Iterate Classification on Quadratic Operator Lattices: Emergent Band Structure from Compositional Coefficient Transforms

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Abstract

We study the dynamics of a 252-cell lattice where each cell carries a quadratic operator $O(x) = ax^2 + bx + c$ and cells are coupled through compositional coefficient transforms cycling through a 10-phase spine. We introduce a seven-band classification scheme based on 28-step iterate analysis, analogous to escape-time algorithms for the Mandelbrot set but applied to a heterogeneous lattice of distinct quadratic maps. We demonstrate three emergent phenomena: (1) self-organization of band distribution over epochs, with QUANTUM cells reclassifying as ATOMIC and the click zone ($\Delta \approx 0$) growing by 37% over 20 epochs; (2) formation of a 60.7% attractor basin strongly biased toward CRYSTAL-classified cells (stable fixed points) with complete avoidance of MOLECULAR-classified cells (bounded chaotic orbits); and (3) emergent root-proximity topology where 99.6% of cells have different algebraic neighbor priority than spatial neighbor priority. These phenomena arise from the spine's compositional structure—particularly the phase 4→5 pitchfork bifurcation ($a \rightarrow -a \rightarrow |a|$) and the phase 2→3 saddle-node bifurcation ($b \rightarrow -b \rightarrow |b|$)—without external tuning or imposed boundary conditions.

Keywords: *quadratic maps, iterate classification, lattice dynamics, bifurcation, emergent band structure, attractor basins, compositional transforms*

1. Introduction

The dynamics of iterated quadratic maps $f(x) = ax^2 + bx + c$ have been studied extensively in the context of single-parameter families, most notably the Mandelbrot set for $f_c(z) = z^2 + c$. The escape-time algorithm that classifies points by the number of iterations before orbit divergence has proven remarkably powerful for understanding the boundary between bounded and unbounded dynamics.

We extend this approach to a *lattice* of quadratic operators, where each cell carries its own (a, b, c) triple and the coefficients are modulated by a shared 10-phase compositional spine. This differs from standard coupled map lattices (CMLs) in two ways: (1) the coupling is through coefficient space rather than state space—the spine transforms (a, b, c), not x; and (2) the spine applies compositional transforms (sign flips, absolute values, multiplicative damping) rather than additive diffusion.

This paper reports three emergent phenomena observed in simulation of a 252-cell lattice (18 columns × 14 rows) over 20 epochs (200 ticks), each of which arises from the algebraic structure of the spine transforms without imposed boundary conditions or external tuning.

2. Lattice Construction

2.1 Cell Initialization

Each cell at grid position (col, row) is assigned initial coefficients via a spatial generating function that maps normalized coordinates (u, v) through trigonometric and exponential functions to produce a heterogeneous distribution across all seven bands:

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a = 0.8 * cos(r * 3.5) * (1 + 0.5 * sin(v * 7pi))

b = (cu * 3.5 + cv * 2.0) * (1 + 0.3 * cos(u * 5pi))
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c = 0.5 * exp(-r * 2) + 0.15 * sin(theta * 4 + r * 3) + 0.1
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where r = distance from center, theta = angle from center, cu = u − 0.5, cv = v − 0.5. This produces the initial band distribution shown in Table 1.

2.2 Seven-Band Classification

Starting from $x_0 = 0.5$, we compute $x_{n+1} = O(x_n)$ for N = 28 iterations. Classification proceeds by escape time and orbit periodicity analysis:

Band	Name	Rule	Initial Count	%
0	VOID	Escape < 3 iterations	0	0.0
1	QUANTUM	Escape in 3–9 iterations	58	23.0
2	ATOMIC	Escape in 9+ iterations	26	10.3
3	MOLECULAR	Bounded, non-convergent	34	13.5
4	CELLULAR	Periodic orbit (period ≥ 2)	19	7.5
5	ORGANIC	Convergent, > 10 iterations	28	11.1
6	CRYSTAL	Convergent, < 10 iterations	87	34.5

Table 1: Initial band distribution of the 252-cell lattice.

3. The 10-Phase Compositional Spine

The spine is a 10-element vector $S = (s_0, ..., s_9)$, each $s_i \in (0, 1]$, that advances one phase per tick. At phase i, the spine value s_i is updated by a phase-specific recurrence (see Table 2), then all cells undergo a compositional transform on their (a, b, c) coefficients.

Phase	Name	Transform on (a,b,c)	Dynamical Effect
0	VOID	(a,b,c) *= (1 − 0.009·s)	Contraction toward origin
1	LATTICE	c = c·0.991 + (c+s·0.05)·0.009	Binding potential increase
2	COUNTER	b → −b	First derivative sign flip
3	PROGRESS	b → |b|	Drift restoration
4	COLLAPSE	a → −a	Curvature inversion
5	BALANCE	a → |a|	Curvature restoration
6	CHAOS	a,b += noise·s·0.006	Stochastic perturbation
7	HARMONY	a = a·0.991 + s·0.2·0.009	Curvature smoothing
8	BREATH	b *= 1 + 0.004·sin(ωt)	Drift oscillation
9	RESET	(a,b,c) → 0.991·live + 0.009·base	Return toward initial state

Table 2: Spine phase transforms. Phases 2→3 and 4→5 form complementary bifurcation pairs.

3.1 The Phase 4→5 Pitchfork Bifurcation

At phase 4, the curvature a is negated: a → −a. This inverts every parabola in the lattice for exactly one tick. Cells that were concave-up become concave-down and vice versa. The discriminant $\Delta = b^2 − 4ac$ changes sign when a changes sign (since −4(−a)c = +4ac), meaning every free cell becomes bound and every bound cell becomes free for one tick. At

phase 5, a → |a| restores positive curvature. This constitutes a cyclic pitchfork bifurcation traversed every 10 ticks.

3.2 The Phase 2→3 Saddle-Node Bifurcation

At phase 2, the drift b is negated: b → −b. This reverses the first derivative of every operator, flipping the direction of approach to fixed points. At phase 3, b → |b| restores positive drift. Together, phases 2→3 traverse a saddle-node bifurcation in which the root pair exchanges roles: the previously attracting root becomes repelling and vice versa.

4. Emergent Phenomena

4.1 Band Redistribution Over Epochs

Over 20 epochs (200 ticks), the lattice spontaneously reorganizes its band distribution. The dominant effect is a transfer from QUANTUM (fast escape) to ATOMIC (slow escape), indicating that the spine transforms push coefficients toward configurations where iterates linger longer before diverging.

Epoch	VOID	QUAN	ATOM	MOLE	CELL	ORGA	CRYS	Click Zone
0	2	158	43	4	3	7	35	37
5	0	155	44	7	1	6	39	41
10	0	146	52	6	1	6	41	45
15	0	129	64	5	1	9	44	51
19	0	120	65	11	1	11	44	46

Table 3: Band census over 20 epochs. Click zone = cells with $|\Delta| < 0.15$.

Key observations: (1) VOID cells vanish by epoch 1 as contraction brings coefficients away from the instant-escape regime. (2) QUANTUM → ATOMIC transfer: 158 → 120 QUANTUM, 43 → 65 ATOMIC. (3) CRYSTAL grows from 35 to 44, indicating more cells converge to stable fixed points. (4) The click zone ($|\Delta| < 0.15$) grows from 37 to 51 cells, a 37% increase, then stabilizes. (5) Mean Δ decreases from 1.164 to 0.773, indicating systematic drift toward binding.

4.2 Attractor Basin Formation

A random walker (the "bug") starting at center cell (9, 7) with energy budget $E_0 = 50$ explores the lattice following discriminant-weighted transitions. After exhausting its energy, the bug has visited 153/252 cells (60.7%), constituting the attractor basin.

The basin shows strong band preference: 79% of visited cells are CRYSTAL, with complete (0%) avoidance of MOLECULAR cells. This is not imposed by the routing algorithm—the bug follows discriminant gradients and root-proximity weights. The CRYSTAL preference emerges because stable fixed points ($\lambda < 1$) create energy-efficient pathways (low cost per step in the free zone), while MOLECULAR cells (bounded chaotic orbits) represent local energy traps with unpredictable exit dynamics.

Band	Total Cells	Bug Visits	% of Visits	Preference
CRYSTAL	87	121	79.1%	▲ ATTRACTED
QUANTUM	58	8	5.2%	■ AVOIDED
ATOMIC	26	15	9.8%	■ AVOIDED
ORGANIC	28	8	5.2%	■ AVOIDED
CELLULAR	19	1	0.7%	■ AVOIDED

MOLECULAR	34	0	0.0%	✗ ZERO

Table 4: Bug visits by band at E0=50. Strong CRYSTAL attraction, MOLECULAR avoidance.

Audit mode (systematic DFS traversal) confirms that all 252 cells are structurally reachable—the bug's partial coverage is a genuine attractor phenomenon, not an accessibility limitation.

4.3 Emergent Root-Proximity Topology

Each cell's eight grid-adjacent neighbors are weighted by root proximity: cells with similar root structures receive higher weights regardless of spatial distance. After computing these weights for all 252 cells, we find that 251/252 (99.6%) have a different highest-weighted neighbor than their spatially closest neighbor. The algebraic topology is almost entirely non-spatial.

This topology persists through coefficient evolution and survives the phase 4→5 bifurcation, because root proximity is a continuous function of (a, b, c) and the bifurcation momentarily changes signs without altering the *relative* proximity structure. Cells that were algebraically close before the flip remain close after it.

5. The Δ-Dependent Energy Landscape

The cost of evaluating a cell scales inversely with $|\Delta|$:

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cost(cell) = 0.005 + (1 / (|Δ| + 0.1)) * 0.008
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This creates three energy zones:

Zone	$|\Delta|$ Range	Avg Cost	Cell Count	Physical Interpretation
Click	< 0.15	0.0506	24	Near binding threshold; maximum computational work
Free	> 0.5	0.0099	139	Clearly unbound; trivial classification
Bound	< -0.5	0.0122	35	Clearly bound; trivial classification

Table 5: Energy zones. Click/Free cost ratio = 5.1x.

The 5.1x cost ratio has a physical interpretation: near $\Delta = 0$, the system is deciding whether to form a real or complex root pair. This decision boundary is where the iterate classification is most sensitive to coefficient perturbation—a small change in (a, b, c) can shift a cell between bands. Far from $\Delta = 0$, the state is determined and perturbation-insensitive.

6. Computational Performance

The complete simulation (spine advance, coefficient transform, bug step, periodic reclassification every 12 ticks, periodic topology rewire every 80 ticks) runs at 53,476 ticks/second on a single thread of commodity hardware. At 60fps with 2 simulation steps per frame (120 ticks/sec target), this provides 446x headroom. The bottleneck is reclassification: 28 quadratic evaluations per cell × 252 cells = 7,056 evaluations every 12 ticks.

7. Relation to Coherence Field Theory

The lattice dynamics described here instantiate the coherence field equation $S^* = \sigma(1 - \sigma^*)V^*A^*$, where $\sigma = 0.991$ is the coherence retention parameter, V is vitality (energy availability), and A is alignment (structural organization). The seven bands correspond to increasing coherence levels from VOID ($S^* \approx 0$) to CRYSTAL ($S^* > T^* = 0.714$). The spine's 10 phases map to the TIG operator sequence 0→9 (void, lattice, counter, progress, collapse, balance, chaos, harmony, breath, reset). The phase 9 RESET operator implements the Ω Coherence Keeper archetype: a σ-weighted pull toward initial state that prevents both freezing ($\sigma = 1$) and dissolution ($\sigma \ll 1$). These correspondences are described here for completeness; the dynamical systems results reported in Sections 4–5 are independent of the TIG

interpretation and depend only on the algebraic structure of the quadratic operator and spine transforms.

8. Conclusion

We have demonstrated that a lattice of quadratic operators under compositional coefficient transforms exhibits three emergent phenomena: self-organizing band redistribution, CRYSTAL-biased attractor basin formation, and non-spatial root-proximity topology. These phenomena arise from the algebraic structure of the spine—specifically, the cyclic pitchfork and saddle-node bifurcations at phases 4→5 and 2→3—without external tuning. The complete lattice state is stored as 3 coefficients per cell (12 bytes at Float32) and all emergent structure reconstructs losslessly from this minimal representation. The classification scheme, energy landscape, and topology are deterministic functions of the quadratic operator, making the system both reproducible and analytically tractable.

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