ROTATIONAL MOTION

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↳ FLOW

Types of Motion
(3) Translational, Rotational, BOTH

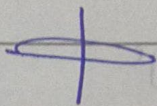
Moment of Inertia (kgm^2)

$I = \sum m_i r_i^2$

resistance shown by body towards rotational motion

* MOI of standard bodies,

① Circular ring:



$I = MR^2$

② uniform rod

$I = \frac{ML^2}{12}$

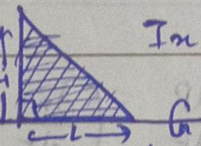
③ uniform disc

$I = \frac{MR^2}{2}$ $\quad K = \frac{R}{\sqrt{2}}$

* Parallel axis theorem

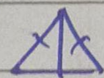
$I' = I_{com} + md^2$

④



$I_x = \frac{MH^2}{6}$ $\quad I_y = \frac{ML^2}{6}$

⑤ Isosceles.

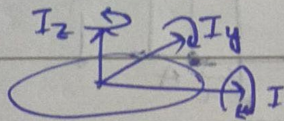


$I_x = \frac{MH^2}{6}$

$I_y = 0$

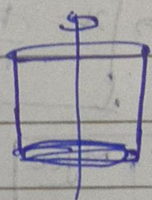
* Radius of Gyration (k)

* ⊥ Axis Theorem. → valid only for 1D & 2D bodies



$I_z = I_x + I_y$

⑥ Thin cylinder shell.

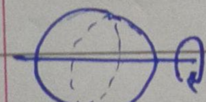


$I_{\text{Hollow cyl}} = I_{Ring} = MR^2$

⑦ Solid cylinder.

$I_{\text{solid cyl}} = I_{disc} = \frac{MR^2}{2}$.

⑧ Spherical shell



$I = \frac{2}{3} MR^2$

⑨ Solid sphere.

$I = \frac{2}{5} MR^2$

⑩ Thin circular cone.

$I = \frac{MR^2}{2}$

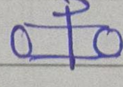
⑪ Solid circular cone.

$I = \frac{3}{10} MR^2$.

⑫ Solid cylinder.

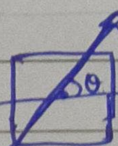
$I = \frac{MR^2}{4} + \frac{ML^2}{12}$.

⑬ Cylinderical shell.



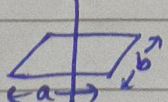
$I = \frac{ML^2}{12} + \frac{MR^2}{2}$.

⑭ Square plate:



At any θ $I = \frac{ML^2}{12}$.

⑮

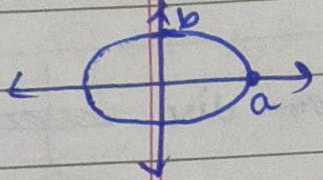


$I = \frac{M}{12}(a^2 + b^2)$

(16) Ellipse plate. $I_x = \frac{Mb^2}{4}$

$I_y = \frac{Ma^2}{4}$

$I_z = \frac{M}{4}(a^2+b^2)$



* MOI of object w/ cavity.

I_1 = MOI of object w/ cavity.

I = MOI of complete object.

I_2 = MOI of cavity

$\boxed{I_1 = I - I_2}$

THRUST.

$F_{thrust} = \frac{dP}{dt} = \left(\frac{dm}{dt}\right) v_0$

v_0 → velocity of body

$\frac{dm}{dt}$ → rate of change of mass

Torque.

$\boxed{\tau = \vec{r} \times \vec{F}}$

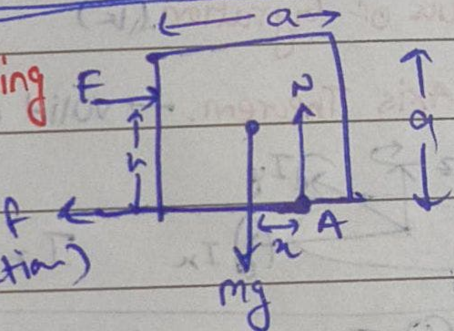
At equillibrium:

Translational Eq $\Rightarrow \Sigma \vec{F} = 0$

Rotational Eq $\Rightarrow \Sigma \vec{\tau} = 0$

τ abt any point.

Toppling



At A: $\tau = 0$

$\therefore Fh = mgx$.

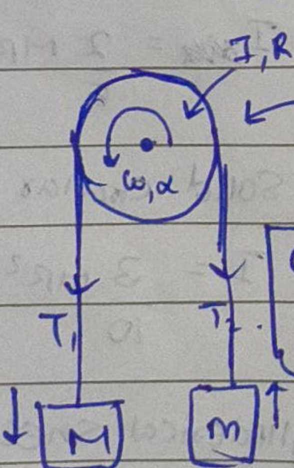
$\boxed{x = \frac{Fh}{mg}}$

Min Force for topple $\Rightarrow \boxed{F = \frac{Mga}{h}}$

Rotational Dynamics

$\boxed{\tau = I\alpha}$

$KE = \frac{1}{2} I\omega^2$



← How to avoid slipping?

$\boxed{a = \frac{(M-m)g}{M+m+I/R^2}}$

Angular Momentum

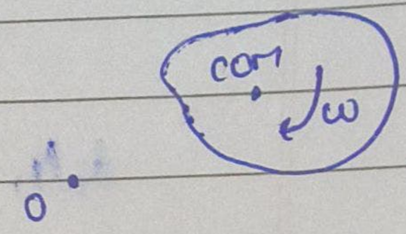
$\boxed{L = \vec{r} \times \vec{p}}$ $\boxed{\vec{L} = I \cdot \vec{\omega}}$

If $\vec{\tau} = 0$, then $\vec{L}$ is conserved

$J_{net} = \int_0^t \tau_{net} \cdot dt = \Delta \vec{L}$

↳ impulse

Angular Momentum in combined motion:



$\vec{L}_{Body,O} = \vec{L}_{Body,COM} + \vec{L}_{COM,O}$

$\boxed{\vec{L}_{Body,O} = I_{COM}\vec{\omega} + \vec{R}_{COM} \times P_{COM}}$

★ ICOR:

↳ A point where there is zero velocity at given instant.

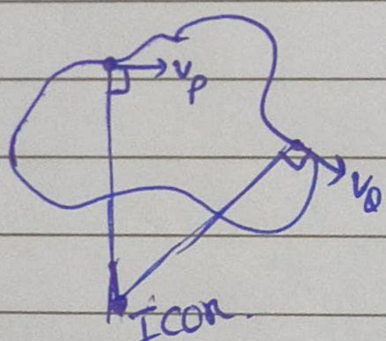
The body seems to be in pure rotation

$\vec{L}_{Body} = I_{ICOR}\vec{\omega}$ $K_{Body} = \frac{1}{2} I_{ICOR} \cdot \omega^2$.

How to find ICOR?

If v_P & $v_Q \neq \parallel$

then their ⊥'s intersection is ICOR.



If v_P & $v_Q = \parallel$

then → $v_P = (d+x)\omega$, $v_Q = x\omega$. → Find x :D

If antiparallel:

$v_P = x\omega$, $v_Q = (d-x)\omega$. → Find x.