Quadratic Lattice Compression: Lossless State Reconstruction from Minimal Coefficient Storage

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Abstract

We present a compression scheme for dynamical lattice systems in which the complete state of each cell is encoded as three scalar coefficients (a, b, c) of a quadratic operator $O(x) = ax^2 + bx + c$. From these three numbers, the reader deterministically reconstructs the discriminant, root structure, iterate-based band classification (7 classes derived from 28-step orbit analysis), fixed point locations and stability eigenvalues, neighborhood topology weights via root proximity, and full cobweb visualization paths. We demonstrate 100% fidelity round-trip at both Float32 (4 bytes/coefficient) and Float64 (8 bytes/coefficient) precision on a 252-cell lattice. At Float32, the collapsed state occupies 3,076 bytes, yielding a 22x compression ratio over flat storage of equivalent derived quantities. The decompressor is mathematically identical to the physics engine itself, requiring only the 20-line quadratic class definition. We show that 479 complete lattice snapshots fit on a standard 1.44 MB floppy disk, and a single floppy can store a lattice of 122,880 cells. This work establishes that quadratic coefficient storage constitutes a natural lossless codec for systems whose dynamics are governed by second-order polynomial operators.

Keywords: *lossless compression, quadratic operators, lattice dynamics, deterministic reconstruction, discriminant classification, dynamical systems codec*

1. Introduction

State compression in computational lattice systems typically relies on general-purpose algorithms (gzip, LZ4, Zstandard) that treat the data as an opaque byte stream. These achieve compression ratios of 2x-10x on floating-point simulation data but produce compressed representations that cannot be queried, classified, or analyzed without full decompression. The decompressor has no knowledge of the underlying physics.

We observe that in systems governed by quadratic operators $O(x) = ax^2 + bx + c$, the entire dynamical state of each lattice cell is *determined* by the three coefficients (a, b, c). Every derived quantity—the discriminant $\Delta = b^2 - 4ac$, root structure, iterate orbit classification, fixed point stability, and inter-cell topology—is a pure function of these coefficients. Storing only (a, b, c) therefore constitutes a lossless codec in which the decompressor is the mathematical operator itself.

This paper makes three contributions: (1) We define the quadratic lattice codec formally and enumerate the 15+ quantities reconstructed from 3 stored values. (2) We measure compression ratio, fidelity, and Float32 degradation on a 252-cell lattice after 500 ticks of spine-driven coefficient evolution. (3) We demonstrate that the reconstruction process is computationally cheaper than storing and loading the equivalent derived state, because classification and topology emerge from the same operator used for simulation.

2. The Quadratic Operator as Universal State Object

$$O(x) = ax^2 + bx + c$$

The operator O encapsulates all physics in a single algebraic object. The following quantities are deterministically computable from (a, b, c) with no additional stored state:

#	Quantity	Formula	Bytes if Stored
1	Discriminant Δ	$b^2 - 4ac$	8

2	Root type	sign(Δ)	1
3-4	Roots r1, r2	(–b ± √Δ) / 2a	16
5-6	Vertex (vx, vy)	(–b/2a, O(vx))	16
7	Fixed point x*	ax*² + (b–1)x* + c = 0	8
8	Stability λ	|O'(x*)| = |2ax* + b|	8
9	Stable?	λ < 1	1
10	Band (7-class)	28-iterate orbit analysis	4
11	Orbit (28 values)	O(O(O(...x0)))	224
12	Curvature	O''(x)/2 = a	8
13	O'(x)	2ax + b	8
14	Neighbor weights (8)	rootDist(Oi, Oj)	64
15	Cobweb path	Interleaved (x, O(x)) pairs	~288

Table 1: Quantities reconstructed from (a, b, c). Total if stored independently: ~654 bytes/cell.

The stored representation requires 12 bytes per cell (3 × Float32) or 24 bytes (3 × Float64). The flat representation storing all derived quantities requires approximately 654 bytes per cell. This yields a compression ratio of 654/12 = **54.5x at Float32** or 654/24 = **27.3x at Float64** against full derived-state storage. Against minimal derived storage (omitting orbit and cobweb), the ratio is 273/12 = **22.8x**.

3. Iterate-Based Band Classification

Band classification follows a Mandelbrot-style iterate analysis. Starting from $x_0 = 0.5$, we compute $x_{n+1} = O(x_n)$ for 28 iterations and classify the orbit:

Band	Name	Classification Rule	Initial %
0	VOID	Escape in < 3 iterations	0.0%
1	QUANTUM	Escape in 3–9 iterations	23.0%
2	ATOMIC	Escape in 9+ iterations	10.3%
3	MOLECULAR	Bounded, non-convergent (chaotic)	13.5%
4	CELLULAR	Periodic orbit (period ≥ 2)	7.5%
5	ORGANIC	Slow convergence to fixed point	11.1%
6	CRYSTAL	Fast convergence (< 10 iterations)	34.5%

Table 2: Seven-band classification by iterate dynamics. Percentages from initial 252-cell lattice.

Critically, band classification is not a lookup table on Δ. A cell with Δ > 0 can be any band from VOID to CRYSTAL depending on the specific values of (a, b, c) and the resulting iterate dynamics. The classification is a *functional* property of O(x), not a direct consequence of any single derived quantity.

4. Root-Proximity Topology

Inter-cell connectivity is weighted by root proximity rather than grid adjacency. For cells i and j with operators O_i and O_j, the weight is:

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w(i,j) = 1 / (1 + rootDist(O_i, O_j) * 0.5)
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where rootDist compares root sets: minimum pairwise distance for real-real pairs, center-plus-imaginary distance for complex-complex pairs, and vertex distance for mixed pairs. This topology is fully determined by (a, b, c) and requires no stored adjacency matrix.

Measurement: In 99.6% of cells (251/252), root-priority neighbor ordering differs from grid-priority ordering. The topology is emergent from the operator, not imposed by the spatial grid. After collapse and expansion, topology reconstruction achieves **100% fidelity**—the same neighbor ordering is recovered in every cell.

5. Experimental Results

5.1 Round-Trip Fidelity

We evolved a 252-cell lattice for 500 ticks under deterministic spine transforms (CHAOS noise disabled for reproducibility). After evolution, we collapsed the lattice to (a, b, c) per cell, then expanded by recomputing all derived quantities from coefficients alone.

Quantity	Float64 Fidelity	Float32 Fidelity
Discriminant Δ	252/252 (100.0%)	252/252 (100.0%)
Roots	252/252 (100.0%)	252/252 (100.0%)
Band classification	252/252 (100.0%)	252/252 (100.0%)
Fixed points	252/252 (100.0%)	252/252 (100.0%)
Orbit convergence	252/252 (100.0%)	252/252 (100.0%)
Topology (top-3 neighbors)	252/252 (100.0%)	252/252 (100.0%)

Table 3: Round-trip fidelity. All quantities reconstruct perfectly at both precisions.

5.2 Storage Capacity

Format	Per Snapshot	Floppy (1.44 MB)	Max Cells/Floppy
Float32	3,076 bytes	479 snapshots	122,880
Float64	6,152 bytes	239 snapshots	61,440
Flat (minimal derived)	~68,796 bytes	21 snapshots	5,440
Flat (full derived)	~164,808 bytes	8 snapshots	2,267

Table 4: Storage comparison on 1.44 MB medium. Codec achieves 22–54x capacity gain.

5.3 Reconstruction Cost

Reconstruction of all derived quantities for 252 cells takes < 1ms on commodity hardware (measured within the 53,476 tick/sec benchmark that includes full reclassify and rewire passes). The reconstruction cost is amortized because the same operations (classify, wireNeighbors) are required during simulation. Loading collapsed state and reconstructing is therefore *faster* than deserializing pre-computed derived state, because the codec expansion path executes fewer memory allocations and no format parsing.

6. Comparison with General-Purpose Compression

General-purpose compressors (gzip, LZ4, Zstandard) applied to the flat derived-state representation achieve 3–10x compression, reducing the 68,796-byte flat state to approximately 7,000–23,000 bytes. However, the compressed output is opaque: querying any derived quantity requires full decompression. The quadratic codec at 3,076 bytes

(Float32) outperforms even the best general-purpose compression while maintaining the property that any derived quantity can be computed on-demand from the stored representation without a decompression pass. The codec and the query engine are the same 20 lines of code.

7. Discussion

7.1 The Decompressor Is the Physics Engine

The central insight is that $O(x) = ax^2 + bx + c$ serves simultaneously as: (1) the simulation operator that evolves cell state, (2) the classification function that determines band membership, (3) the topology generator that defines inter-cell connectivity, and (4) the codec that compresses and reconstructs state. No other compression scheme shares its decompressor with its physics engine. This identity—codec = engine—is the reason the compression is lossless: there is no encoding/decoding boundary where information could be lost.

7.2 Generalization

The approach generalizes to any lattice system where cell state is governed by a parameterized operator family. Cubic operators (4 coefficients) would yield higher-order dynamics with similar codec properties. The key requirement is that all derived quantities are deterministic functions of the stored parameters, with no hidden state or history dependence in the classification algorithm.

7.3 Limitations

The codec does not preserve transient quantities such as visit counts, coherence scores accumulated during bug traversal, or the bug's path history. These are session-specific metadata rather than intrinsic cell state. The codec stores the lattice's *constitution*—what each cell is—not its *history*—what has happened to it. For applications requiring full session replay, the collapsed state would need to be supplemented with an event log.

8. Conclusion

We have demonstrated that quadratic operator lattices admit a natural lossless codec in which three coefficients per cell encode the complete dynamical state. The codec achieves 22–54x compression over flat storage with 100% round-trip fidelity at both Float32 and Float64. The decompressor is mathematically identical to the physics engine, eliminating the encoding/decoding boundary entirely. 479 complete 252-cell lattice snapshots fit on a 1.44 MB floppy disk. A single floppy can store a lattice of 122,880 cells. The quadratic is the codec.

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