Tutorial -4

(DAA)

* Master Theorem

1. $T(n) = 3T(n/2) + n^2$

$a = 3, b = 2$

$c = \log_2 3 = 1.5$

$n^2 > n^{1.5}$

$\therefore T(n) = \Theta(n^2)$

2. $T(n) = 4T(n/2) + n^2$

$a = 4, b = 2$

$c = \log_2 4$

$c = 2$

$\therefore f(n) = n^c$

$n^2 = n^2$

$T(n) = \Theta(n^2 \log n)$

3. $T(n) = T(n/2) + 2^n$

$a = 1, b = 2$

$c = \log_2 1$

$c = 0 \quad (n^c = 1)$

$2^n > 1$

$T(n) = \Theta(2^n)$

4. $T(n) = 2^n T(n/2) + 2^n$

$a = 2^n, b = 2 ; c = \log_2 2^n$

$n^c = n^n$

$T(n) = \Theta(n^n)$

5) $T(n) = 16T(n/4) + n$

$a = 16, b = 4$

$c = \log_4 4^2$

$c = 2$

Comparing ; $f(n) < n^c$

$n < n^2$

$T(n) = \Theta(n^2)$

6) $T(n) = 2T(n/2) + n \log n$

$c = \log_2 2 = 1$

$f(n) > n^c$

$n \log n > n$

$T(n) = \Theta(n \log n)$

7) $T(n) = 2T(n/2) + n/\log n$

$a = 2, b = 2$

$c = \log_2 2$

$c = 1$

$\frac{n}{\log n} < n^1$

$f(n) < n^c$

$T(n) = \Theta(n)$

8) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2, b = 4$

$c = \log_4 2, c = 1/2$

$f(n) = n^c$

$n^{0.5} = n^{0.5}$

$T(n) = \Theta(n^{0.5} \log n)$

9) $T(n) = 16T(n/4) + n!$

$a = 16, b = 4$

$c = \log_4 16 ; c = 2$

if $(n > 3) \Rightarrow n! > n^2$

$\Theta(n!)$

if $(n < 3) \Rightarrow n^2 > n!$

$\Theta(n^2)$

10) $T(n) = 0.5T(n/2) + 1/n$

$c = \log \frac{1}{2} = -1$

$n^{-1} = n^{-1}$

$T(n) = \Theta(n^{-1} \log n)$

11) $T(n) = 4T(n/2) + \log n$

$a = 4, b = 2$

$c = \log_2 4 = 2$

$\log n < n^2$

$T(n) = \Theta(n^2)$

12) $T(n) = \text{sqrt}(n)\, T(n/2) + \log n$

$a = \sqrt{n}, b = 2$

$c = \log_2 n^{1/2}$

13) $T(n) = 3T(n/2) + n$

$a = 3, b = 2$

$c = \log_2 3 = 1.5$

$f(n) < n^c$

$n < n^{1.5}$

$T(n) = \Theta(n^{1.5})$

14) $T(n) = 3T(n/3) + \text{sqrt}(n)$

$a = 3, b = 3$

$c = \log_b a = 1$

$f(n) = n^{1/2}$

$n^c = n^1$

$f(n) < n^c$

$T(n) = \Theta(n)$

15) $T(n) = 4T(n/2) + cn$

$a = 4, b = 2$

$c = \log_2 2^2$

$c = 2$

$f(n) < n^c$

$T(n) = \Theta(n^2)$

16) $T(n) = 3T(n/4) + n\log n$

$a = 3, b = 4$

$c = \log_4 3$

$f(n) = n \log n$

$n^c = n^{0.7}$

$f(n) > n^c$

$T(n) = n\log n$

17) $T(n) = 3T(n/3) + n/2$

$a = 3, b = 3$

$c = \log_b a = 1$

$f(n) = n/2$

$n^{c'} = n$

$f(n) < n^c$

$T(n) = n$

18) $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, b = 3$

$c = \log_3 6 = 1.6$

$f(n) > n^c$

$T(n) = \Theta(n^2 \log n)$

19) $T(n) = 4T(n/2) + n/\log n$

$a = 4, b = 2$

$c = \log_2 4 = 2$

$f(n) = n\log n$

$n^c = n^2$

$T(n) = \Theta(n^2)$

20) $T(n) = 64T(n/8) - n^2 \log n$

$a = 64, b = 8$

$c = \log_8 64 = 2$

$f(n) > n^c$

$T(n) = \Theta(n^2 \log n)$

21) $T(n) = 7T(n/3) + n^2$

$a = 7, b = 3$

$c = \log_3 7 = 1.77$

$f(n) > n^c$

$n^2 > n^{1.77}$

$T(n) = \Theta(n^2)$

22) $T(n) = T(n/2) + n(2 - \cos n)$

$a = 1, b = 2$

$\log_2 1 = 0$

$f(n) > n^c$

$n(2 - \cos n) > n^0$

$T(n) = \Theta(n(2 - \cos n))$