MAGNETISM

MAGNETIC FIELD —

* Biot - Savarts Law :

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \times \frac{i\,d\vec{l}\times\vec{r}}{r^3}}$$

OR $$d\vec{B} = \frac{\mu_0}{4\pi} \times \frac{i\,dl\sin\theta}{r^2}$$

* Due to moving charge

$$\boxed{\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{q\vec{V}\times\vec{r}}{r^3}}$$

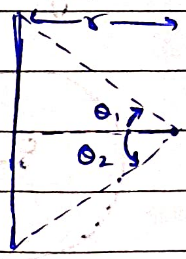
Note :

$$\frac{\mu_0}{4\pi} = 10^{-7}$$

μ_0 = permeability of free space

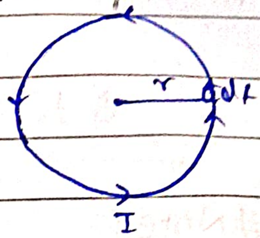
$\boxed{\mu = \mu_0\mu_r}$ → relative permeability of a medium

* Due to current carrying straight wire section —



$$\boxed{B = \frac{\mu_0 i}{4\pi r_0}(\sin\theta_1 + \sin\theta_2)}$$

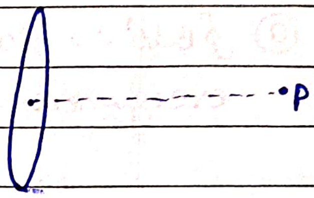
* Circular loop at it's center



$$\boxed{B_{centre} = \frac{\mu_0 N I}{2R}}$$

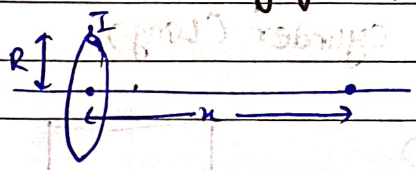
$N \in R$ → can be non integer

* Circular coil at it's axis.



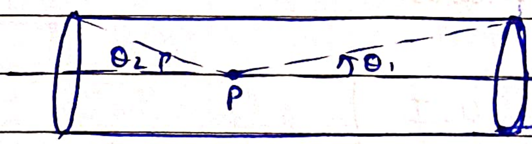
$$B_{axis} = \frac{\mu_0 N I R^2}{2(R^2+x^2)^{3/2}}$$

Current carrying loop as Magnet :



$$\boxed{\vec{M} = IN\vec{A}}$$

Solenoid at it's axis -



$$n = \frac{N}{L}$$

$$B_{axis} = \frac{\mu_0 n i}{2}(\cos\theta_1 + \cos\theta_2)$$

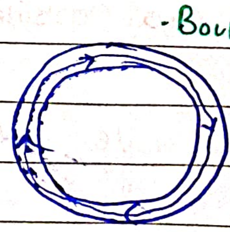
For long solenoid :

$B = \mu_0 n i$ → n = no. of turns per unit length

* Thin Toroid :

$$B_{in} = \mu_0\left(\frac{N}{2\pi R}\right)i$$

$B_{out} = 0$



Ampere's Law –

$$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{in}}$$

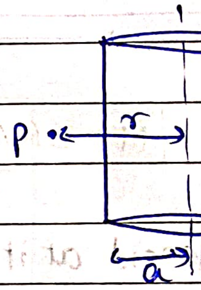
#Note:

1. Shape of Ampere loop doesn't matter
2. position of wire doesn't matter
3. $\oint B d\vec{l} = 0$ doesn't imply $B = 0$ everywhere on the loop.

Mag Field Part II:

* Thin Hollow Cylinder (Long):

1. $r < a$ (inside)



$$\boxed{B_{inside} = 0}$$

2. $r > a$ (outside)

$$\boxed{B_{outside} = \frac{\mu_0 I}{2\pi r}}$$

* Long Solid Cylinder – (radius $= a$)
(dist $= r$)

1. $r < a$ (inside)

$$\boxed{B_{inside} = 0}$$

2. $r > a$ (outside)

$$\boxed{B_{outside} = \frac{\mu_0 I}{2\pi r}}$$

→ If $\vec{J}$ is given:

1. Inside:

$$\vec{B_{in}} = \frac{\mu_0 (\vec{J} \times \vec{r})}{2}$$

2. Outside

$$\vec{B_{out}} = \frac{\mu_0 a^2 (\vec{J} \times \vec{r})}{2r^2}$$

Force on Moving charge:

velocity of pt. charge

$$\boxed{\vec{F} = q\vec{V} \times \vec{B}}$$

w/ sign

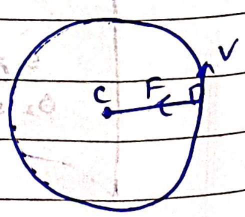
#Note:

1. $\vec{F} \perp \vec{V}$
2. Power $= \vec{F}\cdot\vec{V} = 0$
3. work done by Mag force on pt. charge $= 0$
4. $\Delta KE = 0$
5. Mag force does not affect magnitude of force.

Motion of point charge:

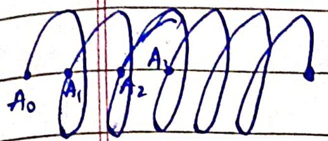
If $\vec{V} \perp \vec{B}$ (1)

$\otimes \vec{B}$



$T = \dfrac{2\pi m}{qB}$, $\omega = \dfrac{qB}{m}$ } independent of V

② If $\vec{v}$ is neither ⊥ nor ∥ to $\vec{B}$



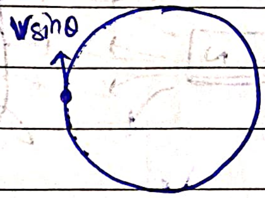
← Helical Path

$A_0A_1 = A_1A_2 = A_2A_3 =$ pitch (P)

pitch $= V_x \cdot T$

$= v\cos\theta \cdot \frac{2\pi m}{qB}$

In y-z plane:



$R = \frac{m(v\sin\theta)}{qB}$.

$T = \frac{2\pi m}{qB}$ $\omega = \frac{qB}{m}$

③ $\vec{V} \parallel \vec{E} \parallel \vec{B}$

Mag force = 0

$F = q\vec{E}$

④ $\vec{B} \parallel \vec{E}$, $\vec{V}$ is neither ⊥ or ∥ to $\vec{B} \parallel \vec{E}$.

Helical path w/ increased pitch.

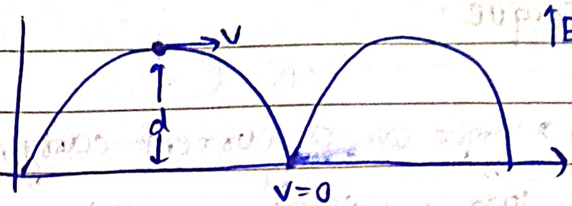
x dirⁿ: $a_x = \frac{qE}{m}$

$v_x = u\cos\theta + \frac{qE}{m} t$

$x = u\cos\theta\, t + \frac{1}{2} \cdot \frac{qE}{m} \cdot t^2$

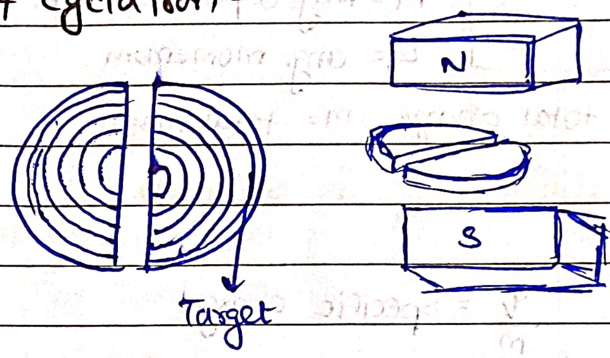
y-z dirⁿ → same as ③

⑤ $\vec{E} \perp \vec{B}$ & charge is released from rest



$d = \frac{2mE}{qB^2}$

Cyclotron –



$T = \frac{2\pi m}{qB}$ $f = \frac{1}{T}$ $R = \frac{mv}{qB}$.

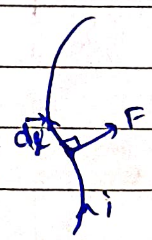
final speed $= \frac{qBR_0}{m}$.

final KE $= \frac{1}{2} \cdot m \cdot \left(\frac{qBR_0}{m}\right)^2$

MAGNETIC FORCE

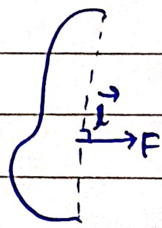
* current carrying small wire:

$\boxed{d\vec{F} = i(d\vec{l} \times \vec{B})}$



If $\vec{B}$ is uniform:

$\boxed{\vec{F} = i(\vec{l} \times \vec{B})}$



* Torque -

$$\boxed{\tau = \vec{M} \times \vec{B}}$$

Torque on a current carrying loop in uniform mag field.

Note:

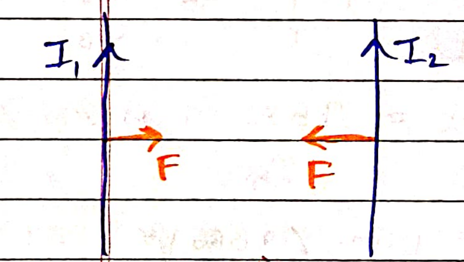
For any shape (rotating)

$\boxed{\frac{M}{L} = \frac{q}{2m}}$. M = Mag. dipole moment.
, L = ang. momentum.

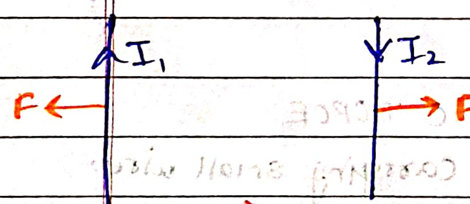
q = total charge m = total mass

Condition : $\frac{q}{m}$ is uniform

$\frac{q}{m}$ = Specific charge



attraction



repulsion

MAGNETIC PROP

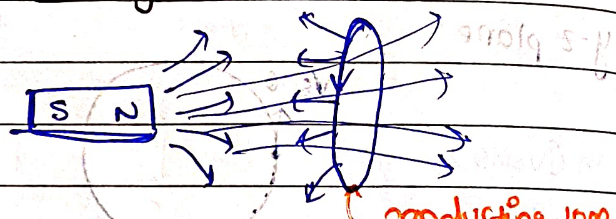
* $\vec{M}$ due to orbital motion of e^-

$$M = \frac{e}{2m} \times \frac{nh}{2\pi}$$

For n=1

$M = \frac{eh}{4m\pi}$ = Bohr's Magneton

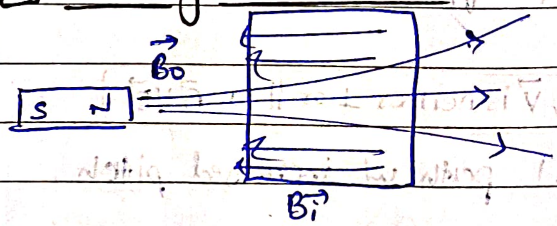
* Diamagnetism -



Lenz Law:

A current is indeed in the loop such that it opposes the change in magnetic field.

1 Diamagnetic Material -



$$\vec{M} = \Sigma \vec{M_i} \neq 0$$

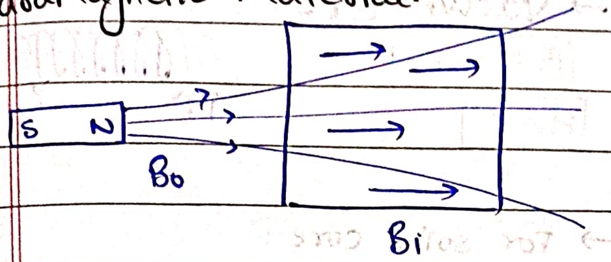
$$B_{net} = B_0 - B_i$$

B_{net} = Resultant mag field.
B_0 = external mag induction.
B_i = internal mag induction.

Super conductors have $B_{net} = 0$

$\therefore B_0 = B_i$

2 Paramagnetic Material.



$\vec{M} = \sum \vec{m}_i \neq 0$

$B_{net} = B_0 + B_i$

* Intensity Magnetism $(\vec{I})$:-

$\boxed{\vec{I} = \frac{\vec{M}}{V}}$

$\boxed{SI = Am^{-1}}$

$\boxed{B_i = \mu_0 \vec{I}}$

* Magnetic field Vectors $(\vec{B}$ & $\vec{H})$ -

$\vec{H}$ = Mag field Intensity
= Mag intensity = Mag field strength
↳ does not depend on Material.

$\boxed{\vec{B_0} = \mu_0 \vec{H}}$ → $\boxed{\text{unit} = Am^{-1}}$
↳ unit = Tesla

* Magnetic Susceptibility (χ)

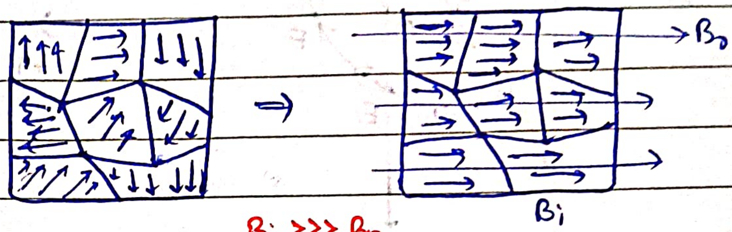
ease with which the material can be Magnetised when subjected to $\vec{B_0}$

$\boxed{\chi = \vec{I}/H} \Rightarrow \boxed{\vec{I} = \chi \vec{H}}$
↳ unitless

diamag → χ = -ve
paramag → $\vec{I} \parallel \vec{H}$ → χ = +ve

$\boxed{\mu_r = 1 + \chi}$ para → $\mu_r > 1$
dia → $\mu_r < 1$

3 FerroMagnetic Material -



$B_i >>> B_0$

$I >> H$

$\chi >> 1 \rightarrow \mu_r >> 1$

Curie's Law :-

For Paramagnetic :

Temp ↑ ⇒ thermal agitation ↑ ⇒ Randomness ↑
⇓
$\chi \downarrow \Leftarrow \vec{I} \downarrow \Leftarrow \vec{M} \downarrow$

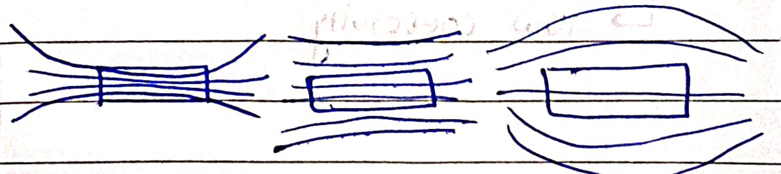
$\boxed{\chi \propto \frac{1}{T}} \Rightarrow \boxed{\chi = \frac{c}{T}}$ C = Curie's constant

For FerroMagnetic

when ferromagnetic material is heated, it becomes paramagnetic above a certain temp called Curie temperature

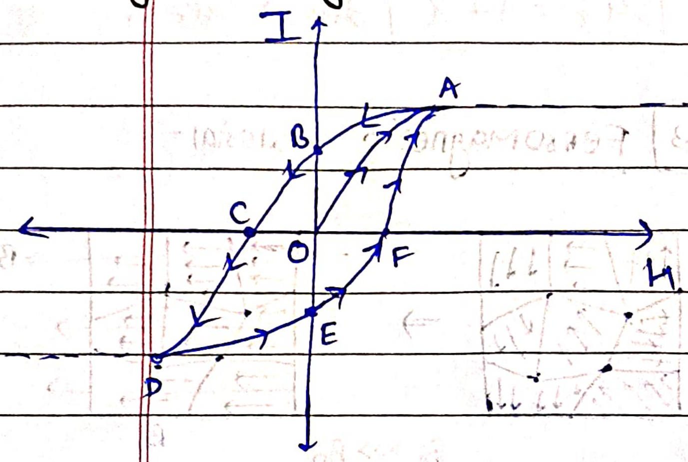
above curie temp:

$\boxed{\chi = \frac{c}{T - T_c}}$ $T_c = 1043K$
↳ for iron



ferromag paramag diamag

Magnetic Hysteresis -



OB = Retentivity or Residual Magnetism or Remanence or Magnetic Inertia

OC = coercivity

Area $\propto$ Heat loss in one cycle in unit volume of the material

- Coercivity of steel is much higher than soft iron.
- Retentivity of soft iron is slightly higher than steel

For permanent Mag :
↳ high retentivity
↳ high coercivity

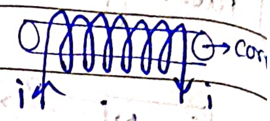
For electromagnets :
↳ low retentivity
↳ low coercivity

* For long solenoid

→ vaccum core

$\boxed{B = \mu_0 n i}$

$\boxed{H = ni}$



→ For solid core :

Method 1

$B = \mu_r B_i$

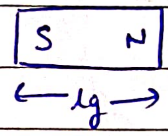
$= \mu_r \mu_0 n i$

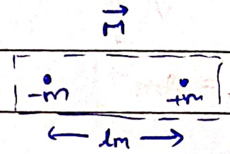
Method 2.

$B = B_0 + B_i$

$= \mu_0 H + \mu_0 I$

BAR MAGNET





l_g = geometric length

l_m = effective magnetic length.

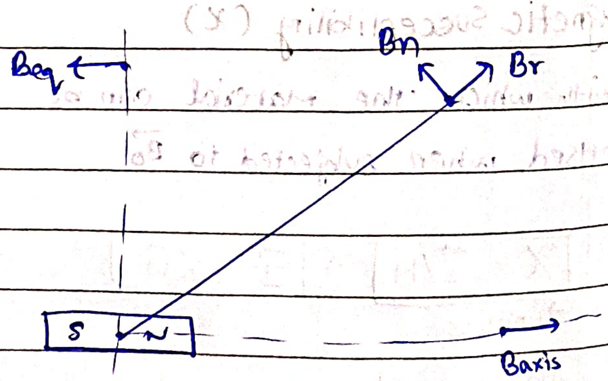
$(l_m \approx \frac{5}{6} l_g)$

$\boxed{\vec{M} = m l_m}$

dir of $\vec{M}$ = -ve (S) to +ve (N)

→ Magnetic Field

$\boxed{B = \frac{F}{m}}$



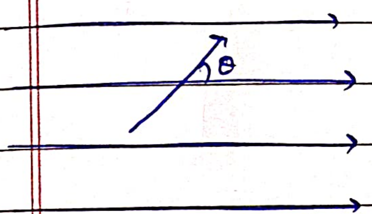
$$B_{eq} = \left(\frac{\mu_0}{4\pi}\right)\frac{M}{r^3}$$

$$B_{axis} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M}{r^3}$$

$$B_r = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\cos\theta}{r^2}$$

$$B_n = \left(\frac{\mu_0}{4\pi}\right)\frac{M\sin\theta}{r^3}$$

→ Bar Magnet in uniform B -



$$\boxed{\vec{\tau} = \vec{M}\times\vec{B}}$$

$$\boxed{PE = -\vec{M}\cdot\vec{B}}$$

at $\theta = 0$, $PE = \min$, $KE = \max$

→ Bar Magnet in non uniform B -

$$F = \left| M \cdot \frac{dB}{dx} \right|$$

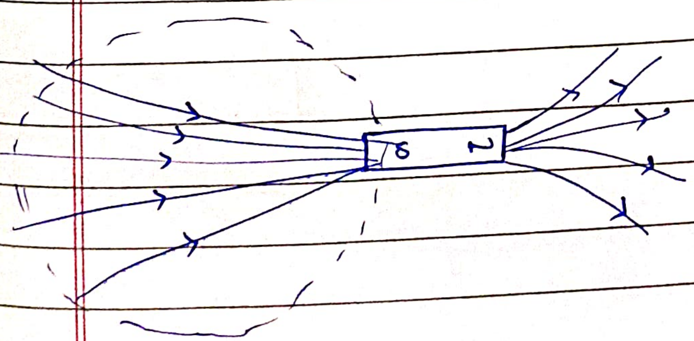
Magnetic Flux -

$$\boxed{\phi = \int \vec{B}\cdot d\vec{s}}$$

$$\boxed{\phi = \vec{B}\cdot\vec{S}} \rightarrow \vec{B} \text{ is uniform}$$

$$\oint \vec{B}\cdot d\vec{s} = 0$$

closed surface



Reason:

① no. of field lines exiting = no. of field lines entering.