Set - 4 : Mathematical Modelling of Tumor and Population Dynamics

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CS302, Modelling and Simulation

I. GOMPERTZ MODEL FOR TUMOR GROWTH

A. Model

The Gompertz equation models tumour growth as

$$\dot{x} = -axln(bx) \tag{1}$$

Where $a, b > 0$.
Rescale $X = x/b^{-1}$ and T = at,

$$\dot{X} = -XlnX \tag{2}$$

The integral solution of above equation is,

$$X = e^{[ln(X_{in})e^{-T}]} \tag{3}$$

Relative error between numerical and analytical solution is,

$$relativeerror = \frac{numericalsolution - analyticalsolution}{Analyticalsolution} \tag{4}$$

B. Results

Fig. 1 shows $\dot{x}$ versus x.

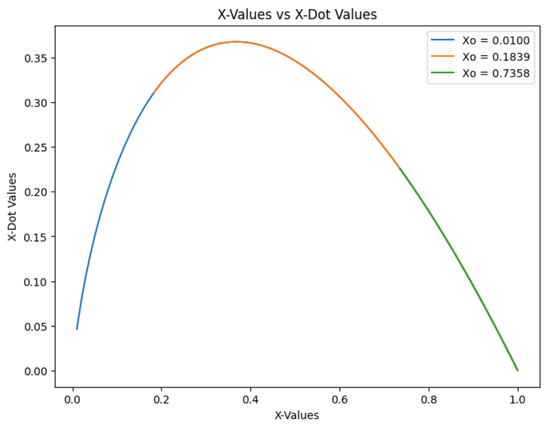

FIG. 1: Here the initial values of x are 0.01, 0.1838 and 0.763 $\Delta x = 0.01\,unit$

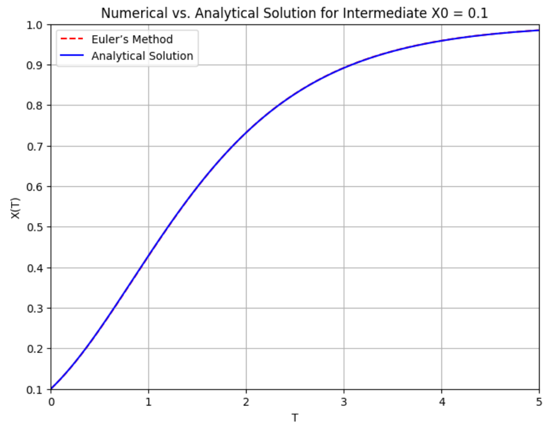

FIG. 2: Here the initial value of x(0) = 0.1. $\Delta t = 0.01\,unit$

Fig. 2 shows tumor growth wrt time.
Fig. 3 shows relative error between analytical and numerical solution.

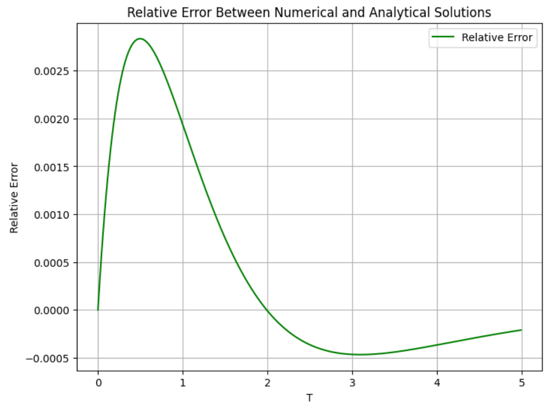

FIG. 3: relative error between the analytical solution and the numerical solution . The graph has zero value at t=0 and 1.27. $\Delta t = 0.01\,unit$

II. ALLEY'S EQUATION MODEL FOR POPULATION FOR MODEL GROWTH

A. Model

The Allee effect models high growth rate of a population when the initial population size has an intermediate

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value.The model equation is,

$$\dot{x} = x[r - a(x - b)^2] \tag{5}$$

Where $a, b, r > 0$.The model is effective only when $r < ab^2$.

B. Results

Fig. 4 shows $\dot{x}$ versus x for an Allee effect.

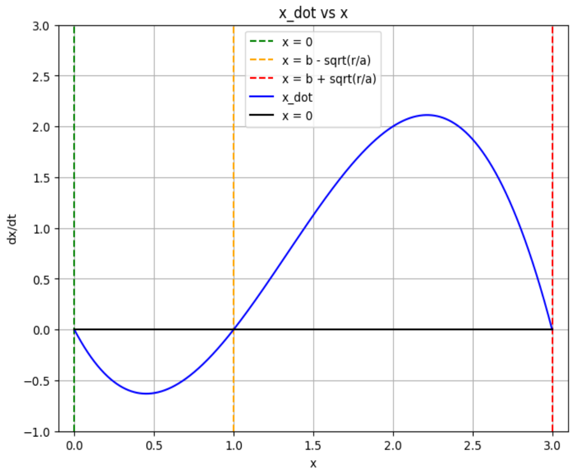

FIG. 4: Here a=1,b=2,r=1. Graph value becomes zero at t=0,1 and 3 units. $\Delta t = 0.01\,unit$

Fig. 5 shows the numerical solution vs time for allee effect.

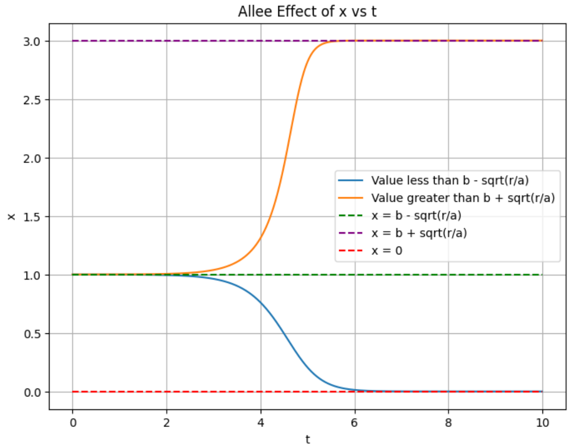

FIG. 5: The two initial values taken are 0.99 and 1.01. $\Delta x = 0.01\,unit$