Set - 3 : Modifications to the logistic equation

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I. EQUATIONS

A. Human Population Model

The logistic growth model is described by the differential equation:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \tag{1}$$

where: $P(t)$ is the population at time t,
r is the growth rate,
K is the carrying capacity.

The solution to this equation is:

$$P(t) = \frac{K}{1 + \left(\frac{K-P_0}{P_0}\right)e^{-rt}} \tag{2}$$

where P_0 is the initial population at $t = 0$.

B. Spread of Agricultural Innovation

The dynamical equation for the spread of agricultural innovations among farmers through personal communications is given by

$$\dot{x} = Cx(n - x) \tag{3}$$

similarly, dynamical equation for both through personal and impersonal communications is given by

$$\dot{x} = (Cx + C')(n - x) \tag{4}$$

The x(t) for the spread of agricultural innovations among farmers through personal and impersonal communications is given by

$$x = \frac{NC'[1 - e^{-(CN+C')t}]}{C' + CNe^{-(CN+C')t}} \tag{5}$$

$$\text{Defining } X = \frac{x}{N}, \quad T = cNt, \quad A = \frac{C'}{CN}, \quad \dot{X} \equiv \frac{dX}{dT}$$

The revised dynamical equation for the spread of agricultural innovations among farmers through personal communications is given by

$$\dot{X} = X(1 - X) \tag{6}$$

X(T) for the spread of agricultural innovations among farmers through personal communications is given by

$$X = \frac{1}{1 + A^{-1}e^{-T}} \tag{7}$$

Recasting dynamical equation,

$$\dot{X} = (X + A)(1 - X) \tag{8}$$

X(T) for the spread of agricultural innovations among farmers through impersonal communications is given by

$$X = \frac{1 - e^{-(1+A)T}}{1 + A^{-1}e^{-(1+A)T}} \tag{9}$$

C. Harvesting Model

The logistic equation is modified as:

$$\dot{x} = f(x) = rx\left(1 - \frac{x}{k}\right) - h \tag{10}$$

where: $\dot{x}$ is the rate of change,
r is the growth rate,
k is the carrying capacity,
h is the harvesting rate.

In General

- The $x(t)$ of Euler's method is given by

$$x(n+1) = x(n) + f(x(n))\Delta t \tag{11}$$

- The relative error between the analytical solution and the numerical solution is given by

$$\text{relative error} = \frac{\text{numerical sol} - \text{analytical sol}}{\text{analytical sol}} \tag{12}$$

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II. GRAPHS

A. Question 1

Fig. 1 shows Global Population Growth as per the global census data of 1961.

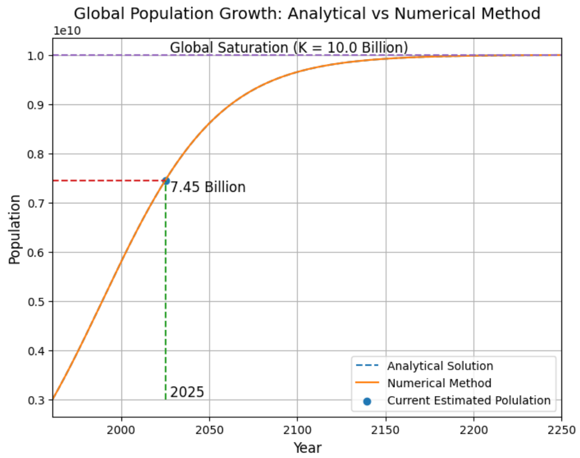

FIG. 1: Included Values: $a = 0.03$, $b = 3{\times}10^{-12}$, $x_0 = 3{\times}10^9$, and $\Delta t = 0.001$ unit.

Fig. 2 shows the relative error between numerical and analytical estimation of human population growth.

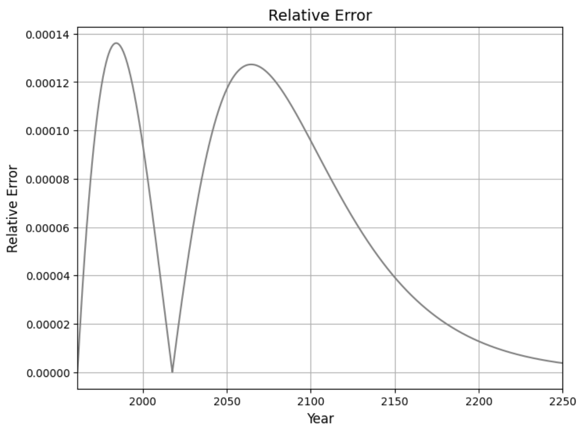

FIG. 2: plot of relative error Δt= 0.001 unit

B. Question 2

Fig. 3 shows $\dot{X}$ versus X for A = 0, 0.2, 0.5.

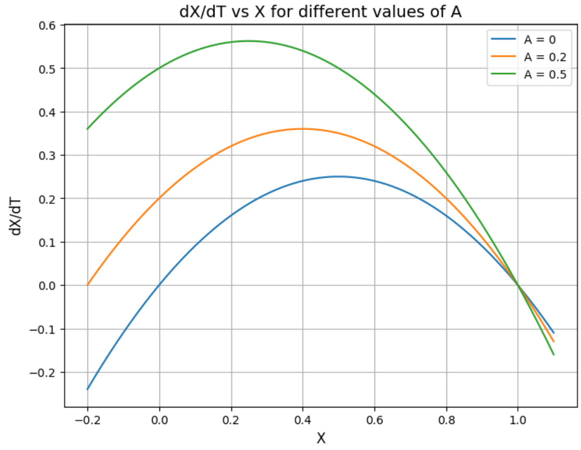

FIG. 3: plot $\dot{X}$ versus X for A = 0, 0.2, 0.5. Δx =0.001 unit

Fig. 4 shows the integral solution X(T) for A = 0, 0.2, 0.5.

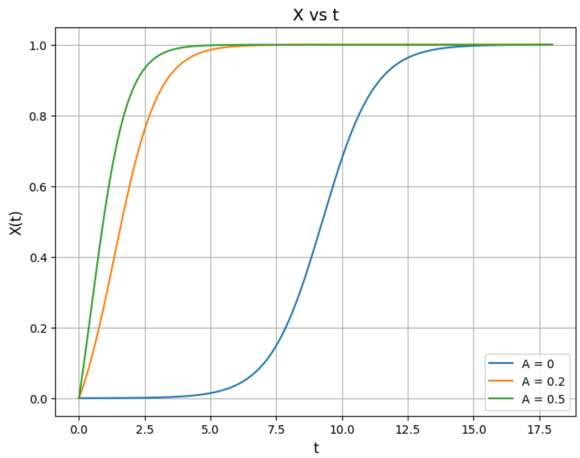

FIG. 4: Plot of the integral solution X(T) for A = 0, 0.2, 0.5 taking initial value X(0) = 0.1, Δt= 0.001 unit

C. Question 3

Fig. 5 shows $\dot{x}$ versus x with h = 0, 100, 500.

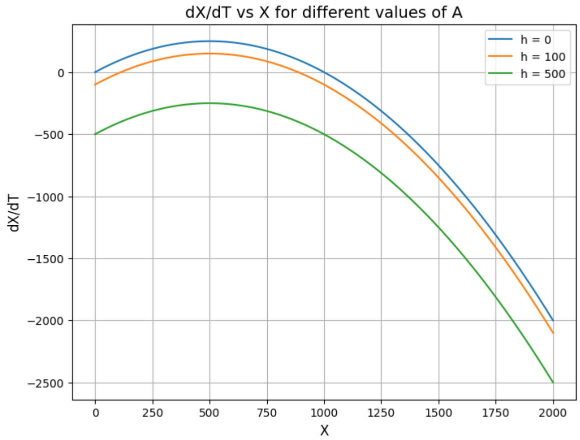

FIG. 5: Here $r = 1$, $k = 1000$ and Δx=0.001 unit

Fig. 6 shows x (by Euler's method) versus t with h = 0, 100, 500.

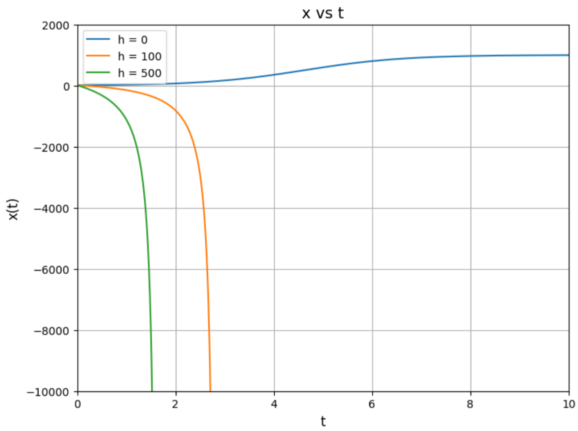

FIG. 6: The initial value are $r = 1$, $k = 1000$, $x_0 = 10$ and Δt=0.001 unit

Fig. 7 shows comparison between the analytical solution and the numerical solution for h=0.

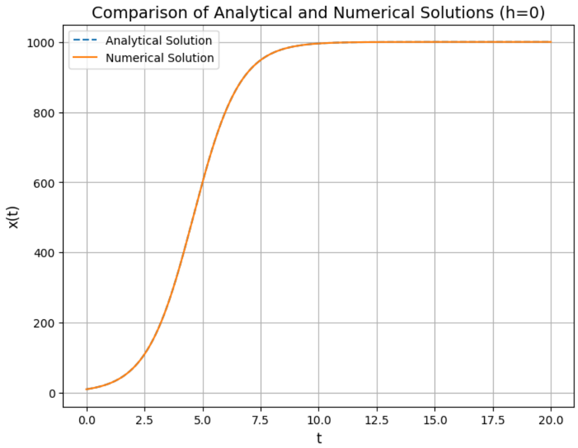

FIG. 7: Here shows comparison between the analytical solution and the numerical solution for h=0. Δt = 0.01 unit

Fig. 8 The relative error between the analytical solution and the numerical solution for h=0.

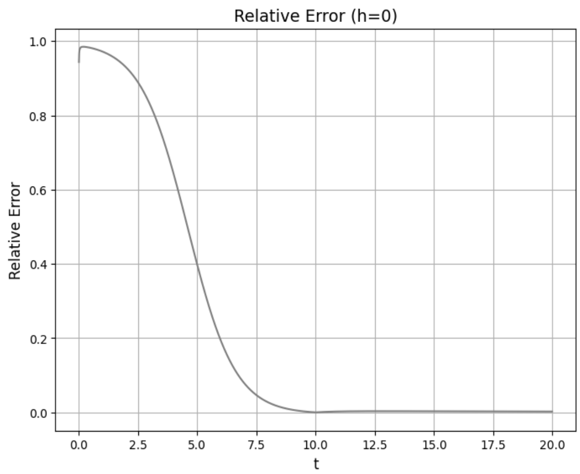

FIG. 8: Here The relative error between the analytical solution and the numerical solution for h=0. Δt = 0.001 unit