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Difference event $A - B = \{x \in S : x \in A, x \notin B\}$

Mutually Exclusive event if $A \cap B = \phi$ then A & B are mutually exclusive events.

Exhaustive event if $A \cup B = S$

Equally likely event - let set $S = \{A_1, A_2 \ldots A_n\}$ be a finite sample space if probability of set A_1 = probability of set A_2 = probability of set A_n
$P(\{A_1\}) = P(\{A_2\}) = \ldots = P(\{A_n\})$
If we take 2 events and banne ni probability same hoi to.

Favourable event - No. of cases favourable to an event in an experiment is the no. of outcomes which the happening of event.
Eg: Throwing a fair dice. (standard shape) getting 5:
Getting $5 = \{(1,4), (2,3), (3,2), (4,1)\}$

Probability
- Classical/Mathematical
- Statistical probability

Classical probability - let S be sample space, A be an event, m be favourable outcomes, n be total outcomes

$$\therefore P(A) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}} = \frac{m}{n}$$

$$^nC_R = \frac{n!}{(n-R)!\,R!}$$

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⇒7. There are 5 yellow, 2 red and 3 white balls in the box. 3 balls are randomly selected from the box. Find probability of following events

$1/4$ ① All are of diff. color ② 2 yellow & 1 red $\frac{1}{6}$

$11/120$ ③ All are of same color

Total outcomes (n)

There are total 10 outcomes. From that we have to select 3. So $^{10}C_3 = \frac{10!}{(10-3)!\,3!} = \frac{10\times 9\times 8}{3\times 2\times 1} = 120$

① All are of diff color : A

$$P(A) = \frac{^5C_1 * {^2C_1} * {^3C_1}}{^{10}C_3 \text{ (Total outcomes)}} = \frac{30}{120} = \boxed{\frac{1}{4}}$$

$\frac{1}{4}$

② 2 yellow & 1 red : B

$$P(B) = \frac{^5C_2 * {^2C_1}}{120} = \frac{20}{120} = \boxed{\frac{1}{6}}$$

$\frac{5\times 4\times 3\times 2\times 1}{3\times 2\times 1 \times 2\times 1} = 10$

③ All are of same color : C

$$P(C) = \frac{^5C_3 + {^3C_3}}{120}$$

Here $^2C_3$ won't come. As we have to select 3 balls and there are only 2 red balls.

$$= \frac{10+1}{120} = \frac{11}{120}$$

$\frac{5\times 4\times 3\times 2\times 1}{2\times 3\times 2\times 1}$

3×2

$$\frac{3!}{3!\,(3-3)!} = \frac{3!}{3!\cdot 0!}$$

$0! = 1$

3×2 $\boxed{6}$

Some Important Result.

1. Probability of any event lies between 0 & 1.
2. Total probability is 1.
3. For any pair of mutually exclusive events A & B $(A \cap B = \phi)$ ~~Prob~~ $P(A \cup B) = P(A) + P(B)$
4. $P(A') = 1 - P(A)$
5. $P(\phi) = 0$ (Impossible event)
6. Let A and B be any 2 events of a sample space with $A \subset B$

   (i) $P(B - A) = P(B) - P(A)$

   (ii) $P(B) \geq P(A)$

   $P(A \cap B') = 0$

   if A and B are arbitary events i.e (we don't know whether there is common or not) then.

   $P(A \cap B') = P(A) - P(A \cap B)$
7. Addition rule for abitary events

   (i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

   (ii) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Addition rule for mutually exclusive events.

$P(A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n) = P(A_1) + P(A_2) + \dots + P(A_n)$ $A \cap B$

→ There are 3 red and 3 white balls in a box

Case-1 Pick - Again place inside - Pick

$P(R_1) = \frac{3}{6}$ $P(R_2) = \frac{3}{6}$ Independent event

Case-2: Pick - (Keep outside only) - Pick

$P(R_1) = \frac{3}{6}$ $P(R_2) = \frac{2}{5}$ Dependent event

Independent event - let A and B be the 2 events of a sample space S, then A and B are called independent if

$$P(A \cap B) = P(A) \cdot P(B) \longrightarrow (i)$$

This means that the P(A) doesnot depend on the occurence or non occurence of event B ~~are~~ and vice-versa.

Conditional Probability - let A and B be any 2 events of a sample space S, then probability of occurence of event A when it is given that B has already occured is expressed by the symbol

$$P(A/\underset{\text{already occured}}{B}) = \frac{P(A \cap B)}{P(B)} \longrightarrow (ii)$$

If A and B are independent, then (i) = (ii)

⇒ A problem of statistics is given to 3 students X, Y and Z whose chances of solving it are: 1/3, 1/4 and 1/2 respectively. What is the probability that the problem will be solved.
This is independent as there no effect of 1 on others. Also even if 1 will get ans; it will be solved, 2. then also solved, 3 then also solved

A = Probability problem solved by student X
B = " " " " Y
C = " " " " Z

If Any 1 student out of 3, solve the problem then it is said that problem is solved.

$\therefore P(A \cup B \cup C) = 1 - P(A \cup B \cup C)'$

$= 1 - P(A' \cap B' \cap C')$ $(P(A) = 1 - P(A'))$

$= 1 - P(A') \cdot P(B') \cdot P(C')$ (Independent event formula)

$= 1 - [1 - P(A)][1 - P(B)][1 - P(C)]$

$= 1 - \left[\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{2}\right)\right]$

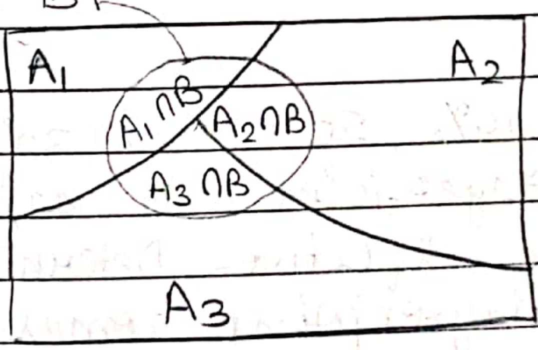
$= 1 - \left[\left(\frac{3-1}{3}\right)\left(\frac{4-1}{4}\right)\left(\frac{2-1}{2}\right)\right]$

$= 1 - \frac{2 \cdot 3 \cdot 1}{24} = 1 - \frac{\cancel{6}}{\cancel{24}_4}$

$= 1 - \frac{1}{4} = \frac{4-1}{4} = \boxed{\frac{3}{4}}$

BAYES THEOREM - is a special case of conditional probability

event B



A_1, A_2, A_3 are mutually exclusive event.

By conditional probability

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \rightarrow (i)$$

$$\therefore P(B/A) = \frac{P(A \cap B)}{P(A)} \rightarrow (ii)$$

From above diagram

$P(B) = P(A_1 \cap B) + P(A_2 \cap B) + P(A_3 \cap B)$

From (i) and (ii)

$P(A \cap B) = P(A/B) \cdot P(B) = P(B/A) \cdot P(A)$

$$\therefore P(B/A) = \frac{P(A/B) \cdot P(B)}{P(A)} \longrightarrow \text{Bayes theorem}$$

Also $P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}$

Bayes Theorem

Let $A_1, A_2, \ldots A_n$ be n mutually exclusive and exhaustive events of a sample space S let B be any event, then

$$P(A_i/B) = \frac{P(A_i) \cdot P(B/A_i)}{P(B)} \quad (P(B) > 0)$$

where $i = 1, 2, \ldots n$

$P(B) > 0$

$P(B) = \sum_{i=1}^{n} P(B/A_i) \cdot P(A_i)$

⇒ 3 boxes contains 10%, 20% and 30% defective finger joints. A fingerjoint is selected at random which is defective. Determine the probability that defective fingerjoint comes from 1) 1^{st} box 2) 2^{nd} box 3) 3^{rd} box

Here pehla it is defective item and then from which box.

B = defective items selection

~~A_1, A_2, A_3 = be item from 1^{st}, 2^{nd} and~~

A_1 = defective fingerjoint is from box 1

A_2 = " " " box 2

A_3 = " " " box 3

$P(B/A_1) = 10\% = 0.1$ Je box che emathi

$P(B/A_2) = \quad = 0.2$ ketli defective che

$P(B/A_3) \quad = 0.3$ eni probability

$P(A_1) = P(A_2) = P(A_3) = \frac{1}{3}$ 3 box mathi koi ek box laie e

Here Pehla box pachi item che in above formula $P(B/A_1)$ but in question it is ~~pt~~ defective item first and then box

$\therefore P(A_1/B)$

Let S be a sample space and R be a set of Real numbers, then a Random variable is a f^n which can take any value from the sample space and having range of some set of real numbers.

$X : S \rightarrow R$

$S = \{HH, HT, TH, TT\}$

X = No. of tails

$X = \{x_1, x_2, x_3, x_4\} = \{0, 1, 1, 2\}$

$\therefore X = \{0, 1, 2\}$

There are 2 types of Random Variable
1) Discrete
2) Continuous

Discrete Random Variable - These are the random variables which can take only finite no. of values in a finite observation interval. For eg: no. of mistakes in a page, no. of telephone calls received per day

Continuous Random Variable - A random variable that takes an infinite number of values is known as continuous Random variable.
[ ] Any closed or open interval, Real, Rational, are uncountable sets.
For eg: height, age, temperature.

DISCRETE PROBABILITY DISTRIBUTION

When 2 dice are thrown, what is the probability of sum of nos. appearing on the top?

X = sum

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

Discrete Probability Distribution - If a random variable X can assume a discrete set of values $x_1, x_2, \ldots x_n$ with respect to probabilities $P_1, P_2, \ldots P_n$, such that sum of the probabilities is always 1 $\left(\sum P_i = 1\right)$

PROBABILITY MASS FUNCTION (PMF)

For a discrete random variable X with a possible values $x_1, x_2, \ldots x_n$ a probability Mass Function is a fn such that

1) $f(x_i) = P(X = x_i) = P(x_i)$ → Symbol

1) $P(x_i) \geq 0$
2) $\sum_{i=1}^{n} P(x_i) = 1$

} Conditions

COMMULATIVE DISTRIBUTION FUNCTION (CDF)

If X is a random variable then $P(X \leq x)$ is called CDF. and it is denoted by $F(x)$

$\therefore F(x) = P(X \leq x) = \sum_{x_i \leq x} P(x_i)$

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PROPERTIES OF CDF

1. $0 \leq F(x) \leq 1$
2. If $x < y$, then $F(x) < F(y)$

MEAN & VARIANCE (OF DISCRETE RANDOM VAR.)

Mean - It is denoted by μ or $E(X)$

$$\mu = E(X) = \sum_{i=1}^{n} P(x_i)\, x_i$$

Variance - It is denoted by σ^2 or $V(X)$

$$\sigma^2 = V(X) = E(X-\mu)^2$$
$$= \sum P(x_i)(x_i - \mu)^2$$
$$= \sum P(x-\mu)^2 \quad (P(x_i) = P)$$
$$= \sum [Px^2 + P\mu^2 - 2Px\mu]$$
$$= \sum Px^2 + \sum P\mu^2 - 2\sum Px\mu$$
$$= \sum Px^2 + \mu^2 - 2\mu^2 \quad \left(\text{Taking } \mu \text{ out}, \ \sum Px = \mu, \ 2x\mu = 2\mu^2\right)$$
$$= \sum Px^2 - \mu^2$$
$$= E(x^2) - [E(X)]^2$$

$$\sigma^2 = E(X^2) - [E(X)]^2$$

Standard Deviation (σ)

$$\sigma = \sqrt{V(X)}$$

(Square root of Variance)

=) Random variable X has the foll. distribution

X	-2	-1	0	1	2	3
P(X)	0.1	K	0.2	2K	0.3	K

1) Determine value of k
2) Mean
3) Variance

We know that $\sum p_i = 1$

$\therefore 0.1 + K + 0.2 + 2K + 0.3 + K = 1$

$\therefore 0.6 + 4K = 1$

$\therefore 4K = 1 - 0.6 \qquad \therefore 4K = 0.4$

$\therefore K = \frac{0.4}{4} = \frac{4}{10 \times 4} = \frac{1}{10} = \boxed{0.1}$

Mean.

$\mu = (-2)(0.1) + (-1)(0.1) + 0(0.2) + 1(0.2) + 2(0.3) + 3(0.1)$

$= \cancel{-0.2} - 0.1 + \cancel{0.2} + 0.6 + 0.3$

$= 0.8$

Variance

$= \sum P(X^2) - [E(X)]^2 \quad 0 + 1(0.2) + 4(0.3) + 9(0.1)$

$= [(-2)(-2)(0.1) + (-1)(-1)(0.1) + 0(0.2) + (1)(1)(0.2) + (2)(2)(0.3)$

$+ (3)(3)(0.1)] - (0.8)^2$

$= 4(0.1) + 1(0.1) + 0 + 1(0.2) + 4(0.3) + 9(0.9) - 0.64$

$= (0.4 + 0.1 + 0.2 + 1.2 + 0.9) - 0.64$

$= 2.8 - 0.64$

$= 2.16$

6
0.8
×0.8
0.64

⇒ No. of emails received / hour has foll. distributio

X = no. of emails.

X	10	11	12	13	14	15
P(X)	0.08	0.15	0.30	0.20	0.20	0.07

Determine Mean & Standard Deviation

Mean:

$\mu = 10(0.08) + 11(0.15) + 12(0.30) + 13(0.20) + 14(0.20)$
$+ 15(0.07)$
$= 0.8 + 1.65 + 3.6 + 2.6 + 2.8 + 1.05$
$= \underline{12.5}$

1.85 Variance: $\Sigma(x^2) - [\Sigma x]^2$
$= [100(0.08) + 121(0.15) + 144(0.30) + 169(0.20)$
$+ 196(0.20) + 225(0.07)] - (12.5)^2$

158.10
-156.25
001.85

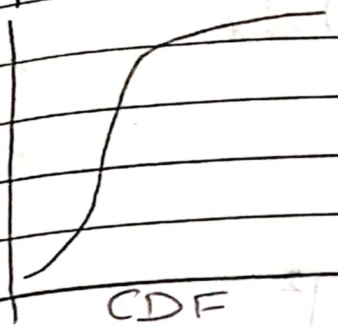
$= 8 + 18.15 + 43.20 + 33.8 + 39.2 + 15.75$
$- 156.25$
$= 158.1 - 156.25 \quad = \underline{1.85}$

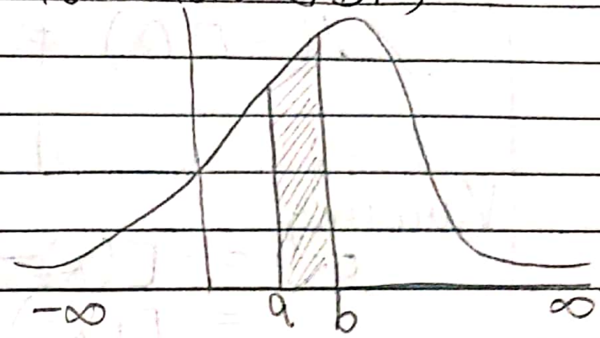
$\therefore \sigma^2 = 1.85$

$\therefore \sigma =$

here there is integration bcoz there is continuous

PROBABILITY DENSITY FUNCTION (PDF)





A f^n f is said to be a PDF of a continuous random variable x where x satisfies the following condition.

1.) $f(x) \geq 0 \quad ; \forall x$

2.) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Note: Probability at any fixed point in the density f^n is always equal to 0

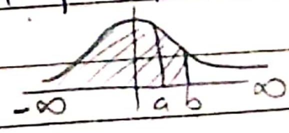
i.e. $P(X = x) = 0$ at any point

Probability of Density f^n between any two points A and B, $P(a \leq x \leq b)$

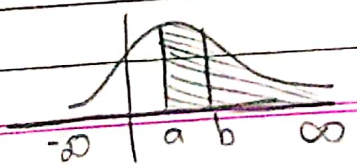
$$P(a \leq x \leq b) = \int_a^b f(x)\,dx$$

DISTRIBUTION FUNCTION

If X is a continuous random variable with PDF f then the distribution f^n F is defined as : $\underset{P(X \leq b)}{F(x)} = \int_{-\infty}^{b} f(x)\,dx$



$P(X \geq a)$ or $F(x) = \int_a^{\infty} f(x)\,dx$



Mean

$$\mu = E(X) = \int_{-\infty}^{\infty} x \cdot \overset{P_i(x)}{f(x)}\, dx$$

Variance

$$\sigma^2 = E(X-\mu)^2$$
$$= E(X^2) - [E(X)]^2$$
$$= \int_{-\infty}^{\infty} x^2 f(x)\, dx - \left[\int_{-\infty}^{\infty} x f(x)\, dx\right]^2$$

Standard Deviation $SD = \sigma$ $\boxed{\sqrt{\sigma^2}}$

⇒ Check whether the f^n defined as

$$f(x) = \begin{cases} 0 & ; x < 2 \\ \frac{1}{18}(3+2x) & ; 2 \leq x \leq 4 \\ 0 & ; x > 4 \end{cases}$$

is a PDF ? If yes then find $P(2 \leq X \leq 3)$

Here ① $f(x) \geq 0$; $\forall x$ is satisfied

② $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx + \int_{4}^{\infty} f(x)\,dx$

$$= 0 + \int_{2}^{4} f(x)\,dx + 0$$

$$= \int_{2}^{4} \frac{1}{18}(3+2x)\,dx$$

$3 \cdot x + 2 \cdot \frac{x^2}{2}$

$$= \frac{1}{18}\left[3x + \frac{2x^2}{2}\right]_2^4$$

$\leq 1.$

Here the f^n satisfies both the condition of PDF and $\therefore$ given f^n is PDF.

$$P(2 \leq X \leq 3) = \int_2^3 f(x)\,dx \qquad \int_2^3 \frac{1}{18}(3+2x)\,dx$$

$$= \frac{1}{18}\left[3x + \frac{2x^2}{2}\right]_2^3$$

$$=$$

BINOMIAL DISTRIBUTION for discrete data

It is possible if the 2 outcomes which are complementary of each other.

Possible only for 2 outcomes like- (succes, failure), (head, tail), (odd, even), (true, fail).

Success and failure

p - success

q - failure

$\therefore p + q = 1$

The problem is to find probability of success which occurs r times and failure which occurs (n-r) times from the total n experiment.

$\therefore$ Probability $= p^r q^{(n-r)}$

for the particular distribution.

Since there are ${}^nC_r$ diff " possible, then

Probability of success $\boxed{P(X = \cancel{x}\,r) = {}^nC_r\, p^r q^{(n-r)}}$

It is called a Binomial distribution.

A,B,C : 3 students.
x : Student is success in interview.
Outcomes: YYY, YYN, YNY, NYY, NNY, NYN, YNN, NNN,

$P(x=2) = \frac{3}{8}$

$2^3 = 8$ sample size (A,B,C), outcomes (Y or N)

Binomial distribution - statisfies both the properties of Mass Function.
So binomial distribution - is Mass function.

Mean $\mu = E(x) = np$

Variance $\sigma^2 = V(x) = npq$

⇒ If 20% of bolt product by a machine are defective. Determine the probability that out of 4 bolts chosen atmost 2 bolts will be defective.
Here 20% defective is given & we have to find defective. So defective will be success.

p : Item is defective $= 20\% = \frac{20}{100} = \boxed{0.2}$

$p+q = 1 \quad \therefore q = 1-0.2 \quad \boxed{\therefore q = 0.8}$

We pick 4 bolts $\therefore n = 4$
we need atmost 2 defective : $P(X \leq 2)$
$= P(X=0) + P(X=1) + P(X=2)$

$p = 0.2, \quad q = 0.8, \quad n = 4, \quad P(X \leq 2)$

$1(0.2)^0\cdot(0.8)^4 + 4\cdot(0.2)^1\cdot(0.8)^3 + 6\cdot(0.2)^2\cdot(0.8)^2$

$1\cdot1\cdot0.4096 + 4(0.2)(0.5120) + 6(0.04)(0.64)$

$0.4096 + 0.1536 + 0.1536$

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0.97$

A multiple choice test in Maths with 40 questions each having 5 options is given to a student. If the student attempt all 40 questions, what are the mean and S.D. of the no. of correct answers.

We have to find no. of correct answer

$\therefore$ p: No of correct answers $= \frac{1}{5}$

$q = 1 - \frac{1}{5} = \frac{4}{5}$

Mean (μ) $= np = 40 \times \frac{1}{5} = 8$

Variance $\sigma^2 = npq = \cancel{40}^{8} \times \frac{1}{\cancel{5}} \times \frac{4}{5} = \frac{32}{5} = 6.4$

S.D. $= \sqrt{6.4}$

A dice is thrown 6 times. If getting odd no. is success. Find probability of

1) 5 success
2) atleast 5 success
3) atmost 5 success $1 - P(X=6)$
4) Mean
5) Variance

p: Getting odd no. $= \frac{3}{6} = \frac{1}{2}$

$q = 1 - \frac{1}{2} = \frac{1}{2}$

Dice is thrown 6 times $\therefore n = 6$

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A, B, C : 3 students.
X : Student is success in interview.
Outcomes: YYY, YYN, YNY, NYY, NNY, NYN, YNN, NNN
$P(X=2) = \frac{3}{8}$

$2^3 = 8$ → sample size (A, B, C)
→ outcomes (Y or N)

Binomial distribution · statisfies both the properties of Mass Function.
So binomial distribution is Mass function.

Mean $\mu = E(x) = np$

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We pick 4 bolts $\therefore n = 4$
we need atmost 2 defective : $P(X \leq 2)$
$= P(X=0) + P(X=1) + P(X=2)$

$p = 0.2, \quad q = 0.8, \quad n = 4, \quad P(X \leq 2)$

$1(0.2)^0 \cdot (0.8)^4 + 4 \cdot (0.2)^1 \cdot (0.8)^3 + 6 \cdot (0.2)^2 \cdot (0.8)^2$

$1 \cdot 1 \cdot 0.4096 + 4(0.2)(0.5120) + 6(0.04)(0.64)$

$0.4 + 0.4096 + 0.1536 + 0.1536$

$\frac{4!}{3!}$ $\frac{4!}{2!\,2!}$

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$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0.97$

⇒ A multiple choice test in Maths with 40 questions each having 5 options is given to a student. If the student attempt all 40 questions, what are the mean and S.D. of the no. of correct answers.

We have to find no. of correct answer

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$q = 1 - \frac{1}{5} = \frac{4}{5}$

Mean (μ) $= np = 40 \times \frac{1}{5} = 8$

Variance $\sigma^2 = npq = \cancel{40}^{8} \times \frac{1}{\cancel{5}} \times \frac{4}{5} = \frac{32}{5} = 6.4$

S.D. $= \sqrt{6.4}$

⇒ A dice is thrown 6 times. If getting odd no. is success. Find probability of

7/32 1) 5 success

7/64 2) atleast 5 success

63/64 3) atmost 5 success $1 - P(X=6)$

4) Mean

5) Variance

P: Getting odd no. $= \frac{3}{6} = \frac{1}{2}$

$q = 1 - \frac{1}{2} = \frac{1}{2}$

Dice is thrown 6 times $\therefore n = 6$

$\frac{6!}{6!} \quad 1 \times \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^0$

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Mean (μ) $= np = 6 \times \frac{1}{2} = 3$

Variance $\sigma^2 = npq = \cancel{6}^3 \times \frac{1}{\cancel{2}} \times \frac{1}{2} = \frac{3}{2} = 1.5$

Getting 5 success $= P(x=5)$

$= \frac{6!}{5! \cdot 1!} \cdot \left(\frac{1}{2}\right)^5 \cdot \left(\frac{1}{2}\right)^1 = \cancel{6}^3 \cdot \frac{1}{\cancel{2}} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{3}{32}$

Getting atleast 5 success $= P(X \geq 5)$

$= P(x=5) + P(X=6)$

$= \frac{3}{32} + \left(\frac{6!}{(6-6)! \cdot 6!} \cdot \left(\frac{1}{2}\right)^6 \cdot \left(\frac{1}{2}\right)^0\right)$

$= \frac{3}{32} + \left(1 \cdot \frac{1}{64} \cdot 1\right) = \frac{3}{32} + \frac{1}{64} = \frac{6+1}{64} = \boxed{\frac{7}{64}}$

Getting atmost 5 success $= P(X \leq 5)$

$= P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=4)$

Or

$1 - P(X=6)$

$= 1 - \frac{1}{64} = \frac{64-1}{64} = \boxed{\frac{63}{64}}$

⇒ If the probability of ~~th~~ defective items is 0.1 find the mean and S.D. of the distribution of defective items of 400.

P: Defective items $= 0.1 = \frac{1}{10}$

n: 400, q: $1 - 0.1 = 0.9$

Mean (μ) $= np = \frac{1}{10} \times 400 = 40$

~~S.D.~~ (σ^2) $= npq = \frac{1}{\cancel{10}} \times \frac{9}{\cancel{10}} \times 4\cancel{00} = 36$

S.D $= \sqrt{\sigma^2} = \sqrt{36} = \boxed{6}$

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⇒ If in a manufacturing company it is found that there are a small chance of ~~17~~ 1 over 500 to be defective. The items are supplied in a packet of 30. Use the binomial distribution to find total no. of packets containing
1) no defective items
2) 2 defective items
In the order of 10,000 packets.

$p = \frac{1}{500}$ (given)

Here $n \to \infty$ and $p \to 0$ (almost)
So here binomial distribution is not possible.

If n is very large ($n \to \infty$) and p is almost near to 0 ($p \to 0$) then we have to use Poission Distribution

Po Distribution

$$P(X=r) = \frac{e^{-m} m^r}{r!}$$

Note: Possion Distribution is a Mass Function.

$P(0) = \frac{1}{e^m}$, $P(1) = \frac{m}{1!\,e^m}$, $P(2) = \frac{m^2}{2!\,e^m}$

$\Sigma P = P(0) + P(1) + \ldots$

$= \frac{1}{e^m}\left(1 + m + \frac{m^2}{2!} \ldots\right)$ This is formula of e^m

$= \frac{1}{e^m}(e^m) = \boxed{1}$ $\therefore \Sigma P = 1$

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Mean (μ)

$$\mu = m = np$$ → +ve finite no.

Variance (σ^2)

$$\sigma^2 = m = \mu$$

In Po Distribution, Mean = Variance.

⇒ Solution of problem ahead.

$p = \frac{1}{500}$, $N = 10,000$ (Not given. Assum bulk = N)

$n = 30$, Chances are 0, 1, 2, . . . , 30 defective.

$m = np = \frac{30}{500} = \frac{3}{50}$

① No defective (of 1 box) $= P(X=0)$

$$= \frac{e^{-m} m^r}{r!}$$

$$= \frac{e^{-\frac{3}{50}} \cdot \left(\frac{3}{50}\right)^0}{0!} = \frac{e^{-\frac{3}{50}} \cdot 1}{1} = 0.94176$$

So the total no. of packets having no defective items = total packets $\cdot$ $P(X=0)$

$= 10,000 \times 0.94176$

$= 9417.6$ ≈ 9418 boxes have no defective items

② 2 Defective items. $= P(X=2)$

$$= \frac{e^{-m} m^r}{r!} = \frac{e^{-\frac{3}{50}} \cdot \left(\frac{3}{50}\right)^2}{2!}$$

$= \frac{0.94176 \times 0.00366}{2} = \frac{0.0034}{2}\ \frac{0.0035}{2}\ 0.0017$

So total no. of packets having 2 defective items

$= 10000 \times 0.0017$

$= \underline{17}$ boxes have 2 defective item.

⇒ An insurance company has discovered that only 0.1 % of the population is involved in a certain type accident every year. If its 1000 policy holders are selected at random from the population what is the probability that not more than 5 of its clients are involved in such accident next year. ($e^{-1} = \frac{1}{e} = 0.3668$)

0.9997

Here $P = 0.1\% = \frac{0.1}{100} = \underline{0.001}$

1000 policy holders are selected $\therefore \frac{1000 \times 0.1}{100} =$

$n = 1000$, $m = np = 1000 * 0.001 = 1$.

Probability of not more than 5 $\therefore \mu \leq 5$.

$$\frac{e^{-m} \cdot m^r}{r!} = \frac{e^{-1}(1)^0}{0!} + \frac{e^{-1}(1)^1}{1!} + \frac{e^{-1}(1)^2}{2!} \cdots \frac{e^{-1}(1)^5}{5!}$$

$$= \frac{1}{e}\left[1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!}\right]$$

$$= \frac{1}{e}\left[1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120}\right]$$

$= 0.3668(2 + 0.5 + 0.1667 + 0.0417 . 0.0083)$
$= 0.3668(2.7167) = 0.9965$
$= \underline{0.997}$

⇒ Potholes on a highway can be a serious problem. The past experience suggest that there are on an average 2 potholes/mile after a certain amount of usage. It is assumed that the poission process applies to the random variable, no. of potholes, what is the probability that no more than 4 potholes occurs in a given section of 5 miles.

$\frac{1}{e} -$

We have to find for 5 miles and per mile value is given 2.

$\therefore \underline{m = 10}$

Probability of no more than 4. $(x \leq 4)$

$$= \frac{e^{-m} m^r}{r!} = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$$

$$= \frac{e^{-10}(10)^0}{0!} + \frac{e^{-10}(10)^1}{1!} + \frac{e^{-10}(10)^2}{2!} + \frac{e^{-10}(10)^3}{3!} + \frac{e^{-10}(10)^4}{4!}$$

$$= \frac{1}{e^{10}}\left(1 + 10 + 50 + \frac{5000}{33} + \frac{12500}{33}\right)$$

$$= \frac{1}{e^{10}}(614166.66 + 416.66) 416.6 = \frac{1}{e^{10}}(644.3267)$$

$$= \frac{1}{e}(644.3267)$$

NORMAL DISTRIBUTION / GAUSSIAN DISTRIBUTION

A continuous random variable X is said to be a normal distribution if its probability density function is given by

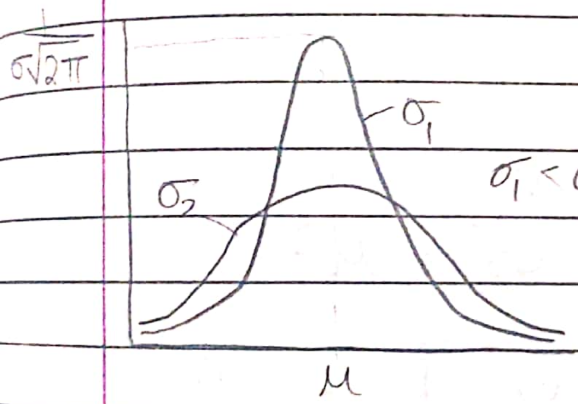
$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

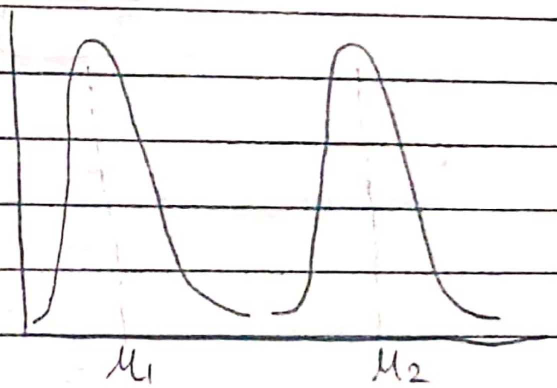
where $-\infty < x \leq \infty$, $\sigma > 0$

x - denotes value of random variable X

μ - expected values of X.

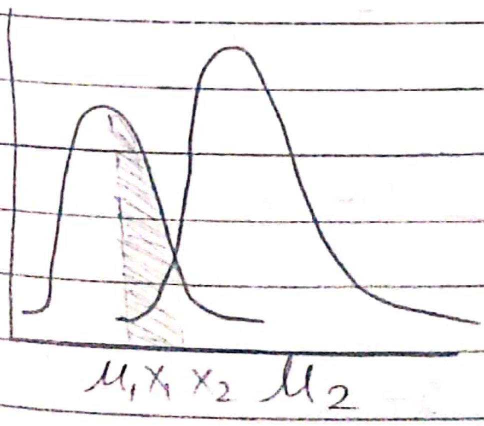
σ - S.D. of X.





Ahiya banne nu μ same che. Pan σ alag.

Ahiya banne nu σ same che pan μ alag.



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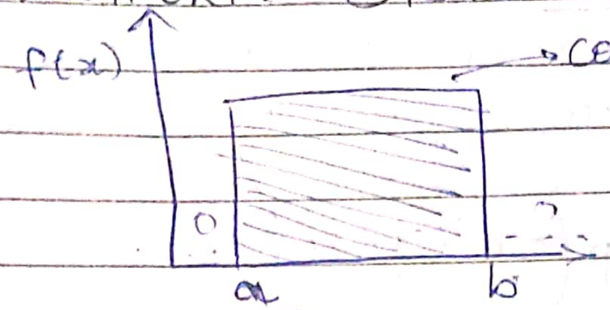
Standard normal distribution

UNIFORM DISTRIBUTIONS



$$f(x) = \frac{1}{b-a}$$

$f(x) = k \; , \; a < x < b$

$= 0 \; , \;$ otherwise

$$\mu = E(x) = \int_{-\infty}^{\infty} f(x)\, x\, dx$$

constant $\leftarrow \dfrac{1}{b-a} \left(\int_a^b \right($

$$= \boxed{\frac{a+b}{2}}$$

Variance $\sigma^2 = E(X^2) - [E(X)]^2$

$$= \int_{-\infty}^{\infty} f(x)\, x^2\, dx - \mu^2$$

$$= \frac{1}{b-a} \int_a^b x^2\, dx - \left[\frac{(b+a)}{2}\right]^2$$

$$= \frac{1}{b-a} \left[\frac{x^3}{3}\right]_a^b - \frac{(b+a)^2}{4}$$

$$= \frac{1}{b-a} \left[\frac{b^3 - a^3}{3}\right] - \frac{(b+a)^2}{4}$$

$$= \frac{1}{\cancel{b-a}} \frac{\cancel{(b-a)}(b^2+ab+a^2)}{3} - \frac{(b^2+2ab+a^2)}{4}$$

$$= \frac{1}{12}(4b^2 + 4ab + 4a^2 - 3b^2 - 6ab - 3a^2)$$

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$= \frac{1}{12}(b^2 - 2ab + a^2)$

$= \boxed{\frac{1}{12}(b-a)^2}$

Standard

For Normal Distribution in 3rd case.

$z = \frac{x-\mu}{\sigma}$

$z_1 = \frac{x_1-\mu}{\sigma}$, $z_2 = \frac{x_2-\mu}{\sigma}$

$\mu = 0$ & $\sigma = 1$

$z = \frac{x-\mu}{\sigma}$

So now $f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}(z)^2}$

Continuous Uniform Distribution

A continuous random variable x with probability density function $f(x) = \frac{1}{b-a}$

$a < x < b$ is a continuous uniform distribution.

Mean $(\mu) = E(x) = \frac{a+b}{2}$

Variance $\sigma^2 = \frac{(b-a)^2}{12}$

Standard deviation $\sigma = \sqrt{\frac{(b-a)^2}{12}}$

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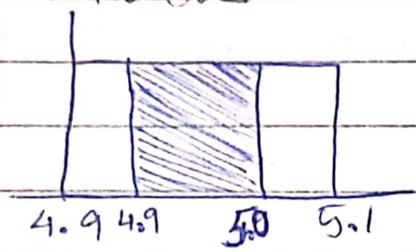
⇒ Random variable X has a continuous uniform distribution on [4.9, 5.1] the probability density function x is $f(x) = 5$. What is the probability that a measurement of current is between 4.95 and 5.0 milli mamp



$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} f(x)\,dx$$

$$P(4.95 < x < 5) = \int_{4.95}^{5} 5\,dx$$

$$= \underline{\underline{0.25}}$$

Mean $(\mu) = 5$

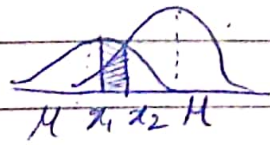
Variance $(\sigma^2) = 0.0033$ milliamp^2

Standard deviation (σ)

Standard Normal Distribution.

Probability for random variable X between points x_1 and x_2 is $P(x_1 < x < x_2) = \int_{x_1}^{x_2} f(x)\,dx$

We need standardization for 3rd case because for 1st area is different and for 2nd area is different.



To standardize the data with $\mu = 0$, & $\sigma = 1$ and take $z = \dfrac{x - \mu}{\sigma}$, we get $f(x) = f(z) = \phi(z)$

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} z^2}, \quad -\infty < z < \infty$$

∴ Probability distribution between z_1 and z_2

$$P(z_1 < z < z_2) = \frac{1}{\sqrt{2\pi}} \int_{z_1}^{z_2} e^{-\frac{1}{2} z^2} \, dz$$

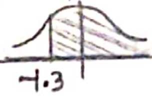
$$z_1 = \frac{x_1 - \mu}{\sigma}, \quad z_2 = \frac{x_2 - \mu}{\sigma}$$

⇒ In the continuous random variable z has the standard normal distribution. Calculate the probability of the following.

1) $z < 1.3$

2) $z > 1.3$

3) $z > -1.3$



4) $-1.37 \leq z \leq 2.01$

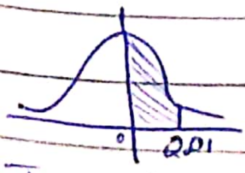
5) $|z| \leq 0.5$

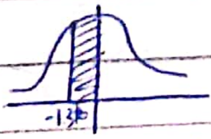
1) $P(z < 1.3) = 0.90320$

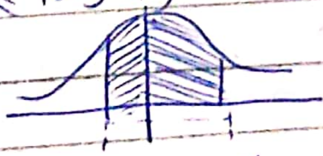
2) $P(z > 1.3) = 1 - P(z < 1.3) = 1 - 0.90320$
$= 0.09680$

3) $P(z > -1.3) = P(z < 1.3) = 0.90320$

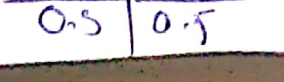
4) $P(-1.37 \leq z \leq 2.01)$
$= P(-1.37 \leq z \leq 0) + P(0 \leq z \leq 2.01)$
$= P(0 \leq z \leq 1.37) + P(0 \leq z \leq 2.01)$







In table we have of whole +ve & -ve together. So we need to subtract 0.5 from value as sum of all = 1 and divided in 2 equal parts so

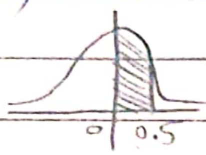


$= (0.91466 - 0.5) + (0.97778 - 0.5)$

$= 0.41466 + 0.47778$

$= 0.89244$

5) $P(|z| \leq 0.5) = P(-0.5 \leq z \leq 0.5)$



$= 2P(0 \leq z \leq 0.5)$

$= 2(0.69146 - 0.5)$

$= 2(0.19146)$

$= 0.38292$

⇒ The compressive strength of sample of cement can be modeled by a normal distribution with mean $\mu = 6000$ kilo/cm^2 and S.D $\sigma = 100$ kg/cm^2

1) What is Probability that a sample strength is less than 6250 kg/cm^2.

2) What is probability that a smample strength is between 5800 & 5900 kg/cm^2.

3) What strength exceed by 95% of the samples

Let x be the compressive strength of sample of cement

$X \sim N(\mu, \sigma)$

$X \sim N(6000, 100)$

1) $P(X \leq 6250)$ Here we have x, so to convert in z

$\Rightarrow P\left(\frac{X-\mu}{\sigma} \leq \frac{6250-\mu}{\sigma}\right)$

$\Rightarrow P\left(Z \leq \frac{6250-6000}{100}\right)$

$= P\left(Z \leq \frac{250}{100}\right) = P(Z \leq 2.5) = 0.99379$

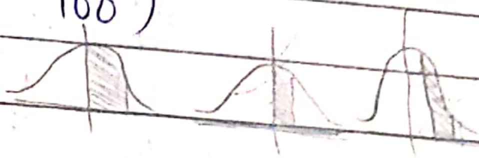
0.13591

2) $P(5800 \leq X \leq 5900)$

$= P\left(\frac{5800-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{5900-\mu}{\sigma}\right)$

$= P\left(\frac{5800-6000}{100} \leq Z \leq \frac{5900-6000}{100}\right)$

$= P\left(\frac{-200}{100} \leq Z \leq \frac{-100}{100}\right)$



$= P(-2 \leq Z \leq -1)$

$= P(-2 \leq Z \leq 0) - P(-1 \leq Z \leq 0)$

$= P(0 \leq Z \leq 2) - P(0 \leq Z \leq 1)$

$= (0.97725 - 0.5) - (0.84134 - 0.5)$

$= 0.47725 - 0.34134$

$= \underline{0.13591}$

3) $P(X \leq x) = P\left(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}\right)$

$= P\left(Z \leq \frac{x-6000}{100}\right) \quad = P\left(Z \leq \frac{x-6000}{100}\right)$

Now 95% is given $P(z) = 0.95$

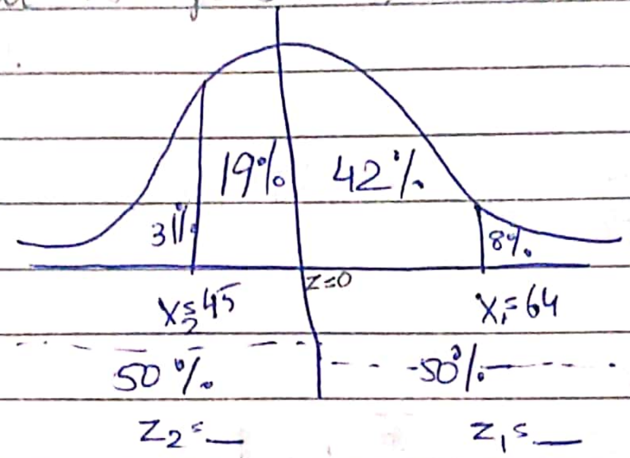
So in table see for 0.95

$= 0.82894$

$= \boxed{0.95}$

⇒ In a normal distribution 31% items are under 45 and 8% are ~~under~~ over 64. Find the mean and S.D of the distribution.

Here it is given in Z, we have to find μ & σ for X.



$\mu = 50$, $\sigma = 10$

$Z_1 = \frac{X_1 - \mu}{\sigma}$, $-Z_2 = \frac{X_2 - \mu}{\sigma}$ → -ve value

or $|Z_2| = \frac{|X_2 - \mu|}{\sigma}$

1.405 ← $Z_1 = \frac{64 - \mu}{\sigma}$, -0.495 $-Z_2 = \frac{45 - \mu}{\sigma}$

$19 + 42 + 8 = 69\% = 0.69$ in z table $0.495 = Z_2$

$31 + 19 + 42 = 92\% = 0.92$ in z table $1.405 = Z_1$

$1.405 = \frac{64 - \mu}{\sigma}$ → (i) $\qquad -0.495 = \frac{45 - \mu}{\sigma}$ → (ii)

By elimination

$1.405\sigma = 64 - \cancel{\mu}$

$-0.495\sigma = 45 - \cancel{\mu}$

$\sigma = 1.900 \times 100 = 19 = 10$

$\therefore \sigma = \frac{19}{1.9} = \frac{19}{19} \times 10$ $\therefore$ $\boxed{\sigma = 10}$

Now $1.405(10) = 64 - \mu$

$\therefore \mu = 64 - 14.05$

$\therefore$ $\boxed{\mu = 50.95 \approx 50}$

Descriptive Statistics

Numerical Summaries of data

Location
Measure of central tendency
(Mean, Median, Mode, Quantiles, Percentiles)

Shape
Variance, S.D., Range

Eg: $-10, 0, 10, 20, 30$	$8, 9, 10, 11, 12$
$\mu = \overline{X} = 10$	$\mu = \overline{X} = 10$
Range $= 30 - (-10) = 40$	Range $= 12 - 8 = 4$
$\sigma^2 = \frac{(-10-10)^2 + (0-10)^2 + (10-10)^2 + (20-10)^2 + (30-10)^2}{5}$	$\sigma^2 = \frac{(8-10)^2 + (9-10)^2 + (10-10)^2 + (11-10)^2 + (12-10)^2}{5}$
$= 200$	$= 2$
More dispersed	

Sample Mean if n obs. in a sample are denoted by $x_1, x_2, \ldots, x_n$ then a sample mean is ~~X~~

$$\overline{X} = \frac{\sum x_i}{n} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

For a finite population with capital N equally likely values, the Probability Mass Function is

$$F(x) = \frac{1}{N}, \quad \mu = \sum x_i \cdot f(x_i) = \frac{\sum x_i}{N}$$

Moreover sample mean is useful, but it does not convey all info. about sample of the data. The variability of the data is described by the sample variance or sample S.D.

Sample Mean $\bar{x}$
Population Mean μ

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$\therefore$ Sample variance $\sigma^2 = \dfrac{\Sigma (x_i - \bar{X})^2}{N-1}$ if sample, N if population

Population variance $\sigma^2 = \dfrac{\Sigma (x_i - \bar{X})^2}{N}$

A computational formula for σ^2
let $x_1, x_2, \ldots x_n$ be a random sample of size n from the distribution X. Then sample variance is $\sigma^2 = \dfrac{n \cdot \Sigma x_i^2 - (\Sigma x_i)^2}{n(n-1)}$

$\Rightarrow$ A population consisting of 5 members (3,5,7,9,11). If a random sample of size $n=2$ is selected find the sampling distribution of sample mean ($\bar{x}$). Also find mean of sample mean, variance of sample mean, population mean and variance.

Population of 5 (3,5,7,9,11)
Sample of 2
So total samples possibe $^5C_2 = 10$

3 5	$\bar{x}_1 = 4$
3 7	$\bar{x}_2 = 5$
3 9	$\bar{x}_3 = 6$
3 11	$\bar{x}_4 = 7$
5 7	6
5 9	7
5 11	8
7 9	8
7 11	9
9 11	10

Mean of sample mean

$\bar{x} = \dfrac{\Sigma \bar{x}_i}{10} = \dfrac{4+5+6+7+6+7+8+8+9+10}{10}$

$= \dfrac{70}{10} = \boxed{7}$

Population mean (μ)

$= \dfrac{3+5+7+9+11}{5} = \dfrac{35}{5} = \boxed{7}$

Sample variance to proper shape of Graph Unbiased

Normal Distribution = Gaussian Distribution

Variance of sample mean

$$S^2 = \frac{(-3)^2+(-2)^2+(-1)^2+(0)^2+(-1)^2+(0)^2+(1)^2+(1)^2+(2)^2+(3)^2}{N-1 = 10-1 = 9}$$

$$= \frac{9+4+1+1+1+1+1+1+9+4}{9} = \frac{300}{99} = \frac{100}{3} = \boxed{3.33}$$

Population variance

$$\sigma^2 = \frac{(-4)^2+(-2)^2+(0)^2+(2)^2+(4)^2}{N=5}$$

$$= \frac{16+4+4+16}{5} = \frac{40}{5} = \boxed{8}$$

Central limit theorem

If Sample size $n \geq 30$

and mean is same, variance is different.

$GD(\mu, \sigma^2)$

Here $\frac{8}{2} = 4$ is near to 3.33

If $n \geq 30$, $GD(\bar{X}, \frac{\sigma^2}{n})$