Set - 10 : Modelling epidemics and endemic breakouts of infectious diseases

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CS302, Modelling and Simulation

I. EPIDEMICS

A. Model

Consider a population size of N, through which an infection spreads. The population is divided into three classes - the infected class x(t), the susceptible class y(t), and the recovered class z(t), so that

$$x(t) + y(t) + z(t) = N(constant) \tag{1}$$

The coupled dynamics of these variables is given by

$$\dot{x} = AxyBx \tag{2}$$

$$\dot{y} = -Axy \tag{3}$$

$$\dot{z} = Bx \tag{4}$$

In which A is the infection rate and B is the removal rate $(A, B > 0)$.

At t = 0, $x(0) = x_0$ and $z(0) = 0$. Hence, $y(0) = y_0 = N - x_0$. The value of x_0 is small $(x_0 << N)$.

B. Results

1. A

Fig. 1 shows infected class x(t) and susceptible class y(t) vs number of days.

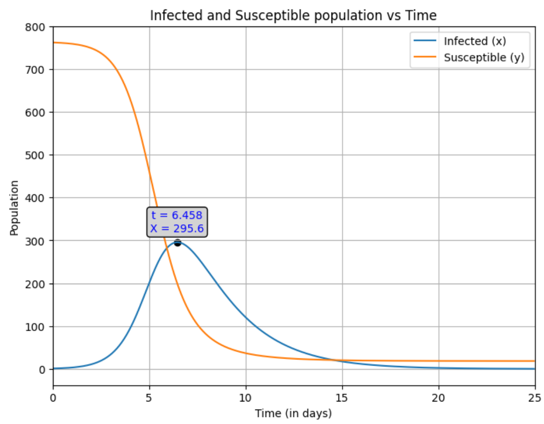

FIG. 1: Here $A = 2.18 * 10^{-3} day^{-1}$, $B = 0.44 day^{-1}$, N = 763, t = 25 days.

The time at which x is maximum : 6.45 days

Fig. 2 shows infected class and susceptible class(logarithmic scale) vs number of days.

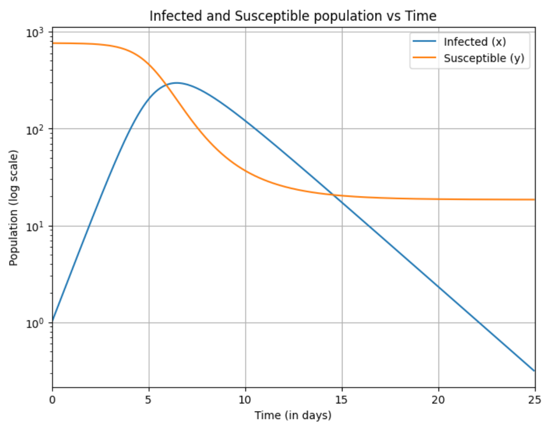

FIG. 2: Here $A = 2.18 * 10^{-3} day^{-1}$, $B = 0.44 day^{-1}$, N = 763, t = 25 days.

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2. B

Fig. 3 shows recovered class z(t) vs number of days.

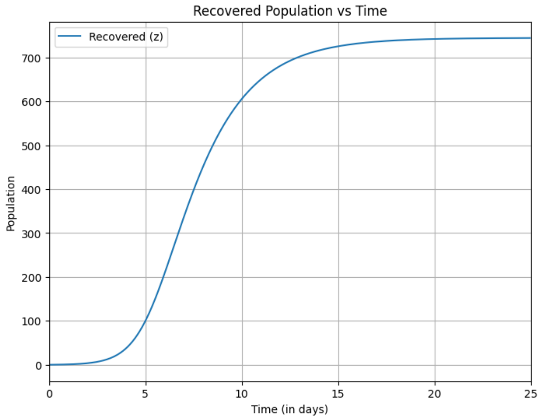

FIG. 3: Here $A = 2.18 * 10^{-3} day^{-1}$, $B = 0.44 day^{-1}$, N = 763, t = 25 days.

Fig. 4 shows recovered class(logarithmic scale) vs number of days.

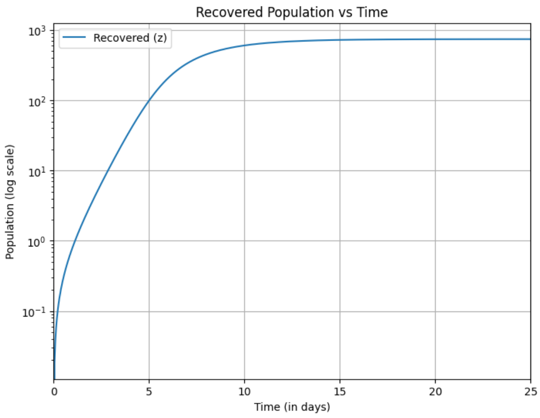

FIG. 4: Here $A = 2.18 * 10^{-3} day^{-1}$, $B = 0.44 day^{-1}$, N = 763, t = 25 days.

3. C

Fig. 5 shows infected class x(t) vs susceptible class y(t).

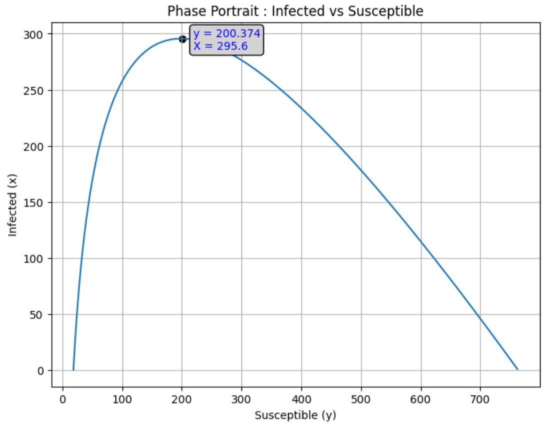

FIG. 5: Here $A = 2.18 * 10^{-3} day^{-1}$, $B = 0.44 day^{-1}$, N = 763, t = 25 days.y(0)=762

The threshold value : 200.37
The value of R : 3.749796
Here $R > 1$ hence an epidemic break out.

II. ENDEMIC DISEASES

A. Model

Endemic diseases persist in a population and break out from time to time. In this case N = N(t), i.e. the total population size changes. If the per capita death rate is a and the per capita birth rate is b $(a, b > 0)$, then the relevant coupled system of equations is given by

$$\dot{x} = Axy - Bx - ax \tag{5}$$

$$\dot{y} = bN - Axy - ay \tag{6}$$

$$\dot{z} = Bx - az \tag{7}$$

$$\dot{N} = (b - a)N \tag{8}$$

B. Results

1. A

Fig. 6 shows infected class x(t) vs number of years.

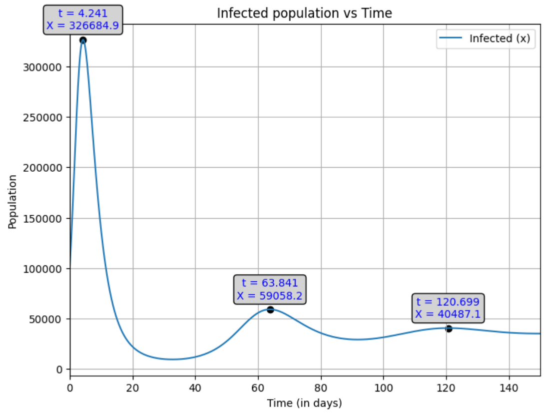

FIG. 6: Here $a = b = 0.02 year^{-1}$ so that $\dot{N} = 0$, i.e. N is fixed. Take $A = 10^{-6} year^{-1}$, $B = 0.333 year^{-1}$, $N = 10^6$, $x_0 = 10^5$ and $y_0 = 9 * 10^5$.

The times when x reaches its peaks : 4.4438 years, 63.726 years, 120.674 years.
These are the times when endemic breakouts occur.

2. B

Fig. 7 shows susceptible class y(t) vs number of days.

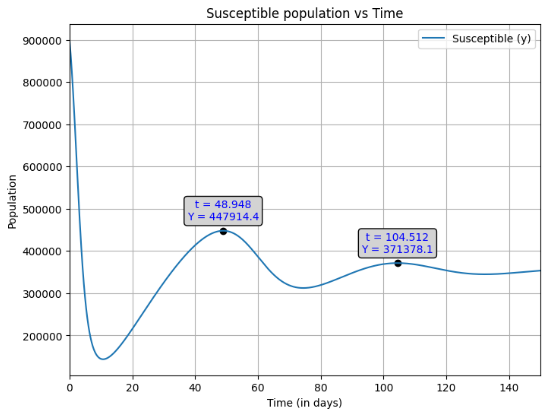

FIG. 7: Here $a = b = 0.02 year^{-1}$ so that $\dot{N} = 0$, i.e. N is fixed. Take $A = 10^{-6} year^{-1}$, $B = 0.333 year^{-1}$, $N = 10^6$, $x_0 = 10^5$ and $y_0 = 9 * 10^5$.

Fig. 8 shows recovered class z(t) vs number of days.

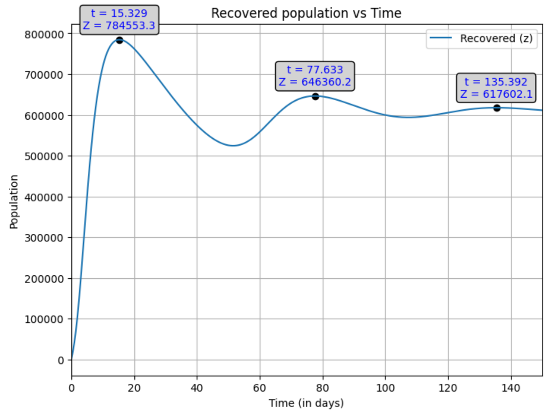

FIG. 8: Here $a = b = 0.02 year^{-1}$ so that $\dot{N} = 0$, i.e. N is fixed. Take $A = 10^{-6} year^{-1}$, $B = 0.333 year^{-1}$, $N = 10^6$, $x_0 = 10^5$ and $y_0 = 9 * 10^5$.