Machine Learning Lab Assignment - 6

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Question 1:

Generate the set of points A and B in R2, each consisting of 2000 data points from a bi-variate normal distribution. The set A and B has been drawn from the N (μ1, Σ1) and N(μ2, Σ2). Let us fix the μ1 = [-1,-1] and μ2 = [2,1]. Separate the 500 data points from each class as a testing set. Plot the optimal Bayesian decision boundary and compute the testing accuracy on test set for three following cases

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

In [2]:
```
mu1 = [-1, -1]
mu2 = [2, 1]
train_size, test_size = 1500, 500
```

In [3]:
```
sigma_cases = {
    'A': (np.eye(2), np.eye(2)),
    'B': (np.array([[1, 0.6], [0.6, 1]]), np.array([[1, 0.6], [0.6, 1]])),
    'C': (np.array([[1, 0.8], [0.8, 1]]), np.array([[1, 0.1], [0.1, 1]]))
}
```

In [4]:
```
def bayesian_decision_boundary(mu1, mu2, sigma1, sigma2):
    inv_sigma1 = np.linalg.inv(sigma1)
    inv_sigma2 = np.linalg.inv(sigma2)

    A = inv_sigma1 - inv_sigma2
    B = 2 * (np.dot(mu2, inv_sigma2) - np.dot(mu1, inv_sigma1))
    C = (np.dot(np.dot(mu1, inv_sigma1), mu1) - np.dot(np.dot(mu2, inv_sigma2),

    return A, B, C
```

In [5]:
```
def generate_data(mu, sigma, num_points):
    return np.random.multivariate_normal(mu, sigma, num_points)

def classify_points(X, a, b):
    return np.sign(np.dot(X, a) + b)

def accuracy(predictions, labels):
    return np.mean(predictions == labels)
```

In [6]:
```
def plot_boundary(a, b, X1_train, X2_train, case):
    plt.figure()
    x_min, x_max = -4, 6
    y_min, y_max = -4, 6

    plt.scatter(X1_train[:, 0], X1_train[:, 1], color='blue', label="Class A", a
    plt.scatter(X2_train[:, 0], X2_train[:, 1], color='red', label="Class B", al

    if np.abs(a[1]) < 1e-5:
```

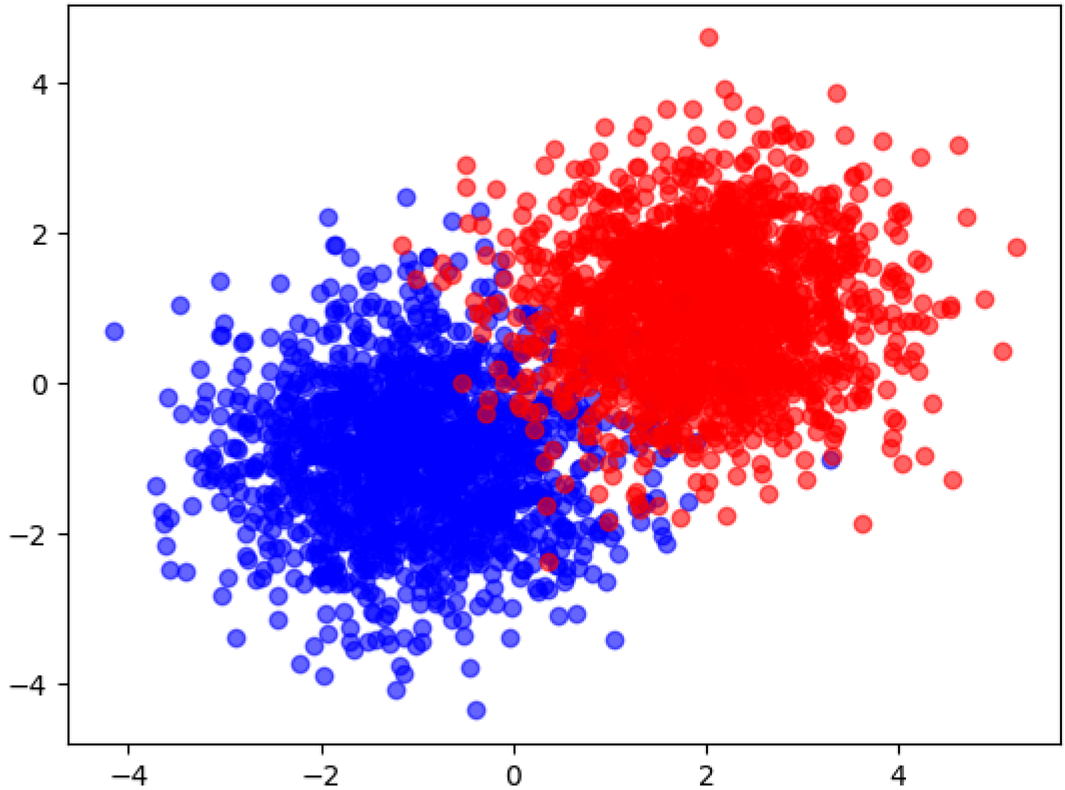
```
        if np.abs(a[0]) > 1e-5:
            x_vals = np.full(100, -b / a[0])
            y_vals = np.linspace(y_min, y_max, 100)
        else:
            print(f"Both a[0] and a[1] are nearly zero for case {case}. Boundary
            return
    else:
        x_vals = np.linspace(x_min, x_max, 100)
        y_vals = -(a[0] * x_vals + b) / a[1]

    plt.plot(x_vals, y_vals, color='green', label="Decision Boundary", linewidth

    plt.xlim(x_min, x_max)
    plt.ylim(y_min, y_max)
    plt.legend()
    plt.title(f"Decision Boundary for Case {case}")
    plt.show()

def plot_quadratic_boundary(A, B, C, X1_train, X2_train, case):
    plt.figure()
    x_min, x_max = -4, 6
    y_min, y_max = -4, 6

    plt.scatter(X1_train[:, 0], X1_train[:, 1], color='blue', label="Class A", a
    plt.scatter(X2_train[:, 0], X2_train[:, 1], color='red', label="Class B", al
    x_vals = np.linspace(x_min, x_max, 300)
    y_vals = np.linspace(y_min, y_max, 300)
    X, Y = np.meshgrid(x_vals, y_vals)

    Z = np.zeros(X.shape)
    for i in range(X.shape[0]):
        for j in range(X.shape[1]):
            point = np.array([X[i, j], Y[i, j]])
            Z[i, j] = np.dot(point.T, np.dot(A, point)) + np.dot(B, point) + C

    plt.contour(X, Y, Z, levels=[0], colors='green')
    plt.xlim(x_min, x_max)
    plt.ylim(y_min, y_max)
    plt.legend()
    plt.title(f"Quadratic Decision Boundary for Case {case}")
    plt.show()
```

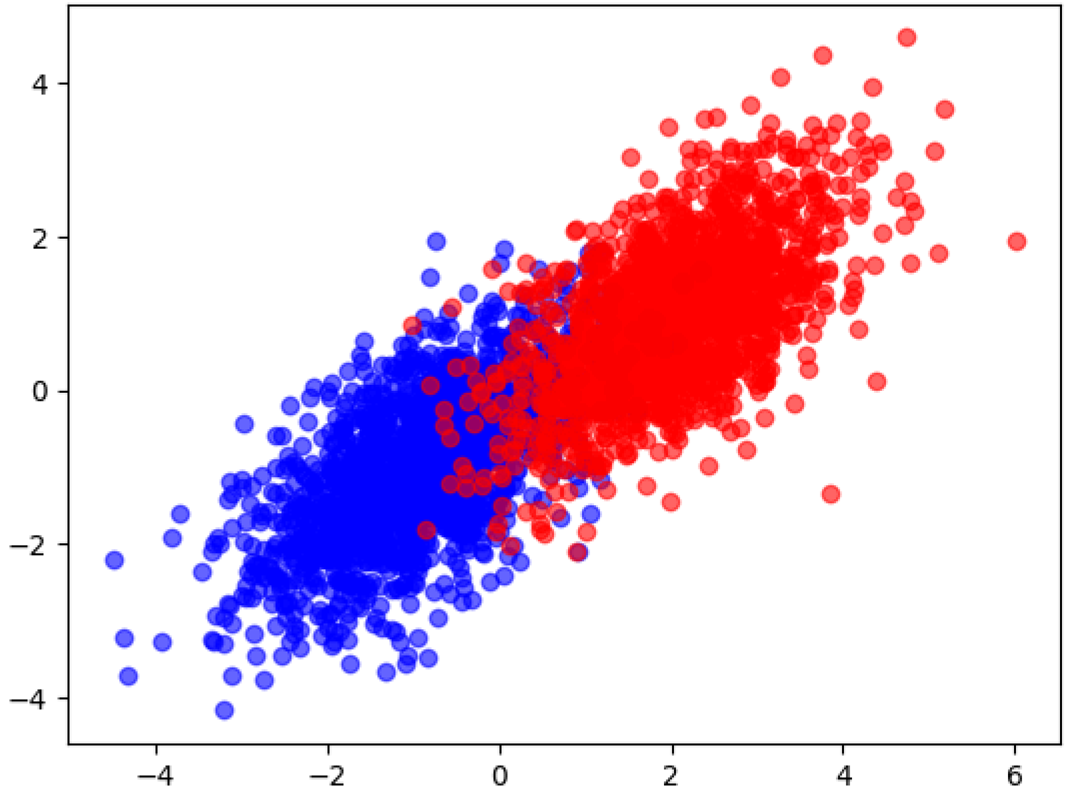
In [7]:

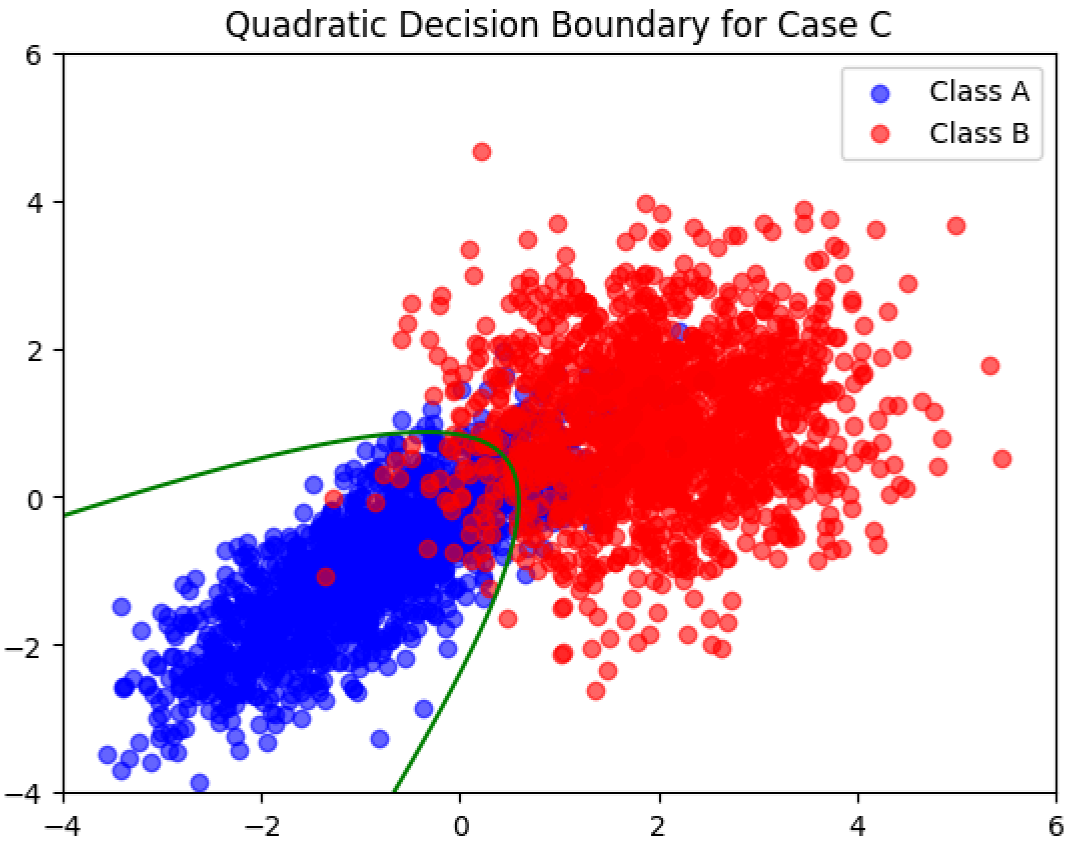
```
for case, (sigma1, sigma2) in sigma_cases.items():
    X1 = generate_data(mu1, sigma1, train_size + test_size)
    X2 = generate_data(mu2, sigma2, train_size + test_size)

    X1_train, X1_test = X1[:train_size], X1[train_size:]
    X2_train, X2_test = X2[:train_size], X2[train_size:]

    X_train = np.vstack((X1_train, X2_train))
    X_test = np.vstack((X1_test, X2_test))
    y_train = np.hstack((np.ones(train_size), -1*np.ones(train_size)))
    y_test = np.hstack((np.ones(test_size), -1*np.ones(test_size)))

    if case == 'C':
        A, B, C = bayesian_decision_boundary(mu1, mu2, sigma1, sigma2)
        plot_quadratic_boundary(A, B, C, X1_train, X2_train, case)
    else:
        inv_sigma1, inv_sigma2 = np.linalg.inv(sigma1), np.linalg.inv(sigma2)
        a = np.dot(inv_sigma1 - inv_sigma2, np.array(mu2) - np.array(mu1))
```

```
        b = 0.5 * (np.dot(np.dot(mu1, inv_sigma1), mu1) - np.dot(np.dot(mu2, inv
        plot_boundary(a, b, X1_train, X2_train, case)
```

```
Both a[0] and a[1] are nearly zero for case A. Boundary can't be computed.
Both a[0] and a[1] are nearly zero for case B. Boundary can't be computed.
```







Consider the label of all points in set A as 1 and label of all points in set B as -1. Write a function implementing the logistic regression model using the gradient descent method. Obtain the best accuracy on the test set. Plot the decision boundary obtained by the logistic regression on test set.

In [8]:
```python
def sigmoid(z):
    return 1 / (1 + np.exp(-z))

def compute_cost(X, y, w):
    m = X.shape[0]
    h = sigmoid(np.dot(X, w))
    cost = -(1/m) * np.sum(y * np.log(h) + (1 - y) * np.log(1 - h))
    return cost
```

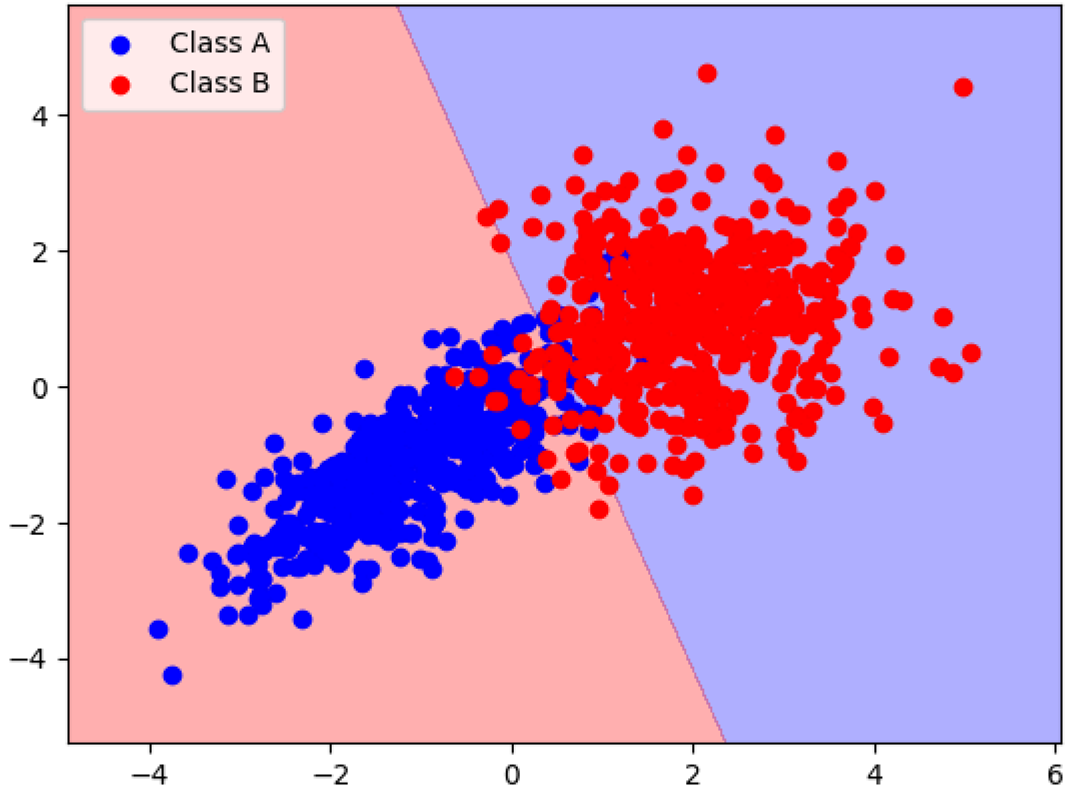
In [9]:
```python
def gradient_descent(X, y, w, learning_rate, num_iterations):
    m = X.shape[0]
    costs = []

    for i in range(num_iterations):
        h = sigmoid(np.dot(X, w))
        gradient = (1/m) * np.dot(X.T, (h - y))
        w = w - learning_rate * gradient

        cost = compute_cost(X, y, w)
        costs.append(cost)

    return w, costs
```

In [10]:
```python
def plot_decision_boundary(X, y, w):
    x_min, x_max = X[:, 1].min() - 1, X[:, 1].max() + 1
    y_min, y_max = X[:, 2].min() - 1, X[:, 2].max() + 1
```

```
    xx, yy = np.meshgrid(np.linspace(x_min, x_max, 500), np.linspace(y_min, y_ma

    grid = np.c_[xx.ravel(), yy.ravel()]
    grid_with_bias = np.c_[np.ones(grid.shape[0]), grid]

    z = np.dot(grid_with_bias, w)
    predictions = sigmoid(z) >= 0.5
    predictions = predictions.reshape(xx.shape)

    plt.contourf(xx, yy, predictions, alpha=0.3, levels=[0, 0.5, 1], colors=['bl
    plt.scatter(X[y == 1][:, 1], X[y == 1][:, 2], color='blue', label='Class A')
    plt.scatter(X[y == 0][:, 1], X[y == 0][:, 2], color='red', label='Class B')
    plt.legend()
    plt.show()
```

In [11]:
```
def prepare_data(X1_train, X2_train, X1_test, X2_test):
    y_train = np.hstack((np.ones(X1_train.shape[0]), np.zeros(X2_train.shape[0])
    y_test = np.hstack((np.ones(X1_test.shape[0]), np.zeros(X2_test.shape[0])))

    X_train = np.vstack((X1_train, X2_train))
    X_train = np.c_[np.ones(X_train.shape[0]), X_train]
    X_test = np.vstack((X1_test, X2_test))
    X_test = np.c_[np.ones(X_test.shape[0]), X_test]

    return X_train, y_train, X_test, y_test
```

In [12]:
```
def logistic_regression(X1_train, X2_train, X1_test, X2_test, learning_rate=0.01
    X_train, y_train, X_test, y_test = prepare_data(X1_train, X2_train, X1_test,

    w = np.zeros(X_train.shape[1])
    w, costs = gradient_descent(X_train, y_train, w, learning_rate, num_iteratic

    plot_decision_boundary(X_test, y_test, w)

    test_predictions = sigmoid(np.dot(X_test, w)) >= 0.5
    accuracy = np.mean(test_predictions == y_test)
    print(f"Accuracy on the test set: {accuracy * 100:.2f}%")

logistic_regression(X1_train, X2_train, X1_test, X2_test, learning_rate=0.01, nu
```




```
Accuracy on the test set: 93.60%
```