\arcsin \frac{33}{35\sqrt{2}} = 41.8^\circ$

(d) $\frac{12}{7\sqrt{2}}$

(e) $\frac{\sqrt{29}}{5}$

$$\begin{aligned}
15.7 \quad A^T &= \begin{pmatrix} 1 & 0 \\ -1 & i \end{pmatrix} & A^{-1} &= \begin{pmatrix} 1 & -i \\ 0 & -i \end{pmatrix} & AB &= \begin{pmatrix} 2 & -2 & -6 \\ 0 & 3i & 5i \end{pmatrix} \\
\bar{A} &= \begin{pmatrix} 1 & -1 \\ 0 & -i \end{pmatrix} & B^T A^T &= (AB)^T & B^T AC &= \begin{pmatrix} 2 & & \\ 1-3i & 1 & \\ -1-5i & -1 & \end{pmatrix} \\
A^\dagger &= \begin{pmatrix} 1 & 0 \\ -1 & -i \end{pmatrix} & B^T C &= \begin{pmatrix} 0 & 2 \\ -3 & 1 \\ -5 & -1 \end{pmatrix} & C^{-1}A &= \begin{pmatrix} 0 & -i \\ 1 & -1 \end{pmatrix} \\
&\quad A^T B^T, BA^T, ABC, AB^T C, B^{-1}C, \text{ and } CB^T \text{ are meaningless.}
\end{aligned}$$

$$15.8 \quad A^\dagger = \begin{pmatrix} 1 & -i & 1 \\ 0 & -3 & 0 \\ -2i & 0 & -i \end{pmatrix} \quad A^{-1} = \frac{1}{3i} \begin{pmatrix} -3i & 0 & 6i \\ 1 & -i & -2 \\ 3 & 0 & -3 \end{pmatrix}$$

$$15.9 \quad A = \begin{pmatrix} 1 + \frac{(n-1)d}{nR_2} & -(n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1 R_2} \right] \\ \frac{d}{n} & 1 - \frac{(n-1)d}{nR_1} \end{pmatrix}, \quad \frac{1}{f} = -A_{12}$$

$$15.10 \quad M = \begin{pmatrix} 1 - \frac{d}{f_2} & -\frac{1}{f_1} - \frac{1}{f_2} + \frac{d}{f_1 f_2} \\ d & 1 - \frac{d}{f_1} \end{pmatrix}, \quad \frac{1}{f} = \frac{f_1 + f_2 - d}{f_1 f_2}, \quad \det M = 1$$

$$15.13 \quad \text{Area} = \frac{1}{2} \left| \vec{PQ} \times \vec{PR} \right| = 7/2$$

$$15.14 \quad x'' = -x, y'' = -y, \quad 180^\circ \text{ rotation}$$

$$15.15 \quad x'' = -y, y'' = x, \quad 90^\circ \text{ rotation of vectors or } -90^\circ \text{ rotation of axes}$$

$$15.16 \quad x'' = y, y'' = -x, z'' = z, \quad 90^\circ \text{ rotation of } (x, y) \text{ axes about the } z \text{ axis,} \\ \text{or } -90^\circ \text{ rotation of vectors about the } z \text{ axis}$$

$$15.17 \quad x'' = x, y'' = -y, z'' = -z, \quad \text{rotation of } \pi \text{ about the } x \text{ axis}$$

$$15.18 \quad \begin{pmatrix} 1 & (1, 1) \\ -2 & (0, 1) \end{pmatrix} \quad 15.19 \quad \begin{pmatrix} 6 & (1, 1) \\ 1 & (1, -4) \end{pmatrix} \quad 15.20 \quad \begin{pmatrix} 1 & (1, 1) \\ 9 & (1, -1) \end{pmatrix}$$

$$15.21 \quad \begin{pmatrix} 0 & (1, -2) \\ 5 & (2, 1) \end{pmatrix} \quad 15.22 \quad \begin{pmatrix} 1 & (1, 0, 1) \\ 4 & (0, 1, 0) \\ 5 & (1, 0, -1) \end{pmatrix} \quad 15.23 \quad \begin{pmatrix} 1 & (1, 1, -2) \\ 3 & (1, -1, 0) \\ 4 & (1, 1, 1) \end{pmatrix}$$

$$15.24 \quad \begin{pmatrix} 2 & (0, 4, 3) \\ 7 & (5, -3, 4) \\ -3 & (5, 3, -4) \end{pmatrix}$$

$$15.25 \quad C = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 1 & \sqrt{2} \end{pmatrix}, \quad C^{-1} = \begin{pmatrix} \sqrt{2} & 0 \\ -1 & 1 \end{pmatrix}$$

$$15.26 \quad C = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{17} \\ 1/\sqrt{2} & -4/\sqrt{17} \end{pmatrix}, \quad C^{-1} = \frac{1}{5} \begin{pmatrix} 4\sqrt{2} & \sqrt{2} \\ \sqrt{17} & -\sqrt{17} \end{pmatrix}$$

$$15.27 \quad 3x'^2 - y'^2 - 5z'^2 = 15, \quad d = \sqrt{5}$$

$$15.28 \quad 9x'^2 + 4y'^2 - z'^2 = 36, \quad d = 2$$

$$15.29 \quad 3x'^2 + 6y'^2 - 4z'^2 = 54, \quad d = 3$$

$$15.30 \quad 7x'^2 + 20y'^2 - 6z'^2 = 20, \quad d = 1$$

$$15.31 \quad \omega = (k/m)^{1/2}, \quad (7k/m)^{1/2}$$

$$15.32 \quad \omega = 2(k/m)^{1/2}, \quad (3k/m)^{1/2}$$

Chapter 4

- 1.1 $\partial u/\partial x = 2xy^2/(x^2 + y^2)^2$, $\partial u/\partial y = -2x^2y/(x^2 + y^2)^2$
 1.2 $\partial s/\partial t = ut^{u-1}$, $\partial s/\partial u = t^u \ln t$
 1.3 $\partial z/\partial u = u/(u^2 + v^2 + w^2)$
 1.4 At (0, 0), both = 0; at (-2/3, 2/3), both = -4
 1.5 At (0, 0), both = 0; at (1/4, $\pm 1/2$), $\partial^2 w/\partial x^2 = 6$, $\partial^2 w/\partial y^2 = 2$
 1.7 $2x$ 1.8 $-2x$ 1.9 $2x(1 + 2 \tan^2 \theta)$
 1.10 $4y$ 1.11 $2y$ 1.12 $2y(\cot^2 \theta + 2)$
 1.13 $4r^2 \tan \theta$ 1.14 $-2r^2 \cot \theta$ 1.15 $r^2 \sin 2\theta$
 1.16 $2r(1 + \sin^2 \theta)$ 1.17 $4r$ 1.18 $2r$
 1.19 0 1.20 $8y \sec^2 \theta$ 1.21 $-4x \csc^2 \theta$
 1.22 0 1.23 $2r \sin 2\theta$ 1.24 0
 1.7' $-2y^4/x^3$ 1.8' $-2r^4/x^3$ 1.9' $2x \tan^2 \theta \sec^2 \theta$
 1.10' $2y + 4y^3/x^2$ 1.11' $2yr^4/(r^2 - y^2)^2$ 1.12' $2y \sec^2 \theta$
 1.13' $2x^2 \sec^2 \theta \tan \theta (\sec^2 \theta + \tan^2 \theta)$
 1.14' $2y^2 \sec^2 \theta \tan \theta$ 1.15' $2r^2 \tan \theta \sec^2 \theta$ 1.16' $2r \tan^2 \theta$
 1.17' $4r^3/x^2 - 2r$ 1.18' $-2ry^4/(r^2 - y^2)^2$
 1.19' $-8r^3y^3/(r^2 - y^2)^3$ 1.20' $4x \tan \theta \sec^2 \theta (\tan^2 \theta + \sec^2 \theta)$
 1.21' $4y \sec^2 \theta \tan \theta$ 1.22' $-8r^3/x^3$
 1.23' $4r \tan \theta \sec^2 \theta$ 1.24' $-8y^3/x^3$
- 2.1 $y + y^3/6 - x^2y/2 + x^4y/24 - x^2y^3/12 + y^5/120 + \dots$
 2.2 $1 - (x^2 + 2xy + y^2)/2 + (x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4)/24 + \dots$
 2.3 $x - x^2/2 - xy + x^3/3 + x^2y/2 + xy^2 \dots$
 2.4 $1 + xy + x^2y^2/2 + x^3y^3/3! + x^4y^4/4! \dots$
 2.5 $1 + \frac{1}{2}xy - \frac{1}{8}x^2y^2 + \frac{1}{16}x^3y^3 - \frac{5}{128}x^4y^4 \dots$
 2.6 $1 + x + y + (x^2 + 2xy + y^2)/2 \dots$
 2.8 $e^x \cos y = 1 + x + (x^2 - y^2)/2 + (x^3 - 3xy^2)/3! \dots$
 $e^x \sin y = y + xy + (3x^2y - y^3)/3! \dots$
- 4.2 2.5×10^{-13} 4.3 14.8 4.4 12.2 4.5 14.96
 4.6 9% 4.7 15% 4.8 5% 4.10 4.28 nt
 4.11 3.95 4.12 2.01 4.13 5/3 4.14 0.005
 4.15 8×10^{23}
- 5.1 $e^{-y} \sinh t + z \sin t$ 5.2 $w = 1$, $dw/dp = 0$
 5.3 $2r(q^2 - p^2)$ 5.4 $(4ut + 2v \sin t)/(u^2 - v^2)$
 5.6 $5(x + y)^4(1 + 10 \cos 10x)$ 5.7 $(1 - 2b - e^{2a}) \cos(a - b)$
- 6.1 $dv/dp = -v/(ap)$, $d^2v/dp^2 = v(1 + a)/(a^2p^2)$
 6.2 $y' = 1$, $y'' = 0$ 6.3 $y' = 4(\ln 2 - 1)/(2 \ln 2 - 1)$

- 6.4 $y' = y(x-1)/[x(y-1)], y'' = (y-x)(y+x-2)y/[x^2(y-1)^3]$
- 6.5 $2x + 11y - 24 = 0$ 6.6 $1800/11^3$
- 6.7 $y' = 1, x - y - 4 = 0$ 6.8 $-8/3$
- 6.9 $y = x - 4\sqrt{2}, y = 0, x = 0$ 6.10 $x + y = 0$
- 6.11 $y'' = 4$
- 7.1 $dx/dy = z - y + \tan(y+z), d^2x/dy^2 = \frac{1}{2}\sec^3(y+z) + \frac{1}{2}\sec(y+z) - 2$
- 7.2 $[2e^r \cos t - r + r^2 \sin^2 t]/[(1-r) \sin t]$
- 7.3 $\partial z/\partial s = z \sin s, \partial z/\partial t = e^{-y} \sinh t$
- 7.4 $\partial w/\partial u = -2(rv+s)w, \partial w/\partial v = -2(ru+2s)w$
- 7.5 $\partial u/\partial s = (2y^2 - 3x^2 + xyt)u/(xy), \partial u/\partial t = (2y^2 - 3x^2 + xys)u/(xy)$
- 7.6 $\partial^2 w/\partial r^2 = f_{xx} \cos^2 \theta + 2f_{xy} \sin \theta \cos \theta + f_{yy} \sin^2 \theta$
- 7.7 $(\partial y/\partial \theta)_r = x, (\partial y/\partial \theta)_x = r^2/x, (\partial \theta/\partial y)_x = x/r^2$
- 7.8 $\partial x/\partial s = -19/13, \partial x/\partial t = -21/13, \partial y/\partial s = 24/13, \partial y/\partial t = 6/13$
- 7.10 $\partial x/\partial s = 1/6, \partial x/\partial t = 13/6, \partial y/\partial s = 7/6, \partial y/\partial t = -11/6$
- 7.11 $\partial z/\partial s = 481/93, \partial z/\partial t = 125/93$
- 7.12 $\partial w/\partial s = w/(3w^3 - xy), \partial w/\partial t = (3w - 1)/(3w^3 - xy)$
- 7.13 $(\partial p/\partial q)_m = -p/q, (\partial p/\partial q)_a = 1/(a \cos p - 1),$
 $(\partial p/\partial q)_b = 1 - b \sin q, (\partial b/\partial a)_p = (\sin p)(b \sin q - 1)/\cos q$
 $(\partial a/\partial q)_m = [q + p(a \cos p - 1)]/(q \sin p)$
- 7.14 13
- 7.15 $(\partial x/\partial u)_v = (2yv^2 - x^2)/(2yv + 2xu),$
 $(\partial x/\partial u)_y = (x^2u + y^2v)/(y^2 - 2xu^2)$
- 7.16 (a) $\frac{dw}{dt} = \frac{3(2x+y)}{3x^2+1} + \frac{4x}{4y^3+1} + \frac{10z}{5z^4+1}$
 (b) $\frac{dw}{dx} = 2x + y - \frac{xy}{3y^2+x} + \frac{2z^2}{3z^2-x}$
 (c) $\left(\frac{\partial w}{\partial x}\right)_y = 2x + y - \frac{2z(y^3 + 3x^2z)}{x^3 + 3yz^2}$
- 7.17 $(\partial p/\partial s)_t = -9/7, (\partial p/\partial s)_q = 3/2$
- 7.18 $(\partial b/\partial m)_n = a/(a-b), (\partial m/\partial b)_a = 1$
- 7.19 $(\partial x/\partial z)_s = 7/2, (\partial x/\partial z)_r = 4, (\partial x/\partial z)_y = 3$
- 7.20 $(\partial u)/(\partial x)_y = 4/3, (\partial u/\partial x)_v = 14/5, (\partial x/\partial u)_y = 3/4, (\partial x/\partial u)_v = 5/14$
- 7.21 $-1, -15, 2, 15/7, -5/2, -6/5$
- 7.26 $dy/dx = -(f_1g_3 - f_3g_1)/(f_2g_3 - g_2f_3)$
- 8.3 $(-1, 2)$ is a minimum point. 8.4 $(-1, -2)$ is a saddle point.
- 8.5 $(0, 1)$ is a maximum point. 8.6 $(0, 0)$ is a saddle point.
 $(-2/3, 2/3)$ is a maximum point.
- 8.8 $\theta = \pi/3$; bend up 8 cm on each side.
- 8.9 $l = w = 2h$ 8.10 $l = w = 2h/3$
- 8.11 $\theta = 30^\circ, x = y\sqrt{3} = z/2$ 8.12 $d = 3$
- 8.13 $(4/3, 5/3)$ 8.15 $(1/2, 1/2, 0), (1/3, 1/3, 1/3)$
- 8.16 $m = 5/2, b = 1/3$
- 8.17 (a) $y = 5 - 4x$ (b) $y = 0.5 + 3.35x$ (c) $y = -3 - 3.6x$
- 9.1 $s = l, \theta = 30^\circ$ (regular hexagon)
- 9.2 $r : l : s = \sqrt{5} : (1 + \sqrt{5}) : 3$ 9.3 36 in by 18 in by 18 in
- 9.4 $4/\sqrt{3}$ by $6/\sqrt{3}$ by $10/\sqrt{3}$ 9.5 $(1/2, 3, 1)$

- 9.6 $V = 1/3$
9.8 $(8/13, 12/13)$
9.10 $d = 5/\sqrt{2}$
9.12 Let legs of right triangle be a and b , height of prism $= h$; then $a = b$, $h = (2 - \sqrt{2})a$.

10.1 $d = 1$
10.3 $2, \sqrt{14}$
10.5 $d = 1$
10.7 $\frac{1}{2}\sqrt{11}$
10.9 $\max T = 4$ at $(-1, 0)$
 $\min T = -\frac{16}{5}$ at $(\frac{1}{5}, \pm\frac{2}{5}\sqrt{6})$
10.11 $\max T = 14$ at $(-1, 0)$
 $\min T = 13/2$ at $(1/2, \pm 1)$
10.13 Largest sum $= 3 \arcsin(1/\sqrt{3}) = 105.8^\circ$, smallest sum $= 90^\circ$

11.1 $z = f(y + 2x) + g(y + 3x)$
11.2 $z = f(5x - 2y) + g(2x + y)$
11.3 $w = (x^2 - y^2)/4 + F(x + y) + G(x - y)$
11.6 $\frac{d^2y}{dz^2} + \frac{dy}{dz} - 5y = 0$
11.10 $f = u - Ts$ $h = u + pv$ $g = u + pv - Ts$
 $df = -p dv - s dT$ $dh = T ds + v dp$ $dg = v dp - s dT$
11.11 $H = p\dot{q} - L$
11.13 (a) $(\partial s/\partial v)_T = (\partial p/\partial T)_v$ (b) $(\partial T/\partial p)_s = (\partial v/\partial s)_p$
(c) $(\partial v/\partial T)_p = -(\partial s/\partial p)_T$

12.1 $\frac{\sin x}{2\sqrt{x}}$
12.2 $\frac{\partial s}{\partial v} = \frac{1 - e^v}{v} \rightarrow -1$; $\frac{\partial s}{\partial u} = \frac{e^u - 1}{u} \rightarrow 1$
12.3 $dz/dx = -\sin(\cos x) \tan x - \sin(\sin x) \cot x$
12.4 $(\sin 2)/2$
12.5 $\partial u/\partial x = -4/\pi$, $\partial u/\partial y = 2/\pi$, $\partial y/\partial x = 2$
12.6 $\partial w/\partial x = 1/\ln 3$, $\partial w/\partial y = -6/\ln 3$, $\partial y/\partial x = 1/6$
12.7 $(\partial u/\partial x)_y = -e^4$, $(\partial u/\partial y)_x = e^4/\ln 2$, $(\partial y/\partial x)_u = \ln 2$
12.8 $dx/du = e^{x^2}$ 12.9 $(\cos \pi x + \pi x \sin \pi x - 1)/x^2$
12.10 $dy/dx = (e^x - 1)/x$ 12.11 $3x^2 - 2x^3 + 3x - 6$
12.12 $(2x + 1)/\ln(x + x^2) - 2/\ln(2x)$ 12.13 0
12.14 $\pi/(4y^3)$
12.16 $n = 2, I = \frac{1}{4}\sqrt{\pi}a^{-3/2}$ $n = 4, I = \frac{1 \cdot 3}{8}\sqrt{\pi}a^{-5/2}$
 $n = 2m, I = \frac{1 \cdot 3 \cdot 5 \cdots (2m-1)}{2^{m+1}}\sqrt{\pi}a^{-(2m+1)/2}$

13.2 (a) and (b) $d = 4/\sqrt{13}$
13.3 $\sec^2 \theta$
13.4 $-\csc \theta \cot \theta$
13.5 $-6x, 2x^2 \tan \theta \sec^2 \theta, 4x \tan \theta \sec^2 \theta$
13.6 $2r \sin^2 \theta, 2r^2 \sin \theta \cos \theta, 4r \sin \theta \cos \theta, 0$
13.7 5%

13.8 $\pi^{-1}\text{ft} \cong 4 \text{ inches}$

13.9 $dz/dt = 1 + t(2 - x - y)/z, z \neq 0$

13.10 $(x \ln x - y^2/x)x^y$ where $x = r \cos \theta, y = r \sin \theta$

13.11 $\frac{dy}{dx} = -\frac{b^2x}{a^2y}, \frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$

13.12 13

13.13 -1

13.14 $(\partial w/\partial x)_y = (\partial f/\partial x)_{s, t} + 2(\partial f/\partial s)_{x, t} + 2(\partial f/\partial t)_{x, s} = f_1 + 2f_2 + 2f_3$

13.15 $(\partial w/\partial x)_y = f_1 + 2xf_2 + 2yf_3$

13.17 $\sqrt{19}$

13.18 $\sqrt{26/3}$

13.19 $1/27$

13.20 At $x = -1, y = 20$; at $x = 1/2, y = -1/4$

13.21 $T(2) = 4, T(5) = -5$

13.22 $T(5, 0) = 10, T(2, \pm \sqrt{2}) = -4$

13.23 $t \cot t$

13.24 0

13.25 $-e^x/x$

13.26 $3 \sin x^3/x$

13.29 $dt = 3.9$

13.30 $2f(x, x) + \int_0^x \frac{\partial}{\partial x} [f(x, u) + f(u, x)] du$

Chapter 5

- 2.1 3 2.2 -18 2.3 4 2.4 $8/3$
2.5 $\frac{e^2}{4} - \frac{5}{12}$ 2.6 2.35 2.7 $5/3$ 2.8 $1/2$
2.9 6 2.10 5π 2.11 36 2.12 2
2.13 $7/4$ 2.14 $4 - e(\ln 4)$ 2.15 $3/2$ 2.16 $(\ln 3)/6$
2.17 $(\ln 2)/2$ 2.18 $(8\sqrt{2} - 7)/3$ 2.19 32 2.20 16
2.21 $131/6$ 2.22 $5/3$ 2.23 $9/8$ 2.24 $9/2$
2.25 $3/2$ 2.26 $4/3$ 2.27 $32/5$ 2.28 $1/3$
2.29 2 2.30 $1 - e^{-2}$ 2.31 6 2.32 $e - 1$
2.33 $16/3$ 2.34 $8192k/15$ 2.35 $216k$ 2.36 $1/6$
2.37 $7/6$ 2.38 -20 2.39 70 2.40 $3/2$
2.41 5 2.42 4 2.43 $9/2$ 2.44 $7k/3$
2.45 $46k/15$ 2.46 $8k$ 2.47 $16/3$ 2.48 $16\pi/3$
2.49 $1/3$ 2.50 $64/3$
- 3.2 (a) ρl (b) $MI^2/12$ (c) $MI^2/3$
3.3 (a) $M = 140$ (b) $\bar{x} = 130/21$ (c) $I_m = 6.92M$ (d) $I = 150M/7$
3.4 (a) $M = 3l/2$ (b) $\bar{x} = 4l/9$ (c) $I_m = \frac{13}{162}MI^2$ (d) $I = 5MI^2/18$
3.5 (a) $Ma^2/3$ (b) $Ma^2/12$ (c) $2Ma^2/3$
3.6 (a) (2, 2) (b) $6M$ (c) $2M$
3.7 (a) $M = 9$ (b) $(\bar{x}, \bar{y}) = (2, 4/3)$
(c) $I_x = 2M, I_y = 9M/2$ (d) $I_m = 13M/18$
3.8 $2Ma^2/3$
3.9 (a) $1/6$ (b) $(1/4, 1/4, 1/4)$ (c) $M = 1/24, \bar{z} = 2/5$
3.10 (a) $s = 2 \sinh 1$ (b) $\bar{y} = (2 + \sinh 2)/(4 \sinh 1) = 1.2$
3.11 (a) $M = (5\sqrt{5} - 1)/6 = 1.7$
(b) $\bar{x} = 0, M\bar{y} = (25\sqrt{5} + 1)/60 = 0.95, \bar{y} = (313 + 15\sqrt{5})/620 = 0.56$
3.14 $V = 2\pi^2 a^2 b, A = 4\pi^2 ab$, where a = radius of revolving circle,
 b = distance to axis from center of this circle.
3.15 For area, $(\bar{x}, \bar{y}) = (0, \frac{4}{3}r/\pi)$; for arc, $(\bar{x}, \bar{y}) = (0, 2r/\pi)$
3.17 $4\sqrt{2}/3$ 3.18 $s = [3\sqrt{2} + \ln(1 + \sqrt{2})]/2 = 2.56$
3.19 2π 3.20 $13\pi/3$
3.21 $s\bar{x} = [51\sqrt{2} - \ln(1 + \sqrt{2})]/32 = 2.23, s\bar{y} = 13/6, s$ as in Problem 3.18;
then $\bar{x} = 0.87, \bar{y} = 0.85$
3.22 $(4/3, 0, 0)$
3.23 $(149/130, 0, 0)$
3.24 $2M/5$
3.25 I/M has the same numerical value as \bar{x} in 3.21.
3.26 $2M/3$ 3.27 $\frac{149}{130}M$ 3.28 $13/6$ 3.29 2 3.30 $32/5$

- 4.1 (b) $\bar{x} = \bar{y} = 4a/(3\pi)$
 (c) $I = Ma^2/4$
 (e) $\bar{x} = \bar{y} = 2a/\pi$
- 4.2 (c) $\bar{y} = 4a/(3\pi)$
 (d) $I_x = Ma^2/4, I_y = 5Ma^2/4, I_z = 3Ma^2/2$
 (e) $\bar{y} = 2a/\pi$
 (f) $\bar{x} = 6a/5, I_x = 48Ma^2/175, I_y = 288Ma^2/175, I_z = 48Ma^2/25$
 (g) $A = (\frac{2}{3}\pi - \frac{1}{2}\sqrt{3})a^2$
- 4.3 (a), (b), or (c) $\frac{1}{2}Ma^2$
- 4.4 (a) $4\pi a^2$ (b) $(0, 0, a/2)$ (c) $2Ma^2/3$
 (d) $4\pi a^3/3$ (e) $(0, 0, 3a/8)$
- 4.5 $7\pi/3$ 4.6 $\pi \ln 2$
- 4.7 (a) $V = 2\pi a^3(1 - \cos \alpha)/3$ (b) $\bar{z} = 3a(1 + \cos \alpha)/8$
- 4.8 $I_z = Ma^2/4$
- 4.10 (a) $V = 64\pi$ (b) $\bar{z} = 231/64$
- 4.11 12π
- 4.12 (c) $M = (16\rho/9)(3\pi - 4) = 9.64\rho$
 $I = (128\rho/15^2)(15\pi - 26) = 12.02\rho = 1.25M$
- 4.13 (b) $\pi a^2(z_2 - z_1) - \pi(z_2^3 - z_1^3)/3$ (c) $\frac{\frac{1}{2}a^2(z_2^2 - z_1^2) - \frac{1}{4}(z_2^4 - z_1^4)}{a^2(z_2 - z_1) - (z_2^3 - z_1^3)/3}$
- 4.14 $\pi(1 - e^{-1})/4$ 4.16 $u^2 + v^2$
 4.17 $a^2(\sinh^2 u + \sin^2 v)$ 4.19 $\pi/4$
 4.20 $1/12$ 4.22 $12(1 + 36\pi^2)^{1/2}$
- 4.23 Length = $(R \sec \alpha)$ times change in latitude
- 4.24 $\rho G\pi a/2$
- 4.26 (a) $7Ma^2/5$ (b) $3Ma^2/2$
- 4.27 $2\pi ah$ (where h = distance between parallel planes)
- 4.28 $(0, 0, a/2)$
- 5.1 $9\pi\sqrt{30}/5$ 5.2 $\pi\sqrt{7/5}$
 5.3 $\pi(37^{3/2} - 1)/6 = 117.3$ 5.4 $\pi/\sqrt{6}$
 5.5 8π for each nappe 5.6 4
 5.7 4 5.8 $[3\sqrt{6} + 9\ln(\sqrt{2} + \sqrt{3})]/16$
 5.9 $\pi\sqrt{2}$ 5.10 $2\pi a^2(\sqrt{2} - 1)$
 5.11 $(\bar{x}, \bar{y}, \bar{z}) = (1/3, 1/3, 1/3)$ 5.12 $M = \sqrt{3}/6, (\bar{x}, \bar{y}, \bar{z}) = (1/2, 1/4, 1/4)$
 5.13 $\bar{z} = \frac{\pi}{4(\pi - 2)}$ 5.14 $M = \frac{\pi}{2} - \frac{4}{3}$
- 5.15 $I_z/M = \frac{2(3\pi - 7)}{9(\pi - 2)} = 0.472$ 5.16 $\bar{x} = 0, \bar{y} = 1, \bar{z} = \frac{32}{9\pi}\sqrt{\frac{2}{5}} = 0.716$
- 6.1 $7\pi(2 - \sqrt{2})/3$ 6.2 $45(2 + \sqrt{2})/112$ 6.3 $15\pi/8$
- 6.4 (a) $\frac{1}{2}MR^2$ (b) $\frac{3}{2}MR^2$
- 6.5 cone: $2\pi ab^2/3$; ellipsoid: $4\pi ab^2/3$; cylinder: $2\pi ab^2$
- 6.6 (a) $\frac{4\pi - 3\sqrt{3}}{6}$ (b) $\bar{x} = \frac{5}{4\pi - 3\sqrt{3}}, \bar{y} = \frac{6\sqrt{3}}{4\pi - 3\sqrt{3}}$
- 6.7 $\frac{8\pi - 3\sqrt{3}}{4\pi - 3\sqrt{3}}M$ 6.8 (a) $5\pi/3$ (b) $27/20$
- 6.9 $(\bar{x}, \bar{y}) = (0, 3c/5)$
- 6.10 (a) $(\bar{x}, \bar{y}) = (\pi/2, \pi/8)$ (b) $\pi^2/2$ (c) $3M/8$
- 6.11 $\bar{z} = 3h/4$ 6.12 $(abc)^2/6$
- 6.13 $8a^2$ 6.14 $16a^3/3$
- 6.15 $I_x = 8Ma^2/15, I_y = 7Ma^2/15$ 6.16 $\bar{x} = \bar{y} = 2a/5$

- 6.17 $Ma^2/6$ 6.18 $(0, 0, 5h/6)$
6.19 $I_x = I_y = 20Mh^2/21, I_z = 10Mh^2/21, I_m = 65Mh^2/252$
6.20 (a) $\pi(5\sqrt{5} - 1)/6$ (b) $3\pi/2$
6.21 $\pi G\rho h(2 - \sqrt{2})$
6.22 $I_x = Mb^2/4, I_y = Ma^2/4, I_z = M(a^2 + b^2)/4$
6.23 (a) $(0, 0, 2c/3)$ (b) $(0, 0, 5c/7)$
6.24 $(0, 0, 2c/3)$
6.25 $\pi/2$
6.26 $\frac{1}{2} \sinh 1$
6.27 $e^2 - e - 1$

Chapter 6

- 3.1 $(\mathbf{A} \cdot \mathbf{B})\mathbf{C} = 6\mathbf{C} = 6(\mathbf{j} + \mathbf{k})$, $\mathbf{A}(\mathbf{B} \cdot \mathbf{C}) = -2\mathbf{A} = -2(2, -1, -1)$,
 $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = -8$, $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = 4(\mathbf{j} - \mathbf{k})$,
 $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = -4(\mathbf{i} + 2\mathbf{k})$
- 3.2 $\mathbf{B} \cdot \mathbf{C} = -16$
- 3.3 $(\mathbf{A} + \mathbf{B}) \cdot \mathbf{C} = -5$
- 3.4 $\mathbf{B} \times \mathbf{A} = -\mathbf{i} + 7\mathbf{j} + 3\mathbf{k}$, $|\mathbf{B} \times \mathbf{A}| = \sqrt{59}$, $(\mathbf{B} \times \mathbf{A}) \cdot \mathbf{C}/|\mathbf{C}| = -8/\sqrt{26}$
- 3.5 $\boldsymbol{\omega} = 2\mathbf{A}/\sqrt{6}$, $\mathbf{v} = \boldsymbol{\omega} \times \mathbf{C} = (2/\sqrt{6})(-3\mathbf{i} + 5\mathbf{j} + \mathbf{k})$
- 3.6 $\mathbf{v} = (2/\sqrt{6})(\mathbf{A} \times \mathbf{B}) = (2/\sqrt{6})(\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$,
 $\mathbf{r} \times \mathbf{F} = (\mathbf{A} - \mathbf{C}) \times \mathbf{B} = 3\mathbf{i} + 3\mathbf{j} - \mathbf{k}$,
 $\mathbf{n} \cdot \mathbf{r} \times \mathbf{F} = [(\mathbf{A} - \mathbf{C}) \times \mathbf{B}] \cdot \mathbf{C}/|\mathbf{C}| = 8/\sqrt{26}$
- 3.7 (a) $11\mathbf{i} + 3\mathbf{j} - 13\mathbf{k}$ (b) 3 (c) 17
- 3.8 $4\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}$, 4, -8, 4
- 3.9 $-9\mathbf{i} - 23\mathbf{j} + \mathbf{k}$, $1/\sqrt{21}$
- 3.12 $A^2 B^2$
- 3.15 $\mathbf{u}_1 \cdot \mathbf{u} = -\mathbf{u}_3 \cdot \mathbf{u}$, $n_1 \mathbf{u}_1 \times \mathbf{u} = n_2 \mathbf{u}_2 \times \mathbf{u}$
- 3.16 $\mathbf{L} = m[r^2 \boldsymbol{\omega} - (\boldsymbol{\omega} \cdot \mathbf{r})\mathbf{r}]$
For $\mathbf{r} \perp \boldsymbol{\omega}$, $v = |\boldsymbol{\omega} \times \mathbf{r}| = \omega r$, $L = m|r^2 \boldsymbol{\omega}| = mvr$
- 3.17 $\mathbf{a} = (\boldsymbol{\omega} \cdot \mathbf{r})\boldsymbol{\omega} - \omega^2 \mathbf{r}$; for $\mathbf{r} \perp \boldsymbol{\omega}$, $\mathbf{a} = -\omega^2 \mathbf{r}$, $|\mathbf{a}| = v^2/r$.
- 3.19 (a) $16\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$ (b) $8/\sqrt{6}$
- 3.20 (a) $13/5$ (b) 12
- 4.2 (a) $t = 2$
(b) $\mathbf{v} = 4\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$, $|\mathbf{v}| = 2\sqrt{14}$
(c) $(x - 4)/4 = (y + 4)/(-2) = (z - 8)/6$, $2x - y + 3z = 36$
- 4.3 $t = -1$, $\mathbf{v} = 3\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$, $(x - 1)/3 = (y + 1)/3 = (z - 5)/(-5)$,
 $3x + 3y - 5z + 25 = 0$
- 4.5 $|\mathbf{dr}/dt| = \sqrt{2}$; $|d^2 \mathbf{r}/dt^2| = 1$; path is a helix.
- 4.8 $d\mathbf{r}/dt = \mathbf{e}_r(dr/dt) + \mathbf{e}_\theta(r d\theta/dt)$,
 $d^2 \mathbf{r}/dt^2 = \mathbf{e}_r[d^2 r/dt^2 - r(d\theta/dt)^2] + \mathbf{e}_\theta[r d^2 \theta/dt^2 + 2(dr/dt)(d\theta/dt)]$.
- 4.10 $\mathbf{V} \times d\mathbf{V}/dt$
- 6.1 $-16\mathbf{i} - 12\mathbf{j} + 8\mathbf{k}$ 6.2 $-\mathbf{i}$
- 6.3 0 6.4 $\pi e/(3\sqrt{5})$
- 6.5 $\nabla \phi = \mathbf{i} - \mathbf{k}$; $-\nabla \phi$; $d\phi/ds = 2/\sqrt{13}$
- 6.6 $6x + 8y - z = 25$, $(x - 3)/6 = (y - 4)/8 = (z - 25)/(-1)$
- 6.7 $5x - 3y + 2z + 3 = 0$, $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + (5\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})t$
- 6.8 (a) $7/3$ (b) $5x - z = 8$; $\frac{x-1}{5} = \frac{z+3}{-1}$, $y = \pi/2$
- 6.9 (a) $2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ (b) $5/\sqrt{6}$ (c) $\mathbf{r} = (1, 1, 1) + (2, -2, -1)t$
- 6.10 \mathbf{j} , 1, $-4/5$

- 6.11 $\nabla\phi = 2x\mathbf{i} - 2y\mathbf{j}$, $\mathbf{E} = -2x\mathbf{i} + 2y\mathbf{j}$
 6.12 (a) $2\sqrt{5}$, $-2\mathbf{i} + \mathbf{j}$ (b) $3\mathbf{i} + 2\mathbf{j}$ (c) $\sqrt{10}$
 6.13 (a) $\mathbf{i} + \mathbf{j}$, $|\nabla\phi| = e$ (b) $-1/2$ (c) \mathbf{i} , $|\mathbf{E}| = 1$ (d) e^{-1}
 6.14 (b) Down, at the rate $11\sqrt{2}$
 6.15 (a) $4\sqrt{2}$, up (b) 0, around the hill
 (c) $-4/\sqrt{10}$, down (d) $8/5$, up
 6.17 \mathbf{e}_r 6.18 \mathbf{i} 6.19 \mathbf{j} 6.20 $2r\mathbf{e}_r$
 7.1 $\nabla \cdot \mathbf{r} = 3$, $\nabla \times \mathbf{r} = 0$ 7.2 $\nabla \cdot \mathbf{r} = 2$, $\nabla \times \mathbf{r} = 0$
 7.3 $\nabla \cdot \mathbf{V} = 1$, $\nabla \times \mathbf{V} = 0$ 7.4 $\nabla \cdot \mathbf{V} = 0$, $\nabla \times \mathbf{V} = -(\mathbf{i} + \mathbf{j} + \mathbf{k})$
 7.5 $\nabla \cdot \mathbf{V} = 2(x + y + z)$, $\nabla \times \mathbf{V} = 0$
 7.6 $\nabla \cdot \mathbf{V} = 5xy$, $\nabla \times \mathbf{V} = xz\mathbf{i} - yz\mathbf{j} + \mathbf{k}(y^2 - x^2)$
 7.7 $\nabla \cdot \mathbf{V} = 0$, $\nabla \times \mathbf{V} = x\mathbf{i} - y\mathbf{j} - x \cos y\mathbf{k}$
 7.8 $\nabla \cdot \mathbf{V} = 2 + x \sinh z$, $\nabla \times \mathbf{V} = 0$ 7.9 $6y$
 7.10 0 7.11 $-(x^2 + y^2)/(x^2 - y^2)^{3/2}$
 7.12 $4(x + y)^{-3}$ 7.13 $2xy$
 7.14 0 7.15 0
 7.16 $2(x^2 + y^2 + z^2)^{-1}$ 7.18 $2\mathbf{k}$
 7.19 $2/r$ 7.20 0
 8.1 $-11/3$
 8.2 (a) -4π (b) -16 (c) -8
 8.3 (a) $5/3$ (b) 1 (c) $2/3$
 8.4 (a) 3 (b) $8/3$
 8.5 (a) $86/3$ (b) $-31/3$
 8.6 (a) 3 (b) 3 (c) 3
 8.7 (a) -2π (b) 0 (c) -2 (d) 2π
 8.8 $yz - x$ 8.9 $3xy - x^3yz - z^2$
 8.10 $\frac{1}{2}kr^2$ 8.11 $-y \sin^2 x$
 8.12 $-(xy + z)$ 8.13 $-z^2 \cosh y$
 8.14 $-\arcsin xy$ 8.15 $-(x^2 + 1) \cos^2 y$
 8.16 (a) \mathbf{F}_1 ; $\phi_1 = y^2z - x^2$
 (b) For \mathbf{F}_2 : (1) $W = 0$ (2) $W = -4$ (3) $W = 2\pi$
 8.17 \mathbf{F}_2 conservative, $W = 0$; for \mathbf{F}_1 , $W = 2\pi$
 8.18 (a) $\pi + \pi^2/2$ (b) $\pi^2/2$
 8.20 $\phi = mgz$, $\phi = -C/r$
 9.2 40 9.3 $14/3$ 9.4 $-3/2$
 9.5 20 9.7 πab 9.8 24π
 9.9 $(\bar{x}, \bar{y}) = (1, 1)$ 9.10 -20 9.11 2
 9.12 $29/3$
 10.1 4π 10.2 3 10.3 9π
 10.4 36π 10.5 $4\pi \cdot 5^5$ 10.6 1
 10.7 48π 10.8 80π 10.9 16π
 10.10 27π
 10.12 $\phi = \begin{cases} 0, & r \leq R_1 \\ (k/2\pi\epsilon_0) \ln(R_1/r), & R_1 \leq r \leq R_2 \\ (k/2\pi\epsilon_0) \ln(R_1/R_2), & r \geq R_2 \end{cases}$

11.1	$-3\pi a^2$	11.2	$2ab^2$	11.3	0
11.4	-12	11.5	36	11.6	45π
11.7	0	11.8	0	11.9	$32\pi/3$
11.10	-6π	11.11	24	11.12	18π
11.13	0	11.14	-8π	11.15	$-2\pi\sqrt{2}$

In the answers for Problems 18 to 22, u is arbitrary.

$$11.18 \quad \mathbf{A} = (xz - yz^2 - y^2/2)\mathbf{i} + (x^2/2 - x^2z + yz^2/2 - yz)\mathbf{j} + \nabla u$$

$$11.19 \quad \mathbf{A} = (y^2z - xy^2/2)\mathbf{i} + xz^2\mathbf{j} + x^2y\mathbf{k} + \nabla u$$

$$11.20 \quad \mathbf{A} = \mathbf{i} \sin zx + \mathbf{j} \cos zx + \mathbf{k} e^{zy} + \nabla u$$

$$11.21 \quad \mathbf{A} = \mathbf{i}y + \nabla u$$

$$11.22 \quad \mathbf{A} = \mathbf{i}(xz - y^3/3) + \mathbf{j}(-yz + x^3/3) + \mathbf{k}(x + y)z + \nabla u$$

$$12.1 \quad \sin \theta \cos \theta \mathbf{C}$$

$$12.2 \quad \frac{1}{2}|\mathbf{B} \times \mathbf{A}|$$

$$12.5 \quad \text{(a) } -4\pi \quad \text{(b) } -16 \quad \text{(c) } -8$$

$$12.6 \quad 5\mathbf{i} - 8\mathbf{j} - 6\mathbf{k}$$

$$12.7 \quad \text{(a) } 9\mathbf{i} + 5\mathbf{j} - 3\mathbf{k} \quad \text{(b) } 29/3$$

$$12.8 \quad 2/\sqrt{5}$$

$$12.9 \quad 24$$

$$12.10 \quad \text{(a) } 2\mathbf{i} + \mathbf{j} \quad \text{(b) } 11/5 \quad \text{(c) } 2x + y = 4$$

$$12.11 \quad \text{(a) } \text{grad } \phi = -3y\mathbf{i} - 3x\mathbf{j} + 2z\mathbf{k}$$

$$\text{(b) } -\sqrt{3}$$

$$\text{(c) } 2x + y - 2z + 2 = 0, \mathbf{r} = (1, 2, 3) + (2, 1, -2)t$$

$$12.12 \quad \text{(a) } 3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$$

$$\text{(b) } 3$$

$$\text{(c) } 3x + 2y + z = 4, \mathbf{r} = (0, 1, 2) + (3, 2, 1)t$$

$$\text{(d) } (3\mathbf{i} + 2\mathbf{j} + \mathbf{k})/\sqrt{14}$$

$$12.13 \quad \text{(a) } 6\mathbf{i} - \mathbf{j} - 4\mathbf{k}$$

$$\text{(b) } 53^{-1/2}(6\mathbf{i} - \mathbf{j} - 4\mathbf{k})$$

$$\text{(c) } \text{same as (a)}$$

$$\text{(d) } 53^{1/2}$$

$$\text{(e) } 53^{1/2}$$

$$12.14 \quad \text{(a) } -3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$$

$$\text{(b) } 0$$

$$12.15 \quad \phi = -y^2 \cosh^2 xz$$

$$12.16 \quad \text{(a) } \mathbf{F}_1 \text{ is conservative.}$$

$$\text{(b) } W_1 = x^2z + \frac{1}{2}y^2$$

$$\text{(c) and (d) } 1$$

$$12.17 \quad 6$$

$$12.18 \quad \text{Not conservative}$$

$$\text{(a) } 1/2$$

$$\text{(b) } 4/3$$

$$12.19 \quad \text{(a) } \mathbf{F}_1 \text{ is conservative; } \mathbf{F}_2 \text{ is not conservative } (\nabla \times \mathbf{F}_2 = \mathbf{k})$$

$$\text{(b) } 2\pi$$

$$\text{(c) } \text{For } \mathbf{F}_1, V_1 = 2xy - yz - \frac{1}{2}z^2$$

$$\text{(d) } W_1 = 45/2$$

$$\text{(e) } W_2 = 2\pi$$

$$12.20 \quad \pi \quad 12.21 \quad 4 \quad 12.22 \quad 108\pi \quad 12.23 \quad 192\pi$$

$$12.24 \quad 54\pi \quad 12.25 \quad -18\pi \quad 12.26 \quad 0 \quad 12.27 \quad 4$$

$$12.28 \quad -2\pi \quad 12.29 \quad 10 \quad 12.30 \quad 4 \quad 12.31 \quad 29/3$$

Chapter 7

	amplitude	period	frequency	velocity amplitude
2.1	3	$2\pi/5$	$5/(2\pi)$	15
2.2	2	$\pi/2$	$2/\pi$	8
2.3	$1/2$	2	$1/2$	$\pi/2$
2.4	5	2π	$1/(2\pi)$	5
2.5 $s = \sin 6t$	1	$\pi/3$	$3/\pi$	6
2.6 $s = 6 \cos \frac{\pi}{8} \sin 2t$	$6 \cos \frac{\pi}{8} = 5.54$	π	$1/\pi$	$12 \cos \frac{\pi}{8} = 11.1$
2.7	5	2π	$1/(2\pi)$	5
2.8	2	4π	$1/(4\pi)$	1
2.9	2	2	$1/2$	2π
2.10	4	π	$1/\pi$	8
2.11 q	3	$1/60$	60	
I	360π	$1/60$	60	
2.12 q	4	$1/15$	15	
I	120π	$1/15$	15	

2.13 $A = \text{maximum value of } \theta, \omega = \sqrt{g/l}.$

2.14 $t = 12$

2.15 $t = 3\pi$

2.16 $t \cong 4.91 \cong 281^\circ$

2.18 $A = 2, T = 1, f = 1, v = 3, \lambda = 3$

2.19 $A = 1, T = 4, f = 1/4, v = 1/4, \lambda = 1$

2.20 $A = 3, T = 4, f = 1/4, v = 1/2, \lambda = 2$

2.21 $y = 20 \sin \frac{\pi}{2}(x - 6t), \frac{\partial y}{\partial t} = -60\pi \cos \frac{\pi}{2}(x - 6t)$

2.22 $y = 4 \sin 2\pi(\frac{x}{3} - \frac{t}{6})$

2.23 $y = \sin 880\pi(\frac{x}{350} - t)$

2.24 $y = \sin \frac{200\pi}{153}(x - 1530t)$

2.25 $y = 10 \sin \frac{\pi}{250}(x - 3 \cdot 10^8 t)$

3.6 $\sin(2x + \frac{\pi}{3})$

3.7 $-\sqrt{2} \sin(\pi x - \frac{\pi}{4})$

4.3 0

4.4 e^{-1}

4.5 $1/\pi + 1/2$

4.6 $2/\pi$

4.7 $\pi/12 - 1/2$

4.8 0

4.9 $1/2$

4.10 0

4.11 $1/2$

4.12 $1/2$

4.14 (a) $2\pi/3$

(b) π

4.15 (a) $3/2$

(b) $3/2$

4.16 (a) π/ω

(b) 1

5.1 to 5.11 The answers for Problems 5.1 to 5.11 are the sine-cosine series in Problems 7.1 to 7.11.

$x \rightarrow$	-2π	$-\pi$	$-\pi/2$	0	$\pi/2$	π	2π
6.1	$1/2$	$1/2$	1	$1/2$	0	$1/2$	$1/2$
6.2	$1/2$	0	0	$1/2$	$1/2$	0	$1/2$
6.3	0	$1/2$	0	0	$1/2$	$1/2$	0
6.4	-1	0	-1	-1	0	0	-1
6.5	$-1/2$	$1/2$	0	$-1/2$	0	$1/2$	$-1/2$
6.6	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$
6.7	0	$\pi/2$	0	0	$\pi/2$	$\pi/2$	0
6.8	1	1	$1 - \frac{\pi}{2}$	1	$1 + \frac{\pi}{2}$	1	1
6.9	0	π	$\pi/2$	0	$\pi/2$	π	0
6.10	π	0	$\pi/2$	π	$\pi/2$	0	π
6.11	0	0	0	0	1	0	0

6.13 and 6.14 At $x = \pi/2$, same series as in the example.

$$7.1 \quad f(x) = \frac{1}{2} + \frac{i}{\pi} \sum_{\substack{-\infty \\ \text{odd } n}}^{\infty} \frac{1}{n} e^{inx} = \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n} \sin nx$$

$$7.2 \quad a_n = \frac{1}{n\pi} \sin \frac{n\pi}{2}, \quad b_n = \frac{1}{n\pi} (1 - \cos \frac{n\pi}{2}), \quad a_0/2 = c_0 = \frac{1}{4},$$

$$c_n = \frac{i}{2n\pi} (e^{-in\pi/2} - 1), \quad n > 0; \quad c_{-n} = \bar{c}_n$$

$$f(x) = \frac{1}{4} + \frac{1}{2\pi} [(1-i)e^{ix} + (1+i)e^{-ix} - \frac{2i}{2}(e^{2ix} - e^{-2ix})$$

$$\quad - \frac{1+i}{3}e^{3ix} - \frac{1-i}{3}e^{-3ix} + \frac{1-i}{5}e^{5ix} + \frac{1+i}{5}e^{-5ix} \dots]$$

$$= \frac{1}{4} + \frac{1}{\pi} (\cos x - \frac{1}{3} \cos 3x + \frac{1}{5} \cos 5x \dots)$$

$$+ \frac{1}{\pi} (\sin x + \frac{2}{2} \sin 2x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \frac{2}{6} \sin 6x \dots)$$

$$7.3 \quad a_n = -\frac{1}{n\pi} \sin \frac{n\pi}{2}, \quad a_0/2 = c_0 = \frac{1}{4}$$

$$b_n = \frac{1}{n\pi} (\cos \frac{n\pi}{2} - \cos n\pi) = \frac{1}{n\pi} \{1, -2, 1, 0, \text{ and repeat}\}$$

$$c_n = \frac{i}{2n\pi} (e^{-in\pi} - e^{-in\pi/2}), \quad n > 0; \quad c_{-n} = \bar{c}_n$$

$$f(x) = \frac{1}{4} + \frac{1}{2\pi} [-(1+i)e^{ix} - (1-i)e^{-ix} + \frac{2i}{2}(e^{2ix} - e^{-2ix})$$

$$+ \frac{1-i}{3}e^{3ix} + \frac{1+i}{3}e^{-3ix} - \frac{1+i}{5}e^{5ix} - \frac{(1-i)}{5}e^{-5ix} \dots]$$

$$= \frac{1}{4} - \frac{1}{\pi} (\cos x - \frac{1}{3} \cos 3x + \frac{1}{5} \cos 5x \dots)$$

$$+ \frac{1}{\pi} (\sin x - \frac{2}{2} \sin 2x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x \dots)$$

7.4 $c_0 = a_0/2 = -1/2$; for $n \neq 0$, coefficients are 2 times the coefficients in Problem 7.3.

$$f(x) = -\frac{1}{2} - \frac{1}{\pi} [(1+i)e^{ix} + (1-i)e^{-ix} - \frac{2i}{2}(e^{2ix} - e^{-2ix})$$

$$- \frac{1-i}{3}e^{3ix} - \frac{1+i}{3}e^{-3ix} + \frac{1+i}{5}e^{5ix} + \frac{1-i}{5}e^{-5ix} \dots]$$

$$= -\frac{1}{2} - \frac{2}{\pi} (\cos x - \frac{1}{3} \cos 3x + \frac{1}{5} \cos 5x \dots)$$

$$+ \frac{2}{\pi} (\sin x - \frac{2}{2} \sin 2x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x - \frac{2}{6} \sin 6x \dots)$$

$$\begin{aligned}
7.5 \quad a_n &= -\frac{2}{n\pi} \sin \frac{n\pi}{2} & a_0/2 = c_0 = 0 \\
b_n &= \frac{1}{n\pi} (2 \cos \frac{n\pi}{2} - 1 - \cos n\pi) = -\frac{4}{n\pi} \{0, 1, 0, 0, \text{ and repeat} \} \\
c_n &= \frac{1}{2in\pi} (2e^{-in\pi/2} - 1 - e^{-in\pi}) = \frac{1}{n\pi} \{-1, 2i, 1, 0, \text{ and repeat} \}, n > 0 \\
c_{-n} &= \bar{c}_n
\end{aligned}$$

$$\begin{aligned}
f(x) &= -\frac{1}{\pi} [e^{ix} + e^{-ix} - \frac{2i}{2} (e^{2ix} - e^{-2ix}) - \frac{1}{3} (e^{3ix} + e^{-3ix}) \\
&\quad + \frac{1}{5} (e^{5ix} + e^{-5ix}) - \frac{2i}{6} (e^{6ix} - e^{-6ix}) \dots] \\
&= -\frac{2}{\pi} (\cos x - \frac{1}{3} \cos 3x + \frac{1}{5} \cos 5x \dots) \\
&\quad - \frac{4}{\pi} (\frac{1}{2} \sin 2x + \frac{1}{6} \sin 6x + \frac{1}{10} \sin 10x \dots)
\end{aligned}$$

$$\begin{aligned}
7.6 \quad f(x) &= \frac{1}{2} + \frac{2}{i\pi} \sum \frac{1}{n} e^{inx} & (n = \pm 2, \pm 6, \pm 10, \dots) \\
&= \frac{1}{2} + \frac{4}{\pi} \sum \frac{1}{n} \sin nx & (n = 2, 6, 10, \dots)
\end{aligned}$$

$$\begin{aligned}
7.7 \quad f(x) &= \frac{\pi}{4} - \sum_{\substack{-\infty \\ \text{odd } n}} \left(\frac{1}{n^2\pi} + \frac{i}{2n} \right) e^{inx} + \sum_{\substack{-\infty \\ \text{even } n \neq 0}} \frac{i}{2n} e^{inx} \\
&= \frac{\pi}{4} - \sum_1 \frac{(-1)^n}{n} \sin nx - \frac{2}{\pi} \sum_1 \frac{1}{n^2} \cos nx
\end{aligned}$$

$$7.8 \quad f(x) = 1 + \sum_{\substack{-\infty \\ n \neq 0}} (-1)^n \frac{i}{n} e^{inx} = 1 + 2 \sum_1 (-1)^{n+1} \frac{1}{n} \sin nx$$

$$7.9 \quad f(x) = \frac{\pi}{2} - \frac{2}{\pi} \sum_{\substack{-\infty \\ \text{odd } n}} \frac{e^{inx}}{n^2} = \frac{\pi}{2} - \frac{4}{\pi} \sum_1 \frac{\cos nx}{n^2}$$

$$7.10 \quad f(x) = \frac{\pi}{2} + \frac{2}{\pi} \sum_{\substack{-\infty \\ \text{odd } n}} \frac{e^{inx}}{n^2} = \frac{\pi}{2} + \frac{4}{\pi} \sum_1 \frac{\cos nx}{n^2}$$

$$\begin{aligned}
7.11 \quad f(x) &= \frac{1}{\pi} + \frac{e^{ix} - e^{-ix}}{4i} - \frac{1}{\pi} \sum_{\substack{-\infty \\ \text{even } n \neq 0}} \frac{e^{inx}}{n^2 - 1} \\
&= \frac{1}{\pi} + \frac{1}{2} \sin x - \frac{2}{\pi} \sum_2 \frac{\cos nx}{n^2 - 1}
\end{aligned}$$

$$7.13 \quad a_n = 2 \operatorname{Re} c_n, b_n = -2 \operatorname{Im} c_n, c_n = \frac{1}{2}(a_n - ib_n), c_{-n} = \frac{1}{2}(a_n + ib_n)$$

$$8.1 \quad f(x) = \frac{1}{2} + \frac{i}{\pi} \sum_{\substack{-\infty \\ \text{odd } n}} \frac{1}{n} e^{in\pi x/l} = \frac{1}{2} - \frac{2}{\pi} \sum_1 \frac{1}{n} \sin \frac{n\pi x}{l}$$

$$\begin{aligned}
8.2 \quad a_n &= \frac{1}{n\pi} \sin \frac{n\pi}{2}, \quad b_n = \frac{1}{n\pi} (1 - \cos \frac{n\pi}{2}), \quad a_0/2 = c_0 = \frac{1}{4} \\
c_n &= \frac{i}{2n\pi} (e^{-in\pi/2} - 1) \\
&= \frac{1}{2n\pi} \{1 - i, -2i, -(1 + i), 0, \text{ and repeat}\}, \quad n > 0; c_{-n} = \bar{c}_n \\
f(x) &= \frac{1}{4} + \frac{1}{2\pi} [(1 - i)e^{i\pi x/l} + (1 + i)e^{-i\pi x/l} - \frac{2i}{2}(e^{2i\pi x/l} - e^{-2i\pi x/l}) \\
&\quad - \frac{1+i}{3}e^{3i\pi x/l} - \frac{1-i}{3}e^{-3i\pi x/l} + \frac{1-i}{5}e^{5i\pi x/l} + \frac{1+i}{5}e^{-5i\pi x/l} \dots] \\
&= \frac{1}{4} + \frac{1}{\pi} (\cos \frac{\pi x}{l} - \frac{1}{3} \cos \frac{3\pi x}{l} + \frac{1}{5} \cos \frac{5\pi x}{l} \dots) \\
&\quad + \frac{1}{\pi} (\sin \frac{\pi x}{l} + \frac{2}{2} \sin \frac{2\pi x}{l} + \frac{1}{3} \sin \frac{3\pi x}{l} + \frac{1}{5} \sin \frac{5\pi x}{l} + \frac{2}{6} \sin \frac{6\pi x}{l} \dots)
\end{aligned}$$

- 8.3 $a_n = -\frac{1}{n\pi} \sin \frac{n\pi}{2}$, $a_0/2 = c_0 = \frac{1}{4}$
 $b_n = \frac{1}{n\pi} (\cos \frac{n\pi}{2} - \cos n\pi) = \frac{1}{n\pi} \{1, -2, 1, 0, \text{ and repeat}\}$
 $c_n = \frac{i}{2n\pi} (e^{-in\pi} - e^{-in\pi/2})$
 $= \frac{1}{2n\pi} \{-(1+i), 2i, 1-i, 0, \text{ and repeat}\}, n > 0; c_{-n} = \bar{c}_n$
 $f(x) = \frac{1}{4} + \frac{1}{2\pi} \left[-(1+i)e^{i\pi x/l} - (1-i)e^{-i\pi x/l} + \frac{2i}{2}(e^{2i\pi x/l} - e^{-2i\pi x/l}) \right.$
 $\left. + \frac{1-i}{3}e^{3i\pi x/l} + \frac{1+i}{3}e^{-3i\pi x/l} - \frac{1+i}{5}e^{5i\pi x/l} - \frac{1-i}{5}e^{-5i\pi x/l} \dots \right]$
 $= \frac{1}{4} - \frac{1}{\pi} (\cos \frac{\pi x}{l} - \frac{1}{3} \cos \frac{3\pi x}{l} + \frac{1}{5} \cos \frac{5\pi x}{l} \dots)$
 $+ \frac{1}{\pi} (\sin \frac{\pi x}{l} - \frac{2}{2} \sin \frac{2\pi x}{l} + \frac{1}{3} \sin \frac{3\pi x}{l} + \frac{1}{5} \sin \frac{5\pi x}{l} - \frac{2}{6} \sin \frac{6\pi x}{l} \dots)$
- 8.4 $c_0 = a_0/2 = -1/2$; for $n \neq 0$, coefficients are 2 times the coefficients in Problem 8.3.
 $f(x) = -\frac{1}{2} - \frac{1}{\pi} \left[(1+i)e^{i\pi x/l} + (1-i)e^{-i\pi x/l} - \frac{2i}{2}(e^{2i\pi x/l} - e^{-2i\pi x/l}) \right.$
 $\left. - \frac{1-i}{3}e^{3i\pi x/l} - \frac{1+i}{3}e^{-3i\pi x/l} + \frac{1+i}{5}e^{5i\pi x/l} + \frac{1-i}{5}e^{-5i\pi x/l} \dots \right]$
 $= -\frac{1}{2} - \frac{2}{\pi} (\cos \frac{\pi x}{l} - \frac{1}{3} \cos \frac{3\pi x}{l} + \frac{1}{5} \cos \frac{5\pi x}{l} \dots)$
 $+ \frac{2}{\pi} (\sin \frac{\pi x}{l} - \frac{2}{2} \sin \frac{2\pi x}{l} + \frac{1}{3} \sin \frac{3\pi x}{l} + \frac{1}{5} \sin \frac{5\pi x}{l} - \frac{2}{6} \sin \frac{6\pi x}{l} \dots)$
- 8.5 $a_n = -\frac{2}{n\pi} \sin \frac{n\pi}{2}$, $a_0 = 0$,
 $b_n = \frac{1}{n\pi} (2 \cos \frac{n\pi}{2} - 1 - \cos n\pi) = -\frac{4}{n\pi} \{0, 1, 0, 0, \text{ and repeat}\}$
 $c_n = \frac{1}{2in\pi} (2e^{-in\pi/2} - 1 - e^{-in\pi}) = \frac{1}{n\pi} \{-1, 2i, 1, 0, \text{ and repeat}\}, n > 0$
 $c_{-n} = \bar{c}_n, c_0 = 0$
 $f(x) = -\frac{1}{\pi} \left[e^{i\pi x/l} + e^{-i\pi x/l} - \frac{2i}{2}(e^{2i\pi x/l} - e^{-2i\pi x/l}) \right.$
 $\left. - \frac{1}{3}(e^{3i\pi x/l} + e^{-3i\pi x/l}) \right.$
 $\left. + \frac{1}{5}(e^{5i\pi x/l} + e^{-5i\pi x/l}) - \frac{2i}{6}(e^{6i\pi x/l} - e^{-6i\pi x/l}) \dots \right]$
 $= -\frac{2}{\pi} (\cos \frac{\pi x}{l} - \frac{1}{3} \cos \frac{3\pi x}{l} + \frac{1}{5} \cos \frac{5\pi x}{l} \dots)$
 $- \frac{4}{\pi} (\frac{1}{2} \sin \frac{2\pi x}{l} + \frac{1}{6} \sin \frac{6\pi x}{l} + \frac{1}{10} \sin \frac{10\pi x}{l} \dots)$
- 8.6 $f(x) = \frac{1}{2} + \frac{2}{i\pi} \sum \frac{1}{n} e^{in\pi x/l} \quad (n = \pm 2, \pm 6, \pm 10, \dots)$
 $= \frac{1}{2} + \frac{4}{\pi} \sum \frac{1}{n} \sin \frac{n\pi x}{l} \quad (n = 2, 6, 10, \dots)$
- 8.7 $f(x) = \frac{l}{4} + \frac{il}{2\pi} \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \frac{(-1)^n}{n} e^{in\pi x/l} - \sum_{\substack{-\infty \\ \text{odd } n}}^{\infty} \frac{l}{n^2 \pi^2} e^{in\pi x/l}$
 $= \frac{l}{4} - \frac{2l}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos \frac{n\pi x}{l} - \frac{l}{\pi} \sum_1^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi x}{l}$
- 8.8 $f(x) = 1 + \frac{il}{\pi} \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \frac{(-1)^n}{n} e^{in\pi x/l} = 1 - \frac{2l}{\pi} \sum_1^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi x}{l}$
- 8.9 $f(x) = \frac{l}{2} - \frac{2l}{\pi^2} \sum_{\substack{-\infty \\ \text{odd } n}}^{\infty} \frac{e^{in\pi x/l}}{n^2} = \frac{l}{2} - \frac{4l}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{\cos n\pi x/l}{n^2}$
- 8.10 (a) $f(x) = i \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \frac{(-1)^n}{n} e^{inx} = -2 \sum_1^{\infty} \frac{(-1)^n}{n} \sin nx$
(b) $f(x) = \pi + \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \frac{i}{n} e^{inx} = \pi - 2 \sum_1^{\infty} \frac{\sin nx}{n}$

- 8.11 (a) $f(x) = \frac{\pi^2}{3} + 2 \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \frac{(-1)^n}{n^2} e^{inx} = \frac{\pi^2}{3} + 4 \sum_1^{\infty} \frac{(-1)^n}{n^2} \cos nx$
- (b) $f(x) = \frac{4\pi^2}{3} + 2 \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \left(\frac{1}{n^2} + \frac{i\pi}{n} \right) e^{inx} = \frac{4\pi^2}{3} + 4 \sum_1^{\infty} \frac{\cos nx}{n^2} - 4\pi \sum_1^{\infty} \frac{\sin nx}{n}$
- 8.12 (a) $f(x) = \frac{\sinh \pi}{\pi} \sum_{-\infty}^{\infty} \frac{(-1)^n (1 + in)}{1 + n^2} e^{inx}$
 $= \frac{\sinh \pi}{\pi} + \frac{2 \sinh \pi}{\pi} \sum_1^{\infty} \frac{(-1)^n}{1 + n^2} (\cos nx - n \sin nx)$
- (b) $f(x) = \frac{e^{2\pi} - 1}{2\pi} \sum_{-\infty}^{\infty} \frac{1 + in}{1 + n^2} e^{inx}$
 $= \frac{e^{2\pi} - 1}{\pi} \left[\frac{1}{2} + \sum_1^{\infty} \frac{1}{1 + n^2} (\cos nx - n \sin nx) \right]$
- 8.13 (a) $f(x) = 2 + \frac{2}{i\pi} \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \frac{(-1)^n}{n} e^{in\pi x/2} = 2 + \frac{4}{\pi} \sum_1^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi x}{2}$
- (b) $f(x) = \frac{2}{i\pi} \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \frac{1}{n} e^{in\pi x/2} = \frac{4}{\pi} \sum_1^{\infty} \frac{1}{n} \sin \frac{n\pi x}{2}$
- 8.14 (a) $f(x) = \frac{8}{\pi} \sum_1^{\infty} \frac{n(-1)^{n+1}}{4n^2 - 1} \sin 2n\pi x = \frac{4i}{\pi} \sum_{-\infty}^{\infty} \frac{n(-1)^n}{4n^2 - 1} e^{2in\pi x}$
- (b) $f(x) = \frac{2}{\pi} - \frac{4}{\pi} \sum_1^{\infty} \frac{\cos 2n\pi x}{4n^2 - 1} = -\frac{2}{\pi} \sum_{-\infty}^{\infty} \frac{1}{4n^2 - 1} e^{2in\pi x}$
- 8.15 (a) $f(x) = \frac{i}{\pi} \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \frac{(-1)^n}{n} e^{in\pi x} = \frac{2}{\pi} \sum_1^{\infty} (-1)^{n+1} \frac{\sin n\pi x}{n}$
- (b) $f(x) = \frac{4}{\pi^2} \sum_{\substack{-\infty \\ \text{odd } n}}^{\infty} \frac{1}{n^2} e^{in\pi x} = \frac{8}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{\cos n\pi x}{n^2}$
- (c) $f(x) = \frac{-4i}{\pi^3} \sum_{\substack{-\infty \\ \text{odd } n}}^{\infty} \frac{1}{n^3} e^{in\pi x} = \frac{8}{\pi^3} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{\sin n\pi x}{n^3}$
- 8.16 $f(x) = 1 - \frac{2}{\pi} \sum_1^{\infty} \frac{\sin n\pi x}{n} = 1 - \frac{1}{i\pi} \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \frac{1}{n} e^{in\pi x}$
- 8.17 $f(x) = \frac{3}{4} - \frac{1}{\pi} \left(\cos \frac{\pi x}{2} - \frac{1}{3} \cos \frac{3\pi x}{2} + \frac{1}{5} \cos \frac{5\pi x}{2} \cdots \right)$
 $+ \frac{1}{\pi} \left(\sin \frac{\pi x}{2} + \frac{2}{2} \sin \pi x + \frac{1}{3} \sin \frac{3\pi x}{2} + \frac{1}{5} \sin \frac{5\pi x}{2} + \frac{2}{6} \sin 3\pi x \cdots \right)$
- 8.18 $f(x) = \frac{100}{3} + \frac{100}{\pi^2} \sum_1^{\infty} \frac{1}{n^2} \cos \frac{n\pi x}{5} - \frac{100}{\pi} \sum_1^{\infty} \frac{1}{n} \sin \frac{n\pi x}{5}$
 $= \frac{100}{3} + 50 \sum_{\substack{-\infty \\ n \neq 0}}^{\infty} \left(\frac{1}{n^2 \pi^2} - \frac{1}{in\pi} \right) e^{in\pi x/5}$
- 8.19 $f(x) = \frac{1}{8} - \frac{1}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos 2n\pi x + \frac{1}{2\pi} \sum_1^{\infty} \frac{(-1)^{n+1}}{n} \sin 2n\pi x$

$$8.20 \quad f(x) = \frac{2}{3} + \sum_1^{\infty} a_n \cos \frac{2n\pi x}{3} + \sum_1^{\infty} b_n \sin \frac{2n\pi x}{3}, \text{ where}$$

$$a_n = \begin{cases} 0, & n = 3k \\ -\frac{9}{8n^2\pi^2}, & \text{otherwise} \end{cases} \quad b_n = \begin{cases} -\frac{1}{n\pi}, & n = 3k \\ -\frac{1}{n\pi} - \frac{3\sqrt{3}}{8n^2\pi^2}, & n = 3k+1 \\ -\frac{1}{n\pi} + \frac{3\sqrt{3}}{8n^2\pi^2}, & n = 3k+2 \end{cases}$$

$$9.1 \quad (\text{a}) \cos nx + i \sin nx$$

$$(\text{b}) x \sinh x + x \cosh x$$

$$9.2 \quad (\text{a}) \frac{1}{2} \ln |1 - x^2| + \frac{1}{2} \ln \left| \frac{1-x}{1+x} \right|$$

$$(\text{b}) (\cos x + x \sin x) + (\sin x + x \cos x)$$

$$9.3 \quad (\text{a}) (-x^4 - 1) + (x^5 + x^3)$$

$$(\text{b}) (1 + \cosh x) + \sinh x$$

$$9.5 \quad f(x) = \frac{4}{\pi} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n} \sin nx$$

$$9.6 \quad f(x) = \frac{4}{\pi} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n} \sin \frac{n\pi x}{l}$$

$$9.7 \quad a_n = \frac{2}{n\pi} \sin \frac{n\pi}{2}, a_0/2 = 1/2$$

$$f(x) = \frac{1}{2} + \frac{2}{\pi} \left(\cos \frac{\pi x}{2} - \frac{1}{3} \cos \frac{3\pi x}{2} + \frac{1}{5} \cos \frac{5\pi x}{2} \dots \right)$$

$$9.8 \quad f(x) = \sum_1^{\infty} \frac{(-1)^{n+1}}{n} \sin 2nx$$

$$9.9 \quad f(x) = \frac{1}{12} + \frac{1}{\pi^2} \sum_1^{\infty} \frac{(-1)^n}{n^2} \cos 2n\pi x$$

$$9.10 \quad f(x) = \frac{\pi}{4} - \frac{2}{\pi} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos 2nx$$

$$9.11 \quad f(x) = \frac{2 \sinh \pi}{\pi} \left(\frac{1}{2} + \sum_1^{\infty} \frac{(-1)^n}{n^2 + 1} \cos nx \right)$$

$$9.12 \quad f(x) = -\frac{2}{\pi} \sum_1^{\infty} \frac{1}{n} \sin n\pi x$$

$$9.15 \quad f_c(x) = \frac{1}{2} - \frac{4}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{\cos n\pi x}{n^2} \quad f_s(x) = \frac{2}{\pi} \sum_1^{\infty} \frac{(-1)^{n+1} \sin n\pi x}{n}$$

$$9.16 \quad f_s = \frac{8}{\pi} \sum_{\substack{1 \\ \text{odd } n=1}}^{\infty} \frac{\sin nx}{n(4-n^2)} \quad f_c = f_p = (1 - \cos 2x)/2$$

$$9.17 \quad f_c(x) = \frac{4}{\pi} \left(\cos \pi x - \frac{1}{3} \cos 3\pi x + \frac{1}{5} \cos 5\pi x \dots \right)$$

$$f_s(x) = f_p(x) = \frac{4}{\pi} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n} \sin 2n\pi x$$

$$9.18 \quad \text{Even function: } a_0/2 = 1/3,$$

$$a_n = \frac{2}{n\pi} \sin \frac{n\pi}{3} = \frac{\sqrt{3}}{n\pi} \{1, 1, 0, -1, -1, 0, \text{ and repeat}\}$$

$$f_c(x) = \frac{1}{3} + \frac{\sqrt{3}}{\pi} \left(\cos \frac{\pi x}{3} + \frac{1}{2} \cos \frac{2\pi x}{3} - \frac{1}{4} \cos \frac{4\pi x}{3} - \frac{1}{5} \cos \frac{5\pi x}{3} + \frac{1}{7} \cos \frac{7\pi x}{3} \dots \right)$$

$$\text{Odd function: } b_n = \frac{2}{n\pi} (1 - \cos \frac{n\pi}{3}) = \frac{1}{n\pi} \{1, 3, 4, 3, 1, 0, \text{ and repeat}\}$$

$$f_s(x) = \frac{1}{\pi} \left(\sin \frac{\pi x}{3} + \frac{3}{2} \sin \frac{2\pi x}{3} + \frac{4}{3} \sin \frac{3\pi x}{3} \right. \\ \left. + \frac{3}{4} \sin \frac{4\pi x}{3} + \frac{1}{5} \sin \frac{5\pi x}{3} + \frac{1}{7} \sin \frac{7\pi x}{3} \dots \right)$$

9.18 continued

Function of period 3:

$$\begin{aligned}
 a_n &= \frac{1}{n\pi} \sin \frac{2n\pi}{3} = \frac{\sqrt{3}}{2n\pi} \{1, -1, 0, \text{and repeat}\}, \quad a_0/2 = 1/3 \\
 b_n &= \frac{1}{n\pi} (1 - \cos \frac{2n\pi}{3}) = \frac{3}{2n\pi} \{1, 1, 0, \text{and repeat}\} \\
 f_p(x) &= \frac{1}{3} + \frac{\sqrt{3}}{2\pi} \left(\cos \frac{2\pi x}{3} - \frac{1}{2} \cos \frac{4\pi x}{3} + \frac{1}{4} \cos \frac{8\pi x}{3} - \frac{1}{5} \cos \frac{10\pi x}{3} \dots \right) \\
 &\quad + \frac{3}{2\pi} \left(\sin \frac{2\pi x}{3} + \frac{1}{2} \sin \frac{4\pi x}{3} + \frac{1}{4} \sin \frac{8\pi x}{3} + \frac{1}{5} \sin \frac{10\pi x}{3} \dots \right)
 \end{aligned}$$

$$9.19 \quad f_c(x) = f_p(x) = \frac{2}{\pi} - \frac{4}{\pi} \sum_1^{\infty} \frac{(-1)^n \cos 2nx}{4n^2 - 1}$$

$$\text{For } f_s, \quad b_n = \frac{2}{\pi} \begin{cases} 0, & n \text{ even} \\ \frac{2}{n+1}, & n = 1 + 4k \\ \frac{2}{n-1}, & n = 3 + 4k \end{cases}$$

$$f_s(x) = \frac{2}{\pi} (\sin x + \sin 3x + \frac{1}{3} \sin 5x + \frac{1}{3} \sin 7x + \frac{1}{5} \sin 9x + \frac{1}{5} \sin 11x \dots)$$

$$\begin{aligned}
 9.20 \quad f_c(x) &= \frac{1}{3} + \frac{4}{\pi^2} \sum_1^{\infty} \frac{(-1)^n}{n^2} \cos n\pi x \\
 f_s(x) &= \frac{2}{\pi} \sum_1^{\infty} \frac{(-1)^{n+1}}{n} \sin n\pi x - \frac{8}{\pi^3} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^3} \sin n\pi x \\
 f_p(x) &= \frac{1}{3} + \frac{1}{\pi^2} \sum_1^{\infty} \frac{1}{n^2} \cos 2n\pi x - \frac{1}{\pi} \sum_1^{\infty} \frac{1}{n} \sin 2n\pi x
 \end{aligned}$$

$$\begin{aligned}
 9.21 \quad f_c(x) &= f_p(x) = \frac{1}{2} - \frac{4}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos n\pi x \\
 f_s(x) &= \frac{8}{\pi^2} \left(\sin \frac{\pi x}{2} - \frac{1}{3^2} \sin \frac{3\pi x}{2} + \frac{1}{5^2} \sin \frac{5\pi x}{2} \dots \right); \quad b_n = \frac{8}{n^2 \pi^2} \sin \frac{n\pi}{2}
 \end{aligned}$$

$$\begin{aligned}
 9.22 \quad \text{Even function: } a_n &= -\frac{20}{n\pi} \sin \frac{n\pi}{2} \\
 f_c(x) &= 15 - \frac{20}{\pi} \left(\cos \frac{\pi x}{20} - \frac{1}{3} \cos \frac{3\pi x}{20} + \frac{1}{5} \cos \frac{5\pi x}{20} \dots \right)
 \end{aligned}$$

Odd function:

$$\begin{aligned}
 b_n &= \frac{20}{n\pi} (\cos \frac{n\pi}{2} + 1 - 2 \cos n\pi) = \frac{20}{n\pi} \{3, -2, 3, 0, \text{and repeat}\} \\
 f_s(x) &= \frac{20}{\pi} \left(3 \sin \frac{\pi x}{20} - \frac{2}{2} \sin \frac{2\pi x}{20} + \frac{3}{3} \sin \frac{3\pi x}{20} + \frac{3}{5} \sin \frac{5\pi x}{20} \dots \right)
 \end{aligned}$$

Function of period 20:

$$f_p(x) = 15 - \frac{20}{\pi} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n} \sin \frac{n\pi x}{10}$$

$$9.23 \quad f(x, 0) = \frac{8h}{\pi^2} \left(\sin \frac{\pi x}{l} - \frac{1}{3^2} \sin \frac{3\pi x}{l} + \frac{1}{5^2} \sin \frac{5\pi x}{l} \dots \right)$$

$$9.24 \quad f(x, 0) = \frac{8h}{\pi^2} \sum_1^{\infty} \frac{\lambda_n}{n^2} \sin \frac{n\pi x}{l} \text{ where}$$

$$\begin{aligned}
 \lambda_1 &= \sqrt{2} - 1, \quad \lambda_2 = 2, \quad \lambda_3 = \sqrt{2} + 1, \quad \lambda_4 = 0, \quad \lambda_5 = -(\sqrt{2} + 1), \\
 \lambda_6 &= -2, \quad \lambda_7 = -\sqrt{2} + 1, \quad \lambda_8 = 0, \quad \dots, \quad \lambda_n = 2 \sin \frac{n\pi}{4} - \sin \frac{n\pi}{2}
 \end{aligned}$$

$$9.26 \quad f(x) = \frac{1}{2} - \frac{48}{\pi^4} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{\cos n\pi x}{n^4} \qquad 9.27 \quad f(x) = \frac{8\pi^4}{15} - 48 \sum_1^{\infty} (-1)^n \frac{\cos nx}{n^4}$$

- 10.1 $p(t) = \sum_1^{\infty} a_n \cos 220n\pi t$, $a_0 = 0$
 $a_n = \frac{2}{n\pi}(\sin \frac{n\pi}{3} + \sin \frac{2n\pi}{3}) = \frac{2}{n\pi}\{\sqrt{3}, 0, 0, 0, -\sqrt{3}, 0, \text{and repeat}\}$
Relative intensities = $1 : 0 : 0 : 0 : \frac{1}{25} : 0 : \frac{1}{49} : 0 : 0 : 0$
- 10.2 $p(t) = \sum_1^{\infty} b_n \sin 262n\pi t$, where
 $b_n = \frac{2}{n\pi}(1 - \cos \frac{n\pi}{3} - 3 \cos n\pi + 3 \cos \frac{2n\pi}{3})$
 $= \frac{2}{n\pi}\{2, -3, 8, -3, 2, 0, \text{and repeat}\}$
Relative intensities = $4 : \frac{9}{4} : \frac{64}{9} : \frac{9}{16} : \frac{4}{25} : 0$
- 10.3 $p(t) = \sum b_n \sin 220n\pi t$
 $b_n = \frac{2}{n\pi}(3 - 5 \cos \frac{n\pi}{2} + 2 \cos n\pi) = \frac{2}{n\pi}\{1, 10, 1, 0, \text{and repeat}\}$
Relative intensities = $1 : 25 : \frac{1}{9} : 0 : \frac{1}{25} : \frac{25}{9} : \frac{1}{49} : 0 : \frac{1}{81} : 1$
- 10.4 $V(t) = \frac{200}{\pi} \left[1 + \sum_{\substack{2 \\ \text{even } n}}^{\infty} \frac{2}{1-n^2} \cos 120n\pi t \right]$
Relative intensities = $0 : 1 : 0 : \frac{1}{25} : 0 : (\frac{3}{35})^2$
- 10.5 $I(t) = \frac{5}{\pi} \left[1 + \sum_{\substack{2 \\ \text{even } n}}^{\infty} \frac{2}{1-n^2} \cos 120n\pi t \right] + \frac{5}{2} \sin 120\pi t$
Relative intensities = $(\frac{5}{2})^2 : (\frac{10}{3\pi})^2 : 0 : (\frac{2}{3\pi})^2 : 0 : (\frac{2}{7\pi})^2$
 $= 6.25 : 1.13 : 0 : 0.045 : 0 : 0.008$
- 10.6 $V(t) = 50 - \frac{400}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos 120n\pi t$
Relative intensities = $1 : 0 : (\frac{1}{3})^4 : 0 : (\frac{1}{5})^4$
- 10.7 $I(t) = -\frac{20}{\pi} \sum_1^{\infty} \frac{(-1)^n}{n} \sin 120n\pi t$
Relative intensities = $1 : \frac{1}{4} : \frac{1}{9} : \frac{1}{16} : \frac{1}{25}$
- 10.8 $I(t) = \frac{5}{2} - \frac{20}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos 120n\pi t - \frac{10}{\pi} \sum_1^{\infty} \frac{(-1)^n}{n} \sin 120n\pi t$
Relative intensities = $\left(1 + \frac{4}{\pi^2}\right) : \frac{1}{4} : \frac{1}{9} \left(1 + \frac{4}{9\pi^2}\right) : \frac{1}{16} : \frac{1}{25} \left(1 + \frac{4}{25\pi^2}\right)$
 $= 1.4 : 0.25 : 0.12 : 0.06 : 0.04$
- 10.9 $V(t) = \frac{400}{\pi} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n} \sin 120n\pi t$
Relative intensities = $1 : 0 : \frac{1}{9} : 0 : \frac{1}{25}$
- 10.10 $V(t) = 75 - \frac{200}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos 120n\pi t - \frac{100}{\pi} \sum_1^{\infty} \frac{1}{n} \sin 120n\pi t$
Relative intensities as in problem 10.8
- 11.5 $\pi^2/8$ 11.6 $\pi^4/90$ 11.7 $\pi^2/6$
11.8 $\pi^4/96$ 11.9 $\frac{\pi^2}{16} - \frac{1}{2}$

$$12.2 \quad f_s(x) = \frac{2}{\pi} \int_0^\infty \frac{1 - \cos \alpha}{\alpha} \sin \alpha x \, d\alpha$$

$$12.3 \quad f(x) = \int_{-\infty}^\infty \frac{1 - \cos \alpha \pi}{i\alpha \pi} e^{i\alpha x} \, d\alpha$$

$$12.4 \quad f(x) = \int_{-\infty}^\infty \frac{\sin \alpha \pi - \sin(\alpha \pi/2)}{\alpha \pi} e^{i\alpha x} \, d\alpha$$

$$12.5 \quad f(x) = \int_{-\infty}^\infty \frac{1 - e^{-i\alpha}}{2\pi i \alpha} e^{i\alpha x} \, d\alpha$$

$$12.6 \quad f(x) = \int_{-\infty}^\infty \frac{\sin \alpha - \alpha \cos \alpha}{i\pi \alpha^2} e^{i\alpha x} \, d\alpha$$

$$12.7 \quad f(x) = \int_{-\infty}^\infty \frac{\cos \alpha + \alpha \sin \alpha - 1}{\pi \alpha^2} e^{i\alpha x} \, d\alpha$$

$$12.8 \quad f(x) = \int_{-\infty}^\infty \frac{(i\alpha + 1)e^{-i\alpha} - 1}{2\pi \alpha^2} e^{i\alpha x} \, d\alpha$$

$$12.9 \quad f(x) = \frac{2}{\pi} \int_{-\infty}^\infty \frac{i - \cos \alpha a}{\alpha^2} e^{i\alpha x} \, d\alpha$$

$$12.10 \quad f(x) = 2 \int_{-\infty}^\infty \frac{\alpha a - \sin \alpha a}{i\pi \alpha^2} e^{i\alpha x} \, d\alpha$$

$$12.11 \quad f(x) = \frac{1}{\pi} \int_{-\infty}^\infty \frac{\cos(\alpha \pi/2)}{1 - \alpha^2} e^{i\alpha x} \, d\alpha$$

$$12.12 \quad f(x) = \frac{1}{\pi i} \int_{-\infty}^\infty \frac{\alpha \cos(\alpha \pi/2)}{1 - \alpha^2} e^{i\alpha x} \, d\alpha$$

$$12.13 \quad f_c(x) = \frac{2}{\pi} \int_0^\infty \frac{\sin \alpha \pi - \sin(\alpha \pi/2)}{\alpha} \cos \alpha x \, d\alpha$$

$$12.14 \quad f_c(x) = \frac{2}{\pi} \int_0^\infty \frac{\cos \alpha + \alpha \sin \alpha - 1}{\alpha^2} \cos \alpha x \, d\alpha$$

$$12.15 \quad f_c(x) = \frac{4}{\pi} \int_0^\infty \frac{1 - \cos \alpha a}{\alpha^2} \cos \alpha x \, d\alpha$$

$$12.16 \quad f_c(x) = \frac{2}{\pi} \int_0^\infty \frac{\cos(\alpha \pi/2)}{1 - \alpha^2} \cos \alpha x \, d\alpha$$

$$12.17 \quad f_s(x) = \frac{2}{\pi} \int_0^\infty \frac{1 - \cos \pi \alpha}{\alpha} \sin \alpha x \, d\alpha$$

$$12.18 \quad f_s(x) = \frac{2}{\pi} \int_0^\infty \frac{\sin \alpha - \alpha \cos \alpha}{\alpha^2} \sin \alpha x \, d\alpha$$

$$12.19 \quad f_s(x) = \frac{4}{\pi} \int_0^\infty \frac{\alpha a - \sin \alpha a}{\alpha^2} \sin \alpha x \, d\alpha$$

$$12.20 \quad f_s(x) = \frac{2}{\pi} \int_0^\infty \frac{\alpha \cos(\alpha \pi/2)}{1 - \alpha^2} \sin \alpha x \, d\alpha$$

$$12.21 \quad g(\alpha) = \frac{\sigma}{\sqrt{2\pi}} e^{-\alpha^2 \sigma^2/2}$$

$$12.24 \quad (c) \quad g_c(\alpha) = \sqrt{\frac{\pi}{2}} e^{-|\alpha|}$$

$$12.25 \quad (a) \quad f(x) = \frac{1}{2\pi} \int_{-\infty}^\infty \frac{1 + e^{-i\alpha \pi}}{1 - \alpha^2} e^{i\alpha x} \, d\alpha$$

$$12.27 \quad (a) \quad f_c(x) = \frac{2}{\pi} \int_0^\infty \frac{\sin(\alpha \pi/2)}{\alpha} \cos \alpha x \, d\alpha$$

$$(b) \quad f_s(x) = \frac{2}{\pi} \int_0^\infty \frac{1 - \cos(\alpha \pi/2)}{\alpha} \sin \alpha x \, d\alpha$$

$$12.28 \quad (a) \quad f_c(x) = \frac{4}{\pi} \int_0^\infty \frac{\cos 3\alpha \sin \alpha}{\alpha} \cos \alpha x \, d\alpha$$

$$(b) \quad f_s(x) = \frac{4}{\pi} \int_0^\infty \frac{\sin 3\alpha \sin \alpha}{\alpha} \sin \alpha x \, d\alpha$$

$$12.29 \quad (a) \quad f_c(x) = \frac{2}{\pi} \int_0^\infty \frac{\sin 3\alpha - 2 \sin 2\alpha}{\alpha} \cos \alpha x \, d\alpha$$

$$(b) \quad f_s(x) = \frac{2}{\pi} \int_0^\infty \frac{2 \cos 2\alpha - \cos 3\alpha - 1}{\alpha} \sin \alpha x \, d\alpha$$

$$12.30 \quad (a) \quad f_c(x) = \frac{1}{\pi} \int_0^\infty \frac{1 - \cos 2\alpha}{\alpha^2} \cos \alpha x \, d\alpha$$

$$(b) \quad f_s(x) = \frac{1}{\pi} \int_0^\infty \frac{2\alpha - \sin 2\alpha}{\alpha^2} \sin \alpha x \, d\alpha$$

$$13.2 \quad f(x) = \frac{i}{2\pi} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{1}{n} e^{2in\pi x}$$

$$13.4 \quad (c) \quad q(t) = CV \left[1 - 2(1 - e^{-1/2}) \sum_{-\infty}^{\infty} (1 + 4in\pi)^{-1} e^{4in\pi t/(RC)} \right]$$

$$13.6 \quad f(t) = \sum_{-\infty}^{\infty} \frac{(-1)^n \sin \omega \pi}{\pi(\omega - n)} e^{int}$$

$$13.7 \quad f(x) = \frac{\pi}{2} - \frac{4}{\pi} \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \frac{1}{n^2} \cos nx$$

$$13.8 \quad (a) \quad 1/2 \qquad (b) \quad 1$$

$$13.9 \quad (b) \quad -1/2, 0, 0, 1/2 \qquad (c) \quad 13/6$$

$$13.10 \quad (c) \quad 0, -1/2, -2, -2 \qquad (d) \quad -1, -1/2, -2, -1$$

$$13.11 \quad \text{Cosine series: } a_0/2 = -3/4,$$

$$\begin{aligned} a_n &= \frac{4}{n^2\pi^2} \left(\cos \frac{n\pi}{2} - 1 \right) + \frac{6}{n\pi} \sin \frac{n\pi}{2} \\ &= \frac{4}{n^2\pi^2} \{-1, -2, -1, 0, \text{ and repeat}\} + \frac{6}{n\pi} \{1, 0, -1, 0, \text{ and repeat}\} \\ f_c(x) &= -\frac{3}{4} + \left(-\frac{4}{\pi^2} + \frac{6}{\pi} \right) \cos \frac{\pi x}{2} - \frac{2}{\pi^2} \cos \pi x \\ &\quad - \left(\frac{4}{9\pi^2} + \frac{2}{\pi} \right) \cos \frac{3\pi x}{2} + \left(\frac{-4}{25\pi^2} + \frac{6}{5\pi} \right) \cos \frac{5\pi x}{2} \cdots \end{aligned}$$

Sine series:

$$\begin{aligned} b_n &= \frac{4}{n^2\pi^2} \sin \frac{n\pi}{2} + \frac{1}{n\pi} \left(4 \cos n\pi - 6 \cos \frac{n\pi}{2} \right) \\ &= \frac{4}{n^2\pi^2} \{1, 0, -1, 0, \text{ and repeat}\} + \frac{1}{n\pi} \{-4, 10, -4, -2, \text{ and repeat}\} \\ f_s(x) &= \left(\frac{4}{\pi^2} - \frac{4}{\pi} \right) \sin \frac{\pi x}{2} + \frac{5}{\pi} \sin \pi x - \left(\frac{4}{9\pi^2} + \frac{4}{3\pi} \right) \sin \frac{3\pi x}{2} \\ &\quad - \frac{1}{2\pi} \sin 2\pi x + \left(\frac{4}{25\pi^2} - \frac{4}{5\pi} \right) \sin \frac{5\pi x}{2} + \frac{5}{3\pi} \sin 3\pi x \cdots \end{aligned}$$

Exponential series of period 2:

$$f_p(x) = -\frac{3}{4} - \sum_{\substack{n=-\infty \\ \text{odd}}}^{\infty} \left(\frac{1}{n^2\pi^2} + \frac{5i}{2n\pi} \right) e^{in\pi x} + \frac{i}{2\pi} \sum_{\substack{n=-\infty \\ \text{even } n \neq 0}}^{\infty} \frac{1}{n} e^{in\pi x}$$

$$13.12 \quad f = 90$$

$$13.13 \quad (a) \quad f_s(x) = \sum_1^{\infty} \frac{\sin nx}{n} \qquad (b) \quad \pi^2/6$$

$$13.14 \quad (a) \quad f(x) = \frac{1}{3} + \frac{4}{\pi^2} \sum_1^{\infty} \frac{\cos n\pi x}{n^2} \qquad (b) \quad \pi^4/90$$

$$13.15 \quad g(\alpha) = \frac{\cos 2\alpha - 1}{i\pi\alpha}, \quad f(x) = \frac{2}{\pi} \int_0^\infty \frac{\cos 2\alpha - 1}{\alpha} \sin \alpha x \, d\alpha, \quad -\pi/4$$

$$13.16 \quad f(x) = \frac{8}{\pi} \int_0^\infty \frac{\cos \alpha \sin^2(\alpha/2)}{\alpha^2} \cos \alpha x \, d\alpha, \quad \pi/8$$

$$13.19 \quad \int_0^\infty |f(x)|^2 \, dx = \int_0^\infty |g_c(\alpha)|^2 \, d\alpha = \int_0^\infty |g_s(\alpha)|^2 \, d\alpha$$

$$13.20 \quad g(\alpha) = \frac{2 \sin^2 \alpha a}{\pi \alpha^2}, \quad \pi a^3/3 \qquad 13.23 \quad \pi^2/8$$

Chapter 8

- 1.4 $x = k^{-1}gt + k^{-2}g(e^{-kt} - 1)$
 1.5 $x = -A\omega^{-2}\sin\omega t + v_0t + x_0$
 1.6 (a) 15 months (b) $t = 30(1 - 2^{-1/3}) = 6.19$ months
 1.7 $x = (c/F)[(m^2c^2 + F^2t^2)^{1/2} - mc]$
- 2.1 $y = mx, m = 3/2$ 2.2 $(1 - x^2)^{1/2} + (1 - y^2)^{1/2} = C, C = \sqrt{3}$
 2.3 $\ln y = A(\csc x - \cot x), A = \sqrt{3}$ 2.4 $x^2(1 + y^2) = K, K = 25$
 2.5 $y = axe^x, a = 1/e$ 2.6 $2y^2 + 1 = A(x^2 - 1)^2, A = 1$
 2.7 $y^2 = 8 + e^{K-x^2}, K = 1$ 2.8 $y(x^2 + C) = 1, C = -3$
 2.9 $ye^y = ae^x, a = 1$ 2.10 $y + 1 = ke^{x^2/2}, k = 2$
 2.11 $(y - 2)^2 = (x + C)^3, C = 0$ 2.12 $xye^y = K, K = e$
 2.13 $y \equiv 1, y \equiv -1, x \equiv 1, x \equiv -1$ 2.14 $y \equiv 0$
 2.15 $y \equiv 2$ 2.16 $4y = (x + C)^2, C = 0$
 2.17 $x = (t - t_0)^2/4$
 2.19 (a) $I/I_0 = e^{-0.5} = 0.6$ for $s = 50$ ft
 Half value thickness $= (\ln 2)/\mu = 69.3$ ft
 (b) Half life $T = (\ln 2)/\lambda$
 2.20 (a) $q = q_0e^{-t/(RC)}$ (b) $I = I_0e^{-(R/L)t}$ (c) $\tau = RC, \tau = L/R$
 Corresponding quantities are $a, \lambda = (\ln 2)/T, \mu, 1/\tau$.
 2.21 $N = N_0e^{Kt}$
 2.22 $N = N_0e^{Kt} - (R/K)(e^{Kt} - 1)$ where N_0 = number of bacteria at $t = 0$,
 KN = rate of increase, R = removal rate.
 2.23 $T = 100[1 - (\ln r)/(\ln 2)]$
 2.24 $T = 100(2r^{-1} - 1)$
 2.26 (a) k = weight divided by terminal speed.
 (b) $t = g^{-1} \cdot (\text{terminal speed}) \cdot (\ln 100)$; typical terminal
 speeds are 0.02 to 0.1 cm/sec, so t is of the order of 10^{-4} sec.
 2.27 $t = 10(\ln \frac{5}{13})/(\ln \frac{3}{13}) = 6.6$ min
 2.28 66° 2.29 $t = 100 \ln \frac{9}{4} = 81.1$ min
 2.30 $A = Pe^{It/100}$ 2.31 $ay = bx$
 2.32 $x^2 + 2y^2 = C$ 2.33 $x^2 + ny^2 = C$
 2.34 $x^2 - y^2 = C$ 2.35 $x(y - 1) = C$
- 3.1 $y = \frac{1}{2}e^x + Ce^{-x}$ 3.2 $y = 1/(2x) + C/x^3$
 3.3 $y = (\frac{1}{2}x^2 + C)e^{-x^2}$ 3.4 $y = \frac{1}{3}x^{5/2} + Cx^{-1/2}$
 3.5 $y(\sec x + \tan x) = x - \cos x + C$ 3.6 $y = (x + C)/(x + \sqrt{x^2 + 1})$
 3.7 $y = \frac{1}{3}(1 + e^x) + C(1 + e^x)^{-2}$ 3.8 $y = \frac{1}{2}\ln x + C/\ln x$
 3.9 $y(1 - x^2)^{1/2} = x^2 + C$ 3.10 $y \cosh x = \frac{1}{2}e^{2x} + x + C$
 3.11 $y = 2(\sin x - 1) + Ce^{-\sin x}$ 3.12 $x = (y + C)\cos y$

- 3.13 $x = \frac{1}{2}e^y + Ce^{-y}$
- 3.14 $x = y^{2/3} + Cy^{-1/3}$
- 3.15 $S = \frac{1}{2} \times 10^7 [(1 + 3t/10^4) + (1 + 3t/10^4)^{-1/3}]$, where S = number of pounds of salt, and t is in hours.
- 3.16 $I = Ae^{-Rt/L} + V_0(R^2 + \omega^2 L^2)^{-1}(R \cos \omega t + \omega L \sin \omega t)$
- 3.17 $I = Ae^{-t/(RC)} - V_0 \omega C(\sin \omega t - \omega RC \cos \omega t)/(1 + \omega^2 R^2 C^2)$
- 3.18 RL circuit: $I = Ae^{-Rt/L} + V_0(R + i\omega L)^{-1}e^{i\omega t}$
 RC circuit: $I = Ae^{-t/RC} + i\omega V_0 C(1 + i\omega RC)^{-1}e^{i\omega t}$
- 3.19 $N_2 = N_0 \lambda t e^{-\lambda t}$
- 3.20 $N_3 = c_1 e^{-\lambda_1 t} + c_2 e^{-\lambda_2 t} + c_3 e^{-\lambda_3 t}$, where
 $c_1 = \frac{\lambda_1 \lambda_2 N_0}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1)}$, $c_2 = \frac{\lambda_1 \lambda_2 N_0}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2)}$, $c_3 = \frac{\lambda_1 \lambda_2 N_0}{(\lambda_1 - \lambda_3)(\lambda_2 - \lambda_3)}$
- 3.21 $N_n = c_1 e^{-\lambda_1 t} + c_2 e^{-\lambda_2 t} + \dots$, where
 $c_1 = \frac{\lambda_1 \lambda_2 \dots \lambda_{n-1} N_0}{(\lambda_2 - \lambda_1)(\lambda_3 - \lambda_1) \dots (\lambda_n - \lambda_1)}$, $c_2 = \frac{\lambda_1 \lambda_2 \dots \lambda_{n-1} N_0}{(\lambda_1 - \lambda_2)(\lambda_3 - \lambda_2) \dots (\lambda_n - \lambda_2)}$,
 etc. (all λ 's different).
- 3.22 $y = x + 1 + Ke^x$
- 3.23 $x = 2\pi^{-1/2} e^{-y^2} \int_k^y e^{u^2} du$
- 4.1 $y^{1/3} = x - 3 + Ce^{-x/3}$
- 4.2 $y^{1/2} = \frac{1}{3}x^{5/2} + Cx^{-1/2}$
- 4.3 $y^3 = 1/3 + Cx^{-3}$
- 4.4 $x^2 e^{3y} + e^x - \frac{1}{3}y^3 = C$
- 4.5 $x^2 - y^2 + 2x(y + 1) = C$
- 4.6 $4 \sin x \cos y + 2x - \sin 2x - 2y - \sin 2y = C$
- 4.7 $x = y(\ln x + C)$
- 4.8 $y^2 = 2Cx + C^2$
- 4.9 $y^2 = Ce^{-x^2/y^2}$
- 4.10 $xy = Ce^{x/y}$
- 4.11 $\tan \frac{1}{2}(x + y) = x + C$
- 4.12 $x \sin(y/x) = C$
- 4.13 $y^2 = -\sin^2 x + C \sin^4 x$
- 4.14 $y = -x^{-2} + K(x - 1)^{-1}$
- 4.15 $y = -x^{-1} \ln(C - x)$
- 4.16 $y^2 = C(C \pm 2x)$
- 4.17 $3x^2 y - y^3 = C$
- 4.18 $x^2 + (y - k)^2 = k^2$
- 4.19 $r = Ae^{-\theta}$, $r = Be^{\theta}$
- 4.25 (a) $y = \frac{C + x^2}{x^2(C - x^2)}$ (b) $y = \frac{x(C + e^{4x})}{C - e^{4x}}$ (c) $y = \frac{e^x(C - e^{2x})}{C + e^{2x}}$
- 5.1 $y = Ae^x + Be^{-2x}$
- 5.2 $y = (Ax + B)e^{2x}$
- 5.3 $y = Ae^{3ix} + Be^{-3ix}$ or other forms as in (5.24)
- 5.4 $y = e^{-x}(Ae^{ix} + Be^{-ix})$ or equivalent forms (5.17), (5.18)
- 5.5 $y = (Ax + B)e^x$
- 5.6 $y = Ae^{4ix} + Be^{-4ix}$ or other forms as in (5.24)
- 5.7 $y = Ae^{3x} + Be^{2x}$
- 5.8 $y = A + Be^{-5x}$
- 5.9 $y = Ae^{2x} \sin(3x + \gamma)$
- 5.10 $y = A + Be^{2x}$
- 5.11 $y = (A + Bx)e^{-3x/2}$
- 5.12 $y = Ae^{-x} + Be^{x/2}$
- 5.19 $y = Ae^{-ix} + Be^{-(1+i)x}$
- 5.20 $y = Ae^{-x} + Be^{ix}$
- 5.22 $y = Ae^x + Be^{-3x} + Ce^{-5x}$
- 5.23 $y = Ae^{ix} + Be^{-ix} + Ce^x + De^{-x}$
- 5.24 $y = Ae^{-x} + Be^{x/2} \sin(\frac{1}{2}x\sqrt{3} + \gamma)$
- 5.25 $y = A + Be^{2x} + Ce^{-3x}$
- 5.26 $y = Ae^{5x} + (Bx + C)e^{-x}$
- 5.27 $y = Ax + B + (Cx + D)e^x + (Ex^2 + Fx + G)e^{-2x}$
- 5.28 $y = e^x(A \sin x + B \cos x) + e^{-x}(C \sin x + D \cos x)$
- 5.29 $y = (A + Bx)e^{-x} + Ce^{2x} + De^{-2x} + E \sin(2x + \gamma)$

$$5.30 \quad y = (Ax + B) \sin x + (Cx + D) \cos x + (Ex + F)e^x + (Gx + H)e^{-x}$$

$$5.34 \quad \theta = \theta_0 \cos \omega t, \quad \omega = \sqrt{g/l}$$

$$5.35 \quad T = 2\pi\sqrt{R/g} \cong 85 \text{ min.}$$

$$5.36 \quad \omega = 1/\sqrt{LC}$$

$$5.38 \quad \text{overdamped: } R^2C > 4L; \text{ critically damped: } R^2C = 4L;$$

$$\text{underdamped: } R^2C < 4L.$$

$$5.40 \quad \ddot{y} + \frac{16\pi}{15}\dot{y} + \frac{4\pi^2}{9}y = 0, \quad y = e^{-8\pi t/15} \left(A \sin \frac{2\pi t}{5} + B \cos \frac{2\pi t}{5} \right)$$

$$6.1 \quad y = Ae^{2x} + Be^{-2x} - \frac{5}{2}$$

$$6.2 \quad y = (A + Bx)e^{2x} + 4$$

$$6.3 \quad y = Ae^x + Be^{-2x} + \frac{1}{4}e^{2x}$$

$$6.4 \quad y = Ae^{-x} + Be^{3x} + 2e^{-3x}$$

$$6.5 \quad y = Ae^{ix} + Be^{-ix} + e^x$$

$$6.6 \quad y = (A + Bx)e^{-3x} + 3e^{-x}$$

$$6.7 \quad y = Ae^{-x} + Be^{2x} + xe^{2x}$$

$$6.8 \quad y = Ae^{4x} + Be^{-4x} + 5xe^{4x}$$

$$6.9 \quad y = (Ax + B + x^2)e^{-x}$$

$$6.10 \quad y = (A + Bx)e^{3x} + 3x^2e^{3x}$$

$$6.11 \quad y = e^{-x}(A \sin 3x + B \cos 3x) + 8 \sin 4x - 6 \cos 4x$$

$$6.12 \quad y = e^{-2x}[A \sin(2\sqrt{2}x) + B \cos(2\sqrt{2}x)] + 5(\sin 2x - \cos 2x)$$

$$6.13 \quad y = (Ax + B)e^x - \sin x$$

$$6.14 \quad y = e^{-2x}(A \sin 3x + B \cos 3x) - 3 \cos 5x$$

$$6.15 \quad y = e^{-6x/5}[A \sin(8x/5) + B \cos(8x/5)] - 5 \cos 2x$$

$$6.16 \quad y = A \sin 3x + B \cos 3x - 5x \cos 3x$$

$$6.17 \quad y = A \sin 4x + B \cos 4x + 2x \sin 4x$$

$$6.18 \quad y = e^{-x}(A \sin 4x + B \cos 4x) + 2e^{-4x} \cos 5x$$

$$6.19 \quad y = e^{-x/2}(A \sin x + B \cos x) + e^{-3x/2}(2 \cos 2x - \sin 2x)$$

$$6.20 \quad y = Ae^{-2x} \sin(2x + \gamma) + 4e^{-x/2} \sin(5x/2)$$

$$6.21 \quad y = e^{-3x/5}[A \sin(x/5) + B \cos(x/5)] + (x^2 - 5)/2$$

$$6.22 \quad y = A + Be^{-x/2} + x^2 - 4x$$

$$6.23 \quad y = A \sin x + B \cos x + (x - 1)e^x$$

$$6.24 \quad y = (A + Bx + 2x^3)e^{3x}$$

$$6.25 \quad y = Ae^{3x} + Be^{-x} - \left(\frac{4}{3}x^3 + x^2 + \frac{1}{2}x\right)e^{-x}$$

$$6.26 \quad y = A \sin x + B \cos x - 2x^2 \cos x + 2x \sin x$$

$$6.33 \quad y = A \sin(x + \gamma) + x^3 - 6x - 1 + x \sin x + (3 - 2x)e^x$$

$$6.34 \quad y = Ae^{3x} + Be^{2x} + e^x + x$$

$$6.35 \quad y = A \sinh x + B \cosh x + \frac{1}{2}x \cosh x$$

$$6.36 \quad y = A \sin x + B \cos x + x^2 \sin x$$

$$6.37 \quad y = (A + Bx)e^x + 2x^2e^x + (3 - x)e^{2x} + x + 1$$

$$6.38 \quad y = A + Be^{2x} + (3x + 4)e^{-x} + x^3 + 3(x^2 + x)/2 + 2xe^{2x}$$

$$6.41 \quad y = e^{-x}(A \cos x + B \sin x) + \frac{1}{4}\pi + \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{4(n^2 - 2) \cos nx - 8n \sin nx}{\pi n^2(n^4 + 4)}$$

$$6.42 \quad y = A \cos 3x + B \sin 3x + \frac{1}{36} + \frac{2}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{\cos n\pi x}{n^2(n^2\pi^2 - 9)} + \frac{1}{\pi} \sum_{\substack{1 \\ \text{all } n}}^{\infty} \frac{(-1)^n \sin n\pi x}{n(n^2\pi^2 - 9)}$$

$$7.1 \quad y = 2A \tanh(Ax + B), \text{ or } y = 2A \tanh(B - Ax),$$

$$\text{or } y(x + a) = 2, \text{ or } y = C.$$

$$(a) \quad y \equiv 5$$

$$(b) \quad y(x + 1) = 2$$

$$(c) \quad y = \tan\left(\frac{\pi}{4} - \frac{x}{2}\right) = \sec x - \tan x$$

$$(d) \quad y = 2 \tanh x$$

$$7.3 \quad y = a(x + b)^2, \text{ or } y = C$$

$$7.4 \quad x^2 + (y - b)^2 = a^2, \text{ or } y = C$$

$$7.5 \quad y = b + k^{-1} \cosh k(x - a)$$

$$7.8 \quad v = (v_0^2 - 2gR + 2gR^2/r)^{1/2}, \quad r_{\max} = 2gR^2/(2gR - v_0^2),$$

$$\text{escape velocity} = \sqrt{2gR}$$

- 7.10 $x = \sqrt{1+t^2}$
 7.11 $x = (1-3t)^{1/3}$
 7.12 $t = \int_1^x u^2(1-u^4)^{-1/2} du$
 7.13 $t = (\omega\sqrt{2})^{-1} \int (\cos \theta)^{-1/2} d\theta$
 7.16 (a) $y = Ax + Bx^{-3}$ (b) $y = Ax^2 + Bx^{-2}$
 (c) $y = (A + B \ln x)/x^3$ (d) $y = Ax \cos(\sqrt{5} \ln x) + Bx \sin(\sqrt{5} \ln x)$
 7.17 $y = Ax^4 + Bx^{-4} + x^4 \ln x$
 7.18 $y = Ax + Bx^{-1} + \frac{1}{2}(x + x^{-1}) \ln x$
 7.19 $y = x^3(A + B \ln x) + x^3(\ln x)^2$
 7.20 $y = x^2(A + B \ln x) + x^2(\ln x)^3$
 7.21 $y = A\sqrt{x} \sin\left(\frac{\sqrt{3}}{2} \ln x + \gamma\right) + x^2$
 7.22 $y = A \cos \ln x + B \sin \ln x + x$
 7.23 $R = Ar^n + Br^{-n}$, $n \neq 0$; $R = A \ln r + B$, $n = 0$
 $R = Ar^l + Br^{-l-1}$
 7.25 $x^{-1} - 1$ 7.26 $x^2 - 1$ 7.27 $x^3 e^x$
 7.28 $x^{1/3} e^x$ 7.29 $x e^{1/x}$ 7.30 $(x-1) \ln x - 4x$
 8.8 $e^{-2t} - t e^{-2t}$ 8.9 $e^t - 3e^{-2t}$
 8.10 $\frac{1}{3} e^t \sin 3t + 2e^t \cos 3t$ 8.11 $\frac{4}{7} e^{-2t} + \frac{3}{7} e^{t/3}$
 8.12 $3 \cosh 5t + 2 \sinh 5t$ 8.13 $e^{-2t}(2 \sin 4t - \cos 4t)$
 8.17 $2a(3p^2 - a^2)/(p^2 + a^2)^3$ 8.21 $2b(p+a)/[(p+a)^2 + b^2]^2$
 8.22 $[(p+a)^2 - b^2]/[(p+a)^2 + b^2]^2$ 8.23 $y = t e^{-2t}(\cos t - \sin t)$
 8.25 $e^{-p\pi/2}/(p^2 + 1)$ 8.26 $\cos(t - \pi)$, $t > \pi$; $0, t < \pi$
 8.27 $-v(p^2 + v^2)^{-1} e^{-px/v}$
 9.2 $y = e^t(3 + 2t)$ 9.3 $y = e^{-2t}(4t + \frac{1}{2}t^2)$
 9.4 $y = \cos t + \frac{1}{2}(\sin t - t \cos t)$ 9.5 $y = -\frac{1}{2}t \cos t$
 9.6 $y = \frac{1}{6}t^3 e^{3t} + 5te^{3t}$ 9.7 $y = 1 - e^{2t}$
 9.8 $y = t \sin 4t$ 9.9 $y = (t+2) \sin 4t$
 9.10 $y = 3t^2 e^{2t}$ 9.11 $y = t e^{2t}$
 9.12 $y = \frac{1}{2}(t^2 e^{-t} + 3e^t - e^{-t})$ 9.13 $y = \sinh 2t$
 9.14 $y = t e^{2t}$ 9.15 $y = 2 \sin 3t + \frac{1}{6}t \sin 3t$
 9.16 $y = \frac{1}{6}t \sin 3t + 2 \cos 3t$ 9.17 $y = 2$
 9.18 $y = 2e^{-2t} - e^{-t}$ 9.19 $y = e^{2t}$
 9.20 $y = 2t + 1$ 9.21 $y = e^{3t} + 2e^{-2t} \sin t$
 9.22 $y = 2 \cos t + \sin t$ 9.23 $y = \sin t + 2 \cos t - 2e^{-t} \cos 2t$
 9.24 $y = (5-6t)e^t - \sin t$ 9.25 $y = (3+t)e^{-2t} \sin t$
 9.26 $y = t e^{-t} \cos 3t$ 9.27 $y = t + (1 - e^{4t})/4$, $z = \frac{1}{3} + e^{4t}$
 9.28 $\begin{cases} y = t \cos t - 1 \\ z = \cos t + t \sin t \end{cases}$ 9.29 $\begin{cases} y = e^t \\ z = t + e^t \end{cases}$
 9.30 $y = t - \sin 2t$ 9.31 $y = t$
 $z = \cos 2t$ $z = e^t$
 9.32 $\begin{cases} y = \sin 2t \\ z = \cos 2t - 1 \end{cases}$ 9.33 $\begin{cases} y = \sin t - \cos t \\ z = \sin t \end{cases}$
 9.34 $3/13$ 9.35 $10/26^2$
 9.36 $\arctan(2/3)$ 9.37 $15/8$
 9.38 $4/5$ 9.39 $\ln 2$
 9.40 1 9.41 $\arctan(1/\sqrt{2})$
 9.42 $\pi/4$

- 10.3 $\frac{1}{2}t \sinh t$ 10.4 $\frac{e^{-at} + e^{-bt}[(a-b)t - 1]}{(b-a)^2}$
- 10.5 $\frac{b(b-a)te^{-bt} + a[e^{-bt} - e^{-at}]}{(b-a)^2}$ 10.6 $\frac{e^{-at} - \cosh bt + (a/b) \sinh bt}{a^2 - b^2}$
- 10.7 $\frac{a \cosh bt - b \sinh bt - ae^{-at}}{a^2 - b^2}$
- 10.8 $\frac{e^{-at}}{(b-a)(c-a)} + \frac{e^{-bt}}{(a-b)(c-b)} + \frac{e^{-ct}}{(a-c)(b-c)}$
- 10.9 $(2t^2 - 2t + 1 - e^{-2t})/4$ 10.10 $(1 - \cos at - \frac{1}{2}at \sin at)/a^4$
- 10.11 $\frac{\cos at - \cos bt}{b^2 - a^2}$ 10.12 $\frac{1}{b^2 - a^2} \left(\frac{\cos bt}{b^2} - \frac{\cos at}{a^2} \right) + \frac{1}{a^2 b^2}$
- 10.13 $(e^{-t} + \sin t - \cos t)/2$ 10.14 $e^{-3t} + (t-1)e^{-2t}$
- 10.15 $\frac{1}{14}e^{3t} + \frac{1}{35}e^{-4t} - \frac{1}{10}e^t$ 10.17 $y = \begin{cases} (\cosh at - 1)/a^2, & t > 0 \\ 0, & t < 0 \end{cases}$
- 11.1 $y = \begin{cases} t-2, & t > 2 \\ 0, & t < 2 \end{cases}$ 11.7 $y = \begin{cases} (t-t_0)e^{-(t-t_0)}, & t > t_0 \\ 0, & t < t_0 \end{cases}$
- 11.8 $y = \begin{cases} e^{-2(t-t_0)} \sin(t-t_0), & t > t_0 \\ 0, & t < t_0 \end{cases}$
- 11.9 $y = \begin{cases} \frac{1}{3}e^{-(t-t_0)} \sin 3(t-t_0), & t > t_0 \\ 0, & t < t_0 \end{cases}$
- 11.10 $y = \begin{cases} \frac{1}{3} \sinh 3(t-t_0), & t > t_0 \\ 0, & t < t_0 \end{cases}$
- 11.11 $y = \begin{cases} \frac{1}{2}[\sinh(t-t_0) - \sin(t-t_0)], & t > t_0 \\ 0, & t < t_0 \end{cases}$
- 11.13 (a) $5\delta(x-2) + 3\delta(x+7)$ (b) $3\delta(x+5) - 4\delta(x-10)$
- 11.15 (a) 1 (b) 0 (c) -3 (d) $\cosh 1$
- 11.21 (a) 8 (b) $\phi(|a|)/(2|a|)$ (c) $1/2$ (d) 1
- 11.23 (a) $\delta(x+5)\delta(y-5)\delta(z), \delta(r-5\sqrt{2})\delta(\theta-\frac{3\pi}{4})\delta(z)/r,$
 $\delta(r-5\sqrt{2})\delta(\theta-\frac{\pi}{2})\delta(\phi-\frac{3\pi}{4})/(r \sin \theta)$
 (b) $\delta(x)\delta(y+1)\delta(z+1), \delta(r-1)\delta(\theta-\frac{3\pi}{2})\delta(z+1)/r,$
 $\delta(r-\sqrt{2})\delta(\theta-\frac{3\pi}{4})\delta(\phi-\frac{3\pi}{2})/(r \sin \theta)$
 (c) $\delta(x+2)\delta(y)\delta(z-2\sqrt{3}), \delta(r-2)\delta(\theta-\pi)\delta(z-2\sqrt{3})/r,$
 $\delta(r-4)\delta(\theta-\frac{\pi}{6})\delta(\phi-\pi)/(r \sin \theta)$
 (d) $\delta(x-3)\delta(y+3)\delta(z+\sqrt{6}), \delta(r-3\sqrt{2})\delta(\theta-\frac{7\pi}{4})\delta(z+\sqrt{6})/r,$
 $\delta(r-2\sqrt{6})\delta(\theta-\frac{2\pi}{3})\delta(\phi-\frac{7\pi}{4})/(r \sin \theta)$
- 11.25 (a) and (b) $F'''(x) = \delta(x) - 2\delta'(x)$ (c) $G'''(x) = \delta(x) + 5\delta'(x)$
- 12.2 $y = \frac{\sin \omega t - \omega t \cos \omega t}{2\omega^2}$ 12.3 $y = \frac{\sin \omega t - \omega \cos \omega t + \omega e^{-t}}{\omega(1+\omega^2)}$
- 12.6 $y = (\cosh at - 1)/a^2, t > 0$ 12.7 $y = \frac{a(\cosh at - e^{-t}) - \sinh at}{a(a^2-1)}$
- 12.8 $y = \begin{cases} 1 - e^{-t} - te^{-t}, & 0 < t < a \\ (t+1-a)e^{a-t} - (t+1)e^{-t}, & t > a \end{cases}$
- 12.11 $y = -\frac{1}{3} \sin 2x$ 12.12 $y = \cos x \ln \cos x + (x - \frac{\pi}{2}) \sin x$

- 12.13 $y = \begin{cases} x - \sqrt{2} \sin x, & x < \pi/4 \\ \frac{\pi}{2} - x - \sqrt{2} \cos x, & x > \pi/4 \end{cases}$
- 12.15 $y = x \sinh x - \cosh x \ln \cosh x$
- 12.16 $y = -x \ln x - x - x(\ln x)^2/2$
- 12.17 $y = -\frac{1}{4} \sin^2 x$
- 12.18 $y = x^2/2 + x^4/6$
- 13.1 $y = -\frac{1}{3}x^{-2} + Cx$ linear 1st order
- 13.2 $(\ln y)^2 - (\ln x)^2 = C$ separable
- 13.3 $y = A + Be^{-x} \sin(x + \gamma)$ 3rd order linear
- 13.4 $r = (A + Bt)e^{3t}$ 2nd order linear, $a = b$
- 13.5 $x^2 + y^2 - y \sin^2 x = C$ exact
- 13.6 $y = Ae^{-x} \sin(x + \gamma) + 2e^x + 3xe^{-x} \sin x$ 2nd order linear, complex a, b, c
- 13.7 $3x^2y^3 + 1 = Ax^3$ Bernoulli, or integrating factor $1/x^4$
- 13.8 $y = x(A + B \ln x) + \frac{1}{2}x(\ln x)^2$ Cauchy
- 13.9 $y(e^{3x} + Ce^{-2x}) + 5 = 0$ Bernoulli
- 13.10 $u - \ln u + \ln v + v^{-1} = C$ separable
- 13.11 $y = 2x \ln x + Cx$ linear 1st order, or homogeneous
- 13.12 $y = A \ln x + B + x^2$ y missing, or Cauchy
- 13.13 $y = Ae^{-2x} \sin(x + \gamma) + e^{3x}$ 2nd order linear, complex a, b
- 13.14 $y = Ae^{-2x} \sin(x + \gamma) + xe^{-2x} \sin x$ 2nd order linear, complex a, b, c
- 13.15 $y = (A + Bx)e^{2x} + 3x^2e^{2x}$ 2nd order linear, $c = a = b$
- 13.16 $y = Ae^{2x} + Be^{3x} - xe^{2x}$ 2nd order linear, $c = a \neq b$
- 13.17 $y^2 + 4xy - x^2 = C$ exact, or homogeneous
- 13.18 $x = (y + C)e^{-\sin y}$ linear 1st order for $x(y)$
- 13.19 $(x + y) \sin^2 x = K$ separable with $u = x + y$
- 13.20 $y = Ae^x \sin(2x + \gamma) + x + \frac{2}{5} + e^x(1 - x \cos 2x)$ 2nd order linear, complex a, b, c
- 13.21 $x^2 + \ln(1 - y^2) = C$ separable, or Bernoulli
- 13.22 $y = (A + Bx)e^{2x} + C \sin(3x + \gamma)$ 4th order linear
- 13.23 $r = \sin \theta [C + \ln(\sec \theta + \tan \theta)]$ 1st order linear
- 13.24 $y^2 = ax^2 + b$ separable after substitution
- 13.25 $x^3y = 2$
- 13.26 $y = x^2 + x$
- 13.27 $y = 2e^{2x} - 1$
- 13.28 $y^2 + 4(x - 1)^2 = 9$
- 13.29 62 min more
- 13.30 $y = \frac{1}{6}g[(1 + t)^2 + 2(1 + t)^{-1} - 3]$; at $t = 1$, $y = g/3$, $v = 7g/12$, $a = 5g/12$
- 13.31 $v = \sqrt{2k/(ma)}$
- 13.32 1:23 p.m.
- 13.33 In both (a) and (b), the temperature of the mixture at time t is $T_a(1 - e^{-kt}) + (n + n')^{-1}(nT_0 + n'T'_0)e^{-kt}$.
- 13.36 (a) $v = u \ln(m_0/m)$
- 13.38 $\ln \sqrt{a^2 + p^2} - \ln p$
- 13.39 $2pa/(p^2 - a^2)^2$
- 13.40 $5/27$
- 13.41 $(\tanh 1 - \operatorname{sech}^2 1)/4 = 0.0854$
- 13.42 $te^{-at}(1 - \frac{1}{2}at)$
- 13.43 $(\sin at + at \cos at)/(2a)$
- 13.44 $(3 \sin at - 3at \cos at - a^2t^2 \sin at)/(8a^5)$
- 13.46 e^{-x} : $g_s(\alpha) = \sqrt{\frac{2}{\pi}} \frac{\alpha}{1 + \alpha^2}$, $g_c(\alpha) = \sqrt{\frac{2}{\pi}} \frac{1}{1 + \alpha^2}$
- xe^{-x} : $g_s(\alpha) = \sqrt{\frac{2}{\pi}} \frac{2\alpha}{(1 + \alpha^2)^2}$, $g_c(\alpha) = \sqrt{\frac{2}{\pi}} \frac{1 - \alpha^2}{(1 + \alpha^2)^2}$
- 13.47 $y = A \sin t + B \cos t + \sin t \ln(\sec t + \tan t) - 1$
- 13.48 $y = A \sin t + B \cos t + (t \sin t - t^2 \cos t)/4$

Chapter 9

- 2.1 $(y - b)^2 = 4a^2(x - a^2)$ 2.2 $x^2 + (y - b)^2 = a^2$
 2.3 $ax = \sinh(ay + b)$ 2.4 $ax = \cosh(ay + b)$
 2.5 $y = ae^x + be^{-x}$ or $y = A \cosh(x + B)$, etc.
 2.6 $x + a = \frac{4}{3}(y^{1/2} - 2b)(b + y^{1/2})^{1/2}$
 2.7 $e^x \cos(y + b) = C$ 2.8 $K^2 x^2 - (y - b)^2 = K^4$
 2.9 $x = ay^2 + b$ 2.10 $y = Ax^{3/2} - \ln x + B$
- 3.1 $dx/dy = C(y^3 - C^2)^{-1/2}$ 3.2 $dx/dy = Cy^2(1 - C^2y^4)^{-1/2}$
 3.3 $x^4 y'^2 = C^2(1 + x^2 y'^2)^3$ 3.4 $\frac{dx}{dy} = \frac{C}{y(y^4 - C^2)^{1/2}}$
 3.5 $y^2 = ax + b$ 3.6 $x = ay^{3/2} - \frac{1}{2}y^2 + b$
 3.7 $y = K \sinh(x + C) = ae^x + be^{-x}$, etc., as in Problem 2.5
 3.8 $r \cos(\theta + \alpha) = C$ 3.9 $\cot \theta = A \cos(\phi - \alpha)$
 3.10 $s = be^{at}$ 3.11 $a(x + 1) = \cosh(ay + b)$
 3.12 $(x - a)^2 + y^2 = C^2$ 3.13 $(x - a)^2 = 4K^2(y - K^2)$
 3.14 $r = be^{c\theta}$
 3.15 $r \cos(\theta + \alpha) = C$, in polar coordinates; or, in rectangular coordinates, the straight line $x \cos \alpha - y \sin \alpha = C$.
 3.17 Intersection of the cone with $r \cos \left(\frac{\theta + C}{\sqrt{2}} \right) = K$
 3.18 Geodesics on the sphere: $\cot \theta = A \cos(\phi - \alpha)$. (See Problem 3.9)
 Intersection of $z = ax + by$ with the sphere: $\cot \theta = a \cos \phi + b \sin \phi$.
- 4.4 42.2 min; 5.96 min
 4.5 $x = a(1 - \cos \theta)$, $y = a(\theta - \sin \theta) + C$
 4.6 $x = a(\theta - \sin \theta) + C$, $y = 1 + a(1 - \cos \theta)$
 4.7 $x = a(1 - \cos \theta) - \frac{5}{2}$, $y = a(\theta - \sin \theta) + C$
- 5.2 $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2) - V(r, \theta, z)$
 $m(\ddot{r} - r\dot{\theta}^2) = -\partial V/\partial r$
 $m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = -(1/r)(\partial V/\partial \theta)$
 $m\ddot{z} = -\partial V/\partial z$
 Note: The equations in 5.2 and 5.3 are in the form
 $m\mathbf{a} = \mathbf{F} = -\nabla V$.
- 5.3 $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2) - V(r, \theta, \phi)$
 $m(\ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\dot{\phi}^2) = -\partial V/\partial r$
 $m(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\sin\theta\cos\theta\dot{\phi}^2) = -(1/r)(\partial V/\partial \theta)$
 $m(r\sin\theta\ddot{\phi} + 2r\cos\theta\dot{\theta}\dot{\phi} + 2\sin\theta\dot{r}\dot{\phi}) = -(1/r\sin\theta)(\partial V/\partial \phi)$
- 5.4 $L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1 - \cos \theta)$
 $l\ddot{\theta} + g \sin \theta = 0$

- 5.5 $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$
 $m\ddot{x} + kx = 0$
- 5.6 $L = \frac{1}{2}m(r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2) - mgr\cos\theta$
 $\begin{cases} a\ddot{\theta} - a\sin\theta\cos\theta\dot{\phi}^2 - g\sin\theta = 0 \\ (d/dt)(\sin^2\theta\dot{\phi}) = 0 \end{cases}$
- 5.8 $L = \frac{1}{2}m(2\dot{r}^2 + r^2\dot{\theta}^2) - mgr$
 $2\ddot{r} - r\dot{\theta}^2 + g = 0$
 $(d/dt)(r^2\dot{\theta}) = 0$
- 5.9 $L = \frac{1}{2}m(2\dot{r}^2 + r^2\dot{\theta}^2) - mgr$
 $2\ddot{r} - r\dot{\theta}^2 + g = 0$
 $r^2\dot{\theta} = \text{const.}$
- 5.10 $L = \frac{1}{2}m_1(4\dot{r}^2 + r^2\dot{\theta}^2) + 2m_2\dot{r}^2 - m_1gr\sqrt{3} + m_2g(l - 2r)$
 $4(m_1 + m_2)\ddot{r} - m_1r\dot{\theta}^2 + m_1g\sqrt{3} + 2m_2g = 0$
 $r^2\dot{\theta} = \text{const.}$
- 5.11 $L = \frac{1}{2}(m + Ia^{-2})\dot{z}^2 - mgz$ (If z is taken as positive down,
 $(ma^2 + I)\ddot{z} + mga^2 = 0$ change the signs of z and \ddot{z} .)
- 5.12 $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - [\frac{1}{2}k(r - r_0)^2 - mgr\cos\theta]$
 $\ddot{r} - r\dot{\theta}^2 + \frac{k}{m}(r - r_0) - g\cos\theta = 0, \frac{d}{dt}(r^2\dot{\theta}) + gr\sin\theta = 0$
- 5.13 $L = \frac{1}{2}m[\dot{r}^2(1 + 4r^2) + r^2\dot{\theta}^2] - mgr^2$
 $\ddot{r}(1 + 4r^2) + 4r\dot{r}^2 - r\dot{\theta}^2 + 2gr = 0, r^2\dot{\theta} = \text{const.}$
 If $z = \text{const.}$, then $r = \text{const.}$, so $\dot{\theta} = \sqrt{2g}$
- 5.14 $L = M\dot{x}^2 + Mgx\sin\alpha, \quad 2M\ddot{x} - Mg\sin\alpha = 0$
- 5.15 $L = \frac{1}{2}(M + Ia^{-2})\dot{x}^2 + Mgx\sin\alpha, (M + Ia^{-2})\ddot{x} - Mg\sin\alpha = 0$
 Since smaller I means greater acceleration, objects reach the bottom in order of increasing I .
- 5.16 $L = \frac{1}{2}m(l + a\theta)^2\dot{\theta}^2 - mg[a\sin\theta - (l + a\theta)\cos\theta]$
 $(l + a\theta)\ddot{\theta} + a\dot{\theta}^2 + g\sin\theta = 0$
- 5.17 $L = \frac{1}{2}(M + m)\dot{X}^2 + \frac{1}{2}m(l^2\dot{\theta}^2 + 2l\cos\theta\dot{X}\dot{\theta}) + mgl\cos\theta$
 $(M + m)\dot{X} + ml\cos\theta\dot{\theta} = \text{const.}$
 $\frac{d}{dt}(l\dot{\theta} + \cos\theta\dot{X}) + g\sin\theta = 0$
- 5.18 $x + y = x_0 + y_0 + a\theta, L = m(\dot{x}^2 + \dot{y}^2 + \dot{x}\dot{y}) + mgy$
 $\dot{x} = -\frac{1}{3}gt, \dot{y} = \frac{2}{3}gt, a\dot{\theta} = \frac{1}{3}gt$
- 5.19 $x = y$ with $\omega = \sqrt{g/l}$; $x = -y$ with $\omega = \sqrt{3g/l}$
- 5.20 $x = y$ with $\omega = \sqrt{g/l}$; $x = -y$ with $\omega = \sqrt{7g/l}$
- 5.21 $L = ml^2[\dot{\theta}^2 + \frac{1}{2}\dot{\phi}^2 + \dot{\theta}\dot{\phi}\cos(\theta - \phi)] + mgl(2\cos\theta + \cos\phi)$
 $2\ddot{\theta} + \ddot{\phi}\cos(\theta - \phi) + \dot{\phi}^2\sin(\theta - \phi) + \frac{2g}{l}\sin\theta = 0$
 $\ddot{\phi} + \ddot{\theta}\cos(\theta - \phi) - \dot{\theta}^2\sin(\theta - \phi) + \frac{g}{l}\sin\phi = 0$
- 5.22 $L = l^2[\frac{1}{2}M\dot{\theta}^2 + \frac{1}{2}m\dot{\phi}^2 + m\dot{\theta}\dot{\phi}\cos(\theta - \phi)] + gl(M\cos\theta + m\cos\phi)$
 $M\ddot{\theta} + m\ddot{\phi}\cos(\theta - \phi) + m\dot{\phi}^2\sin(\theta - \phi) + \frac{Mg}{l}\sin\theta = 0$
 $\ddot{\phi} + \ddot{\theta}\cos(\theta - \phi) - \dot{\theta}^2\sin(\theta - \phi) + \frac{g}{l}\sin\phi = 0$
- 5.23 $\phi = 2\theta$ with $\omega = \sqrt{2g/(3l)}$; $\phi = -2\theta$ with $\omega = \sqrt{2g/l}$
- 5.24 $\phi = \frac{3}{2}\theta$ with $\omega = \sqrt{3g/(5l)}$; $\phi = -\frac{3}{2}\theta$ with $\omega = \sqrt{3g/l}$
- 5.25 $\phi = \sqrt{M/m}\theta$ with $\omega^2 = \frac{g}{l} \frac{1}{1 + \sqrt{m/M}}$
 $\phi = -\sqrt{M/m}\theta$ with $\omega^2 = \frac{g}{l} \frac{1}{1 - \sqrt{m/M}}$

- | | | | | | |
|-----|----------|-----|--------|-----|-------------------|
| 6.1 | catenary | 6.2 | circle | 6.3 | circular cylinder |
| 6.4 | catenary | 6.5 | circle | 6.6 | circle |

- 8.2 $I = \int \frac{x^2 y'^2}{\sqrt{1+y'^2}} dx, \quad x^2(2y' + y'^3) = K(1+y'^2)^{3/2}$
- 8.3 $I = \int \frac{y dy}{\sqrt{x'^2+1}}, \quad x'^2 y^2 = C^2(1+x'^2)^3$
- 8.4 $I = \int \sqrt{r^2 + r^4 \theta'^2} dr, \quad \frac{dr}{d\theta} = Kr\sqrt{r^4 - K^2}$
- 8.5 $y = ae^{bx}$
- 8.6 $(x-a)^2 + (y+1)^2 = C^2$
- 8.7 $(y-b)^2 = 4a^2(x+1-a^2)$
- 8.8 Intersection of $r = 1 + \cos \theta$ with $z = a + b \sin(\theta/2)$
- 8.9 Intersection of the cone with $r \cos(\theta \sin \alpha + C) = K$
- 8.10 Intersection of $y = x^2$ with $az = b[2x\sqrt{4x^2+1} + \sinh^{-1} 2x] + c$
- 8.11 $r = K \sec^2 \frac{\theta+c}{2}$
- 8.12 $e^y \cos(x-a) = K$
- 8.13 $(x+\frac{3}{2})^2 + (y-b)^2 = c^2$
- 8.14 $(x-a)^2 = 4K^2(y+2-K^2)$
- 8.15 $y+c = \frac{3}{2}K \left[x^{1/3} \sqrt{x^{2/3} - K^2} + K^2 \cosh^{-1}(x^{1/3}/K) \right]$
- 8.16 Hyperbola: $r^2 \cos(2\theta + \alpha) = K$ or $(x^2 - y^2) \cos \alpha - 2xy \sin \alpha = K$
- 8.17 $K \ln r = \cosh(K\theta + C)$
- 8.18 Parabola: $(x-y-C)^2 = 4K^2(x+y-K^2)$
- 8.19 $m(\ddot{r} - r\dot{\theta}^2) + kr = 0, \quad r^2\dot{\theta} = \text{const.}$
- 8.20 $m(\ddot{r} - r\dot{\theta}^2) + K/r^2 = 0, \quad r^2\dot{\theta} = \text{const.}$
- 8.21 $\ddot{r} - r\dot{\theta}^2 = 0, \quad r^2\dot{\theta} = \text{const.}, \quad \ddot{z} + g = 0$
- 8.22 $\frac{1}{r} \cdot m(r^2\ddot{\theta} + 2r\dot{r}\dot{\theta} - r^2 \sin \theta \cos \theta \dot{\phi}^2) = -\frac{1}{r} \frac{\partial V}{\partial \theta} = F_\theta = ma_\theta$
 $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} - r \sin \theta \cos \theta \dot{\phi}^2$
- 8.23 $L = \frac{1}{2}ma^2\dot{\theta}^2 - mga(1 - \cos \theta), \quad a\ddot{\theta} + g \sin \theta = 0,$
 θ measured from the downward direction.
- 8.25 $l = 2\sqrt{\pi A}$
- 8.26 $r = Ae^{b\theta}$
- 8.27 $\frac{dr}{d\theta} = r\sqrt{K^2(1+\lambda r)^2 - 1}$
- 8.28 $r^2\dot{\theta} = \text{const.}, \quad |\mathbf{r} \times m\mathbf{v}| = mr^2\dot{\theta} = \text{const.}, \quad \frac{dA}{dt} = \frac{1}{2}r^2\dot{\theta} = \text{const.}$

Chapter 10

- 4.4 $I = \frac{2}{15} \begin{pmatrix} \pi & -1 & 0 \\ -1 & \pi & 0 \\ 0 & 0 & \pi \end{pmatrix}$ Principal moments: $\frac{2}{15}(\pi - 1, \pi, \pi + 1)$; principal axes along the vectors: $(1, 1, 0), (0, 0, 1), (1, -1, 0)$.
- 4.5 $I = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & -2 \\ 0 & -2 & 4 \end{pmatrix}$ Principal moments: $(2, 4, 6)$; principal axes along the vectors: $(0, 1, 1), (1, 0, 0), (0, 1, -1)$.
- 4.6 $I = \begin{pmatrix} 9 & 0 & -3 \\ 0 & 6 & 0 \\ -3 & 0 & 9 \end{pmatrix}$ Principal moments: $(6, 6, 12)$; principal axes along the vectors: $(1, 0, -1)$ and any two orthogonal vectors in the plane $z = x$, say $(0, 1, 0)$ and $(1, 0, 1)$.
- 4.7 $I = \frac{1}{120} \begin{pmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{pmatrix}$ Principal moments: $\left(\frac{1}{60}, \frac{1}{24}, \frac{1}{24}\right)$; principal axes along the vectors: $(1, 1, 1)$ and any two orthogonal vectors in the plane $x + y + z = 0$, say $(1, -1, 0)$ and $(1, 1, -2)$.
- 5.5 1 if $j = k = m = n$ (6 cases); -1 if $j = k = n = m$ (6 cases); 0 otherwise
- 5.6 (a) 3 (b) 0 (c) 2 (d) -2 (e) -1 (f) -1
- 5.7 (a) $\delta_{kq}\delta_{ip} - \delta_{kp}\delta_{iq}$ (b) $\delta_{ap}\delta_{bq} - \delta_{aq}\delta_{bp}$
- 6.9 to 6.14 $\mathbf{r}, \mathbf{v}, \mathbf{F}, \mathbf{E}$ are vectors; $\boldsymbol{\omega}, \boldsymbol{\tau}, \mathbf{L}, \mathbf{B}$ are pseudovectors; T is a scalar.
- 6.15 (a) vector (b) pseudovector (c) vector
- 6.16 vector (if \mathbf{V} is a vector); pseudovector (if \mathbf{V} is a pseudovector)
- 8.1 $h_r = 1, \quad h_\theta = r, \quad h_\phi = r \sin \theta$
 $d\mathbf{s} = \mathbf{e}_r dr + \mathbf{e}_\theta r d\theta + \mathbf{e}_\phi r \sin \theta d\phi$
 $dV = r^2 \sin \theta dr d\theta d\phi$
 $\mathbf{a}_r = \dot{\mathbf{i}} \sin \theta \cos \phi + \dot{\mathbf{j}} \sin \theta \sin \phi + \dot{\mathbf{k}} \cos \theta = \mathbf{e}_r$
 $\mathbf{a}_\theta = \dot{\mathbf{i}} r \cos \theta \cos \phi + \dot{\mathbf{j}} r \cos \theta \sin \phi - \dot{\mathbf{k}} r \sin \theta = r \mathbf{e}_\theta$
 $\mathbf{a}_\phi = -\dot{\mathbf{i}} r \sin \theta \sin \phi + \dot{\mathbf{j}} r \sin \theta \cos \phi = r \sin \theta \mathbf{e}_\phi$
- 8.2 $d^2\mathbf{s}/dt^2 = \mathbf{e}_r(\ddot{r} - r\dot{\theta}^2) + \mathbf{e}_\theta(r\ddot{\theta} + 2\dot{r}\dot{\theta}) + \mathbf{e}_z\ddot{z}$
- 8.3 $d\mathbf{s}/dt = \mathbf{e}_r\dot{r} + \mathbf{e}_\theta r\dot{\theta} + \mathbf{e}_\phi r \sin \theta \dot{\phi}$
 $d^2\mathbf{s}/dt^2 = \mathbf{e}_r(\ddot{r} - r\dot{\theta}^2 - r \sin^2 \theta \dot{\phi}^2)$
 $\quad + \mathbf{e}_\theta(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r \sin \theta \cos \theta \dot{\phi}^2)$
 $\quad + \mathbf{e}_\phi(r \sin \theta \ddot{\phi} + 2r \cos \theta \dot{\theta} \dot{\phi} + 2 \sin \theta \dot{r} \dot{\phi})$
- 8.4 $\mathbf{V} = -r\mathbf{e}_\theta + \mathbf{k}$

- 8.5 $\mathbf{V} = \mathbf{e}_r \cos \theta - \mathbf{e}_\theta \sin \theta - \mathbf{e}_\phi r \sin \theta$
- 8.6 $h_u = h_v = (u^2 + v^2)^{1/2}, \quad h_z = 1$
 $d\mathbf{s} = (u^2 + v^2)^{1/2}(\mathbf{e}_u du + \mathbf{e}_v dv) + \mathbf{e}_z dz$
 $dV = (u^2 + v^2) du dv dz$
 $\mathbf{a}_u = \mathbf{i}u + \mathbf{j}v = (u^2 + v^2)^{1/2}\mathbf{e}_u$
 $\mathbf{a}_v = -\mathbf{i}v + \mathbf{j}u = (u^2 + v^2)^{1/2}\mathbf{e}_v$
 $\mathbf{a}_z = \mathbf{k} = \mathbf{e}_z$
- 8.7 $h_u = h_v = a(\sinh^2 u + \sin^2 v)^{1/2}, \quad h_z = 1$
 $d\mathbf{s} = a(\sinh^2 u + \sin^2 v)^{1/2}(\mathbf{e}_u du + \mathbf{e}_v dv) + \mathbf{e}_z dz$
 $dV = a^2(\sinh^2 u + \sin^2 v) du dv dz$
 $\mathbf{a}_u = \mathbf{i}a \sinh u \cos v + \mathbf{j}a \cosh u \sin v = h_u \mathbf{e}_u$
 $\mathbf{a}_v = -\mathbf{i}a \cosh u \sin v + \mathbf{j}a \sinh u \cos v = h_v \mathbf{e}_v$
 $\mathbf{a}_z = \mathbf{k} = \mathbf{e}_z$
- 8.8 $h_u = h_v = (u^2 + v^2)^{1/2}, \quad h_\phi = uv$
 $d\mathbf{s} = (u^2 + v^2)^{1/2}(\mathbf{e}_u du + \mathbf{e}_v dv) + uv \mathbf{e}_\phi d\phi$
 $dV = uv (u^2 + v^2) du dv d\phi$
 $\mathbf{a}_u = \mathbf{i}v \cos \phi + \mathbf{j}v \sin \phi + \mathbf{k}u = h_u \mathbf{e}_u$
 $\mathbf{a}_v = \mathbf{i}u \cos \phi + \mathbf{j}u \sin \phi - \mathbf{k}v = h_v \mathbf{e}_v$
 $\mathbf{a}_\phi = -\mathbf{i}uv \sin \phi + \mathbf{j}uv \cos \phi = h_\phi \mathbf{e}_\phi$
- 8.9 $h_u = h_v = a(\cosh u + \cos v)^{-1}$
 $d\mathbf{s} = a(\cosh u + \cos v)^{-1}(\mathbf{e}_u du + \mathbf{e}_v dv)$
 $dA = a^2(\cosh u + \cos v)^{-2} du dv$
 $\mathbf{a}_u = (h_u^2/a)[\mathbf{i}(1 + \cos v \cosh u) - \mathbf{j} \sin v \sinh u] = h_u \mathbf{e}_u$
 $\mathbf{a}_v = (h_v^2/a)[\mathbf{i} \sinh u \sin v + \mathbf{j}(1 + \cos v \cosh u)] = h_v \mathbf{e}_v$
- 8.11 $d\mathbf{e}_u/dt = (u^2 + v^2)^{-1}(uv - vu)\mathbf{e}_v$
 $d\mathbf{e}_v/dt = (u^2 + v^2)^{-1}(v\dot{u} - u\dot{v})\mathbf{e}_u$
 $d\mathbf{s}/dt = (u^2 + v^2)^{1/2}(\mathbf{e}_u \dot{u} + \mathbf{e}_v \dot{v}) + \mathbf{e}_z \dot{z}$
 $d^2\mathbf{s}/dt^2 = \mathbf{e}_u(u^2 + v^2)^{-1/2}[(u^2 + v^2)\ddot{u} + u(\dot{u}^2 - \dot{v}^2) + 2v\dot{u}\dot{v}]$
 $+ \mathbf{e}_v(u^2 + v^2)^{-1/2}[(u^2 + v^2)\ddot{v} + v(\dot{v}^2 - \dot{u}^2) + 2u\dot{u}\dot{v}] + \mathbf{e}_z \ddot{z}$
- 8.12 $d\mathbf{e}_u/dt = (\sinh^2 u + \sin^2 v)^{-1}(\dot{v} \sinh u \cosh u - \dot{u} \sin v \cos v)\mathbf{e}_v$
 $d\mathbf{e}_v/dt = (\sinh^2 u + \sin^2 v)^{-1}(\dot{u} \sin v \cos v - \dot{v} \sinh u \cosh u)\mathbf{e}_u$
 $d\mathbf{s}/dt = a(\sinh^2 u + \sin^2 v)^{1/2}(\mathbf{e}_u \dot{u} + \mathbf{e}_v \dot{v}) + \mathbf{e}_z \dot{z}$
 $d^2\mathbf{s}/dt^2 = \mathbf{e}_u a(\sinh^2 u + \sin^2 v)^{-1/2}$
 $\times [(\sinh^2 u + \sin^2 v)\ddot{u} + (\dot{u}^2 - \dot{v}^2) \sinh u \cosh u + 2\dot{u}\dot{v} \sin v \cos v]$
 $+ \mathbf{e}_v a(\sinh^2 u + \sin^2 v)^{-1/2}[(\sinh^2 u + \sin^2 v)\ddot{v}$
 $+ (\dot{v}^2 - \dot{u}^2) \sin v \cos v + 2\dot{u}\dot{v} \sinh u \cosh u] + \mathbf{e}_z \ddot{z}$
- 8.13 $d\mathbf{e}_u/dt = (u^2 + v^2)^{-1}(u\dot{v} - v\dot{u})\mathbf{e}_v + (u^2 + v^2)^{-1/2}v\dot{\phi}\mathbf{e}_\phi$
 $d\mathbf{e}_v/dt = (u^2 + v^2)^{-1}(v\dot{u} - u\dot{v})\mathbf{e}_u + (u^2 + v^2)^{-1/2}u\dot{\phi}\mathbf{e}_\phi$
 $d\mathbf{e}_\phi/dt = -(u^2 + v^2)^{-1/2}(v\mathbf{e}_u + u\mathbf{e}_v)\dot{\phi}$
 $d\mathbf{s}/dt = (u^2 + v^2)^{1/2}(\mathbf{e}_u \dot{u} + \mathbf{e}_v \dot{v}) + \mathbf{e}_\phi uv \dot{\phi}$
 $d^2\mathbf{s}/dt^2 = \mathbf{e}_u(u^2 + v^2)^{-1/2}[(u^2 + v^2)\ddot{u} + u(\dot{u}^2 - \dot{v}^2) + 2v\dot{u}\dot{v} - uv^2\dot{\phi}^2]$
 $+ \mathbf{e}_v(u^2 + v^2)^{-1/2}[(u^2 + v^2)\ddot{v} + v(\dot{v}^2 - \dot{u}^2) + 2u\dot{u}\dot{v} - u^2v\dot{\phi}^2]$
 $+ \mathbf{e}_\phi(uv\ddot{\phi} + 2v\dot{u}\dot{\phi} + 2u\dot{v}\dot{\phi})$
- 8.14 $d\mathbf{e}_u/dt = -(\cosh u + \cos v)^{-1}(\dot{u} \sin v + \dot{v} \sinh u)\mathbf{e}_v$
 $d\mathbf{e}_v/dt = (\cosh u + \cos v)^{-1}(\dot{u} \sin v + \dot{v} \sinh u)\mathbf{e}_u$
 $d\mathbf{s}/dt = a(\cosh u + \cos v)^{-1}(\mathbf{e}_u \dot{u} + \mathbf{e}_v \dot{v})$
 $d^2\mathbf{s}/dt^2 = \mathbf{e}_u a(\cosh u + \cos v)^{-2}[(\cosh u + \cos v)\ddot{u} + (\dot{v}^2 - \dot{u}^2) \sinh u + 2\dot{u}\dot{v} \sin v]$
 $+ \mathbf{e}_v a(\cosh u + \cos v)^{-2}[(\cosh u + \cos v)\ddot{v} + (\dot{v}^2 - \dot{u}^2) \sin v - 2\dot{u}\dot{v} \sinh u]$

9.3 See 8.2

$$\begin{aligned}
 9.5 \quad \nabla U &= \mathbf{e}_r \frac{\partial U}{\partial r} + \mathbf{e}_\theta \left(\frac{1}{r} \frac{\partial U}{\partial \theta} \right) + \mathbf{e}_\phi \left(\frac{1}{r \sin \theta} \frac{\partial U}{\partial \phi} \right) \\
 \nabla \cdot \mathbf{V} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 V_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta V_\theta) + \frac{1}{r \sin \theta} \frac{\partial V_\phi}{\partial \phi} \\
 \nabla^2 U &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial U}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial U}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 U}{\partial \phi^2} \\
 \nabla \times \mathbf{V} &= \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta V_\phi) - \frac{\partial V_\theta}{\partial \phi} \right] \mathbf{e}_r \\
 &\quad + \frac{1}{r \sin \theta} \left[\frac{\partial V_r}{\partial \phi} - \sin \theta \frac{\partial}{\partial r} (r V_\phi) \right] \mathbf{e}_\theta + \frac{1}{r} \left[\frac{\partial}{\partial r} (r V_\theta) - \frac{\partial V_r}{\partial \theta} \right] \mathbf{e}_\phi
 \end{aligned}$$

9.6 See 8.11 9.7 See 8.12 9.8 See 8.13 9.9 See 8.14

9.10 Let $h = (u^2 + v^2)^{1/2}$ represent the u and v scale factors.

$$\begin{aligned}
 \nabla U &= h^{-1} \left(\mathbf{e}_u \frac{\partial U}{\partial u} + \mathbf{e}_v \frac{\partial U}{\partial v} \right) + \mathbf{k} \frac{\partial U}{\partial z} \\
 \nabla \cdot \mathbf{V} &= h^{-2} \left[\frac{\partial}{\partial u} (h V_u) + \frac{\partial}{\partial v} (h V_v) \right] + \frac{\partial V_z}{\partial z} \\
 \nabla^2 U &= h^{-2} \left(\frac{\partial^2 U}{\partial u^2} + \frac{\partial^2 U}{\partial v^2} \right) + \frac{\partial^2 U}{\partial z^2} \\
 \nabla \times \mathbf{V} &= \left(h^{-1} \frac{\partial V_z}{\partial v} - \frac{\partial V_v}{\partial z} \right) \mathbf{e}_u \\
 &\quad + \left(\frac{\partial V_u}{\partial z} - h^{-1} \frac{\partial V_z}{\partial u} \right) \mathbf{e}_v + h^{-2} \left[\frac{\partial}{\partial u} (h V_v) - \frac{\partial}{\partial v} (h V_u) \right] \mathbf{e}_z
 \end{aligned}$$

9.11 Same as 9.10 with $h = a(\sinh^2 u + \sin^2 v)^{1/2}$

9.12 Let $h = (u^2 + v^2)^{1/2}$

$$\begin{aligned}
 \nabla U &= h^{-1} \left(\mathbf{e}_u \frac{\partial U}{\partial u} + \mathbf{e}_v \frac{\partial U}{\partial v} \right) + (uv)^{-1} \frac{\partial U}{\partial \phi} \mathbf{e}_\phi \\
 \nabla \cdot \mathbf{V} &= \frac{1}{uh^2} \frac{\partial}{\partial u} (uh V_u) + \frac{1}{vh^2} \frac{\partial}{\partial v} (vh V_v) + \frac{1}{uv} \frac{\partial V_\phi}{\partial \phi} \\
 \nabla^2 U &= \frac{1}{h^2 u} \frac{\partial}{\partial u} \left(u \frac{\partial U}{\partial u} \right) + \frac{1}{h^2 v} \frac{\partial}{\partial v} \left(v \frac{\partial U}{\partial v} \right) + \frac{1}{u^2 v^2} \frac{\partial^2 U}{\partial \phi^2} \\
 \nabla \times \mathbf{V} &= \left[\frac{1}{hv} \frac{\partial}{\partial v} (v V_\phi) - \frac{1}{uv} \frac{\partial V_v}{\partial \phi} \right] \mathbf{e}_u \\
 &\quad + \left[\frac{1}{uv} \frac{\partial V_u}{\partial \phi} - \frac{1}{hu} \frac{\partial}{\partial u} (u V_\phi) \right] \mathbf{e}_v + \frac{1}{h^2} \left[\frac{\partial}{\partial u} (h V_v) - \frac{\partial}{\partial v} (h V_u) \right] \mathbf{e}_\phi
 \end{aligned}$$

9.13 Same as 9.10 if $h = a(\cosh u + \cos v)^{-1}$ and terms involving either z derivatives or V_z are omitted. Note, however, that $\nabla \times \mathbf{V}$ has only a z component if $\mathbf{V} = \mathbf{e}_u V_u + \mathbf{e}_v V_v$ where V_u and V_v are functions of u and v .

$$\begin{aligned}
 9.14 \quad h_u &= [(u+v)/u]^{1/2}, \quad h_v = [(u+v)/v]^{1/2} \\
 \mathbf{e}_u &= h_u^{-1} \mathbf{i} + h_v^{-1} \mathbf{j}, \quad \mathbf{e}_v = -h_v^{-1} \mathbf{i} + h_u^{-1} \mathbf{j} \\
 m [h_u \ddot{u} - h_u^{-1} (u\dot{v} - v\dot{u})^2 / (2u^2 v)] &= -h_u^{-1} \partial V / \partial u = F_u \\
 m [h_v \ddot{v} - h_v^{-1} (u\dot{v} - v\dot{u})^2 / (2uv^2)] &= -h_v^{-1} \partial V / \partial v = F_v
 \end{aligned}$$

$$\begin{aligned}
 9.15 \quad h_u &= 1, \quad h_v = u(1-v^2)^{-1/2} \\
 \mathbf{e}_u &= \mathbf{i} + \mathbf{j}(1-v^2)^{1/2}, \quad \mathbf{e}_v = \mathbf{i}(1-v^2)^{1/2} - \mathbf{j}v \\
 m [\ddot{u} - u\dot{v}^2 / (1-v^2)] &= -\partial V / \partial u = F_u \\
 m [(u\ddot{v} + 2\dot{u}\dot{v})(1-v^2)^{-1/2} + uv\dot{v}^2(1-v^2)^{-3/2}] &= -h_v^{-1} \partial V / \partial v = F_v
 \end{aligned}$$

$$9.16 \quad r^{-1}, \quad 0, \quad 0, \quad r^{-1} \mathbf{e}_z$$

$$9.17 \quad 2r^{-1}, \quad r^{-1} \cot \theta, \quad r^{-1} \mathbf{e}_\phi, \quad r^{-1} (\mathbf{e}_r \cot \theta - \mathbf{e}_\theta)$$

$$9.18 \quad -r^{-1}\mathbf{e}_\theta, \quad r^{-1}\mathbf{e}_r, \quad 3$$

$$9.19 \quad 2\mathbf{e}_\phi, \quad \mathbf{e}_r \cos \theta - \mathbf{e}_\theta \sin \theta, \quad 3$$

$$9.20 \quad r^{-1}, \quad r^{-3}, \quad 0$$

$$9.21 \quad 2r^{-1}, \quad 6, \quad 2r^{-4}, \quad -k^2 e^{ikr \cos \theta}$$

$$11.4 \quad \text{Vector}$$

$$11.5 \quad ds^2 = du^2 + h_v^2 dv^2, \quad h_u = 1, \quad h_v = u(2v - v^2)^{-1/2}, \\ dA = u(2v - v^2)^{-1/2} du dv, \quad d\mathbf{s} = \mathbf{e}_u du + h_v \mathbf{e}_v dv, \\ \mathbf{e}_u = \mathbf{i}(1 - v) + \mathbf{j}(2v - v^2)^{1/2}, \quad \mathbf{e}_v = -\mathbf{i}(2v - v^2)^{1/2} + \mathbf{j}(1 - v) \\ \mathbf{a}_u = \mathbf{e}_u = \mathbf{a}^u, \quad \mathbf{a}_v = h_v \mathbf{e}_v, \quad \mathbf{a}^v = \mathbf{e}_v / h_v$$

$$11.6 \quad m \left(\ddot{u} - \frac{u\dot{v}^2}{v(2-v)} \right) = -\frac{\partial V}{\partial u} = F_u \\ m \left(\frac{u\ddot{v} + 2\dot{u}\dot{v}}{[v(2-v)]^{1/2}} + \frac{u\dot{v}^2(v-1)}{[v(2-v)]^{3/2}} \right) = -u^{-1}[v(2-v)]^{1/2} \frac{\partial V}{\partial v} = F_v$$

$$11.7 \quad \nabla U = \mathbf{e}_u \partial U / \partial u + \mathbf{e}_v u^{-1} \sqrt{v(2-v)} \partial U / \partial v \\ \nabla \cdot \mathbf{V} = u^{-1} \partial(uV_u) / \partial u + u^{-1} \sqrt{v(2-v)} \partial V_v / \partial v \\ \nabla^2 U = \frac{1}{u} \frac{\partial}{\partial u} \left(u \frac{\partial U}{\partial u} \right) + \frac{1}{u^2} \sqrt{v(2-v)} \frac{\partial}{\partial v} \left(\sqrt{v(2-v)} \frac{\partial U}{\partial v} \right)$$

$$11.8 \quad u^{-1}, \quad u^{-1}\mathbf{k}, \quad 0$$

Chapter 11

3.2	$3/2$	3.3	$9/10$	3.4	$25/14$
3.5	$32/35$	3.6	72	3.7	8
3.8	$\Gamma(5/3)$	3.9	$\Gamma(5/4)$	3.10	$\Gamma(3/5)$
3.11	1	3.12	$\Gamma(2/3)/3$	3.13	$3^{-4}\Gamma(4) = 2/27$
3.14	$-\Gamma(4/3)$	3.15	$\Gamma(2/3)/4$	3.17	$\Gamma(p)$

7.1	$\frac{1}{2}B(5/2, 1/2) = 3\pi/16$	7.2	$\frac{1}{2}B(5/4, 3/4) = \pi\sqrt{2}/8$
7.3	$\frac{1}{3}B(1/3, 1/2)$	7.4	$\frac{1}{2}B(3/2, 5/2) = \pi/32$
7.5	$B(3, 3) = 1/30$	7.6	$\frac{1}{3}B(2/3, 4/3) = 2\pi\sqrt{3}/27$
7.7	$\frac{1}{2}B(1/4, 1/2)$	7.8	$4\sqrt{2}B(3, 1/2) = 64\sqrt{2}/15$
7.10	$\frac{4}{3}B(1/3, 4/3)$	7.11	$2B(2/3, 4/3)/B(1/3, 4/3)$
7.12	$(8\pi/3)B(5/3, 1/3) = 32\pi^2\sqrt{3}/27$		
7.13	$I_y/M = 8B(4/3, 4/3)/B(5/3, 1/3)$		

8.1	$B(1/2, 1/4)\sqrt{2l/g} = 7.4163\sqrt{l/g}$	(Compare $2\pi\sqrt{l/g}$.)
8.2	$\frac{1}{4}\sqrt{35/11}B(1/2, 1/4) = 2.34 \text{ sec}$	
8.3	$t = \pi\sqrt{a/g}$	

10.2	$\Gamma(p, x) \sim x^{p-1}e^{-x}[1 + (p-1)x^{-1} + (p-1)(p-2)x^{-2} + \dots]$
10.3	$\operatorname{erfc}(x) = \Gamma(1/2, x^2)/\sqrt{\pi}$
10.5	(a) $E_1(x) = \Gamma(0, x)$
	(b) $\Gamma(0, x) \sim x^{-1}e^{-x}[1 - x^{-1} + 2x^{-2} - 3!x^{-3} + \dots]$
10.6	(a) $\operatorname{Ei}(\ln x)$ (b) $\operatorname{Ei}(x)$ (c) $-\operatorname{Ei}(\ln x)$

11.4	$1/\sqrt{\pi}$	11.5	1	11.10	e^{-1}
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12.1	$K = F(\pi/2, k) = (\pi/2) \left[1 + \left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^2 k^4 + \dots \right]$
	$E = E(\pi/2, k) = (\pi/2) \left[1 - \left(\frac{1}{2}\right)^2 k^2 - \left(\frac{1}{2 \cdot 4}\right)^2 \cdot 3k^4 - \left(\frac{1 \cdot 3}{2 \cdot 4 \cdot 6}\right)^2 \cdot 5k^6 \dots \right]$

Caution : For the following answers, see the text warning about elliptic integral notation just after equations (12.3) and in Example 1.

12.4	$K(1/2) \cong 1.686$	12.5	$E(1/3) \cong 1.526$
12.6	$\frac{1}{3}F\left(\frac{\pi}{3}, \frac{1}{3}\right) \cong 0.355$	12.7	$5E\left(\frac{5\pi}{4}, \frac{1}{5}\right) \cong 19.46$
12.8	$7E\left(\frac{\pi}{3}, \frac{2}{7}\right) \cong 7.242$	12.9	$F\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right) \cong 0.542$
12.10	$\frac{1}{2}F\left(\frac{\pi}{4}, \frac{1}{2}\right) \cong 0.402$	12.11	$F\left(\frac{3\pi}{8}, \frac{3}{\sqrt{10}}\right) + K\left(\frac{3}{\sqrt{10}}\right) \cong 4.097$
12.12	$10E\left(\frac{\pi}{6}, \frac{1}{10}\right) \cong 5.234$	12.13	$3E\left(\frac{\pi}{6}, \frac{2}{3}\right) + 3E\left(\arcsin \frac{3}{4}, \frac{2}{3}\right) \cong 3.96$

$$12.14 \quad 12E\left(\frac{\sqrt{5}}{3}\right) \cong 15.86$$

$$12.15 \quad 2\left[E\left(\frac{\sqrt{3}}{2}\right) - E\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)\right] \cong 0.585$$

$$12.16 \quad 2\sqrt{2}E(1/\sqrt{2}) \cong 3.820$$

$$12.23 \quad T = 8\sqrt{\frac{a}{5g}}K(1/\sqrt{5}); \text{ for small vibrations, } T \cong 2\pi\sqrt{\frac{2a}{3g}}$$

$$13.7 \quad \Gamma(4) = 3!$$

$$13.8 \quad \frac{\sqrt{\pi}}{2} \operatorname{erf}(1)$$

$$13.9 \quad 2E(\sqrt{3}/2) \simeq 2.422$$

$$13.10 \quad \sqrt{2}K(2^{-1/2}) \simeq 2.622$$

$$13.11 \quad \frac{1}{5}F(\arcsin \frac{3}{4}, 4/5) \cong 0.1834$$

$$13.12 \quad 2^{-1/2}K(2^{-1/2}) \cong 1.311$$

$$13.13 \quad -\operatorname{sn} u \operatorname{dn} u$$

$$13.14 \quad \sqrt{\pi}/2 \operatorname{erfc}(1/\sqrt{2})$$

$$13.15 \quad \Gamma(7/2) = 15\sqrt{\pi}/8$$

$$13.16 \quad \sqrt{\pi}$$

$$13.17 \quad \frac{1}{2}B(5/4, 7/4) = 3\pi\sqrt{2}/64$$

$$13.18 \quad \Gamma(3/4)$$

$$13.19 \quad \frac{1}{2}\sqrt{\pi} \operatorname{erfc} 5$$

$$13.20 \quad \frac{1}{2}B(1/2, 7/4)$$

$$13.21 \quad 5^4 B(2/3, 13/3) = (5/3)^5 (14\pi/\sqrt{3})$$

$$13.22 \quad 4E(1/2) - 2E(\pi/8, 1/2) \cong 5.089$$

$$13.23 \quad 109!!\sqrt{\pi}/2^{55}$$

$$13.24 \quad -2^{55}\sqrt{\pi}/109!!$$

$$13.25 \quad 2^{28}\sqrt{\pi}/55!!$$

Chapter 12

- 1.1 $y = a_1 x e^x$ 1.2 $y = a_0 e^{x^3}$
 1.3 $y = a_1 x$ 1.4 $y = a_0 \cos 2x + a_1 \sin 2x$
 1.5 $y = a_0 \cosh x + a_1 \sinh x$ 1.6 $y = (A + Bx)e^x$
 1.7 $y = Ax + Bx^3$ 1.8 $y = a_0(1 + x) + a_2 x^2$
 1.9 $y = a_0(1 - x^2) + a_1 x$ 1.10 $y = (A + Bx)e^{x^2}$
- 2.1 See Problem 5.3 2.4 $Q_0 = \frac{1}{2} \ln \frac{1+x}{1-x}, \quad Q_1 = \frac{x}{2} \ln \frac{1+x}{1-x} - 1$
- 3.2 $x e^x + 10 e^x$ 3.3 $(30 - x^2) \sin x + 12x \cos x$
 3.4 $-x \sin x + 25 \cos x$ 3.5 $(x^2 - 200x + 9900)e^{-x}$
- 4.3 See Problem 5.3
- 5.1 See Problem 5.3
 5.3 $P_0(x) = 1$ $P_4(x) = (35x^4 - 30x^2 + 3)/8$
 $P_1(x) = x$ $P_5(x) = (63x^5 - 70x^3 + 15x)/8$
 $P_2(x) = (3x^2 - 1)/2$ $P_6(x) = (231x^6 - 315x^4 + 105x^2 - 5)/16$
 $P_3(x) = (5x^3 - 3x)/2$
- 5.8 $5P_0 - 2P_1$ 5.9 $2P_2 + P_1$
 5.10 $\frac{8}{35}P_4 + \frac{4}{7}P_2 + \frac{1}{5}P_0$ 5.11 $\frac{2}{5}(P_1 - P_3)$
 5.12 $\frac{8}{5}P_4 + 4P_2 - 3P_1 + \frac{12}{5}P_0$ 5.13 $\frac{8}{63}P_5 + \frac{4}{9}P_3 + \frac{3}{7}P_1$
- 8.1 $N = \sqrt{\pi/2}, \quad \sqrt{2/\pi} \cos nx$ 8.2 $N = \sqrt{2/5}, \quad \sqrt{5/2} P_2(x)$
 8.3 $N = 2^{1/2}, \quad 2^{-1/2} x e^{-x/2}$ 8.4 $N = \pi^{1/4}, \quad \pi^{-1/4} e^{-x^2/2}$
 8.5 $N = \frac{1}{2} \pi^{1/4}, \quad 2\pi^{-1/4} x e^{-x^2/2}$
- 9.1 $\frac{3}{2}P_1 - \frac{7}{8}P_3 + \frac{11}{16}P_5 \dots$ 9.2 $\frac{1}{4}P_0 + \frac{1}{2}P_1 + \frac{5}{16}P_2 - \frac{3}{32}P_4 \dots$
 9.3 $5P_2 + P_0$ 9.4 $\frac{\pi}{8}(3P_1 + \frac{7}{16}P_3 + \frac{11}{64}P_5 \dots)$
 9.5 $\frac{1}{2}P_0 - \frac{5}{8}P_2 + \frac{3}{16}P_4 \dots$ 9.6 $P_0 + \frac{3}{8}P_1 - \frac{20}{9}P_2 \dots$
 9.7 $\frac{1}{3}P_0 + \frac{2}{5}P_1 - \frac{11}{42}P_2 \dots$
 9.8 $\frac{1}{2}(1 - a)P_0 + \frac{3}{4}(1 - a^2)P_1 + \frac{5}{4}a(1 - a^2)P_2 + \frac{7}{16}(1 - a^2)(5a^2 - 1)P_3 \dots$
 9.9 $P'_n = \sum (2l + 1)P_l$, where the sum is over odd l from 1 to $n - 1$
 when n is even, and over even l from 0 to $n - 1$, when n is odd.
- 9.10 $2P_2 + P_1$ 9.11 $\frac{8}{5}P_4 + 4P_2 - 3P_1 + \frac{12}{5}P_0$
 9.12 $\frac{2}{5}(P_1 - P_3)$ 9.13 $\frac{1}{5}P_0 + \frac{4}{7}P_2 = \frac{3}{35}(10x^2 - 1)$
 9.14 $\frac{1}{2}P_0 + \frac{5}{8}P_2 = \frac{3}{16}(5x^2 + 1)$ 9.15 $-\frac{15}{\pi^2}P_2 = -\frac{15}{2\pi^2}(3x^2 - 1)$

$$21.1 \quad y = Ax + B(x \sinh^{-1} x - \sqrt{x^2 + 1})$$

$$21.2 \quad y = A(1 + x) + Bxe^{1/x}$$

$$21.3 \quad y = A(1 - \frac{x}{2}) + B(1 + \frac{x}{2})e^{-x}$$

$$21.4 \quad y = Ax - Be^x$$

$$21.5 \quad y = A(x - 1) + B[(x - 1) \ln x - 4]$$

$$21.6 \quad y = A\sqrt{x} + B[\sqrt{x} \ln x + x]$$

$$21.7 \quad y = A\frac{x}{1-x} + B[\frac{x}{1-x} \ln x + \frac{1+x}{2}]$$

$$21.8 \quad y = A(x^2 + 2x) + B[(x^2 + 2x) \ln x + 1 + 5x - x^3/6 + x^4/72 + \dots]$$

$$21.9 \quad y = Ax^2 + B[x^2 \ln x - x^3 + x^4/(2 \cdot 2!) - x^5/(3 \cdot 3!) + x^6/(4 \cdot 4!) + \dots]$$

$$21.10 \quad y = Ax^3 + B(x^3 \ln x + x^2)$$

$$22.4 \quad H_0(x) = 1 \qquad H_3(x) = 8x^3 - 12x$$

$$H_1(x) = 2x \qquad H_4(x) = 16x^4 - 48x^2 + 12$$

$$H_2(x) = 4x^2 - 2 \qquad H_5(x) = 32x^5 - 160x^3 + 120x$$

$$22.13 \quad L_0(x) = 1$$

$$L_1(x) = 1 - x$$

$$L_2(x) = (2 - 4x + x^2)/2!$$

$$L_3(x) = (6 - 18x + 9x^2 - x^3)/3!$$

$$L_4(x) = (24 - 96x + 72x^2 - 16x^3 + x^4)/4!$$

$$L_5(x) = (120 - 600x + 600x^2 - 200x^3 + 25x^4 - x^5)/5!$$

Note: The factor $1/n!$ is omitted in most quantum mechanics books but is included as here in most reference books.

$$22.20 \quad L_0^k(x) = 1$$

$$L_1^k(x) = 1 + k - x$$

$$L_2^k(x) = \frac{1}{2}(k+1)(k+2) - (k+2)x + \frac{1}{2}x^2$$

$$22.28 \quad f_1 = xe^{-x/2}$$

$$f_2 = xe^{-x/4}(2 - \frac{x}{2})$$

$$f_3 = xe^{-x/6}(3 - x + \frac{x^2}{18})$$

$$22.30 \quad R_n = -x^n Dx^{-n}, L_n = x^{-n-1} Dx^{n+1}$$

$$23.6 \quad P_{2n+1}^1(0) = (2n+1)P_{2n}(0) = \frac{(-1)^n(2n+1)!!}{2^n n!}$$

$$23.9 \quad \text{For } n \leq l, \int_{-1}^1 x P_l(x) P_n(x) dx = \begin{cases} \frac{2l}{(2l-1)(2l+1)}, & n = l-1 \\ 0, & \text{otherwise} \end{cases}$$

$$23.18 \quad \text{(a) } y = Z_0(e^x) \qquad \text{(b) } y = Z_p(e^{x^2}/2)$$

$$23.23 \quad T_0 = 1, T_1 = x, T_2 = 2x^2 - 1$$

$$23.30 \quad \pi/6$$

Chapter 13

$$2.1 \quad T = \frac{20}{\pi} \sum_1^{\infty} \frac{(-1)^{n+1}}{n} e^{-n\pi y/10} \sin \frac{n\pi x}{10}$$

$$2.2 \quad T = \frac{200}{\pi} \left(\sum_{\substack{1 \\ \text{odd } n}}^{\infty} -2 \sum_{\substack{2 \\ n=2+4k}}^{\infty} \right) \frac{1}{n} e^{-n\pi y/20} \sin \frac{n\pi x}{20}$$

$$2.3 \quad T = \frac{4}{\pi} \sum_{\substack{2 \\ \text{even } n}}^{\infty} \frac{n}{n^2-1} e^{-ny} \sin nx$$

$$2.4 \quad T = \frac{120}{\pi^2} \left(e^{-\pi y/30} \sin \frac{\pi x}{30} - \frac{1}{9} e^{-3\pi y/30} \sin \frac{3\pi x}{30} + \frac{1}{25} e^{-5\pi y/30} \sin \frac{5\pi x}{30} \dots \right)$$

$$2.7 \quad T = \frac{4}{\pi} \sum_{\substack{2 \\ \text{even } n}}^{\infty} \frac{n}{(n^2-1) \sinh n} \sinh n(1-y) \sin nx$$

$$2.8 \quad T = \sum_1^{\infty} \frac{b_n}{\sinh \frac{4n\pi}{3}} \sinh \frac{n\pi}{30} (40-y) \sin \frac{n\pi x}{30}$$

$$\text{where } b_n = \frac{200}{n\pi} \left(1 - \cos \frac{n\pi}{3} \right) = \frac{100}{n\pi} (1, 3, 4, 3, 1, 0, \text{ and repeat})$$

$$2.9 \quad T = \frac{200}{\pi} \left(\sum_{\substack{1 \\ \text{odd } n}}^{\infty} -2 \sum_{\substack{2 \\ n=2+4k}}^{\infty} \right) \frac{1}{n \sinh \frac{n\pi}{2}} \sinh \frac{n\pi}{20} (10-y) \sin \frac{n\pi x}{20}$$

$$2.10 \quad T = \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{400}{n\pi \sinh n\pi} \sinh \frac{n\pi}{10} (10-y) \sin \frac{n\pi x}{10}; \quad T(5,5) \cong 25^\circ$$

$$2.11 \quad T = \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{400}{n\pi \sinh n\pi} \left[\sinh \frac{n\pi}{10} (10-y) \sin \frac{n\pi x}{10} + \sinh \frac{n\pi}{10} (10-x) \sinh \frac{n\pi y}{10} \right]$$

$$2.12 \quad T = \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{400}{n\pi \sinh 3n\pi} \sinh \frac{n\pi}{10} (30-y) \sin \frac{n\pi x}{10} \\ + \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{400}{n\pi \sinh(n\pi/3)} \sinh \frac{n\pi}{30} (10-x) \sin \frac{n\pi y}{30}$$

$$2.13 \quad T(x,y) = \frac{20}{\pi} \sum_1^{\infty} \frac{(-1)^{n+1}}{n \sinh 2n\pi} \sinh \frac{n\pi}{10} (20-y) \sin \frac{n\pi x}{10} \\ + \frac{40}{\pi} \sum_1^{\infty} \frac{(-1)^{n+1}}{n \sinh \frac{n\pi}{2}} \sinh \frac{n\pi}{20} (10-x) \sin \frac{n\pi y}{20}$$

$$2.14 \quad \text{For } f(x) = x - 5: T = -\frac{40}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos \frac{n\pi x}{10} e^{-n\pi y/10}$$

For $f(x) = x$: Add 5 to the answer just given.

$$2.15 \quad \text{For } f(x) = 100, T = 100 - 10y/3$$

$$\text{For } f(x) = x, T = \frac{1}{6}(30 - y) - \frac{40}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2 \sinh 3n\pi} \sinh \frac{n\pi}{10} (30 - y) \cos \frac{n\pi x}{10}$$

$$3.2 \quad u = \frac{400}{\pi} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n} e^{-(n\pi\alpha/10)^2 t} \sin \frac{n\pi x}{10}$$

$$3.3 \quad u = 100 - \frac{100x}{l} - \frac{400}{\pi} \sum_{\substack{2 \\ \text{even } n}}^{\infty} \frac{1}{n} e^{-(n\pi\alpha/l)^2 t} \sin \frac{n\pi x}{l}$$

$$3.4 \quad u = \frac{40}{\pi} \left(\sum_{\substack{1 \\ \text{odd } n}}^{\infty} -2 \sum_{\substack{2 \\ n=2+4k}}^{\infty} \right) \frac{1}{n} e^{-(n\pi\alpha/10)^2 t} \sin \frac{n\pi x}{10}$$

$$3.5 \quad u = 100 + 400 \sum_1^{\infty} b_n e^{-(n\pi\alpha/2)^2 t} \sin \frac{n\pi x}{2} \text{ where } b_n = \begin{cases} 0, & \text{even } n \\ \frac{2}{n^2\pi^2} - \frac{1}{n\pi}, & n = 1 + 4k \\ \frac{-2}{n^2\pi^2} - \frac{1}{n\pi}, & n = 3 + 4k \end{cases}$$

3.6 Add to (3.15): $u_f = 20 + 30x/l$. Note: Any linear function added to both u_0 and u_f leaves the Fourier series unchanged.

$$3.7 \quad u = \frac{l}{2} - \frac{4l}{\pi^2} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \cos \frac{n\pi x}{l} e^{-(n\pi\alpha/l)^2 t}$$

$$3.8 \quad u = 50x + \frac{200}{\pi} \sum_1^{\infty} \frac{(-1)^n}{n} e^{-(n\pi\alpha/2)^2 t} \sin \frac{n\pi x}{2}$$

$$3.9 \quad u = 100 - \frac{400}{\pi} \sum_0^{\infty} \frac{(-1)^n}{2n+1} e^{-[(2n+1)\pi\alpha/4]^2 t} \cos \left(\frac{2n+1}{4} \pi x \right)$$

$$3.11 \quad E_n = \frac{n^2 \hbar^2}{2m}, \quad \Psi(x, t) = \frac{4}{\pi} \sum_{\text{odd } n} \frac{\sin nx}{n} e^{-iE_n t/\hbar}$$

$$3.12 \quad E_n = \frac{n^2 \pi^2 \hbar^2}{2m}, \quad \Psi(x, t) = \frac{8}{\pi} \sum_{\text{odd } n} \frac{\sin n\pi x}{n(4 - n^2)} e^{-iE_n t/\hbar}$$

$$4.2 \quad y = \frac{8h}{\pi^2} \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{l} \cos \frac{n\pi vt}{l}, \text{ where } B_1 = \sqrt{2} - 1, B_2 = \frac{1}{2},$$

$$B_3 = \frac{1}{9}(\sqrt{2} + 1), B_4 = 0, \dots, B_n = (2 \sin n\pi/4 - \sin n\pi/2)/n^2$$

$$4.3 \quad y = \frac{16h}{\pi^2} \sum_1^{\infty} B_n \sin \frac{n\pi x}{l} \cos \frac{n\pi vt}{l} \text{ where } B_n = \left(2 \sin \frac{n\pi}{8} - \sin \frac{n\pi}{4} \right) / n^2$$

$$4.4 \quad y = \frac{8h}{\pi^2} \sum_1^{\infty} \frac{1}{n^2} \sin \frac{n\pi}{2} \sin \frac{2n\pi x}{l} \cos \frac{2n\pi vt}{l}$$

$$4.5 \quad y = \frac{8hl}{\pi^3 v} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^3} \sin \frac{n\pi}{2} \sin \frac{n\pi x}{l} \sin \frac{n\pi vt}{l}$$

$$4.6 \quad y = \frac{4hl}{\pi^2 v} \sum_{\substack{1 \\ \text{odd } n}}^{\infty} \frac{1}{n^2} \sin \frac{n\pi}{2} \sin \frac{n\pi w}{l} \sin \frac{n\pi x}{l} \sin \frac{n\pi vt}{l}$$

$$4.7 \quad y = \frac{9hl}{\pi^3 v} \sum_1^{\infty} \frac{1}{n^3} \sin \frac{n\pi}{3} \sin \frac{n\pi x}{l} \sin \frac{n\pi vt}{l}$$

$$4.8 \quad y = \frac{4l}{\pi^2 v} \left[\frac{1}{3} \sin \frac{\pi x}{l} \sin \frac{\pi vt}{l} + \frac{\pi}{16} \sin \frac{2\pi x}{l} \sin \frac{2\pi vt}{l} - \sum_{n=3}^{\infty} \frac{\sin(n\pi/2)}{n(n^2-4)} \sin \frac{n\pi x}{l} \sin \frac{n\pi vt}{l} \right]$$

- 4.9
1. $n = 1, \nu = v/(2l)$
 2. $n = 2, \nu = v/l$
 3. $n = 3, \nu = 3v/(2l)$, and $n = 4, \nu = 2v/l$, have nearly equal intensity.
 4. $n = 2, \nu = v/l$
 - 5, 6, 7, 8. $n = 1, \nu = v/(2l)$

- 4.11 The basis functions for a string pinned at $x = 0$, free at $x = l$, and with zero initial string velocity, are $y = \sin \frac{(n+\frac{1}{2})\pi x}{l} \cos \frac{(n+\frac{1}{2})\pi vt}{l}$.
The solutions for Problems 2, 3, 4, parts (a) and (b) are:

$$(a) \quad y = \sum_{n=0}^{\infty} a_n \cos \frac{(n+\frac{1}{2})\pi x}{l} \cos \frac{(n+\frac{1}{2})\pi vt}{l}$$

$$(b) \quad y = \sum_{n=0}^{\infty} b_n \sin \frac{(n+\frac{1}{2})\pi x}{l} \cos \frac{(n+\frac{1}{2})\pi vt}{l}$$

where the coefficients are:

$$2(a) \quad a_n = \frac{128h}{(2n+1)^2\pi^2} \sin^2 \frac{(2n+1)\pi}{16} \cos \frac{(2n+1)\pi}{8}$$

$$2(b) \quad b_n = \frac{128h}{(2n+1)^2\pi^2} \sin^2 \frac{(2n+1)\pi}{16} \sin \frac{(2n+1)\pi}{8}$$

$$3(a) \quad a_n = \frac{256h}{(2n+1)^2\pi^2} \sin^2 \frac{(2n+1)\pi}{32} \cos \frac{(2n+1)\pi}{16}$$

$$3(b) \quad b_n = \frac{256h}{(2n+1)^2\pi^2} \sin^2 \frac{(2n+1)\pi}{32} \sin \frac{(2n+1)\pi}{16}$$

$$4(a) \quad a_n = \frac{256h}{(2n+1)^2\pi^2} \sin^2 \frac{(2n+1)\pi}{16} \sin \frac{(2n+1)\pi}{8} \sin \frac{(2n+1)\pi}{4}$$

$$4(b) \quad b_n = \frac{256h}{(2n+1)^2\pi^2} \sin^2 \frac{(2n+1)\pi}{16} \sin \frac{(2n+1)\pi}{8} \cos \frac{(2n+1)\pi}{4}$$

- 4.12 With $b_n = \frac{8}{n^3\pi^3}$, odd n , the six solutions on $(0, 1)$ are:

$$1. \text{ Temperature in semi-infinite plate: } T = \sum b_n e^{-n\pi y} \sin n\pi x$$

$$2. \text{ Temperature in finite plate of height } H:$$

$$T = \sum \frac{b_n}{\sinh(n\pi H)} \sinh n\pi(H-y) \sin n\pi x$$

$$3. \text{ 1-dimensional heat flow: } u = \sum b_n e^{-(n\pi\alpha)^2 t} \sin n\pi x$$

$$4. \text{ Particle in a box: } \Psi = \sum b_n \sin n\pi x e^{-iE_n t/\hbar}, \quad E_n = \frac{\hbar^2 n^2 \pi^2}{2m}$$

$$5. \text{ Plucked string: } y = \sum b_n \sin n\pi x \cos n\pi vt$$

$$6. \text{ String with initial velocity: } y = \sum \frac{b_n}{n\pi v} \sin n\pi x \sin n\pi vt$$

- 4.13 With $b_n = \frac{16}{n\pi(4-n^2)}$, n odd, the six solutions on $(0, \pi)$ are
1. $T = \sum b_n e^{-ny} \sin nx$
 2. $T = \sum \frac{b_n}{\sinh nH} \sinh n(H-y) \sin nx$
 3. $u = \sum b_n e^{(-n\alpha)^2 t} \sin nx$
 4. $\Psi = \sum b_n \sin nx e^{-iE_n t/\hbar}, \quad E_n = \frac{\hbar^2 n^2}{2m}$
 5. $y = \sum b_n \sin nx \cos nvt$
 6. $y = \sum \frac{b_n}{nv} \sin nx \sin nvt$
- 4.14 Same as 4.12 with $b_n = \frac{12(-1)^{n+1}}{n^3 \pi^3}$, all n , on $(0, 1)$
- 5.1 (a) $u \cong 9.76^\circ$ (b) $u \cong 9.76^\circ$
- 5.2 (a) $\sum_{m=1}^{\infty} \frac{2}{k_m J_2(k_m)} J_1(k_m r) e^{-k_m z} \sin \theta$, $k_m = \text{zeros of } J_1$
- (b) $\sum_{m=1}^{\infty} \frac{2a}{k_m J_2(k_m)} J_1(k_m r/a) e^{-k_m z/a} \sin \theta$, $k_m = \text{zeros of } J_1$
 $u(r=1, z=1, \theta=\pi/2) \cong 0.211$
- 5.3 (a) $u = \sum_{m=1}^{\infty} \frac{200}{k_m J_1(k_m) \sinh(10k_m)} J_0(k_m r) \sinh k_m(10-z)$, $k_m = \text{zeros of } J_0$
- (b) $u = \sum_{m=1}^{\infty} \frac{200}{k_m J_1(k_m) \sinh(k_m H/a)} J_0(k_m r/a) \sinh \frac{k_m(H-z)}{a}$,
 $k_m = \text{zeros of } J_0$
- 5.4 $u = \sum_{m=1}^{\infty} \frac{200}{k_m J_1(k_m)} J_0(k_m r/a) e^{-(k_m \alpha/a)^2 t}$, $k_m = \text{zeros of } J_0$
- 5.5 $\sum_{m=1}^{\infty} \frac{200a}{k_m J_2(k_m)} J_1(k_m r/a) e^{-(k_m \alpha/a)^2 t} \sin \theta$, $k_m = \text{zeros of } J_1$
- 5.6 $a_{mn} = \frac{2}{\pi a^2 J_{n+1}^2(k_{mn})} \int_0^a \int_0^{2\pi} f(r, \theta) J_n(k_{mn} r/a) \cos n\theta r dr d\theta$
 $b_{mn} = \frac{2}{\pi a^2 J_{n+1}^2(k_{mn})} \int_0^a \int_0^{2\pi} f(r, \theta) J_n(k_{mn} r/a) \sin n\theta r dr d\theta$
- 5.7 $u = \frac{400}{\pi} + \sum_{\text{odd } n} \frac{1}{n I_0(3n\pi/20)} I_0\left(\frac{n\pi r}{20}\right) \sin \frac{n\pi z}{20}$
- 5.8 $u = 40 + \sum_{m=1}^{\infty} \frac{120}{k_m J_1(k_m)} J_0(k_m r) e^{-k_m^2 \alpha^2 t}$, where $J_0(k_m) = 0$
- 5.9 $u = \frac{1600}{\pi^2} \sum_{\text{odd } m} \sum_{\text{odd } n} \frac{\sin(n\pi x/10) \sin(m\pi y/10) \sinh[\pi(n^2 + m^2)^{1/2}(10-z)/10]}{mn \sinh[\pi(n^2 + m^2)^{1/2}]}$
- 5.10 $u = \frac{6400}{\pi^3} \sum_{\text{odd } n} \sum_{\text{odd } m} \sum_{\text{odd } p} \frac{1}{nmp} \sin \frac{n\pi x}{l} \sin \frac{m\pi y}{l} \sin \frac{p\pi z}{l} e^{-(\alpha\pi/l)^2(n^2+m^2+p^2)t}$
- 5.11 $R = r^n, r^{-n}, n \neq 0; R = \ln r, \text{const.}, n = 0$
 $R = r^l, r^{-l-1}$
- 5.12 $u = 50 + \frac{200}{\pi} \sum_{\text{odd } n} \left(\frac{r}{a}\right)^n \frac{\sin n\theta}{n}$
- 5.13 $u = \frac{400}{\pi} \sum_{\text{odd } n} \frac{1}{n} \left(\frac{r}{10}\right)^{4n} \sin 4n\theta$

- 5.14 $u = \frac{50 \ln r}{\ln 2} + \frac{200}{\pi} \sum_{\text{odd } n} \frac{r^n - r^{-n}}{n(2^n - 2^{-n})} \sin n\theta$
- 5.15 $u = 50 \left(1 - \frac{\ln r}{\ln 2}\right) - \frac{200}{\pi} \sum_{\text{odd } n} \frac{1}{n(2^n - 2^{-n})} \left[\left(\frac{r}{2}\right)^n - \left(\frac{r}{2}\right)^{-n}\right] \sin n\theta$
- 6.2 The first six frequencies are ν_{10} , $\nu_{11} = 1.593\nu_{10}$, $\nu_{12} = 2.135\nu_{10}$, $\nu_{20} = 2.295\nu_{10}$, $\nu_{13} = 2.652\nu_{10}$, $\nu_{21} = 2.917\nu_{10}$.
- 6.4 $\nu_{nm} = \frac{v}{2} \sqrt{\left(\frac{l}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{n}{c}\right)^2}$
- 6.5 $z = \frac{64l^4}{\pi^6} \sum_{\text{odd } m} \sum_{\text{odd } n} \frac{1}{n^3 m^3} \sin \frac{n\pi x}{l} \sin \frac{m\pi y}{l} \cos \frac{\pi v(m^2 + n^2)^{1/2} t}{l}$
- 6.6 $\Psi_n = \sin \frac{n_x \pi x}{l} \sin \frac{n_y \pi y}{l} e^{-iE_n t/\hbar}$, $E_n = \frac{\pi^2 \hbar^2 (n_x^2 + n_y^2)}{2ml^2}$
- 6.7 See Problem 6.3. Some other examples of degeneracy:
 $(n_x, n_y) = (1, 8), (8, 1), (4, 7), (7, 4)$, giving $E_n = 65 \frac{\pi^2 \hbar^2}{2ml^2}$;
 similarly $2^2 + 9^2 = 6^2 + 7^2 = 85$; $2^2 + 11^2 = 5^2 + 10^2 = 125$, etc.
- 6.8 $\Psi_{mn} = J_n(k_{mn}r) \begin{Bmatrix} \sin n\theta \\ \cos n\theta \end{Bmatrix} e^{-iE_{mn}t/\hbar}$, $E_{mn} = \frac{\hbar^2 k_{mn}^2}{2ma^2}$
- 7.1 $u = 7P_0(\cos \theta) + 20r^2 P_2(\cos \theta) + 8r^4 P_4(\cos \theta)$
- 7.2 $u = \frac{2}{5}r P_1(\cos \theta) - \frac{2}{5}r^3 P_3(\cos \theta)$
- 7.3 $u = -2P_0(\cos \theta) + r P_1(\cos \theta) + 2r^2 P_2(\cos \theta)$
- 7.4 $u = -2P_0(\cos \theta) + 3r P_1(\cos \theta) + 2r^2 P_2(\cos \theta) + 2r^3 P_3(\cos \theta)$
- 7.5 $u = \frac{1}{2}P_0(\cos \theta) + \frac{5}{8}r^2 P_2(\cos \theta) - \frac{3}{16}r^4 P_4(\cos \theta) \dots$
- 7.6 $u = \frac{\pi}{8}[3r P_1(\cos \theta) + \frac{7}{16}r^3 P_3(\cos \theta) + \frac{11}{64}r^5 P_5(\cos \theta) \dots]$
- 7.7 $u = \frac{1}{4}P_0(\cos \theta) + \frac{1}{2}r P_1(\cos \theta) + \frac{5}{16}r^2 P_2(\cos \theta) - \frac{3}{32}r^4 P_4(\cos \theta) \dots$
- 7.8 $u = 25[P_0(\cos \theta) + \frac{9}{4}r P_1(\cos \theta) + \frac{15}{8}r^2 P_2(\cos \theta) + \frac{21}{64}r^3 P_3(\cos \theta) \dots]$
- 7.9 $u = r^2 P_2^1(\cos \theta) \sin \phi$
- 7.10 $u = \frac{1}{15}r^3 P_3^2(\cos \theta) \cos 2\phi - r P_1(\cos \theta)$
- 7.11 $u = 200[(3/4)r P_1(\cos \theta) - (7/16)r^3 P_3(\cos \theta) + (11/32)r^5 P_5(\cos \theta) + \dots]$
- 7.12 $u = \frac{3}{4}r P_1(\cos \theta) + \frac{7}{24}r^3 P_3(\cos \theta) - \frac{11}{192}r^5 P_5(\cos \theta) \dots$
- 7.13 $u = E_0(r - a^3/r^2) P_1(\cos \theta)$
- 7.14 $u = 100[(1 - r^{-1})P_0(\cos \theta) + \frac{3}{7}(r - r^{-2})P_1(\cos \theta) - \frac{7}{127}(r^3 - r^{-4})P_3(\cos \theta) \dots]$
- 7.15 $u = 100 + \frac{200a}{\pi r} \sum_1^\infty \frac{(-1)^n}{n} \sin \frac{n\pi r}{a} e^{-(\alpha n\pi/a)^2 t}$
 $= 100 + 200 \sum_{n=1}^\infty (-1)^n j_0(n\pi r/a) e^{-(\alpha n\pi/a)^2 t}$
- 7.17 $\Psi_n = \sin \frac{n_x \pi x}{l} \sin \frac{n_y \pi y}{l} \sin \frac{n_z \pi z}{l} e^{-iE_n t/\hbar}$, $E_n = \frac{\pi^2 \hbar^2 (n_x^2 + n_y^2 + n_z^2)}{2ml^2}$
- 7.19 $\Psi(r, \theta, \phi) = j_l(\beta r) P_l^m(\cos \theta) e^{\pm i m \phi} e^{-i E t/\hbar}$,
 where $\beta = \sqrt{2ME/\hbar^2}$, βa = zeros of j_l , $E = \frac{\hbar^2}{2Ma^2}(\text{zeros of } j_l)^2$.
- 7.20 $\psi_n(x) = e^{-\alpha^2 x^2/2} H_n(\alpha x)$, $\alpha = \sqrt{m\omega/\hbar}$

$$7.21 \quad \psi_n(x) = e^{-\alpha^2(x^2+y^2+z^2)/2} H_{n_x}(\alpha x) H_{n_y}(\alpha y) H_{n_z}(\alpha z), \quad \alpha = \sqrt{m\omega/\hbar},$$

$$E_n = (n_x + \frac{1}{2} + n_y + \frac{1}{2} + n_z + \frac{1}{2})\hbar\omega = (n + \frac{3}{2})\hbar\omega.$$

Degree of degeneracy of E_n is $C(n+2, n) = \frac{(n+2)(n+1)}{2}$, $n = 0$ to ∞ .

$$7.22 \quad \Psi(r, \theta, \phi) = R(r)Y_l^m(\theta, \phi), \quad R(r) = r^l e^{-r/(na)} L_{n-l-1}^{2l+1}\left(\frac{2r}{na}\right), \quad E_n = -\frac{Me^4}{2\hbar^2 n^2}$$

8.3 The second terms in (8.20) and (8.21) are replaced by

$$-q \sum_l \frac{a^l r^l}{R^{2l+1}} P_l(\cos \theta) = \frac{-qR/a}{\sqrt{r^2 - 2(rR^2/a) \cos \theta + (R^2/a)^2}}$$

Image charge $-qR/a$ at $(0, 0, R^2/a)$

8.4 Let K = line charge per unit length. Then

$$V = -K \ln(r^2 + a^2 - 2ra \cos \theta) + K \ln a^2 - K \ln R^2$$

$$+ K \ln \left[r^2 + \left(\frac{R^2}{a} \right)^2 - 2 \frac{R^2}{a} r \cos \theta \right]$$

8.5 K at $(a, 0)$, $-K$ at $(R^2/a, 0)$

$$9.2 \quad u = \frac{200}{\pi} \int_0^\infty k^{-2} (1 - \cos 2k) e^{-ky} \cos kx \, dk$$

$$9.4 \quad u(x, t) = \frac{200}{\pi} \int_0^\infty \frac{1 - \cos k}{k} e^{-k^2 \alpha^2 t} \sin kx \, dk$$

$$9.7 \quad u(x, t) = 100 \operatorname{erf} \left(\frac{x}{2\alpha\sqrt{t}} \right) - 50 \operatorname{erf} \left(\frac{x-1}{2\alpha\sqrt{t}} \right) - 50 \operatorname{erf} \left(\frac{x+1}{2\alpha\sqrt{t}} \right)$$

$$10.1 \quad T = \frac{2}{\pi} \sum_{n=1}^\infty \frac{1}{n \sinh 2n\pi} \sinh n\pi(2-y) \sin n\pi x$$

$$10.2 \quad T = \frac{2}{\pi} \sum_{n=1}^\infty \frac{1}{n \sinh 2n\pi} \sinh n\pi y \sin n\pi x$$

$$10.3 \quad T = \frac{1}{4}(2-y) + \frac{4}{\pi^2} \sum_{\text{odd } n} \frac{1}{n^2 \sinh 2n\pi} \sinh n\pi(2-y) \cos n\pi x$$

$$10.4 \quad T = 20 + \frac{40}{\pi} \sum_{\text{odd } n} \frac{1}{n \sinh \frac{3n\pi}{5}} \sinh \frac{n\pi y}{5} \sin \frac{n\pi x}{5}$$

$$+ \frac{40}{\pi} \sum_{\text{odd } n} \frac{1}{n \sinh \frac{5n\pi}{3}} \sinh \frac{n\pi(5-x)}{3} \sin \frac{n\pi y}{3}$$

$$10.5 \quad u = 20 - \frac{80}{\pi} \sum_{\text{odd } n} \frac{1}{n} e^{-(n\pi\alpha/l)^2 t} \sin \frac{n\pi x}{l}$$

$$10.6 \quad u = 20 - \frac{80}{\pi} \sum_{n=0}^\infty \frac{(-1)^n}{2n+1} e^{-[(2n+1)\pi\alpha/(2l)]^2 t} \cos \left(\frac{2n+1}{2l} \pi x \right)$$

$$10.7 \quad u = \frac{1}{4}y + \frac{4}{\pi^2} \sum_{\text{odd } n} \frac{1}{n^2 \sinh 2n\pi} \sinh n\pi y \cos n\pi x$$

$$10.8 \quad u = 20 - x - \frac{40}{\pi} \sum_{\text{even } n} \frac{1}{n} e^{-(n\pi\alpha/10)^2 t} \sin \frac{n\pi x}{10}$$

$$10.9 \quad y = \frac{8l^2}{\pi^3} \sum_{\text{odd } n} \frac{1}{n^3} \cos \frac{n\pi vt}{l} \sin \frac{n\pi x}{l}$$

$$10.10 \quad u = \frac{1600}{\pi^2} \sum_{\text{odd } n} \sum_{\text{odd } m} \frac{1}{nm I_n(3m\pi/20)} I_n \left(\frac{m\pi r}{20} \right) \sin n\theta \sin \frac{m\pi z}{20}$$

$$10.12 \quad u = \frac{400}{\pi} \sum_{\text{odd } n} \frac{1}{n} \left(\frac{r}{a} \right)^{2n} \sin 2n\theta$$

$$10.14 \quad \text{Same as 9.12}$$

$$10.15 \quad u = \frac{400}{\pi} \sum_{\text{odd } n} \frac{1}{n} \left(\frac{r}{10} \right)^{6n} \sin 6n\theta = \frac{200}{\pi} \arctan \frac{2(10r)^6 \sin 6\theta}{10^{12} - r^{12}}$$

$$10.16 \quad v\sqrt{5}/(2\pi)$$

$$10.17 \quad \nu_{mn}, n \neq 0; \text{ the lowest frequencies are:}$$

$$\nu_{11} = 1.59\nu_{10}, \nu_{12} = 2.14\nu_{10}, \nu_{13} = 2.65\nu_{10}, \nu_{21} = 2.92\nu_{10}, \nu_{14} = 3.16\nu_{10}$$

$$10.18 \quad \nu_{mn}, n = 3, 6, \dots; \text{ the lowest frequencies are:}$$

$$\nu_{13} = 2.65 \nu_{10}, \nu_{23} = 4.06 \nu_{10}, \nu_{16} = 4.13 \nu_{10}, \nu_{33} = 5.4 \nu_{10}$$

$$10.19 \quad u = E_0 \left(r - \frac{a^2}{r} \right) \cos \theta$$

$$10.20 \quad \nu = \frac{v\lambda_l}{2\pi a} \text{ where } \lambda_l \text{ are zeros of } j_l, a = \text{radius of sphere, } v = \text{speed of sound}$$

$$10.21 \quad u = \frac{2}{3}P_0(\cos \theta) + \frac{3}{5}rP_1(\cos \theta) - \frac{2}{3}r^2P_2(\cos \theta) + \frac{2}{5}r^3P_3(\cos \theta)$$

$$10.22 \quad u = 1 - \frac{1}{2}rP_1(\cos \theta) + \frac{7}{8}r^3P_3(\cos \theta) - \frac{11}{16}r^5P_5(\cos \theta) \dots$$

$$10.23 \quad u = 100 \sum_{\text{odd } l} (a_l r^l + b_l r^{-l-1}) P_l(\cos \theta) \text{ where}$$

$$a_l = \frac{2A+1}{2A^2-1} c_l, \quad b_l = -\frac{2A(A+1)}{2A^2-1} c_l, \quad A = 2^l,$$

$$c_l = (2l+1) \int_0^1 P_l(x) dx \quad (\text{Chapter 12, Problem 9.1}).$$

The first few terms are

$$u = (107.1r - 257.1r^{-2})P_1(\cos \theta) - (11.7r^3 - 99.2r^{-4})P_3(\cos \theta) + (2.2r^5 - 70.9r^{-6})P_5(\cos \theta) \dots$$

$$10.24 \quad T = A + \sum_{\text{odd } n} \frac{4(D-A)}{n\pi \sinh(n\pi b/a)} \sinh \frac{n\pi}{a} (b-y) \sin \frac{n\pi x}{a} \\ + \sum_{\text{odd } n} \frac{4(C-A)}{n\pi \sinh(n\pi a/b)} \sinh \frac{n\pi}{b} (a-x) \sin \frac{n\pi y}{b} \\ + \sum_{\text{odd } n} \frac{4(B-A)}{n\pi \sinh(n\pi b/a)} \sinh \frac{n\pi y}{a} \sin \frac{n\pi x}{a}$$

$$10.26 \quad \nu = \frac{v}{2\pi} \sqrt{(k_{mn}/a)^2 + \lambda^2} \text{ where } k_{mn} \text{ is a zero of } J_n$$

$$10.27 \quad \nu = \frac{v}{2} \sqrt{\left(\frac{n}{a}\right)^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{\lambda}{\pi}\right)^2}$$

$$10.28 \quad u(x, y) = \frac{200}{\pi} \int_0^\infty \frac{\sin k}{k \cosh k} \cos kx \cosh ky dk$$

Chapter 14

- 1.1 $u = x^3 - 3xy^2, v = 3x^2y - y^3$ 1.2 $u = x, v = y$
 1.3 $u = x, v = -y$ 1.4 $u = (x^2 + y^2)^{1/2}, v = 0$
 1.5 $u = x, v = 0$ 1.6 $u = e^x \cos y, v = e^x \sin y$
 1.7 $u = \cos y \cosh x, v = \sin y \sinh x$
 1.8 $u = \sin x \cosh y, v = \cos x \sinh y$
 1.9 $u = x/(x^2 + y^2), v = -y/(x^2 + y^2)$
 1.10 $u = (2x^2 + 2y^2 + 7x + 6)/[(x + 2)^2 + y^2], v = y/[(x + 2)^2 + y^2]$
 1.11 $u = 3x/[x^2 + (y - 2)^2], v = (-2x^2 - 2y^2 + 5y - 2)/[x^2 + (y - 2)^2]$
 1.12 $u = x(x^2 + y^2 + 1)/[(x^2 - y^2 + 1)^2 + 4x^2y^2],$
 $v = y(1 - x^2 - y^2)/[(x^2 - y^2 + 1)^2 + 4x^2y^2]$
 1.13 $u = \ln(x^2 + y^2)^{1/2}, v = 0$ 1.14 $u = x(x^2 + y^2), v = y(x^2 + y^2)$
 1.15 $u = e^x \cos y, v = -e^x \sin y$ 1.16 $u = 0, v = 4xy$
 1.17 $u = \cos x \cosh y, v = \sin x \sinh y$
 1.18 $u = \pm 2^{-1/2}[(x^2 + y^2)^{1/2} + x]^{1/2}, v = \pm 2^{-1/2}[(x^2 + y^2)^{1/2} - x]^{1/2},$
 where the \pm signs are chosen so that uv has the sign of y .
 1.19 $u = \ln(x^2 + y^2)^{1/2}, v = \arctan(y/x)$ [angle is in the quadrant
 of the point (x, y)].
 1.20 $u = x^2 - y^2 - 4xy - x - y + 3, v = 2x^2 - 2y^2 + 2xy + x - y$
 1.21 $u = e^{-y} \cos x, v = e^{-y} \sin x$

In 2.1 to 2.24, A = analytic, N = not analytic

- | | | | |
|--|----------------------|----------------------|------------------------|
| 2.1 A | 2.2 A | 2.3 N | 2.4 N |
| 2.5 N | 2.6 A | 2.7 A | 2.8 A |
| 2.9 A, $z \neq 0$ | 2.10 A, $z \neq -2$ | 2.11 A, $z \neq 2i$ | 2.12 A, $z \neq \pm i$ |
| 2.13 N | 2.14 N | 2.15 N | 2.16 N |
| 2.17 N | 2.18 A, $z \neq 0$ | 2.19 A, $z \neq 0$ | 2.20 A |
| 2.21 A | 2.22 N | 2.23 A, $z \neq 0$ | 2.24 N |
| 2.34 $-z - \frac{1}{2}z^2 - \frac{1}{3}z^3 \dots, z < 1$ | | | |
| 2.35 $1 - (z^2/2!) + (z^4/4!) \dots, \text{all } z$ | | | |
| 2.36 $1 + \frac{1}{2}z^2 - \frac{1}{8}z^4 \dots, z < 1$ | | | |
| 2.37 $z - \frac{1}{3}z^3 + \frac{2}{15}z^5 \dots, z < \pi/2$ | | | |
| 2.38 $-\frac{1}{2}i + \frac{1}{4}z + \frac{1}{8}iz^2 - \frac{1}{16}z^3 \dots, z < 2$ | | | |
| 2.39 $(z/9) - (z^3/9^2) + (z^5/9^3) \dots, z < 3$ | | | |
| 2.40 $1 + z + z^2 + z^3 \dots, z < 1$ | | | |
| 2.41 $1 + iz - z^2/2 - iz^3/3! + z^4/4! \dots, \text{all } z$ | | | |
| 2.42 $z + z^3/3! + z^5/5! \dots, \text{all } z$ | | | |
| 2.48 Yes, $z \neq 0$ | 2.49 No | 2.50 Yes, $z \neq 0$ | 2.51 Yes |
| 2.52 No | 2.53 Yes, $z \neq 0$ | 2.54 $-iz$ | 2.55 $-iz^3$ |
| 2.56 $-iz^2/2$ | 2.57 $(1 - i)z$ | 2.58 $\cos z$ | 2.59 e^z |
| 2.60 $2 \ln z$ | 2.61 $1/z$ | 2.62 $-ie^{iz}$ | 2.63 $-i/(1 - z)$ |

- 3.1 $\frac{1}{2} + i$ 3.2 $-(2+i)/3$ 3.3 0 3.4 $i\pi/2$
 3.5 -1 3.6 -1, -1 3.7 $\pi(1-i)/8$ 3.8 $i/2$
 3.9 1 3.10 $(2i-1)e^{2i}$ 3.11 $2\pi i$
 3.12 (a) $\frac{5}{3}(1+2i)$ (b) $\frac{1}{3}(8i+13)$ 3.16 0
 3.17 (a) 0 (b) $i\pi$ 3.18 $i\pi\sqrt{3}/6$ 3.19 $16i\pi$
 3.20 (a) 0 (b) $-17\pi i/4$ 3.22 $-i\pi\sqrt{3}/108$
 3.23 $72i\pi$ 3.24 $-17i\pi/96$
- 4.3 For $0 < |z| < 1$: $\frac{1}{2}z^{-1} + \frac{3}{4} + \frac{7}{8}z + \frac{15}{16}z^2 \dots$; $R(0) = \frac{1}{2}$
 For $1 < |z| < 2$: $-(\dots z^{-4} + z^{-3} + z^{-2} + \frac{1}{2}z^{-1} + \frac{1}{4} + \frac{1}{8}z + \frac{1}{16}z^2 + \frac{1}{32}z^3 \dots)$
 For $|z| > 2$: $z^{-3} + 3z^{-4} + 7z^{-5} + 15z^{-6} \dots$
 4.4 For $0 < |z| < 1$: $-\frac{1}{4}z^{-1} - \frac{1}{2} - \frac{11}{16}z - \frac{13}{16}z^2 \dots$; $R(0) = -\frac{1}{4}$
 For $1 < |z| < 2$: $\dots + z^{-3} + z^{-2} + \frac{3}{4}z^{-1} + \frac{1}{2} + \frac{5}{16}z + \frac{3}{16}z^2 \dots$
 For $|z| > 2$: $z^{-4} + 5z^{-5} + 17z^{-6} + 49z^{-7} \dots$
 4.5 For $0 < |z| < 2$: $\frac{1}{2}z^{-3} - \frac{1}{4}z^{-2} - \frac{1}{8}z^{-1} - \frac{1}{16} - \frac{1}{32}z - \frac{1}{64}z^2$; $R(0) = -\frac{1}{8}$
 For $|z| > 2$: $z^{-3} + z^{-4} + 2z^{-5} + 4z^{-6} + 8z^{-7} \dots$
 4.6 For $0 < |z| < 1$: $z^{-2} - 2z^{-1} + 3 - 4z + 5z^2 \dots$; $R(0) = -2$
 For $|z| > 1$: $z^{-4} - 2z^{-5} + 3z^{-6} \dots$
 4.7 For $|z| < 1$: $2 - z + 2z^2 - z^3 + 2z^4 - z^5 \dots$; $R(0) = 0$
 For $|z| > 1$: $z^{-1} - 2z^{-2} + z^{-3} - 2z^{-4} \dots$
 4.8 For $|z| < 1$: $-5 + \frac{25}{6}z - \frac{175}{36}z^2 \dots$; $R(0) = 0$
 For $1 < |z| < 2$: $-5(\dots + z^{-3} - z^{-2} + z^{-1} + \frac{1}{6}z + \frac{1}{36}z^2 + \frac{7}{216}z^3 \dots)$
 For $2 < |z| < 3$: $\dots + 3z^{-3} + 9z^{-2} - 3z^{-1} + 1 - \frac{1}{3}z + \frac{1}{9}z^2 - \frac{1}{27}z^3 \dots$
 For $|z| > 3$: $30(z^{-3} - 2z^{-4} + 9z^{-5} \dots)$
 4.9 (a) regular (b) pole of order 3
 (c) pole of order 2 (d) pole of order 1
 4.10 (a) simple pole (b) pole of order 2
 (c) pole of order 2 (d) essential singularity
 4.11 (a) regular (b) pole of order 2
 (c) simple pole (d) pole of order 3
 4.12 (a) pole of order 3 (b) pole of order 2
 (c) essential singularity (d) pole of order 1
- 6.1 $z^{-1} - 1 + z - z^2 \dots$; $R = 1$
 6.2 $(z-1)^{-1} - 1 + (z-1) - (z-1)^2 \dots$; $R = 1$
 6.3 $z^{-3} - \frac{1}{6}z^{-1} + \frac{1}{120}z \dots$; $R = -\frac{1}{6}$
 6.4 $z^{-2} + (1/2!) + (z^2/4!) \dots$; $R = 0$
 6.5 $\frac{1}{2}e[(z-1)^{-1} + \frac{1}{2} + \frac{1}{4}(z-1) \dots]$; $R = \frac{1}{2}e$
 6.6 $z^{-1} - (1/3!)z^{-3} + (1/5!)z^{-5} \dots$; $R = 1$
 6.7 $\frac{1}{4} \left[(z - \frac{1}{2})^{-1} - 1 + (1 - \pi^2/2)(z - \frac{1}{2}) + \dots \right]$, $R = \frac{1}{4}$
 6.8 $1/2 - (z - \pi)^2/4! + (z - \pi)^4/6! - \dots$, $R = 0$
 6.9 $-[(z-2)^{-1} + 1 + (z-2) + (z-2)^2 + \dots]$; $R = -1$
 6.14 $R(-2/3) = 1/8$, $R(2) = -1/8$ 6.15 $R(1/2) = 1/3$, $R(4/5) = -1/3$
 6.16 $R(0) = -2$, $R(1) = 1$ 6.17 $R(1/2) = 5/8$, $R(-1/2) = -3/8$
 6.18 $R(3i) = \frac{1}{2} - \frac{1}{3}i$ 6.19 $R(\pi/2) = 1/2$
 6.20 $R(i) = 1/4$ 6.21 $R[\sqrt{2}(1+i)] = \sqrt{2}(1-i)/16$
 6.22 $R(i\pi) = -1$ 6.23 $R(2i/3) = -ie^{-2/3}/12$
 6.24 $R(0) = 2$ 6.25 $R(0) = 2$
 6.26 $R(e^{2\pi i/3}) = \frac{1}{6}(i\sqrt{3} - 1)e^{-\pi\sqrt{3}}$ 6.27 $R(\pi/6) = -1/2$

- 6.28 $R(3i) = -\frac{1}{16} + \frac{1}{24}i$ 6.29 $R(\ln 2) = 4/3$
 6.30 $R(0) = 1/6!$ 6.31 $R(0) = 9/2$
 6.32 $R(2i) = -3ie^{-2}/32$ 6.33 $R(\pi) = -1/2$
 6.34 $R(0) = -7, R(1/2) = 7$ 6.35 $R(i) = 0$
 6.14' $\pi i/4$ 6.15' 0 6.16' $-2\pi i$ 6.17' $\pi i/2$
 6.18' 0 6.19' 0 6.20' 0 6.21' 0
 6.22' 0 6.23' $-\frac{\pi}{3} \sinh \frac{2}{3}$ 6.24' $4\pi i$ 6.25' $4\pi i$
 6.26' $-\frac{2}{3}\pi i(1 + \cosh \pi\sqrt{3} + i\sqrt{3} \sinh \pi\sqrt{3})$
 6.27' $-\pi i$ 6.28' $\frac{1}{4}\pi i$ 6.29' $5\pi i/2$ 6.30' $\pi i/360$
 6.31' $9\pi i$ 6.32' 0 6.33' 0 6.34' 0
 6.35' 0 6.36 $-\frac{1}{4}$ 6.37 $R(-n) = (-1)^n/n!$
- 7.1 $\pi/6$ 7.2 $\pi/2$ 7.3 $2\pi/3$ 7.4 $2\pi/9$
 7.5 $\pi/(1-r^2)$ 7.6 $2\pi/3^{3/2}$ 7.7 $\pi/6$ 7.8 $\pi/18$
 7.9 $2\pi/|\sin \alpha|$ 7.10 π 7.11 $3\pi/32$ 7.12 $\pi\sqrt{2}/8$
 7.13 $\pi/10$ 7.14 $-(\pi/e) \sin 2$ 7.15 $\pi e^{-4/3}/12$ 7.16 $\pi e^{-2/3}/18$
 7.17 $(\pi/e)(\cos 2 + 2 \sin 2)$ 7.18 $\frac{1}{2}\pi e^{-\pi\sqrt{3}/2}$ 7.19 $\pi e^{-3}/54$
 7.20 $\pi e^{-1/3}/9$ 7.22 $-\pi/2$ 7.23 $\pi/8$ 7.24 π
 7.25 $\pi/36$ 7.26 $-\pi/2$ 7.27 $\pi/4$ 7.28 $\pi/4$
 7.29 $\pi/2$ for $a > 0$, 0 for $a = 0$, $-\pi/2$ for $a < 0$
 7.30 $\pi/(2\sqrt{2})$ 7.31 $\pi/3$ 7.32 $\frac{3}{16}\pi\sqrt{2}$ 7.33 $\pi\sqrt{2}/2$
 7.34 $\pi/2$ 7.35 $2\pi(2^{1/3} - 1)/\sqrt{3}$ 7.36 $-\pi^2\sqrt{2}$
 7.38 $\pi \cot p\pi$ 7.39 2 7.40 $\pi^2/4$ 7.41 $(2\pi)^{1/2}/4$
- 7.45 One negative real, one each in quadrants I and IV
 7.46 One negative real, one each in quadrants II and III
 7.47 One negative real, one each in quadrants I and IV
 7.48 Two each in quadrants I and IV
 7.49 Two each in quadrants I and IV
 7.50 Two each in quadrants II and III
 7.51 $4\pi i, 8\pi i$ 7.52 πi
 7.53 πi 7.54 $8\pi i$
 7.55 $\cosh t \cos t$ 7.56 $(\sinh t - \sin t)/2$
 7.57 $1 + \sin t - \cos t$ 7.58 $(\cos 2t + \cosh 2t)/2$
 7.59 $2e^t \cos t\sqrt{3} + e^{-2t}$ 7.60 $t + e^{-t} - 1$
 7.61 $\frac{1}{3}(\cosh 2t + 2 \cosh t \cos t\sqrt{3})$ 7.62 $1 - 4te^{-t}$
 7.63 $(\cosh t - \cos t)/2$ 7.64 $\frac{2}{3} \sinh 2t - \frac{1}{3} \sinh t$
 7.65 $(\cos 2t + 2 \sin 2t - e^{-t})/5$
- 8.3 Regular, $R = -1$ 8.4 Regular, $R = -2$
 8.5 Regular, $R = -1$ 8.6 Simple pole, $R = -5$
 8.7 Simple pole, $R = -2$ 8.8 Regular, $R = 0$
 8.9 Regular, $R = 0$ 8.10 Regular, $R = 2$
 8.11 Regular, $R = -1$ 8.12 Regular, $R = -2$
 8.14 $-2\pi i$ 8.15 πi
- 9.1 $x^2 = \frac{1}{2}[u + (u^2 + v^2)^{1/2}], y^2 = \frac{1}{2}[-u + (u^2 + v^2)^{1/2}]$
 9.2 $u = y/2, v = -(x+1)/2$
 9.3 $u = x/(x^2 + y^2), v = -y/(x^2 + y^2)$
 9.4 $u = e^x \cos y, v = e^x \sin y$
 9.5 $u = (x^2 + y^2 - 1)/[x^2 + (y+1)^2], v = -2x/[x^2 + (y+1)^2]$
 9.7 $u = \sin x \cosh y, v = \cos x \sinh y$
 9.8 $u = \cosh x \cos y, v = \sinh x \sin y$

- 10.4 $T = 200\pi^{-1} \arctan(y/x)$ 10.5 $V = 200\pi^{-1} \arctan(y/x)$
 10.6 $T = 100y/(x^2 + y^2)$; isothermals $y/(x^2 + y^2) = \text{const.}$;
 flow lines $x/(x^2 + y^2) = \text{const.}$
 10.7 Streamlines $xy = \text{const.}$; $\Phi = (x^2 - y^2)V_0$, $\Psi = 2xyV_0$, $\mathbf{V} = (2\mathbf{i}x - 2\mathbf{j}y)V_0$
 10.9 Streamlines $y - y/(x^2 + y^2) = \text{const.}$
 10.10 $\cos x \sinh y = \text{const.}$
 10.11 $(x - \coth u)^2 + y^2 = \text{csch}^2 u$
 $x^2 + (y + \cot v)^2 = \text{csc}^2 v$
 10.12 $T = (20/\pi) \arctan[2y/(1 - x^2 - y^2)]$, \arctan between $\pi/2$ and $3\pi/2$
 10.13 $V = \frac{V_2 - V_1}{\pi} \arctan \frac{2y}{1 - x^2 - y^2} + \frac{3V_1 - V_2}{2}$, \arctan between $\pi/2$ and $3\pi/2$
 10.14 $\phi = \frac{1}{2}V_0 \ln \frac{(x+1)^2 + y^2}{(x-1)^2 + y^2}$
 $\psi = V_0 \arctan \frac{2y}{1 - x^2 - y^2}$, \arctan between $\pi/2$ and $3\pi/2$.
 $V_x = \frac{2V_0(1 - x^2 + y^2)}{(1 - x^2 + y^2)^2 + 4x^2y^2}$, $V_y = \frac{-4V_0xy}{(1 - x^2 + y^2)^2 + 4x^2y^2}$
- 11.1 $\ln(1+z)$ 11.2 $-i \ln(1+z)$
 11.5 $R(i) = (1 - i\sqrt{3})/4$ 11.6 $R(-1/2) = i/(6\sqrt{2})$
 $R(-i) = -1/2$ $R(e^{i\pi/3}/2) = R(e^{5\pi i/3}/2) = -i/(6\sqrt{2})$
 11.7 $R(i) = \pi/4$, $R(-i) = R(e^{3\pi i/2}) = -3\pi/4$
 11.8 $R(1/2) = 1/2$ 11.9 $-1/6$
 11.10 -1 11.12 $1/2$
 11.13 (a) $1/96$ (b) -5 (c) $-1/80$ (d) $1/2$
 11.14 (a) 2 (b) $-\sin 5$ (c) $1/16$ (d) -2π
 11.15 $\pi/6$ 11.16 $-\pi/6$
 11.17 $\pi(e^{-1/2} - \frac{1}{6}e^{-3})/35$ 11.18 $\pi e^{-\pi/2}/4$
 11.19 $3(2^{-1} - e^{-\pi})/(10\pi)$ 11.20 $\pi(e^{-1} + \sin 1)/2$
 11.28 π 11.29 $\pi^3/8$ (Caution: $-\pi^3/8$ is wrong.)
 11.31 One in each quadrant
 11.32 One negative real, one each in quadrants II and III
 11.33 One each in quadrants I and IV, two each in II and III
 11.34 Two each in quadrants I and IV, one each in II and III
 11.40 $\frac{2a^2p}{p^4 + 4a^4}$ 11.41 $\pi^2/8$

Chapter 15

- 1.1 $1/10, 1/9$ 1.2 $3/8, 1/8, 1/4$
 1.3 $1/3, 5/9$ 1.4 $1/2, 1/52, 2/13, 7/13$
 1.5 $1/4, 3/4, 1/3, 1/2$ 1.6 $27/52, 16/52, 15/52$
 1.7 $9/26, 1/2, 1/13$ 1.8 $9/100, 1/10, 3/100, 1/10$
 1.9 $3/10, 1/3$ 1.10 $3/8$
- 2.12 (a) $3/4$ (b) $1/5$ (c) $2/3$ (d) $3/4$ (e) $3/7$
 2.14 (a) $3/4$ (b) $25/36$ (c) $37, 38, 39, 40$
 2.15 (a) $1/6$ (b) $1/2$ (c) $1/3$ (d) $1/3$ (e) $1/9$
 2.17 (a) 3 to 9 with $p(5) = p(7) = 2/9$; others, $p = 1/9$
 (b) 5 and 7 (c) $1/3$
 2.18 (a) $1/2, 1/2$ (b) $1/2, 1/4, 1/4$ (c) Not a sample space
 2.19 $1/3, 1/3; 1/7, 1/7$
- 3.3 $2^{-6}, 2^{-3}, 2^{-3}$
 3.4 (a) $8/9, 1/2$ (b) $3/5, 1/11, 2/3, 2/3, 6/13$
 3.5 $1/33, 2/9$
 3.6 $4/13, 1/52$
 3.10 (a) $1/6$ (b) $2/3$ (c) $P(A) = P(B) = 1/3, P(A + B) = 1/2, P(AB) = 1/6$
 3.11 $1/8$
 3.12 (a) $1/49$ (b) $68/441$ (c) $25/169$ (d) 15 times (e) $44/147$
 3.13 (a) $1/4$ (b) $25/144, 1/16, 1/16$
 3.14 $n > 3.3$, so 4 tries are needed.
 3.15 (a) $1/3$ (b) $1/7$
 3.16 $9/23$
 3.17 (a) $39/80, 5/16, 1/5, 11/16$ (b) $374/819$ (c) $185/374$
 3.18 (a) $15/34$ (b) $2/15$
 3.19 $1/3$
 3.20 $5/7, 2/7, 11/14$
 3.21 $2/3, 1/3$
 3.22 $6/11, 5/11$
- 4.1 (a) $P(10, 8)$ (b) $C(10, 8)$ (c) $1/45$
 4.3 $3, 7, 31, 2^n - 1$
 4.4 $1.98 \times 10^{-3}, 4.95 \times 10^{-4}, 3.05 \times 10^{-4}, 1.39 \times 10^{-5}$
 4.5 $2^8, 2^{-8}, 7/32$ 4.6 15
 4.7 $1/26$ 4.8 $1/221, 1/33, 1/17$

- 4.9 25/102, 25/77, 49/101, 12/25 4.11 0.097, 0.37, 0.67; 13
 4.12 5 4.14 $n!/n^n$
 4.17 MB: 16, FD: 6, BE: 10 4.18 MB: 125, FD: 10, BE: 35
 4.21 $C(n+2, n)$ 4.22 0.135 4.23 0.30
- 5.1 $\mu = 0, \sigma = \sqrt{3}$ 5.2 $\mu = 7, \sigma = \sqrt{35/6}$
 5.3 $\mu = 2, \sigma = \sqrt{2}$ 5.4 $\mu = 1, \sigma = \sqrt{21/2}$
 5.5 $\mu = 1, \sigma = \sqrt{7/6}$ 5.6 $\mu = 3, \sigma = \sqrt{284/13} = 4.67$
 5.7 $\mu = 3(2p-1), \sigma = 2\sqrt{3p(1-p)}$ 5.8 $E(x) = \$12.25$
 5.12 $E(x) = 7$ 5.15 $\bar{x} = 3(2p-1)$
 5.17 Problem 5.2: $E(x^2) = 329/6, \sigma^2 = 35/6$
 Problem 5.6: $E(x^2) = 401/13, \sigma^2 = 284/13$
 Problem 5.7: $E(x^2) = 24p^2 - 24p + 9, \sigma^2 = 12p(1-p)$
- 6.1 (a) $f(x) = \pi^{-1}(a^2 - x^2)^{-1/2}$ (c) $\bar{x} = 0, \sigma = a/\sqrt{2}$
 6.2 $e^{-2} = 0.135$
 6.3 $f(h) = 1/(2\sqrt{l}\sqrt{l-h})$
 6.4 $f(x) = \alpha e^{-\alpha^2 x^2}/\sqrt{\pi}, \bar{x} = 0, \sigma = 1/(\alpha\sqrt{2})$
 6.5 $f(t) = \lambda e^{-\lambda t}, F(t) = 1 - e^{-\lambda t}, \bar{t} = 1/\lambda, \text{ half life} = \bar{t} \ln 2$
 6.6 $F(r) = r^2, f(r) = 2r, \bar{r} = 2/3, \sigma = \sqrt{2}/6$
 6.7 (a) $F(s) = 2[1 - \cos(s/R)], f(s) = (2/R) \sin(s/R)$
 (b) $F(s) = [1 - \cos(s/R)]/[1 - \cos(1/R)] \cong s^2,$
 $f(s) = R^{-1}[1 - \cos(1/R)]^{-1} \sin(s/R) \cong 2s$
 6.8 $f(r) = 3r^2; \bar{r} = 3/4, \sigma = \sqrt{3/80} = 0.19$
 6.9 $f(r) = 4a^{-3}r^2 e^{-2r/a}$

	n	Exactly 7 h	At most 7 h	At least 7 h	Most probable number of h	Expected number of h
7.1	7	0.0078	1	0.0078	3 or 4	7/2
7.2	12	0.193	0.806	0.387	6	6
7.3	15	0.196	0.500	0.696	7 or 8	15/2
7.4	18	0.121	0.240	0.881	9	9

7.5 0.263

8.3 $\mu = 0, \sigma^2 = kT/m, f(v) = \frac{1}{\sqrt{2\pi kT/m}} e^{-mv^2/(2kT)}$

In (8.11) to (8.20), the first number is the binomial result and the second number is the normal approximation using whole steps at the ends as in Example 2.

- 8.11 0.0796, 0.0798 8.12 0.03987, 0.03989
 8.13 0.9598, 0.9596 8.14 0.9546, 0.9546
 8.15 0.03520, 0.03521 8.16 0.4176, 0.4177
 8.17 0.0770, 0.0782 8.18 0.372, 0.376
 8.19 0.0946, 0.0967 8.20 0.462, 0.455
 8.25 C: 38.3%, B and D: 24.2%, A and F: 6.7%
 In $\mu + \frac{1}{2}\sigma$ and $\mu + \frac{3}{2}\sigma$, change $\frac{1}{2}$ to 0.5244, and $\frac{3}{2}$ to 1.2816.

- 9.3 Number of particles: 0 1 2 3 4 5
 Number of intervals: 406 812 812 541 271 108
- 9.4 $P_0 = 0.018, P_1 = 0.073, P_4 = 0.195$
- 9.5 $P_0 = 0.37, P_1 = 0.37, P_2 = 0.18, P_3 = 0.06$
- 9.6 Exactly 5: 64 days. Fewer than 5: 161 days. Exactly 10: 7 days. More than 10: 5 days. Just 1: 12 days. None at all: 2 or 3 days
- 9.7 0.238 9.8 3, 10, 3
- 9.9 $P_2 = 0.022, P_6 = P_7 = 0.149, P_{n>10} = 0.099$
- 9.11 Normal: 0.08, Poisson: 0.0729, (binomial: 0.0732)
- 10.8 $\bar{x} = 5, \bar{y} = 1, s_x = 0.122, s_y = 0.029,$
 $\sigma_x = 0.131, \sigma_y = 0.030, \sigma_{mx} = 0.046, \sigma_{my} = 0.0095,$
 $r_x = 0.031, r_y = 0.0064,$
 $\overline{x+y} = 6$ with $r = 0.03, \overline{xy} = 5$ with $r = 0.04,$
 $\overline{x^3 \sin y} = 105$ with $r = 2.00, \overline{\ln x} = 1.61$ with $r = 0.006$
- 10.9 $\bar{x} = 100$ with $r = 0.47, \bar{y} = 20$ with $r = 0.23,$
 $\overline{x-y} = 80$ with $r = 0.5, \overline{x/y} = 5$ with $r = 0.06,$
 $\overline{x^2 y^3} = 8 \cdot 10^7$ with $r = 2.9 \cdot 10^6, \overline{y \ln x} = 92$ with $r = 1$
- 10.10 $\bar{x} = 6$ with $r = 0.062, \bar{y} = 3$ with $r = 0.067,$
 $\overline{2x-y} = 9$ with $r = 0.14, \overline{y^2 - x} = 3$ with $r = 0.4,$
 $\overline{e^y} = 20$ with $r = 1.3, \overline{x/y^2} = 0.67$ with $r = 0.03$
- 11.1 (a) 11/30 (b) 19.5 cents (c) 6/11 (d) 7/11
- 11.2 (b) $E(x) = 5, \sigma = \sqrt{3}$ (c) 0.0767 (d) 0.0807 (e) 0.0724
- 11.3 20/47
- 11.4 5/8
- 11.6 MB: 25 FD: 10 BE: 15
- 11.7 $\bar{x} = 1/4, \sigma = \sqrt{3}/4$
- 11.8 (b) $\bar{x} = 4/3, \sigma = 2/3$ (c) 1/5
- 11.9 (a) $x:$ 0 1 2
 $p: 55/72 = 0.764 \quad 16/72 = 0.222 \quad 1/72 = 0.139$
 (b) $17/72 = 0.236$
 (c) $6/17 = 0.353$
 (d) $\bar{x} = 1/4, \sigma = \sqrt{31}/12 = 0.463$
- 11.10 (a) 0.7979, 0.7979 (b) 0.9123, 0.9123
- 11.11 (a) 0.0347, 0.0352 (b) 0.559, 0.562
- 11.12 (a) 0.00534, 0.00540 (b) 0.503, 0.500
- 11.13 30, 60 11.14 1
- 11.15 binomial: 0.2241, normal: 0.195, Poisson: 0.2240
- 11.16 (a) binomial: 0.0439, normal: 0.0457, Poisson: 0.0446
 (b) binomial: 0.0946, normal: 0.0967, Poisson: 0.0846
- 11.17 $\bar{x} = 2$ with $r = 0.073, \bar{y} = 1$ with $r = 0.039, \overline{x-y} = 1$ with $r = 0.08,$
 $\overline{xy} = 2$ with $r = 0.11, \overline{x/y^3} = 2$ with $r = 0.25$
- 11.18 $\bar{x} = 5$ with $r = 0.134, \bar{y} = 60$ with $r = 0.335, \overline{x+y} = 65$ with $r = 0.36,$
 $\overline{y/x} = 12$ with $r = 0.33, \overline{x^2} = 25$ with $r = 1.3$