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Imitation of Complex Strategies

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Researchers examining loosely coupled systems, knowledge management, and complementary practices in organizations have proposed, informally, that the complexity of a successful business strategy can deter imitation of the strategy. This paper explores this proposition rigorously. A simple model is developed that parametrizes the two aspects of strategic complexity: the number of elements in a strategy and the interactions among those elements. The model excludes conventional resource-based and game-theoretic barriers to imitation altogether. The model is used to show that complexity makes the search for an optimal strategy intractable in the technical sense of the word provided by the theory of NP-completeness. Consequently, would-be copycats must rely on search heuristics or on learning, not on algorithmic "solutions," to match the performance of superior firms. However, complexity also undermines heuristics and learning. In the face of complexity, firms that follow simple hill-climbing heuristics are quickly snared on low "local peaks," and firms that try to learn and mimic a high performer's entire strategy suffer large penalties from small errors. The model helps to explain why some winning strategies remain unmatched even though they are open to public scrutiny; why certain bundles of organizational practices diffuse slowly even though they lead to superior performance; and why some strategies yield superior returns even after many of their critical ingredients are adopted by competitors. The analysis also suggests roles for management science and managerial choice in a world of complex strategies.
(*Imitation*; *Complexity*; *Interactions*; *Competitive Advantage*)

1. Introduction

The profound influence of imitation on industrial dynamics is, by now, well established. The actions of imitators promote the rapid diffusion of new products, processes, and organizational arrangements. The prospect of imitation may dull the incentive to innovate, but actual imitation may spur past innovators on to fresh invention (Schumpeter 1942). Rapid imitation, it has been argued, reduces the profitability of an industry, the degree to which production is concentrated in a few hands, and the ability of successful firms to maintain productivity advantages (Nelson and Winter 1982, Porter 1980).

Accordingly, the strategists of successful firms seek to deter imitation. Empirical evidence suggests that at least some firms meet this goal. Although information about new products and processes leaks out to rivals within about a year (Mansfield 1985) and superior financial performance at the firm level typically fades within five years (Ghemawat 1991), a handful of firms maintain superior profitability—and presumably defy imitation—over long periods (Mueller 1986). Particularly striking is the ability of some firms to resist imitation despite extensive public scrutiny of their strategies. Firms such as Dell Computer, Southwest Airlines, and Toyota enjoy higher rates of return and faster growth than rivals even though journal articles, case studies, analyst reports, and books by founding executives reveal the ingredients of their successful recipes.

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What prevents the imitation of well-known, winning business strategies? Past research focuses on two classes of barriers to imitation. Proponents of the resource-based view of the firm highlight factors that make imitation difficult: impediments to factor accumulation, social complexity, causal ambiguity, tacit knowledge, economies of scale and scope, adjustment costs, and first-mover advantages (e.g., Barney 1991, Dierickx and Cool 1989, Lippman and Rumelt 1982). Industrial economists, in contrast, emphasize moves by incumbents that make imitation unrewarding, even if it is fully feasible. In many game-theoretic models, for instance, incumbent firms undertake costly commitments that alter their own future incentives, make retaliation threats credible, and thereby fend off copycats.

This paper offers a sharply different explanation for inimitability. I propose, and argue formally, that *the sheer complexity of a strategy can raise a barrier to imitation*. In my formulation, two factors make a strategy complex: the number of decisions that comprise the strategy and the degree of interaction among those decisions (Simon 1962). As the elements of a firm's decision problem grow numerous and interdependent, imitation of a successful strategy becomes very difficult. Indeed it can become "intractable" in a technical sense of the word. Hence, a firm can deter imitation by doing a large number of related things well even if none of its individual actions is inimitable. Because of interactions among the components of a strategy, a would-be imitator could understand most of the ingredients that make up a successful business system yet still fail to grasp the recipe.

Although this paper provides the first formal analysis of complexity as a barrier to imitation, there are ample precedents—theoretical, empirical, and clinical—for the notion that complexity deters imitation. Theoretical precedents come, in part, from the literature on knowledge management. (In this line of research, the object of imitation is typically a product or a process, not a business strategy, but the insights are still relevant.) Conceptual discussions of knowledge management argue that complex knowledge is hard to transfer. Authors differ, however, in how they define "complex knowledge." To Winter (1987), complexity reflects "the amount of information required to characterize the item of knowledge . . . " Zander and Kogut (1995) measure the complexity of an innovation by the number of distinct competencies it combines. Tyre (1991) gauges complexity by "the number, novelty, and technological sophistication of new features and improved concepts introduced" in an innovation, while MacMillan et al. (1985) consider a new product complex if it "requires extensive reorganization of interdependent procedures and/or the coordination of many skills and multiple departments." The definition of complexity which I adopt from Simon (1962) encompasses the two major concepts from the knowledge management literature: the sheer number of elements in an item of knowledge and the degree of interaction among those elements.

The proposal that complexity can deter imitation also draws theoretical support from the extensive, though diffuse, research on loosely and tightly coupled systems. This line of inquiry has focused not on imitation, but on the more general topic of change. Students of both biological systems (e.g., Glassman 1973) and organizations (e.g., Weick 1976) have noted that systems whose elements are tightly coupled to one another find it difficult to learn and adapt to environmental change. Extensive interaction across elements can immobilize systems in general, just as they do the imitators in the analyses presented in this paper. Among the research efforts on change in coupled systems, the recent work of Levinthal (1997) is particularly pertinent because it employs an analytical framework very similar to the one I use below. The concerns motivating this paper and Levinthal's are distinct: Levinthal emphasizes the interplay between organizational adaptation and population-level selection forces while I examine the process of imitation. While I show how tight coupling among elements of a strategy protects a successful firm from imitation, Levinthal identifies a dark side of coupling: successful firms with tightly coupled systems find adaptation difficult in the face of environmental change. Together, the papers illustrate a tension between the protective power and immobilizing influence of tight coupling. I return to this in the concluding section.

Beyond theoretical precedents, a handful of empirical studies support the notion that complexity raises a barrier to imitation. In their analysis of the steel-finishing industry, Ichniowski et al. (1997) document a bundle of interwoven human resource practices that resists imitation. In a study of pharmaceutical research, Cockburn and Henderson (1996) show that many firms are slow to match the organizational practices of rivals that make science-driven R&D effective. They attribute the inertia, in part, to "complex complementarities" among the practices. Examining commercial banking products, MacMillan et al. (1985) find that imitation proceeds slowly when a new product entails interdependent procedures that span department borders. Case studies of individual firms also suggest that highly complex strategies can withstand imitation attempts. In their discussion of "fit" and complementarity, Milgrom and Roberts (1995) suggest informally that rich interactions among Lincoln Electric's many choices may explain why rivals have not replicated that firm's well-documented success. Porter (1996) argues that tight fit among numerous activities helps companies such as Southwest Airlines, Vanguard, and Ikea stave off copycats. More casually, discussions in popular business circles commonly claim that some inability to "get the whole system right" stymies imitators.

Though the proposition that complexity deters imitation has numerous precedents, a formal analysis of the proposal is worthwhile. The analysis highlights, for instance, how interaction among the elements of a strategy—not the mere proliferation of unrelated elements—makes imitation difficult. It pinpoints the type of interaction that stymies copycats: interactions that apply in all business situations (e.g., a complex product line *always* necessitates a highly trained sales force) pose little challenge. Interactions that vary from one situation to another (e.g., a complex product line *sometimes* calls for a multidivisional structure) are far more problematic. The formal analysis identifies general structural features that obstruct strategy making, and it allows us to talk with precision about the computational burden associated with a business strategy. It also allows us to see how great the time required to evaluate a strategy must be before complexity truly trips up copycats.

The rigorous case for complexity as a barrier to imitation begins in § 2. There I construct a model in which (a) a firm must make decisions concerning multiple elements of its strategy, (b) the elements interact, and (c) all other barriers to imitation are absent. The model is an adaptation of a tool developed by Kauffman (1993) in the context of evolutionary biology and introduced into management science by Levinthal (1997). Critically, it parametrizes the two aspects of complexity: the number of elements in the strategy and the degree of interaction among them. This allows one to tune the degree of complexity and analyze how an increase in complexity affects the ease of imitation.

I next show (§ 3) that interactions among decisions render the firm's decision problem NP-complete, i.e., intractable to algorithmic solution. When interactions are pervasive, no general, step-by-step procedure exists that can locate the globally optimal strategy quickly. Hence interactions among decisions play at least a crude role in determining the ease of imitation. When interactions are common and decision problems intractable, a firm that finds a good resolution of its problem can hope that it will be hard for others to discover its formula for success.

If complexity renders global optimization intractable, however, one must presume that thoughtful managers realize the nature of the challenge they face and opt not to attempt global optimization. Rather, they likely employ judgment and heuristics to find good, albeit not necessarily optimal, sets of decisions (March and Simon 1958). Thus it is crucial that I show how complexity hampers not just global optimization, but also the heuristics that imitating firms plausibly employ. Sections 4 and 5 demonstrate how complexity undermines reasonable forms of heuristic search. Section 6 discusses the robustness of the results, and a concluding section summarizes empirical and practical implications.

2. A Model with Tunable Complexity

This section develops a model with "tunable complexity." The model allows one to set the complexity of the strategic problem that firms in an industry face, generate a sample of problems with that level of complexity, examine the ease of imitating a good resolution to each problem, and adjust to a different level of complexity. By repeating this process, one can isolate how complexity affects imitation.

2.1. Formal Model

The Parameters. Two parameters, N and K, govern the complexity of a modeled firm's decision problem. N is the first aspect of complexity, the number of decisions that a firm faces. In the formal model, each of N decisions can be resolved in two ways.[1] Hence a particular strategy $\mathbf{s}$, a configuration of decisions, is a vector $\{s_1, s_2, \ldots, s_N\}$, with each decision s_i set to either 0 or 1. Note that there are 2^N possible configurations of decisions. K controls the second aspect of complexity, the degree to which the decisions interact. The efficacy of each decision is affected not only by the choice (0 or 1) made concerning that decision, but also by the choices regarding K other decisions. In the model, each decision i makes a contribution C_i to overall firm value. C_i depends not only on s_i but also on how K other randomly assigned decisions are resolved: $C_i = C_i(s_i; s_{i1}, \ldots, s_{iK})$. K ranges from 0 to $N - 1$. When $K = 0$, the contribution of each decision depends only on the choice made concerning that decision: $C_i = C_i(s_i)$. When $K = N - 1$, the contribution of each decision is affected by all $N - 1$ others: $C_i = C_i(\mathbf{s})$.

N reflects the reality that managers must make numerous decisions. A firm's strategy is embodied in this nexus of choices. K captures the fact that the choice made concerning one decision may affect the marginal benefit or cost associated with another decision.[2] For instance, investments in machines that permit production of a complex product line are often made more valuable on the margin if a firm also employs a highly trained sales force that can explain the product line to customers. The marginal value of product inspection activities may be reduced by manufacturing activities that eliminate defects in the production process. A large number of recent studies document how choices often complement one another or substitute for each other.[3]

Conceptually, N implies that managers are not searching along a single dimension for an optimal decision, as is sometimes posited in economic models. Rather, they are searching a very-high-dimensional "decision space" for an optimal *combination* of choices. This suggests the following image of a landscape: each of N decisions constitutes a "horizontal" axis in a high-dimensional space, and each decision offers different options. Resulting from each combination of choices is a value for the firm, which is plotted on the vertical axis. The goal of the strategy formulation process is to occupy a high spot on this landscape, i.e., to select a combination of choices that, together, produce a great deal of value for the owners of the firm. As I discuss below, the interactions introduced by K cause the landscape to become rugged and multipeaked, and this makes the search for a high peak profoundly more difficult.

Note that a "strategy" in this formulation is precisely a complete configuration of N decisions. Researchers adopt widely varying definitions of the term "strategy." Whatever definition one adopts, however, a strategy is realized in the marketplace as a set of choices that influence organizational performance. Accordingly, this paper focuses directly on sets of choices. The choices encompassed by the N decisions include elements that many researchers would consider organizational structure, not strategy per se. For that reason, some readers may prefer to interpret this paper as addressing the imitation of entire organizational configurations (Miller et al. 1984) or organizational forms (Levinthal 1997) rather than strategies.

[1] This formulation assumes that the decisions concern discrete, not continuous, variables, i.e., that each can be resolved in a finite or countably infinite number of ways. In reality, many strategic decisions present discrete options. The restriction to binary choice is made purely for convenience and does not affect any of the qualitative results of this paper.

[2] This is an adaptation of Milgrom and Roberts' (1990) definition of "complementarity." Under their definition, complementarity arises when performing *more* of activity A *increases* the marginal benefit of doing *more* of activity B. I use the term "interaction" rather than "complementarity" to encompass both complementarity and substitution between decisions.

[3] See, for instance, Cockburn and Henderson (1996), Hwang and Weil (1997), Ichniowski et al. (1997), MacDuffie (1995), Milgrom and Roberts (1990, 1995), Porter (1996), and Siggelkow (1998).

Landscape Generation. The landscape, or equivalently the decision problem, which firms face in the model is generated by a stochastic process. Although the precise landscape is a product of chance, the parameters that define the complexity of the landscape, N and K, are controlled by the modeler. A payoff is assigned to each of the 2^N combinations of decisions as follows. Recall that the contribution C_i of each decision to overall firm value is affected by K other randomly assigned decisions. For each possible realization of $(s_i; s_{i1}, \ldots, s_{iK})$, a value contribution is drawn at random from a uniform $U[0, 1]$ distribution. The overall value associated with a configuration is the average over the N value contributions:

$$P(\mathbf{s}) = \left[\sum_{i=1}^{N} C_i(s_i; s_{i1}, \ldots, s_{iK}) \right] / N.$$

Note that when $K = 0$, $P(\mathbf{s})$ is the average over N contributions, each of which depends only on a single choice. At the other extreme, when $K = N - 1$, $P(\mathbf{s})$ is the average over N contributions, each of which is affected by all N choices.

Decision problems are generated by a process that is random in two ways: the K decisions that affect a focal decision are assigned by chance, as is the contribution C_i corresponding to a particular configuration of $(s_i;s_{i1}, \ldots, s_{iK})$. At first it may seem odd that decision problems are generated randomly. Recall, however, that the objective is to determine how complexity affects the imitation process *typically*, not for a particular decision problem. The best way to accomplish this is to generate at random a number of decision problems of a particular degree of complexity, determine average behavior on this set, adjust the complexity, and repeat the process.

The role of chance in the modeling procedure reflects an important implicit assumption. The N decisions are choices whose relationships are *not* known a priori. If it is universally true and known that, say, two choices always "go together," then those two choices are treated as a single decision in this paper's model. A key purpose of strategy scholarship is to identify sets of choices that normally accompany one another—and thereby reduce the dimensionality of a firm's decision problem. I discuss this further in the final section.

Imitation. Once generated, a landscape is employed in one of two ways. In §3, I examine the computational burden associated with global optimization on a typical landscape and show how that burden grows with strategic complexity. In §§4 and 5, I consider imitators that use simple heuristics to try to match a benchmark firm. The effectiveness of each heuristic is assessed by a series of Monte Carlo simulations. I begin the simulations by fixing N and K. For a given level of N and K, a number of landscapes are generated. On each landscape, a high point, $\mathbf{s}^\star$, is identified as the best outcome of 100 exploratory searches, and a benchmark firm is assigned this strategy. Five hundred would-be imitators are then released at random initial locations on the landscape. That is, each is assigned an initial decision configuration, or strategy, by chance. Each imitator uses a heuristic to search for higher ground—better sets of decisions—and I record how successful the typical imitator is for that level of complexity. I then adjust N and K and repeat the process. This isolates how the complexity of the underlying decision problem makes the imitation heuristic more or less effective.

Three forms of search heuristic are considered. Under the *incremental improvement* heuristic, an imitating firm considers all alternative strategies that involve changing M or fewer of its current decisions. It accepts an alternative if and only if the alternative generates a higher payoff. Once it has adopted the new strategy, the imitator considers another incremental change, and so forth until the firm reaches a (possibly local) peak. The parameter M governs how local the search is. $M = 1$ implies a polar version of local search, employed by researchers such as Levinthal (1997). $M = N$ constitutes global search.

Under *follow-the-leader imitation*, a firm at $\mathbf{s}$ attempts a major reconfiguration toward $\mathbf{s}^\star$. J of its N decisions are adjusted to match the benchmark firm's choices. The probability that each decision is matched correctly is $\theta \leq 1$. θ is ordinarily less than 1, reflecting the fact that an imitator may not understand the benchmark firm's operations perfectly, does not have full access to the template (Nelson and Winter 1982) provided by the original success, and may not be able to control its own operations precisely.

In contrast to a follow-the-leader imitator, an incremental improver is "imitating" the benchmark firm only in a weak sense of the word. An incremental improver does not use any information about the strategy of the benchmark firm actively. Rather, it operates under the hope that incremental improvement will lead it to rediscover the benchmark's success. Hence the cognitive models underlying the two heuristics are very different.

I also consider a third search heuristic, a *hybrid* in which a firm jumps toward the benchmark in follow-the-leader fashion, tweaks uphill to a local peak, jumps again if necessary, and so forth. This hybrid captures three aspects of reality. It recognizes that firms are capable of both major reorientations and incremental change (Miller et al. 1984). It models imitation as an on-going effort, not a one-shot attempt to mimic success. And it permits rivals to start their imitation attempts (after the initial leap) clustered around the benchmark rather than spread uniformly over the landscape.

As noted above, the benchmark in each simulation is pinpointed by a series of exploratory searches. One hundred firms, released at random locations on the landscape, search for better strategies, and the top performer serves as a standard. In all cases, the search heuristic employed in the exploratory phase is the same as that used by the subsequent imitators. The results are robust with respect to the method used to determine the benchmark (§6).

The primary simulation results of the paper examine how well imitators match a benchmark firm. Additional results show how the benchmark itself evolves over time. Specifically, I release thousands of firms sequentially on unexplored landscapes, allow each firm to seek a successful strategy using one of the heuristics described above, and record when "leadership" in the market changes hands. The first firm released on the landscape sets an initial benchmark for performance. The second firm may match that benchmark, achieve a worse performance, or exceed the benchmark—in which case it sets a new benchmark. By repeating this analysis for various levels of N and K, I isolate the effect of complexity on the evolution of the benchmark. These additional simulations and the primary simulations envision quite different industry settings. The primary simulations reflect a mature business in which an imitator attempts to mimic a well-established leader. The additional simulations apply better to an emerging industry in which the standards for performance are still in flux.

It is important to note what the model *excludes*. Imitators face no costs of adjustment, no threats of retaliation from the incumbent high performer, no fear that colocation with the high performer will lead to intense competition, no second-mover disadvantages, no barriers to modifying individual decisions, no economies of scale or scope, and no impediments to resource accumulation. Search costs are set to zero; indeed, the incremental improver knows the value of all neighboring strategies within a radius of M perfectly without cost or commitment. The real and important fact that some decisions are harder to change than others is purposely suppressed. In short, none of the conventional deterrents to imitation are present. Any barriers that imitators face derive purely from the complexity of the underlying decision problem, coupled with modest limits on what the firm knows and can do.

2.2. Antecedents in Evolutionary Biology

Some readers will recognize the modeling approach as a modification of the NK model developed by evolutionary biologist Stuart Kauffman and his colleagues (Kauffman 1993 and references therein). Kauffman devised the NK model in order to examine how biological entities such as proteins and organisms evolve. Just as managerial decision making can be conceived of as a hill-climbing search in a high-dimensional decision space for an internally consistent set of choices, so evolution has often been seen as a hunt in a high-dimensional genetic space for a peak combination of genes (Wright 1931).

Of course, the biological analogy is not perfect. Managers can scan the strategies adopted by their rivals, learn from them, and purposely reconfigure their choices to match the competition. Mice, on the other hand, cannot modify their genetic makeup to be more elephantine, no matter how much they admire and study elephants. For this reason, I cannot import Kauffman's biological techniques, unmodified, into the realm of management science.

When transporting intellectual goods across discipline borders, as I am doing here, one must declare clearly what is old, what is borrowed, and what is new. The particular process for generating landscapes and the use of simulation are taken, unmodified, from Kauffman (1993). As subsequent references make clear, the structural features of landscapes I discuss below are identified by Kauffman and other users of the NK model. Prior efforts, however, do not explore the collective effect of those features on imitation attempts. Of the search heuristics I explore, the incremental improvement mode is identical to the search process employed by most of Kauffman's biological entities. The informed follow-the-leader heuristic and the hybrid heuristic are new. Indeed, such purposeful search processes make little sense in the context of biological evolution. Similarly, the use of a benchmark toward which followers aim is an innovation. Kauffman (1993) speculates on the computational burden imposed by the NK model but, since the agents in his models cannot devise algorithms, he need not prove the intractability of optimization as I do below. Although developed in the context of evolutionary biology, NK models are beginning to find applications in management science (McKelvey 1999 and references therein). None of these efforts examine imitation.

2.3. Interactions and Landscape Ruggedness

A crucial feature of the modeling approach is that, by means of K, it parametrizes the richness of interaction among the elements of the firm's strategy. Much of the intuition concerning the effects of K depends on the following insight: as the pieces of a strategy become more interdependent, the landscape corresponding to a firm's decision problem becomes increasingly rugged. That is, the correlation between the altitudes of adjacent points on the landscape—points that differ by how one decision is made—falls, and local peaks proliferate. In the biological context, Kauffman (1993, pp. 54–63) goes to great lengths to show that this is the case. Here I give two examples that provide some intuition concerning the link between K and landscape ruggedness.

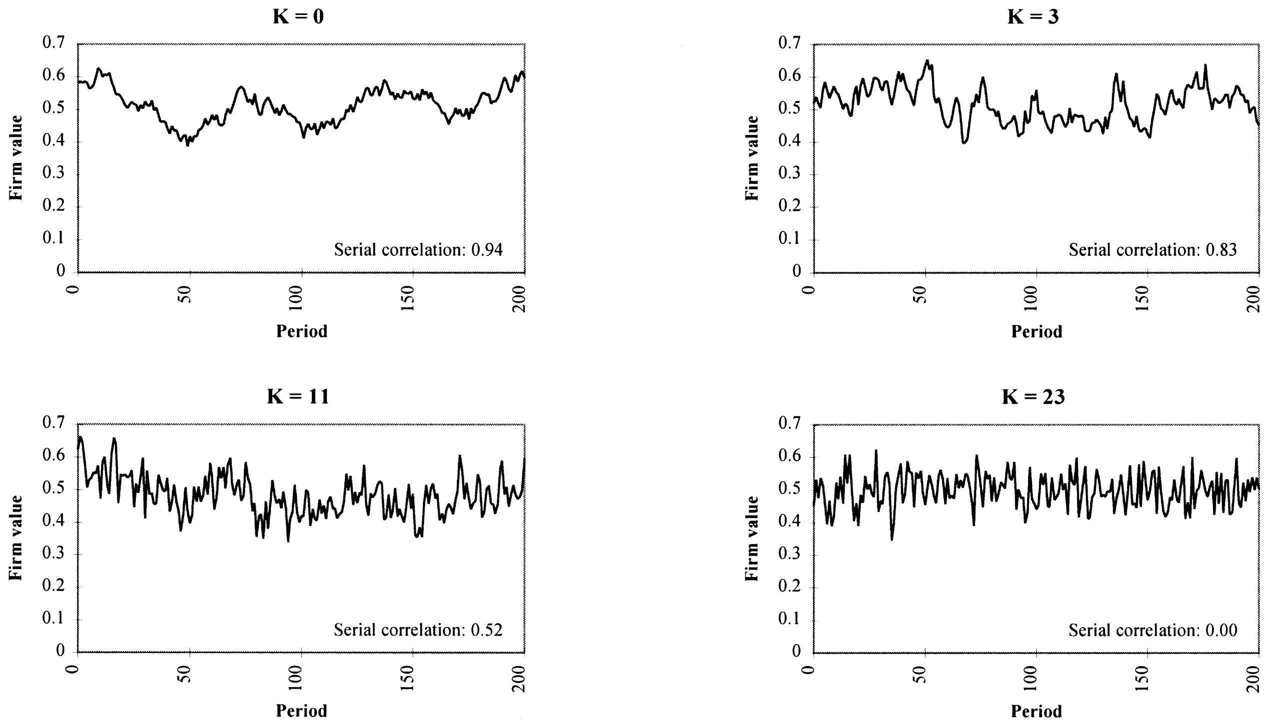
First, consider the extreme cases of $K = 0$ and $K = N - 1$. When $K = 0$, choices make independent contributions to firm value. In this situation, alteration of a single decision changes the contribution of that decision alone. Adjacent locations on the terrain—configurations that differ by how a single decision is resolved—cannot differ in elevation by more than $1/N$, so the landscape is relatively smooth. Moreover, from any initial location on the landscape, firms can rise to the global peak via a series of value-improving, single-decision "tweaks." Management simply adjusts each choice, one by one, to the option that allows that decision to make the greater contribution in isolation. Hence when $K = 0$, one has a smooth landscape with a single peak whose basin of attraction is the entire decision space.

In contrast, $K = N - 1$ implies that every decision influences the contribution of every other choice. Then, a small step on the landscape—a change in a single decision—alters the value contributions of all N elements. Adjacent decision configurations have levels of value (elevations) that are altogether uncorrelated with one another, and the impact of a single decision is bounded above by 1 and below by -1. The landscape is fully random with many local peaks.

The landscapes considered here exist in such high-dimensional spaces that they cannot be depicted directly, but random walks on typical landscapes can convey a sense of ruggedness. Figure 1 does this for $N = 24$ and various values of K. As K rises, the landscape grows rugged, and the elevation at any step becomes a poor predictor of the elevation at the next step.

The intuition of why interactions may deter imitation now begins to become clear. In a richly interactive system, global optimization is difficult. In graphic terms, it is hard to pinpoint and scale the highest peak on rugged terrain. Incremental improvement is ineffective. In enhancing the performance of one element of the system, managers may inadvertently undermine other parts. The firm quickly becomes ensnared in a web of conflicting constraints. Graphically, the firm is trapped on a low local peak. A wholesale reconfiguration might free the firm from the local peak and take it to a much higher location. Unfortunately, interactions also make such large-scale changes risky.

Figure 1 **Random Walks on $N = 24$ Landscapes**



Description. For $N = 24$ and various values of K, landscapes were generated as described in the text. On each landscape, a firm was released at a random initial location and allowed to wander, altering one decision at random each period for 200 periods. The charts show the elevation of each firm over time. The correlation between the height each period and the height in the previous period is also reported.

Interpretation. As K rises, the landscape in decision space becomes rugged.

In a long jump on rugged terrain, a miscalculation on any one of many dimensions can cause a firm to land in a trough instead of atop the intended peak.

The landscape metaphor alone gives us this intuition. The mathematical model of the landscape allows us to deepen our understanding of how complexity deters imitation.

3. Intractability of the Strategy Formulation Problem

Complexity cannot deter imitation unless it first makes global optimization difficult. Suppose that the decision problems posed by the model are easily solved by some algorithm, regardless of N or K. Then a benchmark firm that finds even the best possible set of decisions will see its strategy quickly discovered by others and replicated (unless some other barrier to imitation intercedes). Complexity alone would pose no obstacle to imitation.

This section proves that complexity indeed makes global optimization hard. When decisions are numerous and cross-coupled, the strategy formulation problem defies algorithmic solution. It becomes intractable, in the technical sense of algorithm theory.

3.1. The Theory of NP-Completeness[4]

Algorithm designers in computer science, mathematics, and operations research have developed a language system and a set of ideas concerning the

[4] This subsection draws substantially from Garey and Johnson's (1979) seminal text, especially Chapters 1–3. Algorithm designers draw a distinction between *decision problems* and *search problems*. This section does not emphasize the distinction, which is not relevant in the present context.

tractability of decision problems. The tractability of a particular problem hinges on its *time complexity function*, which relates the size of a specific instance of a problem (n) to the time that a given algorithm requires to solve the problem ($t(n)$). An algorithm is any general, step-by-step procedure for solving a problem. A critical distinction exists between polynomial time algorithms, for which t is some polynomial function of n, and exponential time algorithms, for which t is an exponential function of n. This distinction is crucial because exponential functions grow *far* faster than polynomial functions. Algorithms with exponential time complexity become impractical for large instances while algorithms with polynomial time complexity remain feasible. Problems for which polynomial time algorithms exist are said to fall into the P class and are considered by algorithm designers to be well solved. Problems for which no polynomial time algorithms exist are labeled "intractable."

Definitively proving intractability, however, turns out to be difficult, and this difficulty has led to two additional concepts: the NP class of problems and NP-completeness. Many problems for which no polynomial time algorithm has been found could, in theory, be solved in polynomial time if one had a "nondeterministic computer," a machine with unlimited parallel processing capabilities. Such problems fall into the *nondeterministic polynomial*, *or NP*, *class*. $P \subseteq NP$. Moreover, it is widely believed (but not yet proven) that $P \subset NP$, i.e., there are NP problems with no polynomial time solutions. The *NP-complete class* of problems is a subset of NP problems which have been shown to be as hard as any in the NP class. Transformations have been devised that permit one to convert each NP-complete problem into others in polynomial time. Therefore, if one could find a polynomial time algorithm for any NP-complete problem, one could devise such an algorithm for all NP problems. Conversely, if any problem in the NP class is intractable, all NP-complete problems are intractable. Hundreds of problems have now been shown to fall into the NP-complete class (Garey and Johnson 1979). It is widely believed, though not yet proven, that the NP-complete problems are all intractable. In all likelihood, *no general*, *step-by-step procedure can be devised to solve an NP-complete problem of large size in reasonable time*.

3.2. Strategic Complexity and NP-Completeness

Now consider the implications for the strategy formulation problem as formalized in this paper. Suppose a group of managers faces a decision problem, an NK landscape, whose topography is unknown. Given some time, the managers can correctly evaluate the payoff of any proposed strategy. Can they write down an algorithm that will quickly locate the best set of choices? It turns out that the answer hinges on K. As the degree of interaction among a firm's decisions rises, the NK problem goes from being a member of the P class of problems to being NP-complete. The time required for any algorithm to find the best strategy switches from being a polynomial function of N to being an exponential function of N (under the standard assumption that $P \neq NP$).

To see this, first consider the NK model with no interactions, $K = 0$. Trivially, this can be solved in polynomial time by an algorithm that optimizes each of the N decisions one-by-one, by choosing the better of the two alternatives. The time complexity function associated with this problem is simply a linear function of N.

An appendix available from the author proves, however, that the NK model with $K > 2$ produces an NP-complete problem. The proof proceeds roughly as follows: The NK problem with $K > 2$ is first shown to belong to the NP class. A particular problem that is widely known to be NP-complete, the so-called "K-satisfiability problem," is then chosen. Since the K-satisfiability problem is NP-complete, it is as hard as any in the NP class. It is next shown that any K-satisfiability problem can be transformed in polynomial time into an NK model with $K > 2$. Such an NK problem is therefore at least as hard as the K-satisfiability problem; if the NK problem could be solved in polynomial time, then one could also solve the K-satisfiability problem in polynomial time by transforming it into an NK problem and solving the transformed problem. In sum, the NK problem with $K > 2$ is in the NP class and is at least as hard as the K-satisfiability problem, which itself is at least as hard as any problem in NP. One concludes that the NK

problem is at least as hard as any problem in NP; it is NP-complete. Under the standard assumption that $P \neq NP$, the NK problem is intractable for $K > 2$.

3.3. Implications

The first consequence of NP-completeness, the one most germane to this paper, is that K delineates two categories of decision problems. In the low-K category, decision problems are easy, and great strategies are quickly deciphered and matched. When K is high, problems are complex, and a successful resolution of a decision problem may defy discovery for an exponentially long period. The generality of this result is worth emphasizing. It applies to any step-by-step search procedure that one could write down. It encompasses procedures ranging from simple exhaustive search to sophisticated approaches that send "probes" into the landscape and react to the results.

For $K > 2$, the strategy decision problem is intractable in the technical sense of the word. The problem becomes intractable in the *vernacular* sense, however, only when two further conditions hold: the number of decisions N is substantial, and the time involved in each algorithmic step is not trivial. Some examples make this clear. When $N = 5$ and a proposed strategy can be evaluated in an hour, for example, a very simple algorithm—exhaustive search—will pinpoint the global optimum in a mere $2^5 = 32$ hours. When N grows to 20, however, the solution time associated with exhaustive search rises to 120 years. Table 1 shows the time required for exhaustive search to discover the global optimum under various combinations of N and evaluation time per strategy. (Evaluation time reflects either the time required to conduct a study of an alternative strategy or the time necessary to implement a test and receive feedback.) Only when N is very low and the evaluation time very brief does exhaustive search appear practical.

The NP-completeness result alone does not imply that complexity deters imitation. The result ensures only that complexity undermines *global optimization*. When facing a complex decision problem, thoughtful managers presumably understand the futility of trying to "solve" the problem and match the performance of a benchmark firm by any algorithmic means. Rather, they likely use simple heuristics and attempt to learn directly from the successful firm. The remaining challenge, then, is to show that complexity also undermines the heuristics and attempts at learning. Sections 4 and 5 tackle this challenge.

The NP-completeness result suggests interpretations for two phenomena that are unrelated to imitation. First, it is commonly posited that managers satisfice rather than optimize (e.g., March and Simon 1958, Nelson and Winter 1982). Satisficing seems a sensible course of action if the strategy formulation problem is NP-complete; managers have little choice but to use heuristics and judgment to seek good, albeit not necessarily optimal, configurations of decisions. Second, the NP-completeness finding suggests that in complex settings, some very good strategies may go undiscovered for long periods. Thus a successful new way of doing business may emerge "out of the blue"

Table 1 Time Required for Exhaustive Search

	Time required to evaluate one possible strategy			
	1 second	1 hour	1 day	1 year
$N = 5$	32 seconds	32 hours	32 days	32 years
$N = 10$	17 minutes	43 days	2.8 years	~1,000 years
$N = 20$	12.1 days	120 years	~3,000 years	~1 million years
$N = 30$	34 years	~120,000 years	~3 million years	~1 billion years
$N = 40$	~35,000 years	~125 million years	~3 billion years	~1 trillion years

Description. The table reports the time required to evaluate all 2^N possible strategies, for various combinations of N and evaluation time per strategy.
Interpretation. Exhaustive search is practical only when N is low and evaluation time is brief.

occasionally. Insightful or lucky managers may discover superior strategies—virgin peaks—even though tastes and technologies have not changed.

Finally, the *NP*-completeness of the strategy formulation problem poses a question for the recent, important stream of research on complementarities (e.g., Milgrom and Roberts 1990, 1995; Topkis 1998 and references therein). Like this paper, the complementarity research drops the assumption, often made by economists, that managers must make only a handful of decisions. It also emphasizes the role played by interactions among choice variables. In contrast to this paper, however, the complementarities research is interested primarily in comparative statics: how do optimal sets of decisions concerning complementary variables shift as underlying parameters change? Consider, for instance, the array of decisions which an auto manufacturer must make concerning price, machine flexibility, product variety, the length of production runs, inventory, vertical integration, and human resource practices. How does the best set of decisions shift as exogenous factors change—say, as the costs of data communication, computation, and flexible machines fall?

The power of the complementarity approach is that it identifies the weakest conditions under which one can say anything about such questions. Roughly, Topkis, Milgrom and Roberts, and others show that one can make clean comparative static predictions if and only if the auto maker's profit $\pi(\mathbf{x}, \mathbf{p})$ is supermodular in decision variables $\mathbf{x}$ and outside parameters $\mathbf{p}$. That is, the marginal return to increasing any variable must increase with any other variable, for all values of all other variables. Under those conditions, the optimal choice $\mathbf{x}^{\star}(\mathbf{p})$ is monotone nondecreasing in $\mathbf{p}$. Milgrom and Roberts (1990) use these results to show how declining costs of communication, computers, and flexible machines could prompt an optimizing firm to lower prices, invest in flexible production, broaden product lines, shorten production runs, reduce inventory, buy inputs rather than make them, and adopt team-based human resource practices.

The *NP*-completeness result, however, illustrates a situation in which interactions among numerous decisions make the global optimum difficult to find. This raises a dilemma concerning the comparative static result: is it appropriate to focus on the global optimum when that optimum might be very hard to discover? How relevant are comparative statics when optimization might be a poor model of behavior?

The dilemma is eased, to a certain extent, by the fact that Milgrom and Roberts et al. examine comparative statics only for supermodular functions, in which an increase in each decision variable always raises the incremental returns to all others. (*NK* landscapes are much more general. With a payoff function generated by the *NK* model, altering a decision may increase or decrease the incremental returns to another decision, and the sign of the effect may depend on the choices made concerning other decisions.) Finding a good set of decisions on a supermodular landscape is, in certain senses, easier than finding one on a general *NK* landscape. On a supermodular landscape, for instance, one can test whether a candidate point is a global optimum by surveying just the two orthants above and below the candidate (Milgrom and Roberts 1995, p. 185); on a general landscape, one would have to consider all 2^N orthants. Starting at any point on a supermodular landscape, a firm that scours only those two orthants is sure to get at least half of the gains that an exhaustive search would yield (Milgrom and Roberts 1995, p. 185); on a general landscape, such a limited search might produce none of the gains. For particular restricted classes of problems with supermodular payoff functions, polynomial-time algorithms are known to exist. (Specifically, the ellipsoid method can solve problems involving supermodular set functions in polynomial time (Grötschel et al. 1988, Chapter 10).)

Although the structure imposed by supermodularity makes global optimization easier, it by no means makes it easy. The time required to survey two orthants may easily rise exponentially with the size of a problem. Polynomial-time algorithms are not known to exist for most types of problems with supermodular payoff functions. Even when they do exist, they can be very time-consuming: "theoretically efficient" but not "practically efficient" (Grötschel et al. 1988, Chapter 3).

In sum, before applying comparative static results in situations involving sets of interactive variables, one should be confident that the global optima to which the results apply can indeed be located. This is more likely to be true when the number of decisions involved is modest; when little time is required to evaluate a candidate solution; when interactions are sparse; when search is permitted for a long period; and when a large number of firms are tackling the decision problem. Supermodularity imposes some structure that aids in finding good solutions, but it does not guarantee that the global optimum will be found readily.

4. Imitation by Incremental Improvement

This section and the next turn to heuristic-based imitation. I begin with the plight of the incremental improver. I use simulation to show how strategic complexity undercuts such a firm's ability to approach the benchmark firm. Prior to a full-scale simulation, however, I discuss two structural features of landscapes that bode ill for the incremental improvement effort.

4.1. Structural Deterrents to Incremental Improvement

Consider a rival that hopes to replicate the benchmark firm's success by making a series of incremental, single-decision improvements ($M = 1$). As the underlying decision problem becomes complex—as N and K rise—the rival's prospects worsen for two structural reasons.

Many Peaks. First, the number of local peaks on which the imitator may get trapped rises rapidly with N and K. When $K = 0$, there exists only one peak regardless of N. Facing such a decision problem, the incremental imitator will surely replicate the high performer's strategy eventually. At the other extreme, $K = N - 1$, the elevations of adjacent strategies are fully uncorrelated. Of the 2^N possible strategies, $2^N/(N + 1)$ are expected to be local peaks (Kauffman 1993, p. 47). This number is large for even modest N. For $N = 20$, for instance, the typical $K = N - 1$ landscape has nearly 50,000 local optima. On such terrain, the incremental improver will almost surely get snared on a local peak rather than make its way to the benchmark strategy.

The multiplicity of peaks is not quite so daunting when K is less than $N - 1$, but it remains very large. For various values of N and K, I release 100 firms at random locations on a landscape and allow them to edge uphill by single-decision improvements. Table 2 reports the number of distinct local peaks that the firms discover. When N and K are low, only a handful of peaks exist, and all 100 firms gravitate toward them. When N and K are high, nearly every firm reaches its own local peak. Note that both aspects of complexity are necessary for local peaks to proliferate.

Low Peaks. The multiplicity of peaks on rough terrain would not be harmful to imitators if those peaks were highly profitable. It turns out, however, that not only do local peaks proliferate as K rises, but the "average" peak sinks. For the extreme cases of $K = 0$ and $K = N - 1$, average peak height is easily calculated using order statistics and the Central Limit Theorem. As shown in Table 3, N has no effect on peak elevation when decisions are independent of one another ($K = 0$). In contrast, when interactions are pervasive ($K = N - 1$), increasing N causes the expected peak height to fall. In the large-N limit, firm value on a local peak declines to 0.5, which is no better than the value expected from a randomly selected set of choices. Calculating the heights of local optima is

Table 2 Number of Distinct Peaks Discovered by 100 Firms

	$K = 0$	$K = 1$	$K = 3$	$K = 7$	$K = 11$
$N = 4$	1.0	1.9	3.3		
	(0.0)	(0.2)	(0.4)		
$N = 8$	1.0	2.4	8.6	23.5	
	(0.0)	(0.5)	(0.7)	(0.7)	
$N = 12$	1.0	5.1	20.2	55.6	80.4
	(0.0)	(1.1)	(1.2)	(1.2)	(1.1)
$N = 24$	1.0	15.1	68.8	97.8	99.6
	(0.0)	(1.8)	(1.5)	(0.4)	(0.2)

Description. For various values of N and K, 100 firms were released on a landscape and allowed to move uphill, one decision at a time, until reaching a local peak. This table records the number of distinct local peaks discovered in this fashion. Each data point is the average over ten landscapes, and the standard deviation of the average is shown in parentheses below each data point.
Interpretation. As N and K rise, local peaks proliferate.

Table 3 Expected Height of a Local Peak

$N + 1$	$K = 0$	$K = N - 1$
10	0.667	0.648
20	0.667	0.624
30	0.667	0.610
40	0.667	0.600
50	0.667	0.593
100	0.667	0.573

Description. This table shows the expected height of a local peak for various values of N and for extreme values of K.
Interpretation. As the underlying decision problem grows complex, the typical local peak falls.

more difficult for intermediate levels of K. Weinberger (1991), however, succeeds in doing so via the Central Limit Theorem and a series of approximations. He finds that for values of K that are large but possibly much less than N, the heights of local peaks are asymptotically normally distributed with approximate mean $\mu = \frac{1}{2} + \sqrt{\ln(K + 1)/6(K + 1)}$. Note that $d\mu/dK < 0$ for $K \geq 2$ and $\lim_{K\to\infty} \mu = \frac{1}{2}$. For large K, the typical local optimum on which an incremental imitator gets trapped corresponds to mediocre performance. As the web of conflicting constraints thickens, opportunities for incremental improvement can vanish even at low levels of performance.

4.2. Simulation Results

The structural features suggest that incremental improvement will fail in the face of complexity. Simulations confirm that this is the case. To provide a polar case, I set M, the number of decisions modified in each incremental step, to 1. For various values of N and K, 500 imitators are released on each of ten landscapes and allowed to "tweak" uphill until encountering a (possibly local) peak.

Table 4 reports the results of this process. Panel A records the portion of incremental imitators that meet or exceed the performance of the benchmark strategy. The table also gives the gap between the benchmark strategy and the highest point found by the typical imitator. This gap is measured in terms of both firm value (panel B) and distance in decision space (panel C). As expected, when $K = 0$ incremental improvement always delivers the imitator to the single (global)

Table 4 Results of Incremental Improvement Efforts

Panel A: What portion of imitators match or exceed the benchmark firm's performance eventually?

	$K = 0$	$K = 1$	$K = 3$	$K = 7$	$K = 11$
$N = 4$	100.0%	70.2%	50.1%		
	(0.0%)	(4.6%)	(4.3%)		
$N = 8$	100.0%	63.5%	28.0%	7.6%	
	(0.0%)	(5.9%)	(3.8%)	(0.9%)	
$N = 12$	100.0%	49.1%	17.9%	3.2%	1.3%
	(0.0%)	(7.3%)	(3.0%)	(0.4%)	(0.3%)
$N = 24$	100.0%	22.6%	5.7%	1.4%	0.7%
	(0.0%)	(4.1%)	(1.8%)	(0.4%)	(0.2%)

Panel B: How much worse off is the typical imitator than the benchmark firm, as a percent of value?

	$K = 0$	$K = 1$	$K = 3$	$K = 7$	$K = 11$
$N = 4$	0.0%	2.2%	8.9%		
	(0.0%)	(0.5%)	(1.6%)		
$N = 8$	0.0%	2.6%	5.3%	11.7%	
	(0.0%)	(0.5%)	(0.6%)	(0.9%)	
$N = 12$	0.0%	2.3%	6.4%	11.9%	15.5%
	(0.0%)	(0.5%)	(0.4%)	(0.8%)	(0.8%)
$N = 24$	0.0%	2.5%	7.4%	9.9%	10.8%
	(0.0%)	(0.2%)	(0.5%)	(0.5%)	(0.6%)

Panel C: Once it has reached a local peak, how many decisions does the typical imitator make differently from the benchmark firm?

	$K = 0$	$K = 1$	$K = 3$	$K = 7$	$K = 11$
$N = 4$	0.0	0.7	1.3		
	(0.0)	(0.1)	(0.1)		
$N = 8$	0.0	1.2	3.4	3.8	
	(0.0)	(0.2)	(0.3)	(0.1)	
$N = 12$	0.0	2.8	4.6	5.9	6.0
	(0.0)	(0.4)	(0.3)	(0.1)	(0.0)
$N = 24$	0.0	3.8	9.3	11.9	12.0
	(0.0)	(0.5)	(0.4)	(0.1)	(0.0)

Description. For each level of N and K shown here, ten landscapes were generated. On each landscape, a benchmark strategy was pinpointed as the best of 100 exploratory searches. Five hundred imitators were then released on each landscape with randomly assigned initial strategies and were allowed to improve their strategies incrementally ($M = 1$) until reaching a local peak. The panels address various questions regarding the imitators' performance. Each data point is the average over the ten landscapes, and the standard deviation of the average is shown in parentheses below each data point.
Interpretation. As the underlying decision problem becomes complex, incremental improvement grows less successful.

peak, regardless of N. As interactions proliferate, however, the portion of copycats that match the benchmark strategy declines. By the $N = 24$, $K = 11$ case, only 37 of the 5,000 imitators match the benchmark. Local peaks become numerous and the typical imitator gets stranded well below and far away from the best strategy. Panel B highlights the crucial role played by K, relative to N. It is the interaction among numerous decisions, not the mere proliferation of decisions, that harms the incremental improver.

Table 5 examines the evolution of the benchmark under the incremental improvement heuristic. For each level of N and K, one thousand firms are released sequentially on each of 20 landscapes, and each firm exhausts its opportunities for local improvement. Panel A records the number of times that a new firm establishes a new, higher standard for performance. Panel B reports the number of times that a new firm matches (but does not exceed) the existing benchmark. Panel C analyzes the number of firms released before each new benchmark is matched or exceeded; i.e., it measures how long each new benchmark remains unbeaten. New, superior strategies are discovered more frequently on complex landscapes (panel A), and such strategies are far more rarely matched in complex settings (panel B). Consequently, benchmark firms have much longer runs of success when decisions are numerous and intertwined (panel C).

Table 5 **Evolution of the Benchmark Under the Incremental Improvement Heuristic**

Panel A: How many times is a new benchmark established (maximum = 1,000)?

	$K = 0$	$K = 1$	$K = 3$	$K = 7$	$K = 11$
$N = 4$	1.0	1.2	1.8		
$N = 8$	1.0	1.6	2.1	4.0	
$N = 12$	1.0	1.6	2.6	4.2	5.5
$N = 24$	1.0	2.6	4.3	6.3	7.2

Panel B: How many times is an existing benchmark matched (but not exceeded) by a subsequent firm (maximum = 999)?

	$K = 0$	$K = 1$	$K = 3$	$K = 7$	$K = 11$
$N = 4$	999	710	466		
$N = 8$	999	600	251	62	
$N = 12$	999	507	184	28	6
$N = 24$	999	205	57	7	0

Panel C: How many firms are released on average before a newly established benchmark is matched or exceeded?

	$K = 0$	$K = 1$	$K = 3$	$K = 7$	$K = 11$
$N = 4$	1.0	1.7	1.7		
$N = 8$	1.0	1.3	2.4	13.5	
$N = 12$	1.0	2.4	4.5	13.7	55.6
$N = 24$	1.0	3.0	24.7	66.9	79.8

Description. For each level of N and K shown here, 20 landscapes were generated. On each landscape, 1,000 firms were released sequentially, each at a random location. Each firm improved its strategy incrementally ($M = 1$) until reaching a local peak. Each time a firm exceeded the highest value attained so far, that firm became the new benchmark. The panels address various questions concerning the evolution of the benchmark. Each data point is an average over the 20 landscapes.

Interpretation. As the underlying decision problem becomes complex, new standards for performance are set more often, but those standards are much less readily matched by others.

5. Imitation by Following the Leader

Surely, one might argue, incremental improvement is a poor caricature of the imitation process. An imitator does not try to match a successful rival solely by making uninformed, local "tweaks" to its own choices. Rather, it observes the high performer closely: reverse-engineers its products, interviews its customers and former employees, speaks with its suppliers, dissects its public literature, benchmarks its every activity, and so forth. Once well informed, the imitator changes a large number of its choices—leaps in decision space toward the configuration of the successful firm—in hope of landing somewhere near the profitable strategy. From there, it edges uphill.

Complexity makes this style of imitation less effective, just as it undermines incremental improvement. I discuss two structural reasons for this, then turn to an integrated simulation.

5.1. Structural Deterrents to Follow-the-Leader Imitation

Vanishing Basins of Attraction. Suppose an imitating firm can alter numerous choices at once.

Management cannot guarantee that it will recreate the strategy of a high-performing rival exactly, however, because it does not understand the rival's operations precisely, because management's control is imperfect, or because perfect replication is prohibitively expensive. Nonetheless, the would-be imitator hopes that it will come close enough to the successful set of decisions that subsequent "tweaks" will take it to the peak. Such a wholesale reconfiguration is successful only if the imitator's leap lands in the basin of attraction below the high performer's peak.

Unfortunately for the imitator, complexity undermines its leap. When decisions are numerous and richly cross-coupled, the attraction basins of high peaks become small, and the odds of jumping into them grow correspondingly poor. To see this, consider the extreme cases of $K = 0$ and $K = N - 1$, with $M = 1$. For $K = 0$, of course, the basin of attraction of the global peak is the entire landscape regardless of N, and incremental improvements will take a firm from any set of choices to the best configuration. The accuracy of a long jump is irrelevant. For $K = N - 1$, however, the situation is very different. Consider the global peak, which typically has among the broadest basins of attraction. The portion of points from which a firm can tweak its way to the global peak dwindles rapidly with N. The basin of the global peak covers only 7% of the landscape when $N = 8$, $K = 7$, and less than 0.5% when $N = 16$, $K = 15$.

Spreading Peaks. The problem of the dwindling basins would not be so vexing for would-be copycats if high peaks on the landscape clustered together. If peaks lay in "mountain ranges," then an imitator would, on average, find a long jump toward a high performer beneficial even if it failed to land in precisely the right basin of attraction. The imitator might not wind up atop Mount Everest, but at least it would be in the Himalayas.

Once again, however, interactions work against the imitator. As K increases, *peaks on the landscape spread apart*. This effect is illustrated in Figure 2. For $N = 24$ and various values of K, I release 100 firms and allow them to move uphill incrementally until they discover local peaks. For each peak, Figure 2 plots two items against one another: (1) the height of the peak, and (2) its distance in decision space from the highest peak found on the landscape. For $N = 24$ and $K = 1$, note the strong negative correlation. The higher peaks tend to be near "Everest" and the lower peaks far away. As K rises, the correlation fades and high peaks spread apart. The location of a high peak carries less and less information about where other high summits can be found.

The intuition behind this is straightforward. When choices are loosely connected, the strategy of a successful firm can typically be decomposed into separable subsystems of well-made choices (Simon 1962). Reconfiguring one of the subsystems has limited effect on the others and generates a new local peak that is quite close in decision space to the original one. The mountain range around Everest consists of Everest-like decision configurations with individual subsystems modified. In contrast, when choices are tightly interlocked, the strategy of a successful firm cannot be broken into subsystems. High peaks are then likely to be altogether different ways of doing business rather than slight variations on a theme. When interactivity is strong, slight variations destroy the theme.

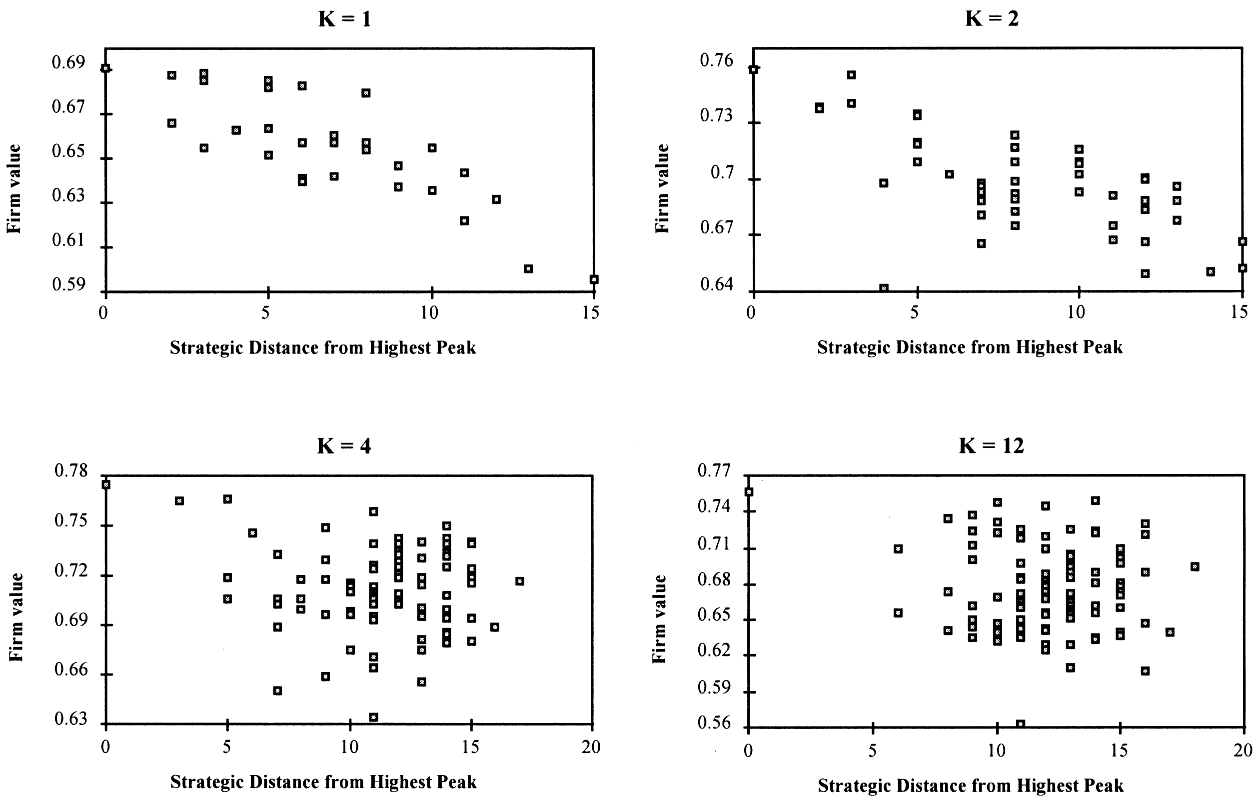
Note the special role played by K in determining these structural features. Never does an increase in N alone, the mere number of decisions a firm faces, impede imitators. Both structural obstacles discussed here (as well as the two considered in §4.1) hinge crucially on K. Interactions, then, are at the core of the imitation barrier proposed in this paper. High values of N are important in deterring imitation largely because they create rich opportunities for interactions.

5.2. Simulation Results

Simulations confirm what the structural features propose: complexity makes follow-the-leader imitation less successful. Recall the key parameters governing the search processes: J, the number of decisions reconfigured in a leap toward the benchmark, and θ, the probability that each of the J decisions will be imitated correctly in the reorientation.

First consider reorientations in isolation. As in §4.2, I release 500 imitators on each of ten landscapes, but now allow them to "leap" in decision space toward the benchmark strategy. Of course, when $J = N$ and $\theta = 1$,

Figure 2 **Spreading of Peaks for $N = 24$**



Description. For $N = 24$ and various values of K, 100 firms were released on a landscape and allowed to tweak uphill, one decision at a time, until reaching a local peak. Each gray point represents a local peak discovered in this fashion. The point shows the height of the peak and the "strategic distance" of the peak from the highest discovered on the landscape. The strategic distance between two peaks is defined as the number of decisions made differently in the two strategies that constitute the peaks. This figure was inspired by Kauffman (1993), fig. 2.7.

Interpretation. For low K, the higher peaks tend to be nearer the highest peak. For high K, this clustering is not evident.

the imitator jumps to precisely the benchmark peak in a single bound. However, even a small departure of θ from 1 to 0.9 creates a substantial problem for the imitator when the underlying decision problem is complex (Table 6). For high N and K, the typical imitator is much worse off than the benchmark firm (panel B). Note the interplay between N and K. As N grows, the likelihood that the imitator will accurately replicate all of the benchmark firm's decisions without a single mistake (θ^N) declines (panel A). As K increases, the impact of each mistake rises (panel B). A high N makes errors likely, while a high K makes errors matter. In a strategy whose pieces are numerous and tightly knit, small probabilities that each element will be replicated incorrectly cumulate to produce a high likelihood that imitators will fare poorly. Once again, panel B emphasizes the role of K, not N alone, in making the imitator worse off.

Next, I couple follow-the-leader and incremental search. The imitator's hope here is that the two modes of search will work together to overcome complexity: a reconfiguration will position the firm in the valley beneath a good strategy, and subsequent incremental improvement will achieve that strategy. Given enough cycles of following the leader and improving incrementally, the imitator will (almost) surely match the benchmark firm. It turns out, however, that the time required to do so may be lengthy. As results available from the author show, the median number of periods required for imitation to succeed is an order of

Table 6 Results of Follow-the-Leader Imitation Efforts

Panel A: What portion of imitators match or exceed the benchmark firm's performance?

	$K = 0$	$K = 1$	$K = 3$	$K = 7$	$K = 11$
$N = 4$	66.1%	66.6%	66.4%		
	(0.8%)	(0.6%)	(0.8%)		
$N = 8$	43.8%	42.7%	42.7%	43.0%	
	(0.4%)	(0.8%)	(0.7%)	(0.6%)	
$N = 12$	28.1%	29.0%	30.9%	28.4%	28.4%
	(0.5%)	(0.9%)	(1.5%)	(0.5%)	(0.4%)
$N = 24$	11.6%	11.2%	10.3%	9.8%	8.9%
	(0.5%)	(0.9%)	(0.8%)	(0.7%)	(0.6%)

Panel B: How much worse off is the typical imitator than the benchmark firm, as a percent of value?

	$K = 0$	$K = 1$	$K = 3$	$K = 7$	$K = 11$
$N = 4$	5.0%	6.7%	9.9%		
	(0.3%)	(0.9%)	(1.4%)		
$N = 8$	4.9%	7.2%	11.3%	15.9%	
	(0.4%)	(0.5%)	(0.4%)	(1.2%)	
$N = 12$	4.6%	7.9%	11.4%	17.8%	18.4%
	(0.3%)	(0.5%)	(0.8%)	(0.7%)	(0.8%)
$N = 24$	4.7%	7.2%	10.6%	15.4%	18.9%
	(0.2%)	(0.3%)	(0.3%)	(0.5%)	(0.6%)

Panel C: How many decisions does the typical imitator make differently from the benchmark firm?

	$K = 0$	$K = 1$	$K = 3$	$K = 7$	$K = 11$
$N = 4$	0.4	0.4	0.4		
	(0.0)	(0.0)	(0.0)		
$N = 8$	0.8	0.8	0.8	0.8	
	(0.0)	(0.0)	(0.0)	(0.0)	
$N = 12$	1.2	1.2	1.2	1.2	1.2
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)
$N = 24$	2.4	2.4	2.4	2.4	2.4
	(0.0)	(0.0)	(0.0)	(0.0)	(0.0)

Description. For each level of N and K shown here, ten landscapes were generated. One each landscape, a benchmark strategy was pinpointed as the best of 100 exploratory searches. Five hundred imitators were then released on each landscape with randomly assigned initial strategies and were allowed to attempt a "long jump." That is, each imitator tried to replicate the N decisions of the benchmark firm, with a $\theta = 90\%$ chance of success on each decision. The panels address various questions regarding the imitators' performance. Each data point is the average over the ten landscapes, and the standard deviation of the average is shown in parentheses below each data point.

Interpretation. Follow-the-leader efforts become ineffective as the underlying decision problem grows complex. High N makes mistakes likely, and high K makes mistakes matter.

Table 7 Evolution of the Benchmark Under the Follow-the-Leader Heuristic

Panel A: How many times is a new benchmark established (maximum = 1,000)?

	$K = 0$	$K = 1$	$K = 3$	$K = 7$	$K = 11$
$N = 4$	1.0	1.3	1.7		
$N = 8$	1.0	1.2	2.2	3.2	
$N = 12$	1.0	1.5	2.3	4.0	5.2
$N = 24$	1.0	2.6	3.8	5.9	7.3

Panel B: How many times is an existing benchmark matched (but not exceeded) by a subsequent firm (maximum = 999)?

	$K = 0$	$K = 1$	$K = 3$	$K = 7$	$K = 11$
$N = 4$	999	927	762		
$N = 8$	999	876	607	326	
$N = 12$	999	771	527	218	99
$N = 24$	999	631	366	98	35

Panel C: How many firms are released on average before a newly established benchmark is matched or exceeded?

	$K = 0$	$K = 1$	$K = 3$	$K = 7$	$K = 11$
$N = 4$	1.0	1.0	1.2		
$N = 8$	1.0	1.3	1.5	2.3	
$N = 12$	1.0	1.7	2.1	4.0	6.7
$N = 24$	1.0	1.7	2.5	6.3	17.9

Description. For each level of N and K shown here, 20 landscapes were generated. On each landscape, 1,000 firms were released sequentially, each at a random location. Each firm reoriented itself toward the current benchmark ($\theta = 0.9$, $J = N/2$), then moved incrementally ($M = 1$) until reaching a local peak. Each time a firm exceeded the highest value attained so far, that firm became the new benchmark. The panels address various questions concerning the evolution of the benchmark. Each data point is the average over the 20 landscapes.

Interpretation. As the underlying decision problem becomes complex, new standards for performance are set more often, but those standards are much less readily matched by others.

magnitude greater when $N = 24$ and $K = 11$ than it is when $N = 4$ and $K = 0$.

A similar pattern emerges when one examines the evolution of the benchmark strategy under the follow-the-leader heuristic (Table 7). Good strategies are matched far less often when decisions are numerous and intertwined. As a result, firms that attain the best performance to date have a much longer run of success when strategic complexity is high.

6. Robustness of the Results

It is notoriously difficult to demonstrate the robustness of simulation results conclusively. This is true because the space of possible model specifications and parameter values is vast, far too large to explore by exhaustive search. Nonetheless, several features of §§3–5 should reassure the reader that the main result, that complexity deters imitation, is robust. First, the analytical techniques employed from algorithm theory do not rely on simulation. Second, the reported results are averages over large numbers of simulation trials. Third, the simulations span the full range of parameter values permitted by a state-of-the-art personal computer. Fourth, the simulations explore a variety of search heuristics. Fifth and finally, the discussion tries to elucidate the structural features that give rise to the simulation results, and the intuition behind those features, before it turns to simulation.

An appendix available on request subjects the model to a further battery of robustness tests. The results are not sensitive to the arbitrary assumption that contributions C_i are drawn from a $U[0, 1]$ distribution. Nor are they sensitive to the assumptions that the K decisions affecting each C_i are assigned at random and that all decisions are equally influential ex ante. I also consider three alternative ways to identify the benchmark strategy: as the globally optimal strategy; as the highest point discovered in an exhaustive search over a fixed fraction of the landscape; and as the highest point discovered in an exhaustive search over a fixed number of strategies. In all cases, complexity makes imitators worse off, and results are largely comparable to those reported here.

The result of virtually any model can be shaken, however, if one alters the model dramatically enough. An important question is, what are the least extreme modifications that undo the main result? In exploring the model under a wide range of conditions, I noted three modifications that neutralize complexity as an obstacle to imitation. First, the barrier to follow-the-leader imitation disappears if imitators understand the high performer's strategy thoroughly and can replicate it precisely ($\theta = 1$). Second, the barrier posed by complexity declines as incremental imitators are allowed to search more broadly ($M > 1$). Though diminished, the barrier remains so long as $M < N$. When imitators are permitted global search ($M = N$), the barrier disappears altogether. Similarly, and as noted in §3.3, the NP-completeness result loses its force if imitators can evaluate alternative strategies very rapidly and N is modest. Exhaustive search then becomes practical. Of these three conditions, the one that seems most likely to arise in reality is that N, the number of decisions over which managers have true latitude, is limited. This might occur, for example, if managerial choice is rigidly constrained by national institutions.

7. Conclusion

The complexity of a strategy, coupled with limits on what managers know about rivals and can implement, raises a barrier to imitation. When the decisions that embody a strategy are numerous and tightly linked to one another, a firm that discovers an effective combination of choices is protected against imitation in three ways. First, interactions among the decisions make the strategy formulation problem intractable. When the number of decisions is large and strategy evaluation is time consuming, this undermines any logical, algorithmic attempt to rediscover the high performer's strategy. Second, a would-be imitator that tries to match the successful strategy by incremental improvement will soon find itself snared in a web of conflicting constraints. Finally, a would-be imitator that attempts to match the high performer's entire system, via a wholesale reconfiguration, is also likely to fail. Suppose it succeeds in imitating most of the high performer's decisions but misses on a handful. Then because the decisions' effects are tightly knit together, the overall imitation attempt will probably be a failure.

Due to the complexity barrier to imitation, successful strategies may remain unmatched even though they are open to public scrutiny and even after many of their critical ingredients are adopted by competitors. This may help to explain the persistent superior performance of certain firms, as mentioned in §1. It may also shed light on the slow diffusion of complex bundles of practices whose superior performance is well documented: e.g., modern manufacturing techniques (Milgrom and Roberts 1990), flexible production systems in the auto industry (MacDuffie 1995), bundles of high-performance work practices in the steel finishing industry (Ichniowski et al. 1997), modern logistical practices in the textile business (Hwang and Weil 1997), and science-driven research in pharmaceuticals (Cockburn and Henderson 1996). There may be some truth in the popular business wisdom that the whole of a strategy can resist imitation more effectively than the sum of its parts.

The analysis is also consistent with Sorenson's (1997) finding that tight coupling confers a contemporaneous advantage on computer workstation makers. Sorenson takes vertical integration as a proxy for the extent of interaction across decisions (K) and finds that more integrated organizations have a lower mortality rate at a point in time. His interpretation is that integration allows firms to realize operating efficiencies, exploit market power, tap synergies across vertical stages, or reduce uncertainty. It is also possible, however, that tight integration is advantageous because it protects a successful workstation maker from copycats.

In addition to illuminating well-established patterns, the analysis suggests new empirical fingerprints that complexity might leave behind. Where decisions are numerous and interconnected, one expects imitation attempts to be less successful, and perhaps less common, than they are where strategies are relatively simple. This suggests that the distribution of firm performance will be broader and convergence to mean performance will be slower in settings with more complex strategies. Indeed, industries differ substantially in the range of performance levels they exhibit and in their speed of convergence (Waring 1996), and conventional industrial organization variables explain these cross-industry differences incompletely. A natural hypothesis is that proxies for industry complexity would help to explain the residual differences.

The impediment to imitation I propose arises purely from the character of the decision problem and the nature of search. It owes nothing to the game-theoretic considerations or resource barriers that have dominated recent analyses of imitation. My claim is *not* that other barriers to imitation are unimportant. Indeed, strategic complexity amplifies many of those barriers (and vice versa). Suppose, for instance, that lack of some piece of tacit knowledge prevents a would-be imitator from matching one of the benchmark firm's decisions. The damage this does to the copycat is greater if the affected decision interacts widely with many other choices than if it is isolated. A small misstep is more dangerous on rugged terrain than on a smooth surface.[5]

Though complementary in some ways to game-theoretic and resource-based views of strategy, a focus on complexity does lead one to an unconventional view of what choices are "strategic." Traditionally, strategy scholars have paid closest attention to inherently irreversible choices with large performance implications (e.g., Ghemawat 1991). In the present paper, however, it is difficult to label any isolated decision as strategic. No single choice is innately sticky. What does appear strategic by traditional standards is the choice of a particular logic of configuration, a local peak. The stickiness of individual choices is then endogenous; once a firm adopts a specific configuration or peak, changes in individual choices become costly. Likewise, the consequences of individual decisions become large because they undermine the effec-

[5] Among the imitation barriers associated with the resource-based view of the firm, the nearest relative to complexity is "causal ambiguity." Lippman and Rumelt (1982) argue on logical grounds that a basic ambiguity concerning causal connections between actions and results can make imitation an uncertain venture. They then show how uncertainty in imitation can lead to persistent performance differences and above-normal rates of return despite free entry, profit-maximizing behavior, no scale economies, and no market power. They never explain, however, the roots of causal ambiguity, nor do they specify *how* it deters imitation. The complexity explored here can certainly underlie causal ambiguity. When a winning strategy consists of numerous, interlocking decisions, it is nearly impossible to assign credit for any part of the success to any isolated choice. Success arises from the whole, not the parts, so cause and effect are blurred. On the other hand, complexity can deter imitation even when causal ambiguity is altogether absent. This arises, for instance, if the managers of the would-be imitator understand causal connections perfectly but have imperfect control over internal operations. Hence, complexity may underpin causal ambiguity, but it may also operate independently of ambiguity.

tiveness of accompanying choices. In such a setting, decisions that seem minor or easily altered can take on surprising salience. Recent research on complementary bundles of organizational practices gives at least some evidence that this occurs.

The world depicted in this paper could lead one to a fatalistic view of management: seemingly minor decisions can be salient, relationships among decisions are thoroughly unknown a priori, algorithmic analysis of decision problems is doomed, and reasonable heuristics leave firms trapped in webs of conflicting constraints. What role is there, then, for management science or managerial choice in a complex world? My belief is that major roles remain. I highlight two.

First, the analysis in this paper should make clear the power and importance of anticipating the relationships among decisions. An imitator who understands that a particular pair of choices should always be adjusted in unison effectively reduces the number of decisions and the number of interactions that she must actively consider. As the numerical results make clear, modest reductions in the dimensionality of the decision problem can make imitation significantly easier. Even an understanding that certain variables usually, though not always, move together can be powerful. This is one reason that recent work on complementarity and supermodularity have generated such interest. As discussed in §3, the structure imposed by supermodularity can greatly simplify managerial search. This also helps to explain recurring efforts to classify strategies into a manageable number of generic types (e.g., Miles and Snow 1978, Porter 1980). Such taxonomies hold out the hope that only a handful of coherent sets of choices exist. If true, this simplifies the managerial search problem dramatically. Empirical support for taxonomies is mixed (e.g., Miller and Dess 1993), suggesting that they may simplify decision problems too much. An important role of management science is to isolate relationships that truly, reliably exist among decisions and thereby to simplify decision problems (without simplifying them falsely).

Analyses in the paper also hint at the potency of *altering* the relationships among decisions. In the formal models described here, managers take N and K as immutable. In reality, managers make investments to build connections across decisions or to sever links, i.e., to modularize (Baldwin and Clark 2000). Hence a successful firm might build new and elaborate connections between, say, its production and marketing operations in order to fend off imitators. Alternatively, an imitator may try to break a successful strategy into discrete pieces and attack each piece in turn. A firm might expand its scope of operations, increasing N, or focus on a portion of the total value chain, reducing N. In altering the complexity of its strategy, a firm will face the tension identified in §1: a more modular, less coupled system adjusts more readily to environmental change and is less subject to inertia (Levinthal 1997, Sorenson 1997), but it is more easily attacked by imitators. Baldwin and Clark (2000) observe precisely this tension in the evolution of the computer industry. By adopting modular architectures in the mainframe industry and later in the personal computer market, IBM made itself and its products far more responsive to market needs. This approach, however, allowed widespread entry by imitators. Loosely coupled organizations, it appears, are responsive to change, but vulnerable to imitation. A natural implication, worthy of further research, is that designers of organizations must decide which they fear more: inertia or imitation.[6]

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