Set-8: Modeling Strategic Conflicts Between Nations

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In this lab, we analyze two historic battles— the Battle of Iwo Jima and the Battle of Trafalgar— using numerical integration through Euler's method to predict outcomes and interpret combat strategies.

I. INTRODUCTION AND MODELING

Combat dynamics between two opposing forces can be modeled mathematically using Lanchester's equations. These equations describe the rate at which each side loses strength based on the enemy's firepower. The simplest form of the model is:

$$\frac{dx}{dt} = -bY \tag{1}$$

$$\frac{dy}{dt} = -aX \tag{2}$$

where, $X(t)$ and $Y(t)$ represent the strength of the two opposing forces at time t, while a and b are the combat effectiveness coefficients of each side. Lanchester's square law states that the outcome of a battle can be predicted by comparing the squared ratio of troop numbers weighted by their effectiveness:

$$X_0^2 b - Y_0^2 a \tag{3}$$

If the result is positive, the American troops are expected to win; otherwise, the Japanese troops prevail.

II. BATTLE OF IWO JIMA

A. Prediction Using Lanchester's Square Law

Going By the Lanchester's Square Law, the Predictions for the outcome of the Battle are:

1. Taro data naakh

B. Numerical Integration of Combat Equations

As per Eq.1 and Eq.2, the combat equations for the battle are:

$$\frac{dJ}{dt} = -aA \tag{4}$$

$$\frac{dA}{dt} = -jJ \tag{5}$$

where, A represents The US Army and J represents The Japanese Imperial Army. $a = 0.0106$, $j = 0.0544$, and initial troop numbers are $J_0 = 18274$, $A_0 = 66454$. Taking a time step $\Delta t = 1$ day, if we integrate Eq.4 and Eq.5 and stop the integration when the number of troops of one army falls below 1, we get:

1. By Lanchester's Law, America will win is our prediction as value of K is positive for America.
2. The plot of $J(t)$ and $A(t)$ along the vertical axis and t along the horizontal axis.

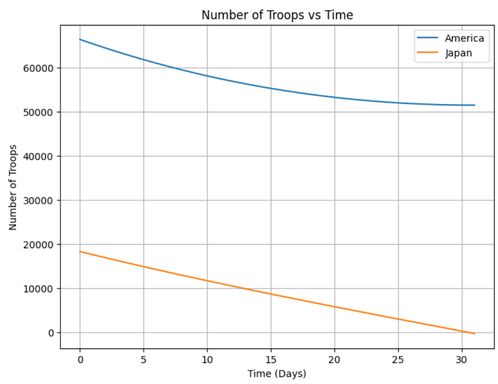

FIG. 1: Time evolution of troop strength in the Battle of Iwo Jima. The numbers decrease over time, with one army being eliminated first as soon as the troops are reduced to zero.
Number of days war lasts : 31
Troops of America : 51526
Casualities of America : 14928

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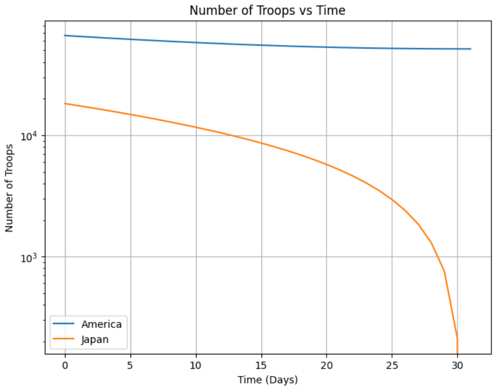

FIG. 2: Troop strength plotted on a logarithmic scale. The slopes indicate the decay rate, which can be compared to $\sqrt{aj}$.

The value of $\sqrt{aj}$ = **0.02401**. The value of the approximate slopes of J and A are as follows:

- Slope of $log(A)$ vs t: **0.0124**
- Slope of $log(J)$ vs t: **0.0305**

3. Plot of A vs. J.

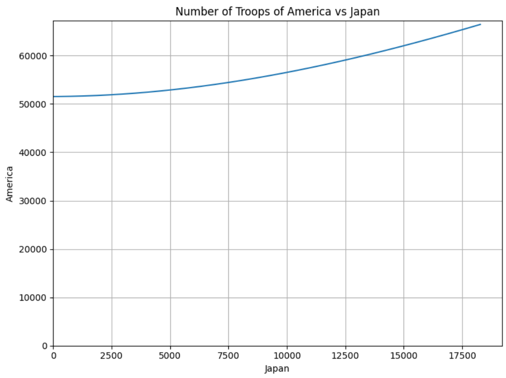

FIG. 3: Phase plot showing the relationship between the two troop sizes during the battle.

III. BATTLE OF TRAFALGAR

Lord Nelson divided his fleet into three stages, engaging enemy forces strategically. The combat equations are:

$$\frac{dF}{dt} = -bB \tag{6}$$

$$\frac{dB}{dt} = -fF \tag{7}$$

where, $b = f = 0.05$. Euler's method is applied with $\Delta t = 1$.

A. Stage 1: Initial Engagement

For the first stage, the British engaged 3 French ships with 13 of their own while keeping 14 in reserve. The system is integrated, stopping when $F < 2$.

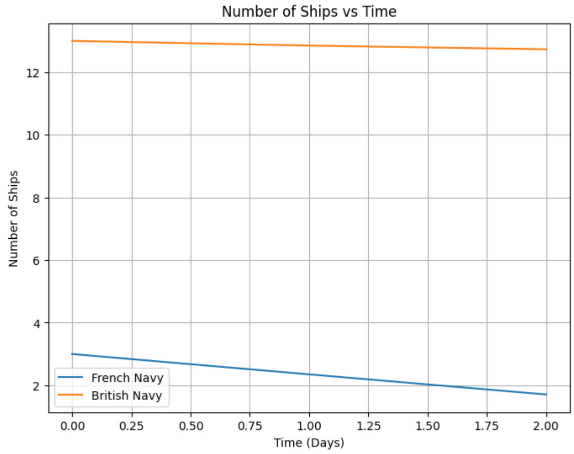

FIG. 4: Time evolution of ships in the first stage. The remaining British ships move to the next phase.
Number of days war lasts : 2
Ships of French : 1.70
Ships of British : 12.73

B. Stage 2: Reinforcements Join Battle

The surviving British ships from Stage 1 join 14 reserve ships, while the remaining French ships join 17 reinforcements. Euler's method is applied again with the same stopping criterion.

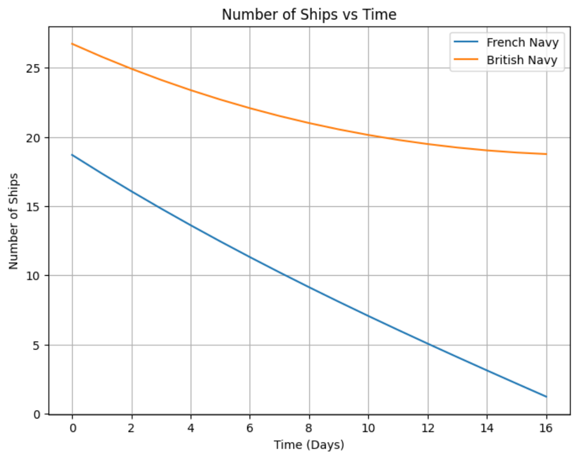

FIG. 5: Time evolution of ships in the second stage, with reinforcements altering the battle outcome.
Number of days war lasts : 16
Ships of French : 1.23
Ships of British : 18.76

C. Stage 3: Final Confrontation

The final battle occurs between the remaining British fleet and the last 13 French ships. The system is integrated again, stopping when $F < 1$.

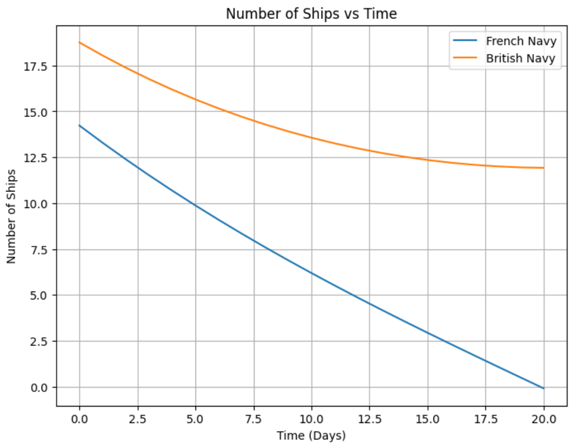

FIG. 6: Final engagement leading to the decisive outcome of the Battle of Trafalgar.
Number of days war lasts : 20
Ships of French : -0.096
Ships of British : 11.92

D. War Game: Alternative Scenario

To simulate an alternate history, we assume Lord Nelson had engaged all 33 French ships with his 27 ships at once. The effectiveness parameters are changed to $b = f = 0.1$, and Euler's method is applied.

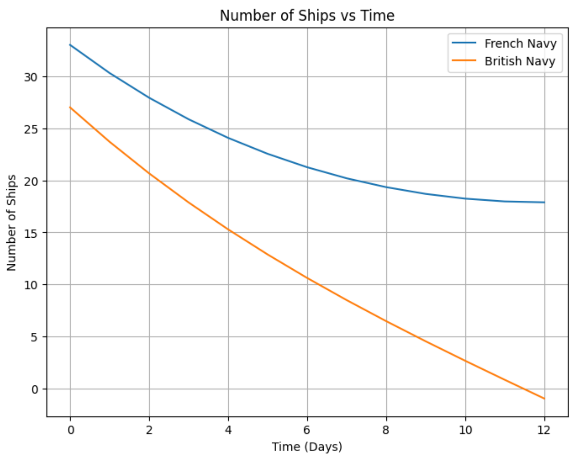

FIG. 7: Simulation of a direct fleet engagement scenario showing a different battle outcome.
Number of days war lasts : 12
Ships of French : 17.88
Ships of British : -0.94

IV. CONCLUSION

1. The application of Lanchester's model in battle simulations highlights the importance of numerical superiority and strategic decision-making. In both battles, outcomes depended on force effectiveness and tactical choices.

2. The staged engagement strategy of the British Navy at Trafalgar demonstrated a superior combat approach compared to a direct engagement. Similarly, the American troop advantage in Iwo Jima played a decisive role in the battle's outcome.

[1] Arnab K. Ray, *Lecture Notes*