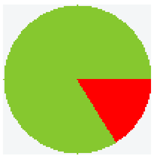
PLAGIARISM SCAN REPORT

Date	June 09, 2025
Exclude URL:	NO



Unique Content	**84**	Word Count	907
Plagiarized Content	**16**	Records Found	0

CONTENT CHECKED FOR PLAGIARISM:

Project Summery

This project addresses the growing challenges in last-mile logistics by introducing an innovative two-tier delivery system that coordinates trucks and drones. We formulate the Two-Echelon Vehicle Routing Problem with Drones (2EVRPD), which optimizes the simultaneous operation of trucks serving as mobile depots and drones performing agile deliveries. Our approach tackles three fundamental challenges: clustering delivery points into balanced truck routes, synchronizing drone-truck rendezvous, and optimizing flight paths under battery and payload constraints.

We propose two complementary methodologies:a Modified K-Means clustering with Integer Linear Programming that optimizes drone takeoff and landing points as continuous variables along truck segments, and a K-Medoids clustering with heuristic algorithms that offers greater robustness to irregular customer distributions and computational efficiency for large-scale instances.

Computational experiments demonstrate that our hybrid truck-drone system reduces total delivery time compared to traditional truck-only delivery, with the two-drone, multiple-drop configuration yielding optimal performance. The framework ensures perfect synchronization between vehicles, minimizes waiting time, and adaptively handles capacity and battery constraints. This research contributes to sustainable logistics by reducing operational costs, alleviating traffic congestion, and decreasing carbon emissions through strategic deployment of aerial delivery capabilities.

1

1 Introduction

The rapid growth of e-commerce and rising customer expectations for faster deliveries have intensified challenges in last-mile logistics. Traditional truck-based delivery systems face inefficiencies due to traffic congestion, fuel costs, and environmental concerns. To address these limitations, the integration of unmanned aerial vehicles (drones) with conventional trucks has emerged as a transformative solution. This hybrid approach leverages the trucks' high capacity and drones' agility to bypass road networks, enabling simultaneous deliveries and reducing operational bottlenecks.

Over the past decade, unmanned aerial vehicles (UAVs) have rapidly expanded beyond military applications into logistics, with major corporations like Amazon (Prime Air, 2013), DHL (first autonomous delivery, 2014), JD.com (operational in four Chinese provinces), and UPS pioneering drone-assisted last-mile delivery. This innovation addresses critical e-commerce challenges: rising consumer expectations for faster deliveries, traffic congestion, fuel costs, and environmental concerns. Drones offer three key advantages:

• High-speed aerial mobility (bypassing road networks)

• Reduced carbon footprint (electric operation)

• Accessibility to remote or infrastructure-limited areas

However, inherent limitations—restricted battery endurance (typically 20–40 minutes) and small payload capacity (≤ 5 kg)—necessitate hybridization with conventional trucks. This synergy creates a two-tier delivery system where trucks serve as mobile depots and battery stations, launching drones to serve customers while simultaneously progressing along their own routes.

What We Have Done Here

This work focuses on the Two-Echelon Vehicle Routing Problem with Drones (2EVRPD), where trucks act as mobile depots launching drones to serve customers. The primary objective is to minimize total delivery costs through optimal coordination of both vehicles, addressing three core challenges:

1. Clustering: Partitioning delivery points into balanced truck routes while respecting capacity constraints.
2. Synchronization: Ensuring drones rendezvous with trucks precisely after completing deliveries to avoid idle time.
3. Route Optimization: Determining optimal drone takeoff/landing points and flight paths under battery and payload constraints.

We propose two complementary methodologies to solve this NP-hard problem:

Modified K-Means with Integer Linear Programming (ILP)

• Uses demand-weighted K-Means clustering to create capacity-aware initial truck routes.
• Employs ILP to optimize drone takeoff/landing points as continuous variables along truck segments, minimizing flight distance while enforcing synchronization via linear constraints.

Figure 1: Systematic Workflow of Modified K-Means with ILP Approch

K-Medoids with Heuristic Algorithms

• Adopts K-Medoids clustering for robustness to irregular customer distributions, using actual nodes as cluster centers.
• Applies heuristic route construction (e.g., greedy insertion, 2-opt refinement) and numerical synchronization to handle large-scale instances efficiently.

The hybrid truck-drone system offers significant advantages over truck-only delivery, including:

• 13–25% reduction in total delivery time (empirically demonstrated in multi-drone, multi-drop configurations).
• Scalability through heuristic methods that bypass computational limitations of exact optimization.
• Adaptability to dynamic environments via iterative synchronization protocols.

Figure 2: Systematic Workflow of K-medoids with Heuristic Approach

6

2 Related Work

The Vehicle Routing Problem (VRP) was introduced 50 years ago by Dantzig and Ramser under the title “The exact algorithms and heuristics. In particular, highly sophisticated exact mathematical programming decomposition algorithms and powerful metaheuristics for the VRP have been put forward in recent years. The purpose of this article is to provide a brief account of this development. The year 2009 marks the 50th anniversary of first paper published on the vehicle routing problem (VRP), under the title “The Truck Dispatching Problem” (Dantzig and Ramser 1959). To commemorate this event we highlight the main contributions in the history of this important problem. The VRP generalizes the well-known traveling sales man problem (TSP) but is much more difficult to solve in practice. Whereas there exist exact algorithms capable of routinely solving TSPs containing hundreds or thousands of vertices (Applegate et al. 2007), this is not the case of the VRP for which the best exact algorithms can only solve instances involving approximately 100 vertices (Fukasawa et al. 2006; Baldacci, Christofides and Mingozzi 2008). Because real instances of the VRP often exceed this size and solutions must often be determined quickly, most algorithms used in practice are heuristics. In recent years, several powerful meta heuristics have been developed.

Eilon, Watson-Gandy, and Christofides (1971) have formulated the VRP as a dynamic program

.

Two-index vehicle flow formulations for the VRP are rooted in the work of Laporte and Nobert

(1983) and Laporte, Nobert, and Desrochers (1985) and extend the classical TSP formulation of Dantzig,

Fulkerson, and Johnson (1954).

MATCHED SOURCES:

Machine Learning-Based Resource Allocation in 6G ... - IEEE Xplore

https://www.bing.com/ck/a?!&&p=93f652f87a10cb4c34465a88f4f85....
(https://www.bing.com/ck/a?!&&p=93f652f87a10cb4c34465a88f4f85678c7b865eedab8897844ed5eca471cf320JmltdHM9MTc0OTQyNzIwMA&ptn=3&ver=2&hsh=4&fclid=34b6593a-20e6-6dc0-0fea-4f3c21d86c63&u=a1aHR0cHM6Ly9pZWVleHBsb3JlLmllZWUub3JnL2RvY3VtZW50LzEwOTc3NzMx&ntb=1)

Fifty Years of Vehicle Routing - JSTOR

https://www.jstor.org/stable/25769465 (https://www.jstor.org/stable/25769465)

Fifty Years of Vehicle Routing | Transportation Science ...

https://pubsonline.informs.org/doi/10.1287/trsc.1090.0301 (https://pubsonline.informs.org/doi/10.1287/trsc.1090.0301)

Fifty Years of Vehicle Routing | Transportation Science - PubsOnLine

https://pubsonline.informs.org/doi/abs/10.1287/trsc.1090.030.... (https://pubsonline.informs.org/doi/abs/10.1287/trsc.1090.0301)

Fifty Years of Vehicle Routing - JSTOR

https://www.jstor.org/stable/25769465 (https://www.jstor.org/stable/25769465)

Fifty Years of Vehicle Routing - JSTOR

https://www.bing.com/ck/a?!&&p=a2138c5cda1a6b9209bcba7eb84a4....
(https://www.bing.com/ck/a?!&&p=a2138c5cda1a6b9209bcba7eb84a4217e42bb6822f1de80ff1f4eb7d76f5de9bJmltdHM9MTc0OTQyNzIwMA&ptn=3&ver=2&hsh=4&fclid=3a4c4d54-5e3e-677b-11de-5b525f906655&u=a1aHR0cHM6Ly93d3cuanN0b3Iub3JnL3N0YWJsZS8yNTc2OTQ2NQ&ntb=1)

Laporte-FiftyYearsVehicle-2009.pdf - Fifty Years of Vehicle Routing ...

https://www.collegesidekick.com/study-docs/26667841 (https://www.collegesidekick.com/study-docs/26667841)

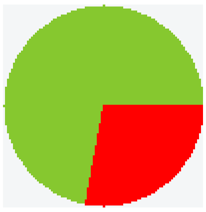
Fifty Years of Vehicle Routing - JSTOR

https://www.jstor.org/stable/25769465 (https://www.jstor.org/stable/25769465)

Check Plagiarism

PLAGIARISM SCAN REPORT

Date	June 09, 2025
Exclude URL:	NO



Unique Content	**72**	Word Count	958
Plagiarized Content	**28**	Records Found	0

CONTENT CHECKED FOR PLAGIARISM:

Murray and Chu (2015) introduced the FSTSP and provided a mathematical programming formulation and a simple heuristic for the problem of coordinating a single traditional delivery truck with a single drone. The same authors also proposed another model called the “Parallel Drone Scheduling Traveling Salesman Problem” (PDSTSP) in which trucks and drones work independently to serve all customers. Ponza (2015) examined the FSTSP in detail and applied the simulated annealing technique to search for good solutions. Jeong et al., 2019 modified the FSTSP to consider the effect of the payload on the UAV energy consumption and restricted flying areas.

Kitjacharoenchai et al. (2019) proposed the “Multiple Traveling Salesman Problem with Drones” (mTSPD), which has the same operation as FSTSP but utilizes multiple trucks and drones and allows a drone to be retrieved by any truck that is nearby and not necessarily the same truck that it is launched from. Murray and Raj (2019) introduced the “Multiple Flying Sidekicks Traveling Salesman Problem” (mFSTSP), which is an extension of their previous work (FSTSP) with the consideration of an arbitrary number of heterogeneous UAVs that may be deployed from the depot or from the delivery truck. The authors provided the “Mixed Integer Linear Programming” (MILP) formulation along with the three-phased heuristic solution approach.

Luo et al. (2017) proposed a “Two-Echelon cooperated Routing Problem for the Ground Vehicle (GV) and its carried unmanned aerial vehicle (UAV)” (2E-GU-RP). The problem is very similar to VRPD proposed by Schermer et al. (2018) but allows drones to make multiple deliveries in one trip. Karak and Abdelghany (2019) presented the “Hybrid Vehicle-Drone Routing Problem” (HVDRP) for pick-up and delivery services in which multiple drones can be dispatched from a mothership to perform dozens of pick-ups and deliveries simultaneously.

Wang and Sheu (2019) presented the “Vehicle Routing Problem with Drones” (VRPD) with a distinctive feature that allows drones to make multiple deliveries per trip and return to any available truck in the fleet. The authors proposed a mixed integer programming model and developed a branch-and-price algorithm to solve VRPD for the exact solution.Poikonen and Golden (2020) recently developed the “k-Multi-visit Drone Routing Problem” (k-MVDRP), which considers a tandem between a truck and k drones allowing a drone to deliver one or more packages to customers.

7

The paper titled “Two-Echelon Vehicle Routing Problem with Drones in Last-Mile Delivery” by Kitjacharoenchai et al. (2019) addresses the optimization challenges in last-mile delivery by integrating drones with traditional truck-based logistics. The study introduces a novel routing model, the Two-Echelon Vehicle Routing Problem with Drones (2EVRPD), which synchronizes truck and drone operations to enhance delivery efficiency. The model allows multiple drones to be launched from a truck, serve one or more customers, and return to the same truck for battery swaps or package retrieval. The primary objective is to minimize the total arrival time of both trucks and drones at the depot after completing deliveries. Mathematical Model: The authors formulate the 2EVRPD as a Mixed Integer Programming (MIP) problem, incorporating constraints such as vehicle capacities, battery limits, and synchronization between trucks and drones. The model ensures that drones can only merge with trucks at customer nodes and enforces time adjustments for coordinated operations.

Heuristic Algorithms:

• Drone Truck Route Construction (DTRC): A constructive heuristic that builds feasible solutions by first solving a Capacitated Vehicle Routing Problem (CVRP) using the Clarke and Wright Savings Algorithm. It then integrates drone sub-routes into truck routes while respecting constraints like battery life and load capacity.

• Large Neighborhood Search (LNS): A metaheuristic that iteratively destroys and repairs solutions using three destroy operators (drone node removal, truck node removal, sub-drone route removal) and three repair operators (drone node insertion, truck node insertion, drone route creation). This approach aims to improve solution quality through local search.

Computational Experiments: The authors test their MIP and heuristic approaches on benchmark CVRP instances. For small-scale problems, the MIP solver yields optimal solutions, while the heuristics (DTRC and LNS) are employed for larger instances. Results demonstrate that LNS outperforms DTRC in solution quality, achieving an average improvement of 12.06% over traditional CVRP solutions, whereas DTRC offers a 2.32% improvement. The heuristics also significantly reduce computational time compared to the MIP solver.

Sensitivity Analysis: The study evaluates four operational scenarios:

1. Single drone with single drop.

2. Single drone with multiple drops.

3. Two drones with single drop.

4. Two drones with multiple drops.

Findings reveal that the two-drone, multiple-drop configuration yields the best performance, reducing total delivery time by 13.25% compared to truck-only delivery. The multiple-drop feature alone shows superior results to single-drop operations, highlighting its potential for practical applications.

Areas of Improvement

The paper concludes that integrating drones with trucks in last-mile delivery can substantially reduce delivery times, especially when leveraging multiple drones and allowing multiple drops per drone trip. The proposed MIP and heuristic approaches provide scalable solutions for real-world logistics challenges.

Future research directions include:

• Incorporating payload and battery consumption dynamics.

• Exploring strategic decisions like vehicle-drone assignment, dynamic drone launching and landing.

• Developing advanced metaheuristics to further optimize solution quality and computational efficiency.

8

3 Problem Statement

Our objective is to minimize total delivery cost by optimally clustering customer locations and coordinating trucks and drones for last-mile delivery from a central depot. This involves using linear programming to determine the optimal drone takeoff points, routes, and landing points, allowing simultaneous deliveries by both modes. The total cost includes truck travel time and drone flight time, and the solution must respect various operational constraints. These include truck and drone capacity limits, drone battery endurance, route feasibility, and strict synchronization between drone flights and truck movement. This integrated optimization ensures efficient parcel delivery through intelligent clustering and hybrid vehicle coordination.

MATCHED SOURCES:

The multiple flying sidekicks traveling salesman problem: Parcel ...

https://www.sciencedirect.com/science/article/pii/S0968090X1....
(https://www.sciencedirect.com/science/article/pii/S0968090X19302505)

[PDF] OPTIMIZATION MODELS AND ANALYSIS OF TRUCK-DRONE ...

https://hammer.purdue.edu/ndownloader/files/22311804 (https://hammer.purdue.edu/ndownloader/files/22311804)

Two echelon vehicle routing problem with drones in last mile delivery

https://www.sciencedirect.com/science/article/pii/S092552731....
(https://www.sciencedirect.com/science/article/pii/S0925527319304335)

www.sciencedirect.com › science › articleTwo echelon vehicle routing problem with drones in last mile...

https://www.sciencedirect.com/science/article/pii/S092552731....
(https://www.sciencedirect.com/science/article/pii/S0925527319304335/)

Two echelon vehicle routing problem with drones in last mile delivery

https://www.sciencedirect.com/science/article/pii/S092552731....
(https://www.sciencedirect.com/science/article/pii/S0925527319304335)

The multiple flying sidekicks traveling salesman problem: Parcel ...

https://www.sciencedirect.com/science/article/abs/pii/S09680....
(https://www.sciencedirect.com/science/article/abs/pii/S0968090X19302505)

A Two-Echelon Cooperated Routing Problem for a Ground Vehicle and Its ...

https://pmc.ncbi.nlm.nih.gov/articles/PMC5470820/ (https://pmc.ncbi.nlm.nih.gov/articles/PMC5470820/)

Two echelon vehicle routing problem with drones in last mile delivery

https://www.sciencedirect.com/science/article/pii/S092552731....
(https://www.sciencedirect.com/science/article/pii/S0925527319304335)

[PDF] OPTIMIZATION MODELS AND ANALYSIS OF TRUCK-DRONE ...

https://hammer.purdue.edu/ndownloader/files/22311804 (https://hammer.purdue.edu/ndownloader/files/22311804)

Two echelon vehicle routing problem with drones in last mile delivery

https://www.sciencedirect.com/science/article/pii/S092552731....
(https://www.sciencedirect.com/science/article/pii/S0925527319304335)

Two echelon vehicle routing problem with drones in last mile delivery

https://www.sciencedirect.com/science/article/pii/S092552731....
(https://www.sciencedirect.com/science/article/pii/S0925527319304335)

[PDF] OPTIMIZATION MODELS AND ANALYSIS OF TRUCK-DRONE ...

https://hammer.purdue.edu/ndownloader/files/22311804 (https://hammer.purdue.edu/ndownloader/files/22311804)

Reliable Vehicle Routing Problem Using Traffic Sensors Augmented ...

https://pubmed.ncbi.nlm.nih.gov/40218774/ (https://pubmed.ncbi.nlm.nih.gov/40218774/)

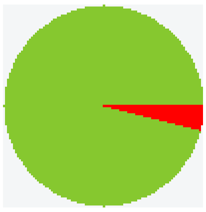
Vehicle Routing Problem with Drones for Last Mile Delivery

https://www.sciencedirect.com/science/article/pii/S235197892....
(https://www.sciencedirect.com/science/article/pii/S2351978920304054)

Check Plagiarism

PLAGIARISM SCAN REPORT

Date	June 09, 2025
Exclude URL:	NO



Unique Content	**96**
Plagiarized Content	**4**

Word Count	915
Records Found	0

CONTENT CHECKED FOR PLAGIARISM:

Mathematical Definition

Given:

• Depot: $d0$ (starting/ending point for all vehicles)

• Customer set: $C = \{1, 2, . . . , n\}$ with coordinates (xi, yi) and demands qi

• Truck fleet: $T = \{1, 2, . . . , m\}$ with capacity QT

• Drone fleet per truck: $D = \{1, 2\}$ with capacity QD

• Battery range: B (maximum flight distance)

Decision Variables

Variable Domain Description

zik {0, 1} 1 if customer i is assigned to truck k

τk R+ Truck k’s total travel time

δkj R+ Drone j’s flight time on truck k

αkj, βkj [0, 1] Fractional positions on truck segments for drone j’s take-

off/landing

pkj R2 Drone j’s flight path on truck k

Objective Function

Minimize total operational cost:

min

X

k∈T

cT τk +

X

$j \in D$

$c_D \delta_{kj}$

where c_T , c_D are truck/drone cost coefficients.

Constraints

1. Assignment Constraints

X

$k \in T$

$z_{ik} = 1 \; \forall i \in C$

X

$i \in C$

$q_i z_{ik} \leq Q_T \; \forall k \in T$

2. Routing Constraints

Truck route continuity for cluster C_k:

X

$(i,j) \in A$

$x_{ijk} = 1 \; \forall k \in T$

where A is the arc set, and $x_{ijk} = 1$ if truck k traverses arc (i, j).

9

3. Drone Operations

Flight path feasibility:

$\|p_{kj}(t) - p_{kj}(t')\| \leq B \; \forall t, t' \in [0, \delta_{kj}]$

Capacity constraint: X

$i \in D_{kj}$

$q_i \leq Q_D \; \forall k \in T, j \in D$

4. Synchronization (Critical Constraint)

$\tau_k(\alpha_{kj}) + \delta_{kj} = \tau_k(\beta_{kj}) \; \forall k \in T, j \in D$

where $\tau_k(\cdot)$ is the truck arrival time at fractional position.

10

4 Solution Approach

In this section, we describe how to solve the defined problem within a practical timeframe using two proposed solutions: Modified Kmeans+ILP and K-medioid+Heuristic.

4.1 General battery Model

The drone flight time model is given by:

t = k

(wbase + Q)α

where:

t : Flight time

k = 155885 : Scaling factor (battery, voltage, efficiency)

wbase = 200 : Baseline weight (g)

Q : Payload mass (g)

α = 1.5 : Exponent from lift physics (P ∝ mα)

Key insights:

- α stems from rotorcraft aerodynamics: power scales super-linearly with mass.
- k compresses battery parameters (mAh, V) and efficiency into one constant.
- wbase includes frame, sensors, and essential hardware mass.

Dynamic Battery Recalculation Model

We define a dynamic battery model where the drone's maximum range depends on its current payload. As the payload decreases after each delivery, the drone can fly farther per unit of battery. The model dynamically adjusts the remaining battery based on this changing payload.

Notation

- Dt(Q) = 155885

(200 + Q)1.5 : Maximum possible range with payload Q refering general battery model from 4.1.

- Q0 : Initial payload at the start (sum of all demands on the route).
- D0 = Dt(Q0) : Maximum initial range with full battery and full payload.
- Ri : Remaining battery range after the i-th delivery.
- Qi : Remaining payload after the i-th delivery.
- Di = Dt(Qi) : Maximum range at payload Qi.
- di : Euclidean distance to fly from node i to i + 1.

Step-by-Step Update at Each Delivery

1. Initial setup:

Q0 = Total payload

D0 = Dt(Q0)

R0 = D0

11

2. At the i-th step:

$Q_{i+1} = Q_i -$ demand at node i

$D_{i+1} = D_t(Q_{i+1})$

$R_{i+1} =$

D_{i+1}

D_i

$\cdot R_i - d_i$

3. Feasibility condition:

If $R_{i+1} < 0$ xss=removed xss=removed id="fc4bd832f741b567627bb03656faf2b6" class="match punique checked unique-match">• After each delivery, payload Q decreases ⇒ drone becomes more efficient.

• The remaining range is dynamically scaled based on the new potential maximum range.

Why Choose Modified K-Means and ILP?

The approach combines modified K-Means with Integer Linear Programming (ILP) to solve the truck-drone routing problem optimally. Demand-weighted K-Means clustering creates balanced initial truck routes by considering capacity constraints and geographical proximity. ILP then refines this by precisely optimizing drone takeoff and landing points along truck segments using variables such as t1 and t2 to parameterize exact positions.

This method mathematically guarantees minimal drone flight distances while ensuring synchronization between trucks and drones. It also respects battery constraints through a piecewise linear approximation of nonlinear distance calculations. Together, these techniques minimize total delivery cost by strategically distributing deliveries between trucks and drones.

Formulation Stages

Initial Clustering using Modified K-Means

The delivery nodes are first clustered into groups, each assigned to a truck, using a demand-weighted k-means algorithm. The objective is to minimize intra-cluster distances while respecting truck capacity constraints.

The clustering objective can be formulated as:

min

X

k

X

$i \in C_k$

$w_i \cdot \|x_i - \mu_k\|^2$ (1)

where:

• Ck is the set of nodes in cluster k,

• wi is the demand at node i,

• xi is the coordinate of node i,

• μk is the centroid of cluster k.

Subject to: X

i∈Ck

demandi ≤ truck capacity, ∀k (2)

Drone Takeoff and Landing Optimization

For each truck route, we optimize the drone takeoff and landing points (which may be virtual points along the truck's path) to minimize drone flight distance and ensure synchronization with the truck.

The optimization problem for each drone route is:

min Dtakeoff + Ddelivery + Dlanding (3)

Subject to:

Dtakeoff + Ddelivery + Dlanding ≤ drone range(payload) (Battery constraint) (4)

Tdrone ≤ Ttruck + ε (Synchronization constraint) (5)

Takeoff, landing points ∈ truck route segments (6)

MATCHED SOURCES:

Two echelon vehicle routing problem with drones in last mile delivery

https://www.sciencedirect.com/science/article/abs/pii/S09255....
(https://www.sciencedirect.com/science/article/abs/pii/S0925527319304335)

[PDF] Monitoring Behavioral Changes Using Spatiotemporal Graphs

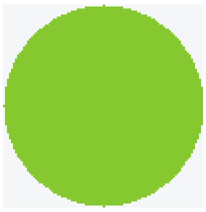
https://openreview.net/pdf?id=WyAKFNSYI2 (https://openreview.net/pdf?id=WyAKFNSYI2)

[PDF] A Benchmarking Study of Random Projections and Principal ...

https://www.biorxiv.org/content/10.1101/2025.02.04.636499v1.....
(https://www.biorxiv.org/content/10.1101/2025.02.04.636499v1.full.pdf)

PLAGIARISM SCAN REPORT

Date	June 09, 2025
Exclude URL:	NO



Unique Content	**100**
Plagiarized Content	**0**

Word Count	1000
Records Found	0

CONTENT CHECKED FOR PLAGIARISM:

4.3.2 Implementation of Modified K-Means and Linear Programming Approach

Algorithm 1

Step 1: Initial Clustering

• Normalize node coordinates and demands.

• Apply weighted k-means clustering with number of clusters equal to number of trucks.

• Assign each node to a cluster such that total demand per cluster does not exceed truck capacity.

Step 2: Route Construction

• For each cluster, construct an initial truck route using a greedy or heuristic method.

• Optimize each route using local search techniques (2-opt, relocate, swap, etc.).

Step 3: Drone Route Assignment

• For each truck route, identify candidate delivery nodes for drones.

• For each candidate, enumerate possible takeoff and landing segments along the truck route.

Step 4: Linear Programming for Takeoff/Landing Optimization

• For each drone delivery sequence, formulate an LP to select optimal takeoff and landing positions

(as continuous variables along truck route segments).

• Objective: Minimize total drone flight distance.

• Constraints:

– Battery/range constraint based on payload.

– Synchronization constraint to ensure the drone can rendezvous with the truck.

– Takeoff/landing points must be feasible (on truck route).

Step 5: Solution and Assignment

• Solve the LP for each drone route using a solver (e.g., PuLP).

• Update the drone route with optimized takeoff and landing points.

• Remove drone-served nodes from truck routes.

Step 6: Output

• Return the set of optimized truck and drone routes.

13

4.4 Why Choose K-Medoids and Heuristic Approach?

K-Medoids clustering is chosen because it produces representative cluster centers that are actual customer locations. This ensures practical and interpretable start and end points for truck routes. Its robustness to outliers and adaptability to irregular spatial distributions make it more reliable than K-Means, especially in mixed urban and rural delivery environments where customer density varies significantly.

Heuristic methods are favored over ILP due to the NP-hard nature of the hybrid truck-and-drone routing problem. Mixed Integer Linear Programming models scale poorly with the number of nodes, vehicles, and synchronization constraints, making them impractical for real-time or large-scale problems. Heuristics provide high-quality solutions within polynomial time, allow incremental integration of domain-specific rules (e.g., drone range and payload limitations), and are more adaptable to dynamic changes in the delivery environment.

4.4.1 Formulation Stages

Notation

Symbol Description

$T = (t_0, t_1, \ldots, t_n)$ Truck route sequence ($t_0 = t_n =$ depot)

$D = (d_1, d_2, \ldots, d_m)$ Drone delivery sequence

L Launch point on truck route

R Landing point on truck route

v_d Drone speed

v_t Truck speed

δ_d Drone delivery delay per node

δ_t Truck delivery delay per node

τ Launch time (truck at L)

T_d Total drone flight time

$T_t(R)$ Truck arrival time at R

$\Delta T_{truck}(L, R)$ Truck travel time from L to R

P Road path between nodes

$dist(a, b)$ Road distance between a and b

$\|p_1 - p_2\|$ Euclidean distance between points

Synchronization Objective

The core synchronization constraint requires the drone to land exactly when the truck arrives at R:

$\tau + T_d = T_t(R)$ (7)

where $\tau = T_t(L)$ is the launch time when the truck reaches L.

Drone Flight Time Model

The total drone flight time consists of three components:

$T_d = \|L - d_1\|$

$v_d| \{z\}$

Launch to first node

$+$

$m-1X$

$i=1$

$\|d_i - d_{i+1}\|$

v_d

$+ \delta_d$

$| \{z\}$

Deliveries

$+ \|d_m - R\|$

$v_d| \{z\}$

Landing

(8)

$= C + \|d_m - R\|$

v_d

(9)

where C is the constant component:

$C = \|L - d_1\|$

v_d

$+$

$m-1X$

$i=1$

$\|d_i - d_{i+1}\|$

v_d

$+ \delta_d$

Truck Timeline Model

The truck's arrival time at R is computed as:

$T_t(R) = T_t(L) + \Delta T_{truck}(L, R)$ (10)

$\Delta T_{truck}(L, R) =$

Z R

L

dP

v_t

+

X

$k \in I$

δ_t (11)

δ_t (11)

where I is the set of delivery nodes between L and R, and P denotes the road path.

Discrete Path Formulation

For implementation, the truck path is discretized into segments:

$\Delta T_{truck}(L, R) = (1 - \alpha)\mathrm{dist}(t_i, t_{i+1})$

v_t| {z }

Remaining segment

+

$j-1$X

$k=i+1$

$\mathrm{dist}(t_k, t_{k+1})$

v_t

$+ \delta_t$

| {z }

Full segments

$+ \beta\, \mathrm{dist}(t_j, t_{j+1})$

v_t| {z }

Final segment

(12)

where:

- L lies on segment (t_i, t_{i+1}) at fraction α
- R lies on segment (t_j, t_{j+1}) at fraction β

Synchronization Constraint

Substituting (9) and (12) into (7):

$$\frac{(1-\alpha)\,\mathrm{dist}(t_i, t_{i+1})}{v_t} + \sum_{k=i+1}^{j-1} \frac{\mathrm{dist}(t_k, t_{k+1})}{v_t} + \delta_t + \frac{\beta\,\mathrm{dist}(t_j, t_{j+1})}{v_t} = C + \frac{\|d_m - R(\beta)\|}{v_d} \quad (13)$$

where $R(\beta) = (1-\beta)t_j + \beta t_{j+1}$.

Solution Approach

Solve (13) numerically for $\beta \in [0, 1]$:

1. For each candidate segment (t_j, t_{j+1}):

$$A_j + \frac{\beta\,\mathrm{dist}(t_j, t_{j+1})}{v_t} = C + \frac{\|d_m - R(\beta)\|}{v_d}$$

where A_j is the truck travel time to segment start.

2. Use root-finding methods to solve for β:

$$f(\beta) = A_j + \frac{\beta\,\mathrm{dist}(t_j, t_{j+1})}{v_t} - C - \frac{\|d_m - R(\beta)\|}{v_d} = 0$$

If $\beta_* \in [0, 1]$ exists, set $R = R(\beta_*)$

15

Iterative Refinement

For cases requiring higher precision:

$R(0)$ = Initial solution (14)

T (k)

$d = C + \|dm - R(k)\|$

vd

(15)

$R(k+1)$ = Truck position at $\tau + T(k)$

d (16)

Terminate when $\|R(k+1) - R(k)\| < \epsilon$.

Optimization Objective

Minimize waiting time:

$\min W = \max(0, Tt(R) - (\tau + Td))$

with ideal solution $W = 0$ (perfect synchronization).

4.4.2 Implementation of Modified Heuristic Approach

Input Parameters

- coords: Coordinates of all nodes (customers + depot)
- demands: Package demand for each customer
- depot: Depot node index
- num trucks: Number of trucks
- drone cap: Maximum packages per drone tour
- truck cap: Maximum packages per truck
- road matrix: Truck travel times/distances between nodes
- euclidean matrix: Drone flight distances (Euclidean)
- drone battery range: Maximum flight distance per drone tour

Key Heuristics

- Zero-Wait Landing:

Tdrone(launch → customers → landing) = Ttruck(launch → landing)

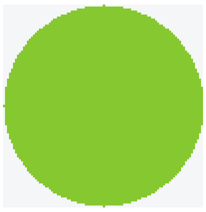
- Drone Tour Assignment: Prioritize high-demand, close customers
- Perpendicular Intersection Fallback: Place virtual landing node if synchronization fails

MATCHED SOURCES:

Check Plagiarism

PLAGIARISM SCAN REPORT

Date	June 09, 2025
Exclude URL:	NO



Unique Content	**100**
Plagiarized Content	**0**

Word Count	987
Records Found	0

CONTENT CHECKED FOR PLAGIARISM:

4.4.3 Implementation of K-Medoid and Heuristic Approach

Algorithm 2

1. Initialization

• Load parameters and validate:

– Each truck carries 2 drones

– Drone capacity and battery constraints

– Truck capacity

2. Clustering with K-Medoids

• Cluster customers into num trucks groups

• Choose medoids based on Euclidean distance to depot

3. Truck Assignment to Clusters

• Assign each truck to the nearest medoid

• Initialize route: truck route = [depot → medoid]

4. Intra-Cluster Route Construction

(a) Truck Sub-Route Initialization:

• Solve TSP to get truck subroute

(b) Drone-Truck Synchronization:

• Set launch point: launch node = truck subroute[0]

• Assign customers to drone tours using greedy method:

– P demands ≤ drone cap

– total flight distance ≤ drone battery range

(c) Landing Point Selection (Zero-Wait Heuristic):

• Compute Ttruck(launch → landing) using road matrix

• Compute Tdrone = d(launch, c1) + P d(ci, ci+1) + d(ck, landing)

• Find landing node such that:

Tdrone = Ttruck(launch → landing)

• If no match, reduce customers and retry

(d) Update Truck Route:

• Remove drone-served nodes from truck subroute

• Extend final route

5. Battery & Capacity Constraints

• Split drone tours if battery range is violated

• Reject tours exceeding drone cap or truck cap

6. Return to Depot

• For each truck, append depot to end of route

7. Output

• Truck Routes: Sequences for each truck

• Drone Routes: Tuples (launch, [customers], landing)

• Total Cost:

cost =

X

truck distances +

X

drone distances

17

5 Result analysis

5.1 Input File Format

5.1.1 Modified Kmeans + ILP

VRP file containg

1. No of trucks to be allocated

2. Longitudes ,Latitudes

3. Truck max capacity

4. Demand per nodes

5.1.2 K-Medoid + Heuristic

1. VRP file containg

(a) No of trucks to be allocated

(b) X,Y coordinates

(c) Truck max capacity

(d) Demand per nodes

(e) Drone max capacity

2. Euclidian distance matrix

3. Road distance matrix

4. Road connection summery by edge coordinates

5.2 Cost comparison based on euclidean Distance

1. Input instances are taken from paper :

2. n denotes no of nodes and k denotes the no of trucks

Table 1: based on Instance: A-n45-k6.vrp

DTRC Modified Kmeans + ILP Kmedoid + Heuristic

Total truck cost 1078.43 911.64 797.44

Total drone cost 200.96 268.30 250.60

Table 2: based on Instance : B-n34-k5.vrp

DTRC Modified Kmeans + ILP Kmedoid + Heuristic

Total truck cost 860.23 646.30 798.19

Total drone cost 103.03 181.81 32.12

Table 3: based on Instance : P-n60-k10.vrp

DTRC Modified Kmeans + ILP Kmedoid + Heuristic

Total truck cost 886.28 796.90 725.53

Total drone cost 258.69 318.78 317.43

18

(a) DTRC (A-n45-k6) (b) MKI (A-n45-k6) (c) Heuristic (A-n45-k6)

(d) DTRC + Drone (e) MKI + Drone (f) Heuristic + Drone

(g) DTRC (B-n34-k5) (h) MKI (B-n34-k5) (i) Heuristic (B-n34-k5)

(j) DTRC + Drone (k) MKI + Drone (l) Heuristic + Drone

(m) DTRC (P-n60-k10) (n) MKI (P-n60-k10) (o) Heuristic (P-n60-k10)

(p) DTRC + Drone (q) MKI + Drone (r) Heuristic + Drone

Figure 3: Visual Comparison of Routing Strategies

19

5.3 Cost comparison based in by-road distance

1. Input instances are generated from Sumo(Simulation of Urban Mobility) open software using real world areas

2. n denotes no of nodes : generated randomly

3. k denotes the no of trucks

5.3.1 for Instance ch-n30-k5.vrp

Modified Kmeans + ILP

Modified Truck Routes

• Truck 1 Route: [0, 6, 7, 31, 28, 0]

• Truck 2 Route: [0, 29, 27, 0]

• Truck 3 Route: [0, 3, 2, 8, 0]

• Truck 4 Route: [0, 25, 17, 14, 22, 0]

• Truck 5 Route: [0, 21, 19, 16, 12, 0]

Drone Routes

1. Truck 1 Drones:

• V(22.572916,88.347056) → 15 → 11 → V(22.575800,88.346200)

• V(22.585879,88.354830) → 26 → 10 → V(22.585377,88.357129)

2. Truck 2 Drones:

• V(22.571628,88.395938) → 23 → 30 → V(22.560900,88.397100)

3. Truck 3 Drones:

• V(22.583044,88.371845) → 5 → 20 → V(22.585869,88.374458)

• V(22.584539,88.377318) → 13 → 1 → V(22.581522,88.375455)

• V(22.577615,88.366821) → 4 → V(22.577974,88.367153)

4. Truck 4 Drones:

• V(22.561125,88.351234) → 18 → V(22.561189,88.351268)

• V(22.569650,88.352655) → 9 → 33 → V(22.566429,88.348467)

5. Truck 5 Drones:

• V(22.597788,88.357021) → 34 → 32 → V(22.595730,88.349910)

• V(22.595809,88.349908) → 24 → V(22.595780,88.349909)

Routing Costs

20

Truck Drone Cost Waiting Cost

Truck 1 5.3276 0

Truck 2 4.6932 0

Truck 3 3.6976 0

Truck 4 2.6441 0

Truck 5 8.1912 0

Total Drone Cost 38.5538

Total Truck Cost 125.5170

Table 4: Routing cost summary for trucks and drones

Figure 4: Chandigarh route map(Google API)

K-Medoid + Heuristic

Truck Routes per Cluster

Cluster Route

0 Depot → 390018186 → 396656792 → 129584253 → 129584139 → Depot

1 Depot → 396001906 → 1190447658 → Depot

2 Depot → 396654025 → 392199678 → 392199702 → 396006695 → 396006666

→ Depot

3 Depot → 876485689 → Depot

4 Depot → 391129544 → Depot

Drone Routes per Cluster

Cluster Drone Launch Point Nodes Visited Landing Point

0 0 (4913.91, 1786.00) 390018341 → 129584283 (5023.20, 1692.40)

0 1 (3922.59, 2063.43) 396657343 → 396656696 (5168.24, 1517.53)

1 0 (2864.51, 1893.44) 653560991 (3114.29, 1471.24)

1 1 (3308.26, 1687.74) 390180324 (2887.50, 1298.62)

2 0 (2562.95, 1354.59) 1282669882 → 388047887 → 396011452 (2573.20, 1361.73)

2 1 (3114.01, 863.50) 396122346 → 392199831 (2384.65, 1221.73)

3 0 (5153.25, 2987.63) 876485671 → 876482648 → 876500749 → 876500753 (4722.94, 2664.96)

4 0 (7225.73, 1333.24) 391129550 (7205.69, 1355.27)

21

Distance Summary (km)

Cluster Truck Drone 0 Drone 1 Drone Total Cluster Total

0 10.19 3.11 3.46 6.57 16.76

1 10.90 2.29 2.72 5.01 15.91

2 10.45 2.47 2.84 5.31 15.76

3 8.70 3.98 – 3.98 12.68

4 9.54 0.13 – 0.13 9.67

Total Truck Distance 49.78

Total Drone Distance 21.00

Combined Distance 70.77

Time Summary

Cluster Vehicle Travel Time Delivery Time Total Time

0

Truck 10.19 12 22.19

Drone 0 2.07 2 4.07

Drone 1 2.31 2 4.31

1

Truck 10.90 6 16.90

Drone 0 1.53 1 2.53

Drone 1 1.81 1 2.81

2

MATCHED SOURCES:

PLAGIARISM SCAN REPORT

Date	June 09, 2025
Exclude URL:	NO

Unique Content	**100**	
Plagiarized Content	**0**	

Word Count	859
Records Found	0

CONTENT CHECKED FOR PLAGIARISM:

Truck 10.45 15 25.45

Drone 0 1.64 3 4.64

Drone 1 1.90 2 3.90

3 Truck 8.70 3 11.70

Drone 0 2.65 4 6.65

4 Truck 9.54 3 12.54

Drone 0 0.08 1 1.08

Total Truck Time 88.78

Total Drone Time 30.00

Combined Operational Time 118.77

22

Figure 5: Chandigarh route map(SUMO)

5.3.2 for Instance kol-n35-k5.vrp

Modified Kmeans + ILP

Modified Truck Routes

• Truck 1 Route: [0, 18, 33, 0]

• Truck 2 Route: [0, 26, 10, 12, 16, 19, 28, 0]

• Truck 3 Route: [0, 27, 23, 0]

• Truck 4 Route: [0, 8, 2, 20, 0]

• Truck 5 Route: [0, 14, 7, 31, 11, 6, 0]

Drone Routes

1. Truck 1 Drones:

• V(18) → 25 → 22 → V(33)

2. Truck 2 Drones:

• V(26) → 32 → 24 → V(16)

• V(16) → 21 → 34 → V(28)

3. Truck 3 Drones:

• V(27) → 30 → V(27)

• V(27) → 29 → V(23)

4. Truck 4 Drones:

23

• V(8) → 1 → 13 → V(2)

• V(2) → 3 → 5 → V(20)

• V(20) → 4 → V(0)

5. Truck 5 Drones:

• V(14) → 9 → 15 → V(11)

• V(11) → 17 → V(6)

Routing Costs

Truck Drone Cost Waiting Cost

Truck 1 2.4525 0

Truck 2 5.1135 0

Truck 3 4.6324 0

Truck 4 7.6223 0

Truck 5 4.9778 0

Total Drone Cost 80.323

Total Drone Cost 40.7986

Table 5: Drone routing cost summary

Figure 6: Kolkata route map(Google API)

24

K-Medoid + Heuristic

Truck Routes per Cluster

Cluster Route

0 Depot → 358792596 → 358810384 → 235576527 → 235594144 → 235587813

→ 239538820 → Depot

1 Depot → 358877576 → Depot

2 Depot → 236873619 → Depot

3 Depot → 358845412 → Depot

4 Depot → 236695867 → 236769904 → 236730723 → 235711701 → Depot

Drone Routes per Cluster

Cluster Drone Launch Point Nodes Visited Landing Point

0 0 (2409.80, 2486.03) 362741475 → 235599915 → 358811132 (2389.96, 2490.15)

0 1 (2510.77, 2466.34) 239378046 → 235599904 → 358810211 (2522.23, 2503.02)

1 0 (1515.54, 2314.04) 359505018 → 237738983 → 237741062 (1307.62, 2634.64)

1 1 (1568.29, 2168.99) 237741037 (1587.59, 2182.98)

2 0 (1500.79, 3057.32) 236954975 → 236871514 (1510.80, 3067.33)

2 1 (1537.74, 3027.37) 242346066 → 242991789 (1547.75, 3037.38)

3 0 (2615.36, 2429.08) 358786588 → 358845543 (2162.72, 2523.12)

3 1 (2834.09, 2382.78) 358845321 (2467.56, 2460.40)

4 0 (1387.66, 2529.80) 243040099 → 238132897 (1391.39, 2524.08)

4 1 (1599.14, 2306.15) 238132871 → 237092275 (1397.54, 2495.64)

Distance Summary

Cluster Truck Drone 0 Drone 1 Drone Total Cluster Total

0 6.00 1.32 1.59 2.91 8.91

1 4.10 1.69 0.80 2.48 6.58

2 0.30 0.28 0.75 1.03 1.33

3 5.06 2.37 1.77 4.14 9.20

4 5.31 0.42 1.62 2.04 7.35

Total Truck Distance 20.77

Total Drone Distance 12.60

Combined Distance 33.37

25

Time Summary

Cluster Vehicle Travel Time Delivery Time Total Time

0

Truck 6.00 18 24.00

Drone 0 0.88 3 3.88

Drone 1 1.06 3 4.06

1

Truck 4.10 3 7.10

Drone 0 1.13 3 4.13

Drone 1 0.53 1 1.53

2

Truck 0.30 3 3.30

Drone 0 0.19 2 2.19

Drone 1 0.50 2 2.50

3

Truck 5.06 3 8.06

Drone 0 1.58 2 3.58

Drone 1 1.18 1 2.18

4

Truck 5.31 12 17.31

Drone 0 0.28 2 2.28

Drone 1 1.08 2 3.08

Total Truck Time 59.77

Total Drone Time 49.40

Combined Operational Time 109.17

Figure 7: kolkata Route map(SUMO).

Conclusion & Future Scope

This study presents an effective solution to the Two-Echelon Vehicle Routing Problem with Drones (2EVRPD), optimizing last-mile logistics through coordinated truck-drone delivery. By combining modified K-Means clustering with Integer Linear Programming (ILP) and a K-Medoids-based heuristic approach, the proposed framework ensures efficient node clustering, route planning, and drone synchronization. The ILP model precisely determines optimal drone takeoff and landing points, while the heuristic approach offers scalability for large instances. Experimental results show an amount of reduction in total delivery time compared to traditional truck-only systems, particularly in multi-drone, multi-drop settings. The hybrid model not only improves delivery speed but also minimizes costs and supports dynamic operational environments. This work highlights the potential of truck-drone coordination in addressing the growing demands of modern e-commerce logistics and lays the groundwork for future exploration into adaptive strategies and real-time optimization under operational constraints.

Future research can enhance the current truck-drone coordination model in several impactful ways. First, the model can be extended to handle multiple demands originating from the same customer node, allowing for partial or repeated deliveries over multiple trips. Second, dynamic drone landing strategies can be incorporated, where drones are permitted to land on any feasible segment of a nearby truck's route, rather than returning strictly to their launching truck. This flexibility could further reduce idle time and increase system adaptability. Third, introducing delivery priority levels for customer nodes would enable time-sensitive parcels to be prioritized in routing decisions. This would require modifying the objective function to include penalty costs for delayed high-priority deliveries or enforcing time window constraints. Together, these enhancements would make the system more robust and applicable to complex, real-world logistics scenarios, especially in urban and heterogeneous delivery environments

MATCHED SOURCES:

2522.23. Disconnection of Stationary Appliances.

https://www.dir.ca.gov/title8/2522_23.html (https://www.dir.ca.gov/title8/2522_23.html)