* Unit :→ 3

Mathematical Probability and method of list square and curve Fitting :→

[A] 1. Binomial Distribution
2. Poisson Distribution
3. Normal Distribution
4. Principle of list square
5. First degree and second degree eqn.

[B] Operation Research linear Programming Problem.

1. Introduction
2. Formulation Technique of L.P.P.
3. Graphical soln of two - Variable Problem.

⊛. THEORETICAL DISTRIBUTIONS ⊛

→ In this chapter we shall study the following univariate probability distributions.

<1> Binomial Distribution
<2> Poisson Distribution
<3> Normal Distribution.

The first two distributions are discrete probability distributions and the third is a continuous probability distribution.

[A] BINOMIAL DISTRIBUTION

Binomial distribution is also known as the Bernoulli distribution. This distribution can be used under the following conditions:

<1> An experiment consists of a finite number of repeated trials.

<2> Each trial has only two possible mutually exclusive out comes which are known as a "success" and a "failure"

<3> All the (Different) trials are independent i.e the result of any trial ; is not affected in any way by the preceding trials

and does not affect the result of succeeding trials.

4) The probability of success in any trial is p and is constant for each trial. The Probability of a failure denoted by q; and is equal to 1 - p.

The sequence of trials under the above assumption is also known as a Bernoulli trials.

for ex: We know that the probability of getting a head or a tail on tossing a coin is $\frac{1}{2}$

If the coin is tossed thrice; the probability of getting one head and two tails can be combined as

H-T-T, T-H-T, T-T-H.

The probability of each one of these being:

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

i.e $\left(\frac{1}{2}\right)^3$

their total probability shall be:

$$3\left(\frac{1}{2}\right)^3$$

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Similarly ; if a trial is repeated n-times and if p is the probability of a success and q that of a failure ; then the probability of r successes and $(n-r)$ failures is given by :

$$p^r \cdot q^{n-r}$$

But these r successes and $(n-r)$ failures can occur in any of the ^nC_r ways in each of which the probability is same.

Thus ; the probability of r successes is

$$^nC_r \, p^r \, q^{n-r}$$

→ Let n be the total numbers of repeated trials ; p be the probability of a success in a trial and q be the probability of its failure.

Let r be a random variable which denotes the number of successes in n trials.
The possible values of r are $0, 1, 2, 3, 4, \ldots, n$.

We are intereseted in finding the Probability of r successes out of n trials

i.e $P(r)$

To find this probability ; we assume that the first r trials are successes and remaining $(n-r)$ trials are failars. Since different trials are assumed to be independent ; the probability of this sequence is ;

$$\underbrace{p \cdot p \cdot p \cdot \ldots p}_{r \text{ times}} \cdot \underbrace{q \cdot q \cdot q \cdot \ldots q}_{(n-r) \text{ times}}.$$

i.e $p^{r} q^{n-r}$

Since out of n trials any r trials can be success ; the number of sequences showing any r trials as success and remaining $(n-r)$ trials as failures is : ${}^{n}C_{r}$; where the probability of r successes in each trial is : $p^{r} q^{n-r}$

Hence the required probability is

$$\boxed{P(r) = {}^nC_r\, p^r q^{n-r}} \quad \text{-----} \quad (*)$$

where $p+q=1$ and $r = 0, 1, 2, \ldots n$.

The distribution $(*)$ is called the binomial probability distribution.

Note :→

The successive probabilities $P(r)$ in $(*)$ for $r = 0, 1, 2, \ldots n$ are:

${}^nC_0\, p^0 q^n$; ${}^nC_1\, p^1 . q^{n-1}$; ${}^nC_2\, p^2 q^{n-2}$; ${}^nC_3\, p^3 q^{n-3}$; ----

${}^nC_n\, p^n q^0$.

Which are the terms in the binomial expansion of $(q+p)^n$.

Hence this distribution is known as a Binomical distribution.

(*) Recurrence formula for the Binomial Distribution :→

In a Binomical distribution

$$\begin{cases} P(r) = {}^nC_r\, q^{n-r} p^r = \dfrac{n!}{(n-r)!\, r!}\, q^{n-r} p^r \\ P(r+1) = {}^nC_{r+1}\, q^{n-r-1} p^{r+1} = \dfrac{n!}{(n-r-1)!\,(r+1)!}\, q^{n-r-1} p^{r+1} \end{cases}$$

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Now ;

$$\frac{P(r+1)}{P(r)} = \frac{(n-r)!}{(n-r-1)!} \times \frac{r!}{(r+1)!} \times \frac{p}{q}$$

$$= \frac{(n-r)(n-r-1)!}{(n-r-1)!} \times \frac{r!}{(r+1)\,r!} \times \frac{p}{q}$$

$$= \frac{(n-r)}{r+1} \times \frac{p}{q}$$

$$\Rightarrow \boxed{P(r+1) = \frac{(n-r)}{r+1} \times \frac{p}{q} \cdot P(r)}$$

which is the required recurrence formula. Applying this formula successively we can find : $P(1)$, $P(2)$, $P(3)$ --- ; if $P(0)$ is known.

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r	$P(r)$	$r \cdot P(r)$	$r^2 P(r)$
0	q^n	0	0
1	${}^nC_1\, q^{n-1}\, p$	${}^nC_1\, q^{n-1}\, p$	$1^2\, {}^nC_1\, q^{n-1}\, p$
2	${}^nC_2\, q^{n-2}\, p^2$	$2\, {}^nC_2\, q^{n-2}\, p^2$	$2^2\, {}^nC_2\, q^{n-2}\, p^2$
3	${}^nC_3\, q^{n-3}\, p^3$	$3\, {}^nC_3\, q^{n-3}\, p^3$	$3^2\, {}^nC_3\, q^{n-3}\, p^3$
|	|	|	|
|	|	|	|
|	|	|	|
|	|	|	|
n	p^n	np^n	$n^2 p^n$

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(*) Mean and Variance of the Binomial Distribution :→

(*) Mean $\mu = \Sigma r \cdot P(r)$

$$= \sum_{r=0}^{n} r \cdot {}^nC_r \, q^{n-r} p^r$$

$$= 0 + 1 \cdot {}^nC_1 \, q^{n-1} p + 2 \, {}^nC_2 \, q^{n-2} p^2 + 3 \, {}^nC_3 \, q^{n-3} p^3 + \cdots + n \, {}^nC_n \, p^n$$

$$= n q^{n-1} p + 2 \frac{n(n-1)}{2!} q^{n-2} p^2 + \frac{3 n(n-1)(n-2)}{3!} q^{n-3} p^3 + \cdots + n p^n$$

$$= n q^{n-1} p + n(n-1) q^{n-2} p^2 + \frac{n(n-1)(n-2)}{2} q^{n-3} p^3 + \cdots + n p^n$$

$$= np \left\{ q^{n-1} + (n-1) q^{n-2} p + \frac{(n-1)(n-2)}{2 \cdot 1} q^{n-3} p^2 + \cdots + p^{n-1} \right\}$$

$$= np \left\{ {}^{n-1}C_0 \, q^{n-1} + {}^{n-1}C_1 \, q^{n-2} p + {}^{n-1}C_2 \, q^{n-3} p^2 + \cdots + {}^{n-1}C_{n-1} \, p^{n-1} \right\}$$

$$= np \, (q + p)^{n-1}$$

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$$\boxed{\text{Mean} = np}$$ $(\because p+q=1)$

* Variance $\sigma^2 = \sum_{r=0}^{n} r^2 p(r) - [\text{Mean}]^2$

$$= \sum_{r=0}^{n} [r + r(r-1)] P(r) - \mu^2$$

$$= \sum_{r=0}^{n} r.P(r) + \sum_{r=0}^{n} r(r-1) P(r) - \mu^2$$

$$= \mu + \sum_{r=0}^{n} r(r-1) P(r) - \mu^2$$

$$= \mu + \sum_{r=2}^{n} r(r-1) P(r) - \mu^2$$

($\because$ contribution due to $r=0$ and $r=1$ is zero)

$$= \mu + \sum_{r=2}^{n} r(r-1)\, {}^nC_r\, q^{n-r} p^r - \mu^2$$

$$= \mu - \mu^2 + \sum_{r=2}^{n} r(r-1)\, {}^nC_r\, q^{n-r} p^r$$

$$= \mu - \mu^2 + \left[2\cdot1\, {}^nC_2\, q^{n-2} p^2 + 3\cdot2\, {}^nC_3\, q^{n-3} . p^3 + \cdots + n(n-1)\, {}^nC_n\, p^n \right]$$

$$= \mu - \mu^2 + \left\{ \frac{2\cdot 1\cdot n(n-1)}{2!} q^{n-2} p^2 + \frac{3\cdot 2\, n(n-1)(n-2)}{3!} q^{n-3} p^3 + \cdots + n(n-1) p^n \right\}$$

$$= \mu - \mu^2 + \left\{ n(n-1) q^{n-2} p^2 + n(n-1)(n-2) q^{n-3} p^3 + \cdots + n(n-1) p^n \right\}$$

$$= \mu - \mu^2 + n(n-1)p^2 \left\{ q^{n-2} + (n-2) q^{n-3} p + \cdots + p^{n-2} \right\}$$

$$= \mu - \mu^2 + n(n-1)p^2 \left\{ {}^{(n-2)}C_0\, q^{n-2} + {}^{(n-2)}C_1\, q^{n-3} p + \cdots + {}^{(n-2)}C_{(n-2)}\, p^{n-2} \right\}$$

$$= \mu - \mu^2 + n(n-1)p^2 [q + p]^{n-2}$$

$$= \mu - \mu^2 + n(n-1)p^2 \qquad (\because p + q = 1)$$

$$= np - n^2p^2 + n(n-1)p^2$$

$$= np \left\{ 1 + (n-1)p - np \right\}$$

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$= np \{1 - p\}$

$$\boxed{\text{Variance} = npq.}$$

where; Variance $= \sigma^2$

→ standard deviation of the binomial distribution is

$$\sqrt{npq} = \sigma.$$

$$\boxed{{}^nC_r = \frac{n!}{(n-r)!\,r!}}$$

Ex-1 Ten unbiased coins are tossed simultaneously. Find the Probability of obtaining.

<1> Exactly 6 heads
<2> Atleast 8 heads
<3> No head
<4> At least one head
<5> Not more than three heads
<6> At least 4 heads.

Sol^n :→ If p denotes the probability of a head; then

$$p = q = \frac{1}{2}$$

Here $n = 10$

If the random variable x denotes the number of heads; then by Binomial Probability law; the Probability of r head is given by:

$$P(r) = P(x = r) = {}^nC_r\, p^r \cdot q^{n-r}$$

$$= {}^{10}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{10-r}$$

$$= {}^{10}C_r \left(\frac{1}{2}\right)^{10}$$

$$\therefore P(r) = \frac{1}{1024}\, {}^{10}C_r \quad \text{------} \quad (*)$$

<1> Required Prob. = $P(6)$

$= \frac{1}{1024} \cdot {}^{10}C_6$

$= \frac{252}{1024}$

$= \frac{63}{256}$

<2> Required Prob. = $P(8) + P(9) + P(10)$
$P(X \geq 8)$

$= \frac{1}{1024} \left\{ {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} \right\}$

$= \frac{1}{1024} \left\{ 45 + 10 + 1 \right\}$

$= \frac{56}{1024}$

$= \frac{7}{128}$

<3> Req. Prob. $P(X=0) = P(0)$

$= \frac{1}{1024} \cdot {}^{10}C_0$

$= \frac{1}{1024}$

<4> Req. Prob. = $P(1) + P(2) + P(3) + \ldots + P(10)$

= P [At least one head]

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$= 1 - P$ [no head]

$= 1 - P(0)$

$= 1 - \frac{1}{1024}$

$= \frac{1023}{1024}$

<5> Req. Prob. $P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$

$= \frac{1}{1024} \{ {}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 \}$

$= \frac{1}{1024} \{ 1 + 10 + 45 + 120 \}$

$= \frac{176}{1024} = \frac{11}{64}$

<6> Req. Prob $P(X \geq 4) = P(4) + P(5) + \dots + P(10)$

$= \frac{1}{1024} \{ {}^{10}C_4 + {}^{10}C_5 + \dots + {}^{10}C_{10} \}$

$= \frac{53}{64}$

OR

$= 1 - P(X \leq 3)$

$= 1 - \{ P(0) + P(1) + P(2) + P(3) \}$

$= 1 - \frac{11}{64}$ (∵ by <5>)

$= \frac{53}{64}$

* die = small cube with six sides marked one to six
  = ૬ બાજુઓ એક થી ૬ સુધીનાં બિન્દુવાળો ઘનાકૃતિ પાસો.
* dice = pl. of die (બહુવચન)

Ex-2 A pair of dice is thrown 10 times. If getting a doublet (same number on both) is considered a success; find the probability of

⟨1⟩ 4 successes.
⟨2⟩ No. success.

Sol^n:-

Here $n = 10$

A doublet can be obtained when a pair of dice is thrown in

$(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)$

i.e. in 6 ways.

$\therefore p = 6/36 = 1/6$

$\therefore q = 1 - p = 1 - 1/6 = 5/6$

since; $P(X = r) = {}^nC_r \, p^r \cdot q^{n-r}$

⟨1⟩ $P(4 \text{ success}) = {}^{10}C_4 \left(\frac{5}{6}\right)^6 \left(\frac{1}{6}\right)^4$

$$= \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} \cdot \frac{(5)^6}{(6)^{10}}$$

$$= 210 \times \frac{(5)^6}{(6)^{10}}$$

$$= 7 \times \frac{(5)^7}{(6)^9}$$

$$= \frac{7}{36} \left(\frac{5}{6}\right)^7 = 0.054$$

(2) $P(\text{no success}) = P(0)$

$$= {}^{10}C_0 \left(\frac{5}{6}\right)^{10}$$

$$= \left(\frac{5}{6}\right)^{10} = 0.1615$$

Ex-3 The probability of a man hitting a target is 1/4. If he fires 7 times; what is the probability of his hitting the target at least twice?

Sol^n:- Here $p = 1/4$

$\therefore q = 1 - 1/4 = 3/4$ } $n = 7$

The probability of hitting the target twice

$$= P(x \geq 2)$$

$$= P(2) + P(3) + P(4) + \cdots + P(7)$$

$$= 1 - P(0) - P(1)$$

$$= 1 - {}^nC_0 \, p^0 q^7 - {}^nC_1 \, p^1 q^6$$

$$= 1 - {}^7C_0 \left(\tfrac{1}{4}\right)^0 \left(\tfrac{3}{4}\right)^7 - {}^7C_1 \left(\tfrac{1}{4}\right)^1 \left(\tfrac{3}{4}\right)^6$$

$$= 1 - \left(\tfrac{3}{4}\right)^7 - 7 \cdot \tfrac{1}{4} \left(\tfrac{3}{4}\right)^6$$

$$= 1 - \frac{3^6}{4^7}(3+7)$$

$$= 1 - \frac{7290}{16384} = \frac{9094}{16384} = 0.5551$$

Ex-4 Eight coins are thrown simultaneously. Find the chance of obtaining at least six heads.

Sol^n :→ when a coin is thrown;

Let p = the Prob. of getting head }
q = " " " tail }

$\therefore p = q = \frac{1}{2}$ and $n = 8$

$\therefore$ P(at least 6 heads) $= P(x \geq 6)$

$= P(6 \text{ heads}) + P(7 \text{ heads}) + P(8 \text{ heads})$

$= 8C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^2 + 8C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right) + 8C_8 \left(\frac{1}{2}\right)^8$

$= \left(\frac{1}{2}\right)^8 \left\{ 8C_6 + 8C_7 + 8C_8 \right\}$

$= \frac{1}{256} \left\{ \frac{8 \times 7}{2 \times 1} + \frac{8}{1} + 1 \right\}$

$= \frac{1}{256} \left\{ 28 + 8 + 1 \right\}$

$= \frac{37}{256}$

$= 0.144$

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Ex-5 If the Probability that a man aged 60 will live to be 70 is 0.65 what is the Probability that out of 10 men now 60 at least 7 will live to be 70?

Sol^n:→ Let p = Prob. of living up to 70

$= 0.65$

$= \frac{65}{100} = \frac{13}{20}$

$\therefore q$ = Prob. of dying $= 1 - \frac{13}{20} = \frac{7}{20}$

Here n = total no. of men $= 10$

and at least 7 will live to be 70

∴ There are following four possibilities.

Req. Prob. $P(x \geq 7) = P(7) + P(8) + P(9) + P(10)$

$= P_1 + P_2 + P_3 + P_4$

<1> P_1 = The prob. of 7 living and 3 dying

$= {}^{10}C_7 \, p^7 q^3$

$= \frac{10!}{7!\,3!} \left(\frac{13}{20}\right)^7 \left(\frac{7}{20}\right)^3$

$= \frac{120 \, (13)^7 (7)^3}{(20)^{10}} = 0.25222$

<2> P_2 = The Prob. of 8 living and 2 dying

$= {}^{10}C_8 \, p^8 q^2$

$= \frac{10!}{8!\,3!} \left(\frac{13}{20}\right)^8 \left(\frac{7}{20}\right)^2$

$= \frac{45 \,(13)^8 (7)^2}{(20)^{10}} = 0.17565$

<3> P_3 = The Prob. of 9 living and 1 dying

$= {}^{10}C_9 \, p^9 q^1$

$= \frac{10!}{9!\,1!} \left(\frac{13}{20}\right)^9 \left(\frac{7}{20}\right)^1$

$= \frac{10 \,(13)^9 (7)^1}{(20)^{10}} = 0.072492$

<4> P_4 = The Prob. of 10 living

$= {}^{10}C_{10} \, p^{10}$

$= 1 \cdot \left(\frac{13}{20}\right)^{10} = 0.013463$

Hence Required Probability $= P_1 + P_2 + P_3 + P_4$

$=$

$= 0.5739$

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Ex-6 A man can kill a bird once in three shots on this assumption he fires three shots what is the chance that a bird is killed ?

Soln :-> Let p = Prob. of killing a bird in a single shot

$= 1/3$

q = Prob. of not killing a bird in a single shot is

$= 1 - p$

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

Here $n = 3$

Req. Prob. $= P(1) + P(2) + P(3)$

$= 3C_1 \, p^1 q^2 + 3C_2 \, p^2 q^1 + 3C_3 \, p^3 q^0$

$= 3 \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 + \frac{3!}{2! \, 1!}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right) + 1\left(\frac{1}{3}\right)^3$

$= \frac{4}{9} + \frac{2}{9} + \frac{1}{27}$

$= \frac{19}{27}$

$= 0.7036$

$$^nC_r = {}^nC_{n-r}$$

Ex-7 What is the probability of guessing correctly at least 6 of 10 answers in a TRUE – FALSE objective test ?

Solⁿ :→ Let p = the Prob. of guessing an answer correct $= \frac{1}{2}$

then $q = 1 - p = \frac{1}{2}$

Here $n = 10$

$$\therefore P(r) = {}^{10}C_r \left(\tfrac{1}{2}\right)^r \left(\tfrac{1}{2}\right)^{10-r}$$

$$= {}^{10}C_r \left(\tfrac{1}{2}\right)^{10}$$

Now the required probability p of gussing correctly at least 6 of 10 answer is :

$$P = P(6) + P(7) + P(8) + P(9) + P(10)$$

$$= \left(\tfrac{1}{2}\right)^{10} \left\{ {}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} \right\}$$

$$= \frac{1}{1024} \left\{ {}^{10}C_4 + {}^{10}C_3 + {}^{10}C_2 + {}^{10}C_1 + 1 \right\}$$

$$= \frac{1}{1024} \left\{ 210 + 120 + 45 + 10 + 1 \right\}$$

$$= \frac{386}{1024}$$

$$= \frac{193}{512} = 0.3769$$

(*) delinquent = કરજ ચૂકનાર કે ગુનો કરનાર વ્યક્તિ

Ex-8 A merchant's file of 20 accounts contains 6 delinquent and 14 non-delinquent accounts. An auditor randomly selects 5 of these accounts for examination

<1> what is the Probability that the auditor finds exactly 2 delinquent accounts ?

<2> Find the expected number of delinquent accounts in the sample selected

Sol^n:→ We have:

p = Prob. of a delinquent account $= \frac{6}{20} = 0.3$

$\therefore q$ = no. delinquent $= 1 - p = 0.7$

$\therefore P(r)$ = Prob. that auditor finds r delinquent accounts in a random choice of $n = 5$ accounts.

$$= nC_r \, p^r \, q^{n-r}$$

$$= 5C_r \, (0.3)^r \, (0.7)^{5-r}$$

<1> The Prob. that the auditor finds exactly 2 delinquent accounts is given by :

$$P(2) = 5C_2 \, (0.3)^2 \, (0.7)^3$$

$$= \frac{5 \times 4}{2} \times 0.09 \times 0.343$$

$$= 0.3087$$

(2) The expected number of delinquent accounts in the selected sample of $n = 5$ is :

$= np$

$= 5 \times 0.3$

$= 1.5$

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EX-9 : If the chance that the vessel arrives safely at a port (બંદર) is $9/10$. Find the chance that out of 5 vessels expected at least 4 will arrive safely.

sol^n :- p = Prob. that a vessel arrives safely at the port

$= 9/10$

$q = 1 - p = 1/10$

$\therefore$ The probability that out of 5 vessels; x vessels arrive safely at the port is

$$P(x) = {}^5C_x \left(\frac{9}{10}\right)^x \left(\frac{1}{10}\right)^{5-x}$$

$$= \frac{1}{10^5} \, {}^5C_x \, 9^x$$

$\therefore$ The prob. of at least 4 vessels arrive safely at the port

$= P(4) + P(5)$

$$= \frac{1}{10^5} \left\{ {}^5C_4 \, 9^4 + {}^5C_5 \, 9^5 \right\}$$

$$= \frac{9^4}{10^5} [5 + 9] = 0.91854$$

Ex-10 Assume that half the Population is vegetarian so that the chance of an individual being a vegetarian is $1/2$. Assuming that 100 investigators each take sample of 10 individuals to see whether they are vegetarians; How-many investigators would you expect to report that three people or less were vegetarian?

Sol^n:- Here $n = 10$

Let p = Prob. that an individual is a vegetarian
$= 1/2$

$\therefore q = 1 - p = 1/2$

Now the probability that there are r vegetarians in a sample of 10 is given by

$$P(r) = {}^{10}C_r \left(\tfrac{1}{2}\right)^r \left(\tfrac{1}{2}\right)^{10-r}$$

$$= {}^{10}C_r \left(\tfrac{1}{2}\right)^{10}$$

$$= \frac{1}{1024}\ {}^{10}C_r$$

Thus; the Prob. that in a sample of 10; three or less people are vegetarian is

$$= P(0) + P(1) + P(2) + P(3)$$

$$= \frac{1}{1024}\left\{ {}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 \right\}$$

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$= \frac{1}{1024} \{ 1 + 10 + 45 + 120 \}$

$= \frac{176}{64} = 0.1718$

Hence; out of 100 investigators; the number of investigators who will report 3 or less vegetarians in a sample of 10 is

$= 100 \times \frac{11}{64}$

$= \frac{275}{16}$

$= 17.2$

$= 17$ investigators.

Ex-11 With the usual notations; find p for a binomial random variable x if $n = 6$ and if $9P(X=4) = P(X=2)$

Soln:- We know

$P(r) = P(X=r) = nC_r \, p^r q^{n-r}$

$= 6C_r \, p^r q^{6-r}$

we are given;

$9P(X=4) = P(X=2)$

$\Rightarrow 9 \, {}^6C_4 \, p^4 q^2 = {}^6C_2 \, p^2 q^4$

$${}^nC_r = {}^nC_{n-r}$$

$\Rightarrow 9p^2 = q^2 \qquad (\because {}^6C_4 = {}^6C_2)$

$\Rightarrow 9p^2 = (1-p)^2$

$\Rightarrow 8p^2 + 2p - 1 = 0 \qquad (\because (1-p)^2 = 1 + p^2 - 2p)$

$\Rightarrow 8p^2 + 4p - 2p - 1 = 0$

$\Rightarrow 4p(2p+1) - 1(2p+1) = 0$

$\Rightarrow (4p-1)(2p+1) = 0$

$\Rightarrow p = 1/4 \quad$ OR $\quad p = -\frac{1}{2}$

since probability cannot be negative

$\therefore \boxed{p = 1/4}$

Ex-12 The mean and variance of a binomial distribution are 3 and 2 respectively. Find the probability that the variate takes values <1> less than or equal to 2
<2> greater than or equal to 7

Solⁿ :→ Here given :

Mean $= np = 3$ ------- (i)

Variance $= npq = 2$ ------- (ii)

dividing (ii) by (i) ; we get

$\boxed{q = 2/3} \qquad \therefore \boxed{p = 1 - q = 1/3}$

Now from (i) ; $np = 3$

$\Rightarrow n \cdot \frac{1}{3} = 3 \Rightarrow \boxed{n = 9}$

$\therefore P(X=r) = {}^nC_r\, p^r q^{n-r}$

$= {}^9C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{9-r}$

<1> The prob. that the variate takes the value less than or equal to 2 is

$= P(0) + P(1) + P(2)$

$= {}^9C_0 \left(\frac{2}{3}\right)^9 + {}^9C_1 \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^8 + {}^9C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^7$

$= \left(\frac{2}{3}\right)^7 \left\{ \left(\frac{2}{3}\right)^2 + 9 \cdot \frac{1}{3} \cdot \frac{2}{3} + \frac{9\times 8}{2} \left(\frac{1}{3}\right)^2 \right\}$

$= \left(\frac{2}{3}\right)^7 \left\{ \frac{4}{9} + 2 + 4 \right\}$

$= \left(\frac{2}{3}\right)^7 \times \frac{58}{9}$

$= 0.0585 \times 6.44$

$= 0.3771$

<2> The Prob. that variate takes the value greater than or equal to 7

$= P(7) + P(8) + P(9)$ $\quad (\because n = 9)$

$= {}^9C_7 \left(\frac{1}{3}\right)^7 \left(\frac{2}{3}\right)^2 + {}^9C_8 \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right) + {}^9C_9 \left(\frac{1}{3}\right)^9$

$= \left(\frac{1}{3}\right)^7 \left\{ \frac{9\times 8}{2} \left(\frac{2}{3}\right)^2 + 9 \times \frac{1}{3} \times \frac{2}{3} + 1 \left(\frac{1}{3}\right)^2 \right\}$

$= \frac{1}{2187} \left\{ 16 + 2 + \frac{1}{9} \right\} = \frac{18.1111}{2187} = 0.0083$

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(*) Fitting of Binomial Distribution :→

Suppose a random experiment consists of n trials ; satisfying the conditions of Binomial distribution and suppose this experiment is repeated N times then the frequency of r successes is given by the formula:

$$N \times P(r) = N \times {}^nC_r\, p^r q^{n-r}$$

where $r = 0, 1, 2, 3, \ldots n$

Putting $r = 0, 1, 2, 3, \ldots n$ we get the expected or theoretical frequencies $f(0), f(1), f(2), \ldots f(n)$ of the Binomial distribution

$$\begin{cases} f(0) = N \cdot q^n \\ f(1) = N\, {}^nC_1\, p\, q^{n-1} \\ f(2) = N \cdot {}^nC_2\, p^2 q^{n-2} \\ \vdots \qquad\qquad \vdots \\ f(n) = N \cdot p^n \end{cases}$$

$\therefore f(0) + f(1) + f(2) + \cdots + f(n) = N \cdot (p+q)^n$

EX-13 Six dice are thrown 729 times How many times do you expect at least three dice to show a five or six ?

$\underline{\underline{\text{Sol}^n}}$:→ Here given ; $n = 6$ and $N = 729$

Let the event of getting 5 or 6 in the throw of a single die be called a success then

p = Prob. of success $= \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

(for 5) (for 6)

$\therefore q = 1 - p = 2/3$

In a single throw of 6 dice ; the Prob. of getting r successes (i.e getting 5 or 6 on r dice) is

$$P(r) = {}^nC_r \, p^r \, q^{n-r}$$

$$= {}^6C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{6-r}$$

$$= \frac{1}{3^6} \, {}^6C_r \, 2^{6-r}$$

$$= \frac{1}{729} \, {}^6C_r \, 2^{6-r}$$

Thus ; the probability that at least 3 dice show a 5 or 6 is

$$= P(3) + P(4) + P(5) + P(6)$$

Hence in 729 throws of 6 dice each ;
the required frequence of getting at least 3 success is

$$= N \times \{ P(3) + P(4) + P(5) + P(6) \}$$

$$= 729 \times \frac{1}{729}\left\{ {}^6C_3\, 2^3 + {}^6C_4\, 2^2 + {}^6C_5 \cdot 2 + {}^6C_6\, 1 \right\}$$

$$= \{ 8 \times 20 + 15 \times 4 + 6 \times 2 + 1 \}$$

$$= 160 + 60 + 12 + 1$$

$$= 233$$

EX-14 <a> 8 coins are tossed (at a time) 256 times. Find the expected frequencies of successes (getting a head) and tabulate the results obtained.
<b> Also obtain the values of the mean and standard derivation of the theoretical (fitted) distribution.

Sol^n :→ Here given;

$$n = 8 \quad ; \quad N = 256$$

p = Prob. of success (head) in a single throw of a coin
$= \frac{1}{2}$

$\therefore\ q = 1 - p = \frac{1}{2}$

$\therefore$ the probability of r success in a toss of 8 coins is given by

$$P(r) = {}^nC_r\, p^r q^{n-r}$$

$$= {}^8C_r \left(\tfrac{1}{2}\right)^r \left(\tfrac{1}{2}\right)^{8-r}$$

$$= {}^8C_r \left(\tfrac{1}{2}\right)^8$$

$$= \frac{1}{256}\, {}^8C_r$$

Hence; in 256 throws of 8 coins; the frequency of r success is

$$f(r) = N \cdot P(r) = 256 \times \frac{1}{256} \cdot {}^{8}C_{r} = {}^{8}C_{r}$$

Thus; the expected frequencies are tabulated below:

No. of heads	Expected Frequency
0	${}^{8}C_{0} = 1$
1	${}^{8}C_{1} = 8$
2	${}^{8}C_{2} = 25$
3	${}^{8}C_{3} = 56$
4	${}^{8}C_{4} = 70$
5	${}^{8}C_{5} = 56$
6	${}^{8}C_{6} = 28$
7	${}^{8}C_{7} = 8$
8	${}^{8}C_{8} = 1$

(b) For the theoretical distribution

$$\text{Mean} = n \cdot p = 8 \cdot \frac{1}{2} = 4$$

$$\text{S.d} = \sqrt{npq} = \sqrt{8 \times \frac{1}{2} \times \frac{1}{2}} = \sqrt{2} = 1.4142$$

Ex-15

Assume that on the average one telephone number out of fifteen called bet^n 2 P.M. and 3 P.M. on week days is busy what is the probability that if 6 randomly selected telephone numbers are called

⟨i⟩ not more than three will be busy
⟨ii⟩ at least three of them will be busy?

sol :→ Let p = the prob. of a telephone number being busy betn 2 p.m and 3 p.m on week days.

$\therefore p = \frac{1}{15}$

$\therefore q = 1 - \frac{1}{15} = \frac{14}{15}$

$n = 6$

Now the probability of the r telephone will be busy is

$$p(r) = {}^nC_r\, p^r q^{n-r}$$

$$= {}^6C_r \left(\frac{1}{15}\right)^r \left(\frac{14}{15}\right)^{6-r}$$

$$= \frac{1}{(15)^6}\, {}^6C_r\, (14)^{6-r}$$

⟨1⟩ The prob. that not more than three will be busy

$$= P(0) + P(1) + P(2) + P(3)$$

$$= \frac{1}{(15)^6}\left\{ {}^6C_0 (14)^6 + {}^6C_1 (14)^5 + {}^6C_2 (14)^4 + {}^6C_3 (14)^3 \right\}$$

$$= \frac{(14)^3}{(15)^6}\left\{ 2744 + 1176 + 210 + 20 \right\}$$

$$= \frac{2744 \times 4150}{(15)^6} = 0.9997$$

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(ii) The prob. that at least three of them will be busy

$$= P(3) + P(4) + P(5) + P(6)$$

$$= \frac{1}{15^6}\left\{ {}^6C_3 (14)^3 + {}^6C_4 (14)^2 + {}^6C_5 (14) + {}^6C_6 \right\}$$

$$= \frac{14}{15^6}\left\{ \right\}$$

$$= 0.005$$

Ex-16 A quiz has 10 multiple choice questions each with 3 alternatives. Find the prob. that a students gets 8 or more correct answers.

Soln :-

Here $n = 10$

$p =$ Prob. of guessing correct answer

$$= \frac{1}{3}$$

$$\therefore q = 1 - \frac{1}{3} = \frac{2}{3}$$

$\therefore$ Required prob. $= P(X \geq 8)$

$$= P(8) + P(9) + P(10)$$

$$= \left[{}^{10}C_8 \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right)^2 + {}^{10}C_9 \left(\frac{1}{3}\right)^9 \left(\frac{2}{3}\right) + {}^{10}C_{10} \left(\frac{1}{3}\right)^{10} \right]$$

$$= 0.0034$$

Ex-17 Assuming that the probability of a child being a boy or a girl is equal find the number of families out of 400 consisting 3 children each having <i> all boys

<ii> two boys and one girl

<iii> at most one boy.

Solⁿ:-

p = Prob. that child is a boy $= \frac{1}{2}$

n = no. of children in a family $= 3$

<i> p (all boys) $= P(3) = \binom{3}{3} \cdot \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$\therefore$ no. of families having all boys

$= 400 \times \frac{1}{8} = 50$

<ii> p (two boys and one girl) $= P(2) = \binom{3}{2}\left(\frac{1}{8}\right) = \frac{3}{8}$

$\therefore$ no. of families having 2 boys and 1 girl

$= 400 \times \frac{3}{8} = 150$

<iii> p (at most one boy) $= P(0) + P(1) = \frac{1}{2}$

$\therefore$ total $= 400 \times \frac{1}{2} = 200$

[B] POISSON DISTRIBUTION

This distribution can be derived as a limiting case of the binomial distribution by making n very large ($n \to \infty$) and p very small ($p \to 0$).

If $n \to \infty$ and $p \to 0$ then np always remains finite ; say m

$$\therefore \quad np = m$$

$$\therefore \quad p = m/n \qquad \therefore \quad q = 1 - p = 1 - m/n$$

Now ; for a Binomial distribution

$$P(X = r) = {}^nC_r \, p^r \, q^{n-r}$$

$$= \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}\left(\frac{m}{n}\right)^r\left[1 - \frac{m}{n}\right]^{n-r}$$

$$= \frac{m^r}{r!} \times \frac{n(n-1)(n-2)\ldots(n-r+1)}{n^r} \times \frac{(1 - m/n)^n}{(1 - m/n)^r}$$

$$= \frac{m^r}{r!}\left(\frac{n}{n}\right)\left(\frac{n-1}{n}\right)\left(\frac{n-2}{n}\right)\ldots\left(\frac{n-r+1}{n}\right)\frac{(1 - m/n)^n}{(1 - m/n)^r}$$

$$= \frac{m^r}{r!}\left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)\ldots\left(1 - \frac{r-1}{n}\right)\frac{\left\{(1 - m/n)^{-n/m}\right\}^{-m}}{(1 - m/n)^r}$$

$$- - - - - - - (*)$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e \quad ; \quad \lim_{n\to\infty}\left(1+\frac{q}{n}\right)^A = 1$$

Now ; as $n \to \infty$; each of the $(r-1)$ factors

$$\left(1-\frac{1}{n}\right), \left(1-\frac{2}{n}\right), \dots \left(1-\frac{r-1}{n}\right) \to 1$$

Also $\left(1-\frac{m}{n}\right)^{r} \to 1$ and $\left\{\left(1-\frac{m}{n}\right)^{-n/m}\right\}^{-m} \to e^{-m}$

Hence ; when $n \to \infty$; from eqn (*) ; we have ;

$$P(r) = \frac{m^r e^{-m}}{r!}$$

Thus ; the probability fns of the Poisson distribution is

$$\boxed{P(r) = \frac{e^{-m} m^r}{r!} = \frac{e^{-\lambda}\lambda^r}{r!}} \quad ; r = 0, 1, 2, \dots$$

where $m = \lambda = np$

Note :- The sum of the Probability is 1 for $r = 0, 1, 2, 3, \dots n$

$$P(0) + P(1) + P(2) + \dots + P(n) + \dots$$

$$= e^{-m} + \frac{m e^{-m}}{1!} + \frac{m^2 e^{-m}}{2!} + \frac{m^3 e^{-m}}{3!} + \dots$$

$$= e^{-m}\left\{1 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \dots\right\}$$

$$\boxed{e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots}$$

$$= e^{-m} e^{m}$$

$$= 1$$

Recurrence Formula for the Poisson Distribution :→

For Poisson Distribution

$$P(r) = \frac{m^r e^{-m}}{r!}$$

$$P(r+1) = \frac{m^{r+1} e^{-m}}{(r+1)!}$$

Hence,

$$\frac{P(r+1)}{P(r)} = \frac{m \cdot r!}{(r+1)!} = \frac{m}{r+1}$$

$$\Rightarrow \boxed{P(r+1) = \frac{m}{r+1} \cdot P(r)}$$

where $r = 0, 1, 2, 3, \ldots$

(*) Mean and Variance of the Poisson Distribution

(*) Constants of Poisson Distribution

r	$P(r)$	$rP(r)$	$r^2P(r)$
0	e^{-m}	0	0
1	me^{-m}	$1\cdot m\cdot e^{-m}$	me^{-m}
2.	$\frac{m^2 e^{-m}}{2!}$	$\frac{2\cdot m^2 e^{-m}}{2!}$	$\frac{2^2 m^2 e^{-m}}{2!}$
3.	$\frac{m^3 e^{-m}}{3!}$	$\frac{3\cdot m^3\cdot e^{-m}}{3!}$	$\frac{3^2 m^2 e^{-m}}{3!}$
4.	$\frac{m^4 e^{-m}}{4!}$	$\frac{4\cdot m^4 e^{-m}}{4!}$	$\frac{4^2 m^2 e^{-m}}{4!}$
⋮	⋮	⋮	⋮

(*) Mean $\mu = \sum_{r=0}^{\infty} r\cdot P(r)$

$$= \sum_{r=0}^{\infty} r\cdot \frac{e^{-m} m^r}{r!}$$

$$= me^{-m} + \frac{2m^2e^{-m}}{2!} + \frac{3m^3e^{-m}}{3!} + \frac{4m^4e^{-m}}{4!}$$

$$+ \cdots\cdots$$

$$= me^{-m}\left\{1+m+\frac{m^2}{2!}+\frac{m^3}{3!}+\cdots\right\}$$

$$= me^{-m}\left\{e^{m}\right\}$$

$$= m$$

(*) Variance $\sigma^2 = \sum_{r=0}^{\infty} r^2 P(r) - \mu^2$

$$= \sum_{r=0}^{\infty} r^2 \cdot \frac{m^r \cdot e^{-m}}{r!} - m^2$$

$$= me^{-m} + \frac{2^2 m^2 e^{-m}}{2!} + \frac{3^2 m^3 e^{-m}}{3!} + \cdots - m^2$$

$$= me^{-m}\left\{1+2m+\frac{3}{2!}m^2+\frac{4}{3!}m^3+\cdots\right\} - m^2$$

$$= me^{-m}\left\{1+\frac{(1+1)m}{1!}+\frac{(1+2)m^2}{2!}+\frac{(1+3)m^3}{3!}+\cdots\right\} - m^2$$

$$= me^{-m}\left\{\left[1+m+\frac{m^2}{2!}+\frac{m^3}{3!}+\cdots\right] + \left[m+\frac{2m^2}{2!}+\frac{3m^3}{3!}+\cdots\right]\right\} - m^2$$

$$= me^{-m}\left\{\left[1+m+\frac{m^2}{2!}+\frac{m^3}{3!}+\cdots\right] + m\left[1+m+\frac{m^2}{2!}+\frac{m^3}{3!}+\cdots\right]\right\} - m^2$$

$$= m e^{-m} \{ e^{m} + m e^{m} \} - m^2$$

$$= m e^{-m} e^{m} \{ 1 + m \} - m^2$$

$$= m (1 + m) - m^2$$

$$= m + m^2 - m^2$$

$$= m$$

Hence ; for the poisson Distribution ;

$$\boxed{\text{Mean} = \text{Variance} = m}$$

Note :→ If p is small and n is large then we use poisson distribution.

6

Ex-1 If the variance of the poisson distribution is 2 ; find the Probabilities for $r = 1,2,3,4$ from the recurrence relation of the Poisson distribution

sol^n :- λ : the Parameter of Poisson dist

= variance

= 2

Recurrence relation for the poisson dist is

$$P(r+1) = \frac{\lambda}{r+1} P(r) = \frac{2}{r+1} P(r)$$

Now $P(r) = \frac{m^r e^{-m}}{r!}$ $\Rightarrow P(0) = \frac{e^{-2}}{0!} = e^{-2} = 0.1353$

Putting $r = 0, 1, 2, 3, \ldots$; we get

$P(1) = 2 \cdot P(0) = 2 \times 0.1353 = 0.2706$

$P(2) = \frac{2}{2} \cdot P(1) = 0.2706$

$P(3) = \frac{2}{3} P(2) = \frac{2}{3} \times 0.2706 = 0.1804$

$P(4) = \frac{2}{4} P(3) = \frac{1}{2} \times 0.1804 = 0.0902$

Ex.-2 Using poissons distribution ; find the Probability that the aces of spades will be drawn from a pack of well-shuffled cards at least once in 104 consecutive trials (given $e^{-2} = 0.1353$)

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કાળીનો એક્કો ace = ગંજીફાનો એક્કો
spade = ગંજીફાનું કાળીની બાજુનું પાનું
shuffle = to mix at random

Sol^n :→ We have

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{x!} \quad ; \quad x = 0, 1, 2, 3, \ldots$$

Here $\lambda = n \cdot p = 104 \times \frac{1}{52} = 2$

∴ Probability of drawing an ace of spades at least once

$= P(1) + P(2) + \cdots + P(52)$

$= 1 - P(0)$

$= 1 - \frac{e^{-2}(1)}{0!}$

$= 1 - e^{-2}$

$= 1 - 0.1353$

$= 0.8647$

Ex-3 :→ If the probability that an individual suffers a bad reaction from a certain injection is 0.001 determine the probability that out of 2000 individuals
(a) exactly 3 and
(b) more than 2 individuals will suffer a bad reaction (given $e^{-2} = 0.136$)

sol:→ Here $P = 0.001$; $n = 2000$

$\lambda = nP = 2000 \times 0.001 = 2$

According to poissons distribution

$$P(r) = \frac{e^{-\lambda} \lambda^{r}}{r!}$$

(a) $P(3) = \frac{e^{-2} (2)^3}{3!} = \frac{8}{6} \times e^{-2} = \frac{4}{3} \times 0.136 = 0.1804$

or $1 - P(r \le 2)$

(b) $P(r > 2) = P(3) + P(4) + P(5)$

$= 1 - [P(0) + P(1) + P(2)]$

$= 1 - \left\{ \frac{e^{-2}}{0!} + \frac{e^{-2} \cdot 2}{1!} + \frac{e^{-2}\, 2^2}{2!} \right\}$

$= 1 - e^{-2} \{1 + 2 + 2\}$

$= 1 - (0.136)(5)$

$= 1 - 0.680$

$= 0.32$

Ex-4 Between the hours 2 P.M. and 4 P.M. The average number of phone calls per minute coming into the switch board of a company is 2.35. Find the probability that during one particular minute; there will be at most 2 phone calls. [given $e^{-2.35} = 0.095374$]
(by using Poisson dist)

Solⁿ :- If the variable x denotes the number of telephone calls per minute; then x will follow Poisson distribution with parametre $m = 2.35$ and probability fⁿ;

$$P(x = r) = \frac{e^{-m} m^r}{r!} = \frac{e^{-2.35} \times (2.35)^r}{r!}$$

The prob. that during one particular minute there will be at most 2 phone calls is given by

$$P(x \le 2) = P(0) + P(1) + P(2)$$

$$= e^{-2.35} \left\{ 1 + 2.35 + \frac{(2.35)^2}{2!} \right\}$$

$$= 0.095374 \{ 1 + 2.35 + 2.76125 \}$$

$$= 0.095374 \times 6.11125$$

$$= 0.5828543$$

Ex-5 It is known from past experience that in a certain plant there are on the average 4 industrial accidents per month. Find the probability that in a given year there will be less than 4 accidents by assuming poisson distribution ($e^{-4} = 0.0183$)

Sol:- In the usual notations; given $m = 4$
If the variable x denotes the number of accidents in the plant per month; then by poisson distribution;

$$P(X = r) = \frac{e^{-m} m^{r}}{r!} = \frac{e^{-4} (4)^{r}}{r!}$$

The required Prob. that there will be less than 4 accidents is given by;

$$P(X < 4) = P(0) + P(1) + P(2) + P(3)$$

$$= e^{-4} \left\{ 1 + 4 + \frac{4^2}{2!} + \frac{4^3}{3!} \right\}$$

$$= e^{-4} \{ 1 + 4 + 8 + 10.67 \}$$

$$= 0.0183 \times 23.67$$

$$= 0.4332$$

Ex-6 If 5% of the electric bulbs manufactured by a company are defective; use poisson distribution to find the probability that in a sample of 100 bulbs.
⟨i⟩ none is defective
⟨ii⟩ 5 bulbs will be defective.

(given : $e^{-5} = 0.007$)

sol^n :→ Here given ; $n = 100$

p = Prob. of a defective bulb = 5% = 0.05

{ since p is small and n is large ; we may approximate the given dist. by poisson dist. }

The parameter m of poisson dist. is :

$$m = np = 100 \times 0.05 = 5$$

Let x denote the number of defective bulbs in a sample of 100 then

$$P(x = r) = \frac{e^{-m} m^r}{r!} \Rightarrow \frac{e^{-5} 5^r}{r!}$$

⟨i⟩ The prob. that none of the bulbs is defective is given by :

$$P(x=0) = e^{-5} = 0.0067$$

⟨ii⟩ The prob. of 5 defective bulbs is given by

$$P(x=5) = \frac{e^{-5}\, 5^5}{5!} = \frac{0.0067 \times 625}{24}$$

$$= \frac{4.375}{24}$$

$$= 0.1754$$

Ex-7 A manufacturer of blades knows that 5% of his product is defective if he sells blades in boxes of 100; and guarantees that not more than 10 blades will be defective; what is the probability that a box will fail to meet the guaranteed quality?

Soln:→ Let p = Prob. of a defective blade = 5% = 0.05

(since the Prob. of a defective blade is small; we use poisson distribution)

In usual notation; given $n = 100$

Hence;

$$m = np = 100 \times 0.05 = 5$$

If x denote the no. of defective blades in a box of 100; then by poisson dist

$$P(x = r) = \frac{e^{-m}\, m^r}{r!} = \frac{e^{-5}\, 5^r}{r!}$$

A box will fail to meet the guaranteed quality if the number of defectives in it is more than 10

∴ the required probability is :

$$P(X>10) = 1 - P(X \le 10)$$

$$= 1 - \sum_{r=0}^{10} P(r)$$

$$= 1 - \sum_{r=0}^{10} \frac{e^{-5}\, 5^{r}}{r!}$$

$$= 1 - e^{-5} \sum_{r=0}^{10} \frac{5^{r}}{r!}$$

$$= 0.01369 \quad \text{OR} \quad 0.0189$$

-8 In a certain factory turning out optical lenses; there is a small chance 1/500 for any one lens to be defective. The lences are supplied in packets of 10

Use poisson distribution to calculate the approximate number of packets containing no defective; one defective, two defective, three defective lenses respectively in a consignment of 20,000 packets

$$\left(\text{given } e^{-0.02} = 0.9802\right)$$

Sol:→ In the usual notations ; given

$N = 20,000$

$n = 10$

p = Prob. of a defective optical lense $= 1/500$

$m = np = 10 \times 1/500 = 1/50 = 0.02$

Let x denote the number of defective optical lenses in a packet of 10

∴ The prob. of r defective lenses in a packet is given by

$$P(x=r) = \frac{e^{-0.02} \times (0.02)^r}{r!}$$

$$= \frac{0.9802 \times (0.02)^r}{r!}$$

Hence in a consignment of 20,000 packets the frequency (number) of packets containing r defective lenses is

$$= N \cdot P(x=r)$$

$$= \frac{20000 \times 0.9802 \times (0.02)^r}{r!} \text{ ------- } (*)$$

Putting : $r = 0, 1, 2, 3, 4$ in eq^n (*)

we get ;

⊛ No. of packets containing no defective lens is

$= 20000 \times 0.9802$

$= 19604$

⊛ No. of packets containing 1 defective lens is

$$= \frac{20000 \times 0.9802 \times (0.02)}{1}$$

$= 19604 \times 0.02$

$= 392.08$

$\simeq 392$

⊛ No. of packets containing 2 defective lences is

$$= \frac{20000 \times 0.9802 \times (0.02)^2}{2!}$$

$$= \frac{392.08 \times 0.02}{2} = 3.9208 \simeq 4$$

⊛ No. of packets containing 3 defective lences is

$$= \frac{20000 \times 0.9802 \times (0.02)^3}{3!}$$

$$= \frac{3.9208 \times 0.02}{3} = 0.026138 \simeq 0$$

Hence the number of Packets containing 0, 1, 2 and 3 defective lenses is respectively 19604, 392, 4, and 0

EX-9 A car hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson variate with mean 1.5. Calculate the proportion of days on which

⟨i⟩ Neither car is used

⟨ii⟩ some demand is refused.

Sol: Let the variable x denote the number of demands for a car on any day.

Then x follows Poisson law with parameter $m = 1.5$

$\therefore P(x = r)$ = Prob. of r demands for a car on any day

$$= \frac{e^{-1.5} \times (1.5)^r}{r!}$$

⟨i⟩ Neither car will be used; if there is no demand for any car.

$\therefore$ The required proportion of days on which no car is used is given by

$$P(x = 0) = e^{-1.5} = 0.2231$$

⟨ii⟩ since the firm has only two cars; some demand will be refused if the number of demands per day is greater than 2

∴ The Required proportion is

$P(X > 2) = 1 - P(X \leq 2)$

$= 1 - \{ P(0) + P(1) + P(2) \}$

$= 1 - \left\{ e^{-1.5} + 1.5\, e^{-1.5} + \frac{(1.5)^2}{2!} e^{-1.5} \right\}$

$= 1 - e^{-1.5} \left\{ 1 + 1.5 + \frac{2.25}{2} \right\}$

$= 1 - 0.2231 \{ 1 + 1.5 + 1.125 \}$

$= 1 - 0.2231 \{ 3.625 \}$

$= 1 - 0.80874$

$= 0.19126$

Ex-10 Assuming that one is 80 births in a case of twins; calculate the probability of 2 or more sets of twins on a day when 30 births occur. Compare the results obtained by using <i> the Binomial distribution
<ii> Poisson approximation.

Sol:→ <i> Using Binomial distribution

p = Prob. of twin births $= \frac{1}{80} = 0.0125$

$q = 1 - p = 0.9875$ and $n = 30$

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If x denotes the number of twin births on a day then ;

$$P(X=r) = {}^{30}C_r\,(0.0125)^r\,(0.9875)^{30-r}$$

∴ Prob. of two or more sets of twins on a day is given by :

$$P(x \geq 2) = 1 - P(x < 2)$$

$$= 1 - [P(0) + P(1)]$$

$$= 1 - \{(0.9875)^{30} + {}^{30}C_1\,(0.9875)^{29}\,(0.0125)\}$$

$$=$$

$$= 1 - \{0.6839 + 0.2597\}$$

$$= 0.0541 \quad (\text{or } 0.0564)$$

<ii> Using poisson distribution

$n = 30$; $p = 0.0125$, $m = np = 0.3750$

By poisson Probability law ;

$$P(X=r) = \frac{e^{-m}\,m^r}{r!}$$

$$= \frac{e^{-0.375} \times (0.375)^r}{r!}$$

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If x denotes the number of twin births on a day then ;

$$P(X = r) = {}^{30}C_r (0.0125)^r (0.9875)^{30-r}$$

$\therefore$ Prob. of two or more sets of twins on a day is given by :

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - [P(0) + P(1)]$$

$$= 1 - \{(0.9875)^{30} + {}^{30}C_1 (0.9875)^{29} (0.0125)\}$$

$$=$$

$$= 1 - \{0.6839 + 0.2597\}$$

$$= 0.0541 \quad (\text{or } 0.0564)$$

(ii) Using poisson distribution

$n = 30$; $p = 0.0125$, $m = np = 0.3750$

By poisson Probability law ;

$$P(X = r) = \frac{e^{-m} m^r}{r!}$$

$$= \frac{e^{-0.375} \times (0.375)^r}{r!}$$

$\therefore$ Required prob. $= P(x \geq 2)$

$$= 1 - \{P(0) + P(1)\}$$

$$= 1 - \{e^{-0.375} + (0.375)e^{-0.375}\}$$

$$= 1 - e^{-0.375}\{1 + 0.375\}$$

$$= 1 - (0.6873)(1.375)$$

$$= 1 - 0.9450$$

$$= 0.05498$$

Ex-11 Assume that the probability of an individual coal miner being killed in a mine accident during a year is $1/2400$ use appropriate statistical distribution to calculate the probability that in a mine employing 200 miners there will be at least one fatal accident in a year.

Sol^n:-

Here ; $n = 200$; $p = 1/2400$

i.e n is very large ; p is very small

$\therefore$ we use poisson distribution

Mean $\lambda = np = 200 \times \frac{1}{2400} = \frac{1}{12} = 0.083$

coal miner – કોલસાની ખાણ
mine – જમીનમાં ગોઠવેલી સુરંગ
fatal – ઘાતક, વિનાશક

Now the prob. of r fatal accidents is

$$P(X = r) = \frac{e^{-\lambda} \lambda^{r}}{r!}$$

$\therefore$ The prob. that no fatal accident occurs

$$= P(0)$$

$$= e^{-0.083}$$

$$= 0.9200$$

$\therefore$ The prob. of at least one fatal accident in a year

$$= 1 - P(0)$$

$$= 1 - 0.9200$$

$$= 0.0796$$

$$= 0.08$$

Ex-12 six coins are tossed 6400 times use the Poisson distribution, determine the approximate probability of getting six heads x times.

Sol^n:- Prob. of getting a head with one coin

$$= \frac{1}{2}$$

$\therefore$ The prob. of getting six heads with six coins $= \left(\frac{1}{2}\right)^{6} = \frac{1}{64}$

∴ Average number of six heads with six coin in 6400 thrown

$= np$

$= 6400 \times \frac{1}{64}$

$= 100$

∴ The mean $\lambda = 100$

Approximate probability of getting six heads x times in poisson dist

$= \frac{e^{-\lambda} \lambda^{x}}{x!}$

$= \frac{e^{-100} \times (100)^{x}}{x!}$

Ex-13. An insurance company finds that 0.005 % of the population dies from a certain kind of accident each year. what is the probability that the company must pay off no more than 3 of 10,000 insured risks against such accidents in a given year?

Sol :→ $p = 0.005\% = \frac{0.005}{100} = 0.00005$

$n = 10{,}000$

$\lambda = np = 10,000 \times 0.00005 = 0.5$

If r is the number of pay off's against insured risks

$$P(r) = \frac{e^{-\lambda} \lambda^r}{r!} = \frac{e^{-0.5} \times (0.5)^r}{r!}$$

Now,

$$P(r>3) = 1 - P(r \le 3)$$

$$= 1 - \{P(0) + P(1) + P(2) + P(3)\}$$

$$= 1 - e^{-0.5} \left\{1 + 0.5 + \frac{(0.5)^2}{2!} + \frac{(0.5)^3}{3!}\right\}$$

$$= 1 - 0.6065 (1.646)$$

$$= 1 - 0.9984$$

$$= 0.0016$$

Ex-14 Find the probability that at most 5 defective components will be found in a lot of 200 if experience shows that 2% of such components are defective. Also find the prob. of more than 5 defective components.

Solⁿ:- Here; $p = 2\% = \frac{2}{100} = 0.02$

$n = 200$

$\therefore \lambda = np = 200 \times 0.02 = 4$

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$\therefore$ P (at most 5 defective components)

$= P(X \le 5)$

$= P(0) + P(1) + P(2) + P(3) + P(4) + P(5)$

$= \frac{e^{-4} 4^0}{0!} + \frac{e^{-4} 4^1}{1!} + \frac{e^{-4} 4^2}{2!} + \frac{e^{-4} 4^3}{3!} + \frac{e^{-4} 4^4}{4!} + \frac{e^{-4} 4^5}{5!}$

$= e^{-4} \left\{ 1 + 4 + 8 + \frac{32}{3} + \frac{32}{2} + \frac{128}{15} \right\}$

$= 0.018 \left\{ \frac{643}{15} \right\}$

$= 0.784$

(*) Also P (more than 5 defective components)

$= P(X > 5)$

$= 1 - P(X \le 5)$

$= 1 - 0.784$

$= 0.216$

(*) Fitting of Poisson Distribution

(pg : 994)

Book : Fundamentals of statistics

→ By S. C. Gupta.

If N is the total observed freq. then the expected or theoretical freq. of the poisson distribution are given by

$$= N \times P(r)$$

$$= N \times \frac{e^{-m} m^{r}}{r!}$$

[C] NORMAL DISTRIBUTION

The above Distributions; Viz. Binomial distribution and poisson distribution are discrete probability distributions; since the variables under study were discrete random variables.

Now we confine the discussion to continuous probability distributions which arise when the underlying variable is a continuous one.

Normal distribution is one of the most important continuous theoretical distributions in statistics.

Def^{n}:→ If x is a continuous random variable following by normal probability distribution with mean μ and standard deviation σ; then its probability density f^{x} is given by:

$$\boxed{P(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}$$

where $-\infty < x < \infty$

Here π and e are absolute constants with values 3.14159 and 2.71828 respectively

→ The mean μ and standard deviation σ

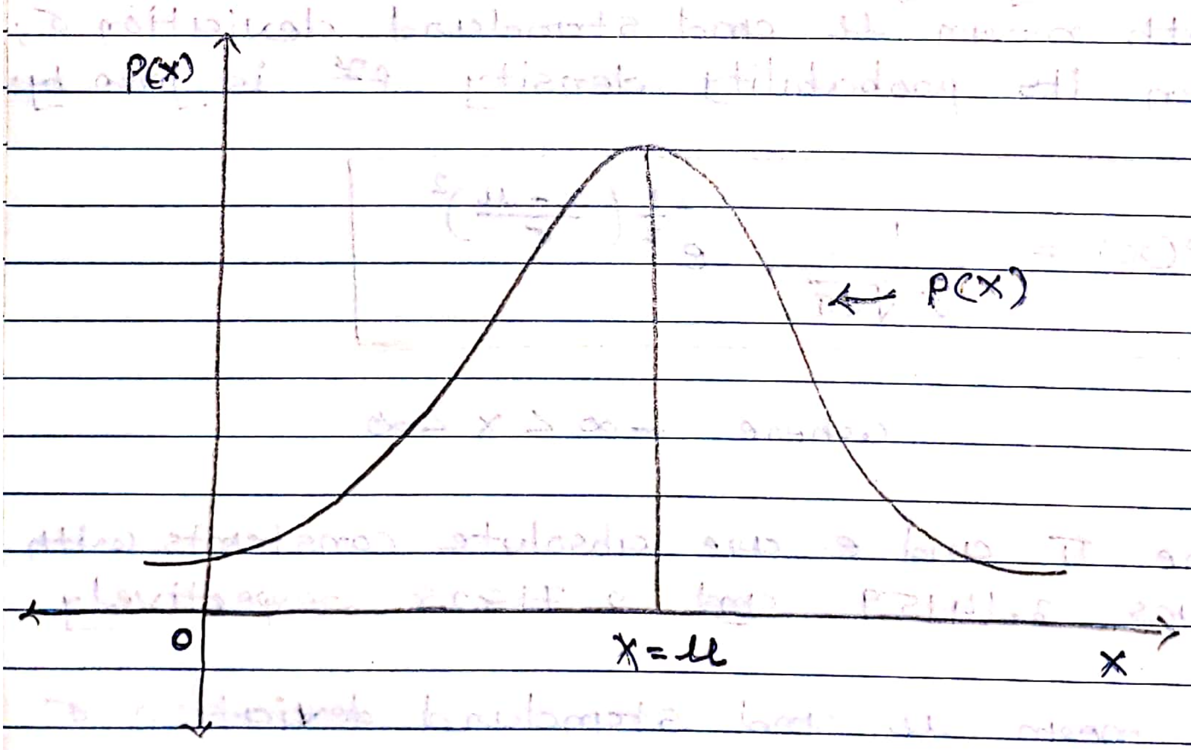
are called the parameters of the Normal distribution.

→ If x is a random variable followed by normal distribution with mean μ and standard deviation σ; then the random variable z defined as:

$$z = \frac{x - \mu}{\sigma}$$

is called the standard normal variate.

→ For given values of the parameters μ and σ; the shape of the curve corresponding to normal probability density f^n p(x) is the famous bell shaped curve as shown in the diagram as under.



* Properties of Normal distribution :

<1> It is perfectly symmetrical about the mean μ and is bell - shaped.

<2> since the distribution is symmetrical, mean; median and mode coincide.

i.e

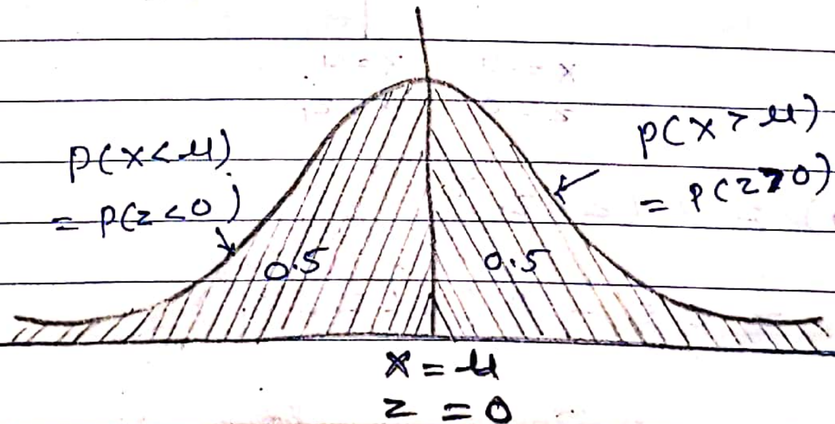
Mean = Median = Mode

<3> Distribution is unimodal; since the only mode occuring at $x = \mu$.

<4> Since Mean = Median = μ ; the ordinate at $x = \mu$ $(z = 0)$ divides the whole (area) region into two equal parts.

Also; since total area under normal probability curve is 1; the area to the right of the ordinate as well as to the left of the ordinate at $x = \mu$ (or $z = 0$) is 0.5

<5> since total probability is always 1; we have the total area under the normal probability curve is 1



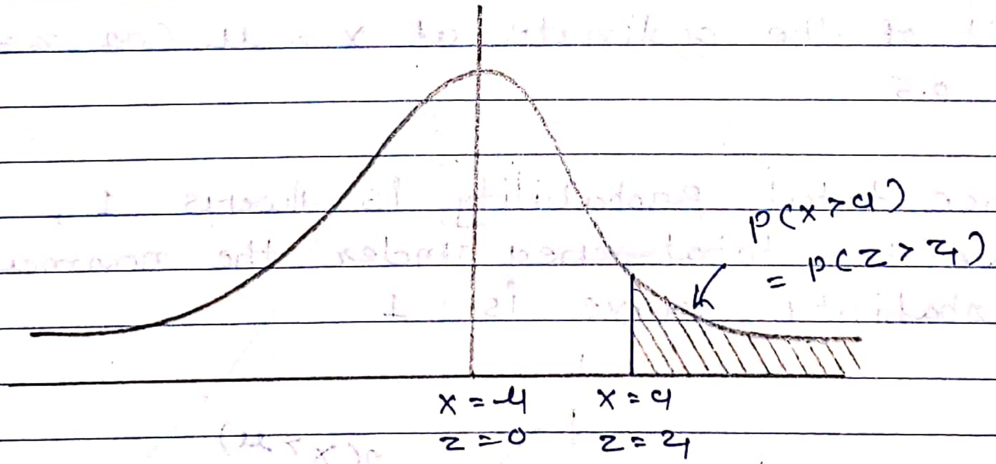
Note:- For Practical problems we don't deal with the variable x but first convert it in to standard normal variate z. Next we try to convert the required area in the form $P(0 < z < z_1)$ by using the following results.

$P(X > \mu) = P(z > 0) = 0.5$
$P(X < \mu) = P(z < 0) = 0.5$
(∵ by above figure)

and making use of the symmetry property of the distribution.

(A) computation of area to the right of the ordinate at $X = a$ i.e to find $\{P(X > a)\}$

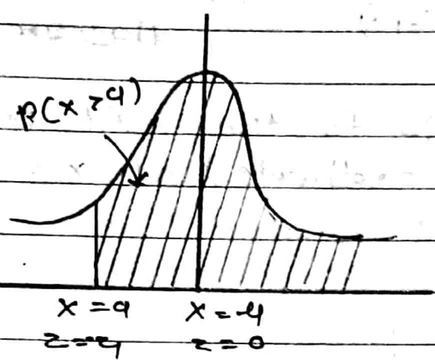
⊛ case → (i) : $a > \mu$; i.e a is to the right of the mean ordinate.

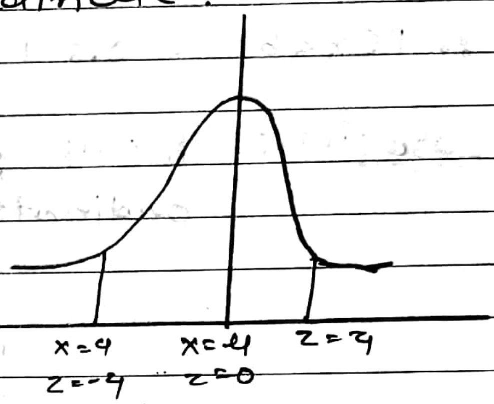


$\therefore P(X > a) = P(z > z_1)$

$= 0.5 - P(0 < z < z_1)$

⊛ case (ii): $a < \mu$; i.e. a is to the left of the mean ordinate.





since $a < \mu$; the value of z corresponding to $x = a$ will be negative

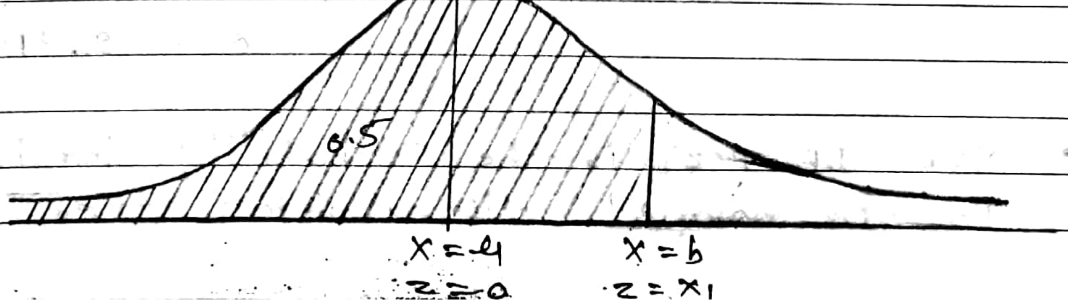
when $x = a$; $z = \frac{a - \mu}{\sigma} = -z_1$ (say)

$\therefore P(X > a) = 0.5 + P(-z_1 < z < 0)$ ($\because$ from diagram)

$= 0.5 + P(0 < z < z_1)$ ($\because$ By symmetry)

(B) Computation of the area to the left of the ordinate $x = b$ i.e. to find $P(X < b)$

⊛ case (i): $b > \mu$; i.e. b is to the right of the ordinate (mean ordinate) at $x = \mu$



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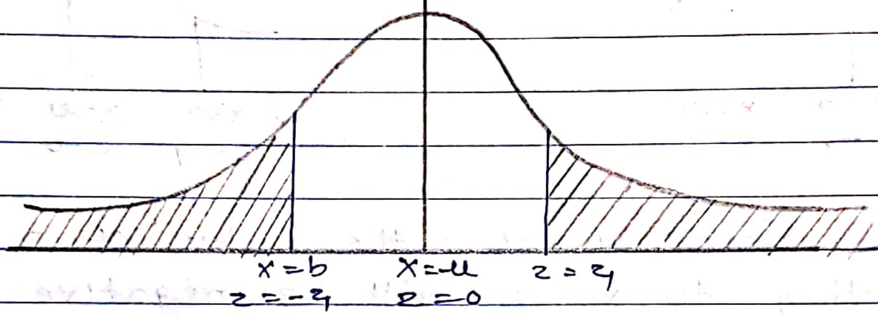
when $x = b$; $z = \frac{b-\mu}{\sigma} = z_1$ (say)

$\therefore P(x<b) = 0.5 + P(0<z<z_1)$ ($\because$ from diagram)

Case (ii) :- $b < \mu$; ie b is to the left of the ordinate (Mean ordinate) at $x = \mu$



$\therefore P(x<b) = P(z<-z_1)$
$= P(z>z_1)$
$= 0.5 - P(0<z<z_1)$ ($\because$ by Symmetry)

Note:→ If $z = \frac{x-\mu}{\sigma}$ is the standard normal variate then ;
The probability density f^{ns} for the normal distribution in standard form is given by

$$\boxed{f(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}}$$

where $\pi = 22/7$; $\sqrt{2\pi} = 2.5066$
$e = 2.71828$

→ Now If $f(z)$ is the probability density f^{ns} for the normal distribution then

$$P(z_1 \le z \le z_2) = \int_{z_1}^{z_2} f(z)\,dz$$

$\rightarrow P(z_1 \le z \le z_2) = P(z_1 \le z < z_2) = P(z_1 < z \le z_2) = P(z_1 < z < z_2)$

$\rightarrow$ See table $\rightarrow$ VI
pg : 1347
Book $\rightarrow$ Fund. of Stat
By S. C. Gupta

Ex-1 Suppose the waist measurements W of 800 girls are normally distributed with mean 66 cms and standard deviation 5 cms. Find the number N of girls with waists.
<i> between 65 and 70 cms;
<ii> greater than or equal to 72 cms.

Sol:→ Here W : waists measurements (in cms) of girls

W (in cms.)	65	70	72
$Z = \frac{W-\mu}{\sigma} = \frac{W-66}{5}$ (standard normal variate)	$\frac{65-66}{5} = -0.2$	$\frac{70-66}{5} = 0.8$	$\frac{72-66}{5} = 1.2$

For table → Book : N. P. Bali (pg : 1651)

<i> The prob. that a girl has waist bet^n 65 cms and 70 cms is given by :

$$P(65 \le W \le 70) = P(-0.2 \le z \le 0.8)$$

$$= P(-0.2 \le z \le 0) + P(0 \le z \le 0.8)$$

$$= P(0 \le z \le 0.2) + P(0 \le z \le 0.8)$$

(∵ by symmetry)

$$= 0.0793 + 0.2881$$

$$= 0.3674$$

see table - VI
pg. : 1347
Book : Fundamental of Satistics
by S.C. Gupta

Hence in a group of 800 girls; the expected number of girls with waists bet^n 65 cms and 70 cms. is given by

$= 800 \times 0.3674$

$= 293.92 \simeq 294$

(ii) The probability that a girl has waist greater than or equal to 72 cms is given by

$P(W \geq 72) = P(Z \geq 1.2)$ ($\because$ by above table)

$= 0.5 - P(0 \leq Z \leq 1.2)$

$= 0.5 - 0.3849$

$= 0.1151$

$\therefore$ In a group of 800 girls; the expected number of girls with waists greater than or equal to 72 cms. is given by

$= 800 \times 0.1151$

$= 92.08$

$\simeq 92$

Ex-2 A sample of 100 dry battery cells tested to find the length of life produced the following results.

$\bar{x} = \mu = 12$ hrs. ; $\sigma = 3$ hrs.

Assuming the data to be normally distributed what percentage of battery cells are expected to have a life

<i> more than 15 hours,

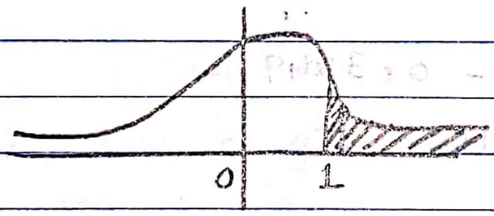
<ii> less than 6 hours.

<iii> between 10 and 14 hours.

Sol^n :→ Let x denotes the length of life of dry battery cells.

Also : $z = \frac{x-\mu}{\sigma} = \frac{x-12}{3}$

<i> When $x = 15$; $z = 1$.



$\therefore P(x \geq 15) = P(z \geq 1)$

$= P(0 < z < \infty) - P(0 < z < 1)$

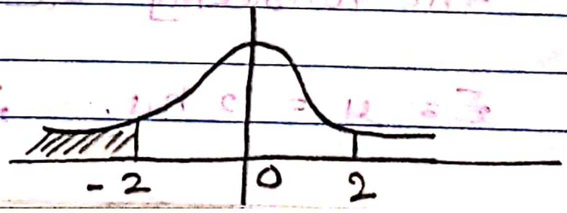
$= 0.5 - P(0 < z < 1)$

$= 0.5 - 0.3413$

$= 0.1587$

$= 15.87\ \%$

<ii> when $x = 6$; $z = -2$



$\therefore P(x < 6) = P(z < -2)$

$= P(z > 2)$ ($\because$ by symmetry)

$= P(0 < z < \infty) - P(0 < z < 2)$

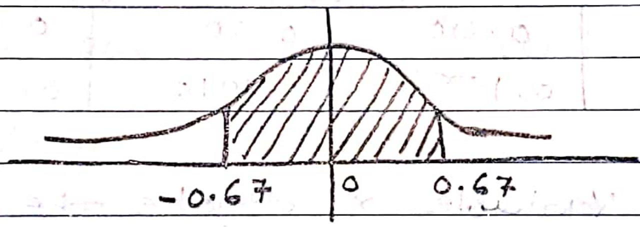
$= 0.5 - 0.4772$

$= 0.0228$

$= 2.28\ \%$

<iii> when $x = 10$; $z = -\frac{2}{3} = -0.67$

when $x = 14$; $z = \frac{2}{3} = 0.67$



$P(10 < x < 14) = P(-0.67 < z < 0.67)$

$= 2\,P(0 < z < 0.67)$ ($\because$ symmetry)

$= 2 \times 0.2487$

$= 0.4974$

$= 49.74\ \%$

0.67 જોવા માટે
0 1 2 3 4 5 6 7
0.6

Ex-3 : A sales Tax officer has reported that the average sales of the 500 business that he has to deal with during a year amount to Rs. 36,000 with a standard deviation of Rs. 10,000. Assuming that the sales in these

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business are normally distributed; find
<i> The number of businesses the sales of which are over Rs. 40,000
<ii> The percentage of businessess; the sales of which are likely to range between Rs. 30,000 and Rs. 40,000
<iii> The probability that the sales of business selected at random will be over Rs. 30000

Proportions of the area under the normal curve:

z	0.25	0.40	0.50	0.60
area	0.0987	0.1554	0.1915	0.2257

sol^n:→ Let the variable x denote the sales (in Rs.) of the business during a year
given:

$\mu = 36{,}000$ and $\sigma = 10{,}000$

<i> The probability that the sales of a business is over Rs. 40,000 is; given by
$P(x > 40{,}000)$

when $x = 40{,}000$; $z = \frac{x-\mu}{\sigma} = \frac{40000 - 36000}{10{,}000}$

$= 0.4$

$\therefore P(x > 40{,}000) = P(z > 0.4)$

$= 0.5 - P(0 \le z \le 0.4)$

$= 0.5 - 0.1554$

$= 0.3446$

∴ in a group of 500 businesses, the expected number of businesses with annual sales over Rs. 40,000 is

$= 500 \times 0.03446$

$= 172.3$

$\simeq 172$

<ii> Required probability p is given by

$p(30{,}000 < x < 40{,}000)$

$= p(-0.6 < z < 0.4)$

∴ when $x = 30{,}000$; $z = \frac{x - \mu}{\sigma} = \frac{30{,}000 - 36{,}000}{10{,}000}$

$= -0.6$

when $x = 40{,}000$; $z = 0.4$

$= p(-0.6 < z < 0.4)$

$= p(-0.6 < z \leq 0) + p(0 \leq z < 0.4)$

$= p(0 \leq z < 0.6) + p(0 \leq z < 0.4)$

(∵ symmetry)

$= 0.2257 + 0.1554$

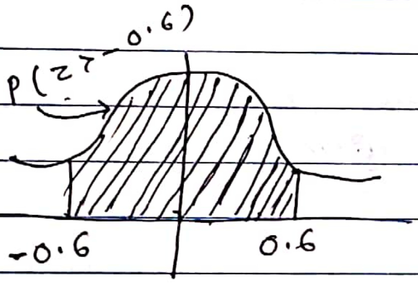
$= 0.3811$

$= 38.11\%$

<iii> The probability that the annual sales of a business selected at random will be over Rs. 30,000 is given by :

$P(X > 30{,}000) = P(z > -0.8)$

when $X = 30{,}000$; $z = \frac{x-\mu}{\sigma} = -0.6$



$= P(-0.6 < z < 0) + 0.5$

$= P(0 < z < 0.6) + 0.5$

$= 0.2257 + 0.5$

$= 0.7257$

Ex-4 Assume the mean height of soldiers to be 68.22 inches with a variance of 10.8 inches. How many soldiers in a regiment of 1000 would you expect to be
<i> over six feet tall and
<ii> below 66 inches.
Assume heights to be normally distributed.

Sol^n:- Let the variable x denote the height (in inches) of the soldiers
Then we are given

mean $\mu = 68.22$
Variance $\sigma^2 = 10.8$

<i> A soldier will be over 6 feet tall if x is greater than 72

($\because$ x is height in inches and 6 feet = 72 inches)

$\therefore$ when $x = 72$

$$z = \frac{x-\mu}{\sigma} = \frac{72 - 68.22}{\sqrt{10.8}} = \frac{3.78}{3.286} = 1.15$$

$\therefore$ The probability that a soldier is over 6 feet tall is given by

$$P(x > 72) = P(z > 1.15)$$
$$= 0.5 - P(0 \le z \le 1.15)$$
$$= 0.5 - 0.3749 \quad (\because \text{by table}).$$
$$= 0.1251$$

$\therefore$ In a regiment of 1,000 soldiers; the number of soldiers over 6 feet tall is

$$= 1000 \times 0.1251$$
$$= 125.1$$
$$\simeq 125$$

(ii) The probability that a soldier is below 66 inches is given by:

$$P(x < 66) = P(z < -0.6756)$$

when $x = 66$; $z = \frac{x-\mu}{\sigma} = -0.6756$

0.6756 જોવા માટે

0 1 2 3 4 5 6 7

0.6   0.67

$$= P(z > 0.6756) \quad (\because \text{symmetry})$$
$$= 0.5 - P(0 < z < 0.6756)$$

$= 0.5 - 0.2501$ (∵ by table)

$= 0.2499$

∴ The number of soldiers over 66 inches in a regiment of 1,000 soldiers is :

$= 1000 \times 0.2499$

$= 249.9$

$\simeq 250$

Ex-5 The average test marks in a particular class is 79. The standard deviation is 5. If the marks are distributed normally; how many students in a class of 200 did not receive marks between 75 and 82?

given :

$$\left.\begin{aligned} Pr.\{0 \le z \le 0.6\} &= 0.2257 \\ Pr.\{0 \le z \le 0.7\} &= 0.2580 \\ Pr.\{0 \le z \le 0.8\} &= 0.2881 \end{aligned}\right\}$$

where z is a standard normal variable.

Solⁿ :→ Let the variable x denotes the marks obtained by the students in the given test betⁿ 75 and 82; then we are given :

$\mu = 79$; $\sigma = 5$

$$\left.\begin{aligned} \text{when } x = 75 \;;\; z &= \frac{x-\mu}{\sigma} = -0.8 \\ x = 82 \;;\; z &= \frac{x-\mu}{\sigma} = 0.6 \end{aligned}\right\}$$

$\therefore$ The probability that a one student gets marks betⁿ 75 and 82 is given by :

$$P(75 < x < 82) = P(-0.8 < z < 0.6)$$

$$= P(-0.8 < z < 0) + P(0 < z < 0.6)$$

$$= P(0 < z < 0.8) + P(0 < z < 0.6)$$

($\because$ symmetry)

$$= 0.2881 + 0.2257$$

$$= 0.5738$$

$\therefore$ The prob. P that a student does not get marks between 75 and 82 is given by :

$$P = 1 - P(\text{student gets marks bet}^{n} \text{ 75 and } 82)$$

$$= 1 - P(75 < x < 82)$$

$$= 1 - 0.5738$$

$$= 0.4862$$

Hence in a class of 200 students ; the number of students who did not receive marks betⁿ 75 and 82 is given by :

$$= 200 \times P$$

$$= 200 \times 0.4862$$

$$= 97.24$$

$$\simeq 97$$

Ex-6 The weekly wages of 1000 workmen are normally distributed around a mean of Rs. 70 and with a standard deviation of Rs. 5 Estimate the number of workers whose weekly wages will be:

<i> between Rs. 70 and 72

<ii> betn Rs. 69 and 72

<iii> more than Rs. 75

<iv> less than Rs. 63

<v> more than Rs. 80.

Also; estimate the lowest weekly wages of the 100 highest paid workers.

Soln :→ Let the variable x denote the weekly Wages in Rupees. Then x is normal variable with mean $\mu = 70$ and $\sigma = 5$

$$\therefore z = \frac{x-\mu}{\sigma} = \frac{x-70}{5}$$

<i> We have to find : $P(70 < X < 72)$

$$\left.\begin{array}{l} \text{when } x = 70 \ ; \ z = 0 \\ \quad\quad\ \ x = 72 \ ; \ z = 0.4 \end{array}\right\}$$

$$\therefore P(70 < x < 72) = P(0 < z < 0.4)$$
$$= 0.1554$$

∴ The number of workers with weekly wages betn Rs. 70 and 72 is

$= 1000 \times 0.1554$
$= 155.4$
$\simeq 155$

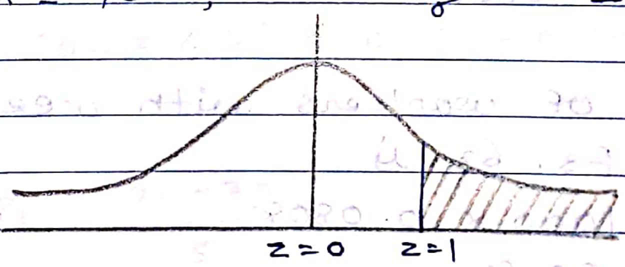
<ii> $P(69 < x < 72) = P(-0.2 < z < 0.4)$

$= P(-0.2 < z < 0) + P(0 < z < 0.4)$
$= P(0 < z < 0.2) + P(0 < z < 0.4)$
$= 0.0793 + 0.1554$
$= 0.2347$

$\therefore$ The required number of workers
$= 1000 \times 0.2347$
$= 234.7$
$\simeq 235$

<iii> we have to find $P(x > 75)$

when $x = 75$; $z = \frac{x-\mu}{\sigma} = 1$



$\therefore P(x > 75) = P(z > 1)$
$= 0.5 - P(0 < z < 1)$
$= 0.5 - 0.3413$
$= 0.1587$

$\therefore$ The number of workers with weekly wages more than the Rs. 75 is

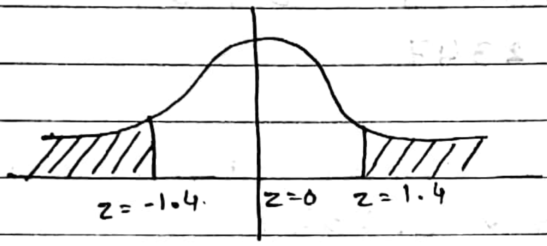
$= 1000 \times 0.587$

$= 158.7$

$\simeq 159$

<iv> $P(X < 63) = P(z < -1.4)$

$\left(\text{when } x = 63 ; \; z = \frac{x - \mu}{\sigma} = -1.4 \right)$



$\therefore P(X < 63) = P(z < -1.4)$

$= P(z > 1.4)$ ($\because$ by symmetry)

$= 0.5 - P(0 < z < 1.4)$

$= 0.5 - 0.4192$

$= 0.0808$

$\therefore$ The number of workers with weekly wages less than Rs. 63 is

$= 1000 \times 0.0808$

$= 80.8$

$\simeq 81$

(v) When $x = 80$; $z = \frac{x - \mu}{\sigma} = 2$

$\therefore P(X > 80) = P(z > 2)$

$= 0.5 - P(0 < z < 2)$

$= 0.5 - 0.4772 = 0.0228$

$\therefore$ The number of worker's with weekly wages over Rs. 80 is

$$= 1000 \times 0.0228$$
$$= 22.8$$
$$\simeq 23.$$

(*) Now from the given 1000 workers; Proportion of the 100 highest Paid workers

$$= \frac{100}{1000} = \frac{1}{10} = 0.10$$

we want to determine $X = x_1$ (say)
s.t $P(X > x_1) = 0.10$

when $X = x_1$; $z = \frac{x_1 - 70}{5} = z_1$ (say)

$\therefore P(X > x_1) = P(z > z_1) = 0.10$

$\Rightarrow P(0 < z < z_1) = 0.50 - 0.10 = 0.40$

Now;

$$z_1 = \frac{x_1 - 70}{5}$$

$$\Rightarrow 1.28 = \frac{x_1 - 70}{5}$$

($\because$ in table; $0.40 = 1.28$)

$$\Rightarrow x_1 = 1.28 \times 5 + 70$$
$$= 6.40 + 70$$
$$= 76.40$$

$\therefore$ the lowest weekly wages of the 100 highest paid workers are Rs. 76.40

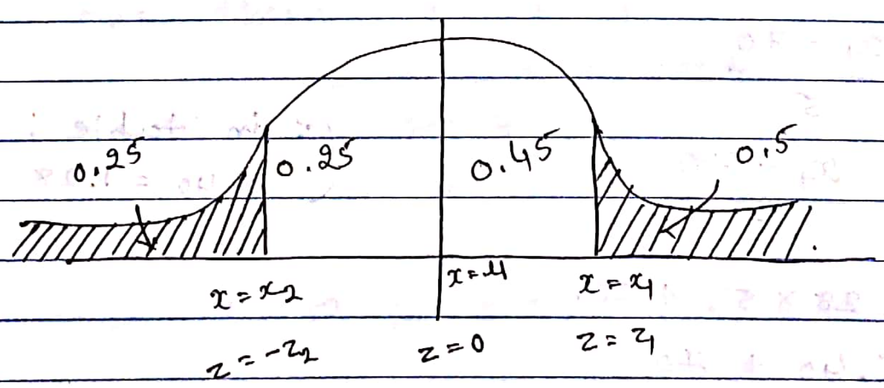
Ex-7 A set of examination marks is approximately normally distributed with a mean of 75 and standard deviation of 5. If the top 5% of students get grade A and the bottom 25% get grade F; what mark is the lowest A and what mark is the highest F?

Sol^n:- Let x denote the marks in exam.
Then x is normally distributed with

$$\left.\begin{array}{ll}\text{mean} & \mu = 75 \\ \text{s.d} & \sigma = 5\end{array}\right\}$$

Let x_1 be the lowest marks for grade A and x_2 be the highest marks for grade F
Then we are give:

$$P(X > x_1) = 0.05$$
$$P(X < x_2) = 0.25$$



Then the standard normal variables corr. to x_1 and x_2 are given by

$$z = \frac{x_1 - \mu}{\sigma} = \frac{x_1 - 75}{5} = z_1 \text{ (say)}$$

$$z = \frac{x_2 - \mu}{\sigma} = \frac{x_2 - 75}{5} = -z_2 \text{ (say)}$$

------ (*)

From the figure ; we get

$P(0 < z < z_1) = 0.45 \Rightarrow z_1 = 1.645$ (from table)

$P(-z_2 < z < 0) = 0.25 \Rightarrow P(0 < z < z_2) = 0.25$

$\Rightarrow z_2 = 0.675$ (from table)

substituting for z_1 and z_2 in (*) ; we get

$x_1 = 75 + 5z_1 = 75 + 5(1.645) = 83.225 \simeq 83$

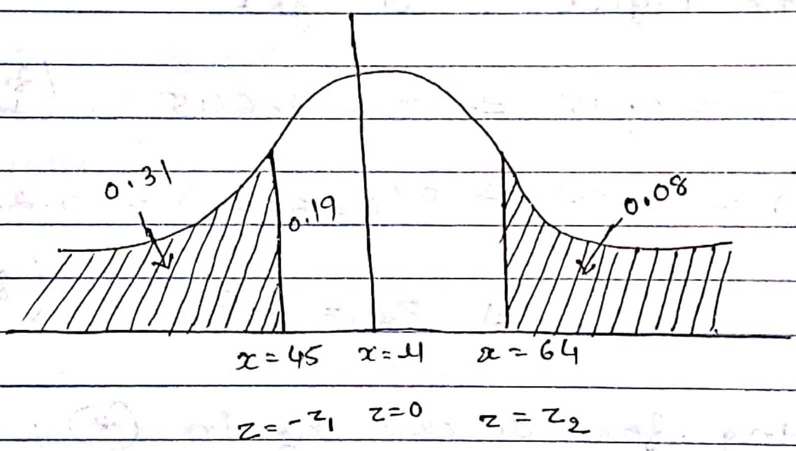
$x_2 = 75 - 5z_2 = 75 - 5(0.675) = 71.625 \simeq 72$

$\therefore$ The lowest marks for grade A is 83 and the highest mark for grade F is 72.

Ex-8 In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

Sol^n:→ Let x be a given variable then we are given;

$$\left.\begin{aligned} P(X<45) &= 0.31 \\ P(X>64) &= 0.08 \end{aligned}\right\}$$



If x has a normal dist. with mean μ and s.d σ then the standard variables corresponding to x = 45 and x = 64 are as given below:

$$\left.\begin{aligned} \text{when } x = 45 ;\quad z &= \frac{45-\mu}{\sigma} = -z_1 \text{ (say)} \\ x = 64 ;\quad z &= \frac{64-\mu}{\sigma} = z_2 \text{ (say)} \end{aligned}\right\}$$

------ (*)

From the above figure it is clear that:

$$\left.\begin{array}{l} P(0<z<z_2) = 0.45 \\ P(-z_1<z<0) = 0.19 \end{array}\right\}$$

Now : $P(0<z<z_2) = 0.45 \Rightarrow \boxed{z_2 = 1.405}$ (from table)

and $P(-z_1<z<0) = 0.19$

$\Rightarrow P(0<z<z_1) = 0.19 \Rightarrow \boxed{z_1 = 0.496}$ (from table)

substituting the values of z_1 and z_2 in ⊛; we get

$$\left.\begin{array}{l} 45-\mu = -0.496\,\sigma \\ 64-\mu = 1.405\,\sigma \end{array}\right\} \begin{array}{l} ----(i) \\ ----(ii) \end{array}$$

Now subtracting (i) from (ii) ; we get

$19 = 1.901\,\sigma$

$\Rightarrow \boxed{\sigma = 10}$ (approx.)

Hence from eqn (i) ;

$\mu = 45 + 0.496(10) = 45 + 4.96 = 49.96$

$\therefore \boxed{\mu \simeq 50}$

Thus; mean is 50 and s.d is 10

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Ex-9 <i> A normal distribution has 77.0 as mean. Find its standard deviation if 20% of the area under the curve lies to the right of 90.0

<ii> A random variable has a normal distribution with 10 as standard deviation. Find its mean if the probability that the random variable takes on a value less than 80.5 is 0.3264

Sol^n :→

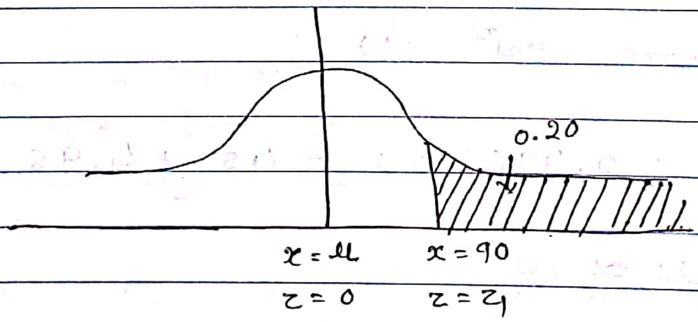
<i> Here given : $\mu = 77.0$ and

$P(x > 90.0) = 0.20$

we have to find s.d σ

when $x = 90$; $z = \dfrac{x-\mu}{\sigma} = \dfrac{90-77}{\sigma} = \dfrac{13}{\sigma} = z_1$ (say)

------ (*)



Thus : $P(x > 90.0) = 0.20$

$\Rightarrow P(z > z_1) = 0.20$

$\Rightarrow P(0 \le z \le z_1) = 0.30$

$\Rightarrow \boxed{z_1 = 0.84}$ (from table)

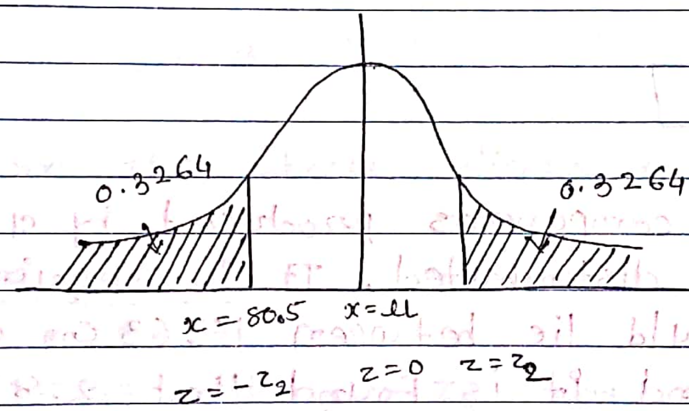
Hence from (*) ;

$$\frac{13}{\sigma} = z_1 = 0.84$$

$$\Rightarrow \boxed{\sigma = \frac{13}{0.84} = 15.48}$$

<ii> we are given :

$$\sigma = 10 \; ; \; P(x < 80.5) = 0.3264$$



since $P(x < 80.5) = 0.3264$; the value of $x = 80.5$ lies to the left of the value $x = \mu$

∴ the value of z corresponding to $x = 80.5$ is negative.

when $x = 80.5$; $z = \frac{x-\mu}{\sigma} = \frac{80.5 - \mu}{10} = -z_2$

------ (**)

Then we are given ;

$$P(x < 80.5) = 0.3264$$

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$\Rightarrow P(z < -z_2) = 0.3264$

$\Rightarrow P(z > z_2) = 0.3264$

$\Rightarrow P(0 < z < z_2) = 0.5 - 0.3264 = 0.1736$

$\Rightarrow \boxed{z_2 = 0.45}$ (by table)

Hence from (**) ;

$$\frac{80.5 - \mu}{10} = -0.45$$

$\Rightarrow \boxed{\mu = 85}$

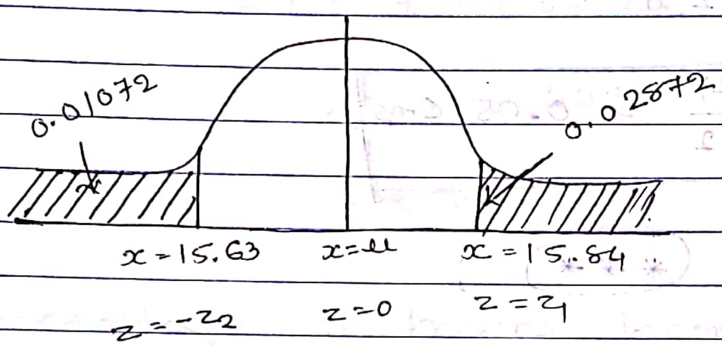
Ex-10 The sizes of components produced by a machine are normally distributed. It is required that the size should lie between 15.63 cm and 15.84 cm. and it is found that 2.872 % of the production is rejected for being over size and 1.072 % of the production is rejected for being undersize. Find the mean and the standard deviation of the distribution of the component sizes.

Solⁿ :→ Let x denote the sizes (in cms) of the components since the size of the components is required to lie betⁿ 15.63 cms and 15.84 cms. the component will be rejected for being over-size if $x > 15.84$ and it will be rejected for being undersize if $x < 15.63$

Thus ; we are given.

$P(X > 15.84) = 2.872\ \% = 0.02872$

$P(X < 15.63) = 1.072\ \% = 0.01072$



From the above diagram ; we get

$$\left.\begin{aligned} \frac{15.84 - \mu}{\sigma} &= z_1 \text{ (say)} \\ \frac{15.63 - \mu}{\sigma} &= -z_2 \text{ (say)} \end{aligned}\right\} \text{-------} \ (*)$$

$$\left.\begin{aligned} \text{and} \quad P(0 < z < z_1) &= 0.47128 \\ P(-z_2 < z < 0) &= 0.48928 \end{aligned}\right\}$$

Now. $P(0 < z < z_1) = 0.47128 \Rightarrow \boxed{z_1 = 1.9}$ (from table)

and $P(-z_2 < z < 0) = 0.48928$

$\Rightarrow P(0 < z < z_2) = 0.48928 \Rightarrow z_2 = 2.3$ (symmetry & from table)

Hence $(*) \Rightarrow$; we get .

$$\left.\begin{aligned} 15.84 - \mu &= 1.9\sigma \\ 15.63 - \mu &= -2.3\sigma \end{aligned}\right\} \begin{aligned} &------ (**) \\ &------ (***) \end{aligned}$$

Subtracting above we get,

$$0.21 = 4.2\sigma$$

$$\Rightarrow \boxed{\sigma = \frac{0.21}{4.2} = 0.05 \text{ cms}}$$

Hence from (***) ;

$$\mu = 15.63 + 2.3(0.05)$$

$$\Rightarrow \boxed{\mu = 15.745 \text{ cms.}}$$

Ex-11: The incomes of a group of 10,000 persons were found to be normally distributed with mean = Rs. 750 per month and s.d = Rs. 50.
Show that of this group about 95% had income exceeding Rs. 668 and only 5% had income exceeding Rs. 832.

Solⁿ:→ Let x be the given variable then

$$\mu = 750 \quad ; \quad \sigma = 50$$

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⟨i⟩ $P(x > 668) = 0.5 + P(668 < x < 750)$

$= 0.5 + P(-1.64 < z < 0)$

$\left(\because z = \frac{x-\mu}{\sigma}\right)$

$= 0.5 + P(0 < z < 1.64)$

$= 0.5 + 0.4495$

$= 0.9495$

$\therefore$ Percentage of having income exceeding Rs. 668

$= 0.9495 \times 100$

$= 94.95\ \%$

⟨ii⟩ $P(x > 832) = 0.5 - P(750 < x < 832)$

$= 0.5 - P(0 < z < 1.64)$

$= 0.5 - 0.4495$

$= 0.0505$

$\therefore$ Percentage of having income exceeding Rs. 832 is

$= 0.0505 \times 100$

$= 5\ \%$

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Ex-12 The average monthly sales of 5000 firms are normally distributed with mean Rs. 36,000 and standard deviation Rs. 10,000

Find: (i) The number of firms with sales over Rs. 40,000

(ii) The percentage of firms with sales betn Rs. 38,500 and Rs. 41,000

(iii) The number of firms with sales betn Rs. 30,000 and Rs. 40,000.

Soln :→ Let x denote the monthly sales of a firm then we are given:

$\mu = 36,000$

$\sigma = 10,000$

(i) Here to find : $P(X > 40,000)$

when $x = 40,000$; $z = \frac{x-\mu}{\sigma} = 0.4$

$\therefore P(x > 40{,}000) = P(z > 0.4)$

$= 0.5 - P(0 \le z \le 0.4)$

$= 0.5 - 0.1554$

$= 0.3446$

∴ The number of firms having sales over Rs. 40,000

$= 0.3446 \times 5000$

$= 1723$

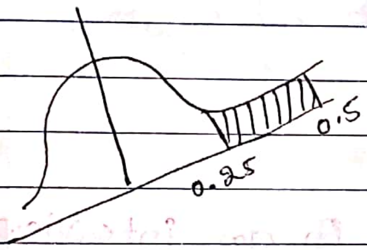
<ii> To find : $P(38500 \le x \le 41{,}000)$

when $x = 38{,}500$; $z = \frac{x-\mu}{\sigma} = 0.25$

when $x = 41{,}000$; $z = \frac{x-\mu}{\sigma} = 0.5$

$\therefore P(38{,}500 \le x \le 41{,}000) = P(0.25 \le z \le 0.5)$

$= P(0 \le z \le 0.5) - P(0 \le z \le 0.25)$



$= 0.1915 - 0.0987$

$= 0.0928$

$\therefore$ The required percentage of firms

$= 0.0928 \times 100$

$= 9.28\ \%$

<iii> To find : $P(30{,}000 \le x \le 40{,}000)$

when $x = 30{,}000$; $z = \frac{x-\mu}{\sigma} = -0.6$

when $x = 40{,}000$; $z = \frac{x-\mu}{\sigma} = 0.4$

$\therefore P(30{,}000 \le x \le 40{,}000) = P(-0.6 \le z \le 0.4)$

$= P(0 \le z \le 0.6) + P(0 \le z \le 0.4)$

$= 0.2258 + 0.1554$

$= 0.3812$

$\therefore$ The required number of firms

$= 0.3812 \times 5000$

$= 1906.$

For more ex. Book → Business Stat. by R.S. Bhardwaj (Pg: 19.29)

Q:- A husband and wife appear in an interview for two vacancies in the same post. The Probability of husband's selection is $1/7$ and that of wife's selection is $1/5$ what is the probability that
<a> both of them will be selected
<b> only one of them will be selected
<c> none of them will be selected.

Solⁿ:- Let A denote husband's selection and
B " wife's "

then;

$$\left.\begin{aligned} P(A) &= 1/7 \\ P(B) &= 1/5 \end{aligned}\right\}$$

Q'

(a) Since events are independent; Prob. that both of them will be selected.

$$= P(A \text{ and } B)$$
$$= P(A) \cdot P(B)$$
$$= \frac{1}{7} \times \frac{1}{5}$$
$$= \frac{1}{35}$$

(b) Prob. that only one of them will be selected

$$= P\{(A \text{ and } \bar{B}) \text{ or } (B \text{ and } \bar{A})\}$$
$$= P(A \text{ and } \bar{B}) + P(B \text{ and } \bar{A})$$
$$= P(A) \cdot P(\bar{B}) + P(B) \cdot P(\bar{A})$$
$$= P(A)\,[1 - P(B)] + P(B)\,[1 - P(A)]$$
$$= \frac{1}{7}\left[1 - \frac{1}{5}\right] + \frac{1}{5} + \left[1 - \frac{1}{7}\right]$$
$$= \left(\frac{1}{7} \times \frac{4}{5}\right) + \left(\frac{1}{5} \times \frac{6}{7}\right)$$
$$= \frac{10}{35}$$

(c) Prob. that none of them will be selected.

$$= P(\bar{A}) \times P(\bar{B})$$
$$= \frac{6}{7} \times \frac{4}{5} = \frac{24}{35}$$