Set - 1 : Logistic Modelling of GDP and Trade

Natansh Shah (202201445)* and Tirth Modi (202201513)†
Dhirubhai Ambani Institute of Information & Communication Technology,
Gandhinagar, Gujarat 382007, India
CS302, Modelling and Simulation

This study aimed to predict how the GDP and trade of the top 6 countries would change over time. We used a mathematical model called the logistic equation to make these predictions. To check the accuracy of our predictions, we compared them to the actual data from the World Bank.

I. MODEL

The logistic equation is a standard example of a first-order autonomous nonlinear dynamical system.First-order autonomous dynamical systems have the general form of $\dot{x} = f(x)$ where x = x(t), with t being time.A basic model of a nonlinear function is given by

$$\dot{x} = ax - bx^2 \tag{1}$$

Under the initial condition of x(0) = x_0, and with the definition of k = a/b, the integral solution of Eq. (1) is

$$x(t) = \frac{kx_0e^{at}}{k + x_0(e^{at} - 1)} \tag{2}$$

The time at which the non-linear effect starts asserting itself is given by

$$t_{nl} = \frac{1}{a}\ln(\frac{k}{x_0} - 1) \tag{3}$$

The linearization of GDP and Trade given the following formula:

$$G(T) \sim T^{\alpha} \tag{4}$$

The time it takes a country to reach any GDP in future can be predicted by the following formula:

$$t = b + \frac{1}{a}\ln[\frac{x(k - x_0)}{x_0(k - x)}] \tag{5}$$

II. RESULTS

A. Plot the GDP and Trade of top countries

Fig. 1 shows the GDP and trade of USA using World Bank data.

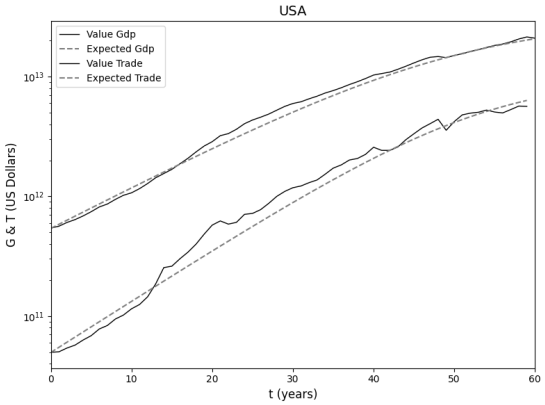

FIG. 1: Modelling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for USA. The dotted curves follow the logistic equation with the parameter values given in the paper referred. The zero year of both plots is 1960. The GDP plot ends in 2020, but the trade plot ends in 2019.

Fig. 2 shows the GDP and trade of China using World Bank data.

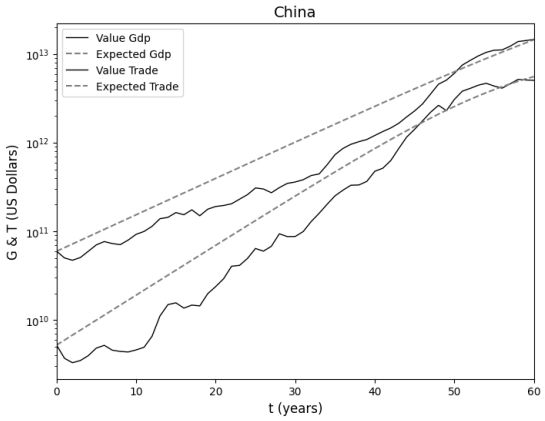

FIG. 2: Modelling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for China. The dotted curves follow the logistic equation with the parameter values given in the paper referred. The zero year of both plots is 1960, and both end in 2020.

*Electronic address: 202201445@daiict.ac.in
†Electronic address: 202201513@daiict.ac.in

Fig. 3 shows the GDP and trade of Japan using World Bank data.

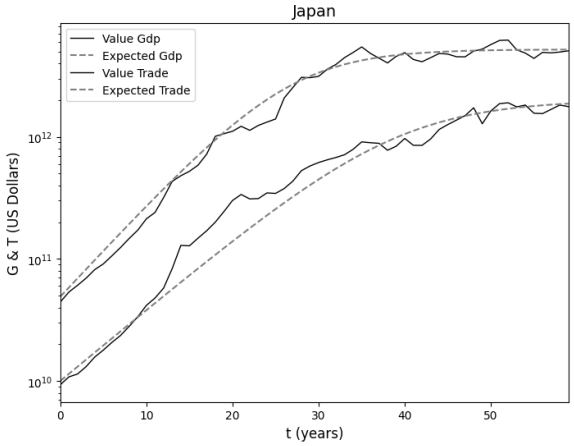

FIG. 3: Modelling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for Japan. The dotted curves follow the logistic equation with the parameter values given in the paper referred. The zero year of both plots is 1960, and both end in 2019.

Fig. 4 shows the GDP and trade of Germany using World Bank data.

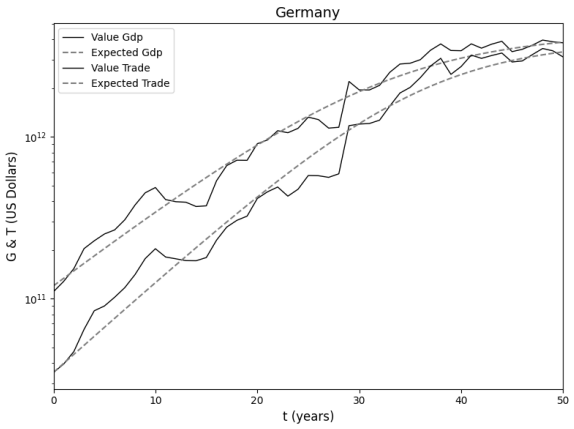

FIG. 4: Modelling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for Germany. The dotted curves follow the logistic equation with the parameter values given in the paper referred. The zero year of both plots is 1970, and both end in 2020.

Fig. 5 shows the GDP and trade of UK using World Bank data.

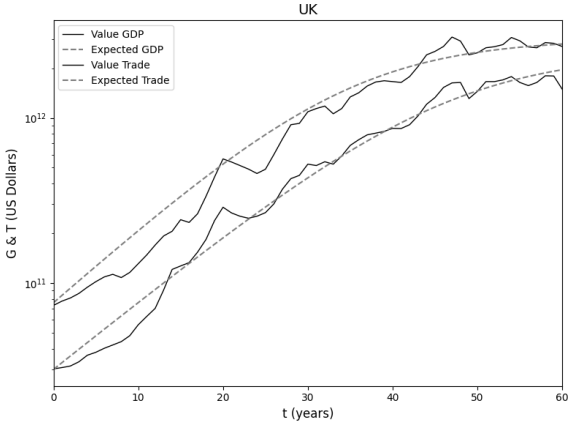

FIG. 5: Modelling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for UK. The dotted curves follow the logistic equation with the parameter values given in the paper referred. The zero year of both plots is 1960, and both end in 2020.

Fig. 6 shows the GDP and trade of India using World Bank data.

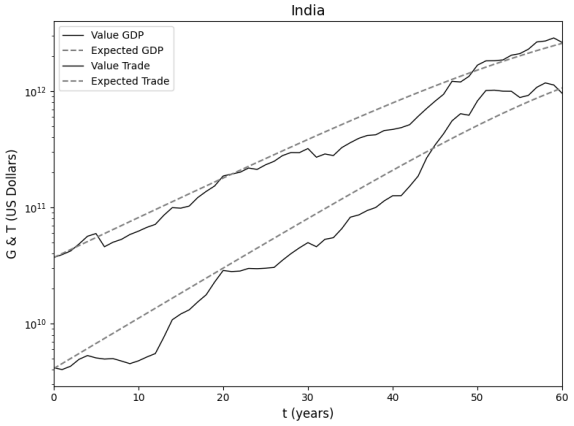

FIG. 6: Modelling the dynamics of GDP (upper plot) and trade (lower plot) using World Bank data for India. The dotted curves follow the logistic equation with the parameter values given in the paper referred. The zero year of both plots is 1960, and both end in 2020.

B. Plot the GDP vs Trade of Top countries

Fig. 7 shows GDP against trade using World Bank data for USA.

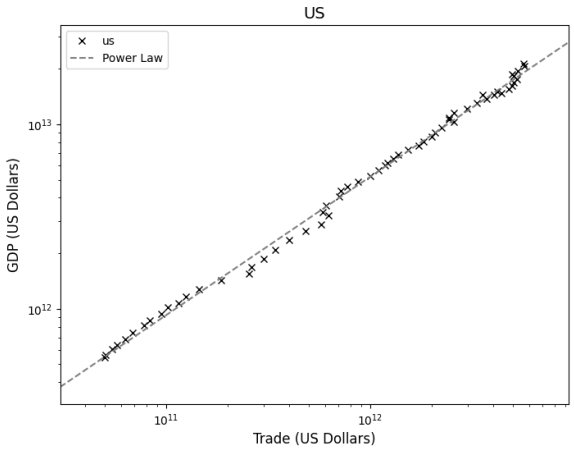

FIG. 7: Plotting GDP against trade using World Bank data for USA. The dotted line follows Eq. (4) with $\alpha = 0.75$. The plot begins in 1960 and ends in 2019.

Fig. 8 shows GDP against trade using World Bank data for China.

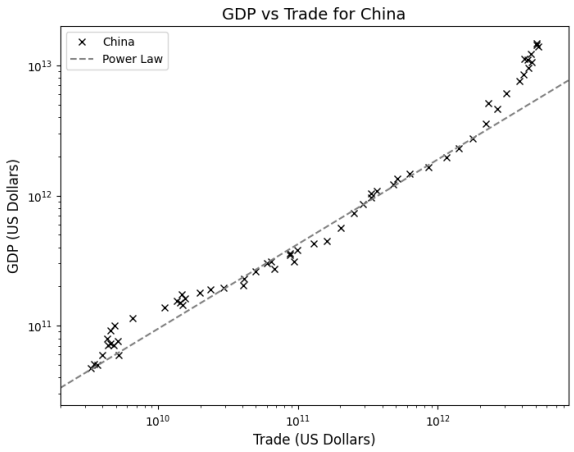

FIG. 8: Plotting GDP against trade using World Bank data for China. The dotted line follows Eq. (4) with $\alpha = 0.65$. The plot begins in 1960 and ends in 2020.

Fig. 9 shows GDP against trade using World Bank data for Japan.

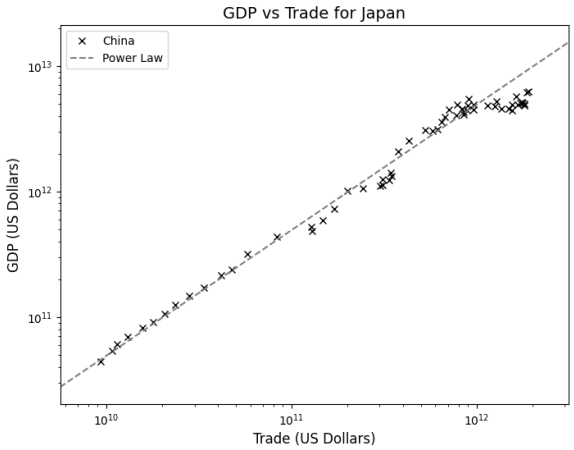

FIG. 9: Plotting GDP against trade using World Bank data for Japan. The dotted line follows Eq. (4) with $\alpha = 1.00$. The plot begins in 1960 and ends in 2019.

Fig. 10 shows GDP against trade using World Bank data for Germany.

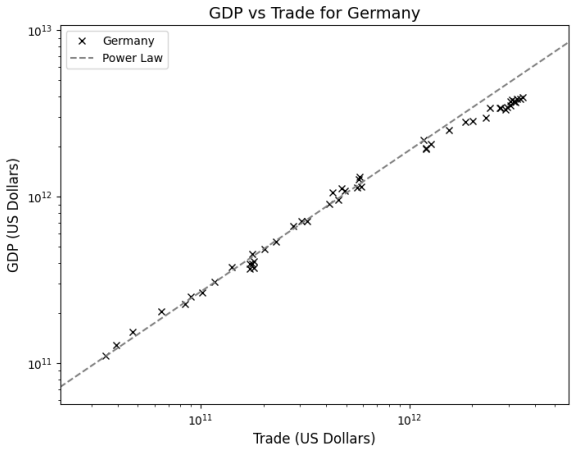

FIG. 10: Plotting GDP against trade using World Bank data for Germany. The dotted line follows Eq. (4) with $\alpha = 0.85$. The plot begins in 1970 and ends in 2020.

Fig. 11 shows GDP against trade using World Bank data for UK.

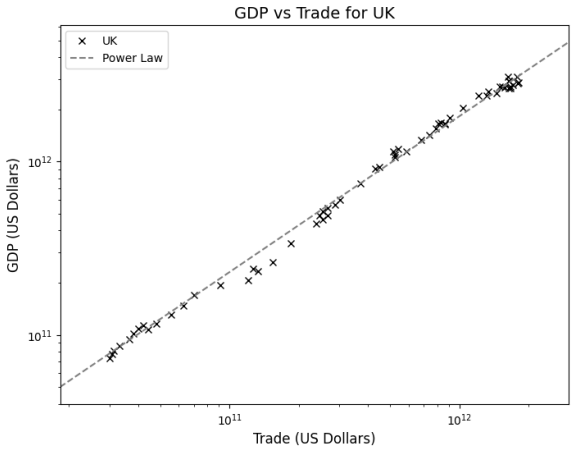

FIG. 11: Plotting GDP against trade using World Bank data for UK. The dotted line follows Eq. (4) with $\alpha = 0.90$. The plot begins in 1960 and ends in 2020.

Fig. 12 shows GDP against trade using World Bank data for India.

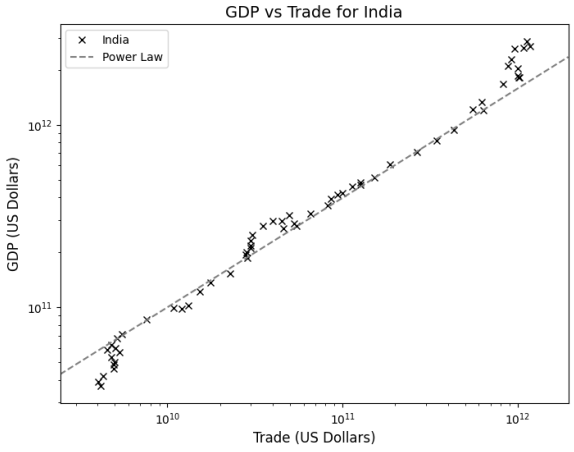

FIG. 12: Plotting GDP against trade using World Bank data for India. The dotted line follows Eq. (4) with $\alpha = 0.60$. The plot begins in 1960 and ends in 2020.

C. Plot the comparison between GDP of top countries

Fig. 13 shows comparison of GDP growth of the USA and China.

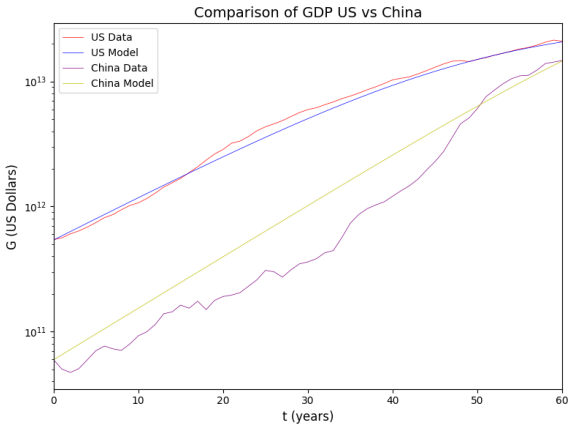

FIG. 13: Comparing the GDP growth of the USA and China, which are, respectively, the countries with the highest and the second highest GDPs in the world. The smooth dotted curves model the GDP growth of both countries according to Eq. (2), with the values of a and k.

Fig. 14 shows comparison of GDP growth of the India and Japan.

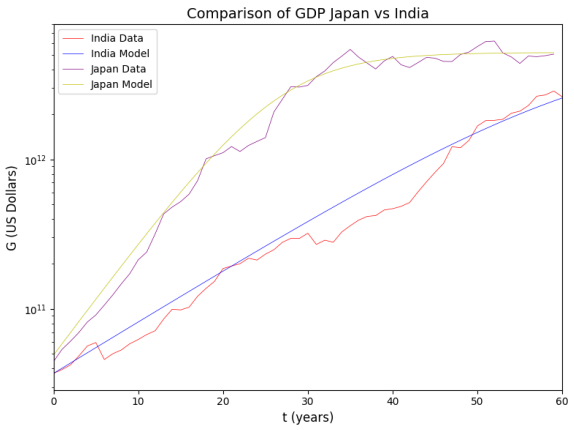

FIG. 14: Comparing the GDP growth of Japan and India, which, after China, are, respectively, the countries with the second and the third highest GDPs in the Indo-Pacific region.The GDP growth of Japan has a steep gradient in the early years, but by the year 2000, the growth has visibly stagnated. Both of these features are modelled closely by the logistic function — the smooth dotted curve. In contrast, the GDP growth of India has been slow but on the whole steady, and by the year 2020, the GDP of India grows with a higher gradient than the GDP of Japan. At this rate, the GDP of India will eventually overtake the GDP of Japan.

Fig. 15 shows comparison of GDP growth of the UK and Germany.

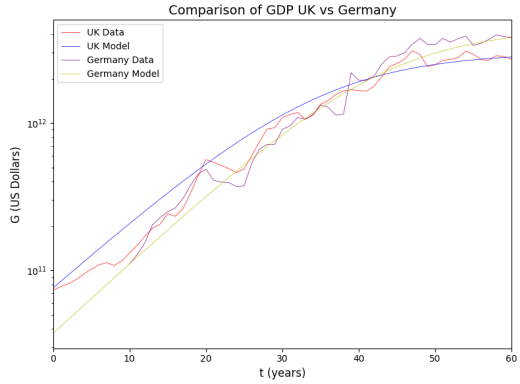

FIG. 15: Comparing the GDP growth of Germany and the UK, which, after the USA, are, respectively. The World Bank data of the annual GDP of the UK start from 1960 (t = 0) and end at 2020. For Germany, however, the GDP data start from 1970 (t = 10 years). Till 1999-2000, both countries ran each other very close in terms of their GDP growth.

D. Prediction

Fig. 16 shows comparison of GDP growth of the Japan, Germany, UK and India.

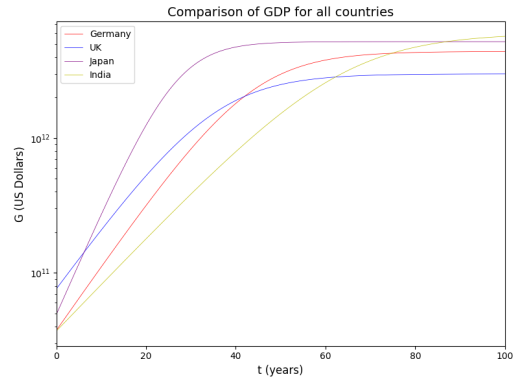

FIG. 16: Two crossings of the logistic functions occur before 2020, one in 1966, when the GDP of Japan overtook the GDP of the UK, and the other in 2000, .The remaining three intersections are to occur after 2020. The overtakes of the GDPs of Germany and Japan by the GDP of India are predicted for the years 2035 and 2047, respectively.

III. PREDICTING WHEN INDIA WILL BECOME A $4 AND $5 TRILLION ECONOMY

Using Equation (5), we can predict when India will become a $ 4 and $ 5 economy respectively.

According to our predictions using the given model and stated equation, India will become a **4 trillion dollar economy in 2032** and a **5 trillion dollar economy in 2043**.

IV. PREDICTING GDP FOR EXPONENTIAL GROWTH TILL 2047

$$x(t) = x_0 e^{at} \tag{6}$$

Using above Equation (6), we can predict India will become a **38.98 trillion dollar economy in 2047.**

V. STATISTICAL ANALYSIS

A. Statistical analysis of GDP

Country Name	Mean	Standard deviation
USA	0.0445	0.0839
China	0.0877	0.2317
Japan	-0.0526	0.1478
Germany	0.0512	0.1720
UK	-0.0970	0.1749
India	-0.1384	0.1666

B. Statistical analysis of Trade

Country Name	Mean	Standard deviation
USA	0.1090	0.1831
China	-0.4012	0.3091
Japan	0.1923	0.3389
Germany	0.0701	0.2148
UK	0.0047	0.1583
India	-0.1599	0.3255

VI. CONCLUSIONS

In conclusion, our exploration involved a basic mathematical model, the logistic equation, to predict the GDP and Trade of top 6 countries.

⇒ Our study reveals that the logistic equation effectively models the growth of GDP and trade under conducive conditions, emphasizing the impact of internal

politics, military engagements, and economic policies. Additionally, a power-law correlation suggests a linked growth of GDP and trade, potentially characterizing economies based on geographical scales and population sizes.

⇒Leveraging the scale-free order observed in international trade, particularly within the North-Atlantic and Indo-Pacific economic regions, can inform globally-coordinated strategies for post-Covid-19 economic recovery.The correlation between GDP and trade highlights the importance of reactivating international trade networks to positively impact the participating countries' economies.

⇒The logistic equation effectively forecasts GDP competitiveness among leading economies, as evidenced by its accurate predictions of GDP milestones. However, the reliability of these forecasts is subject to recalibration due to unforeseen natural, social, economical, and political events, exemplified by recent shocks like the Covid-19 pandemic and the Ukraine war.

[1] Abhin Kakkad, Arnab K. Ray, *Global dynamics of GDP and trade*, DA-IICT (2022).

[2] Arnab K. Ray, *Logistic forecasting of GDP competitiveness*, DA-IICT (2023).