

Todo list

- quoi quest-ce linterferometrie, intérêt par rapport à lobsevation par un télescope mono-pupille en terme de résolution angulaire - nulling, contraintes - schema hi-5, explications - VLTI - bandes dobservation - unités astro - Bases de la BeamPropMethod 1



Simulation and characterization of integrated optics beam combiners for astrointerferometry

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Abstract

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Résumé

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Contents

List of Figures	iii
List of Tables	iv
1 Motivation and scientific background	1
2 Simulation of the DBC	1
2.1 Monochromatic light	2
2.1.1 Mathematical formalism	2
2.1.2 Impact of evanescent coupling on the output power	4
2.1.3 Influence of geometrical parameters	7
2.1.4 Adding a «fan out»	12
2.2 Polychromatic light	16
2.2.1 Mathematical formalism	16
2.2.2 Influence of the bandwidth on the V2PM	17
2.2.3 Retrieving the visibility function	18
3 Laboratory characterization of the DBC	20
3.1 characterization setup and method	20
3.2 Retrieving the visibility function	21
A Appendix	22
A.1 The condition number	22
Bibliography	23

List of Figures

1	The Zig-Zag array's cross section with the numbering convention. The four input waveguides are displayed in orange.	2
2	An hypothetical V2PM matrix for a 3 to 2 beam combiner. All visibility and phases in the matrix are the instrumental ones.	3
3	Example of 3 outputs in phase guiding the same power. The modes are Gaussian perfectly centred on the wave-guide	4
4	Influence of different parameters on the phases and amplitudes of the interferogram in the middle wave-guide. The phases relations between the outputs are highly influenced by the evanescent coupling	6
5	Origin and presence of harmonics in the output calculated power	7
6	Evolution of the throughput over the cross section with P_x and P_y at fixed cross-section and simulation parameters (see text for details). (a) at fixed $P_y = 10.8\mu m$, (b) at fixed $P_x = 24\mu m$	8
7	Throughput over the cross section for different optical index differences	10

8	Condition number of the V2PM matrix for different lengths of the DBC part. The power is calculated either over a 1 by 1 micrometer area or the «1/e» area centered on the cross-section. (see text for details)	11
9	12
10	Effect of adding a «flat» fan-out at the output. The color-scale is the same (0 in blue to 0.6 in red) and the scalar field is displayed.	13
11	Baseline 1-2 Visibilities (photometry corrected) and phases relations (referring to the first output) for 3 different area used to calculate the power at the outputs. The «1/e ³ » correspond to an area of $27 \times 35\mu\text{m}$, the «1/e» to an area of $15.596 \times 20.227\mu\text{m}$ and the 3pixels to an area of $3 \times 3\mu\text{m}$	14
12	Condition number of monochromatic V2PM matrices at different wavelength. (a) for the component without fan-out, (b) for the component with fan-out. The geometry of the component is the optimized one (see text for details) .	15
13	Condition number of the V2PM for different bandwidth using the component with and without fan-out. Both DBC part have the same dimensions. . . .	19
14	Deviation of the retrieved phases and visibilities to the expected ones for the component without fan-out. The deviation is calculated by Eq. 10 . . .	20
15	Deviation of the retrieved phases and visibilities to the expected ones for the component with fan-out. The deviation is calculated by Eq. 10. The blue dashed line is the limit of 5% error.	21

List of Tables

1	Condition number and throughput (in parenthesis) for several x and y spacing. The throughput is calculated by the sum of the power at each outputs normalized by the total input power. The power is calculated by a power integral of the simulated field over the wave-guide's cross-section . . .	8
2	Condition number and throughput (in parenthesis) over the WG's cross-section for several width and height. The throughput is calculated by the sum of the power at each outputs normalized by the total input power. The power is calculated by a power integral of the simulated field over the wave-guide's cross-section	9
3	Condition number and throughput over the WG's cross-section for several values of δn . The throughput is calculated by the sum of the power at each outputs normalized by the total input power. The power is calculated by a power integral of the simulated field over the wave-guide's cross-section . .	10
4	Condition number of the V2PM matrix for 3 different area used to calculate the power at the outputs. The «1/e ³ » correspond to an area of $27 \times 35\mu\text{m}$ (and contain more than 99% of the fundamental's power), the «Cross» is the wave-guide's cross section and the 3pixels to an area of $3 \times 3\mu\text{m}$	13
5	Error on the retrieved parameters from the V2PM calibrated using TE/TM polarized light. The TE/TM data refers as the simulated input fields. . . .	15

6	Optimised set of parameters (distance unit in μm)	16
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Introduction

Since antiquity and down to our time, astronomers always tried to see further and further in space requiring more and more sensitive instruments. Increasing the telescope diameter is one way to reach higher angular resolution but in the same time make it more sensitive to atmospheric turbulence. Therefore even with the recent progress in adaptative optics, today's largest telescopes can only resolve few of the brightest and nearest stars.

Using individual telescopes to form an interferometer, the resolution is determined by the distance between the telescopes. Until recently the instruments combining the light from individual telescopes were based on costly and cumbersome bulk optics. The recent advances in manufacturing integrated-optics and especially in laser processing have resulted in new instruments that are operational on sky and delivering higher quality results.

The purpose of this work is to optimise and characterise the performance of Integrated optic (IO) beam combiners and especially one promising type of component called discrete beam combiner (DBC). Allowing to retrieve the astronomical parameters without scanning the interferogram these components could allow to observe fast varying objects.

This report is organised in three parts. In the first part I will present the motivation and the basis of astronomical interferometry. In the second part will be presented the simulation results of the DBC as well as its optimisation. In the last part will be discussed the experimental characterisation of asymmetric couplers, Multi Mode Interferometer (MMI) and of the DBC.

1 Motivation and scientific background

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2 Simulation of the DBC

The discrete beam combiner is a component made of multiple straight waveguides close to each other. It has been demonstrated that in the case of a N telescope DBC, an array of more than N^2 waveguides is needed for efficient operation of the DBC [Min12]. But at this point it hasn't been studied the impact of other geometrical parameters such as the spacing between each waveguides.

The component studied is formed of 23 outputs and four inputs to combine the light from four individual telescopes. A cross-section of it is shown on Fig. 1. Both the input configuration and the "Zig-Zag" shape have already been optimised. After a brief explanation of the theory behind the DBC we will focus on optimising it regarding P_x ,

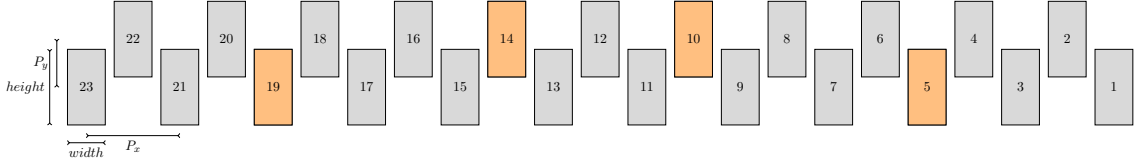


Figure 1: The Zig-Zag array's cross section with the numbering convention. The four input waveguides are displayed in orange.

P_y , $width$, $height$ and the length of the DBC part (the notations refers to Fig. 1) in the case of monochromatic light. In a second part the performances of the optimised component regarding the bandwidth of the input light are simulated. All simulation are performed using the commercial software Beamprop in scalar mode (a full-vectorial mode would have been more accurate but didn't show much different results for both TE and TM polarisation regarding the condition number of the Visibility to Pixel Matrix (V2PM) -see next section-, correlation mode and transparent boundary condition. The grid size was chosen as a balance between computation time and accuracy.

2.1 Monochromatic light

In this section is shown the impact on the performances of the DBC regarding its geometry. Two main parameters are studied, the condition number of the V2PM and the throughput as it hadn't been done before.

2.1.1 Mathematical formalism

As stated before the "Zig-Zag" DBC is composed of 23 outputs and can combine the light from 4 individual telescopes. This component has the particularity that all the information about the coherence function of the studied object is included in the way the 23 outputs are related to each other [TMC⁺07]. In the case of monochromatic light the theory is exact. We will do a brief overview of the theoretical background in this part. For further reading the reader can refer to [TMC⁺07, SMD⁺13, DTL⁺17].

In this part we will consider the light combined from the input A and B at the n^{th} output. In that case the intensity I_n at the output can be expressed as :

$$I_n = \kappa_A I_A + \kappa_B I_B + 2\sqrt{\kappa_A I_A \kappa_B I_B} V_{AB}^{inst} V_{AB}^{obj} \cos(\phi_{AB}^{inst} + \phi_{AB}^{obj}) \quad (1)$$

In this equation κ_i is the transmission coefficient from the input i to the considered output. $inst$ and obj relates the visibility/phases of the instrument and of the observed object. Equation 1 can easily be reduced to Eq.2 in which the produce of the instrumental and object visibility are reduced in V_{AB} .

$$I_n = \kappa_A I_A + \kappa_B I_B + 2\sqrt{\kappa_A I_A \kappa_B I_B} V_{AB} \left(\cos(\phi_{AB}^{inst}) \cos(\phi_{AB}^{obj}) - \sin(\phi_{AB}^{inst}) \sin(\phi_{AB}^{obj}) \right) \quad (2)$$

From Eq.2 the problem of getting the object mutual coherence function can be reduced to the produce of a matrix and a vector. Thus the characteristics of the input fields can be linked to the output intensities by the relation :

$$\vec{I} = V2PM \times \vec{V} \quad (3)$$

In which $\vec{I} = (I_1, \dots, I_M)^T$ represent the intensities at the M outputs,

$\vec{V} = (I_1, \dots, I_M, V_{12}^{obj} \sqrt{I_1 I_2} \cos(\Phi_{21}^{obj}), V_{12}^{obj} \sqrt{I_1 I_2} \sin(\Phi_{21}^{obj}), \dots, V_{N-1,N}^{obj} \sqrt{I_{N-1} I_N} \cos(\Phi_{N,(N-1)}^{obj}), V_{N-1,N}^{obj} \sqrt{I_{N-1} I_N} \sin(\Phi_{N,(N-1)}^{obj}))$ and the visibility to pixel matrix V2PM represent the beam combiner's properties. An example of a V2PM for an hypothetical beam combiner with 3 inputs and 2 outputs is displayed in Figure 2

$$\begin{pmatrix} \kappa_{11} & \kappa_{21} & \kappa_{31} & 2V_{12}^1 \sqrt{\kappa_{11}\kappa_{21}} \cos(\Phi_{12}^{inst}) & -2V_{12}^1 \sqrt{\kappa_{11}\kappa_{21}} \sin(\Phi_{12}^{inst}) & 2V_{13}^1 \sqrt{\kappa_{11}\kappa_{31}} \cos(\Phi_{13}^{inst}) & -2V_{13}^1 \sqrt{\kappa_{11}\kappa_{31}} \sin(\Phi_{13}^{inst}) & 2V_{23}^1 \sqrt{\kappa_{21}\kappa_{31}} \cos(\Phi_{23}^{inst}) & -2V_{23}^1 \sqrt{\kappa_{21}\kappa_{31}} \sin(\Phi_{23}^{inst}) \\ \kappa_{12} & \kappa_{22} & \kappa_{32} & 2V_{12}^2 \sqrt{\kappa_{12}\kappa_{22}} \cos(\Phi_{12}^{inst}) & -2V_{12}^2 \sqrt{\kappa_{12}\kappa_{22}} \sin(\Phi_{12}^{inst}) & 2V_{13}^2 \sqrt{\kappa_{12}\kappa_{32}} \cos(\Phi_{13}^{inst}) & -2V_{13}^2 \sqrt{\kappa_{12}\kappa_{32}} \sin(\Phi_{13}^{inst}) & 2V_{23}^2 \sqrt{\kappa_{22}\kappa_{32}} \cos(\Phi_{23}^{inst}) & -2V_{23}^2 \sqrt{\kappa_{22}\kappa_{32}} \sin(\Phi_{23}^{inst}) \end{pmatrix}$$

Figure 2: An hypothetical V2PM matrix for a 3 to 2 beam combiner. All visibility and phases in the matrix are the instrumental ones.

One can find the Pixel to Visibility Matrix (P2VM) by inverting the V2PM matrix with the relation 4 and then the astronomical information from \vec{V} . To be consistent with the notation introduced in [SMD⁺13], $\vec{V} = (\Gamma_{11}, \dots, \Gamma_{MM}, \mathcal{R}\Gamma_{12}, \mathcal{I}\Gamma_{12}, \dots, \mathcal{R}\Gamma_{N(N-1)}, \mathcal{I}\Gamma_{N(N-1)})$ the object phase and visibility can be extracted by :

$$V_{ij}^{obj} = \sqrt{\frac{(\mathcal{R}\Gamma_{ij})^2 + (\mathcal{I}\Gamma_{ij})^2}{\Gamma_{ii}\Gamma_{jj}}} \quad \Phi_{ij}^{obj} = \arctan\left(\frac{\mathcal{I}\Gamma_{ij}}{\mathcal{R}\Gamma_{ij}}\right) \quad i \neq j$$

$$P2VM = (V2PM^T \times V2PM)^{-1} \times V2PM^T \quad (4)$$

In the light of this formalism the retrieved coherence function from the P2VM can be inaccurate and one has to minimize the condition number of the matrix in order to minimize the possibility of a strong amplification of measure inaccuracy. For further explanation on this subject refers to Annexe A.1.

To characterise the instrumental phase and visibility at an output a coherent source is used (thus the object's visibility is 1 and the phase visibility is 0). A cosine is fitted to the simulated curve of the intensity at an output. Than photocorrection5 is then applied using the intensity simulated with only one input beam used. This process is repeated for all 6 baselines to build all the V2PM.

$$V_n(x) = \frac{I_n(x) - \kappa_A I_A - \kappa_B I_B}{2\sqrt{\kappa_A I_A \kappa_B I_B}} = V_{AB}^{inst} V_{AB}^{obj} \cos(\phi_{AB}^{inst} + \phi_{AB}^{obj}) \quad (5)$$

2.1.2 Impact of evanescent coupling on the output power

The main principle behind the the DBC being the coupling of electromagnetic fields, it is important to understand how much the area chosen to calculate the power at an output could affect the phases relations and visibility. This part focuses on this problem.

Considering only 3 distinct wave-guides of the DBC Zig-Zag component. The cross section of each WG is a rectangle $width \times height$. In such a dielectric wave-guide, there is no analytical solution to the scalar wave equation but according to [Lab08] a good approximation of the transverse field profile is close to a gaussian :

$$\Psi(x, y) \approx \Psi_0 \exp\left(\frac{-x^2}{\omega_x^2} + \frac{-y^2}{\omega_y^2}\right)$$

Using this equation and retrieving ω_x and ω_y from BeamProp simulation by the width of the gaussian at $1/e$ of its maximal amplitude, we can «calculate» the transverse field profile in the wave-guide.

We simulate this behaviour for the following parameters (in μm) :

- Px = 24
- Py = 10.8
- width = 9.5
- height = 17
- $n_{clad} = 2.31$
- $\delta n = 0.005$

In this case we have $\omega_x \approx 7.798$ and $\omega_y \approx 10.114$. The resulting field for 3 outputs in phase and guiding the same power is shown in Fig. 3.

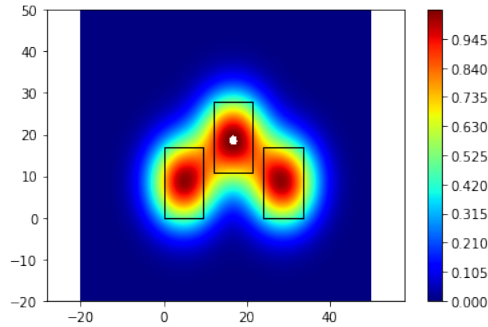


Figure 3: Example of 3 outputs in phase guiding the same power. The modes are Gaussian perfectly centred on the wave-guide

One can see that in this case, where the 3 outputs are in phase, the determination of the power in the middle WG will be badly estimated by a simple power integral as :

$$P \propto \iint_{\mathbb{R}^2} \Psi^*(x, y) \Psi_{sim}(x, y) dx dy \quad (6)$$

where Ψ_{sim} is the simulated output fields as shown on the upper figure. Actually the exact knowledge of the power guided through one individual output is not needed. But if the outputs are not in phase, estimating the "power" by the previous integral would lead to harmonic in the signal vs the optical path difference (OPD). Therefore the integration should be limited to a small area around the WG. In the next paragraph we verify the impact of this choice on the phases relations, instrumental visibility thus the V2PM.

Power of a Gaussian field : For an isolated gaussian field the power P is given by

$$\frac{P}{\Psi_0^2} \propto \iint_{\mathbb{R}^2} \exp\left(\frac{-2x^2}{\omega_x^2} + \frac{-2y^2}{\omega_y^2}\right) dx dy = \frac{\pi}{2} \omega_x \omega_y$$

In the case of our parameters the right hand is equal to $123.88 \mu\text{m}^2$ (a numerical integration using the composite trapezoidal rule lead to 123.76). By integrating the whole field as represented in Fig. 3 over the cross-section we obtain $113.70 \mu\text{m}^2$ but the truly guided power in the central wave-guide calculated over the cross-section should only be $87.39 \mu\text{m}^2$ thus an error of 30 %.

In the opposite case where there is no power in the central wave-guide, and a maximal power in the two surrounding wave-guides we find a guided power in the central wave-guide over the cross-section of $3.10 \mu\text{m}^2$. It can easily be understood that the simulated (ergo the experimental) interferograms will depend of the considered area. The larger this area, the greater the impact of the surrounding wave-guides on the interferogram. In an other hand the smaller this area, the smaller the signal to noise ratio.

Influence on the simulated phases relations : We have seen that the integrating area used to estimate the guided power should have a great impact on the result, we now try to see its impact on the simulated phase relations. To do this the 3 gaussian are multiplied by a cosine to simulate a phase dependency. The left and right outputs are set with phases $\pi/3$ and $2\pi/3$ respectively and the middle one with phase 0. The power is then integrated on different area centred on the waveguide (WG) cross-section.

One can see that in this case, the phase of the output signal is mostly unchanged by the integrating area, but this is only the case for area slightly larger to the WG's cross section. Therefore it might be expected that the phase relation between outputs with comparable power magnitude will stay the same for low variation of the integrating area. The only changed parameter might be the Visibility but one can not conclude as the mean value, ergo the photometries changes too.

To this point we have only seen the impact of the integrating area when all outputs are guiding the same maximal power. It is now studied the impact on a low guided power in the middle WG comparatively to the left and right ones. Same phases are introduced. The power in the left and right WG are the same and 4 times the power in the middle WG. The results to those simulations are shown in Fig. 4

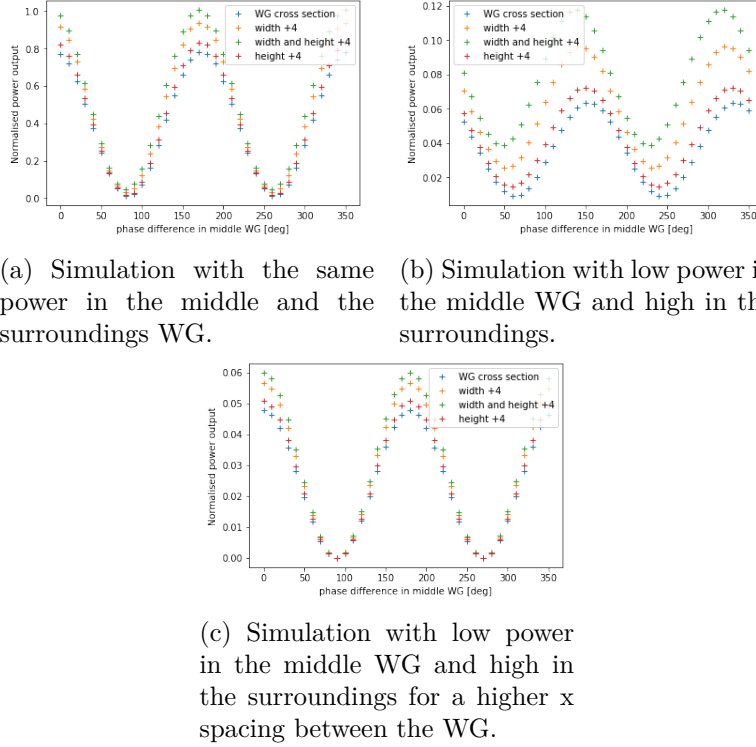


Figure 4: Influence of different parameters on the phases and amplitudes of the interferogram in the middle wave-guide. The phases relations between the outputs are highly influenced by the evanescent coupling

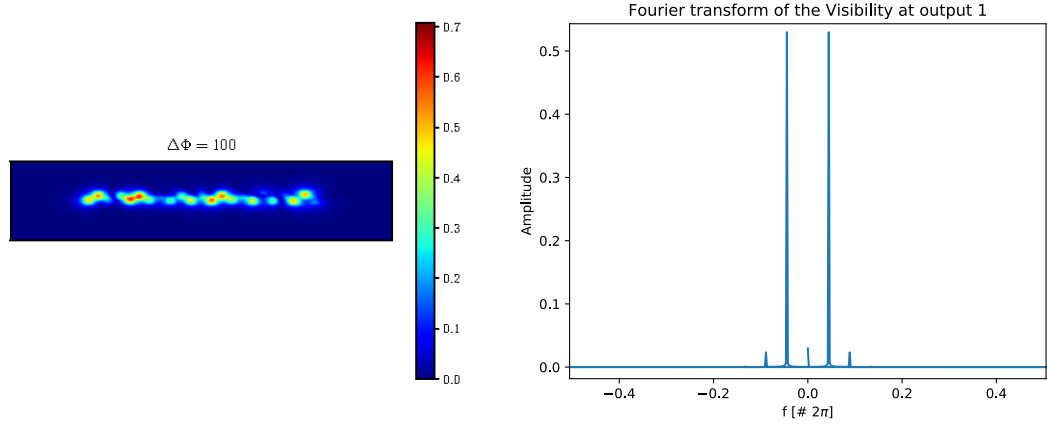
As can be seen, the more the power difference between the middle WG and the sides WG and the larger the integrating area, the more impacted the retrieved phase differences. Therefore it seems that the V2PM matrix of the component should be calculated not only for the isolated component but also with the imaging system. One way to get rid of these dependency would be to have a greater spacing between the WG at the output so that the evanescent coupling is as low as possible (as shown in Fig. 4c). Thus the use of a «fan out» could be an option to calculate the power over a larger area thus have a greater integrated power (ergo a smaller signal to noise ratio (SNR)).

2.1.3 Influence of geometrical parameters

Knowing the problem exposed in the previous section, the "ideal" surface over which the power is calculated has to be estimated. Then the influence of the geometrical parameters on the performances (estimated by the condition number of the V2PM) can be studied.

In all the following, the amplitude of visibility (or visibility) refers to V_{ij} (sometimes to the all function $V_{ij}\cos(\phi_{ij})$ where ϕ_{ij} is a function of the OPD. A good approximation of ϕ_{ij} is $\frac{2\pi x}{\lambda}$ where x is the OPD. V and ϕ are both obtained by fitting the simulated curve with a cosine.

To estimate the critical surface, the power at an output versus the OPD is simulated using the software *BeamPropTM*. It has been found that the presence of harmonics in the signal should not be higher than 2% of the main signal (the cosine). A surface corresponding to the waveguide's cross-section appeared to solve this problem for a wavelength of $3.8\mu\text{m}$ as shown on Fig.5b.



(a) Example of the output simulated field. (b) Spectrum of the visibility at output 1

Figure 5: Origin and presence of harmonics in the output calculated power

All simulation unless explicitly written are performed using the previously determined integrating area. Moreover the base parameters : $P_x = 24\mu\text{m}$, $P_y = 10.8\mu\text{m}$, $width = 9.5\mu\text{m}$, $height = 17\mu\text{m}$, $\delta n = 0.005$, $\lambda = 3.4\mu\text{m}$, length of the DBC's part = 25mm , length of the input = 25mm , together with transparent boundary condition. The grid dimensions and z step are chosen as a balance between computation time and desired accuracy. For general results one can choose a «coarse» grid and then narrow it for more accurate results. Results presented are for scalar solution of the wave equation. Therefore they do not incorporate impacts of polarization effects. Moreover the material are supposedly perfects in the simulation (i.e. isotropic material, no absorption, no scattering...).

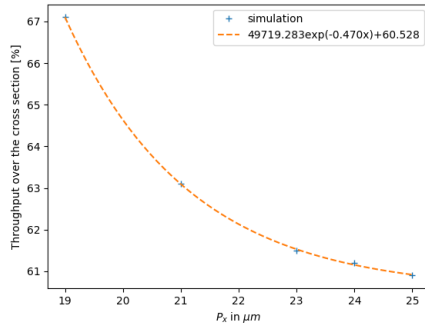
To ensure a good accuracy on the retrieved astronomical parameters, the V2PM's condition

number must be as close to 1 as possible (see A.1). This part will be focused on lowering the V2PM's condition number by using different geometrical parameters. The launch fields are Gaussian of same power and of $1/e$ diameters the width and height of the wave-guide. Thus a coupling loss of approximately 10% occurs. Fresnel losses are not included.

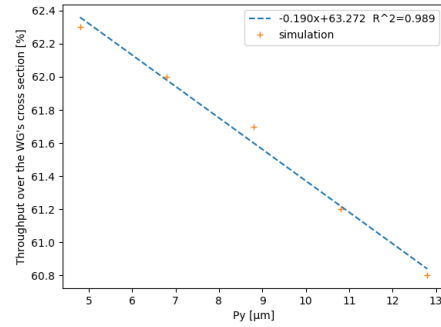
X and Y spacing: Simulations were performed for different values of P_x and P_y to find which parameters minimizes the condition number. The results are shown in table 1. With the few tested parameters one can only conclude that the V2PM condition number seems to be minimum for $P_x \approx 24\mu\text{m}$ and $P_y \approx 10.8\mu\text{m}$. Concerning the throughput, it seems to have a quite linear dependency with P_y and an exponential decay with P_x within the tested range (and only within the tested range as the throughput should stay between 0% and 100%). Fitted results are displayed in figure 6. These results are obtained for one set of parameters and should be valid only within the tested range.

$P_x \backslash P_y [\mu\text{m}]$	4.8	6.8	8.8	10.8	12.8
19				10.68 (67.1%)	
21				10.69 (63.1%)	
23				24.77 (61.5%)	
24	24.05 (62.3%)	32.6 (62.0%)	18.77 (61.7%)	7.16 (61.2%)	16.6 (60.8%)
25				11.7 (60.9%)	

Table 1: Condition number and throughput (in parenthesis) for several x and y spacing. The throughput is calculated by the sum of the power at each outputs normalized by the total input power. The power is calculated by a power integral of the simulated field over the wave-guide's cross-section



(a)



(b)

Figure 6: Evolution of the throughput over the cross section with P_x and P_y at fixed cross-section and simulation parameters (see text for details). (a) at fixed $P_y = 10.8\mu\text{m}$, (b) at fixed $P_x = 24\mu\text{m}$

The cross section: Now that a minimum of the condition number regarding x and y spacing has been found, the wave-guide's dimensions has to be optimized too. In the tested range all dimensions ensure that the wave-guide is mono-mode to have all «ray» traveling at the same speed in z direction (in the fundamental mode, most of the energy is traveling close to the core of the WG). This is to ensure a low modal dispersion and low coupling losses of the input field. These simulations are performed for $P_x = 24\mu\text{m}$ and $P_y = 10.8\mu\text{m}$.

height width	15	16	17	18
7.5			5.07 (45.9%)	
8.5			8.00 (54.1%)	
9.5	7.76 (57.8%)	8.29 (59.5%)	7.16 (61.2%)	10.12 (62.4%)
10.5			14.6 (67.5%)	

Table 2: Condition number and throughput (in parenthesis) over the WG's cross-section for several width and height. The throughput is calculated by the sum of the power at each outputs normalized by the total input power. The power is calculated by a power integral of the simulated field over the wave-guide's cross-section

One more time a minimum of the condition number is found for $width = 7.5\mu\text{m}$ and $height = 17\mu\text{m}$, but as such a width would lead to too much more refinance (which our simulation doesn't take into account) a width of $9.5\mu\text{m}$ will be used instead. The throughput seems to be linear for dimensions of the wave-guides slightly different. But only for dimensions closes to the simulated ones.

Optical index difference: The wave-guides used are rectangular dielectric wave-guides with step index. As a higher optical index difference lead to a stronger mode confinement, a higher throughput over the cross section is to be expected with a higher δn . This paragraph focuses on finding the best δn to have an higher throughput together with a low condition number of the V2PM matrix. In this simulation the previously «optimised» geometrical parameters are used (except for the width of the WGs which is $width = 9.5\mu\text{m}$) and the throughput is still calculated over the wave-guide cross-section. The results are shown in table 3.

It seems that the throughput evolves as the logarithm of δn within the tested range (and only within the tested range)(see fig.7).

The condition number seems to be minimum for $\delta_n = 0.004$ but as it is only of 1 lower than for $\delta_n = 0.005$ and the throughput over the wave-guide's cross-section is 10% greater in this case, $\delta_n = 0.005$ would be used as «optimised» optical index difference.

Lengths: An other parameter that has to be optimized is the length of the DBC's part. To this point all simulations were performed for a length of 25mm. An other length that

δn	condition number	throughput
0.002	61.3	23%
0.003	13.5	40%
0.004	6.1	52%
0.005	7.16	61%
0.006	15.0	68%
0.007	14.6	73%

Table 3: Condition number and throughput over the WG's cross-section for several values of δn . The throughput is calculated by the sum of the power at each outputs normalized by the total input power. The power is calculated by a power integral of the simulated field over the wave-guide's cross-section

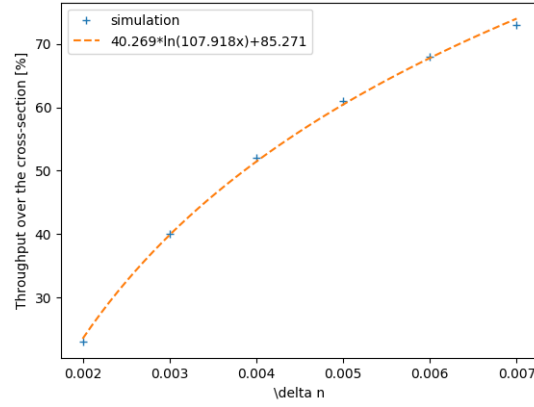


Figure 7: Throughput over the cross section for different optical index differences

could be optimized is the length of the inputs which were also to this point of 25mm, but as the x and y spacing of the inputs ensures that the fields aren't coupled, this shouldn't impact the V2PM condition number. This was verified in the simulation for length greater to 1cm. The results are shown in figure 8. The «1/e» area refers to a rectangle of width and height the 1/e width and height of the fundamental mode of the wave-guide (for $\delta n = 0.005$ which should contains 91% of the mode power)

The simulations were performed for a length greater than 15mm to be sure that each inputs field propagates through the 23 outputs as can be seen in the figure 9a.

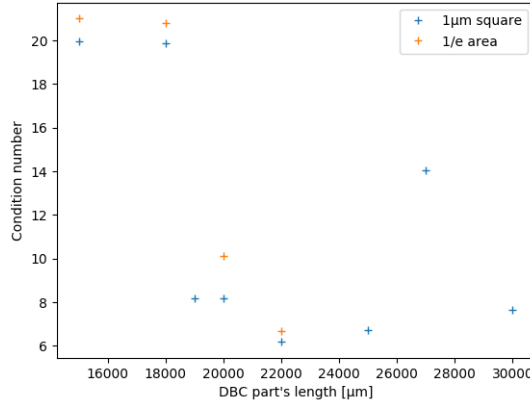
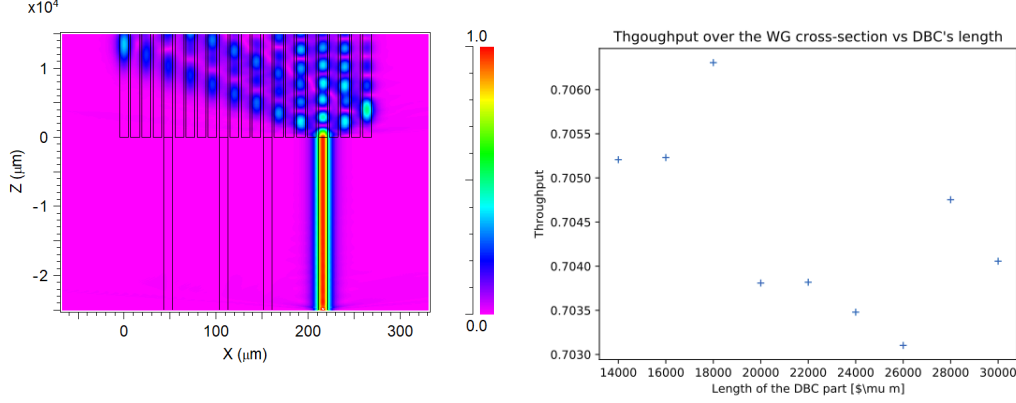


Figure 8: Condition number of the V2PM matrix for different lengths of the DBC part. The power is calculated either over a 1 by 1 micrometer area or the «1/e» area centered on the cross-section. (see text for details)

One can notice that the condition number tend to be higher for a short length of the DBC's part, and have a minimum of 6.15 for a DBC part of 22mm long. Moreover with a higher power integrating area, the condition number seems to behave the same for a large or a small integrating area so that a minimum of the condition number found with an integrating area should remain a minimum for another (if the area isn't too large and doesn't overlap surrounding fields).

Concerning the throughput, it seems to be constant within the tested length (see Fig.9b). Of course our simulated WG doesn't include scattering, surfaces defaults etc. The only kinds of losses that occurs in this simulation are bending losses and radiation in the cladding (absorption is negligible). The reader may have noticed a 10% higher throughput than in the previous sections. This is because the input field in this simulation is a Gaussian of «1/e» width and height the «1/e» width and height of the fundamental mode of the WG. Thus the 10% coupling losses experienced in the previous do not take place in this simulation.

It has be seen that the V2PM matrix is quite dependent of the area over which the power is calculated at the output. One possible way to minimise this dependency would be to



(a) Propagation of the scalar field from the 4th input. A length of 12mm seems to be a minimum to have the field reaching every single output.

(b) Throughput over the cross-section for different lengths of the DBC part. Two area are considered to estimate the power at the outputs (see text for details).

Figure 9

design a «fan-out». By increasing the spacing of the outputs, the field should be more centered and no overlapping would occur at the output. The next section will focus on this component.

2.1.4 Adding a «fan out»

We have seen that both the visibility and phases relations are dependent of the way the power is calculated. This is caused by the fields overlapping at the outputs. One way to get rid of this effect could be to add a «fan out» at the end of the DBC part and then be able to really take into account of all the flux when calculating the power. Simulations with this have been performed with a spacing of the outputs of 2 times the « $1/e^3$ » width of the fundamental mode to be certain that no more overlapping will occur (see Fig. 10). The length of the fan-out is chosen in order to limit bending losses and 1 cm appeared to be enough. A short straight section of 2 mm is added in order that all the flux have enough length to be well centered in the WG. By doing this one can estimate the power in a wave-guide over a finite area and knowing which percentage of the power of a Gaussian field is contained in the same area, retrieve the total guided power (assuming that the fundamental mode is a Gaussien). But the point is that with such a device, the way of calculating the power at the output should no more impact neither the visibility nor phases relations, ergo the V2PM condition number (as it is the case with the previous component for large integrating area). Therefore in these simulation the power is calculated by the power integral of the simulated field over the WG's cross-section. Results show that most of the visibility in that case are between 0.98 and 1.00 (without any polarization effects

they should be all of 1) where they could be below 0.3 without the fan out. Moreover with the simulated design the fan-out lead to less than 3% losses (bending losses and radiation in the cladding) as the throughput over the WG's cross-section is up to 68%. A simulation of the throughput where the power at the end of each output is calculated over a large enough area to consider that all the power is being accounted we obtain 96.7% of throughput. This results could seems high but it is to be known that the simulations doesn't take into account scattering, material absorption (which might be very low for the considered material (GLS) at this wavelength) etc... Losses are only radiations in the cladding, bending losses and coupling losses with the input field (which we managed to get below 1% in this simulation). Moreover one can see that 68 is almost 71% of 96.7 which is the percentage of the true centered Gaussian field's power calculated over the cross-section of the WG. This shows that at the outputs the fields should be mostly Gaussian centered fields thus the «fan out» might be well designed.



(a) Output of the ZigZag DBC without fan-out. Fields are highly overlapping and not centred in the WG. (b) Output of the ZigZag DBC with a «flat» fan-out. Fields are centered in the WG and not overlapping.

Figure 10: Effect of adding a «flat» fan-out at the output. The color-scale is the same (0 in blue to 0.6 in red) and the scalar field is displayed.

With this geometry of the «fan out», the condition number of the V2PM matrix seems to behave the same with the length of the DBC's part. The simulation leads to a condition number of 6.89 for a length of 22mm (11.61 for a length of 20mm and 7.87 for a length of 19mm). Table 4 show for 3 different length of the DBC part and for three area considered to calculate the power, how much the condition number is now «independent» of the considered area. It also seems that the component with the «fan out» behave the same

Area	Length [mm]		
	18	20	22
$1/e^3$	7.77	11.73	6.89
Cross	7.87	11.61	6.90
$3\mu m$	7.92	11.64	6.89

Table 4: Condition number of the V2PM matrix for 3 different area used to calculate the power at the outputs. The « $1/e^3$ » correspond to an area of $27 \times 35\mu m$ (and contain more than 99% of the fundamental's power), the «Cross» is the wave-guide's cross section and the 3pixels to an area of $3 \times 3\mu m$

than the component without it except that the way the power is calculated at the output

doesn't impact much more the V2PM matrix, visibility and phases relations as can be seen on figure 11. Of course the visibilities are still impacted but they still are greater than 95% for the majority (and greater than 0.75). In the case without the fan-out the visibilities were down to 0.4 or less.

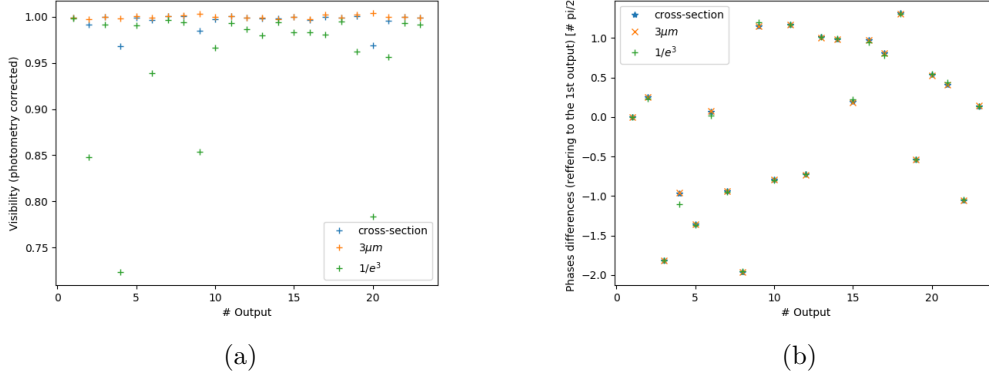


Figure 11: Baseline 1-2 Visibilities (photometry corrected) and phases relations (referring to the first output) for 3 different area used to calculate the power at the outputs. The « $1/e^3$ » correspond to an area of $27 \times 35\mu\text{m}$, the « $1/e$ » to an area of $15.596 \times 20.227\mu\text{m}$ and the 3pixels to an area of $3 \times 3\mu\text{m}$

Effect of polarization on the retrieved parameters To this point no polarization effects has been simulated. In this part the V2PM matrix is calculated using monochromatic TE and TM polarized light. It is then studied the impact of using a TM polarization calibrated V2PM on the TE polarization data to retrieve the astronomical parameters (and vice et versa).

First of all the V2PM calibrated matrix using TE polarized light show a condition number of 6.203 and it is 6.124 for the TM polarized light. In the case of the scalar field (no polarization effect) the condition number was 6.161 which is almost the mean of TE and TM ones. Those two V2PM matrices are used to retrieve the astronomical parameters from the simulated output fields. The results are shown in Tab.5 in which ϵ_i is calculated by $\epsilon_i = \sqrt{\sum \frac{1}{N} (a - \tilde{a})^2}$ where a is the data, \tilde{a} the expected parameter (phase or visibility) and N the number of simulated phase and visibility. One can see that the error on the retrieved parameters are between 30 and 40 times higher when using the V2PM calibrated using polarized light on 90° shifted polarized light. The impact is still very low and justify not taking into account the polarization for the previous simulation.

adaptability to other wavelength Before simulating the DBC using poly-chromatic light the dependence of the design regarding the wavelength is tested. Of course to have

data \ V2PM	TE		TM	
	ϵ_ϕ [rad]	ϵ_V [%]	ϵ_ϕ [rad]	ϵ_V [%]
TE	6.95×10^{-4}	7.14×10^{-4}	2.17×10^{-2}	2.51×10^{-2}
TM	2.08×10^{-2}	2.47×10^{-2}	6.36×10^{-4}	6.77×10^{-4}

Table 5: Error on the retrieved parameters from the V2PM calibrated using TE/TM polarized light. The TE/TM data refers as the simulated input fields.

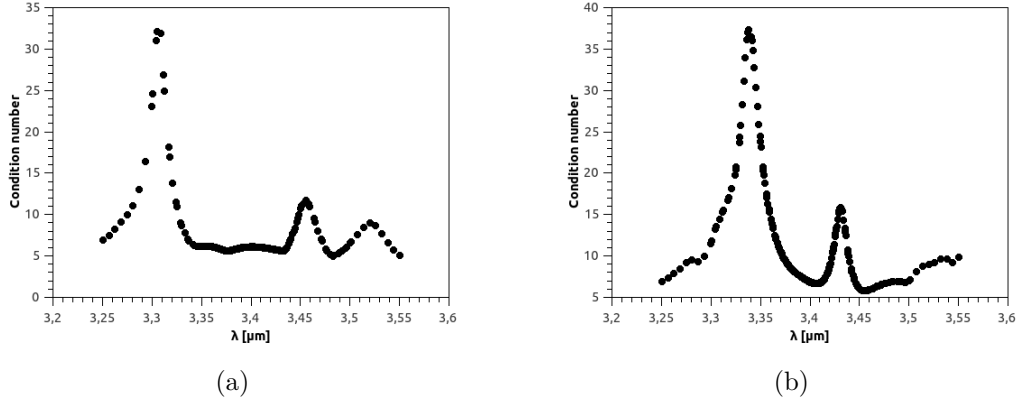


Figure 12: Condition number of monochromatic V2PM matrices at different wavelength. (a) for the component without fan-out, (b) for the component with fan-out. The geometry of the component is the optimized one (see text for details)

a component that behave the same at an other wavelength one can simply multiply its dimensions by the ratio of the wavelength. But as a component will have to be used using poly-chromatic light it is interesting to see how the condition number is influenced for a given design by the wavelength. The design previously "optimized" is used for this simulation. The results of this simulation are shown in Fig.12. As can be seen both of the curves are very similar in first sight. The component without fan-out show a flat curve for lambda ranging from 3.35 to 3.43 μm where for the component with fan-out this "flat zone" almost doesn't exist. This suggest that the component without fan-out could be more stable than the component with. This can be explained by the fact that the geometry of the fan-out act like if the coupling length were longer. This apparent length is strongly dependent of the wavelength because the coupling of the fields between two nearby waveguides are strongly dependents of the wavelength. The zone where the WGs separates will couple a larger wavelength for a longer length because of the larger overlap integral between the two individual WG's modes for higher wavelength. For further explanation on the coupled mode theory the reader can refers to [BEAS07, DM91].

The evolution of the V2PM's condition number regarding the geometrical parameters and the wavelength has been studied and an optimal configuration has been found. This configuration ensures a condition number of approximately 6 which means that an error

on the measured output wouldn't be magnified more than 6 times (in the worst case) by the retrieving algorithm. The evolution of the "usable" throughput through a defined surface has also been studied and result of more than 96% in the case of the component with fan-out and 70% without. This throughput doesn't include the losses from coupling nor Fresnel which should be predominant for this component. The optimized parameters are given in Tab.6. All the previous work has been done using only monochromatic light. But the light from a star is obviously not monochromatic, and it's not wanted to use too narrow-band filters in order to keep high signal to noise ratio (SNR). This is the purpose of the next section.

P_x	2.4×10^1
P_y	1.08×10^1
$width$	9.5
$height$	1.7×10^1
δ	5×10^{-3}
λ	3.4
L_c	2.2000×10^4
L_i	1.0000×10^4
background index	2.31

Table 6: Optimised set of parameters (distance unit in μm)

2.2 Polychromatic light

In order to keep a high enough SNR and also for different needing, the component will be used under poly-chromatic light. In the case of poly-chromatic light, the previous mathematical formalism doesn't hold anymore. In this section will be shown the limitation of the previous formalism, studied the impact of the bandwidth both on the V2MP's condition number and the retrieved mutual coherence function. Experimental results will then be compared to the simulated ones.

2.2.1 Mathematical formalism

Using polychromatic light, the interferogram at the n^{th} output can be expressed as a function of the optical path difference, x , as follow :

$$In(x) = \int_{-\infty}^{+\infty} I_A(\sigma)\kappa_A(\sigma) + I_B(\sigma)\kappa_B(\sigma) + 2\sqrt{I_A(\sigma)\kappa_A(\sigma)I_B(\sigma)\kappa_B(\sigma)} |\mu_{AB}(\sigma)| \cos(\phi_{AB}(\sigma) - 2\pi\sigma x) d\sigma \quad (7)$$

where κ_i relates the transmission from the i^{th} input, I_i the normalized intensity at the i^{th} input, $|\mu_{AB}(\sigma)| = |\mu_{AB}(\sigma)^{inst}| |\mu_{AB}(\sigma)^{obj}|$ the visibility of the interferogram and

$\phi_{AB}(\sigma) = \phi_{AB}^{inst}(\sigma) + \phi_{AB}^{obj}(\sigma)$ the phase of the interferogram. In order to build a V2PM matrix which is independent of the spectrum of the source, it is needed to assume that the spectrum is "flat" within the considered bandwidth. Thus the V2PM matrix will be valid only for quasi-monochromatic light (i.e. a small bandwidth). Doing that the terms I_i are no more wavelength dependant. Eq. 7 becomes :

$$In(x) = t_A \int_{\sigma} I_A d\sigma + t_B \int_{\sigma} I_B d\sigma + 2\sqrt{I_A I_B} \int_{\sigma} \sqrt{\kappa_A(\sigma)\kappa_B(\sigma)} |\mu_{AB}(\sigma)| \cos(\phi_{AB}(\sigma) - 2\pi\sigma x) d\sigma \quad (8)$$

In which $t_i = \frac{\int_{\sigma} I_i(\sigma)\kappa_i(\sigma)}{\int_{\sigma} I_i(\sigma)}$. As our assumption lead us to be limited to quasi-monochromatic light, the visibility should also be relatively independent of the wavelength, as well as the phase if the dispersion of the instrument is negligible. Thus Eq.8 becomes :

$$In(x) = t_A \int_{\sigma} I_A d\sigma + t_B \int_{\sigma} I_B d\sigma + 2\sqrt{I_A I_B} |\mu_{AB}| \int_{\sigma} \sqrt{\kappa_A(\sigma)\kappa_B(\sigma)} \cos(\phi_{AB} - 2\pi\sigma x) d\sigma \quad (9)$$

In order to use the same formalism as in the monochromatic case, we use the so called photocorrection :

$$\begin{aligned} \tilde{I}_n &= I_n - t_A \int_{\sigma} I_A d\sigma - t_B \int_{\sigma} I_B d\sigma \\ V_{AB} &= \frac{2\sqrt{I_A I_B} |\mu_{AB}| \int_{\sigma} \sqrt{\kappa_A(\sigma)\kappa_B(\sigma)} \cos(\phi_{AB} - 2\pi\sigma x) d\sigma}{2\sqrt{t_A \int_{\sigma} I_A d\sigma t_B \int_{\sigma} I_B d\sigma}} \end{aligned}$$

In that case the visibility function is no longer a cosine, but it can be seen as the Fourier transform of the spectral response of the component (in the case of the a flat spectrum signal used at the input). Thus the visibility function is now something like a cardinal sine, and the narrower the bandwidth, the closest to a cardinal sine it gets.

To use the same V2PM format, the amplitude of visibility is taken at the maximum of this interferogram, where $V_{AB} \approx |\mu_{AB}|$. The phase is also taken at the position x_0 of this maximum and reduced using the formula $\Phi_{AB} \approx 2\pi\sigma_0 x_0$ where σ_0 is the middle range frequency of the input signal. Taking the nearest point to the "0" OPD is important as will be further explained in the next section.

2.2.2 Influence of the bandwidth on the V2PM

As stated earlier, in order to have a better SNR one has to integrate more flux. Thus the use of a broader spectrum should improve the performances of the DBC. But as explained in the last section the mathematical formalism doesn't hold in the case of polychromatic light. One has to make the assumption of the input signal being "flat-spectrum" as well as considering the spectral response of the component wavelength independent in order to get it work. The aim of this section is to present the results of a study of the performances of the DBC in the case of polychromatic light.

Using the Beam propagation method to simulate the component behavior, our simulated polychromatic light is composed of the sum of 50 Diracs of same weight in Fourier's space. This should be a good approximation for short bandwidth, less for larger but as those simulations are very time consuming, the number of diracs wasn't increased to keep the same "density of diracs". Nevertheless no sensible differences were seen in the case of more Dirac.

The first studied thing is the impact of the V2PM condition number regarding the bandwidth of the input signal. The simulations are performed on the previously optimized component for a "flat spectrum" centered on 3.4 μm . Same parameters of convergence, boundary condition and Padé order are used than in the First case. The result of these simulation are shown in Fig. 13

The integrating area to estimate the phases and visibilities at each outputs is a 1 by 1 micrometer square in the case of the component without fan-out for reasons explained in previous sections. In the case of the component with a fan-out, the area used is the wave-guide's cross section, but in this case the V2PM matrix is quite independent of this area (as seen in the previous section).

It appears that for the component without the fan-out, the condition number oscillate between 5.5 and 7.5 for bandwidth narrower than 40 nm. It then slowly rises for bandwidth below 150 nm and issues a sharp increase for higher bandwidth (see Fig. 13). A similar dependency is found for the component with a fan-out except that the condition number issues a drop around 200 nm bandwidth. Moreover the conditions number from both component are similar at lower bandwidth but are almost 2 times lower at higher bandwidth for the component with a fan-out. This suggest that the component with a fan-out could be more stable at larger bandwidth. Similar dependency has been reported by Saviuk et al. [SMD⁺13] for experimental data on a square array DBC.

The dependence of the V2PM condition number regarding the bandwidth of the input signal can be interpreted as the degree of validity of the assumption we made in the mathematical formalism. One can notice that for bandwidth lower than approximately 50 to 70 nm the condition number of the V2PM is stable within $\pm 5\%$ in the case of the component without fan-out. This bandwidth is up to 100 nm in the case of the component with fan-out.

However the condition number of the V2PM doesn't give a conclusive statement of the performances of the DBC. Therefore a simulation of the retrieved phase and visibility using the P2VM on simulated data has been performed.

2.2.3 Retrieving the visibility function

In order to have a better idea of the performances of the DBC regarding the bandwidth of the signal, simulation of the retrieved visibilities and phases have been ran. To do that the simulated polychromatic light is injected into 2 of the 4 inputs. At one of the input a phase

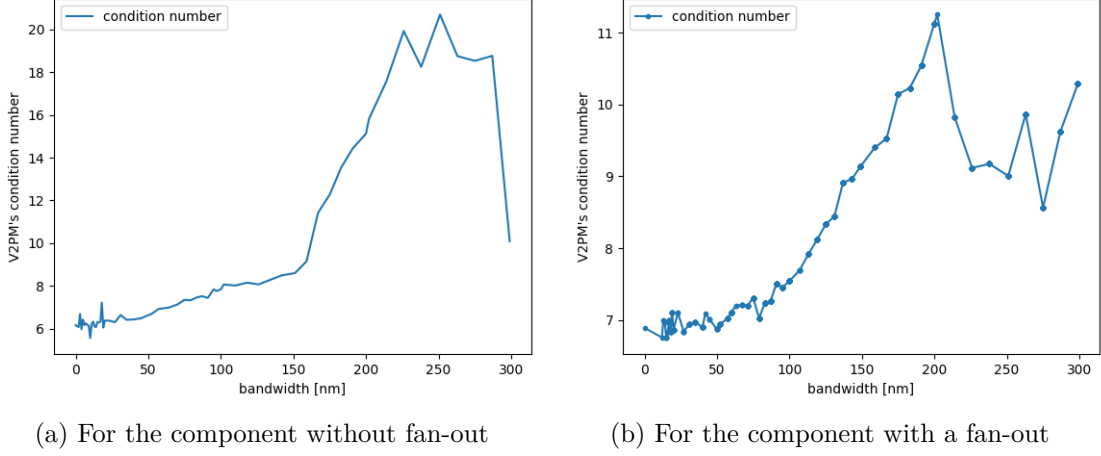


Figure 13: Condition number of the V2PM for different bandwidth using the component with and without fan-out. Both DBC part have the same dimensions.

is added just the same as for constructing the V2PM. At each output an interferogram is reconstructed showing the visibility function versus the OPD. These interferogram (in the shape of a matrix where each column is one simulated phase of the baseline (the distance between two individual telescopes) (BL)) are multiplied by the P2VM to obtain the \vec{V} vector from which the object visibility are calculated as explained in sections 2.1.1 and 2.2.1. The results are presented in Fig.14 and Fig.15

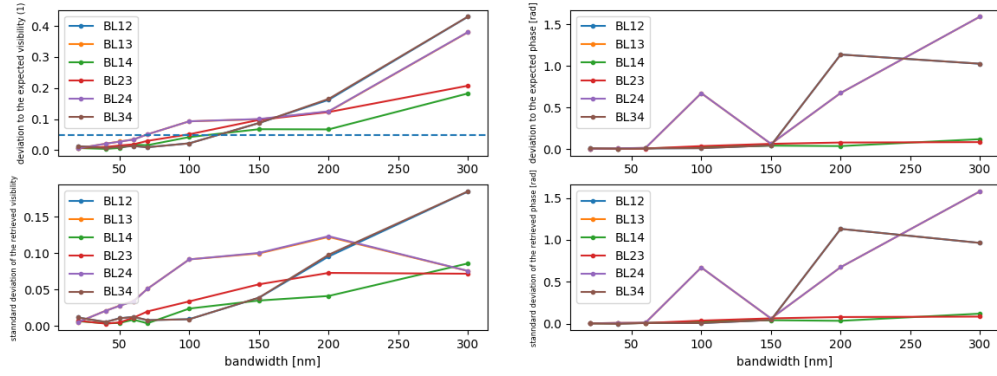
In our simulation no polarization's effects are taken into account so that the visibility of the input source should be 1 at the 0 OPD independently of the phase difference. All simulations were performed with one data point each 12 degrees of phase difference between the inputs from 0 to 360 degrees, for all 50 wavelength evenly spaced in a fixed bandwidth around 3.4 μm and for all 6 baselines.

To estimate the accuracy of the retrieved phases and visibilities, the expected ones are subtracted from the retrieved ones and the deviation ϵ is estimated by the formula :

$$\epsilon = \sqrt{\frac{1}{N} \sum_1^N (a - \tilde{a})^2} \quad (10)$$

where a is the retrieved parameters, \tilde{a} the expected one and N the number of tested retrieved parameters (31 in our case). The result is shown in Fig 14. What can be seen is that the retrieved visibilities are close to the expected ones (at $\pm 5\%$ for bandwidth below 60 nm bandwidth), same as the retrieved phases (at $\pm 0.05\text{rad}$ below 60 nm bandwidth). Meanwhile the standard deviation of both the retrieved phases and visibilities are found to have a similar dependency to the bandwidth. This means that the larger the bandwidth, the more dispersed the retrieved parameters around their mean value. The results for

the component with a "fan-out" is very similar as can be seen in Fig. 15 despite a lower condition number of the V2PM. This clearly shows that the condition number only states the maximum magnification of an error. It also shows that despite the apparent correlation between the retrieved visibilities/phases and the condition number of the V2PM, the impact of this condition number is not the main limiting factor but the approximations of "flat spectral response" is. The broader the bandwidth the more the spatial frequency of the interferogram (i.e the fringe spacing) can get different from $3.4 \mu\text{m}$.



(a) Deviation of the retrieved visibilities to the expected ones

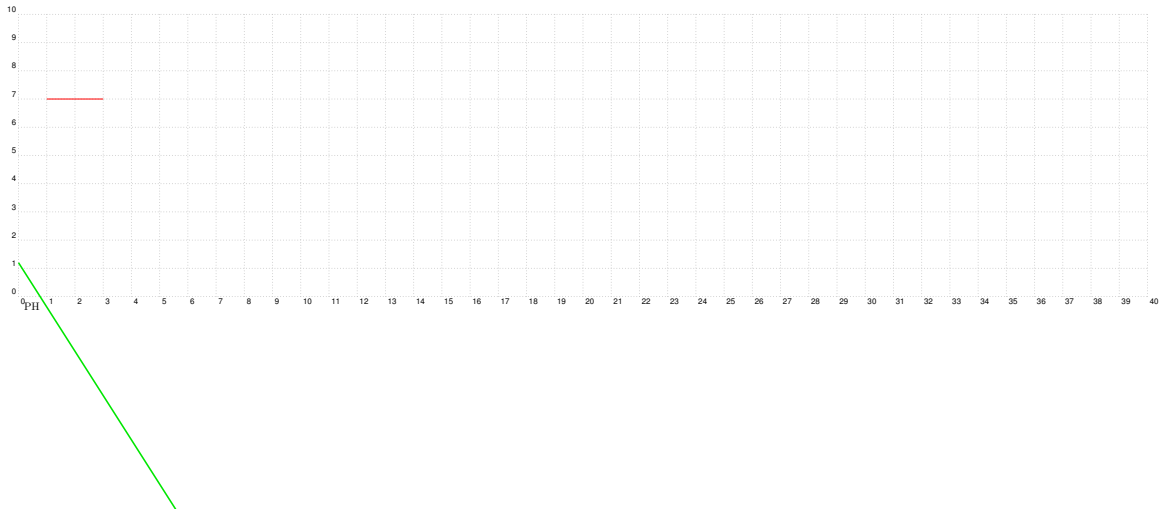
(b) Deviation of the retrieved phases to the expected ones

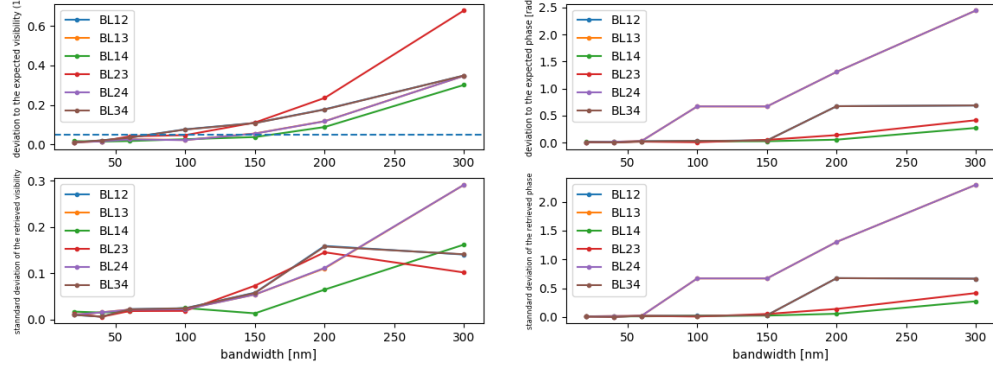
Figure 14: Deviation of the retrieved phases and visibilities to the expected ones for the component without fan-out. The deviation is calculated by Eq. 10

From the results of these simulation, an experimental demonstration of the usability of the DBC with a signal of 70nm bandwidth is still to be done. This is the purpose of the next chapter.

3 Laboratory characterization of the DBC

3.1 characterization setup and method





(a) Deviation of the retrieved visibilities to the expected ones

(b) Deviation of the retrieved phases to the expected ones

Figure 15: Deviation of the retrieved phases and visibilities to the expected ones for the component with fan-out. The deviation is calculated by Eq. 10. The blue dashed line is the limit of 5% error.

3.2 Retrieving the visibility function

Conclusion and Further-work

A Appendix

A.1 The condition number

Considering the following system $A\vec{x} = \vec{b}$ where A is the matrix describing our system (A is a matrix with real coefficients). An error $\vec{\delta x}$ on \vec{x} will lead to an error $\vec{\delta b}$ on \vec{b} . The aim is to know how much bigger or smaller is $\frac{\|\vec{\delta x}\|}{\|\vec{x}\|}$ compared to $\frac{\|\vec{\delta b}\|}{\|\vec{b}\|}$ (i.e how much an error is magnified by the A matrix).

In the case where A is neither symmetric nor square. Then the matrix $A^T A$ is a square symmetric matrix and Then can be diagonalized. Lets call λ_i and \vec{u}_i its eigenvalues and eigenvectors. We can write :

$$A^T A \vec{u}_i = \lambda_i \vec{u}_i$$

Moreover

$$\|A\vec{x}\|^2 = \vec{x}^T A^T A \vec{x} = \|\vec{b}\|^2$$

So $\|\vec{b}\|^2 \leq \max(|\lambda_i|) \|\vec{x}\|^2$ and $\|\vec{\delta b}\|^2 \geq \min(|\lambda_i|) \|\vec{\delta x}\|^2$ and then :

$$\boxed{\frac{\|\vec{\delta x}\|}{\|\vec{x}\|} \leq \frac{\sqrt{\max(|\lambda_i|)}}{\sqrt{\min(|\lambda_i|)}} \frac{\|\vec{\delta b}\|}{\|\vec{b}\|}}$$

The number $\frac{\sqrt{\max(|\lambda_i|)}}{\sqrt{\min(|\lambda_i|)}}$ where $\min(|\lambda_i|)$ is the minimal non zero eigenvalue of $A^T A$, is called the condition number of the A matrix. It means how much an error on the right part of the system can be magnified by the A matrix.

Glossary

DBC discrete beam combiner. 1–3

IO Integrated optic. 1

MMI Multi Mode Interferometer. 1

OPD optical path difference. 5, 7

P2VM Pixel to Visibility Matrix. 3

V2PM Visibility to Pixel Matrix. 2, 5, 7

WG waveguide. 5

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